Photo-production of a heavy-quark pair: interplay between Sudakov and saturation effects

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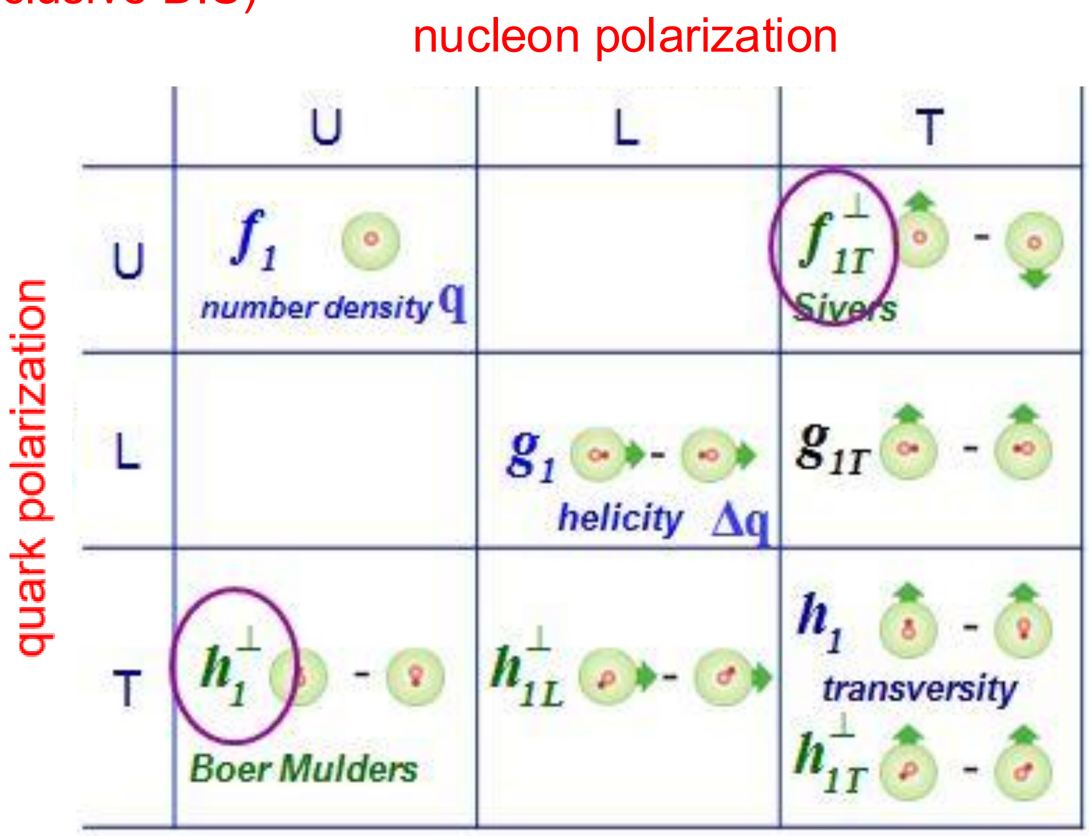
Introduction

TMDs with unpolarized beams

TMDs are crucial to describe hard processes in polarized collisions

(e.g. Drell-Yan and semi-inclusive DIS)

8 leading-twist TMDs



Sivers function

correlation between transverse spin of the nucleon and transverse momentum of the quark

Boer-Mulders function

correlation between transverse spin and transverse momentum of the quark in unpolarized nucleon

