T-matrix approach to quark-gluon plasma at finite baryon density

Going beyond EoS (even) in the equilibrium

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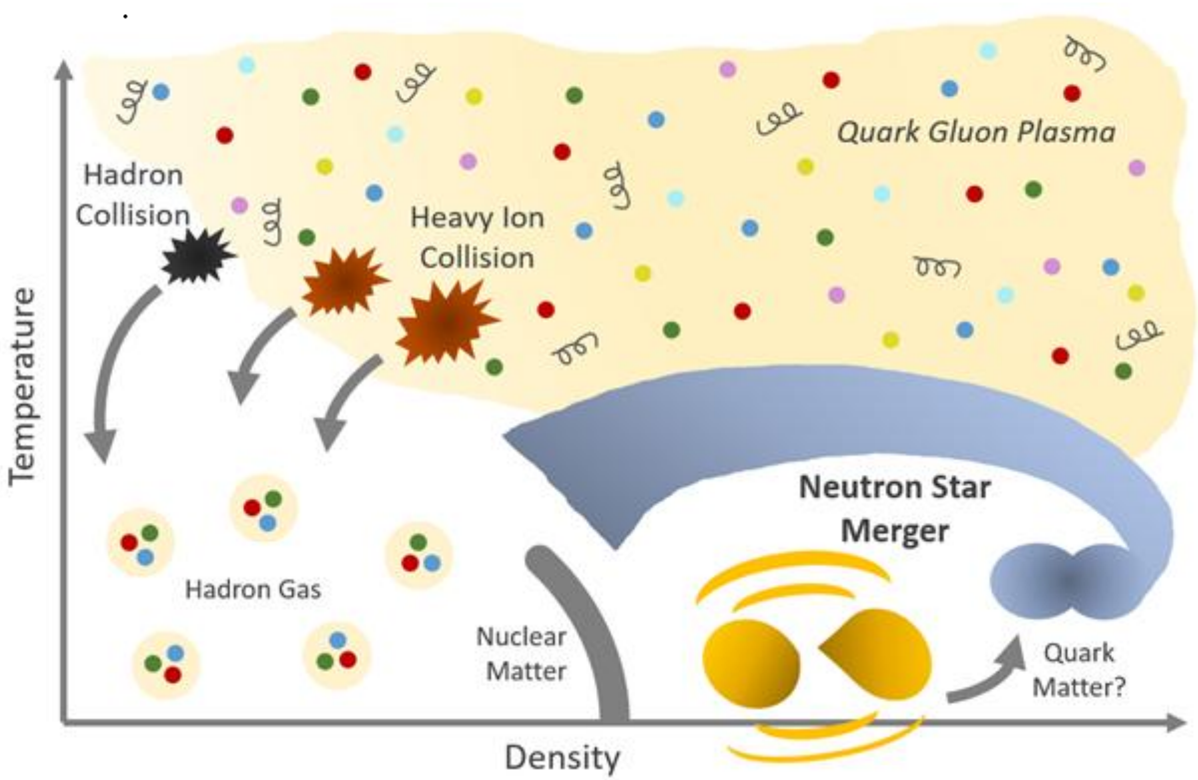
Seattle, WA

Understanding QCD matter



[M. Schwaiger-Haber, WUSTL, & A. Haber, University of Southampton]

- $T > T_c^{\chi}$: still strongly coupled QGP
  - Near perfect fluid
  - Strong jet quenching
  - Rapid heavy-quark thermalization

  [E. Shuryak Rev.Mod.Phys 89 2017]

- Features of a "good" model of QGP:
  - Description of lattice QCD results at zero and low μ_B/T
  - Crossover at $\mu_B = 0$ – transition of dominant degrees of freedom

Green's functions and spectral functions

- Central objects: $G(\omega,\vec{p})$
  - Green's function

  - Spectral function:

$$\rho(\omega,p) = -\frac{1}{\pi}\mathrm{Im}G(\omega,p)$$

- Non-interacting (let $\mu = 0$)

$$G_0(\omega,p) = \frac{1}{\omega - E_p + i0}$$

- Interacting:

$$G(\omega,p) = \frac{1}{G_0^{-1}(\omega,p) - \Sigma(\omega,p)}$$

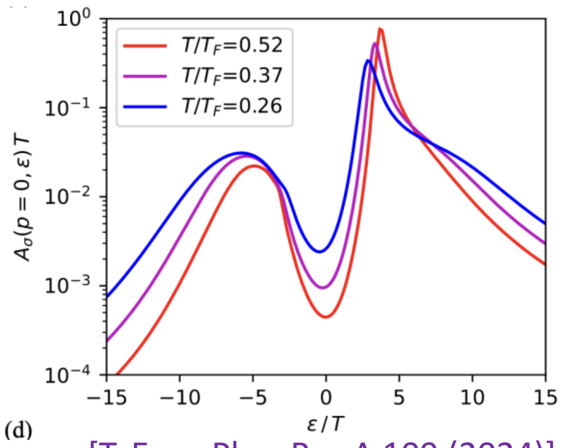
- Choice of diagrams in Σ ↔(largely) many-body framework

$$A(\omega,p) = \frac{-\frac{1}{\pi}\mathrm{Im}[\Sigma(\omega,p)]}{(\omega - E_p - \mathrm{Re}[\Sigma(\omega,p)])^2 + (\mathrm{Im}[\Sigma(\omega,p)])^2}$$

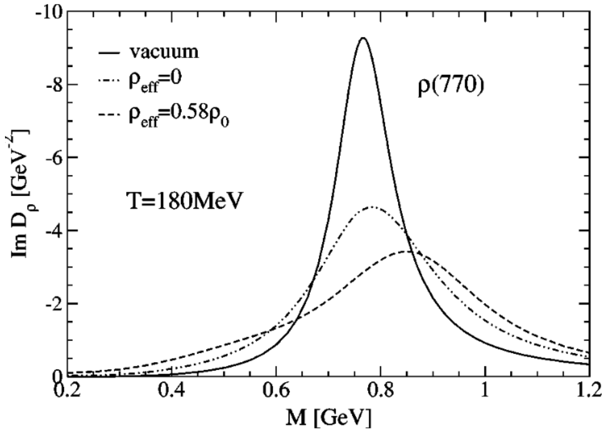
- If we are lucky: $\Gamma \simeq -2\mathrm{Im}\Sigma(E_p^*,p) \to 0$,

$$\rho(\omega,q) = \delta(\omega - E_p^*)$$

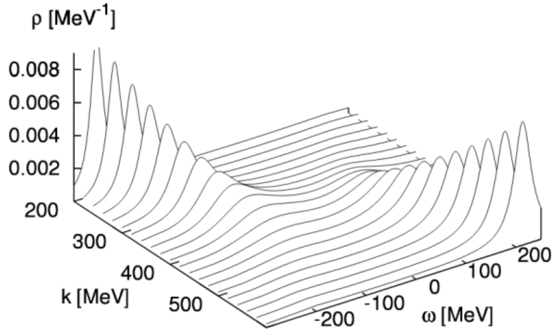
- If not:



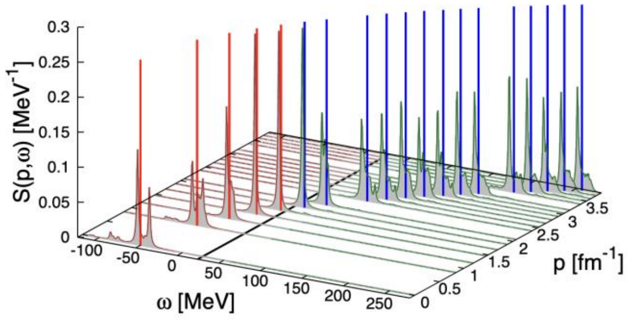
[T. Enns Phys.Rev.A 109 (2024)]



[R.Rapp Phys.Rev.C 63 (2001)]



[M.Kitazawa et al. Phys.Lett. B631 (2005)]

