Sum rules for quark longitudinal and transverse angular momentum (and more)

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INT, “Mass Workshop”
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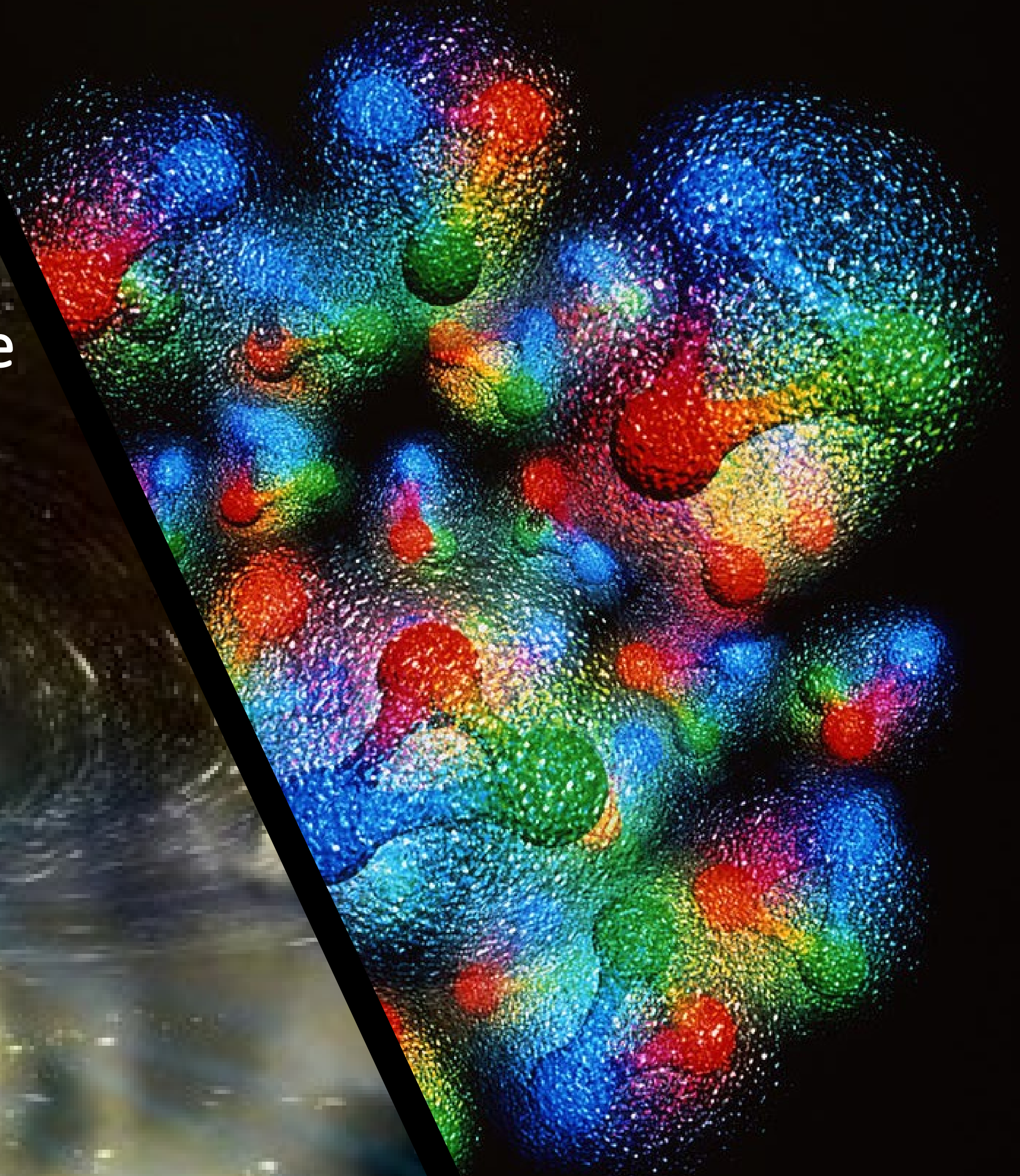
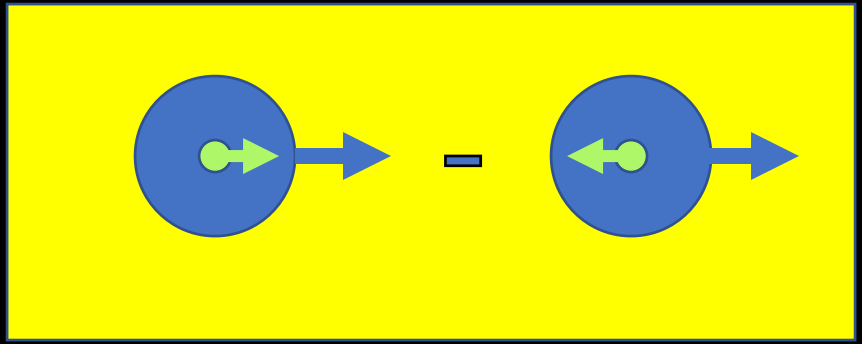
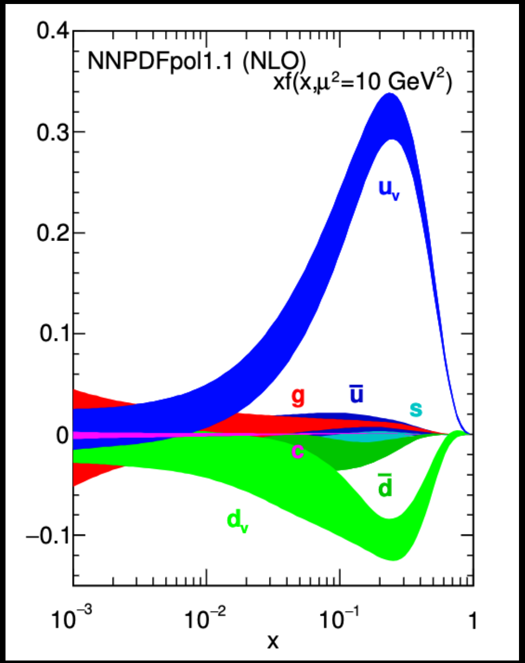


Identifying the Sum Rule elements: quark longitudinal spin, S_z^q

g_1

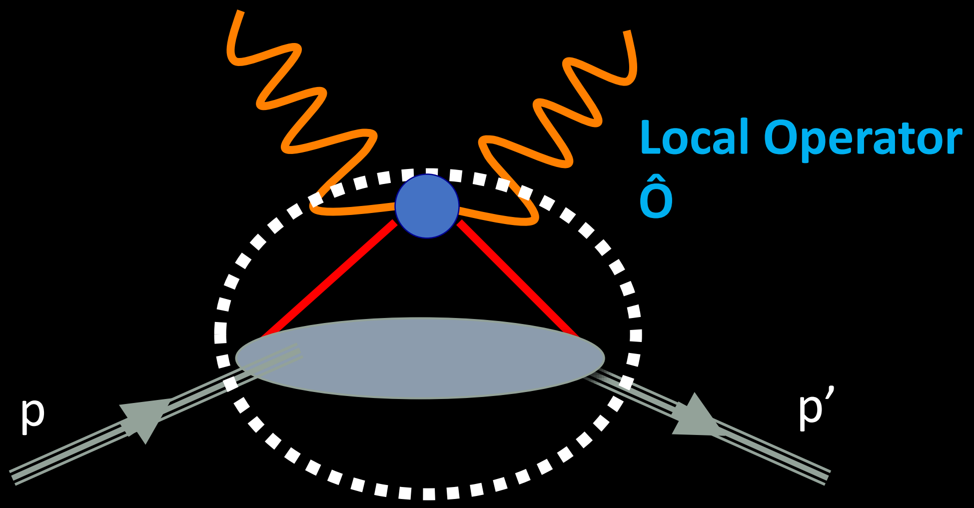




$$S_z^q = \frac{1}{2}\Delta\Sigma_q = \int_0^1 dx\, g_1^q(x)$$

Identifying the Sum Rule elements: quark longitudinal angular momentum, J_z^q

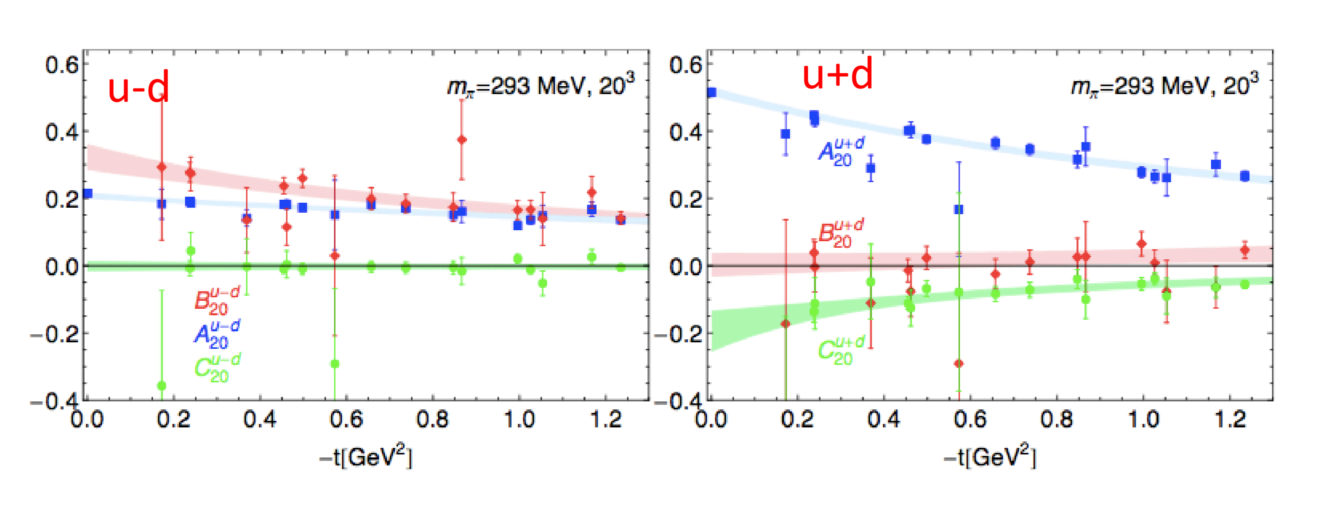
$$J_z^q = \int_0^1 dx\, x\left(H_q + E_q\right)$$