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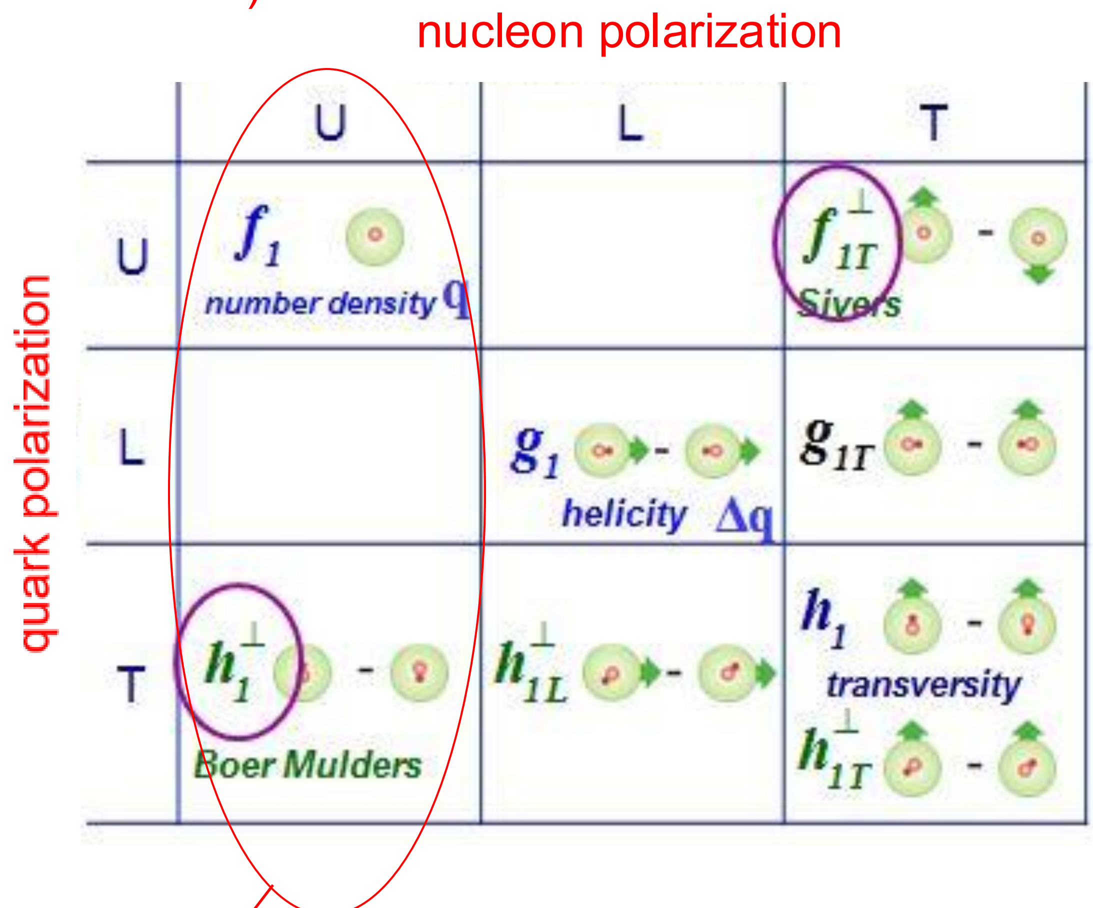
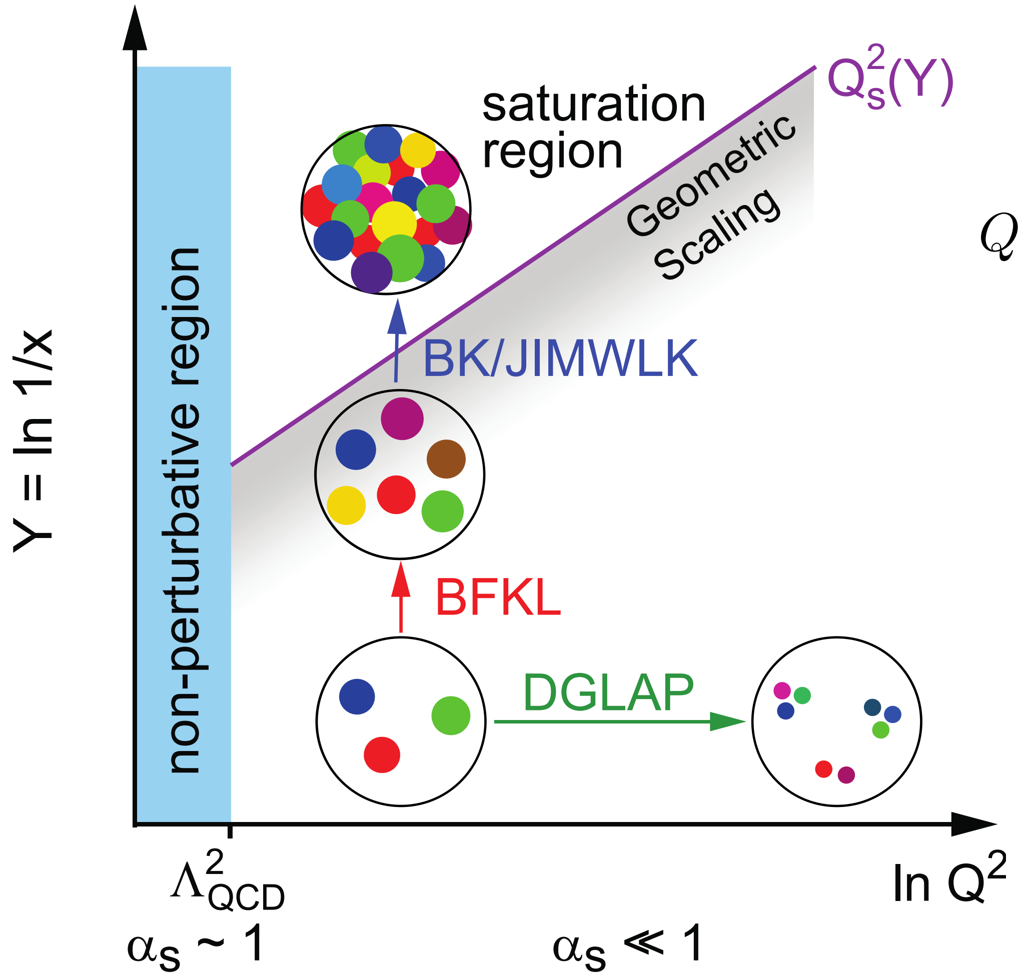


I will discuss those for gluons

→ I consider only hadronic/nuclear states that are *unpolarized*

Small x and saturation



$Q_s^2(x) \sim x^{-0.3} A^{1/3}$

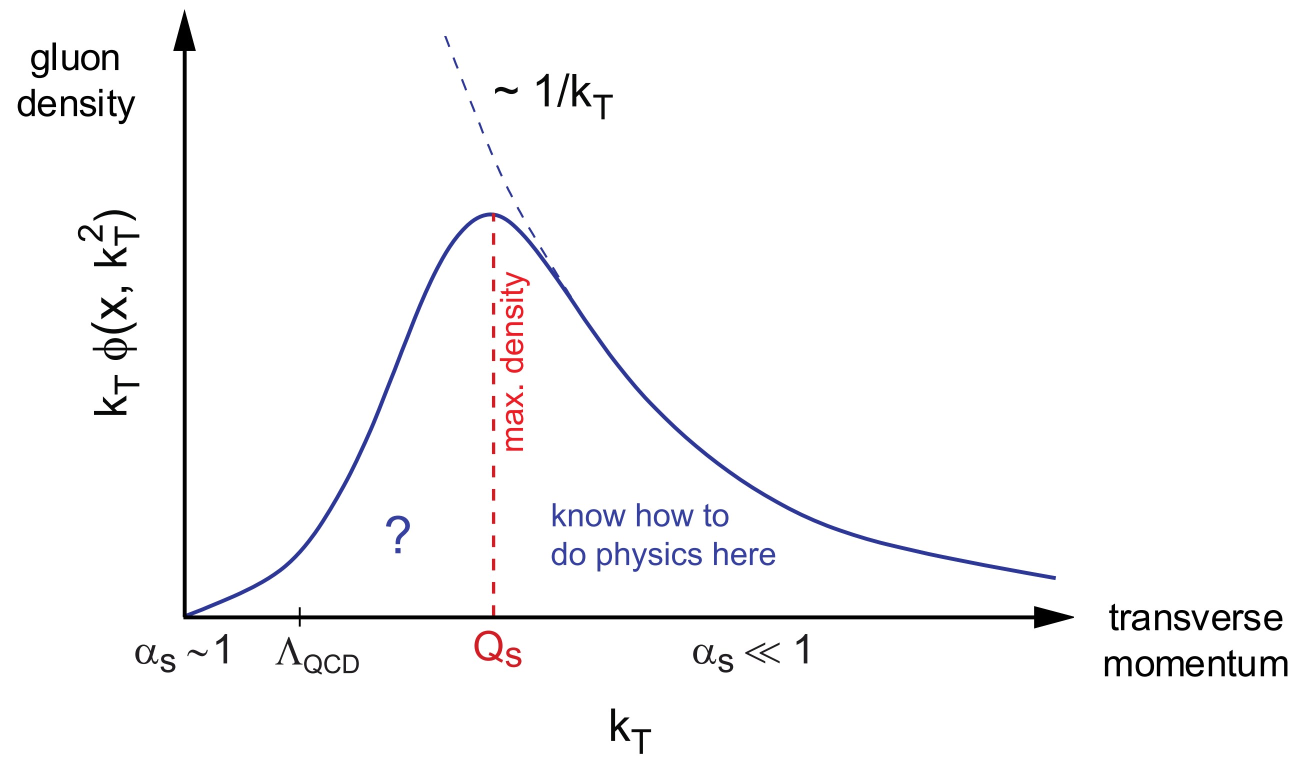
at small-x, the gluon transverse momentum plays an important role

so what does small-x physics have to say about gluon TMDs ?