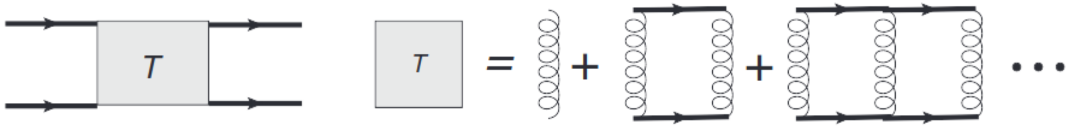


[Barbieri & Carbone "Self-Consistent Green's Function Approaches"]

T-matrix approach to strongly interacting QGP

- Basic d.o.f. – partons: $q, \bar{q}, g; N_f = 3$ (degenerate)
- Start with $\widehat{H}$ with 2-body interaction $V(\vec{q}, \vec{q}\,')$ in various color channels
- Ladder resummation: T-matrix

$$T(E, P; q, q') = V \; + V G_2(E, \ldots) T$$



- Resonances below $2m$ in T-matrix $\leftrightarrow$ bound states

Example: heavy quarkonia $\bar{b}b, \bar{c}c$

[M. Mannarelli, R. Rapp PRC72 (2005), D. Cabrera, R. Rapp PRD76 (2007), F. Riek, R. Rapp PRC82 (2010)]

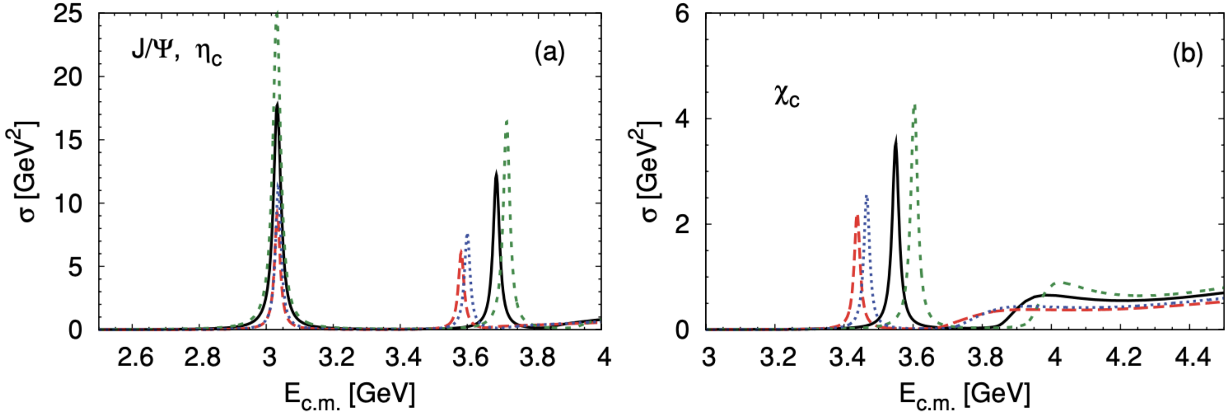
Screened Cornell potential

$$V \sim -\frac{4}{3}\alpha_s \frac{e^{-m_d r}}{r} + \frac{\sigma}{m_s}(1 - e^{-m_s r})$$

Self-consistent evaluation of

Heavy-quark diffusion coefficient

Euclidian-time correlators from lattice QCD

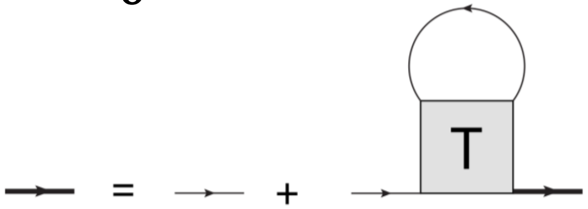


$$\sigma(E_{c.m.}) \sim \mathrm{Im}\, T(E_{c.m.})$$

In-medium: self-consistency

- Interactions ↔self-energies

$$G = G_0 + G_0 \Sigma G, \qquad \Sigma \sim TG$$

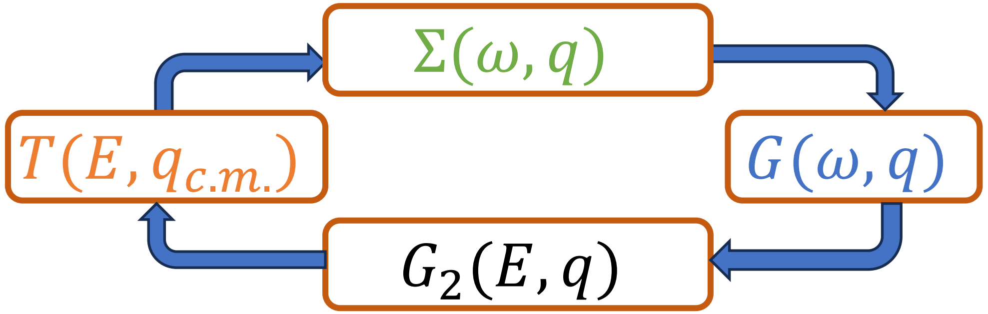


Real-time formulation:

$$\mathrm{Im}\Sigma(\omega, q) \sim \int d\vec{p}_2\, d\omega_2 \mathrm{Im}T(\omega + \omega_2, \vec{p}_1, \vec{p}_2)$$

$$\times\, \rho(\omega_2, \vec{p}_2) \times [n_B(\omega + \omega_2) + n_F(\omega)]$$

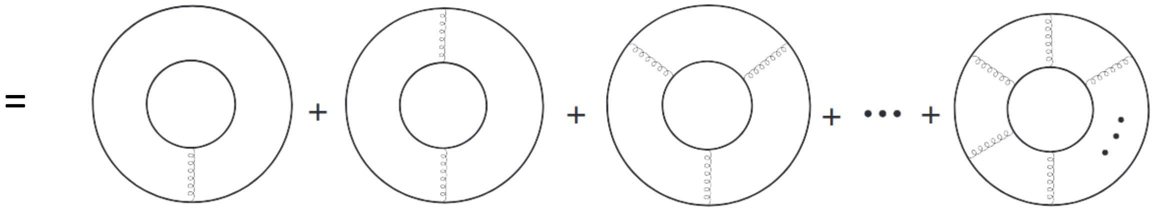
- Self-consistent scheme



Equation of state

$$\Omega(\{\mu\}, T) = \mp T \sum_n \underbrace{\mathrm{Tr}\{\ln(-G^{-1}) + \Sigma G\}}_{\text{1-body}} \pm \underbrace{\Phi[G]}_{\text{2-body}}$$

Luttinger-Ward Functional (LWF) $\Phi[G]$ =



- Self-consistent many-body framework

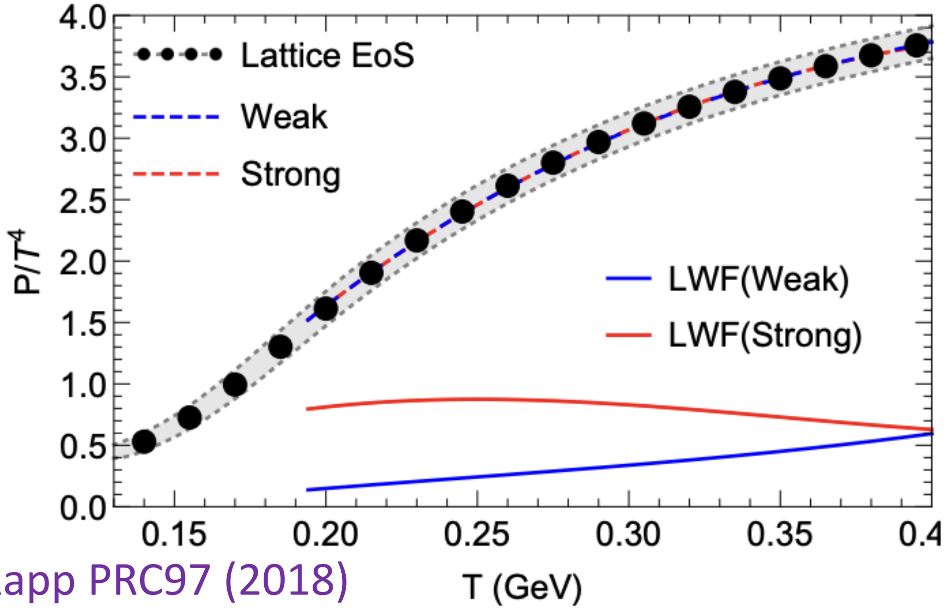
$$\Sigma[\mathrm{G}] = \frac{\delta \Phi[G]}{\delta G}$$