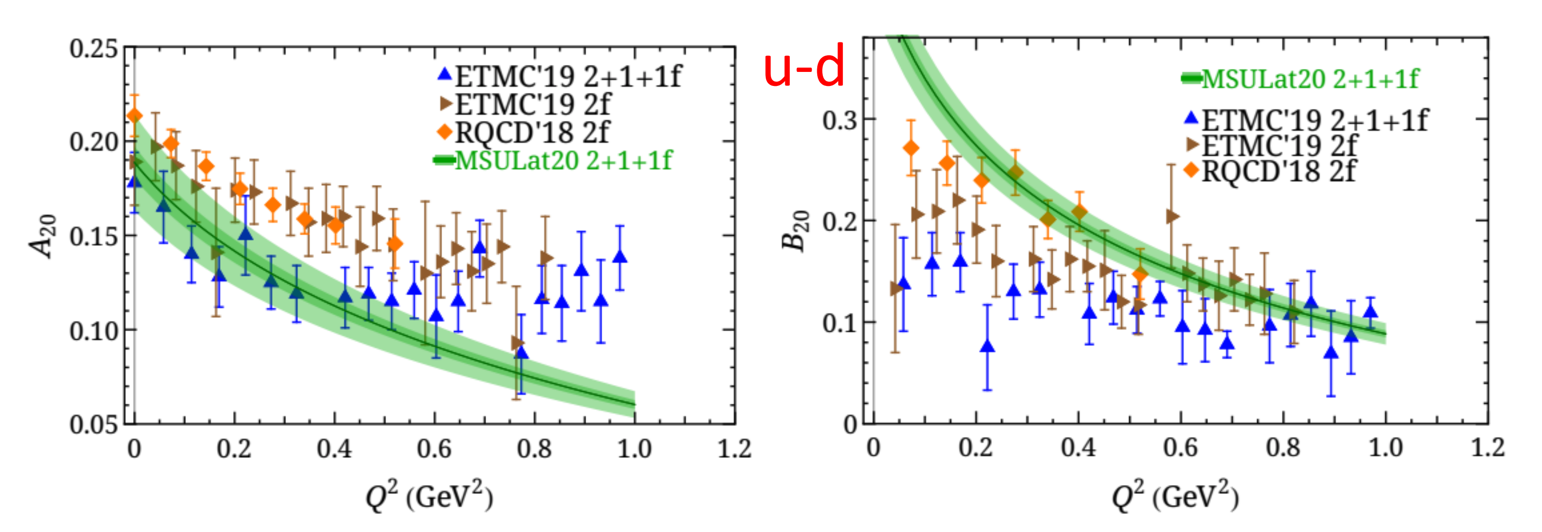
GPD Moments → EMT Form Factors

X. Ji (1997)

Calculable on lattice…



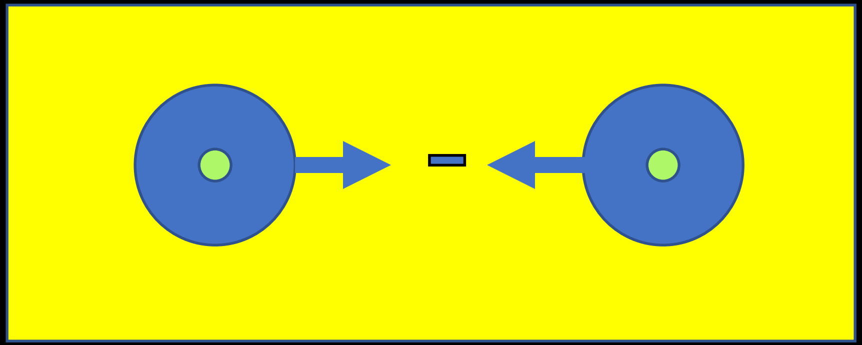
Ph. Haegler, JoP: **295** (2011) 012009



H-W Lin, *Phys.Rev.Lett.* 127 (2021)

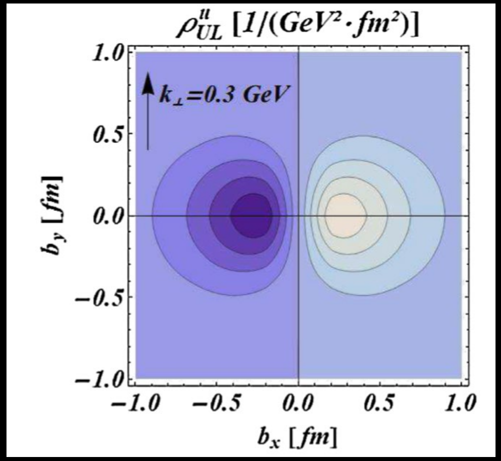
Identifying the Sum Rule elements: quark longitudinal OAM, L_z^q

F_{14}



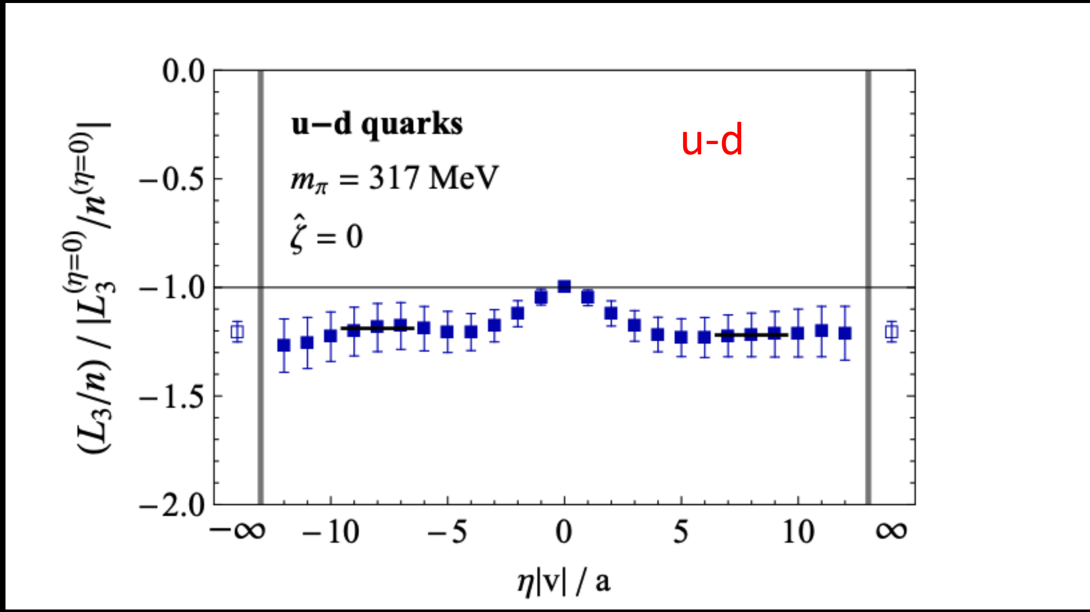
$$L_z^q = \int_0^1 dx \int d^2k_T\ k_T^2 F_{14}(x,0,0,k_T)$$

UL correlation GTMD



Lorce, Pasquini, PRD (2013)