The saturation scale

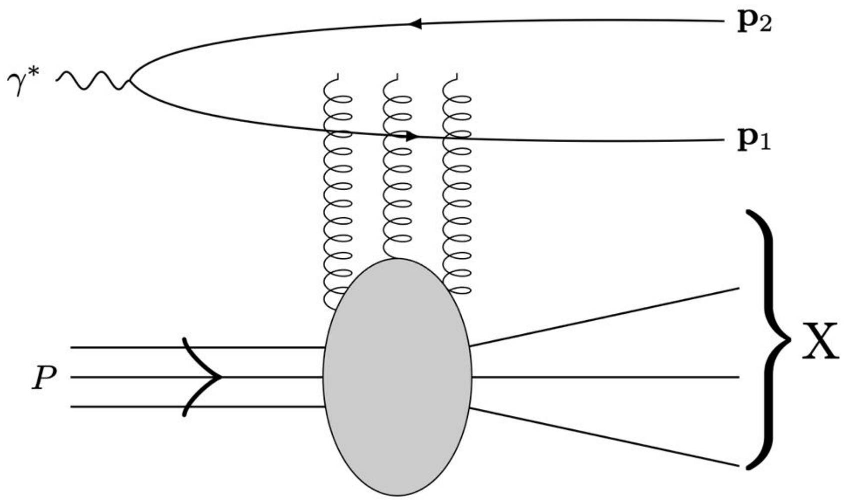
The saturation scale $Q_S(x)$ is the momentum scale which characterizes the transition between the dilute and dense regimes

at small-x, the typical gluon transverse momentum is no more Λ_{QCD} , it is instead $Q_S(x)$



the dynamics is non-linear, but the theory stays weakly coupled $\alpha_s(Q_s) \ll 1$

$Q\bar{Q}$ photo-production at small x



$\langle k_t \rangle \sim Q_s$

$Q_s(x) \gg \Lambda_{QCD}$

(*) the photon may also be virtual, but a large Q^2 value is not needed

The hard scale is: $|p_{1t}|,\ |p_{2t}| \sim \mathbf{P} \gg Q_s$

The semi-hard scale is:

$$|k_t|^2 = |p_{1t} + p_{2t}|^2 = |p_{1t}|^2 + |p_{2t}|^2 + 2|p_{1t}||p_{2t}| cos\Delta\phi$$

→ the small-x gluon's transverse momentum (di-jet imbalance)

The back-to-back regime: TMD factorization

Generic definitions of gluon TMDs

I consider only hadronic/nuclear states that are *unpolarized*

$$2\int \frac{d\xi^+ d^2\boldsymbol{\xi}_t}{(2\pi)^3 p_A^-} e^{ixp_A^-\xi^+ - ik_t\cdot\boldsymbol{\xi}_t} \left\langle A\middle|\mathrm{Tr}\left[F^{i-}\left(\xi^+,\boldsymbol{\xi}_t\right)F^{j-}(0)\right]\middle|A\right\rangle$$

$$= \frac{\delta_{ij}}{2}\mathcal{F}(x,k_t) + \left(\frac{k_i k_j}{k_t^2} - \frac{\delta_{ij}}{2}\right)\mathcal{H}(x,k_t)$$

$\mathcal{F}(x,k_t)$: unpolarized gluon TMD

$\mathcal{H}(x,k_t)$: linearly-polarized gluon TMD

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unpolarized gluon TMD — linearly-polarized gluon TMD

- at small x, $\mathcal{F} = \mathcal{H}$ in the linear (a.k.a. BFKL) regime:

$$\mathcal{F}(x,k_t) = UGD(x,k_t) + \mathcal{O}(Q_s^2/k_t^2)$$

$$\mathcal{H}(x,k_t) = UGD(x,k_t) + \mathcal{O}(Q_s^2/k_t^2)$$

so-called unintegrated gluon distribution

Kotko, Kutak, CM, Petreska, Sapeta, van Hameren (2015)

CM, Roiesnel, Taels (2017)

The back-to-back regime at LO

$$|p_{1t}|, |p_{2t}| \gg |k_t|, Q_s$$

- a factorization can be established in the small *x* limit, for nearly back-to-back di-jets

Dominguez, CM, Xiao and Yuan (2011)

$$d\sigma \propto H^{ij}(\mathbf{P}) \left[\frac{1}{2}\delta^{ij}\mathcal{F}(x,k_t) + \left(\frac{k^i k^j}{k_t^2} - \frac{1}{2}\delta^{ij} \right) \mathcal{H}(x,k_t) \right]$$

↓

hard factors

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↓ hard factors

- gauge links are missing in the previous definition, their structure for this process implies that the gluon TMDs are of the Weizsäcker Williams type, which at small-x gives

$$\mathcal{F}_{WW}(x,k_t) = -\frac{4}{g^2}\int \frac{d^2\mathbf{x}d^2\mathbf{y}}{(2\pi)^3}\, e^{-ik_t\cdot(\mathbf{x}-\mathbf{y})}\left\langle \mathrm{Tr}\left[(\partial_i U_{\mathbf{x}})U_{\mathbf{y}}^\dagger(\partial_i U_{\mathbf{y}})U_{\mathbf{x}}^\dagger\right]\right\rangle_x$$

similarly for H_{WW} with projection onto the other 2d Lorentz structure

ITMD factorization

provides matching with BFKL at $k_t \sim \mathbf{P}$

$$d\sigma \propto H^{ij}(\mathbf{P}, k_t)\left[\frac{1}{2}\delta^{ij}\mathcal{F}(x, k_t) + \left(\frac{k^i k^j}{k_t^2} - \frac{1}{2}\delta^{ij}\right)\mathcal{H}(x, k_t)\right]$$

hard factors

Kotko, Kutak, CM, Petreska, Sapeta, van Hameren (2015 - 2016)
Altinoluk, Boussarie, Kotko (2019)