- Conserving 2PI approximation

[G. Baym PR 127 (1962)]

Light sector: weakly/strongly coupled scenarios

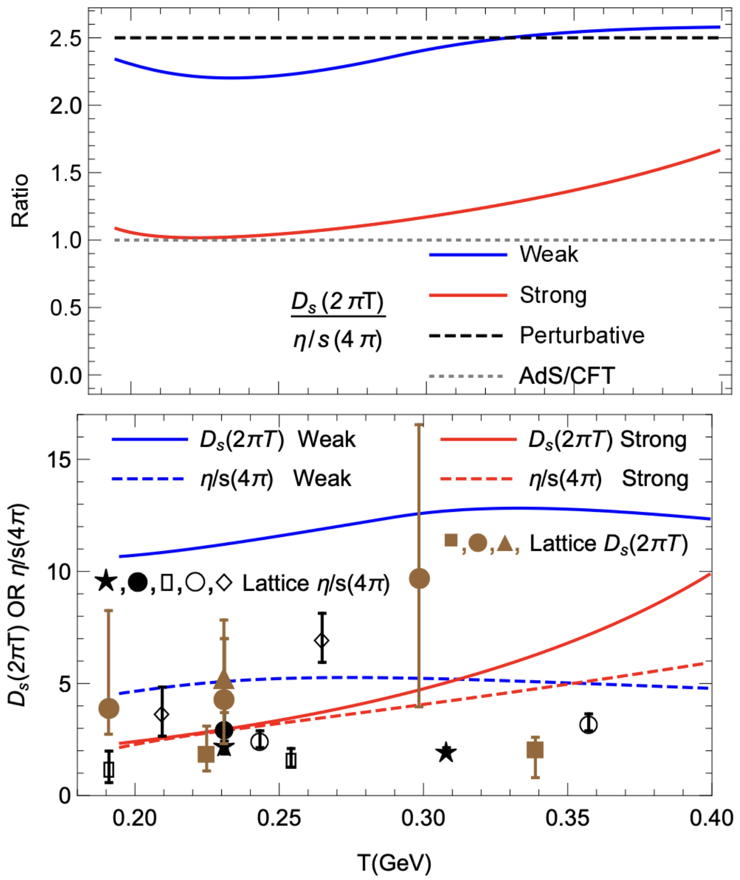
- Parton masses and interaction screening fit lQCD data:
  - $P(T, \mu_B = 0)$
  - Heavy-quark free energy $F(\vec{r})$
  - Heavy-quark correlators
- … due to remnants of the confining force



S. Liu, R. Rapp PRC97 (2018)

- Same EoS, significant difference in transport properties

S. Liu, R. Rapp EPJA 56 (2020)



This work: separable interactions

Technical simplification

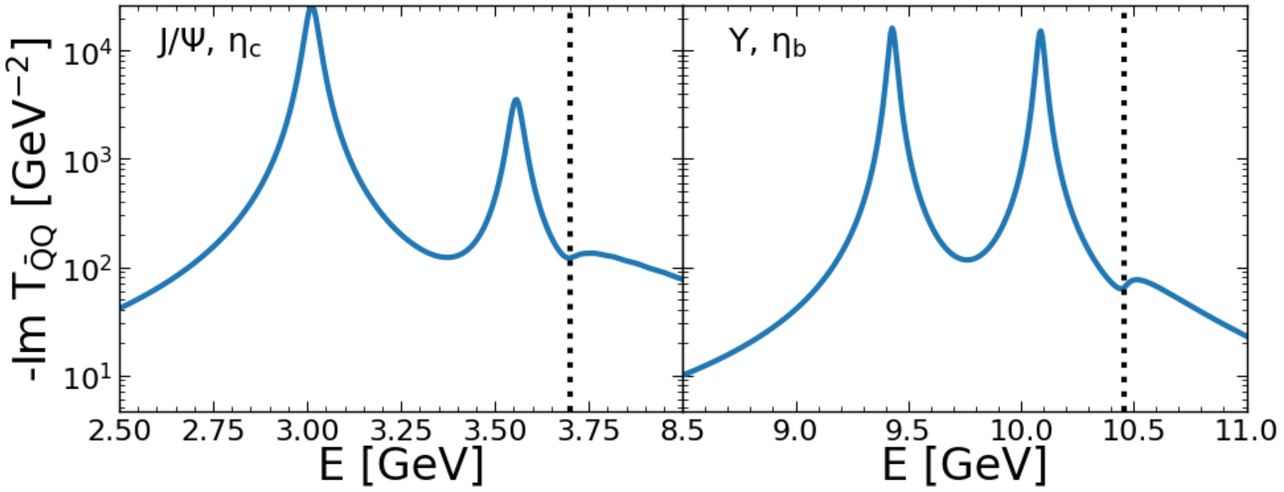
- Solves 3D-reduced BSE and allows to resum LWF semi-analytically
- Rank-2 separable interaction in s- and p-waves:

- $V(q,q') = G_S v_S(q) v_S(q') + G_c v_C(q) v_C(q')$

$$v_S^{l=0}(q) = \frac{\Lambda_S^4}{(q^2+\Lambda_S^2)^2}, \qquad v_C^{l=0}(q) = \frac{\Lambda_C^2}{q^2+\Lambda_C^2}$$

× relativistic corrections

- $\Lambda_S \sim 0.5$ GeV, $\Lambda_C \sim 2.9$ GeV
- Parameters tuned to reasonably describe quarkonium spectroscopy



Can we mimic the strongly coupled QGP with this simplified interaction?

