LFWFs from large momentum effective theory (LaMET)

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The talk is based on the recent theoretical work in :

Computing Light-Front Wave Functions Without Light-Front Quantization: A Large-Momentum Effective Theory Approach, Xiangdong Ji, Yizhuang Liu (Phys. Rev. D 105 (2022), reprint 2106.05310)

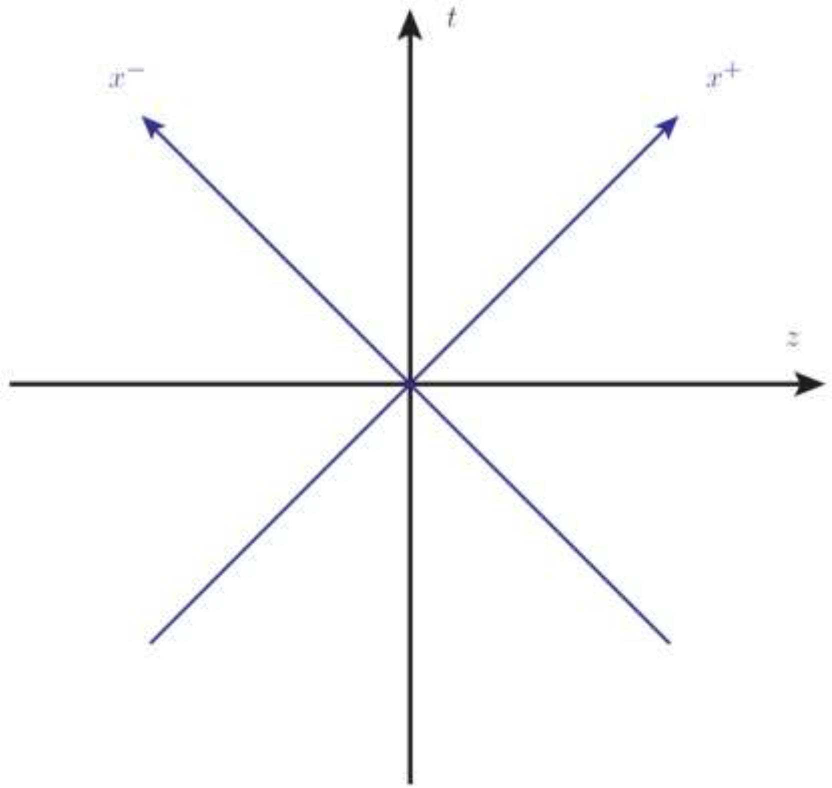
For a review of LaMET, see the review article

Large-momentum effective theory, Rev. Mod. Phys. 93 (2021)

Outline

- Introduction to QFT on the Light-Front
- Conceptual difficulties of LF formulation of QFT
- LFWF amplitudes without LF quantization
- LaMET formulation of LFWF amplitudes
- Example: leading LFWF of pseudo-scalar mesons.

Introduction to QFT on the Light-Front



- Basic information on QFT on LF .

1. Equal time QFT: time-like time t, space like space z.
2. QFT on the LF: light-like time $x^+ = \frac{1}{\sqrt{2}}(t+z)$ and light-like space $x^- = \frac{1}{\sqrt{2}}(t-z)$.
3. Discovered by Dirac as early as 1940s*.

*P. A. M. Dirac, Rev. Mod. Phys. 21, 392 (1949)

Introduction to QFT on the Light-Front

High energy limit of QFT→ QFT on the LF!

1. Large rapidity gap sources the light-cone structure.
2. Natural realization of the IMF for Feynman's parton model.
3. Formal infinite momentum limit in old-fashioned perturbation theory. Simplification occurs: vanishing of backward-moving diagrams.
4. Factorization conjecture: PDFs, DAs are naturally expressed as LF correlators.

