Magnetic monopole in chiral plasma

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ML, J. Reiten and A. V. Sadofyev, PRD 103 (2021) no.12, 125017

The problem: What happens if a single magnetic monopole is inserted into a chiral electrically conducting medium at finite temperature $\mathbf{T}$. The medium is assumed to be electrically neutral while having a finite chiral density $\bar{\mathbf{n}}_5$.

The answer: the radial magnetic field of the monopole electrically polarizes the medium, localizing electrical charge around it, forming a chiral dyon. The electric charge q depends on the chiral asymmetry of the matter, $\mathbf{q} \sim \bar{\mathbf{n}}_5$.

This phenomenon is governed by an interplay between the CME, CSE, and EM field dynamics

Constitutive relations for chiral plasma including CME and CSE

$$\vec{\mathbf{j}} = -\sigma \vec{\nabla}\mu + \mathbf{e}\sigma\vec{\mathbf{E}} + \frac{\mathbf{e}}{2\pi^2}\mu_5\vec{\mathbf{B}}\,,$$

$$\vec{\mathbf{j}}_5 = -\sigma \vec{\nabla}\mu_5 + \frac{\mathbf{e}}{2\pi^2}\mu\vec{\mathbf{B}}\,,$$

μ and μ_5 are the vector and axial chemical potentials.
Einstein: Diffusion $\sim$ conductivity

Electric field $\vec{\mathrm{E}}$ is dynamical - induced by the Gauss law, hence proportional to the charge density.

Magnetic field is static, induced by an external Magnetic Monopole (of a finite size a).

Assumptions:

- Hot matter is in thermal equilibrium (T is the largest energy parameter). Linearized EoS

$$\mathbf{n} = \kappa\,\mu \qquad\qquad \mathbf{n}_5 = \kappa\,\mu_5 \qquad\qquad \kappa \sim \mathbf{T}^2$$

- All the fields and densities are small compared to T (in relevant power). Hence, the constitutive relations are linear in the fields and densities. Higher gradient terms are suppressed too.

Dynamics

$$\partial_t \, \mathbf{n} + \vec{\nabla} \cdot \vec{\mathbf{j}} = \mathbf{0}\,,$$

$$\partial_t \, n_5 + \vec{\nabla} \cdot \vec{j}_5 = \frac{e^2}{2\pi^2} \vec{E} \cdot \vec{B}\,.$$

Stationary case $\partial_t \, \mathbf{n} \;=\; \partial_t \, \mathbf{n_5} \;=\; \mathbf{0}$:

$$-\sigma \boldsymbol{\Delta} \mu + \mathbf{e} \sigma \vec{\nabla} \cdot \vec{\mathbf{E}} + \frac{\mathbf{e}}{\mathbf{2}\pi^{\mathbf{2}}} \vec{\mathbf{B}} \cdot \vec{\nabla} \mu_{\mathbf{5}} = \mathbf{0}\,,$$

$$-\sigma \boldsymbol{\Delta} \mu_{\mathbf{5}} + \frac{\mathbf{e}}{\mathbf{2}\pi^{\mathbf{2}}} \vec{\mathbf{B}} \cdot \vec{\nabla} \mu = \frac{\mathbf{e}^{\mathbf{2}}}{\mathbf{2}\pi^{\mathbf{2}}} \vec{\mathbf{E}} \cdot \vec{\mathbf{B}}$$

Gauss's laws

$$\vec{\nabla} \cdot \vec{\mathbf{B}} = \mathbf{g}\, \delta^{(\mathbf{3})}(\vec{\mathbf{r}}) \quad , \quad \vec{\nabla} \cdot \vec{\mathbf{E}} = \mathbf{e}\, \mathbf{n}(\vec{\mathbf{r}})$$

Dirac quantization: $\mathbf{e}\,\mathbf{g} \;=\; \mathbf{2}\,\pi\,\mathbf{k}\,, \qquad \mathbf{k} = \mathbf{1}$

Stationary solution in 3D spherical symmetry: $\dot{\mathbf{E}} = \mathbf{J} = 0$

Detailed balance between the outflow of chirality and its production due to the anomaly.

$$\left[\Delta - \frac{(a\beta)^2}{r^4}\right] n_5(r) = 0\,, \qquad \beta = \frac{1}{(2\pi)^3}\frac{eg}{a\sigma}$$

Analytical solution

$$n_5(r) = A_{ch}\cosh\frac{a\beta}{r} + A_{sh}\sinh\frac{a\beta}{r}\,,$$

$$\left(\Delta - m^2\right) n(r) = a\beta\,\frac{n_5'(r)}{r^2}$$

massive three-dimensional Klein-Gordon equation with an n_5-dependent source.
$m = e\sqrt{\kappa} \sim e\,T$ **is the thermal or Debye mass**

$$n(r) = \frac{a\,e^{mr}}{r}D_+ + \frac{a\,e^{-mr}}{r}D_- + \frac{a\beta}{mr}\int_a^r dx\,\sinh\left[m(r-x)\right]\frac{n_5'(x)}{x}\,,$$

Boundary Conditions

- No electric current, $\mathbf{J} = 0$
- No flow of axial charge at the monopole surface, $\mathbf{J_5(r = a) = 0}$
- Total electric charge Q and axial charge Q_5 in the system have to be specified.

$$\int \mathrm{d^3r\, n_5(r) = Q_5}\,, \quad \mathrm{n_5'(a) = \frac{\beta}{a} n(a)}, \qquad \int \mathrm{d^3r\, n(r) = Q = 0}\,, \quad \mathrm{n'(a) = \frac{\beta}{a} n_5(a)}$$

Solve in the finite volume R and then carefully study the infinite limit $\mathrm{R} \to \infty$.

$$\mathbf{A_{ch} = \bar{n}_5 = Q_5/V_R}, \qquad \mathbf{V_R = 4\pi R^3/3}$$

$\mathbf{A_{sh}}$, $\mathbf{D_\pm}$ are determined but too messy to display.

The stationary solution is fixed by $\mathbf{\bar{n}_5}$, and dimensionless parameters β and $\gamma \equiv \mathbf{m\,a}$.

Small-β Limit

$$\beta = \frac{1}{(2\pi)^3}\frac{\mathbf{eg}}{\mathbf{a}\sigma} \simeq \frac{\mathbf{0.002}}{\bar{\sigma}\,(\mathbf{aT})}\,, \qquad \bar{\sigma} = \mathbf{e}^2\sigma/\mathbf{T}$$

β appears to be small both in QGP and the Early Universe

$$\mathbf{n}(\bar{\mathbf{r}})/\bar{\mathbf{n}}_5 = -\beta\frac{\mathbf{e}^{\gamma}}{\mathbf{1}+\gamma}\frac{\mathbf{e}^{-\gamma\bar{\mathbf{r}}}}{\bar{\mathbf{r}}} + \mathcal{O}\left(\beta^3\right)\,, \qquad \mathbf{r} = \mathbf{a}\,\bar{\mathbf{r}}$$

$$\mathbf{n}_5(\bar{\mathbf{r}})/\bar{\mathbf{n}}_5 = \mathbf{1} + \beta^2\left(\frac{\mathbf{1}}{\mathbf{2}\bar{\mathbf{r}}^2} - \frac{\gamma}{\mathbf{1}+\gamma}\frac{\mathbf{1}}{\bar{\mathbf{r}}}\right) + \mathcal{O}\left(\beta^3\right)\,.$$

Dyon's charge is localized in a small shell

$$\mathbf{q} = -\left(\frac{\mathbf{4}\pi\mathbf{a}^3}{\gamma^2}\beta\,\bar{\mathbf{n}}_5\right)\mathbf{e} = -\left(\frac{\mathbf{1}}{\mathbf{2}\pi^2}\frac{\mathbf{eg}}{\sigma\mathbf{m}^2}\bar{\mathbf{n}}_5\right)\,\mathbf{e}$$

$\mathbf{q}/\mathbf{e} < \mathbf{1}$ in the Early Universe while $\mathbf{q} \sim \mathbf{e}$ in QGP

Summary

- Magnetic Monopole imbedded into electrically neutral chiral medium becomes a chiral dyon.

- Formation of the dyon generates electric charge asymmetry.