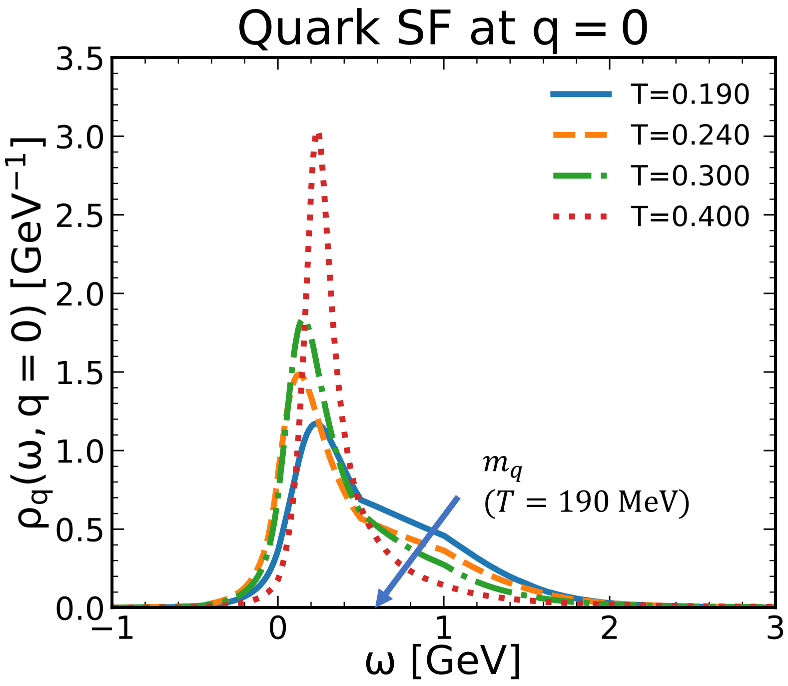
Excitations at finite temperature

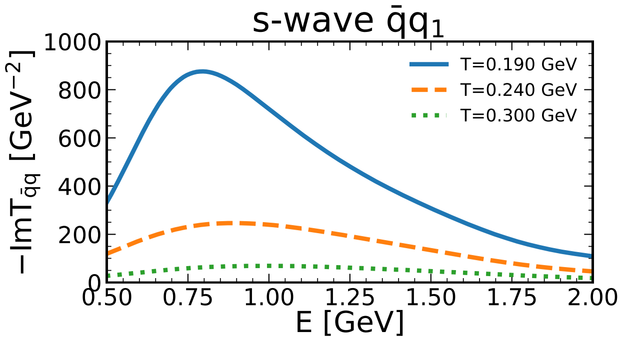
In-medium screening

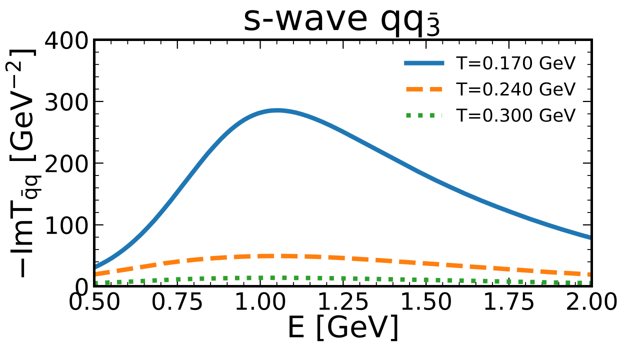
- Assumption: color screening onset at $T > T_0 = 0.15$ GeV
  - $\frac{\Lambda_S^4}{(q^2+\Lambda_S^2)^2} \rightarrow \frac{\Lambda_S^4}{(q^2+\Lambda_S^2+s_S(T^2-T_0^2))^2}$
  - $\frac{\Lambda_C^2}{q^2+\Lambda_C^2} \rightarrow \frac{\Lambda_C^2}{q^2+\Lambda_C^2+s_C(T^2-T_0^2)}$
- $\leftrightarrow$ Debye screening
- $s_S \ll s_C$ - remnants of confining force is still present in deconfined phase

Transition of d.o.f.

- T = 190 MeV: broad partons, well-defined resonances
- T = 300 MeV: narrow partons, dissociated resonances



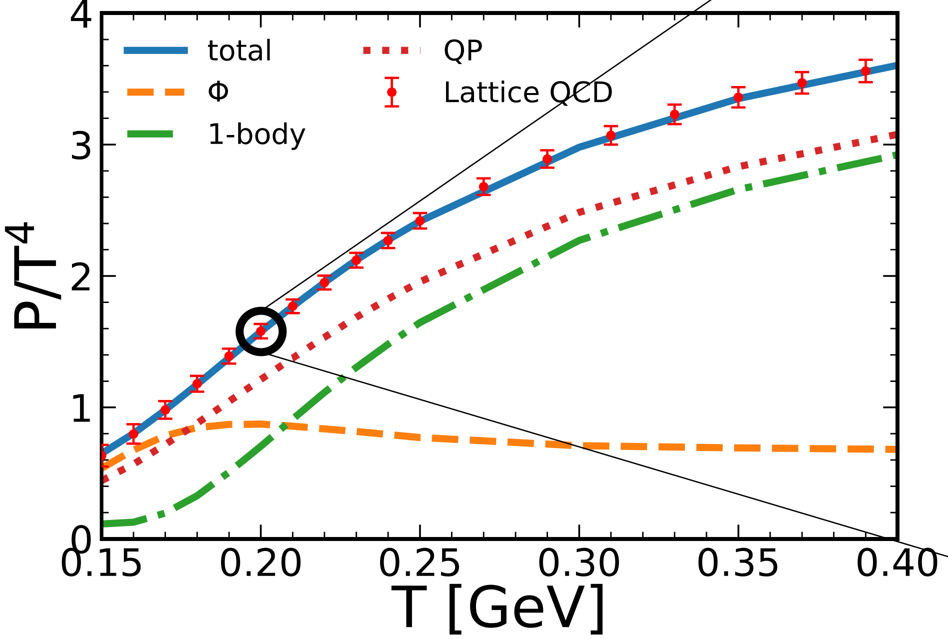


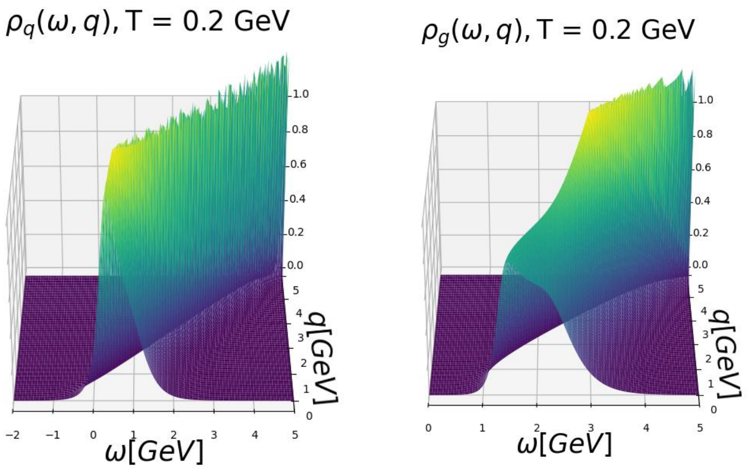


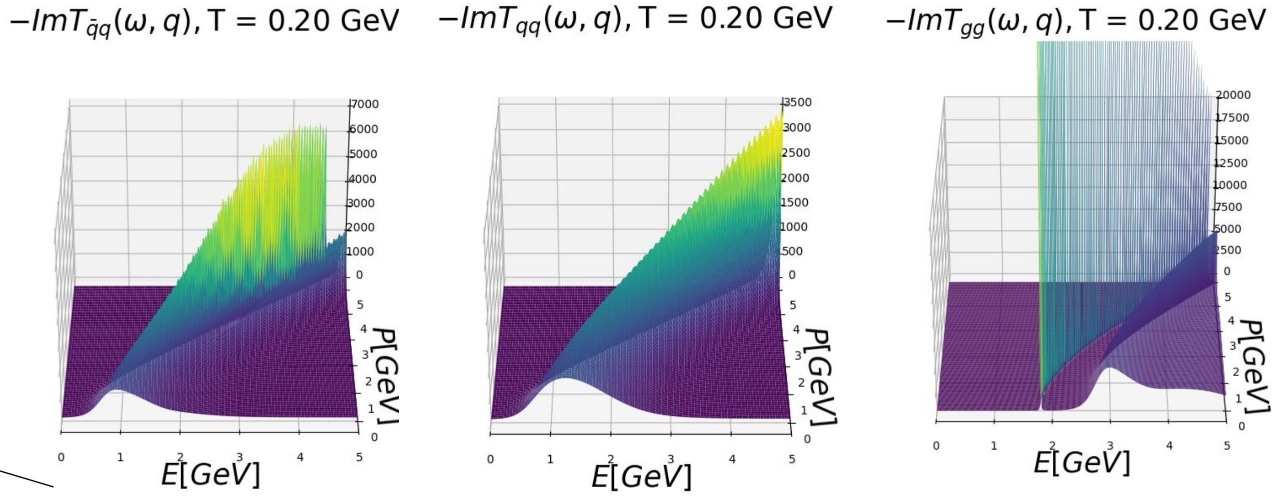
EoS and beyond

- Parton masses fitted to lattice QCD pressure at $\mu_B = 0$

[Borsanyi et al. Phys.Lett.*B* 730 (2014)]







- Main contribution: Φ at low T → partons at large T
- 1-body contribution ≠ QP case

\+ p-wave
\+ many channels:

qq	$q\bar{q}$	$(q/\bar{q})g$	gg
(1/2, 3)	(1, 1)	(9/8, 3)	(9/4, 1)
(−1/4, 6)	(−1/8, 8)	(3/8, 6)	(9/8, 16)
		(−3/8, 15)	(−3/4, 27)

Extension to finite μ_q

- In our formulation μ_q enters the propagators, not thermal distribution functions
- $\omega = 0 \leftrightarrow$ Fermi surface