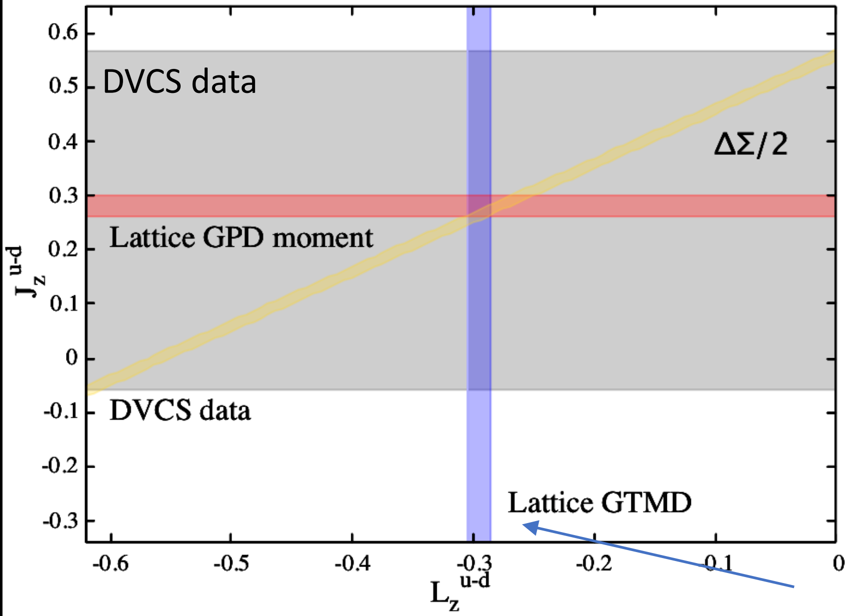
Calculable on lattice…



M. Engelhardt et al. *Phys.Rev. D* (2020)

What we know from measurements and lattice

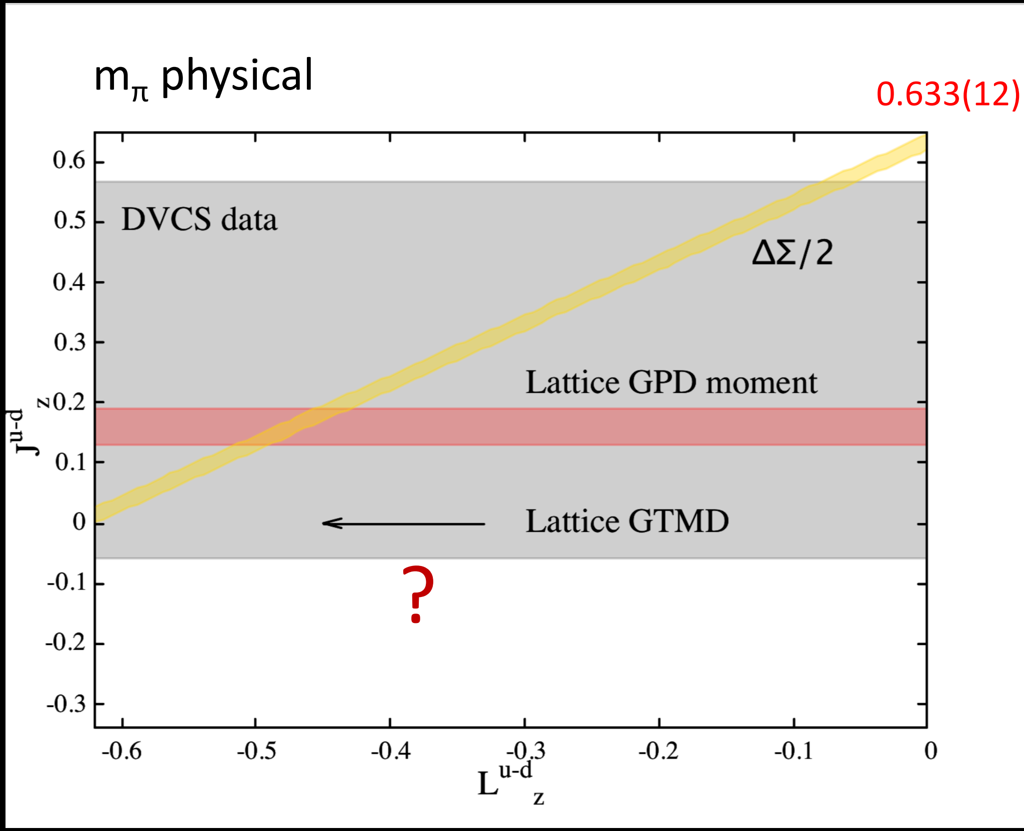
m_π = 320 MeV



Engelhardt et al., Phys. Rev. D102, (2020)

q=u-d

$$J_q = L_q + \frac{1}{2}\Delta\Sigma_q$$



mπ physical
0.633(12)
DVCS data
ΔΣ/2
Lattice GPD moment
Lattice GTMD
?
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
Ju-d z
-0.6
-0.5
-0.4
-0.3
-0.2
-0.1
0
Lu-d z

Do we need to measure a GTMD to learn about OAM?

The other way that OAM is known

Polyakov Kiptily(2004), Hatta(2012)

$$\int_0^1 dx\, x G_2 = -\frac{1}{2}\int_0^1 dx\, x(H+E) + \frac{1}{2}\int_0^1 dx\, \tilde{H}$$

$$J_q = L_q + \frac{1}{2}\Delta\Sigma_q$$

A generalized integrated Wandzura Wilczek relation obtained using OPE for twist 2 and twist 3 operators for the off-forward matrix elements

Connecting the two pictures using **QCD Equations of Motion** and **Lorentz symmetry**

$$\begin{array}{ccccc} J_L & = & L_L & + & S_L \\ \frac{1}{2}\int dx\, x(H+E) & = & \int dx\, x(\widetilde{E}_{2T}+H+E) & + & \frac{1}{2}\int dx\, \widetilde{H} \\ & = & -\int dx\, F_{14}^{(1)} & + & \frac{1}{2}\int dx\, \widetilde{H} \end{array}$$

- A. Rajan, A. Courtoy, M. Engelhardt, S.L., PRD (2016)
- A. Rajan, M. Engelhardt, S.L., PRD (2018)

*Twist 3 GPD notation from Meissner, Metz and Schlegel, JHEP(2009)

- Instead of using the OPE, we derive the different terms, using **nonlocal** matrix elements

- A dynamical picture to understand the origin of quark angular momentum where the role of the transverse momentum of quarks is emphasized and essential

A. Rajan, A. Courtoy, M. Engelhardt, S.L., PRD (2016)

A. Rajan, M. Engelhardt, S.L., PRD (2018)

A. Rajan, O. Alkassasbeh, M. Engelhardt, S.L., *in preparation*

$$0 = \int \frac{d^4x}{(2\pi)^4} e^{ik\cdot x} \langle PS|\overline{\psi}(0)\left[i\not{D}(0) - m\right] i\sigma^{i+}\gamma_5 \psi(x)|PS\rangle$$

Equations of Motion (EoM) relation

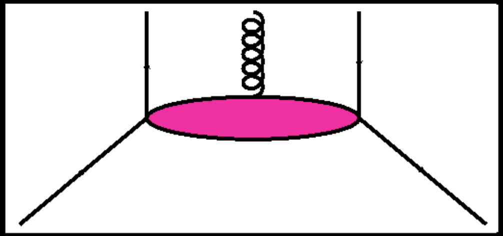
$$0=\int\frac{d^4x}{(2\pi)^4}e^{ik\cdot x}\langle PS|\overline{\psi}(0)\left[i\not{D}(0)-m\right]i\sigma^{i+}\gamma_5\psi(x)|PS\rangle$$

Λ' Λ

$$-\frac{\Delta^+}{2}W^{\gamma^i\gamma^5}_{\Lambda'\Lambda}+ik^+\epsilon^{ij}W^{\gamma^j}_{\Lambda'\Lambda}+\frac{\Delta^i}{2}W^{\gamma^+\gamma^5}_{\Lambda'\Lambda}-i\epsilon^{ij}k^jW^{\gamma^+}_{\Lambda'\Lambda}+\mathcal{M}^{i,S}_{\Lambda'\Lambda}=0$$

$$x\widetilde{E}_{2T}{}^{*} = -\widetilde{H} + \int d^2k_T \frac{k_T^2}{M^2} F_{14} - \mathcal{M}_{F_{14}}$$

Twist 3 GPD — Twist 2 GPD — GTMD — qgq

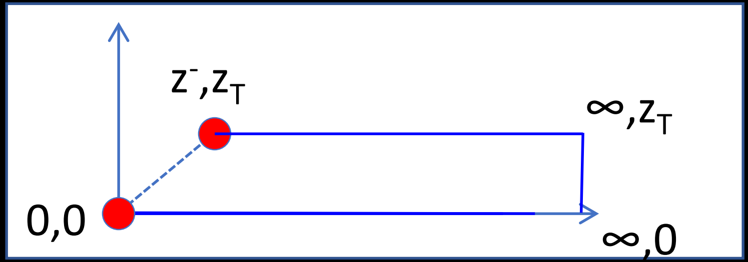


Equations of Motion (EoM) relation

*Twist 3 GPD notation from Meissner, Metz and Schlegel, JHEP(2009)

$$\frac{d}{dx}\int d^2k_T\,\frac{k_T^2}{M^2}\,F_{14}=\tilde{E}_{2T}+H+E+\mathcal{A}$$

GTMD Twist 3 GPD Twist 2 GPD qgq from staple link

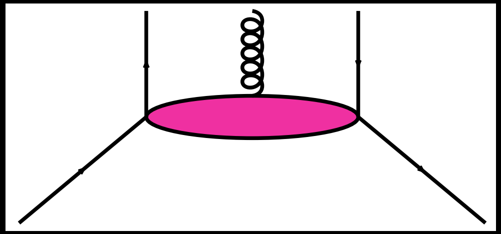


Lorentz Invariance Relation (LIR)

Wandzura Wilczek relation for OAM

Straight gauge link

$$\tilde{E}_{2T} = -\int_x^1 \frac{dy}{y}(H+E) + \left[\frac{\tilde{H}}{x} - \int_x^1 \frac{dy}{y^2}\tilde{H}\right] + \left[\frac{1}{x}\mathcal{M}_{F_{14}} - \int_x^1 \frac{dy}{y^2}\mathcal{M}_{F_{14}}\right]$$



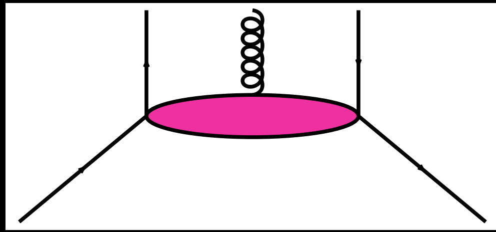
genuine twist 3

- A. Rajan, A. Courtoy, M. Engelhardt, S.L., PRD (2016)
- A. Rajan, M. Engelhardt, S.L., PRD (2018)

Wandzura Wilczek relation for OAM

Staple link

$$\tilde{E}_{2T} = -\int_x^1 \frac{dy}{y}(H+E) + \left[\frac{\tilde{H}}{x} - \int_x^1 \frac{dy}{y^2}\tilde{H}\right] + \left[\frac{1}{x}\mathcal{M}_{F_{14}} - \int_x^1 \frac{dy}{y^2}\mathcal{M}_{F_{14}}\right] - \int_x^1 \frac{dy}{y}\mathcal{A}_{F_{14}}$$