- Monopole-antimonopole will attract and annihilate. Monopole plasma is of interest

- Further directions: check dynamical stability, include higher gradients and non-linear interactions.

- Speculations: The interplay between the chiral effects on the one hand, and presence of magnetic field of the monopole on the other, may affect the evolution of the monopole density in the Early Universe, contribute to the process of baryogenesis (lepton asymmetry), and can also be instrumental for detection of relic monopoles using chiral materials

Anomalous Hydro from Fluid-Gravity correspondence

$\mathbf{U_V(1) \times U_A(1)}$ **Maxwell-Chern-Simons theory in the Schwarzschild-AdS_5.**

$$\mathcal{L} = -\frac{1}{4}(\mathbf{F^V})_{\mathbf{MN}}(\mathbf{F^V})^{\mathbf{MN}} - \frac{1}{4}(\mathbf{F^a})_{\mathbf{MN}}(\mathbf{F^a})^{\mathbf{MN}} + \frac{\kappa\, \epsilon^{\mathbf{MNPQR}}}{\mathbf{2}\sqrt{-\mathbf{g}}}$$

$$\times \left[\mathbf{3A_M(F^V)_{NP}(F^V)_{QR} + A_M(F^a)_{NP}(F^a)_{QR}}\right],$$

Chemical potentials:

$$\mu = \mathbf{A_t(r = \infty) - A_t(r = 1)} = \rho/\mathbf{2} - \mathbf{A_t(r = 1)},$$

$$\mu_{\mathbf{5}} = \mathbf{A^a_t(r = \infty) - A^a_t(r = 1)} = \rho_{\mathbf{5}}/\mathbf{2} - \mathbf{A^a{}_t(r = 1)}$$

$$\mu = \mu[\rho, \vec{\mathbf{E}}, \vec{\mathbf{B}}] \qquad \mu_{\mathbf{5}} = \mu[\rho_{\mathbf{5}}, \vec{\mathbf{E}}, \vec{\mathbf{B}}]$$

$$\partial_\mu \mathbf{J}^\mu = \mathbf{0}, \qquad \partial_\mu \mathbf{J}^\mu_{\mathbf{5}} = \mathbf{12}\kappa \vec{\mathbf{E}} \cdot \vec{\mathbf{B}}$$

Anomaly-induced transport: Anomalous Zoo

General form of constitutive relations

Y. Bu, ML, and A. Sharon, JHEP 1611, 093 (2016); Eur. Phys. J. C 77, no. 3, 194 (2017)
Y. Bu, T. Demircik, ML, JHEP 1901, 078 (2019), Eur. Phys. J. C 79, no. 1, 54 (2019), JHEP 1905, 071 (2019)

$$\vec{\mathbf{J}} \;=\; \vec{\mathbf{J}}[\rho,\rho_5,\mathbf{T},\vec{\mathbf{E}},\vec{\mathbf{B}}]; \qquad \vec{\mathbf{J}}_5 = \vec{\mathbf{J}}_5[\rho,\rho_5,\mathbf{T},\vec{\mathbf{E}},\vec{\mathbf{B}}]$$

- $\vec{\mathbf{J}} \propto \vec{\mathbf{B}}$ CME
- $\vec{\mathbf{J}} \propto \partial_t \vec{\mathbf{B}}$ time relaxation in CME
- $\vec{\mathbf{J}} \propto \mathbf{B}^2\vec{\mathbf{B}}$ & $(\vec{\mathbf{B}}\vec{\mathbf{E}})\vec{\mathbf{B}}$ the first nonlinear corrections to CME
- $\vec{\mathbf{J}} \propto (\vec{\mathbf{B}}\times\vec{\mathbf{E}})\times\vec{\mathbf{E}}$ E^2 correction to CME and chiral electric effect (CEE)
- $\vec{\mathbf{J}} \propto \vec{\mathbf{E}}\times\vec{\mathbf{B}}$ chiral Hall current
- $\vec{\mathbf{J}} \propto \vec{\mathbf{B}}\times(\mu\vec{\nabla}\mu_5+\mu_5\vec{\nabla}\mu)$ Hall diffusion
- $\vec{\mathbf{J}} \propto \vec{\mathbf{E}}\times\vec{\nabla}\mu$ anomalous chiral Hall current

$$\mathbf{J}^{\mathrm{i}}_{\mathrm{diff}} = -\mathcal{D}^{\mathbf{0}}_{\mathrm{ij}}\nabla_{\mathrm{j}}\rho - (\mathcal{D}^{\mathbf{0}}_{\chi})_{\mathrm{ij}}\nabla_{\mathrm{j}}\rho_5,$$

$$\mathcal{D}^{\mathbf{0}}_{\mathrm{ij}} = \frac{\mathbf{1}}{\mathbf{2}}(\sigma^{\mathbf{0}}_{\mathrm{e}}\delta_{\mathrm{ij}} + \mathcal{D}^{\mathbf{0}}_{\mathrm{H}}\epsilon_{\mathrm{ikj}}\mathbf{B}_{\mathrm{k}}\mu), \qquad (\mathcal{D}^{\mathbf{0}}_{\chi})_{\mathrm{ij}} = \frac{\mathbf{1}}{\mathbf{2}}(\sigma^{\mathbf{0}}_{\mathrm{a}\chi\mathrm{H}}\epsilon_{\mathrm{ikj}}\mathbf{E}_{\mathrm{k}} + \mathcal{D}^{\mathbf{0}}_{\mathrm{H}}\epsilon_{\mathrm{ikj}}\mathbf{B}_{\mathrm{k}}\mu_5).$$

- CMW $(\vec{\mathbf{q}}\cdot\vec{\mathbf{B}})$ and chiral Hall density wave (CHDW) $(\vec{\mathbf{q}}\cdot(\vec{\mathbf{E}}\times\vec{\mathbf{B}}))$

I. Linear transport: weak fields

$$\rho(\mathbf{x}_\alpha) = \bar{\rho} + \delta\rho(\mathbf{x}_\alpha), \qquad \rho_{\mathbf{5}}(\mathbf{x}_\alpha) = \bar{\rho}_{\mathbf{5}} + \delta\rho_{\mathbf{5}}(\mathbf{x}_\alpha),$$

$$\mu(\mathbf{x}_\alpha) = \bar{\mu} + \delta\mu(\mathbf{x}_\alpha), \qquad \mu_{\mathbf{5}}(\mathbf{x}_\alpha) = \bar{\mu}_{\mathbf{5}} + \delta\mu_{\mathbf{5}}(\mathbf{x}_\alpha), \qquad \bar{\mu} = \bar{\rho}/\mathbf{2}, \qquad \bar{\mu}_{\mathbf{5}} = \bar{\rho}_{\mathbf{5}}/\mathbf{2}$$

Linear in $\vec{E}\&\vec{B}$ and linear in $\delta\rho$

$$\mathbf{J}^{\mathbf{t}} = \rho, \qquad \vec{\mathbf{J}} = -\mathcal{D}\vec{\nabla}\rho + \sigma_{\mathbf{e}}\vec{\mathbf{E}} + \sigma_{\mathbf{m}}\vec{\nabla}\times\vec{\mathbf{B}} + \sigma_\chi\vec{\mathbf{B}}$$

$$\mathbf{J}_{\mathbf{5}}^{\mathbf{t}} = \rho_{\mathbf{5}}, \qquad \mathbf{J}_{\mathbf{5}}^{\mathbf{i}} = -\mathcal{D}\vec{\nabla}\rho_{\mathbf{5}} + \sigma_{\mathbf{a}}\vec{\nabla}\times\vec{\mathbf{B}} + \sigma_\kappa\vec{\mathbf{B}}. \qquad \partial_\mu\mathbf{J}_{\mathbf{5}}^\mu = \mathbf{0}$$

σ_κ – **CSE**; D. T. Son and A. R. Zhitnitsky, (2004) ; M. A. Metlitski and A. R. Zhitnitsky, (2005)

No linear chiral electric separation effect (CESE) in our model ($\vec{J}_5 \sim \vec{E}$)