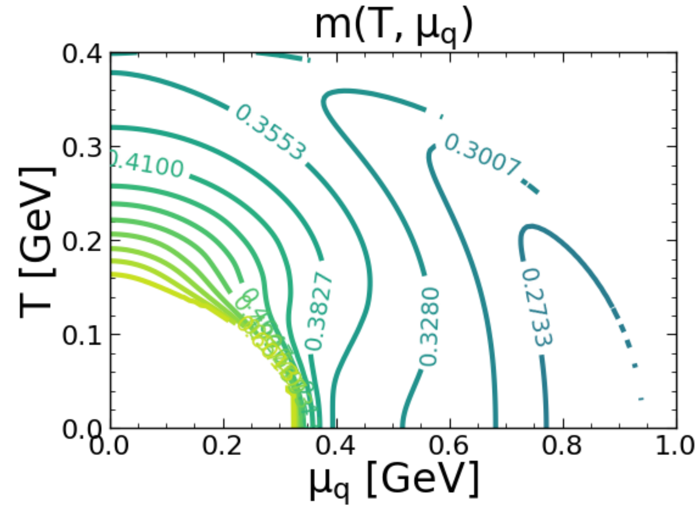
$$G_q(\omega, p; \mu_q) = \frac{1}{\omega + \mu_q - \varepsilon_p - \Sigma(\omega, q)}$$

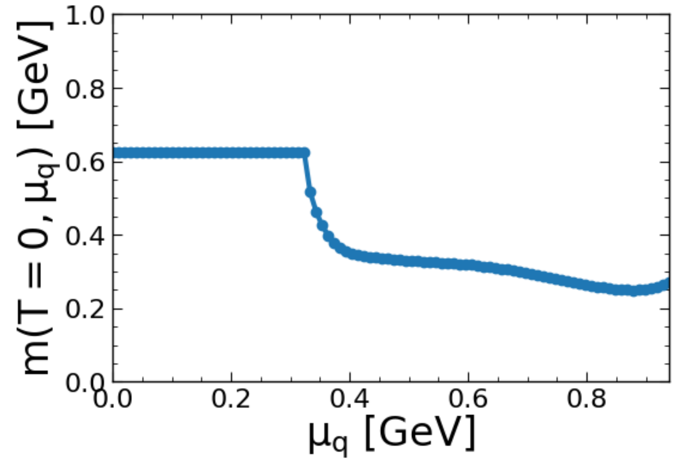
$$G_{\bar{q}}(\omega, p; \mu_q) = \frac{1}{\omega - \mu_q - \varepsilon_p - \Sigma(\omega, q)}$$

- Calibration: lattice QCD susceptibilities

$$P(T, \mu_B) = \sum_n \frac{1}{(2n)!} \chi_B^{(2n)}(T) \left(\frac{\mu_B}{T}\right)^{2n}$$

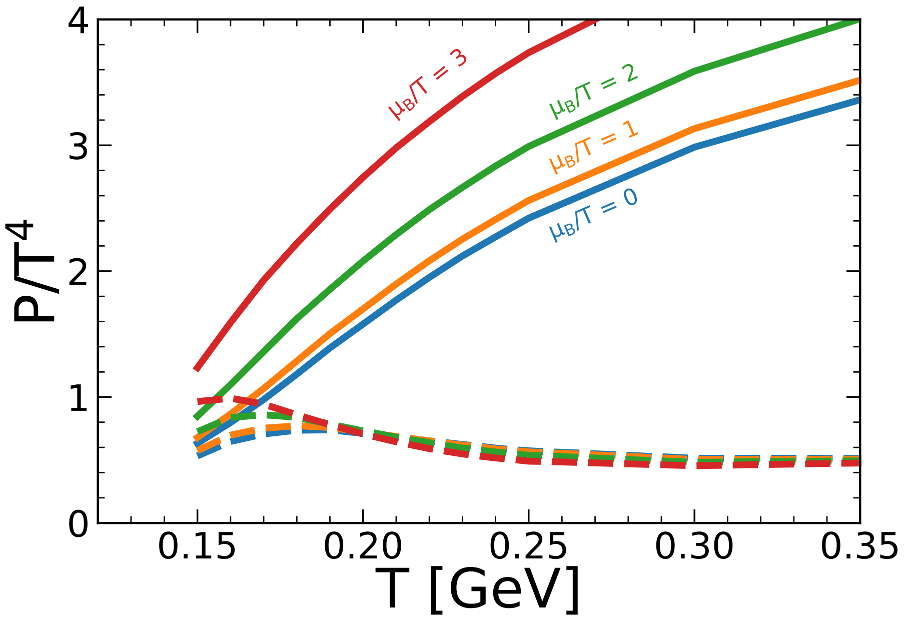
- Extrapolation ansatz for parton masses and screened interaction



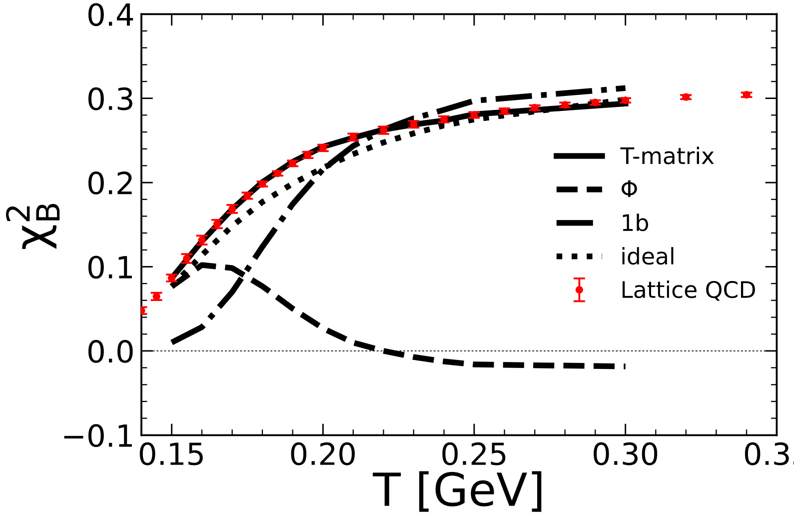


Susceptibilities

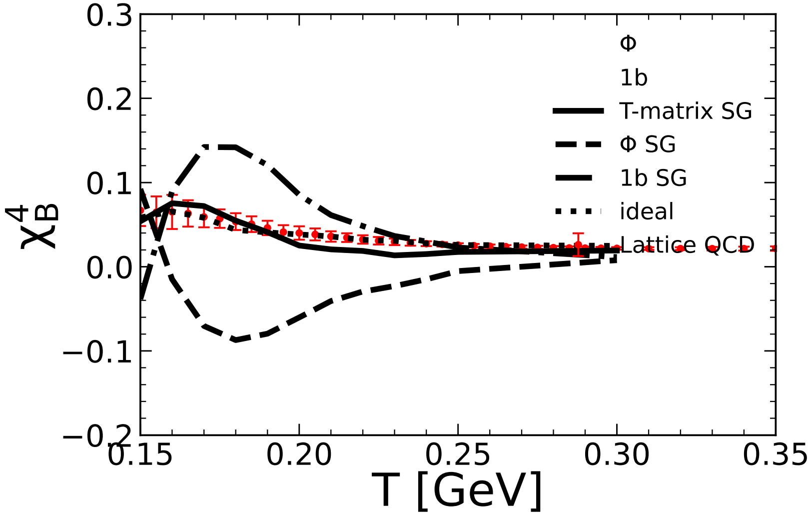
- Polynomial fit to get susceptibilities
  - $P(\mu_B) = P_0 + \frac{\chi_B^2}{2}\mu_B^2 + \frac{\chi_B^4}{4!}\mu_B^4 + \frac{\chi_B^4}{6!}\mu_B^6 + \cdots$
- Reasonable agreement with lattice QCD up to χ_6

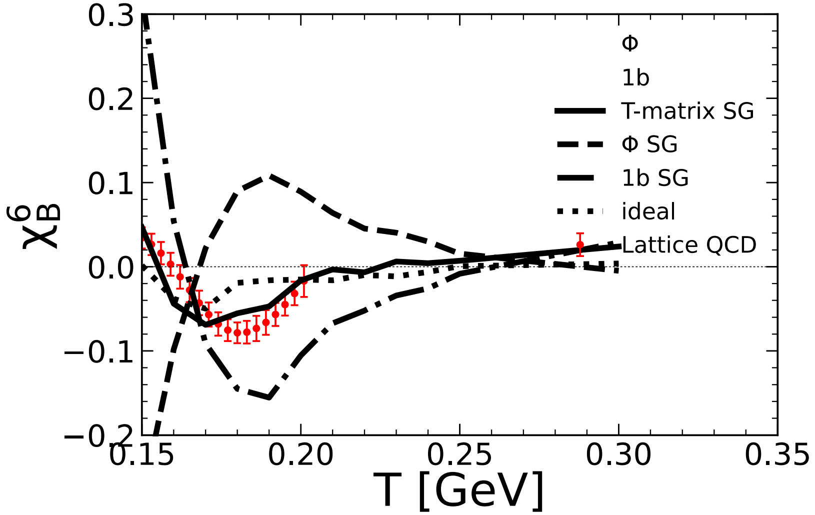


Fit



"Prediction"





