METHODS FOR SYSTEMATIC STUDY OF NUCLEAR STRUCTURE IN HIGH-ENERGY COLLISIONS

OR: CHANGING NUCLEI BY SHIFTING NUCLEONS

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Intersection of nuclear structure and high-energy nuclear collisions
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INTRODUCTION

- Systematic study of nuclear properties requires changing nuclear parameters and studying how observables change
- Small changes in parameters $\Longrightarrow$ small change in observables
- $\Longrightarrow$ Huge statistics required?
- No! It's possible to determine change in observables (or relative observable ratios) much more precisely than absolute value

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Procedure used until now

1. Choose set of nuclear parameters
2. Sample distribution to generate discrete nuclear configurations
3. Collide nuclei and compute observables
4. Choose new set of nuclear parameters
5. Generate new set of nuclear configurations from new distribution
6. Perform collisions and compute observables
7. Take ratios of observables, with independent statistical uncertainty for numerator and denominator

Better procedure

1. Generate discrete nuclear configurations once.
2. For each desired parameter set, modify configurations to obey new distribution by making small shifts to nucleon positions

- Statistical uncertainty in observable ratios can be drastically reduced
- Can be used to systematically study short-range correlations in addition to 1-body distribution

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Outline

1. Preparation of spherical nucleus
2. Modifying 1-body distribution
3. Adding short-range correlations
4. How significant are the benefits?

Outline

Aside: Step + Gauss distribution

- For these numerical results, we use an alternative to a Woods-Saxon
- Not necessary, but has nice properties and makes some things easier

- Nucleon position is sum of two random vectors sampled from:
  1. 3D step $P_s(\mathbf{x}) \sim \Theta(R_s - r)$
  2. 3D Gaussian $P_g(\mathbf{x}) \sim e^{-\frac{r^2}{2w^2}}$
- Rough rule of thumb:

$$R_s(R, a) \simeq R\left[1 + 1.5\left(\frac{a}{R}\right)^{1.8}\right]$$

$$w(R, a) \simeq 1.83\,a$$

$$\rho_c(\mathbf{x}) = \int P_s(\mathbf{z}) P_g(\mathbf{x} - \mathbf{z}) d^3 z$$

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$$\rho_c(\mathbf{x}) \sim \left[\frac{\sqrt{2}w}{r}\left(e^{-\frac{(r+R_s)^2}{2w^2}} - e^{-\frac{(r-R_s)^2}{2w^2}}\right) + \sqrt{\pi}\left\{\mathrm{Erf}\left(\frac{r+R_s}{\sqrt{2}w}\right) - \mathrm{Erf}\left(\frac{r-R_s}{\sqrt{2}w}\right)\right\}\right]$$

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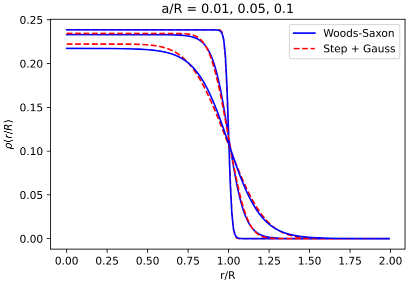

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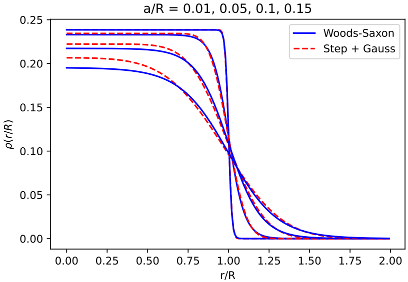

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Step+Gauss distribution advantages

Benefits of Step+Gauss

- Can directly modify Woods-Saxon parameters R, a without using the to-be-described methods
- No need for acceptance/rejection
- Trivial relation between point nucleon density and charge density
- Nice analytic properties — smooth at origin

Outline

- 1-body nucleon distribution parameterized as

$$\rho(r) \propto \frac{1}{1 + e^{\frac{r-R}{a}}}$$

$$\tilde{\rho}(r, \theta, \phi) \propto \frac{1}{1 + e^{\frac{r-R-R\sum \beta_{\ell,m} Y_{\ell,m}}{a}}} = \rho(r - R \sum_{\ell,m} \beta_{\ell,m} Y_{\ell,m})$$

- Define continuous parameter t that takes you from spherical ($t = 0$) to desired deformed distribution ($t = 1$)

$$\rho(\vec{x}, t) \equiv \rho(r - t \sum_{\ell,m} R\beta_{\ell,m} Y_{\ell,m})$$

- Idea: change nuclear properties by shifting the position of nucleons

$$\implies \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \, \vec{v}) = 0$$

- Start with uncorrelated nucleons satisfying $\rho(r)$, end with uncorrelated nucleons satisfying $\rho(r - R \sum_{\ell,m} \beta_{\ell,m})$

$$\rho(\vec{x},t) \equiv \rho(r - t\sum_{\ell,m} R\beta_{\ell,m}Y_{\ell,m})$$

$$0 = \frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\,\vec{v})$$

- One solution (at $t = 0$):