Linear constitutive relations lead to a consistent χMHD as long as the field amplitudes remain weak

$$\sigma_{\mathbf{m}} = \mathbf{72}\kappa^{\mathbf{2}}\left(\bar{\mu}^{\mathbf{2}} + \bar{\mu}_{\mathbf{5}}^{\mathbf{2}}\right)(\mathbf{2}\log\mathbf{2} - \mathbf{1}) + \mathbf{i}\omega\left[\frac{\mathbf{1}}{\mathbf{16}}(\mathbf{2}\pi - \pi^{\mathbf{2}} + \mathbf{4}\log\mathbf{2}) + \mathcal{O}\left(\bar{\mu}^{\mathbf{2}} + \bar{\mu}_{\mathbf{5}}^{\mathbf{2}}\right)\right] + \cdots,$$

$\sigma_{\mathbf{m}}[\mathbf{q} = \mathbf{0}] - \sigma_{\mathbf{m}}[\mathbf{q} = \mathbf{0},\ \bar{\mu} = \bar{\mu}_{\mathbf{5}} = \mathbf{0}]$ **is linear in** $\kappa^2\,(\bar{\mu}^2 + \bar{\mu}_5^2)$

$$\sigma_{\chi} = \mathbf{12}\kappa\bar{\mu}_{\mathbf{5}}\left\{\mathbf{1} + \mathbf{i}\omega\log\mathbf{2} - \frac{\mathbf{1}}{\mathbf{4}}\omega^{\mathbf{2}}\log^{\mathbf{2}}\mathbf{2} - \frac{\mathbf{q}^{\mathbf{2}}}{\mathbf{24}}\left[\pi^{\mathbf{2}} - \mathbf{1728}\kappa^{\mathbf{2}}\left(\bar{\mu}_{\mathbf{5}}^{\mathbf{2}} + \mathbf{3}\bar{\mu}^{\mathbf{2}}\right)(\log\mathbf{2} - \mathbf{1})^{\mathbf{2}}\right]\right\} + \cdots$$

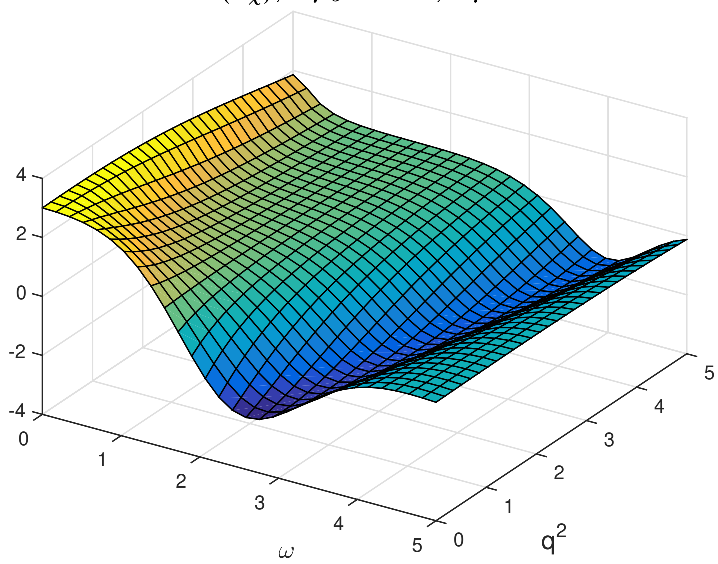
$\sigma_{\chi}^{\mathbf{0}}$ **A. Gynther, K. Landsteiner, F. Pena-Benitez, and A. Rebhan, (2011)**

$\sigma_{\chi}[\mathbf{q} = \mathbf{0}]$ **is linear in** $\kappa\,\mu_5$ **and independent of** μ

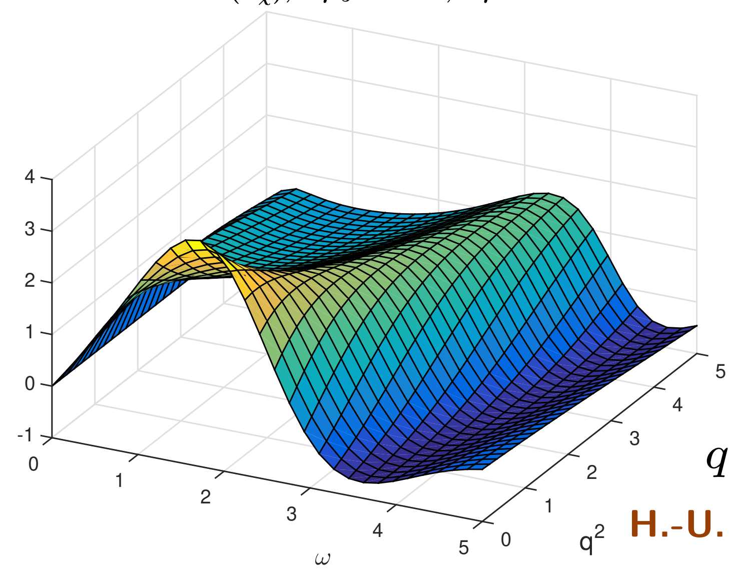
$$\sigma_{\kappa}[\mu,\ \mu_5] \;=\; \sigma_{\chi}[\mu_5,\ \mu]$$

Plus tons of plots for arbitrary $\omega,\ q$ **and** $\mu,\ \mu_5$

$\mathbf{Re}(\sigma_\chi),\ \kappa\bar{\mu}_5 = 0.25,\ \kappa\bar{\mu} = 0$



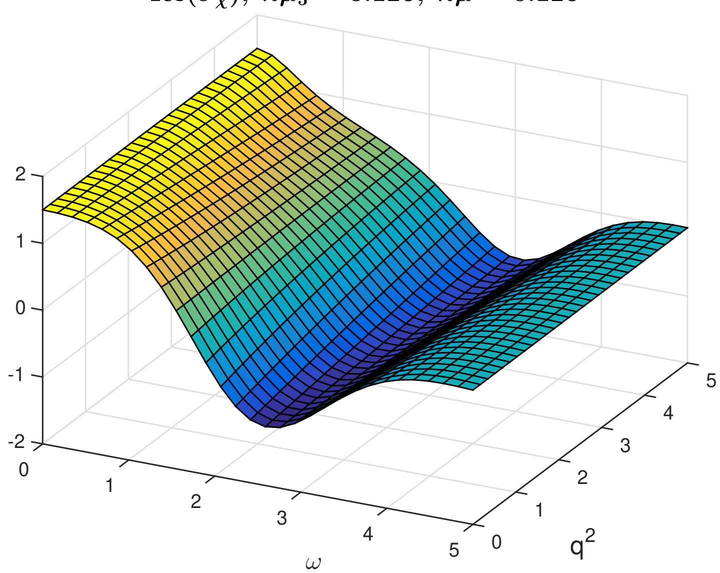
$\mathbf{Im}(\sigma_\chi),\ \kappa\bar{\mu}_5 = 0.25,\ \kappa\bar{\mu} = 0$



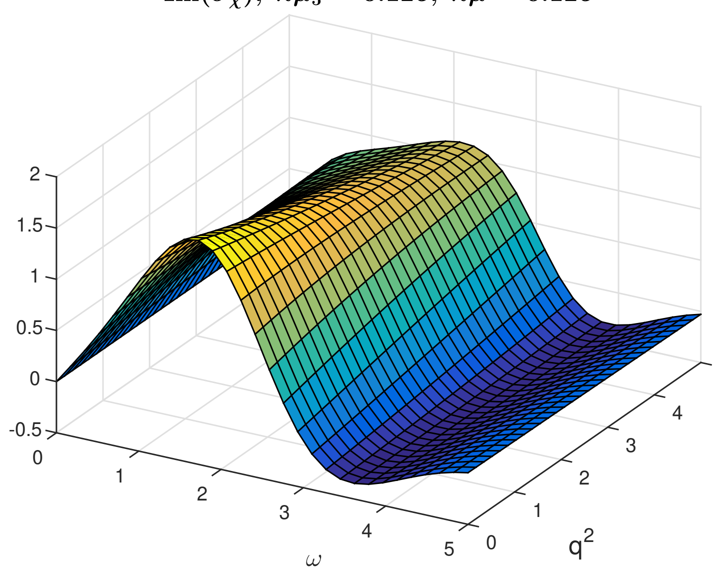
$q = 0$

H.-U. Yee, (2009)