LIR violating term

- A. Rajan, A. Courtoy, M. Engelhardt, S.L., PRD (2016)
- A. Rajan, M. Engelhardt, S.L., PRD (2018)

Integral Relation

$$
\begin{aligned}
J_L &= L_L + S_L \\
\frac{1}{2}\int dx\, x(H+E) &= \int dx\, x(\widetilde{E}_{2T}+H+E) + \frac{1}{2}\int dx\, \widetilde{H} \\
&= -\int dx\, F_{14}^{(1)} + \frac{1}{2}\int dx\, \widetilde{H}
\end{aligned}
$$

- A. Rajan, A. Courtoy, M. Engelhardt, S.L., PRD (2016)
- A. Rajan, M. Engelhardt, S.L., PRD (2018)

*Twist 3 GPD notation from Meissner, Metz and Schlegel, JHEP(2009)

Transverse Angular Momentum Sum Rule

O. Alkassasbeh, M. Engelhardt, SL and A. Rajan, (2022) soon on arXiv

$$\underbrace{\frac{1}{2}\int dx\,x(H+E) - \frac{1}{2}\int dx\,\mathcal{M}_T}_{J_T} = \underbrace{\int dx\,x\Big(\widetilde{E}_{2T} + H + E + \frac{H_{2T}}{\xi}\Big)}_{L_T} + \underbrace{\frac{1}{2}\int dx\,g_T - \frac{1}{2}\int dx\,x\mathcal{A}_T}_{S_T}$$

Twist 3 GPDs Physical Interpretation

GPD	$P_q P_p$	TMD	Ref. 1
$H^{\perp}$	UU	$f^{\perp}$	$2\widetilde{H}_{2T} + E_{2T}$
$\widetilde{H}_L^{\perp}$	LL	$g_L^{\perp}$	$2\widetilde{H}'_{2T} + E'_{2T}$
$H_L^{\perp}$	UL	$f_L^{\perp\,(*)}$	$\widetilde{E}_{2T} - \xi E_{2T}$
$\widetilde{H}^{\perp}$	LU	$g^{\perp\,(*)}$	$\widetilde{E}'_{2T} - \xi E'_{2T}$
$H_T^{(3)}$	UT	$f_T^{(*)}$	$H_{2T} + \tau \widetilde{H}_{2T}$
$\widetilde{H}_T^{(3)}$	LT	g'_T	$H'_{2T} + \tau \widetilde{H}'_{2T}$

→ 1/Q correction to H

→ 1/Q correction to $\widetilde{H}$

NEW!! Orbital Angular Momentum L

NEW!! Spin Orbit correlation L ·S

→ 1/Q correction to E: Transverse OAM, L_T

→ 1/Q correction to $\widetilde{E}$

(*) T-odd

[1] Meissner, Metz and Schlegel, JHEP(2009)

✓ The connection to observables is fundamental

✓ The sum rules, written in terms of twist-2 and twist-3 observables, are an intrinsic feature of (are derived directly from) the QCD EoM, and preserve Lorentz symmetry

➢ Can we extend this to mass?

➢ What are the observables for the mass distribution?

➢ How to include gluons?

Mass Sum Rule

Start from EoM relation at tree level:

$$0 = \int \frac{dz_{in}^- \, d^2 z_{in,T}}{(2\pi)^3} \int \frac{dz_{out}^- \, d^2 z_{out,T}}{(2\pi)^3} e^{ik(z_{in}-z_{out})-i\Delta(z_{in}+z_{out})/2}$$
$$\cdot \langle p', \Lambda' \mid \bar{\psi}(z_{out}) \left[(i\overleftarrow{\not{D}} + m)\Gamma\mathcal{U} \pm \Gamma\mathcal{U}(i\not{D} - m) \right] \psi(z_{in}) \mid p, \Lambda \rangle \Big|_{z_{in}^+ = z_{out}^+ = 0}$$

EoM relation for scalar operator:

$$\Delta^+ W^{[\gamma^-]}_{\Lambda'\Lambda} + \Delta^- W^{[\gamma^+]}_{\Lambda'\Lambda} - \Delta^i_T W^{[\gamma^i_T]}_{\Lambda'\Lambda} - 2\mathcal{M}^{1,S}_{\Lambda'\Lambda} = 0$$
$$k^+ W^{[\gamma^-]}_{\Lambda'\Lambda} + k^- W[\gamma^+]_{\Lambda'\Lambda} - k^i_T W^{[\gamma^i_T]}_{\Lambda'\Lambda} - m W^{[1]}_{\Lambda'\Lambda} - \mathcal{M}^{1,A}_{\Lambda'\Lambda} = 0$$

$$k^2 = m^2 \rightarrow H = \frac{k_T^2 + m^2}{2k^-}$$

A new key to interpret the mass decomposition

$$\Delta^+ W^{[\gamma^-]}_{\Lambda'\Lambda} + \Delta^- W^{[\gamma^+]}_{\Lambda'\Lambda} - \Delta^i_T W^{[\gamma^i_T]}_{\Lambda'\Lambda} - 2\mathcal{M}^{1,S}_{\Lambda'\Lambda} = 0$$

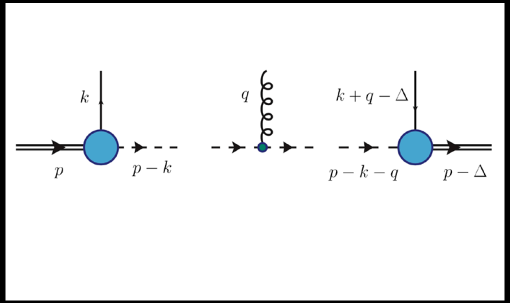
$$k^+ W^{[\gamma^-]}_{\Lambda'\Lambda} + k^- W[\gamma^+]_{\Lambda'\Lambda} - k^i_T W^{[\gamma^i_T]}_{\Lambda'\Lambda} - m W^{[1]}_{\Lambda'\Lambda} - \mathcal{M}^{1,A}_{\Lambda'\Lambda} = 0$$

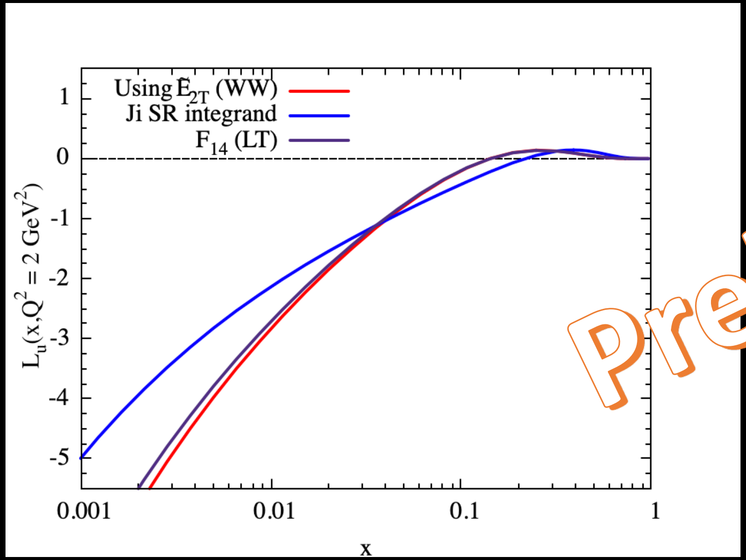
$$H = \frac{k_T^2 + m^2}{2k^-} \rightarrow 2k^- H = k_T^2 + m^2$$

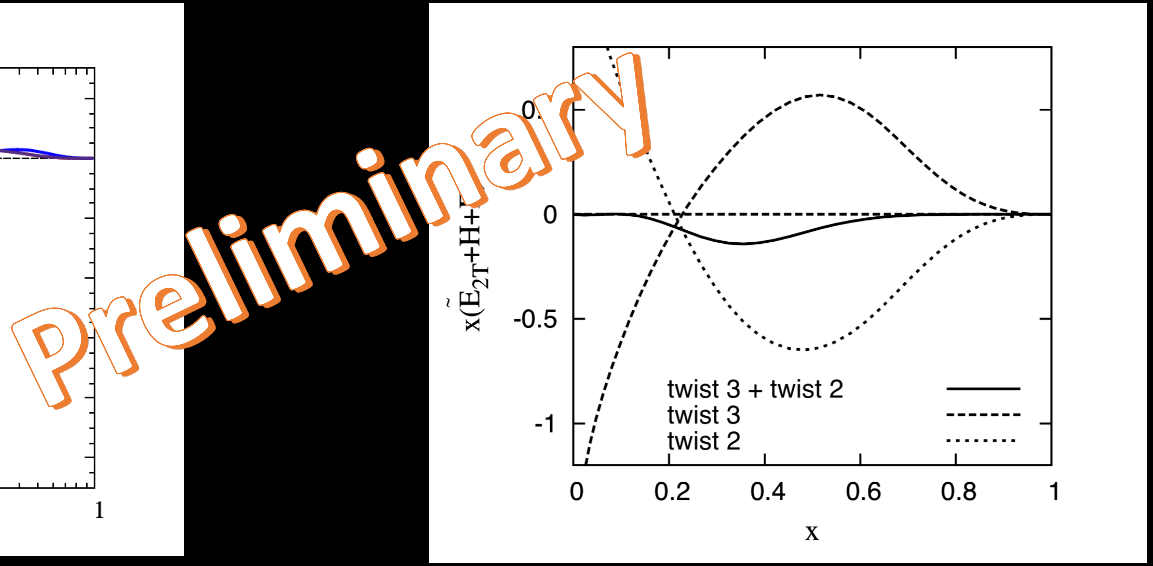
- ✓ At tree level we recover the sum of the trace and traceless part of the quark component in the decomposition
- ✓ Next step: include renormalization and regularization
- ✓ Role of qgq terms?
- ✓ Extension to gluon sector