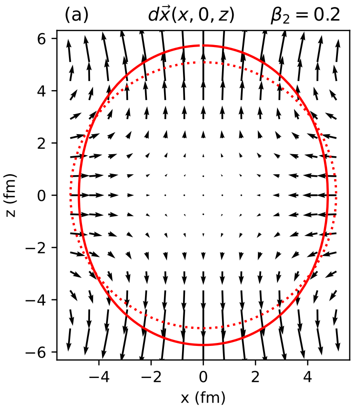
$$\vec{v} = \nabla\Phi(\vec{x})$$

$$\Phi = \sum R\beta_{\ell,m}f_{\ell,m}(r)Y_{\ell,m}$$

$$0 = f''_{\ell,m} + f'_{\ell,m}\left(\frac{2}{r} + \frac{\rho'}{\rho}\right) - \frac{\ell(\ell+1)}{r^2}f_{\ell,m} - \frac{\rho'}{\rho}$$

$$0 = f_{\ell,m}(r\to 0)$$

$$1 = f'_{\ell,m}(r\to\infty)$$



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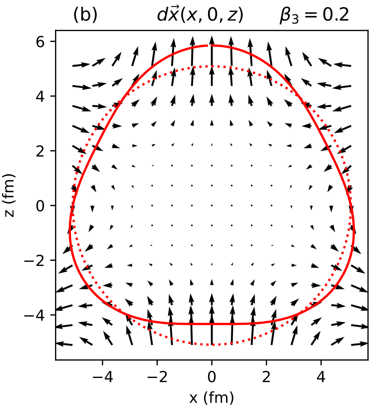
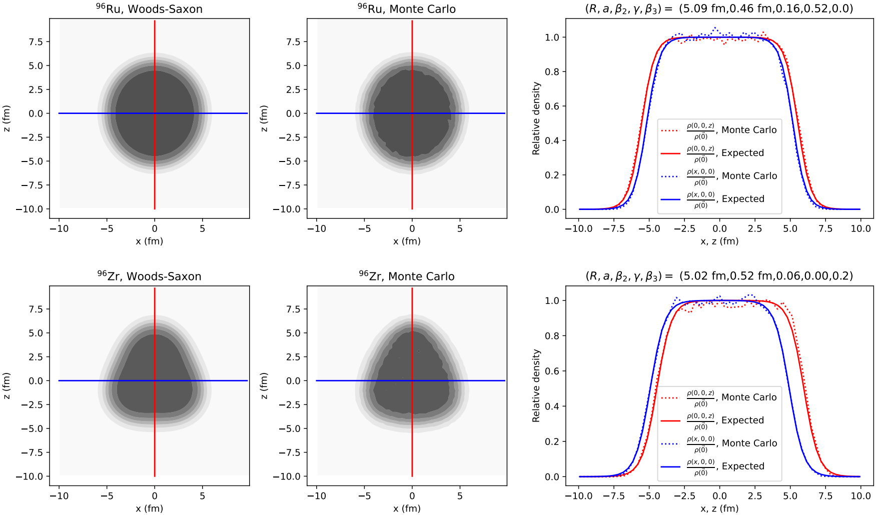


NUMERICAL RESULTS (100K NUCLEI)



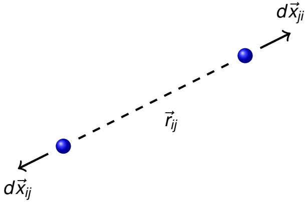
OUTLINE

Short-range correlations

- Short-range interactions cause particles to be correlated

$$\rho_2(\vec{x}_1, \vec{x}_2) = \rho(\vec{x}_1)\rho(\vec{x}_2)\left[1 + C(\vec{r}_{12})\right]$$

- Idea: induce correlation C from uncorrelated set by shifting particles



$$d\vec{x}_i = \sum_{j\neq i} d\vec{x}_{ij} = \sum_{j\neq i} \frac{1}{2}\left(\tilde{r}_{ij} - r_{ij}\right)\hat{r}_{ij}$$

FINDING $dr = \tilde{r} - r$

- Conserve pairs:

$$\int_0^r d^3r' = \int_0^{\tilde{r}} d^3r'(1 + C(\vec{r}'))$$

- Invert relation to solve for $\tilde{r}$
- For simplicity, we implemented a step function correlation function with variable length $C_{\text{length}} \geq 0$ and strength $C_{\text{strength}} \geq -1$

FINDING $dr = \tilde{r} - r$

- Conserve pairs:

$$(r^3 - \tilde{r}^3) = 3\int_0^{\tilde{r}} dr'\, r'^2 C(r')$$

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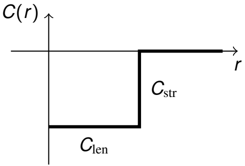


Valid correlation functions

- Note that the number of pairs is fixed:

$$\rho(\vec{x}_1)\rho(\vec{x}_2)\left[1 + C(\vec{r}_{12})\right] = \rho_2(\vec{x}_1, \vec{x}_2)$$

$$\implies \int d^3x_1 d^3x_2 \rho(\mathbf{x}_1)\rho(\mathbf{x}_2)C(\vec{r}_{12}) = 0$$