[R. Bellwied et al Phys. Rev. D 92, 114505 (2015)]

Large μ_B: breaking the Lorentz invariance

- $\mu_B = 0$: vacuum approximation

$$T(E,P;q,q') = T(\sqrt{E^2 - P^2}, 0, q, q')$$

is not too bad ($\sim(10-20)\%$)

- Large μ_B, small T – picture changes
  - $P \ll 2\, p_F$ - particle-particle and hole-hole excitations treated on equal footing (unlike G-matrix approach)
  - $P > 2\, p_F$ - no hole-hole excitations anymore

- Lifesaving choice: separable interaction
- $T(E,P;q,q') \sim \frac{v(q)v(q')}{1-J(E,P)}$

$$\mathrm{Im}\, J(E,P) = \int d\omega \int_{\vec{k}} v_N v_{N'} \{[1-f(\omega)][1-f(E-\omega)] - f(\omega)f(E-\omega)\} \times \rho_i\left(\omega, \left|\frac{\vec{P}}{2} + \vec{k}\right|\right) \rho_j\left(E-\omega, \left|\frac{\vec{P}}{2} - \vec{k}\right|\right)$$

$\mathrm{Im} J$ and $\mathrm{Im}\Sigma$ can be calculated using Fast Fourier Transform

Color superconductivity

- Thouless criterion for superconductivity:

$$\mathrm{Re}\left[\, T_{qq_{\bar{3}}}(E=0, P=0)\right]^{-1} = 0$$

$\vec{p}_1 = -\vec{p}_2$ in the medium frame

- For the separable interaction

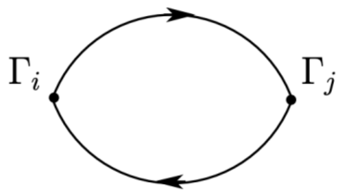
$$\det\nolimits_{\mathrm{sep}}[1 - \mathcal{F}_a \, J_{NN'}(E=0, P=0)] = 0$$

$$\mathcal{F}_a^{qq_{\bar{3}}} = 1/2$$

[T. Alm, G. Röpke et al. PRC 53 (1996)]

- Example: NJL model

$$S_D(\omega, \vec{q}) = \frac{1}{1 \; - 2\, G_D \, \Pi_D(\omega, \vec{q})}$$

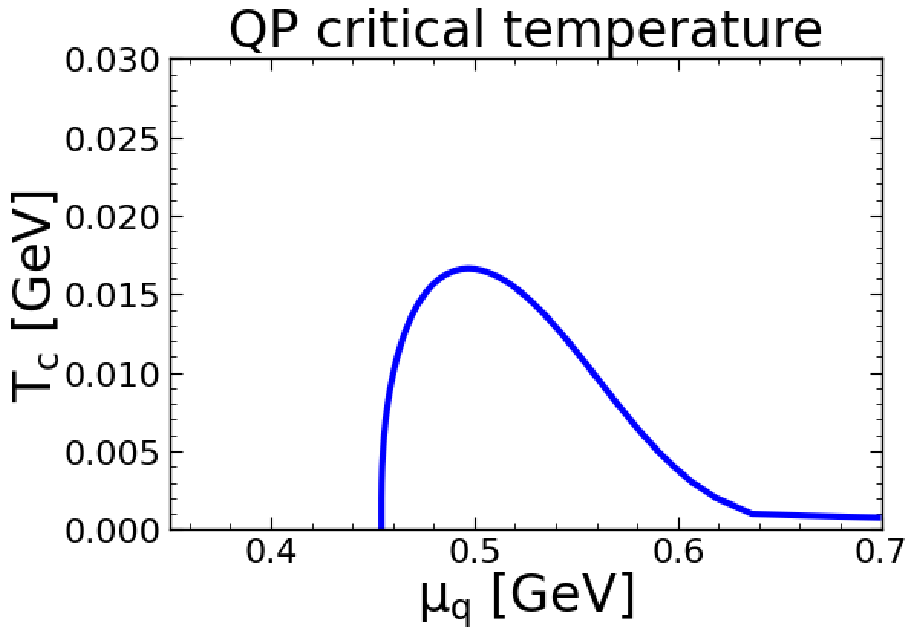
$$\Pi(\omega, \vec{q}) = $$  $$= i\,\mathrm{Tr}\int_k [G\,\Gamma_i\, G\Gamma_j]$$

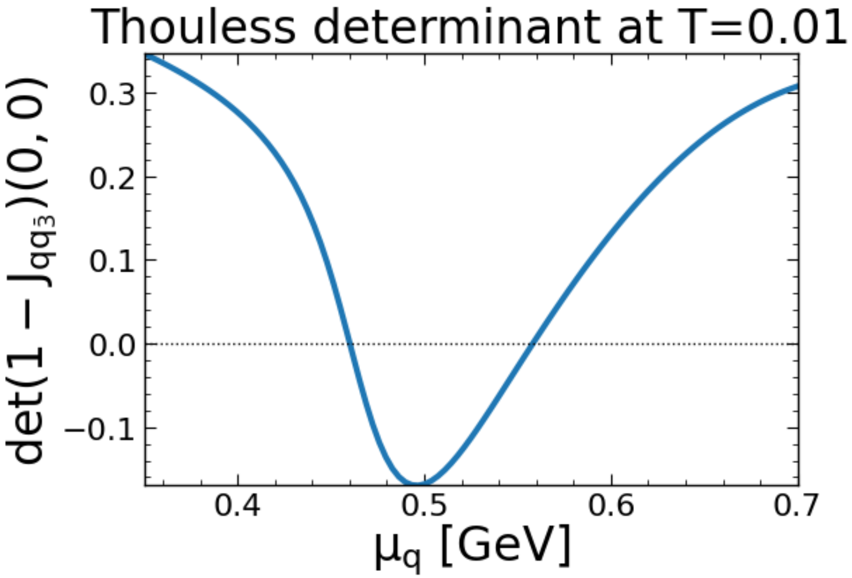
- On-shell fermions:
  Thouless criterion $1 - 2G_D\Pi_D(0{,}0) = 0$ is equivalent to mean-field gap equation at $\Delta = 0$

$$1 = 8\, N_f G_D \int \frac{d^3p}{(2\pi)^3} \frac{1 - 2n_F(\varepsilon_p - \mu, T)}{2\varepsilon_p - 2\mu}$$

Critical temperature: quasiparticle estimate

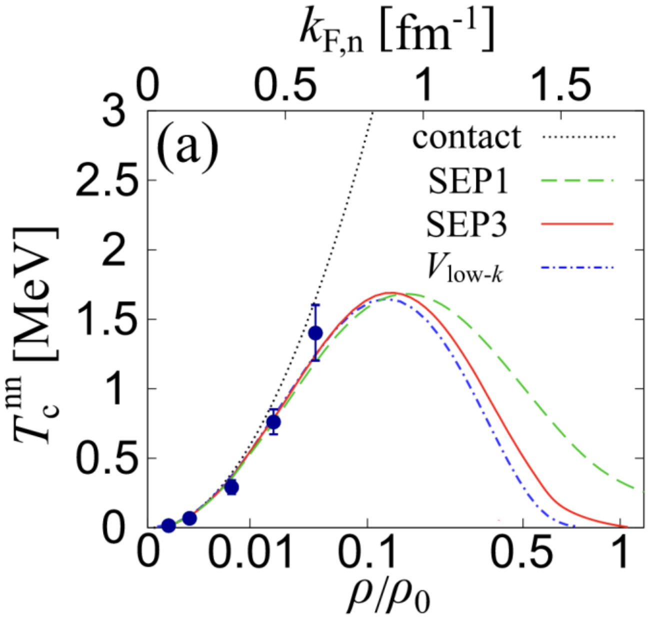
(See next slide)





