Pairing gaps and the speed of sound peak of dense quark matter

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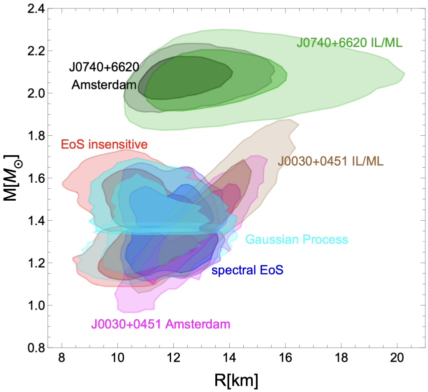
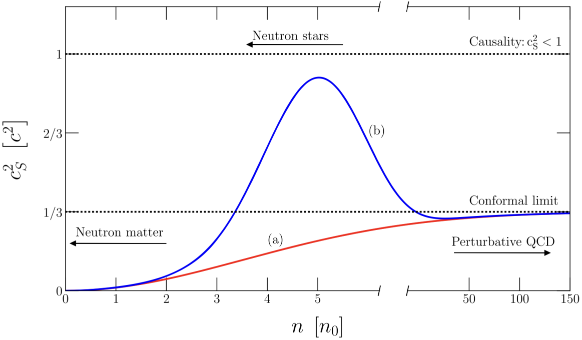
Motivation



MUSES collaboration, Living Rev Relativ 27, 3 (2024).

$$c_s^2 = \partial p / \partial \epsilon$$



I. Tews, J. Carlson, S. Gandolfi, S. Reddy, Astrophys. J. 860, no.2, 149 (2018).

Regularization issues in effective models

- We consider the Nambu–Jona-Lasinio model at finite isospin density, allowing for the development of pion condensation.
- In the presence of the pion condensate Δ, the dispersion relations take the form

$$E_k^{\pm} = \sqrt{\left(E_k \pm \frac{\mu_I}{2}\right)^2 + \Delta^2}\,, \tag{1}$$

with $E_k = \sqrt{k^2 + M^2}$.

- The one-loop contribution to the potential then involves

$$\int \frac{d^3p}{(2\pi)^3} \left(E_k^+ + E_k^-\right) \,. \tag{2}$$

- Ultraviolet divergent: application of a regulator at this stage is referred to as the Traditional Regularization Scheme (TRS).
- Problem: explicit medium dependence (μ_I).

Medium separation scheme (MSS)

- MSS: separation of medium terms from the UV-divergent vacuum.
- Consider the gap equation for the pion condensate: $\Delta = 4GN_c\Delta I_\Delta$, with

$$I_\Delta = \sum_{j=\pm 1} \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{\left(E_k + j\frac{\mu_I}{2}\right)^2 + \Delta^2}} \,. \tag{3}$$

- We rewrite this term such that

$$I_\Delta = 2 \sum_{j=\pm 1} \int_{-\infty}^{+\infty} \frac{dx}{2\pi} \int \frac{d^3k}{(2\pi)^3} \frac{1}{x^2 + \left(E_k + j\frac{\mu_I}{2}\right)^2 + \Delta^2} \,. \tag{4}$$

- Consider the addition and subtraction of $(x^2+k^2+M_0^2)^{-1}$ (M_0 being the effective quark mass in vacuum) through the identity

$$\frac{1}{x^2+\left(E_k+j\frac{\mu_I}{2}\right)^2+\Delta^2}=\frac{1}{x^2+k^2+M_0^2}+\frac{M_0^2-\Delta^2-(\mu_I/2)^2-M^2-j\mu_I E_k}{(x^2+k^2+M_0^2)\left[x^2+\left(E_k+j\frac{\mu_I}{2}\right)^2+\Delta^2\right]}\,.$$

- After some iterations and manipulations

$$I_\Delta^{\mathrm{MSS}}=2I_{\mathrm{quad}}(M_0)-\left(M^2-M_0^2+\Delta^2-\frac{\mu_I^2}{2}\right)I_{\log}(M_0)+\frac{3}{4}\left[\mathcal{M}^2+\mu_I^2M^2-\mu_I^2M_0^2\right]I_1+2I_2\,. \tag{5}$$

- Only the integrals $I_{\mathrm{quad}}(M_0)$ and $I_{\log}(M_0)$ are divergent, and regularized by the 3D cutoff Λ.

$$
\begin{aligned}
I_{\text{quad}}(M_0) &= \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{k^2 + M_0^2}} \,, \\
I_{\text{log}}(M_0) &= \int \frac{d^3k}{(2\pi)^3} \frac{1}{\left(k^2 + M_0^2\right)^{3/2}} \,.
\end{aligned} \tag{6}
$$

- Other terms are finite: integrated over the whole momentum space.
- The MSS thermodynamic potential is written

$$
\begin{aligned}
\Omega_{\text{SU}(2)}^{\text{MSS}} = \frac{\sigma^2 + \Delta^2}{4G} - 2N_c \Bigg[& \tilde{\mathcal{M}} I_{\text{quad}} - \frac{1}{4} \left(\tilde{\mathcal{M}}^2 - \mu_I^2 \Delta^2 \right) I_{\text{log}} \\
& + \int \frac{d^3k}{(2\pi)^3} \left(\frac{\tilde{\mathcal{M}}^2 - \mu_I^2 \Delta^2}{4E_{k,0}^3} - \frac{\tilde{\mathcal{M}}}{E_{k,0}} - 2E_{k,0} + E_k^+ + E_k^- \right) \Bigg] \,,
\end{aligned} \tag{7}
$$

with $\tilde{\mathcal{M}} = \Delta^2 + M^2 - M_0^2$.