$\mathbf{Re}(\sigma_\chi),\ \kappa\bar{\mu}_5 = 0.125,\ \kappa\bar{\mu} = 0.125$



$\mathbf{Im}(\sigma_\chi),\ \kappa\bar{\mu}_5 = 0.125,\ \kappa\bar{\mu} = 0.125$



II. Constant fields: a) zeroth order in gradients

Fields of arbitrary strength. Linearisation in ρ and ρ_5

$$\vec{J}^{\,[0]} = \sigma_e^0\vec{\mathbf{E}} + \sigma_\chi^0\kappa\rho_5\vec{\mathbf{B}} + \delta\sigma_\chi^0\kappa^2(\vec{\mathbf{E}}\cdot\vec{\mathbf{B}})\vec{\mathbf{B}} + \sigma_{\chi H}^0\kappa^2\rho\vec{\mathbf{B}}\times\vec{\mathbf{E}} + \sigma_{\chi e}^0\kappa^3\rho_5(\vec{\mathbf{B}}\cdot\vec{\mathbf{E}})\vec{\mathbf{E}},$$

$$\vec{J}_5^{\,[0]} = \sigma_\chi^0\kappa\rho\vec{\mathbf{B}} + \sigma_{\chi H}^0\kappa^2\rho_5\vec{\mathbf{B}}\times\vec{\mathbf{E}} + \sigma_{\chi e}^0\kappa^3\rho(\vec{\mathbf{B}}\cdot\vec{\mathbf{E}})\vec{\mathbf{E}} + \sigma_s^0\kappa^3(\vec{\mathbf{E}}\cdot\vec{\mathbf{B}})\vec{\mathbf{B}}\times\vec{\mathbf{E}},$$

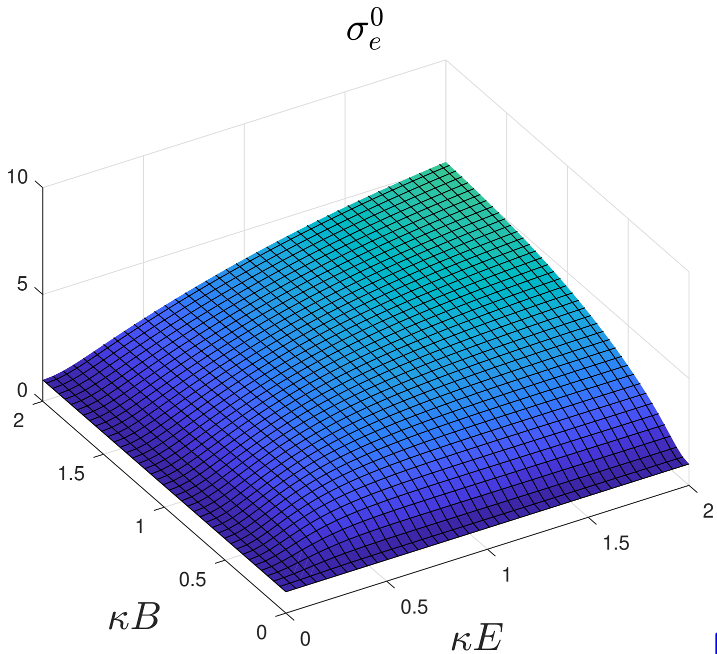
$\sigma^{\mathbf{0}}_{\chi\mathbf{H}}$ – **chiral Hall effect;** $\sigma^{\mathbf{0}}_{\chi\mathbf{e}}$ – **non-linear chiral electric effect /CESE;**

$\delta\sigma^{\mathbf{0}}_\chi$ – $\vec{\mathbf{E}}\cdot\vec{\mathbf{B}}$ **-induced CME;** $\sigma_{\mathbf{s}}$ – $\vec{\mathbf{E}}\cdot\vec{\mathbf{B}}$ **-induced chiral Hall;**

All the transport coefficients $\sigma_{\mathbf{i}} = \sigma_{\mathbf{i}}[\vec{\mathbf{E}}^{\mathbf{2}}, \vec{\mathbf{B}}^{\mathbf{2}}, \vec{\mathbf{E}}\cdot\vec{\mathbf{B}}]$

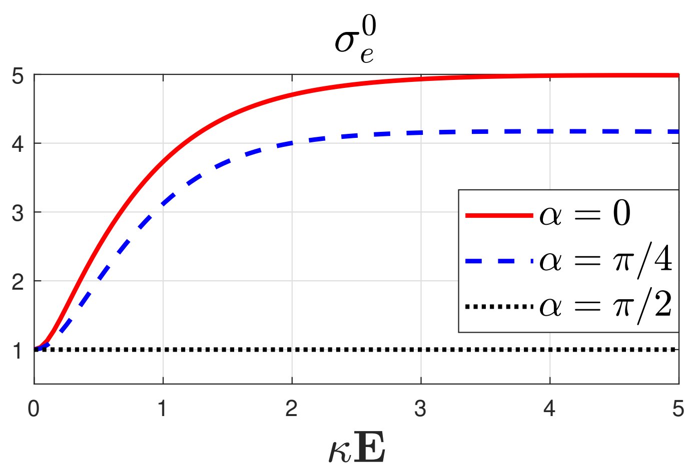
Because fields are generated by the currents, the above constitutive relations are not self-consistent