- Reasons for the small gap:
  - Separable interaction

Approx. gap eq.: $1 \simeq V_s^{\mathrm{SEP}}(k_{\mathrm{F,n}}, k_{\mathrm{F,n}}) \sum_{\boldsymbol{k}} \frac{1}{2\xi_{\boldsymbol{k},\mathrm{n}}} \tanh\left(\frac{\xi_{\boldsymbol{k},\mathrm{n}}}{2T_c^{\mathrm{nn}}}\right).$



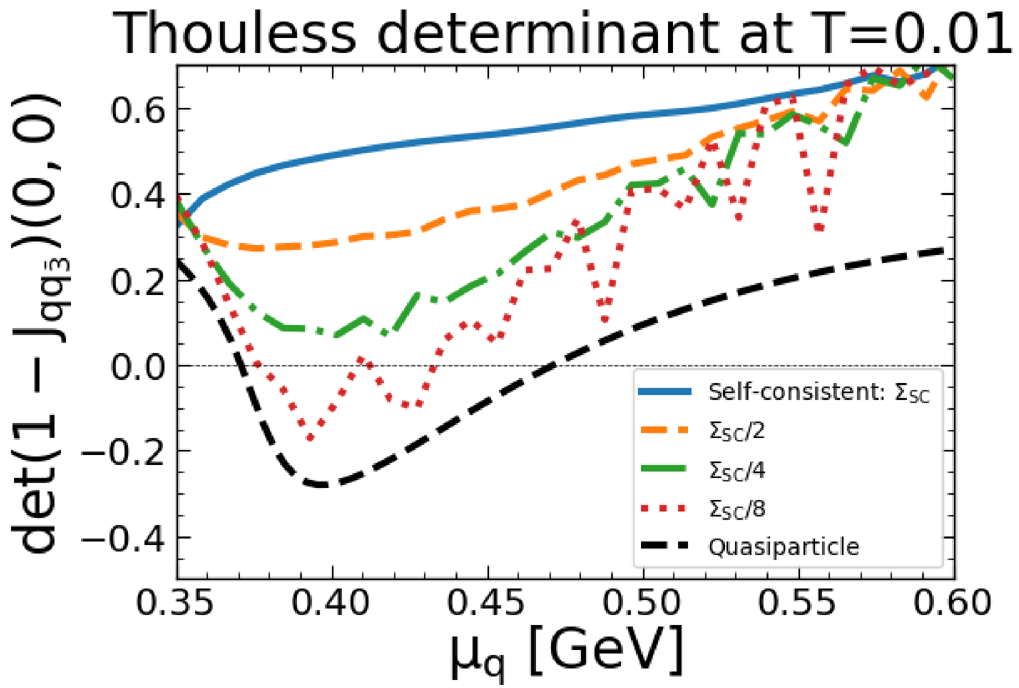
similar in nuclear matter: small at large n_B compared to contact $V(\vec{r}) = a\,\delta(\vec{r})$

[Tajima et al. Scientific Reports (2019)]

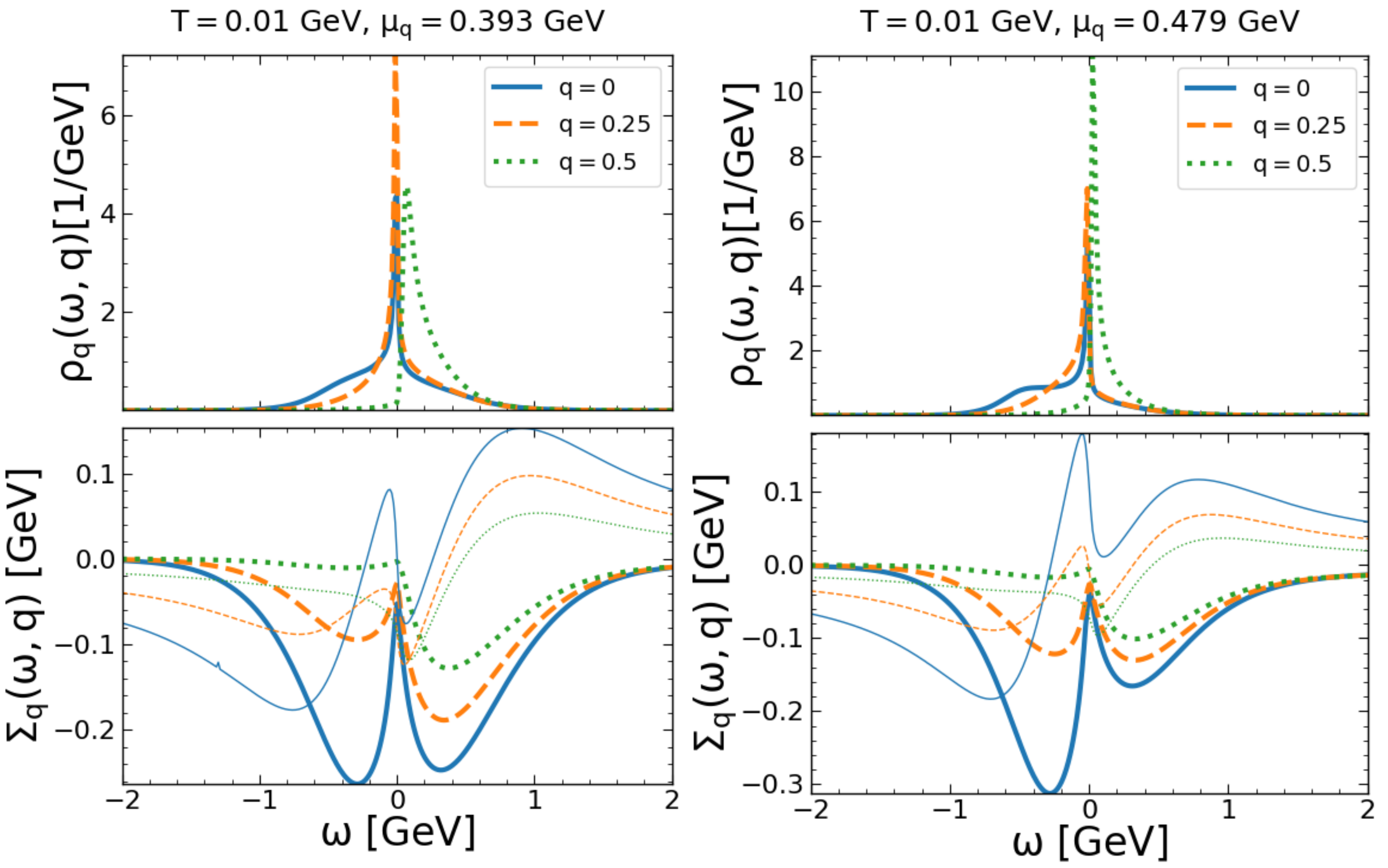
  - Calibration to $\chi_2(T)$ - $qq_{\overline{3}}$ can't be too much bound
  - Screening of $v(q)$ with increasing T, μ_q

Thouless criterion: self-consistent results

I use parameters fitted at $\mu_q = 0$ with Lorentz-invariant kinematics;
recalibration underway



- Self-consistent SFs:

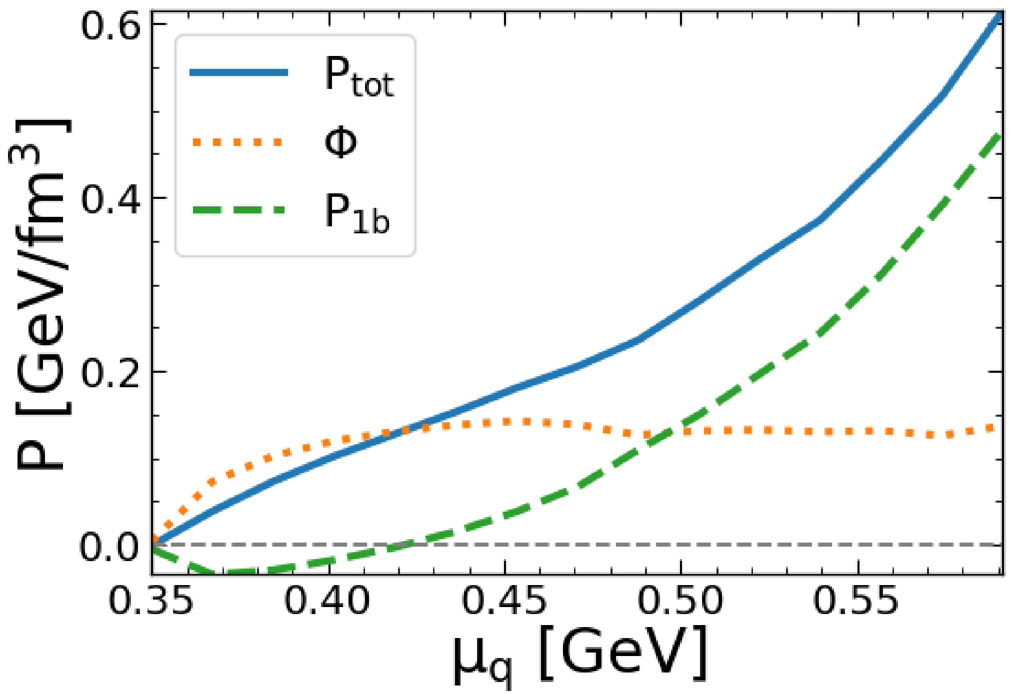


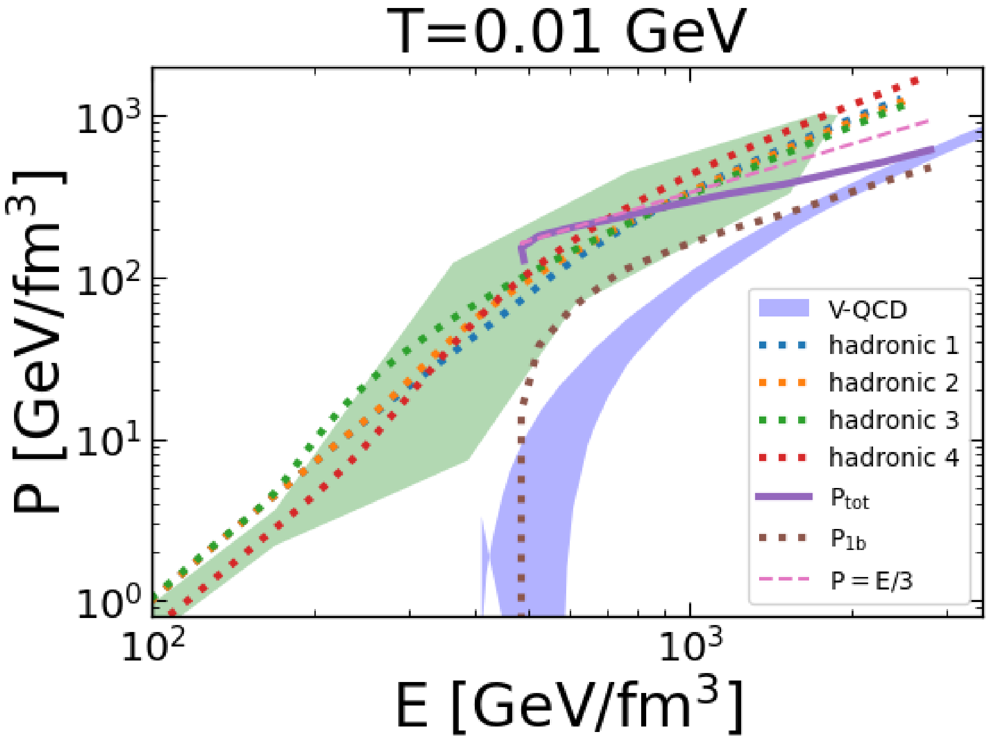
Not unseen before! [H. Müther W. H. Dickhoff PRC 72, (2005)]

Not unseen before! [P. Bożek Phys.Rev. C65 (2002)]

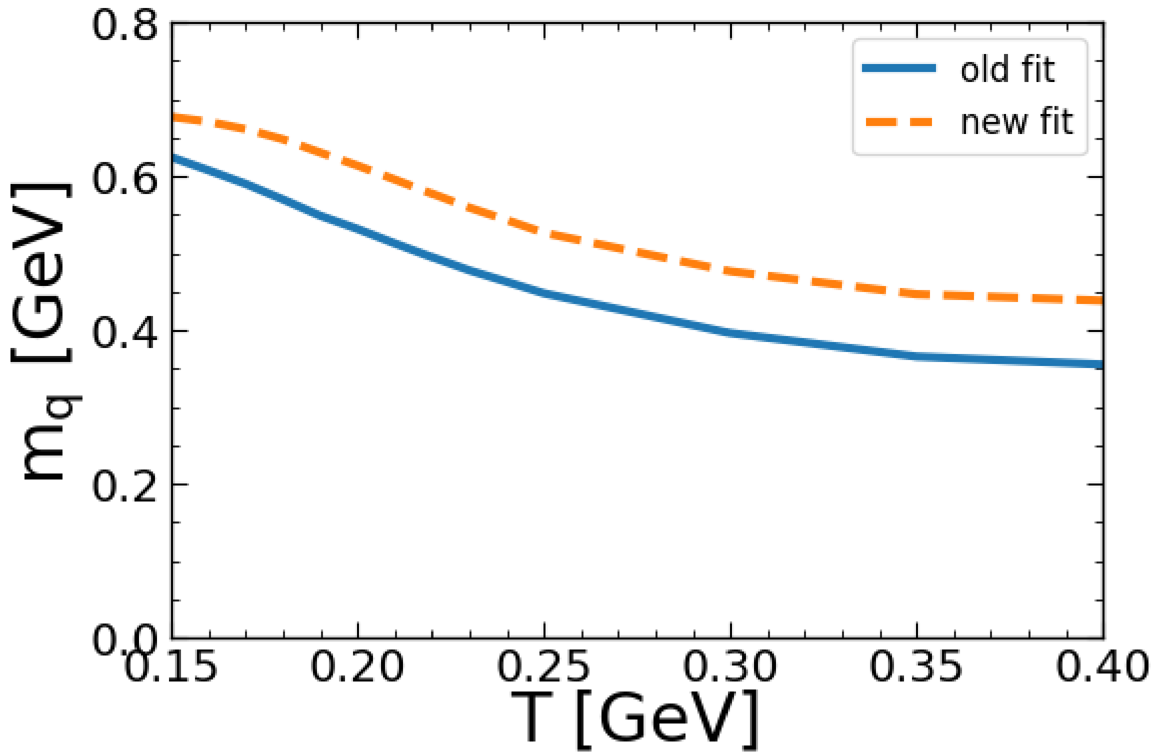
Thermodynamics (quark-diquark model)

- What do we have instead?
- Pressure from precursors of CSC: correlated $qq_{\bar{3}}$ pairs!
- Two-body contribution pushed total EoS into "typically hadronic" region





Work in progress: recalibration with P-dependence



- 10-20 % indeed

Summary $\mu_B = 0$:

- T-matrix model:
  beyond mean-field approach to description of strongly interacting QGP
- Explicit treatment of 2-body mesonic/diquark/glueball correlations
- A rank-2 separable model recovers:
  - quarkonium spectroscopy
  - collisional broadening
  - transition of degrees of freedom
- Model can be calibrated to describe $\mu_B = 0$ lattice QCD EoS and susceptibility

Large μ_B, low T

- Low quasiparticle T_c - artifact of separable interaction
- The very same strong interaction responsible for condensation significantly decreases T_c due to the collisional widths
- Pressure from 2-body diquark excitations brings $P(\mu)$ into typically hadronic region

Backup 1. NJL model with diquark excitations

- Diquark excitations in Generalized Beth-Uhlenbeck approach ("Gaussian fluctuations") [D. Blaschke et al. Annals of Physics 348 (2014)]
- Pressure comparable with quarks near the condensation!

Lead to overshooting the susceptibility (!)