- Observables: L_z in Spectator Model Calculation

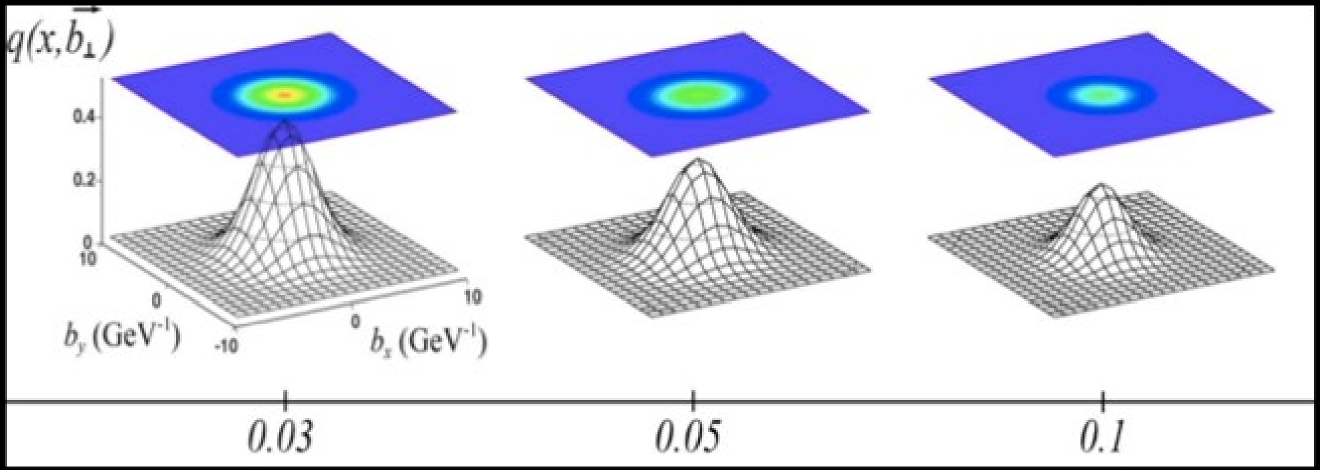
$$\mathcal{M}_{\Lambda\Lambda'} = \mathcal{N} \int \frac{d^2\mathbf{k}_T}{(2\pi)^4} \frac{d^2\mathbf{q}_T}{(2\pi)^4} \left[A_{++,++} - A_{--,--} + A_{+-,+-} - A_{-+,-+}\right] \times \frac{d_{\alpha\beta}(2p - 2k - q)^\beta n^\alpha}{q^2}$$







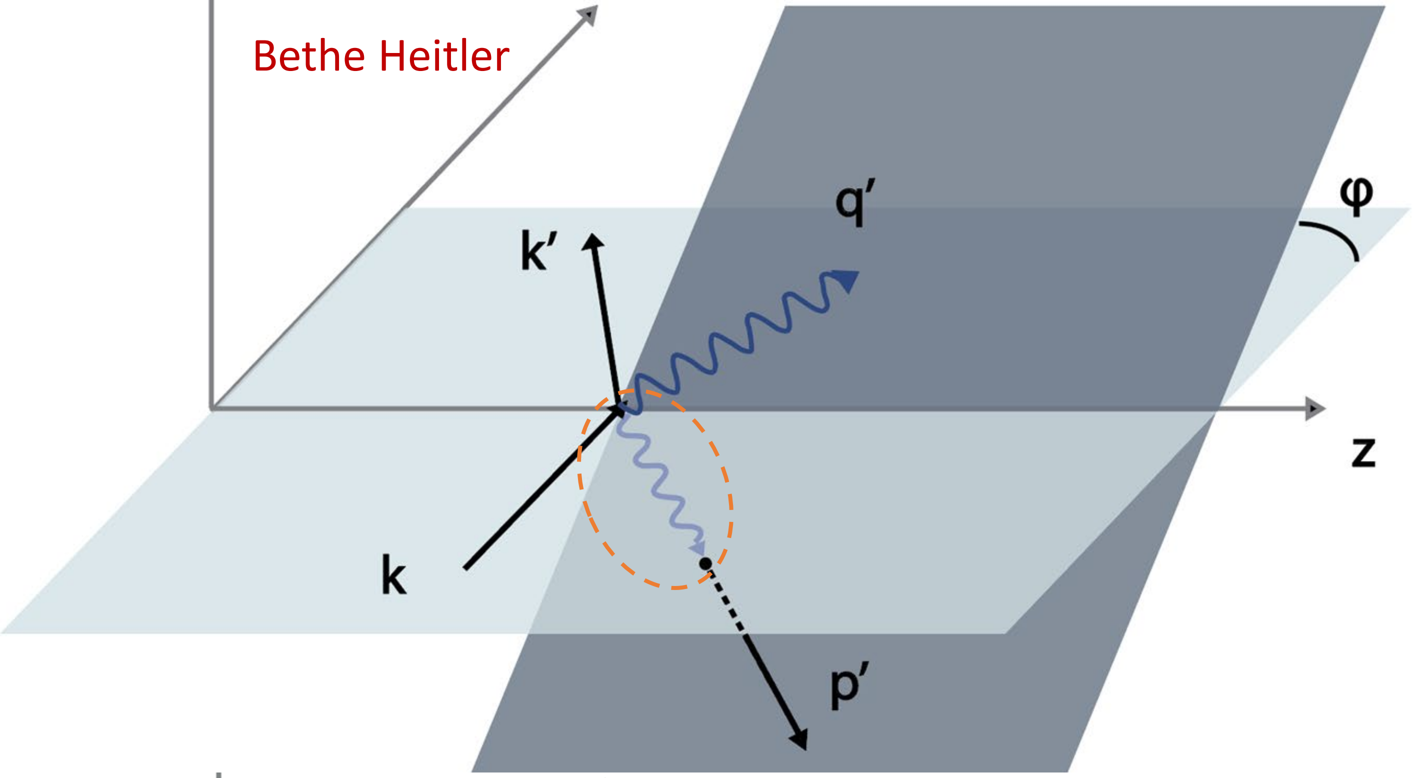
Measuring All This

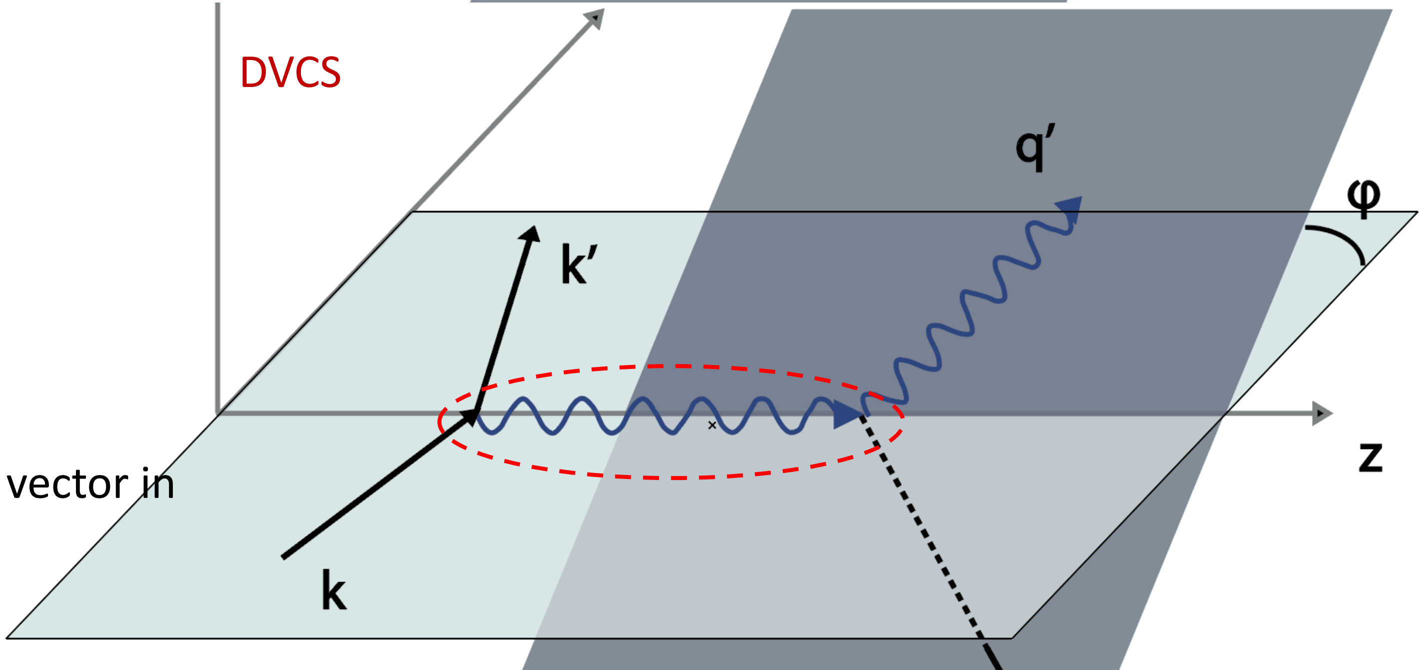


graph from M. Defurne

Demystification of harmonics formalism:

BKM missed a phase

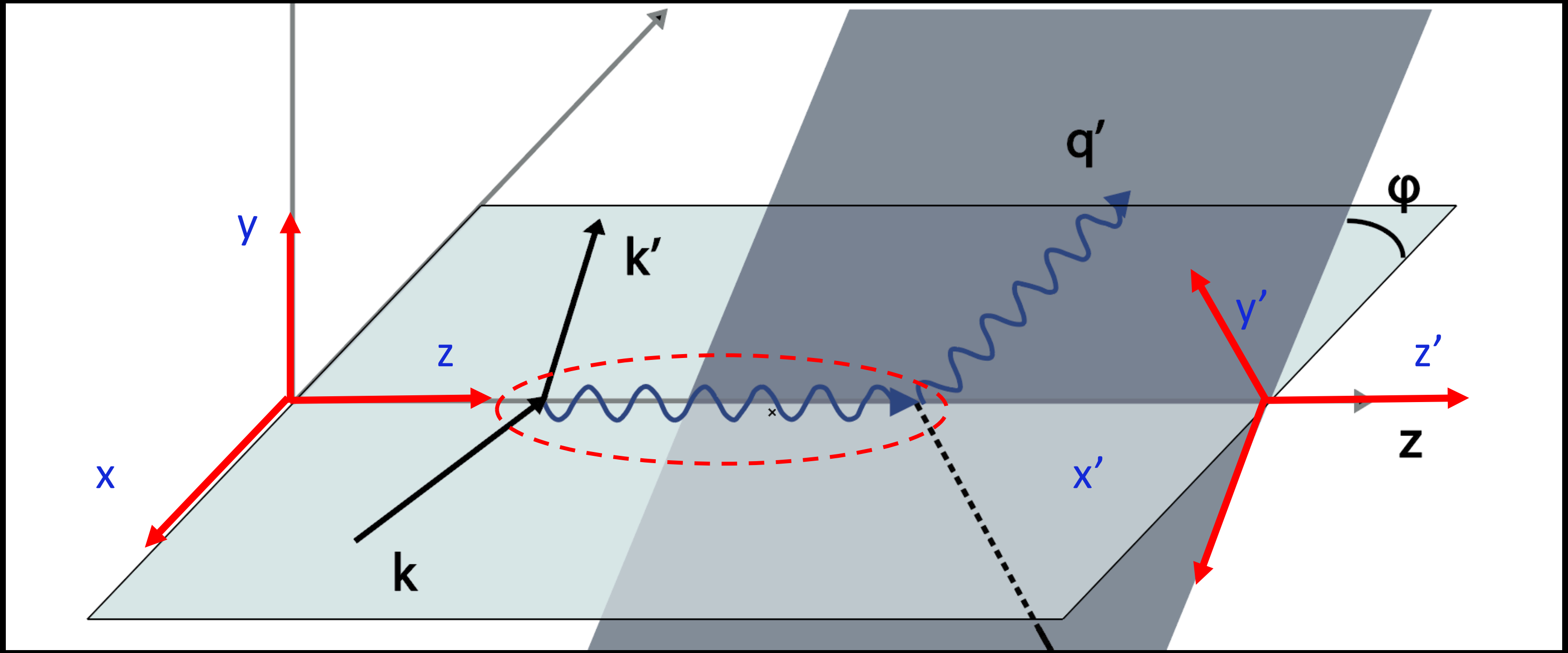




- In DVCS the virtual photon is along the z axis: φ dependence from usual rotation of polarization vector in helicity amp

To understand the cross section we need to understand the ϕ dependence

DVCS



The hadronic tensor is evaluated in the rotated frame

BH

$$\frac{d^5\sigma^{BH}_{unpol}}{dx_{Bj}dQ^2d|t|d\phi d\phi_S} \equiv \frac{\Gamma}{t} F^{BH}_{UU} = \frac{\Gamma}{t}\left[A(y, x_{Bj}, t, Q^2, \phi)\left(F_1^2 + \tau F_2^2\right) + B(y, x_{Bj}, t, Q^2, \phi)\tau G_M^2(t)\right]$$

$$A = \frac{16\, M^2}{t(k\, q')(k'\, q')}\left[4\tau\Big((k\, P)^2 + (k'\, P)^2\Big) - (\tau + 1)\Big((k\, \Delta)^2 + (k'\, \Delta)^2\Big)\right]$$

$$B = \frac{32\, M^2}{t(k\, q')(k'\, q')}\left[(k\, \Delta)^2 + (k'\, \Delta)^2\right],$$

$$_{BH} = \left(1 + \frac{B}{A}(1 + \;)\right)^{-1}$$

…compared to ELASTIC SCATTERING

10/21/21

$$\left(\frac{d\sigma}{d\Omega}\right)_0 = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \frac{\epsilon (G_E^N)^2 + \tau (G_M^N)^2}{\epsilon(1+\tau)},$$

where $N = p$ for a proton and $N = n$ for a neutron, (
the recoil-corrected relativistic point-particle (Mott)
and τ, ϵ are dimensionless kinematic variables:

$$\tau = \frac{Q^2}{4m_N^2}, \quad \epsilon = \left[1 + 2(1+\tau)\tan^2\frac{\theta}{2}\right]^{-1},$$

J. Arrington, G. Cates, S. Riordan, Z. Ye, B. Wojsetowski, A. Puckett …

…compared to BKM, NPB (2001)

$$|\mathcal{T}_{\rm BH}|^2 = \frac{e^6}{x_{\rm B}^2 y^2 (1+\epsilon^2)^2 \Delta^2 \mathcal{P}_1(\phi)\mathcal{P}_2(\phi)} \times \left\{ c_0^{\rm BH} + \sum_{n=1}^{2} c_n^{\rm BH} \cos(n\phi) + s_1^{\rm BH} \sin(\phi) \right\},$$

$$c_{0,\rm unp}^{\rm BH} = 8K^2\left\{(2+3\epsilon^2)\frac{Q^2}{\Delta^2}\left(F_1^2 - \frac{\Delta^2}{4M^2}F_2^2\right) + 2x_{\rm B}^2(F_1+F_2)^2\right\}$$
$$+ (2-y)^2\left\{(2+\epsilon^2)\left[\frac{4x_{\rm B}^2M^2}{\Delta^2}\left(1+\frac{\Delta^2}{Q^2}\right)^2 + 4(1-x_{\rm B})\left(1+x_{\rm B}\frac{\Delta^2}{Q^2}\right)\right]\left(F_1^2 - \frac{\Delta^2}{4M^2}F_2^2\right) + 4x_{\rm B}^2\left[x_{\rm B} + \left(1 - x_{\rm B} + \frac{\epsilon^2}{2}\right)\left(1-\frac{\Delta^2}{Q^2}\right)^2 - x_{\rm B}(1-2x_{\rm B})\frac{\Delta^4}{Q^4}\right](F_1+F_2)^2\right\}$$
$$+ 8(1+\epsilon^2)\left(1 - y - \frac{\epsilon^2y^2}{4}\right) \times \left\{2\epsilon^2\left(1-\frac{\Delta^2}{4M^2}\right)\left(F_1^2 - \frac{\Delta^2}{4M^2}F_2^2\right) - x_{\rm B}^2\left(1-\frac{\Delta^2}{Q^2}\right)^2(F_1+F_2)^2\right\},$$

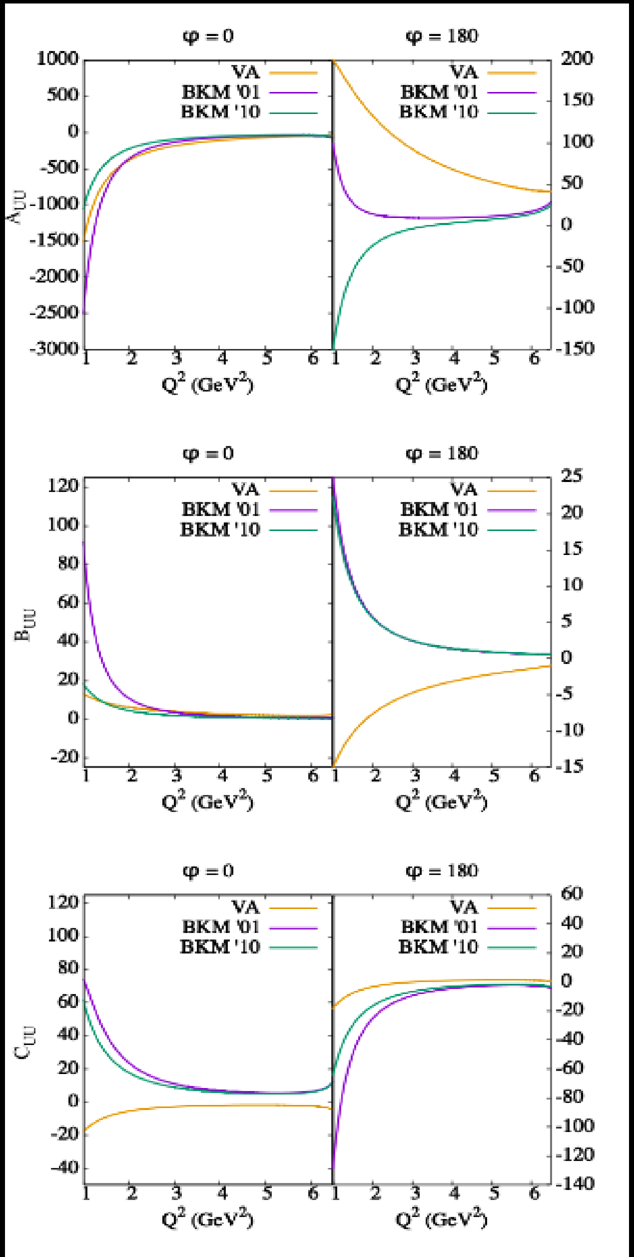
$$c_{1,\rm unp}^{\rm BH} = 8K(2-y)\left\{\left(\frac{4x_{\rm B}^2M^2}{\Delta^2} - 2x_{\rm B} - \epsilon^2\right)\left(F_1^2 - \frac{\Delta^2}{4M^2}F_2^2\right) + 2x_{\rm B}^2\left(1-(1-2x_{\rm B})\frac{\Delta^2}{Q^2}\right)(F_1+F_2)^2\right\},$$

$$c_{2,\rm unp}^{\rm BH} = 8x_{\rm B}^2K^2\left\{\frac{4M^2}{\Delta^2}\left(F_1^2 - \frac{\Delta^2}{4M^2}F_2^2\right) + 2(F_1+F_2)^2\right\}.$$