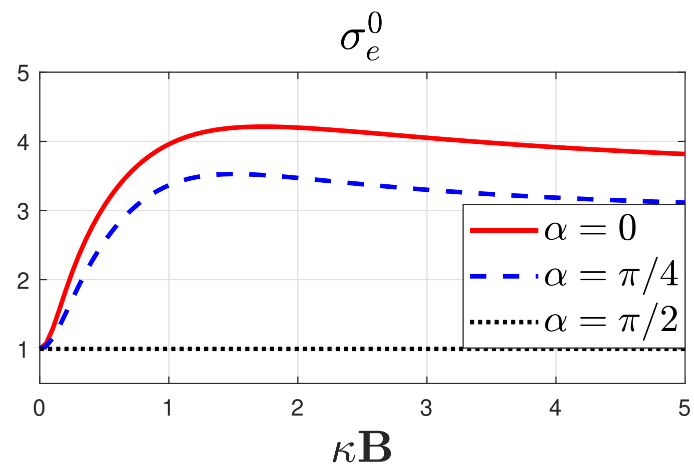
Electric Conductivity



$\vec{\mathbf{E}}||\vec{\mathbf{B}}$

Dependence on the angle between $\vec{\mathrm{E}}$ and $\vec{\mathrm{B}}$





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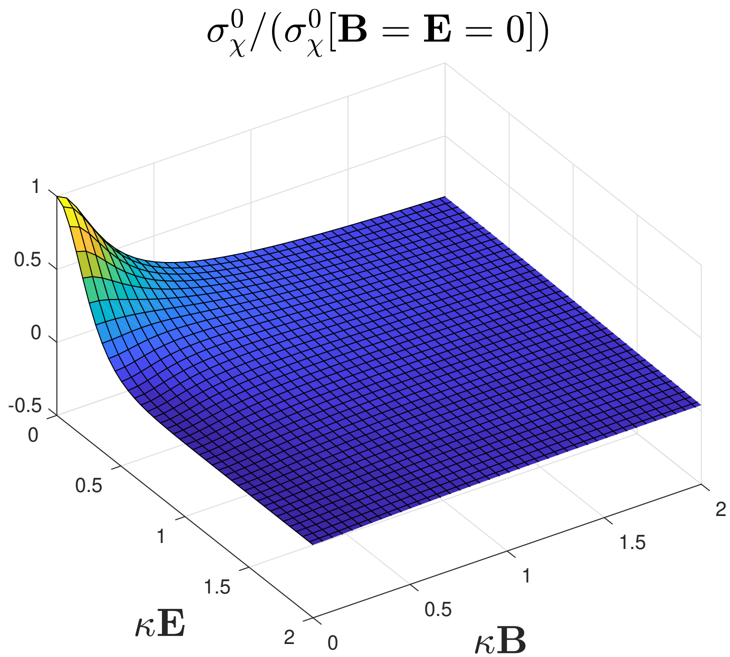
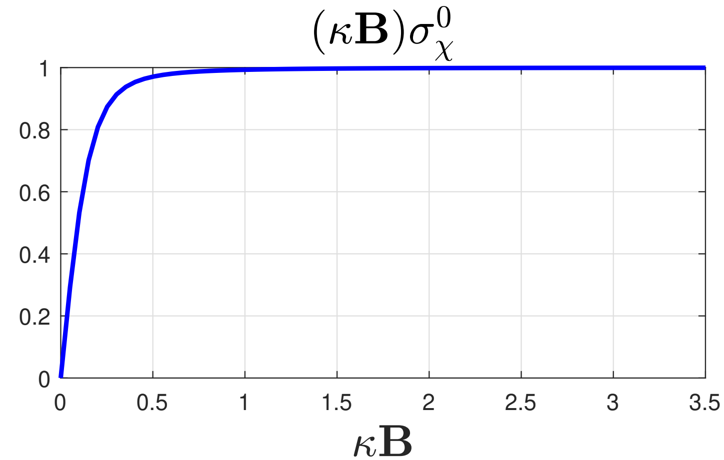
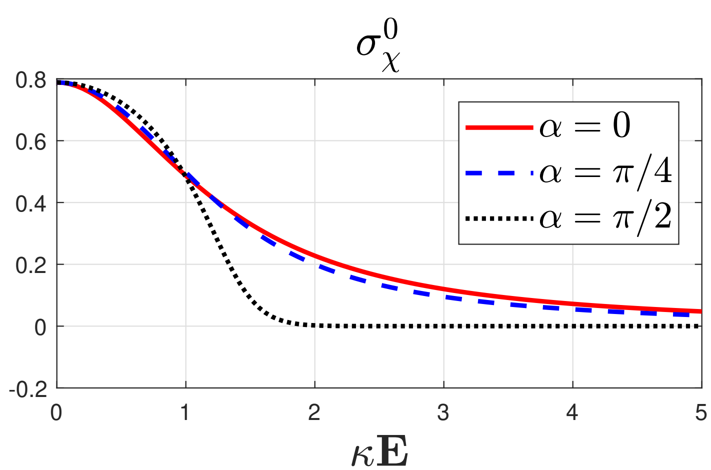
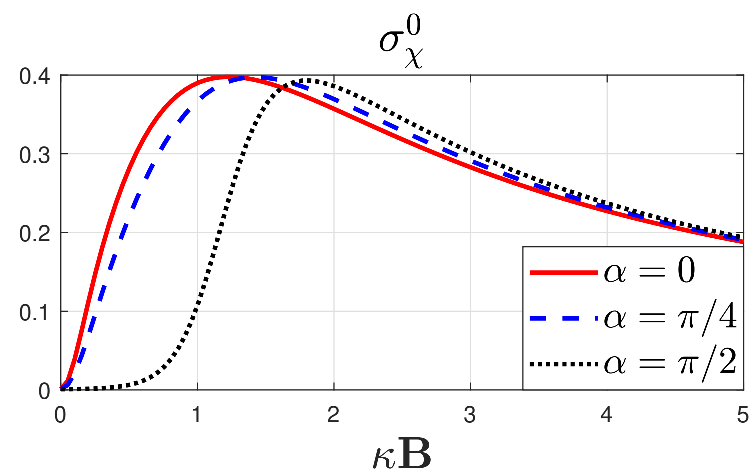
CME Conductivity









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II. Constant fields: b) first order in gradients

Linear in ρ and ρ_5

$\mathbf{E} = \mathbf{0}$

$$\vec{J}^{\,[1]} = -\mathcal{D}_0 \vec{\nabla}\rho + \tau_{\bar{\chi}} \kappa \partial_t \rho_5 \vec{\mathbf{B}} + \mathcal{D}_B^0 \kappa^2 (\vec{\mathbf{B}} \cdot \vec{\nabla}\rho) \vec{\mathbf{B}},$$

$$\vec{J}_5^{\,[1]} = -\mathcal{D}_0 \vec{\nabla}\rho_5 + \tau_{\bar{\chi}} \kappa \partial_t \rho \vec{\mathbf{B}} + \mathcal{D}_B^0 \kappa^2 (\vec{\mathbf{B}} \cdot \vec{\nabla}\rho_5) \vec{\mathbf{B}}$$

$\mathbf{B} = \mathbf{0}$

$$\vec{J}^{\,[1]} = -\mathcal{D}_0 \vec{\nabla}\rho + \sigma_{a\chi H}^0 \kappa \vec{\mathbf{E}} \times \vec{\nabla}\rho_5 + \mathcal{D}_E^0 \kappa^2 (\vec{\mathbf{E}} \cdot \vec{\nabla}\rho) \vec{\mathbf{E}},$$

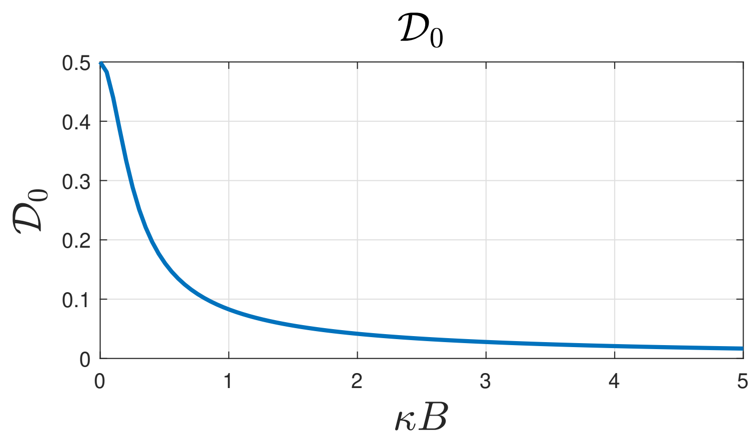
$$\vec{J}_5^{\,[1]} = -\mathcal{D}_0 \vec{\nabla}\rho_5 + \sigma_{a\chi H}^0 \kappa \vec{\mathbf{E}} \times \vec{\nabla}\rho + \mathcal{D}_E^0 \kappa^2 (\vec{\mathbf{E}} \cdot \vec{\nabla}\rho_5) \vec{\mathbf{E}}$$

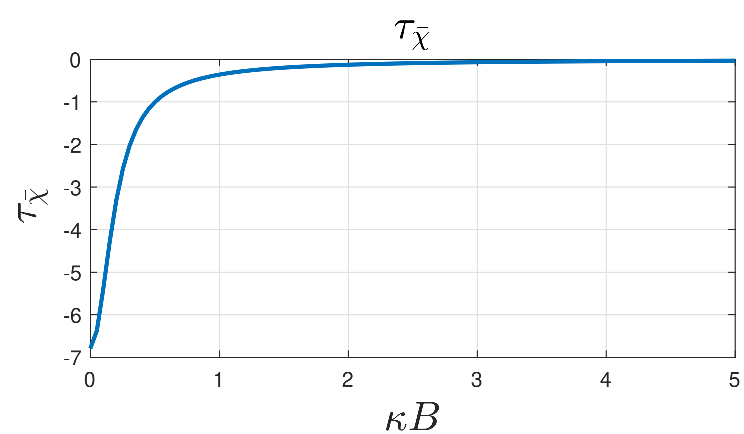
Diffusion

Diffusion constant (weak field expansion)

$$\mathcal{D}_0 = \frac{1}{2} - 18(2\log 2 - 1)\kappa^2 \mathbf{B}^2 - \frac{3}{4}\pi^2\kappa^2\mathbf{E}^2 + \cdots$$

$\mathbf{E} = 0$:





CMW and CHDW

Weak field expansion

$$\omega = \pm\left[\mathbf{1} - \mathbf{36}(\mathbf{2}\log\mathbf{2} - \mathbf{1})\kappa^{\mathbf{2}}\mathbf{B}^{\mathbf{2}} - \frac{\mathbf{3}\pi^{\mathbf{2}}}{\mathbf{2}}\kappa^{\mathbf{2}}\mathbf{E}^{\mathbf{2}}\right]\mathbf{6}\kappa(\vec{\mathbf{q}}\cdot\vec{\mathbf{B}}) \pm \mathbf{9}\pi^{\mathbf{2}}(\vec{\mathbf{E}}\cdot\vec{\mathbf{B}})\kappa^{\mathbf{3}}(\vec{\mathbf{q}}\cdot\vec{\mathbf{E}})$$
$$+(\mathbf{36}\log\mathbf{2})\kappa^{\mathbf{2}}(\vec{\mathbf{q}}\cdot\vec{\mathbf{S}}) - \left[\frac{\mathbf{1}}{\mathbf{2}} + \mathbf{18}(\mathbf{1} - \mathbf{2}\log\mathbf{2})\kappa^{\mathbf{2}}\mathbf{B}^{\mathbf{2}} - \frac{\mathbf{3}\pi^{\mathbf{2}}}{\mathbf{4}}\kappa^{\mathbf{2}}\mathbf{E}^{\mathbf{2}}\right]\mathbf{i}\mathbf{q}^{\mathbf{2}}$$
$$\pm\frac{\mathbf{9}}{\mathbf{2}}\log\mathbf{2}\kappa(\vec{\mathbf{q}}\cdot\vec{\mathbf{B}})\mathbf{q}^{\mathbf{2}} - \frac{\mathbf{i}}{\mathbf{8}}\mathbf{q}^{\mathbf{4}}\log\mathbf{2} - \mathbf{i}\frac{\mathbf{3}}{\mathbf{4}}\pi^{\mathbf{2}}\kappa^{\mathbf{2}}(\vec{\mathbf{q}}\cdot\vec{\mathbf{E}})^{\mathbf{2}} + \mathbf{i}(\mathbf{36}\log\mathbf{2})\kappa^{\mathbf{2}}(\vec{\mathbf{q}}\cdot\vec{\mathbf{B}})^{\mathbf{2}} + \cdots$$

$\vec{S} = \vec{E} \times \vec{B}$

$\omega \sim (\vec{q}\cdot\vec{S})$ **- new gapless excitation - Chiral Hall Density Wave**

II. Constant weak fields: c) gradient resummation

$$\rho(x_\alpha) = \bar{\rho} + \delta\rho(x_\alpha), \qquad\qquad \rho_5(x_\alpha) = \bar{\rho}_5 + \delta\rho_5(x_\alpha)$$

Linear in $\vec{\mathbf{E}}\&\vec{\mathbf{B}}$ and linear in $\delta\rho$

$$\begin{aligned}
\delta\vec{J}^{(1)(1)} =\ & \sigma_{\bar{\chi}}\kappa\vec{\mathbf{B}}\delta\rho_5 - \frac{1}{4}\mathcal{D}_H(\bar{\rho}\vec{\mathbf{B}}\times\vec{\nabla}\delta\rho) - \frac{1}{4}\bar{\mathcal{D}}_H(\bar{\rho}_5\vec{\mathbf{B}}\times\vec{\nabla}\delta\rho_5) \\
& -\frac{1}{2}\sigma_{a\chi H}(\vec{\mathbf{E}}\times\vec{\nabla}\delta\rho_5) - \frac{1}{2}\bar{\sigma}_{a\chi H}(\vec{\mathbf{E}}\times\vec{\nabla}\delta\rho) \\
& +\sigma_1\kappa\left[(\vec{\mathbf{B}}\times\vec{\nabla})\times\vec{\nabla}\right]\delta\rho + \sigma_2\kappa\left[(\vec{\mathbf{B}}\times\vec{\nabla})\times\vec{\nabla}\right]\delta\rho_5 \\
& +\sigma_3\kappa\left[(\vec{\mathbf{E}}\times\vec{\nabla})\times\vec{\nabla}\right]\delta\rho + \bar{\sigma}_3\kappa\left[(\vec{\mathbf{E}}\times\vec{\nabla})\times\vec{\nabla}\right]\delta\rho_5,
\end{aligned}$$

$$\begin{aligned}
\delta\vec{J}_5^{(1)(1)} =\ & \sigma_{\bar{\chi}}\kappa\vec{\mathbf{B}}\delta\rho - \frac{1}{4}\mathcal{D}_H(\bar{\rho}\vec{\mathbf{B}}\times\vec{\nabla}\delta\rho_5) - \frac{1}{4}\bar{\mathcal{D}}_H(\bar{\rho}_5\vec{\mathbf{B}}\times\vec{\nabla}\delta\rho) \\
& -\frac{1}{2}\sigma_{a\chi H}(\vec{\mathbf{E}}\times\vec{\nabla}\delta\rho) - \frac{1}{2}\bar{\sigma}_{a\chi H}(\vec{\mathbf{E}}\times\vec{\nabla}\delta\rho_5) \\
& +\sigma_1\kappa\left[(\vec{\mathbf{B}}\times\vec{\nabla})\times\vec{\nabla}\right]\delta\rho_5 + \sigma_2\kappa\left[(\vec{\mathbf{B}}\times\vec{\nabla})\times\vec{\nabla}\right]\delta\rho \\
& +\sigma_3\kappa\left[(\vec{\mathbf{E}}\times\vec{\nabla})\times\vec{\nabla}\right]\delta\rho_5 + \bar{\sigma}_3\kappa\left[(\vec{\mathbf{E}}\times\vec{\nabla})\times\vec{\nabla}\right]\delta\rho
\end{aligned}$$

In the hydrodynamic limit $\omega, q \ll 1$, the TCFs are analytically computable:

$$\sigma_{\bar{\chi}} = 6+\frac{3}{2}i\omega\left(\pi+2\log 2\right)-\frac{1}{8}\left\{\omega^2\left[\pi^2+6\left(4\mathcal{C}+\log^2 2\right)\right]+q^2\left(12\pi-24\log 2\right)\right\}+\cdots,$$

$$\mathcal{D}_H = \kappa^2\left\{72(3\log 2-2)+i\omega 6\left[\pi(2\pi+3\log 2-6)+(9\log 2-12)\log 2\right]+\cdots\right\},$$

$$\bar{\mathcal{D}}_H = \mathcal{D}_H\left[\bar{\mu}\leftrightarrow\bar{\mu}_5\right],$$

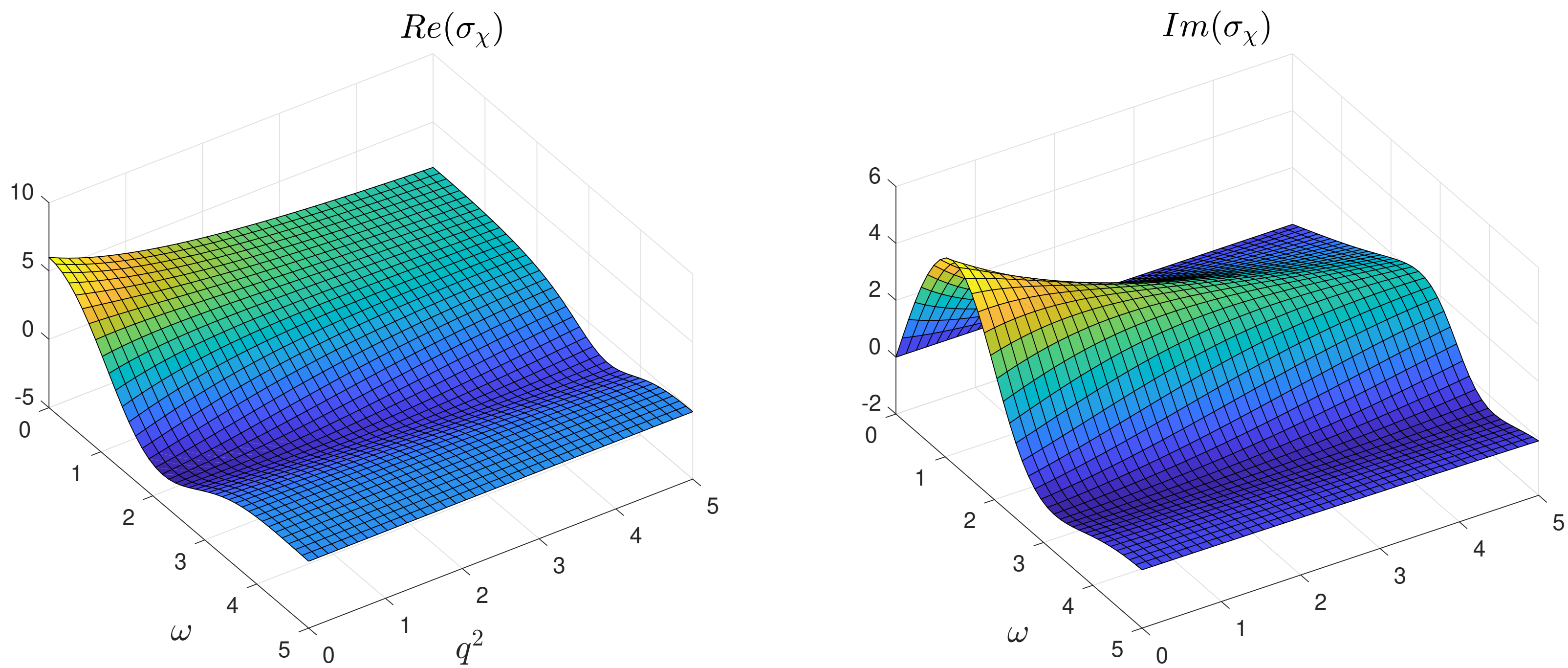
$$\sigma_{a\chi H} = \kappa\left\{6\log 2+i\omega\frac{1}{16}\left(48\mathcal{C}+5\pi^2\right)+\cdots\right\}, \qquad \bar{\sigma}_{a\chi H} = 0+\cdots,$$