- Respecting sum rule important for maintaining fixed 1-body distribution
- If nominal short-range correlation doesn't satisfy, we add constant

$$C(r) = C_{\text{short}}(r) + C_\infty$$

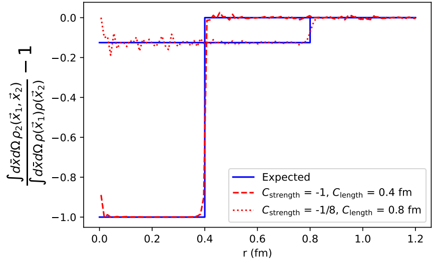
$$C_\infty \simeq -C_{\text{vol}} \int d^3x \rho(\mathbf{x})^2$$

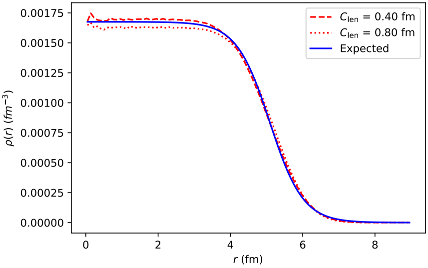
Other benefits

Advantages

Besides statistical speedup:

- Can study correlation of arbitrary shape – not just exclusion distance
- No problems with triaxial nuclei
- Better control over 2-body and 1-body distributions



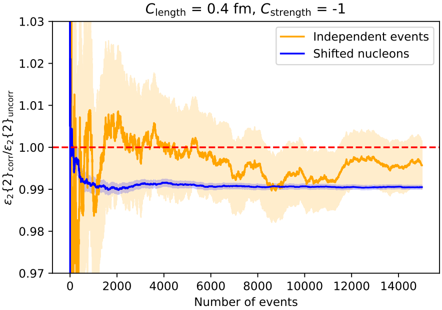


OUTLINE

1. PREPARATION OF SPHERICAL NUCLEUS
2. MODIFYING 1-BODY DISTRIBUTION
3. ADDING SHORT-RANGE CORRELATIONS
4. HOW SIGNIFICANT ARE THE BENEFITS?

HOW MUCH BENEFIT CAN YOU GET?

- Simple benchmark: participant Glauber model at $b = 0$. Ratios of eccentricities $\varepsilon_n\{2\}$, (baseline)/(baseline + change in 1 parameter)
- Compare our method to naive method with independent nuclei
- Question: If I get a statistical uncertainty with N events in the naive case, how many events N/F do I need in order to get at least as small statistical uncertainty, when using this method? F = improvement factor.



Preliminary benchmarks

Par.	Param. Change	$\varepsilon_2\{2\}$ Change	Improv. Factor	Avg. Shift
$C^3_{\rm len}$	$(0.2\ \mathrm{fm})^3$	0.13%	2900	0.002 fm
$C^3_{\rm len}$	$\times 2$	0.27%	1100	0.005 fm
$C^3_{\rm len}$	$\times 4$	0.53%	350	0.009 fm
$C^3_{\rm len}$	$(0.4\ \mathrm{fm})^3$	1.1%	180	0.017 fm
$C^3_{\rm len}$	$\times 2$	2.0%	98	0.032 fm
$C^3_{\rm len}$	$\times 4$	3.8%	54	0.059 fm
$C^3_{\rm len}$	$(0.8\ \mathrm{fm})^3$	7.3%	25	0.11 fm
$C^3_{\rm len}$	$\times 2$	14%	13	0.19 fm

Takeaways

- Significant improvement possible
- Main limitation: nucleon shift can change participant $\leftrightarrow$ spectator
- Larger differences in nuclei $\Longrightarrow$ reduced improvement factor
- Exact numbers will depend on centrality, model, etc.

Preliminary benchmarks

Par.	Param. Change	$\varepsilon_n\{2\}$ Change	Improv. Factor	Avg. Shift
β_2	0.005	0.02%	170	0.008 fm
β_2	0.01	0.10%	100	0.02 fm
β_2	0.02	0.39%	42	0.03 fm
β_2	0.05	2.3%	12	0.08 fm
β_2	0.1	8.8%	4.7	0.17 fm
β_2	0.2	31%	2.1	0.33 fm
β_3	0.01	0.05%	79	0.01 fm
β_3	0.05	1.6%	13	0.06 fm
β_3	0.1	6.3%	5.0	0.12 fm
β_3	0.2	23%	2.2	0.25 fm

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Summary

- Can significantly reduce statistical demands by correlating statistical fluctuations — change nuclear properties by shifting nucleons
- Allows for efficient systematic study of nuclear structure
- Allows for arbitrary Woods-Saxon parameters $(R, a, \{\beta_{\ell,m}\})$ and short-range correlation function $C(\vec{r})$
- Statistical improvements depend on context — better improvement for smaller changes in nuclei — but always an improvement over standard method
- Article and Python code to generate nuclei to appear soon
- Warning: must synchronize other fluctuations in collision model — impact parameter, orientation of nuclei, etc.

Extra Slides