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↓

hard factors

Kotko, Kutak, CM, Petreska, Sapeta, van Hameren (2015 - 2016)
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- TMD factorization involves $H^{ij}(\mathbf{P}, k_t = 0)$

  needs $\mathbf{P} \gg k_t,\ Q_s$

- improved TMD (ITMD) factorization involves

$$H^{ij}(\mathbf{P}, k_t) = H^{ij}(\mathbf{P}, k_t = 0) + \sum_n c_n (k_t/\mathbf{P})^n$$

all-order resummation of higher “kinematic” twists

also valid away from $\Delta\Phi = \pi$, when $k_t \sim \mathbf{P}$

Processes sensitive to $\mathcal{H}$

- factorization may be rewritten

$$d\sigma \propto H^{ns}(\mathbf{P},k_t)\mathcal{F}(x,k_t) + H^{h}(\mathbf{P},k_t)\Big(\mathcal{H}(x,k_t) - \mathcal{F}(x,k_t)\Big)$$

= 0 in BFKL regime

projections onto
“non-sense” polarization

$$H^{ns} = H^{ij}k^ik^j/k_t^2$$

projections onto linear polarization

$$H^{h} = H^{ij}(k^ik^j/k_t^2 - \delta^{ij}/2)$$

emergence, due to non-linear effects, of
small-x gluons which are not fully linearly polarized

Processes sensitive to $\mathcal{H}$

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- processes for which the H^h hard factors are non-zero:

  - dijets in deep inelastic scattering (e+p or e+A)
  - heavy-quark pair production (in photo-production or p+A collisions)
  - trijets or more

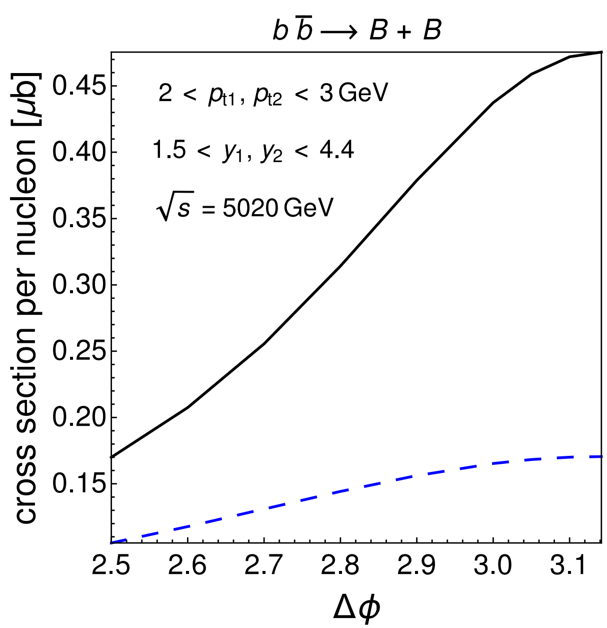
Altinoluk, Boussarie, CM, Taels (2019 - 2020)
Altinoluk, CM, Taels (2021)

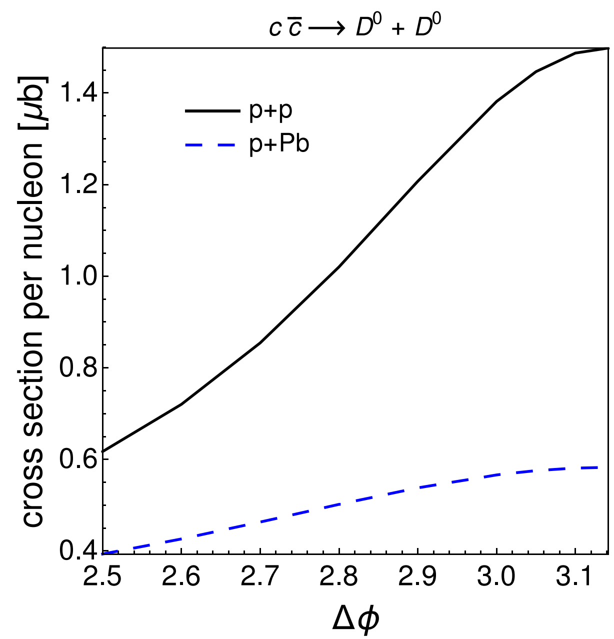
the polarized gluons come with a cos(2ϕ) modulation
(at small k_t / **P**) where ϕ is the angle between k_t and **P**

Forward $Q\bar{Q}$ pair in p+A collisions

- preliminary study performed for HL-LHC yellow report

CM, Giacalone (2018)





soft-gluon resummation needs to be implemented as well

important near $\Delta\Phi = \pi$, when $\log(\mathbf{P}/k_t)$ becomes large

NLO corrections and QCD evolution: di-jet case

Resumming large logarithms

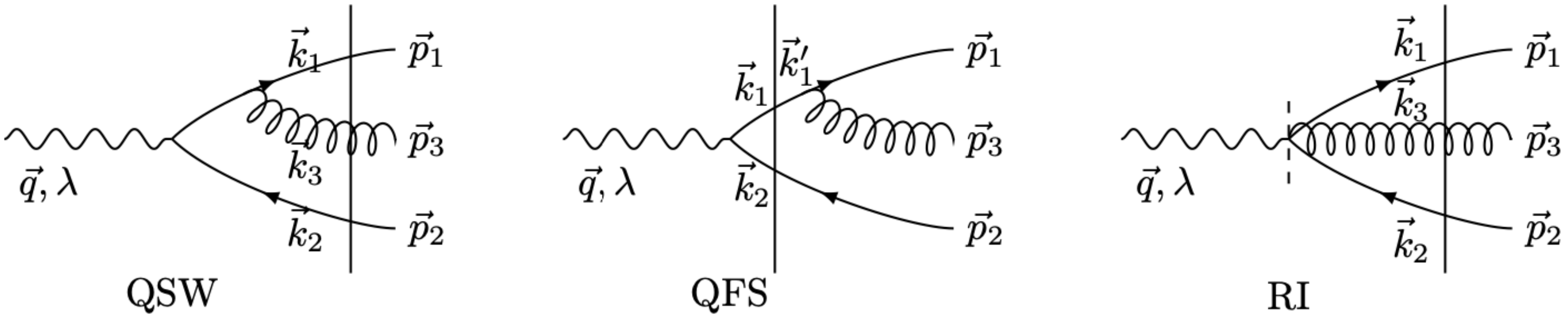
Simultaneous resummation of high-energy $\ln(1/x)$ and Sudakov $\ln(Q^2/\mathbf{k}_\perp^2)$ logarithms?