S. J. Brodsky, H.-C. Pauli, and S. S. Pinsky, Phys. Rept. 301, 299 (1998)

Introduction to QFT on the Light-Front

- Many body Hamiltonian for LF quantization:

1. $P^-|P^+\rangle = \frac{m^2}{2P^+}|P^+\rangle$: P^- is LF Hamiltonian and P^+ is LF momentum.
2. $P^- \equiv H_{LF} = H_{LF}^0 + V$, expressed in terms of LF free-fields.
3. k^+ is supported in $[0, \infty]$ → vanishing of vacuum diagrams→ formal equality between interacting and free vacuum $|0_{int}\rangle = |0_{free}\rangle$.
4. Partons ($k^+ > 0$) well separated from zero modes ($k^+ = 0$).

S. J. Brodsky, H.-C. Pauli, and S. S. Pinsky, Phys. Rept. 301, 299 (1998)

Introduction to QFT on the Light-Front

- Expansion in the free Fock-basis:
  $|P^+\rangle = \sum_{N=0}^{\infty} \int d\Gamma_N \ \psi_N(k_i^+ = x_i P^+, k_{i\perp}) \prod a^\dagger(k_i^+, k_{i\perp})|0\rangle.$

1. The $\psi_N(x_i, k_{i\perp})$ is called LFWF amplitudes.
2. They can be expressed as matrix elements
   $\psi_N(k_i^+ = x_i P^+, k_{i\perp}) \sim \langle 0|\prod a(k_i^+, k_{i\perp})|P^+\rangle.$
3. Formal normalizability: $\sum_N \int d\Gamma_N |\psi_N|^2 = 1.$
4. LF gauge $A^+ = 0$ for gauge theory.

S. J. Brodsky, H.-C. Pauli, and S. S. Pinsky, Phys. Rept. 301, 299 (1998)

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Conceptual difficulties of LF formulation of QFT

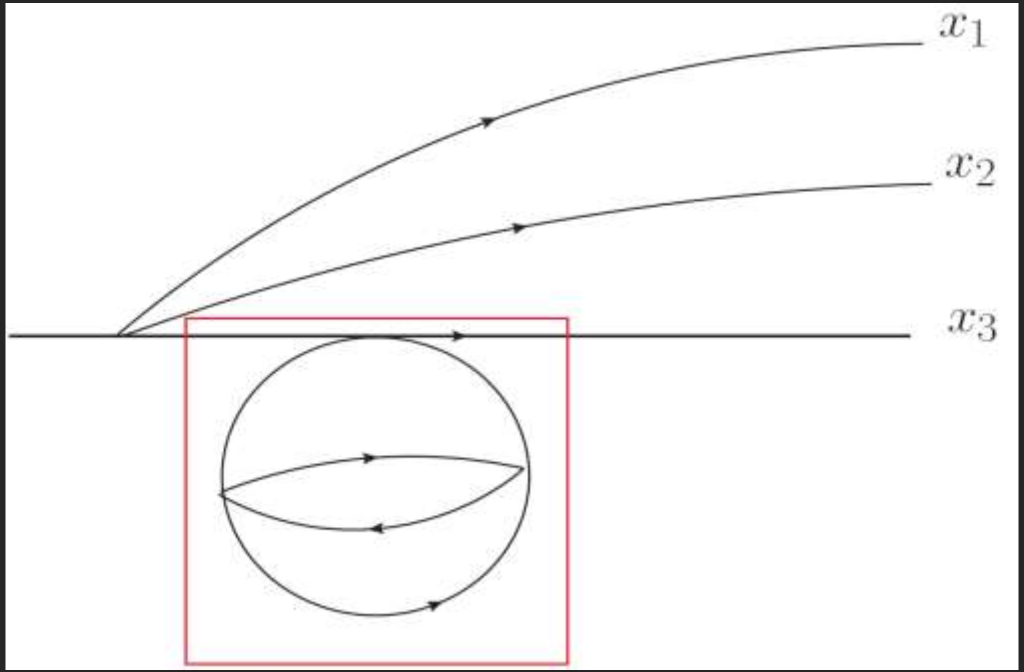
Non-Euclidean high energy limit of QFT is already not easy

1. Above threshold, the formidable sign problem.
2. Below threshold, rapidity logs are hard (space-like sudakov form factor). Lack of systematic multi-scale analysis in rapidity space.
3. Below threshold without rapidity logs ($F(x_B, \frac{Q^2}{m^2})$ when $x_B > 1$). $\ln \frac{Q^2}{m^2}$ is easy but $\ln \frac{Q^2}{\mu^2} + \ln \frac{\mu^2}{m^2}$ is hard.