Results

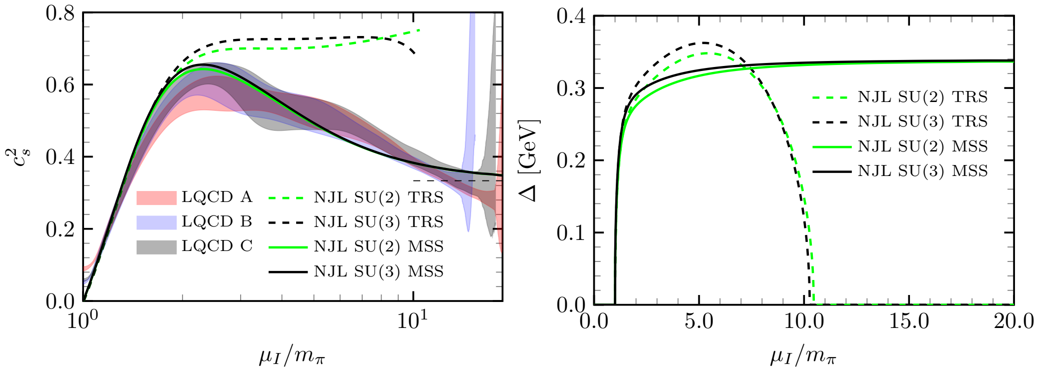


Figure: Speed of sound c_s^2 and pion condensate Δ as functions of the normalized isospin chemical potential μ_I/m_π. [1]

[1] B. S. Lopes, D. C. Duarte, R. L. S. Farias and R. O. Ramos, Phys. Rev. D 112, L091903 (2025).
LQCD data: NPLQCD Collaboration, Phys. Rev. D 108 (2023) 11, 114506.

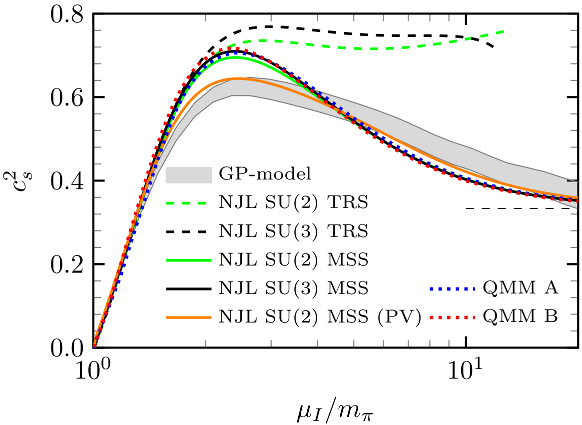
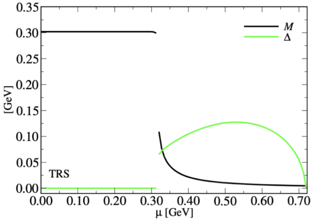


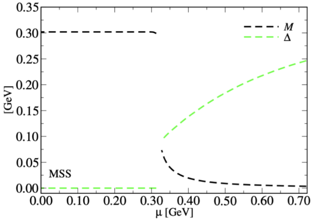
Figure: Speed of sound c_s^2 as a function of the normalized isospin chemical potential μ_I/m_π. [2]

[2] B. S. Lopes, D. C. Duarte, R. L. S. Farias and R. O. Ramos, Phys. Rev. D 112, L091903 (2025).
GP model: NPLQCD Collaboration, Phys. Rev. Lett. 134 (2025) 1, 1.
QMM A: J. Andersen, M. Nodtvedt, Phys. Rev. D 113, 014026 (2026).
QMM B: B. Brandt et al., Phys. Rev. D 112 (2025) 5, 054038.

2SC CSC NJL+MSS X NJL + RG X QMM



(a)



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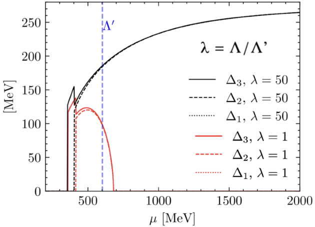

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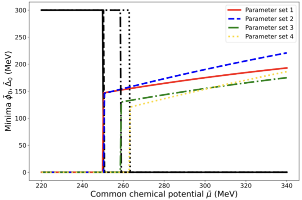

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·Ricardo L. S. Farias @ CSQCD, Barcelona, 2026.

Conclusions

- MSS results are consistent with QMM and LQCD calculations (finite isospin density), with no additional parameter tuning.
- Also in agreement with FRG methods for the color superconducting phase, and RG-consistent NJL treatments.
- Proper disentanglement of vacuum and medium contributions seems to be essential for the description of dense quark matter.

Acknowledgments

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Thank you!