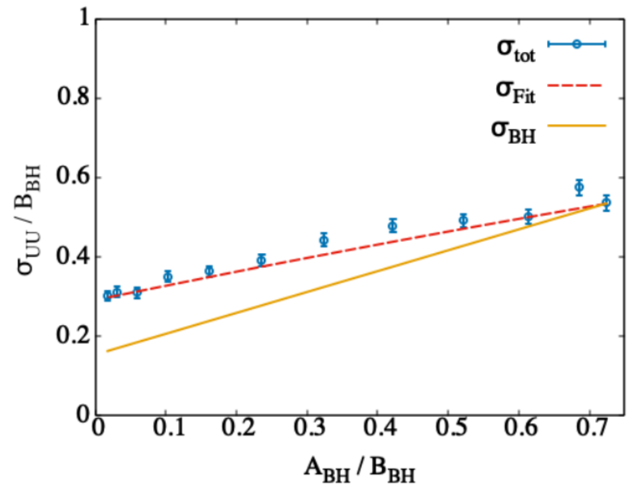
BH-DVCS interference

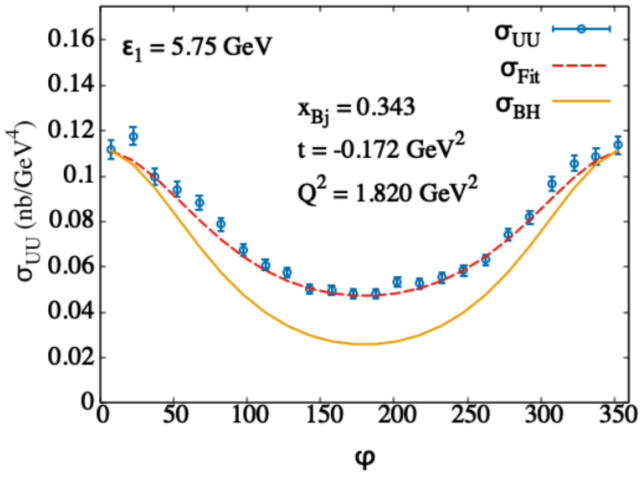
$$F_{UU}^{\mathcal{I},tw2} = A_{UU}^{\mathcal{I}} \Re e \left(F_1 \mathcal{H} + \tau F_2 \mathcal{E}\right) + B_{UU}^{\mathcal{I}} G_M \Re e \left(\mathcal{H} + \mathcal{E}\right) + C_{UU}^{\mathcal{I}} G_M \Re e \widetilde{\mathcal{H}}$$

A_{UU}^{I} B_{UU}^{I} C_{UU}^{I} are φ dependent coefficients

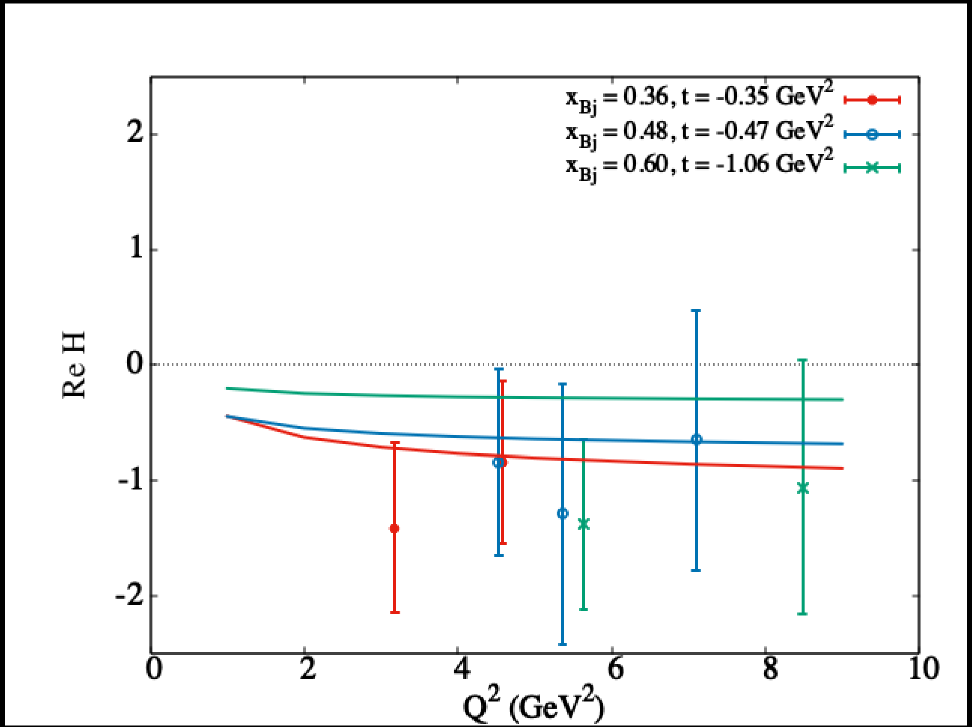


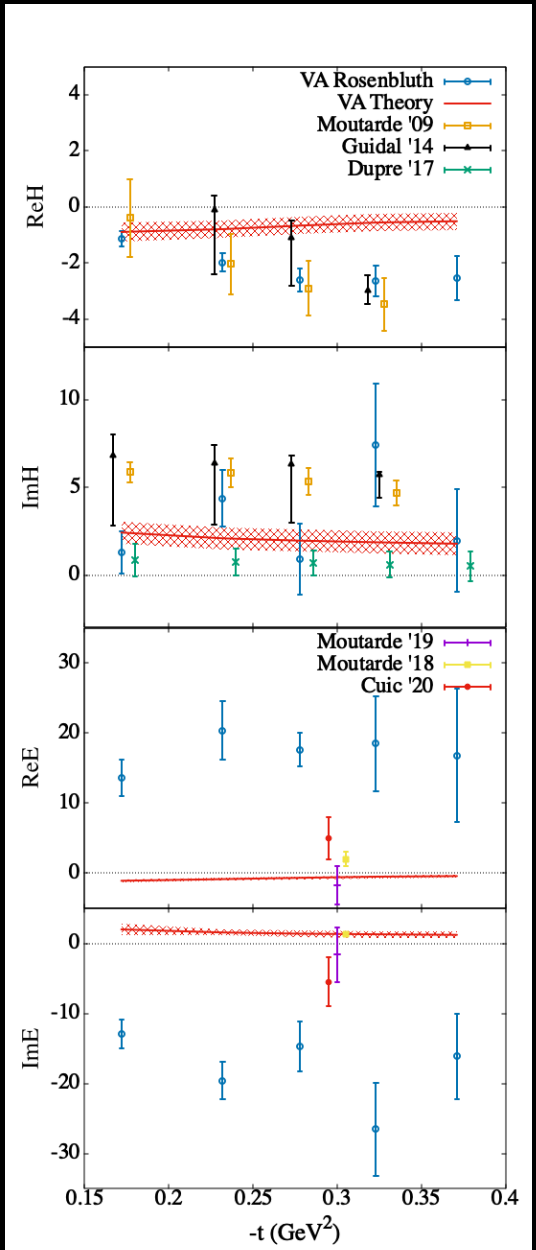
- Rosenbluth Separated BH-DVCS interference data





Compton Form Factor Extraction





Q^2 dependence

Twist 3 BH-DVCS interference

$$F_{UU}^{\mathcal{I}} = F_{UU}^{\mathcal{I},tw2} + \frac{K}{\sqrt{Q^2}} F_{UU}^{\mathcal{I},tw3}$$

$$F_{UU}^{\mathcal{I},tw3} = A_{UU}^{(3)\mathcal{I}} \left[F_1 \left(\Re e(2\widetilde{\mathcal{H}}_{2T} + \mathcal{E}_{2T}) - \Re e(2\widetilde{\mathcal{H}}'_{2T} + \mathcal{E}'_{2T}) \right) + F_2 \left(\Re e(\mathcal{H}_{2T} + \tau \widetilde{\mathcal{H}}_{2T}) - \Re e(\mathcal{H}'_{2T} + \tau \widetilde{\mathcal{H}}'_{2T}) \right) \right]$$

$$+ B_{UU}^{(3)\mathcal{I}} G_M \left(\Re e \widetilde{\mathcal{E}}_{2T} - \Re e \widetilde{\mathcal{E}}'_{2T} \right)$$

Orbital Angular Momentum

$$+ C_{UU}^{(3)\mathcal{I}} G_M \left[2\xi (\Re e \mathcal{H}_{2T} - \Re e \mathcal{H}'_{2T}) - \tau \left(\Re e(\widetilde{\mathcal{E}}_{2T} - \xi \mathcal{E}_{2T}) - \Re e \left(\widetilde{\mathcal{E}}'_{2T} - \xi \mathcal{E}'_{2T} \right) \right) \right]$$