LF QFT in Hamiltonian formalism can only be more challenging.

Conceptual difficulties of LF formulation of QFT

1. The zero-mode problem: $k^+ = 0$ modes fail to decouple completely. Vacuum is not trivial at all !
2. Exists even in 2-D scalar field theory. Partially solved in Discrete LF quantization for 2-D models**.



- The part in the red blob can not be calculated in naïve implementation of LFPT*.
- Corresponds to vacuum condensate $\langle 0|\phi^2|0\rangle$.

*J. Collins, (2018), arXiv:1801.03960
**H. C. Pauli and S. J. Brodsky, Phys. Rev. D32, 2001(1985)

Conceptual difficulties of LF formulation of QFT

3. Besides the zero-mode problem, there are additional LF divergences of LF PT when $k^+ \to 0$. Caused by phase space measure + polarization enhancement+ instantaneous diagram.

4. In 4D gauge theory, the divergence can be more than logarithmic and splits over many different diagrams. No simple pattern.

5. Very hard to regulate in a way consistent with gauge invariance and Lorentz invariance. No consistent regulator known up to now.

6. UV divergence usually become more complicate. Example: the benign 2D massive Gross-Neveu.

Xiangdong Ji, Yizhuang Liu (2106.05310)

Conceptual difficulties of LF formulation of QFT

- Why such difficulties ? Because QFT on LF is an effective theory in the non-trivial large rapidity (large momentum/LF) limit.
- More transcendental than $\xi = \frac{1}{m} \to \infty$. LF QFT ≠ Euclidean QFT.
- Constructing LF QFT from Euclidean QFT:

1. Rapidity renormalization: remove small k^+ contribution causing LF divergence.
2. UV matching: hard scale induced by the large rapidity limit.

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LFWF amplitudes without LF quantization

- LFWF amplitudes as LF correlation functions.

$$\psi_N^{\pm}(x_i, b_{i\perp}, \mu) = \int \prod d\lambda_i e^{i\lambda_i x_i} \langle 0|P_N \prod \Phi_i^{\pm}(\lambda_i n + b_{i\perp})|P\rangle$$

1. $\Phi_i^{\pm}(\xi) = Pexp\left[-i\int_0^{\pm\infty} d\lambda\, n\cdot A(\lambda n + \xi)\right]\phi_i(\xi)$. Gauge invariant fields with light-like gauge links attached.
2. Equivalent to $A^+ = 0$ gauge without gauge links.
3. Transverse gauge link at infinity required in singular gauges.