Longstanding problem, studied using many different approaches, including recently:

SW: Balitsky, Tarasov (2015)
RO: Balitsky (2021-2023)
HEF: Deak, Hautmann, Jung, Kutak, van Hameren, Sapeta, Hentschinski (2016-2021)
BFKL: Nefedov (2021)
PB: Hautmann, Hentschinski, Keersmaekers, Kusina, Kutak, Lelek (2022)
CGC: Mueller, Xiao, Yuan (2011); Hatta, Xiao, Yuan, Zhou (2017-2021); Stasto, Wei, Xiao, Yuan (2018); PT, Altinoluk, Beuf, Marquet (2022); Caucal, Salazar, Schenke, Venugopalan (2022-2023)

Real emission diagrams

Altinoluk, Boussarie, CM and Taels (2020)



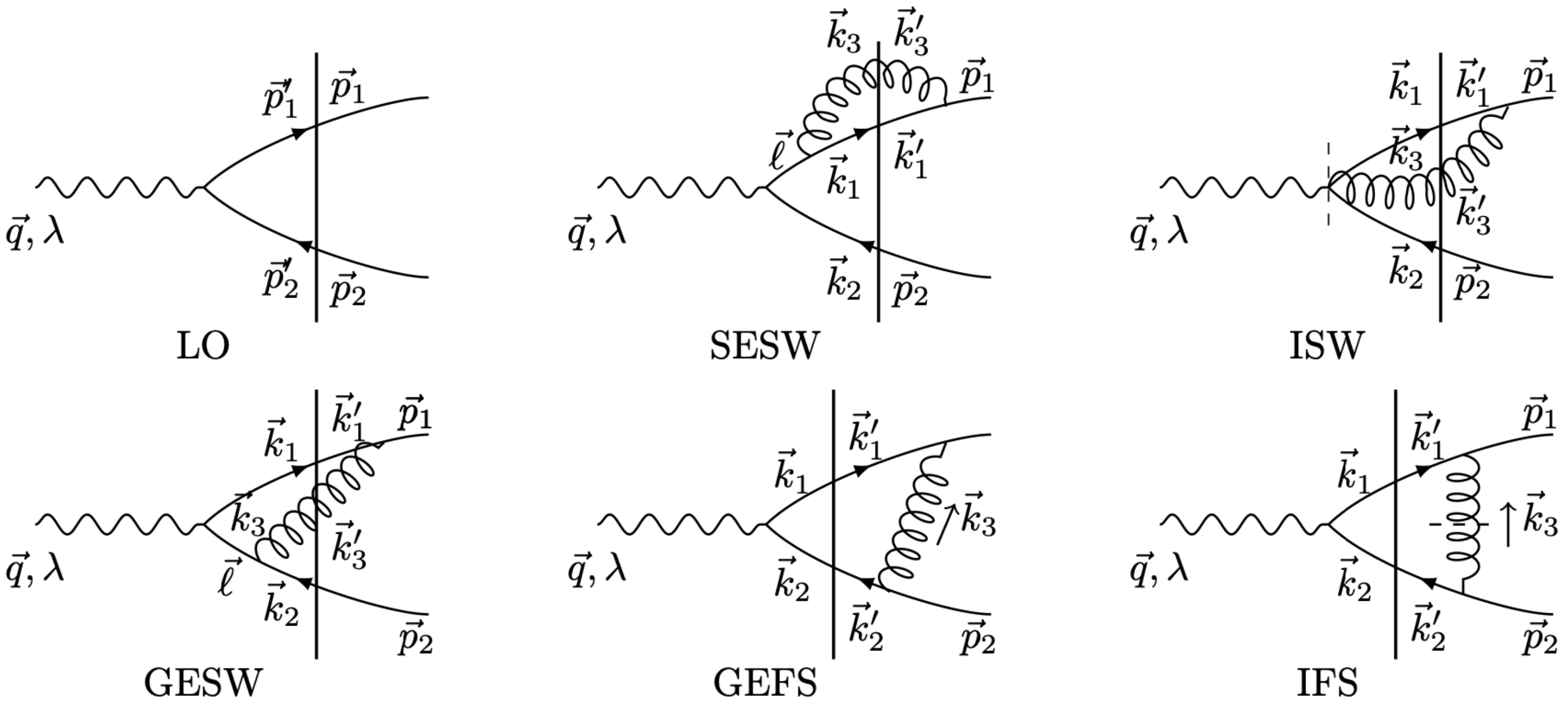
linearly-polarized gluon TMD involved at NLO, even for photo-production

see also

Caucal, Salazar and Venugopalan (2021)
Bergabo and Jalilian-Marian (2022)
Iancu and Mulian (2023)

Virtual diagrams

Caucal, Salazar and Venugopalan (2021)



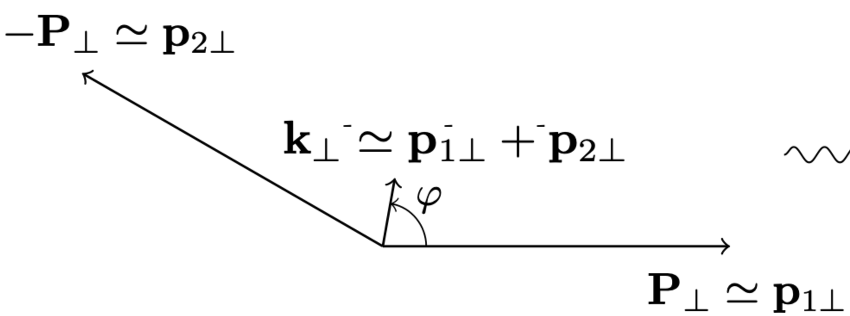
full NLO CGC is UV, soft, collinear finite,
rapidity divergences give small-x evolution