Twist 3 GPDs Physical Interpretation

GPD	$P_q P_p$	TMD	Ref. 1
$H^{\perp}$	UU	$f^{\perp}$	$2\widetilde{H}_{2T} + E_{2T}$
$\widetilde{H}_L^{\perp}$	LL	$g_L^{\perp}$	$2\widetilde{H}'_{2T} + E'_{2T}$
$H_L^{\perp}$	UL	$f_L^{\perp\,(*)}$	$\widetilde{E}_{2T} - \xi E_{2T}$
$\widetilde{H}^{\perp}$	LU	$g^{\perp\,(*)}$	$\widetilde{E}'_{2T} - \xi E'_{2T}$
$H_T^{(3)}$	UT	$f_T^{(*)}$	$H_{2T} + \tau \widetilde{H}_{2T}$
$\widetilde{H}_T^{(3)}$	LT	g'_T	$H'_{2T} + \tau \widetilde{H}'_{2T}$

→ 1/Q correction to H

→ 1/Q correction to $\widetilde{\mathrm{H}}$

NEW!! Orbital Angular Momentum L

NEW!! Spin Orbit correlation $\mathrm{L} \cdot \mathrm{S}$

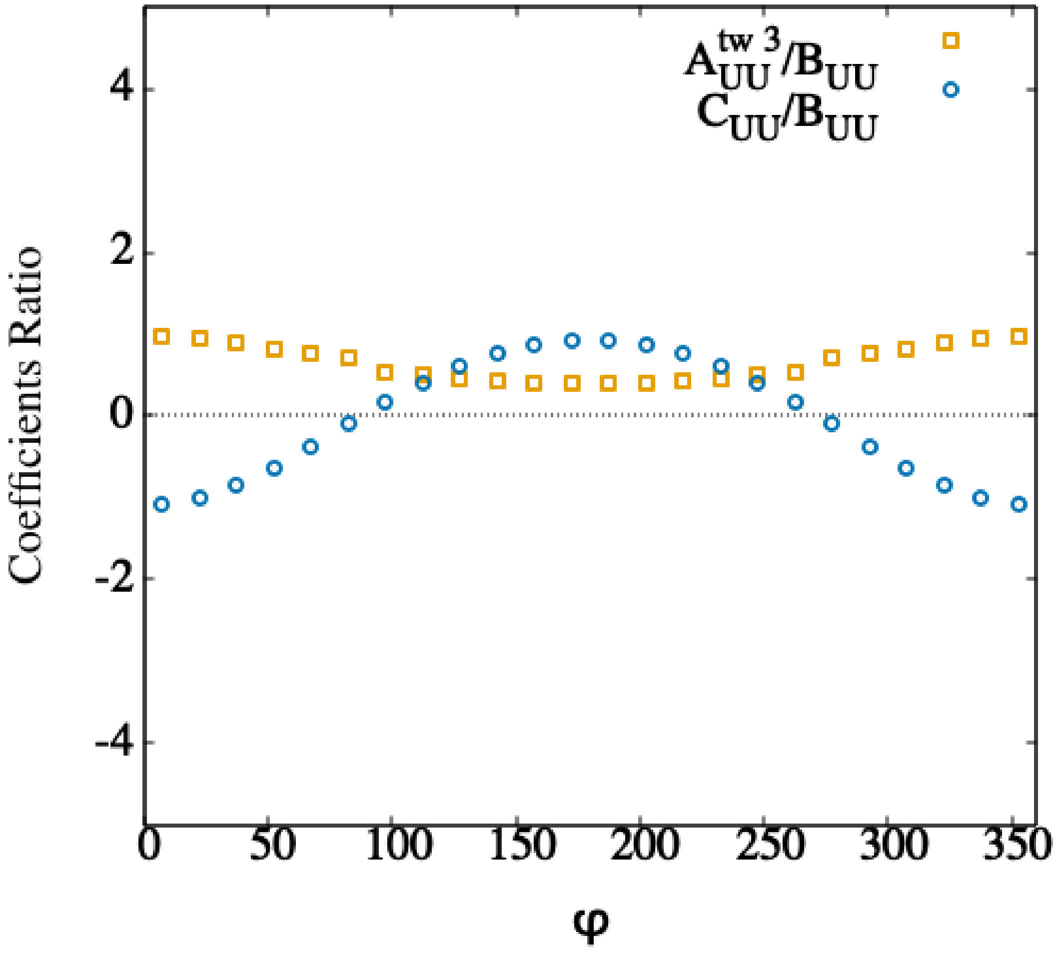
→ 1/Q correction to E

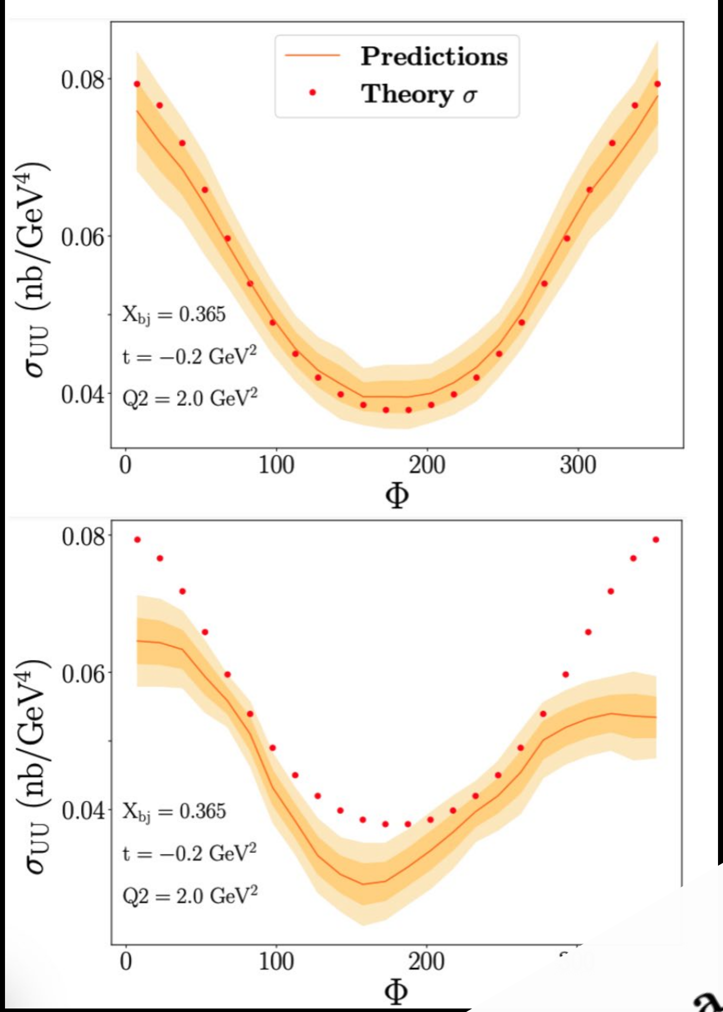
→ 1/Q correction to $\widetilde{\mathrm{E}}$

(*) T-odd

[1] Meissner, Metz and Schlegel, JHEP(2009)

Twist 3 seems small



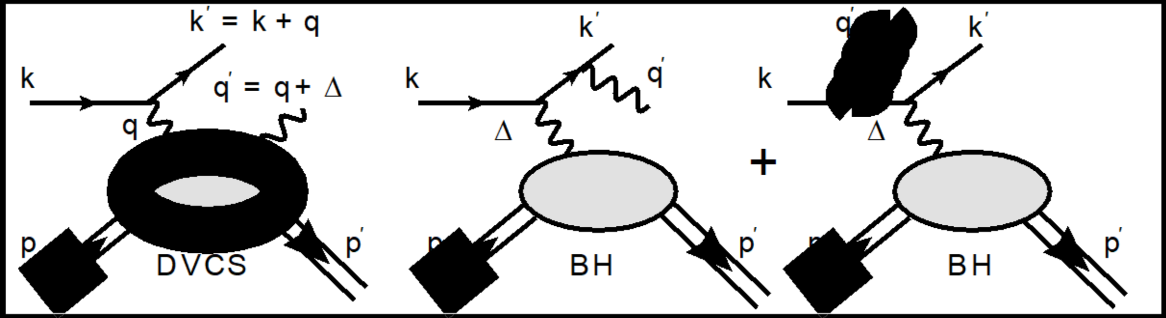


Benchmarks for a Global Extraction of Information from Deeply Virtual Exclusive Scattering Experiments

Manal Almaeen,[1,*] Jake Grigsby,[2,†] Joshua Hoskins,[3,‡] Brandon Kriesten,[4,§] Yaohang Li,[1,¶] Huey-Wen Lin,[5,6,**] and Simonetta Liuti[3,††]

… to appear soon…

We need a robust framework for DVES processes cross section, where kinematic limits are under control



- To observe, evaluate and interpret GPDs and Wigner distributions at the subatomic level requires stepping up data analyses from the standard methods and developing new numerical/analytic/quantum computing methods

DVCS formalism

- B. Kriesten et al, *Phys.Rev. D* 101 (2020)
- B. Kriesten and S. Liuti, *Phys.Rev. D105 (2022),* arXiv 2004.08890
- B. Kriesten and S. Liuti, Phys. Lett. B829 (2022), arXiv:2011.04484

ML

- J. Grigsby, B. Kriesten, J. Hoskins, S. Liuti, P. Alonzi and M. Burkardt, Phys. Rev. D104 (2021)

GPD Parametrization for global analysis

- B. Kriesten, P. Velie, E. Yeats, F. Y. Lopez and S. Liuti,

Phys. Rev D 105 (2022), arXiv:2101.01826