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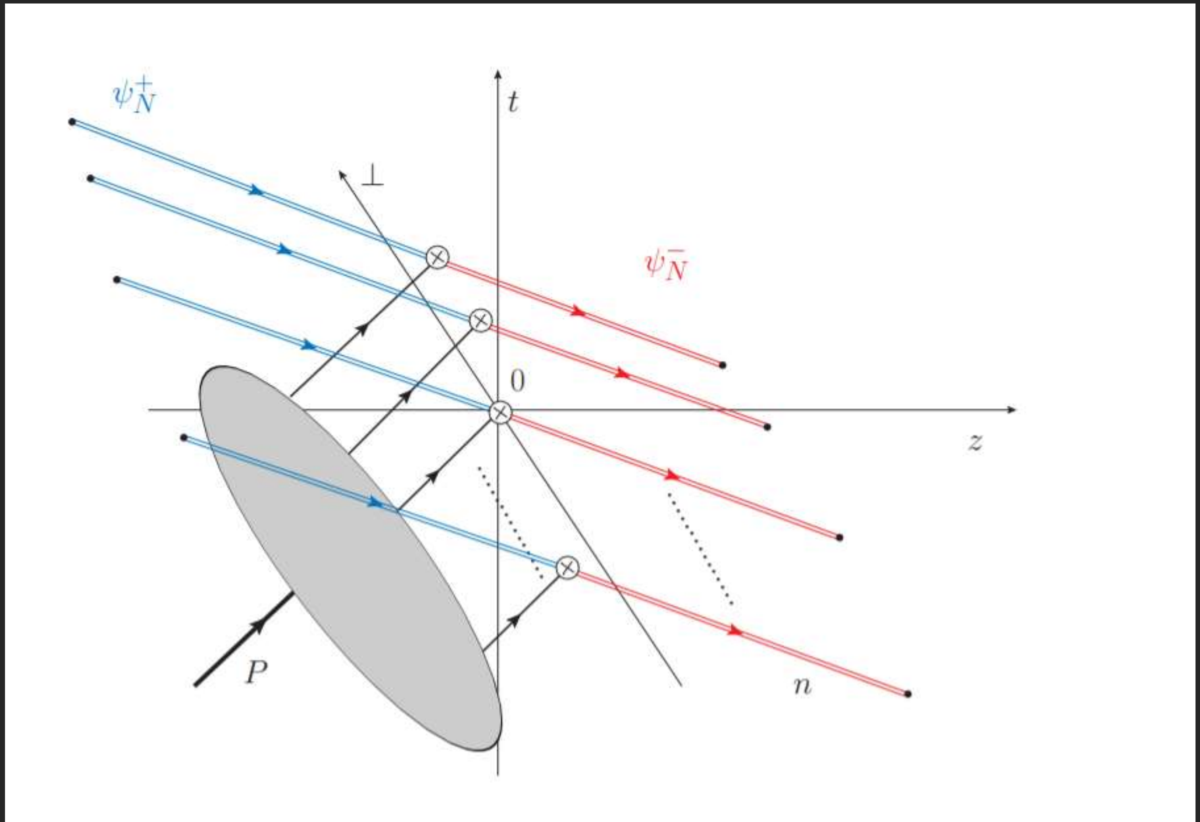
LFWF without LF quantization



Rapidity divergences of the form $\int \frac{dk^+}{k^+}$. Non-cancelling LF divergence.

The LFWFs ψ_N^+ (blue) and ψ_N^- (red). The gauge links disappear in $A^+ = 0$ gauge.

LFWF without LF quantization

1: Nucl. Phys. B193, 381 (1981), J.C. Collins and D
2: Phys. Rev. D 71, 034005(2005), X.D Ji, J.P. Ma,

- Rapidity divergences for infinitely long gauge links. Regulator needed.
- Off light-cone (1,2) : $n \to n \pm e^{-2Y} p$. Gauge link tilted away from LF. Approaches LF as $Y \to \infty$.
- On light-cone : Gauge link along light-cone.

1. The *delta-regulator* (a) : $e^{i\int dx^- A^+(x^-)} \to e^{i\int dx^- A^+(x^-) e^{-\delta^-|x^-|}}$.
2. The *LF length regulator* (b) : $e^{i\int dx^- A^+(x^-)} \to e^{i\int_0^{L^-} dx^- A^+(x^-)}$.
3. The *exponential regulator* (c) *and many others*. Defined through final-state cuts, not applicable for LFWFs.

a: Phys. Rev. D 93, 011502 (2016). M.G. Echevarria, I.Scimemi, and A. Vladimirov
b: Phys. Rev. Lett. 125, 192002 (2020) , A. Vladimirov
c: Nucl. Phys. B 960, 115193 (2020) , Y.Li, D.Neill, and H.X.Zhu

LFWF without LF quantization

- Rapidity divergence factorizes*: $\exp[K_N(b_{i\perp},\mu)\ln\frac{{p^+}^2}{\mp\delta^+\delta^- - i0}]$.

1. K_N: generalized Collins-Soper kernels or rapidity anomalous dimensions. Known to order α_s^2 for $N = 2$(double-parton)**.
2. Constant part is scheme dependent . Must be renormalized in a scheme independent way.
3. For $N = 1$, reduces to the case of TMDPDFs***.