$$\sigma_1 = 162\kappa^2\bar{\mu}\bar{\mu}_5\left[6+\log 2(5\log 2-12)\right]+\cdots,$$

$$\sigma_2 = \frac{1}{8}(6\pi-\pi^2-12\log 2)+108\kappa^2(\bar{\mu}^2+\bar{\mu}_5^2)\left[6+\log 2(5\log 2-12)\right]+\cdots,$$

$$\sigma_3 = 9\kappa\bar{\mu}\log^2 2+\cdots, \qquad \bar{\sigma}_3 = \sigma_3\left[\bar{\mu}\leftrightarrow\bar{\mu}_5\right].$$

CME/CSE TCF

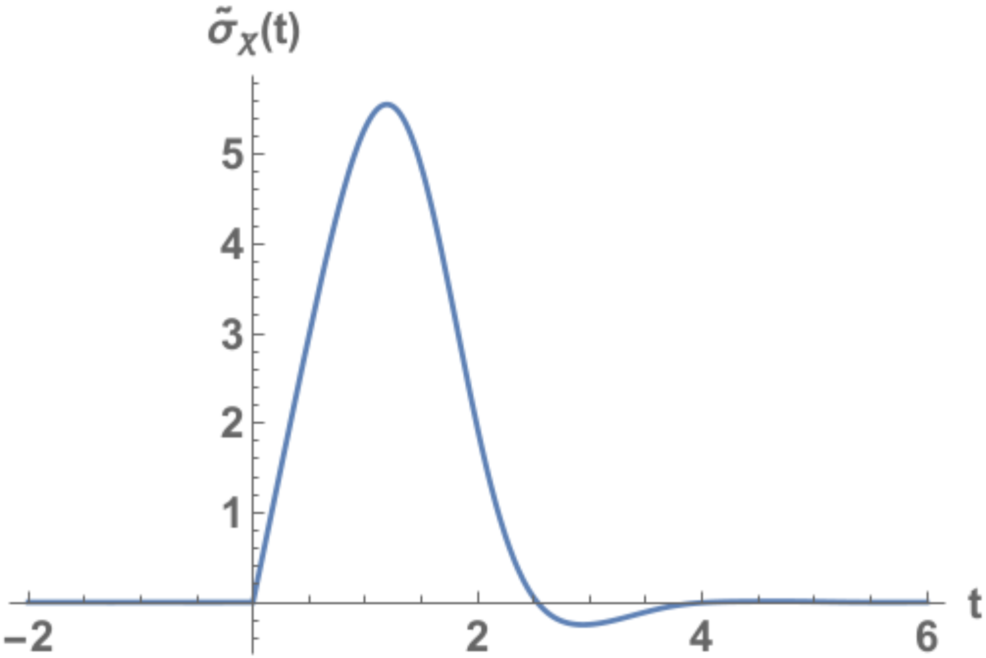


$Re(\sigma_\chi)$
10
5
0
-5
0
1
2
3
4
5
ω
0
1
2
3
4
5
q^2
$Im(\sigma_\chi)$
6
4
2
0
-2
0
1
2
3
4
5
ω
0
1
2
3
4
5

CME/CSE memory function

Via inverse Fourier transform, the memory function

$$\tilde{\sigma}_{\bar{\chi}}(\mathbf{t}) \equiv \frac{\mathbf{1}}{\sqrt{\mathbf{2}\pi}} \int_{-\infty}^{+\infty} \mathbf{d}\omega \mathbf{e}^{-\mathbf{i}\omega\mathbf{t}} \sigma_{\bar{\chi}}(\omega, \mathbf{q} = \mathbf{0}).$$



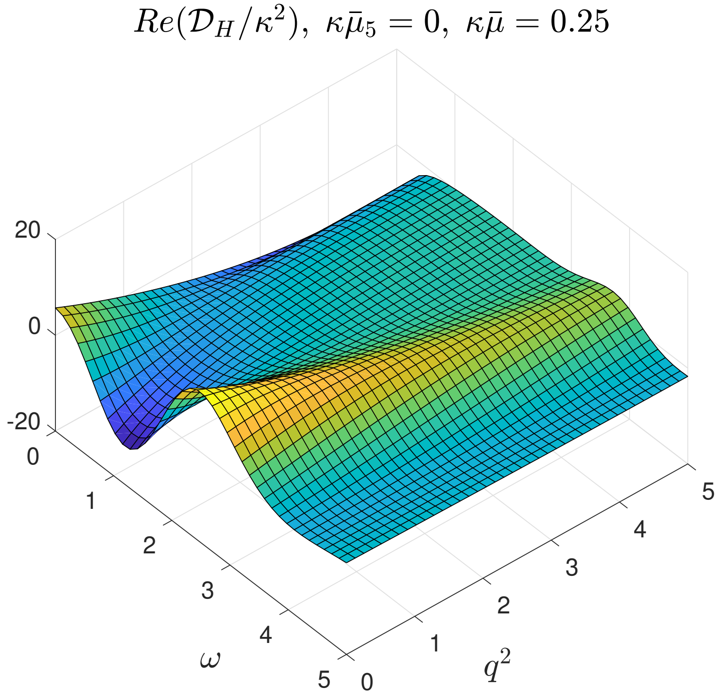
No instantaneous CME response!