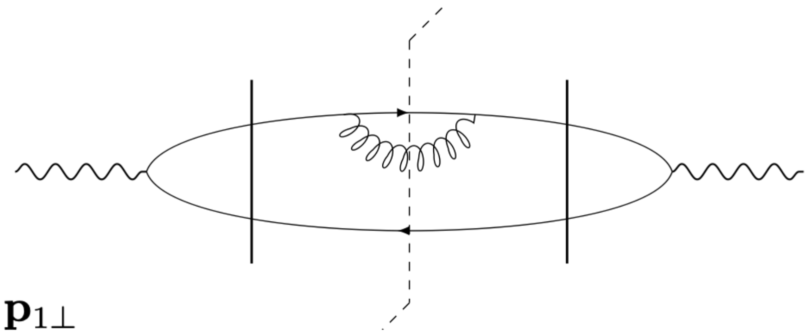
see also Taels, Altinoluk, Beuf and CM (2022)
Bergabo and Jalilian-Marian (2022)

The back-to-back regime at NLO

full NLO + TMD limit

Taels, Altinoluk, Beuf and CM (2022)





Remnants of soft-collinear generate Sudakov double log with wrong sign!

$$\mathrm{d}\sigma_{\mathrm{NLO}}^{\mathrm{TMD}} = \mathrm{d}\sigma_{\mathrm{LO}}^{\mathrm{TMD}} \times \frac{\alpha_s N_c}{4\pi} \ln\left(\frac{\mathbf{P}_\perp^2(\mathbf{b}-\mathbf{b}')^2}{c_0^2}\right)^2$$

$$\mathbf{P}_\perp^2 \sim \mu^2$$
$$(\mathbf{b}-\mathbf{b}')^2 \sim 1/\mathbf{k}_\perp^2$$

this is due to an over-subtraction of the small-x rapidity logarithms

Sudakov and small-x logs aren't completely separated in phase space!

Kinematically-constrained evolution

Taels, Altinoluk, Beuf and CM (2022)

To obtain $d\sigma^{\text{NLO}}_{\text{TMD}} \;\;\text{"}=\text{"}\;\; d\sigma^{\text{LO}}_{\text{TMD}} \times \left(-\frac{\alpha_s N_c}{4\pi}\right) \ln^2(\mathbf{P}^2|\mathbf{x}-\mathbf{y}|^2)$

and then write

$$\mathcal{F}_{WW}(x, k_t; P) = -\frac{4}{g^2}\int \frac{d^2\mathbf{x}d^2\mathbf{y}}{(2\pi)^3}\, e^{-ik_t\cdot(\mathbf{x}-\mathbf{y})} e^{-S_{sud}(\mathbf{P},\mathbf{x}-\mathbf{y})} \left\langle \text{Tr}\left[(\partial_i U_{\mathbf{x}})U^\dagger_{\mathbf{y}}(\partial_i U_{\mathbf{y}})U^\dagger_{\mathbf{x}}\right]\right\rangle_x$$

, re-summing the small-x logs and Sudakov logs separately, the rapidity subtraction must be altered

this leads to a kinematically-constrained small-x evolution

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Taels, Altinoluk, Beuf and CM (2022)

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→ in the small-x evolved LO contribution, the kernel of the JIMWLK equation now contains an extra theta term $\theta\left[(k_g^+/k_f^+)\mathbf{P}^2 - \mathbf{k}_g^2\right]$

confirmed beyond large Nc and double logs in

Caucal, Salazar, Schenke, Venugopalan (2022)

Asymmetry supressed by evolution

- without Sudakov resummation, the CGC predicts sizable <cos(2ϕ)>

$$\mathrm{d}\sigma \propto H_F(\mathbf{P})\mathcal{F}(x,k_t) + \cos(2\phi)H_H(\mathbf{P})\mathcal{H}(x,k_t)$$

$$< \cos(2\phi) > \propto \frac{\mathcal{H}(x,k_t)}{\mathcal{F}(x,k_t)}$$

with $\mathcal{F}$ & $\mathcal{H}$ of similar magnitude

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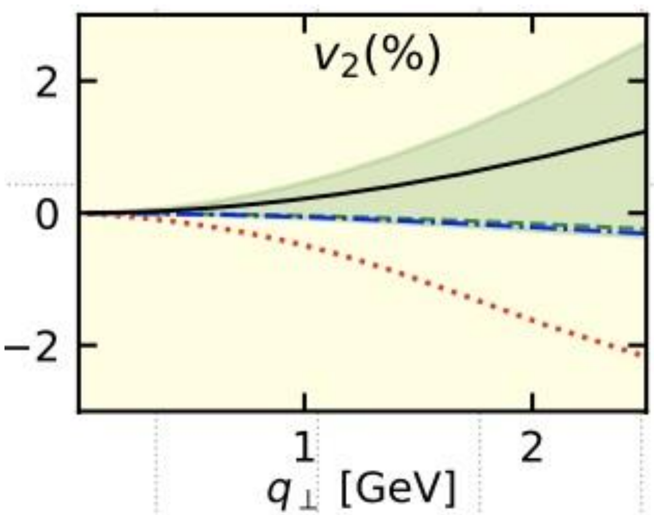
- however TMD evolution suppresses the asymmetry

Boer, Mulders, Zhou and Zhou (2017)



full NLO

LO + Sudakov & BK

LO

Caucal, Salazar, Schenke, Stebel and Venugopalan (2024)

Heavy quark-antiquark pair

Asymetry with heavy $Q\bar{Q}$ pair

CM, Y. Shi and C. Zhang, in preparation

- we computed the Sudakov factor for heavy quark production

following the method in Hatta, Xiao, Yuan, Zhou (2021), we obtain:

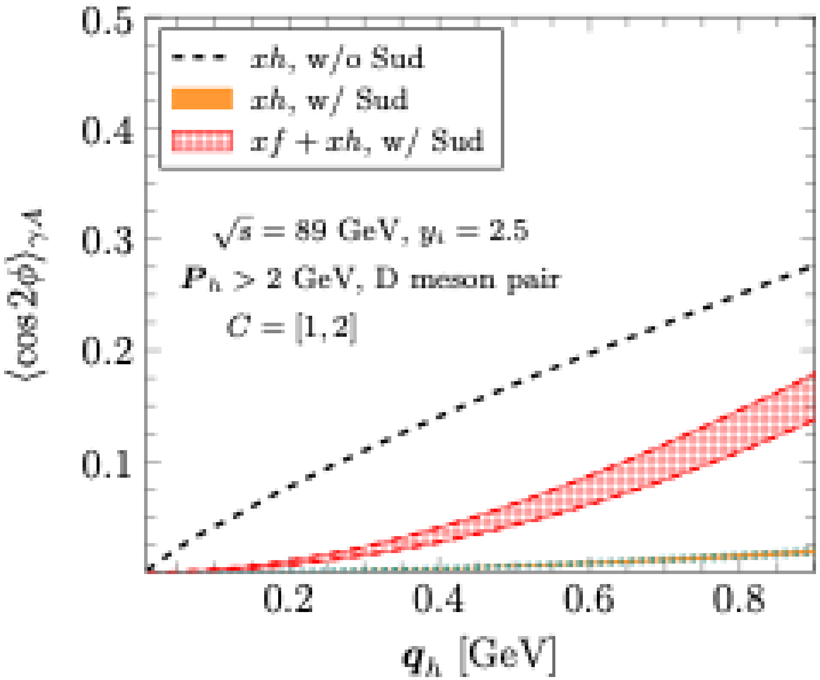
$$\mathcal{F}_{WW}(x,k_t;P) \longrightarrow -\frac{4}{g^2}\int \frac{d^2\mathbf{x}d^2\mathbf{y}}{(2\pi)^3}\, e^{-ik_t\cdot(\mathbf{x}-\mathbf{y})}e^{-S_{sud}(\mathbf{P},\mathbf{x}-\mathbf{y})}\left\langle \mathrm{Tr}\left[(\partial_i U_{\mathbf{x}})U^\dagger_{\mathbf{y}}(\partial_i U_{\mathbf{y}})U^\dagger_{\mathbf{x}}\right]\right\rangle_x \times\left[1-\alpha_s\sum_{n>0}c_{2n}\cos(2n\phi_{\mathbf{P},\mathbf{x}-\mathbf{y}})\right]$$

additional cos(2nϕ) factor implies non-zero asymmetry for the $\mathcal{F}$ term

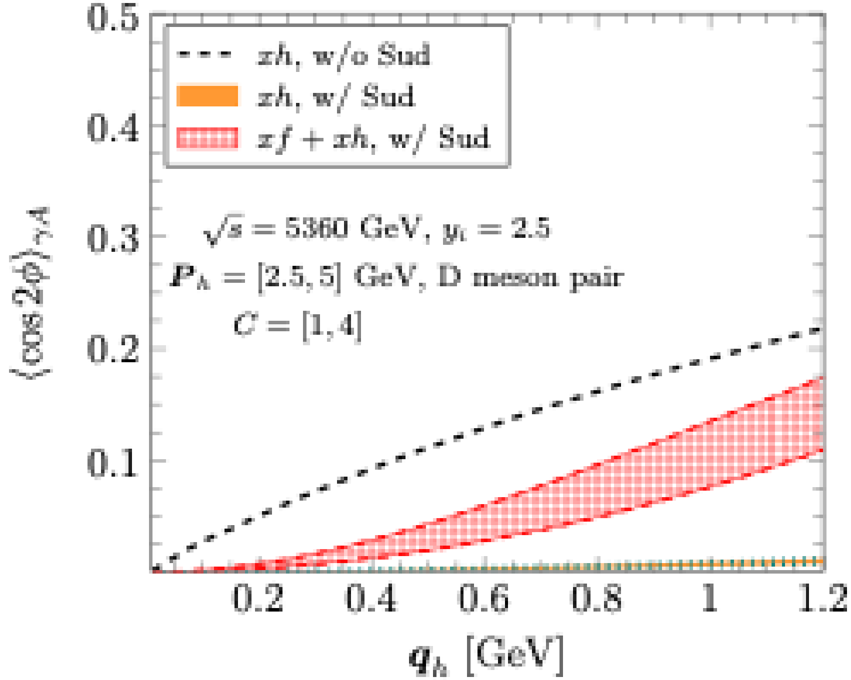
Asymetry with heavy $Q\bar{Q}$ pair

CM, Y. Shi and C. Zhang, in preparation

EIC γ+A



UPC γ+A

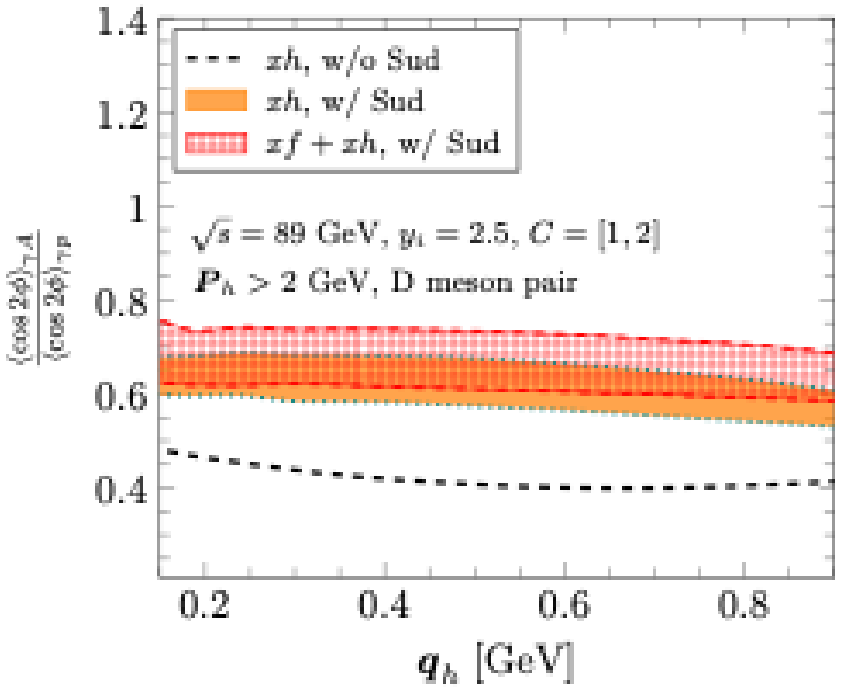


with Sudakov factor: the contribution of linearly-polarized gluons is negligible

Asymetry with heavy $Q\bar{Q}$ pair

CM, Y. Shi and C. Zhang, in preparation

EIC γ+A / γ+p



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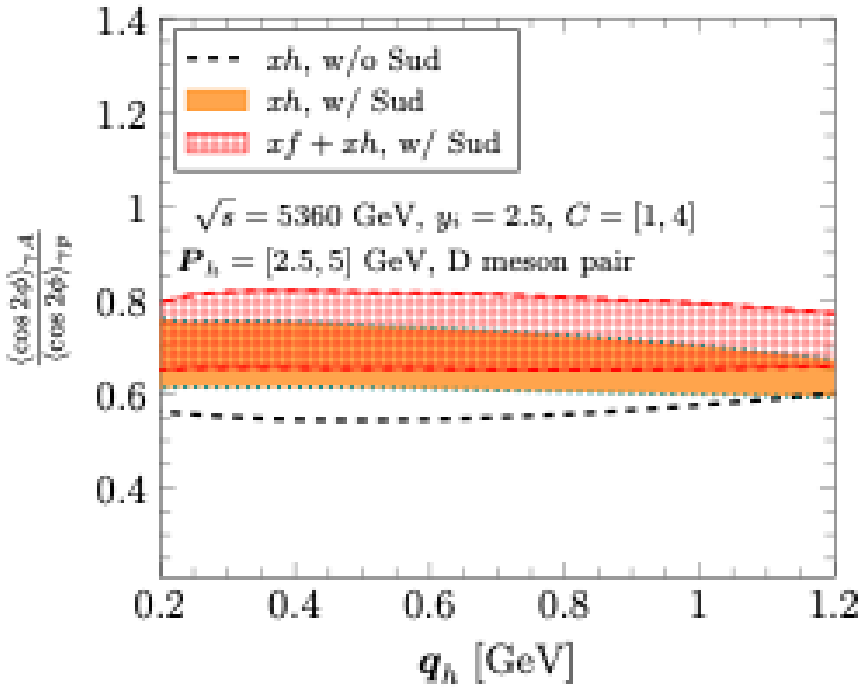
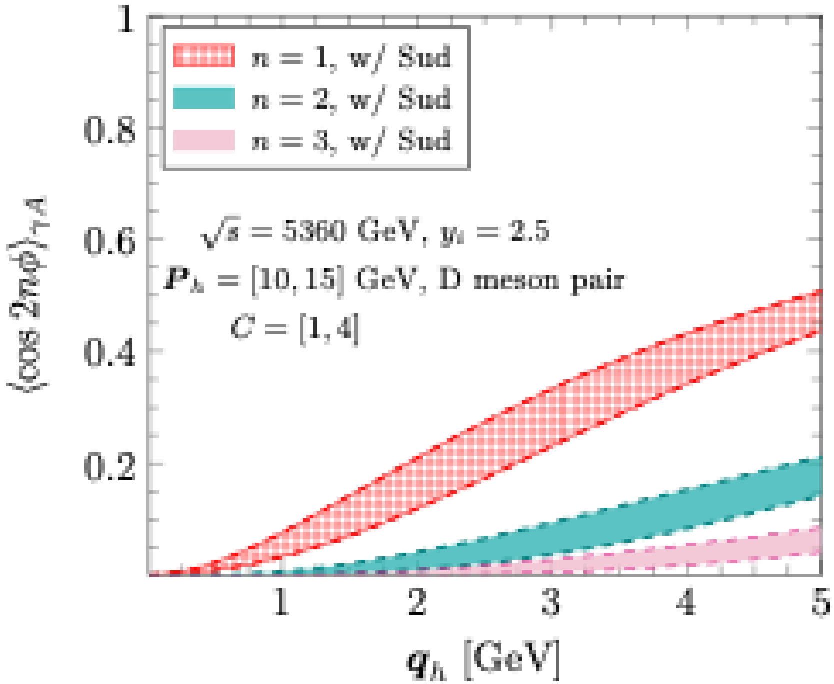


the γ+A / γ+p suppression survives after the Sudakov factor is included

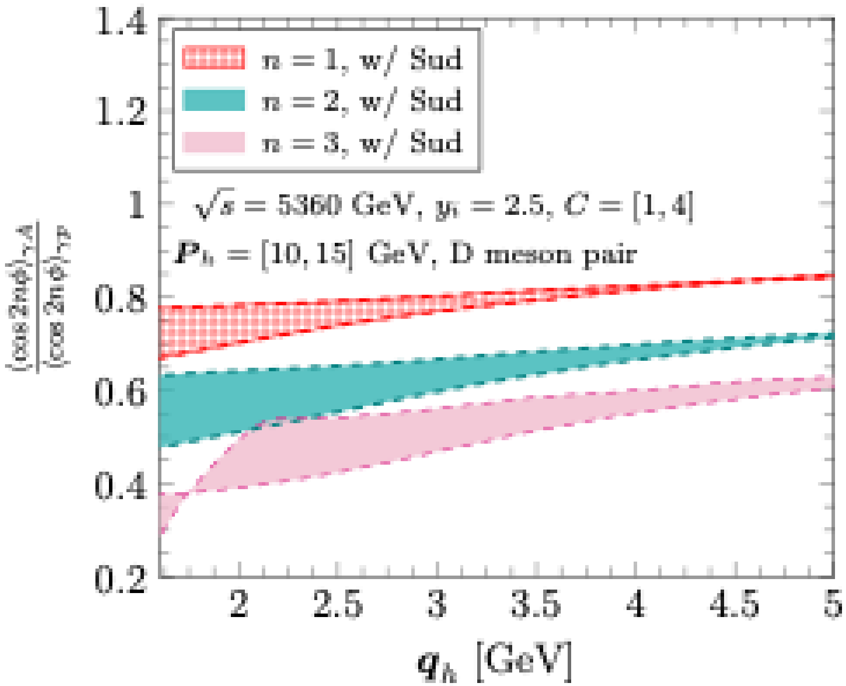
Asymetry with heavy $Q\bar{Q}$ pair

CM, Y. Shi and C. Zhang, in preparation

UPC γ+A



UPC γ+A / γ+p



higher harmonics are even more sensitive to non-linear effects

Conclusions

- to match collinear physics and small-x physics in the linear BFKL regime, the necessity of a kinematical constraint in the small-x evolution was recognized a long time ago (led to CCFM equation)

  Ciafaloni ('88); Andersson, Gustafson, Samuelsson ('96);
  Kwiecinski, Martin, Sutton ('96); Salam ('98)

- more recently, that necessity also emerged in CGC calculations, often in connection with the issue of negative NLO cross sections

  Beuf (2014); Hatta, Iancu (2016);
  Iancu, Madrigal, Mueller, Soyez, Triantafyllopoulos (2019)

- now it also appears in the context of two-scale processes and TMD physics

- heavy-quark photo-production provides a good testing ground for these theoretical developments, UPC measurements will be attempted at the LHC, and then we'll have the EIC