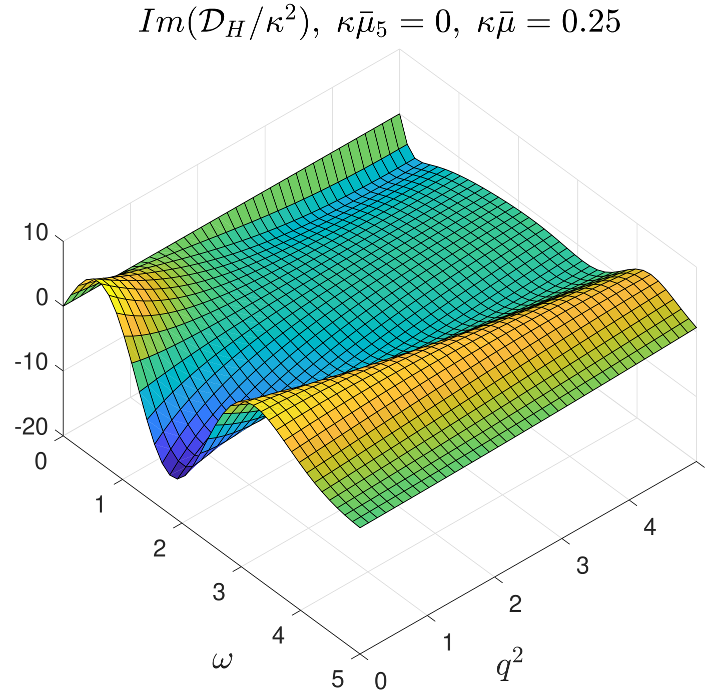
CME response is delayed by a time

of order temperature

Hall diffusion TCF

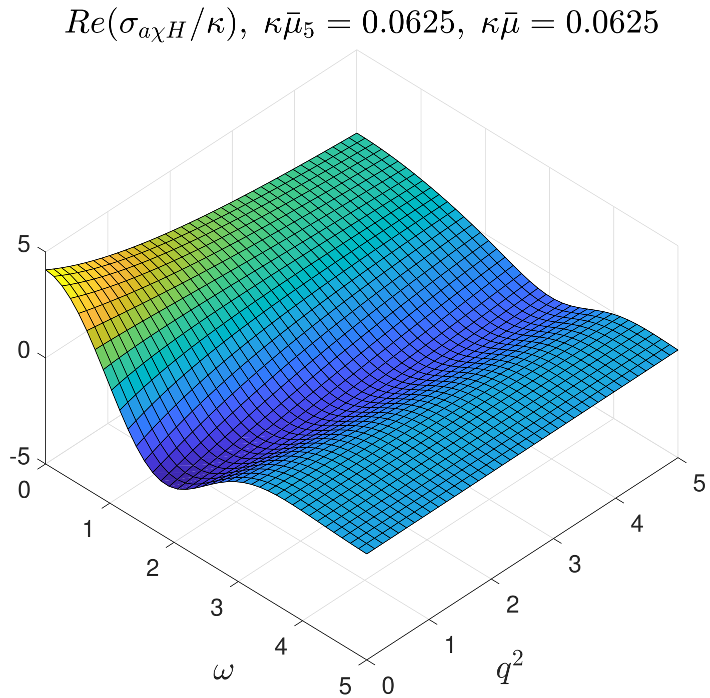
$$\vec{J} \sim \mathcal{D}_H(\bar{\rho}\vec{\mathbf{B}} \times \vec{\nabla}\delta\rho)$$
$$\vec{J}_5 \sim \mathcal{D}_H(\bar{\rho}\vec{\mathbf{B}} \times \vec{\nabla}\delta\rho_5)$$

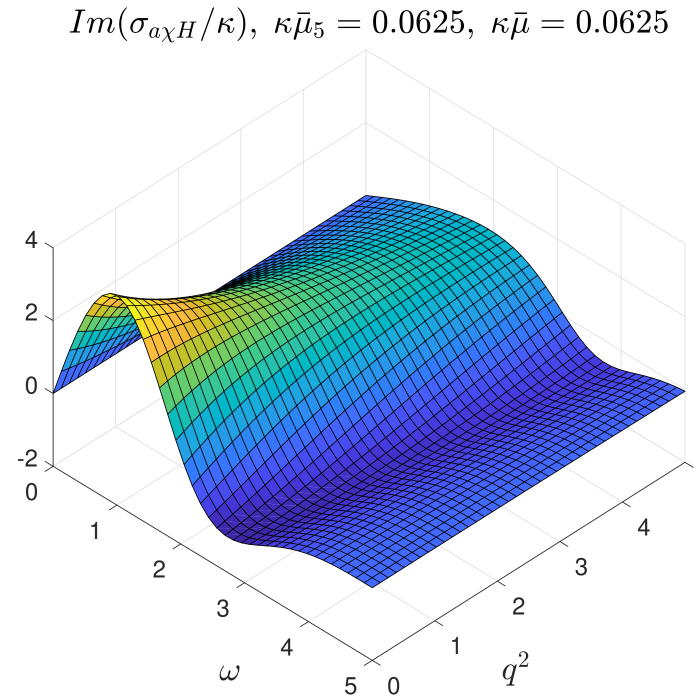




Anomalous chiral Hall TCF

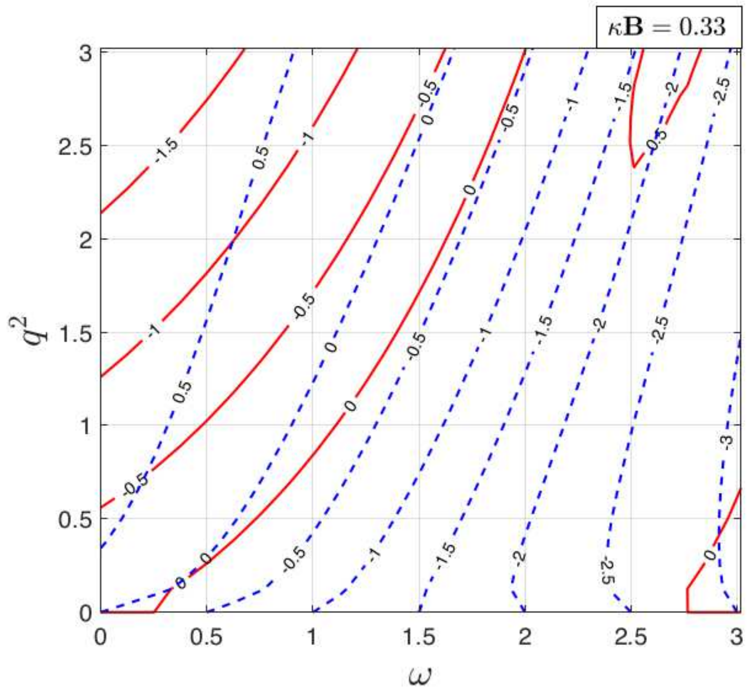
$$\vec{J} \sim \sigma_{a\chi H}(\vec{\mathbf{E}} \times \vec{\nabla}\delta\rho_5)$$
$$\vec{J}_5 \sim \sigma_{a\chi H}(\vec{\mathbf{E}} \times \vec{\nabla}\delta\rho)$$

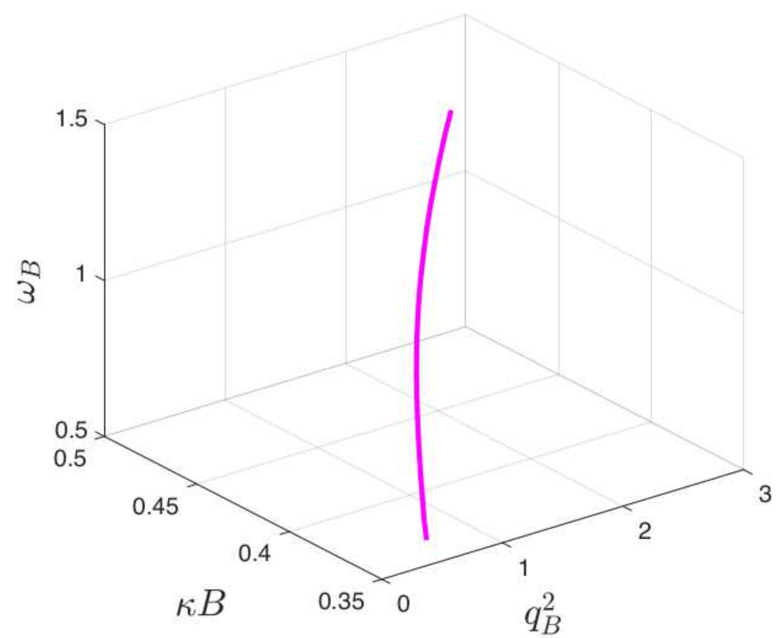




Non-dissipative CMW modes

$$\omega = \pm\left(\bar{\sigma}_{\bar{\chi}} - q^2\mathcal{D}_\chi\right)\kappa\vec{\mathbf{B}}\cdot\vec{q} - i\left(q^2\mathcal{D} - \mathcal{D}_B(\kappa\vec{\mathbf{B}}\cdot\vec{q})^2\right)$$





Conclusions

- **Memory function is an important ingredient of causal relativistic hydrodynamics. Fluid-gravity correspondence provides a calculational framework to rigorously address transports In QFT, including all order resummation. Unfortunately not in QCD ...**

- **We have re-examined transport coefficients induced by the chiral anomaly. We seem to be able to rediscover all known anomaly-induced effects within one and the same holographic model. We have got a zoo of new transport phenomena and corresponding transport coefficients (over 50 computed analytically)**

- **We have discovered a new dissipation-less CMW**

- **We propose to use generalised and more correct constitutive relations (memory functions and non-linear effects) for practical simulations of relativistic hydrodynamics and particularly of χMHD.**