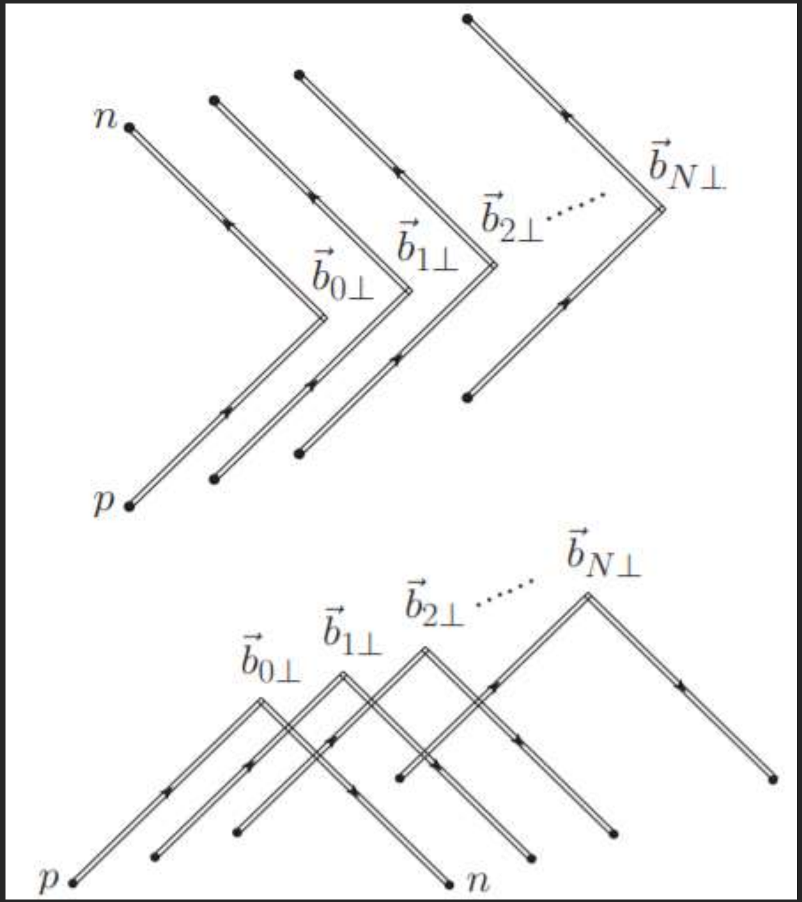
*A. Vladimirov, JHEP 04, 045 (2018)
**A. Vladimirov, JHEP 12, 038 (2016)
***J. C. Collins and D. E. Soper, Nucl. Phys. B193 (1981)

LFWF without LF quantization

A. Vladimirov, JHEP 12, 038 (2016)
Xiangdong Ji, Yizhuang Liu (2106.05310)

- One needs generalized soft functions to renormalize the rapidity divergence.

1. $S_N^{\pm}(b_{i\perp}, \mu, \delta^+, \delta^-) = \langle 0|\mathcal{T}\prod \mathcal{C}^{\pm}(b_{i\perp}, \delta^+, \delta^+)|0\rangle$ defined with $N+1$ Wilson-line cusps.
2. See the figure for the Shape of $\mathcal{C}^+$(upper) and $\mathcal{C}^-$ (lower).



LFWF without LF quantization

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- With the help of generalized soft function $S_N^{\pm}$, one can define the rapidity renormalized LFWF amplitudes.

$$\psi_N^{\pm}(x_i, b_{i\perp}, \mu, \zeta_i) = \lim_{\delta^- \to 0} \frac{\psi_N^{\pm}(x_i, b_{i\perp}, \mu, \delta^-)}{\sqrt{S_N^{\pm}(b_{i\perp}, \mu, \delta^- e^{2y_n}, \delta^-)}}.$$

1. Rapidity divergence cancels. Scheme independent.
2. The rapidity scales $\zeta_i = 2x_i^2 {P^+}^2 e^{2y_n}$ as a result of rapidity renormalization.
3. Rapidity evolution in ζ_i is controlled by K_N again.

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LaMET formulation of LFWFs

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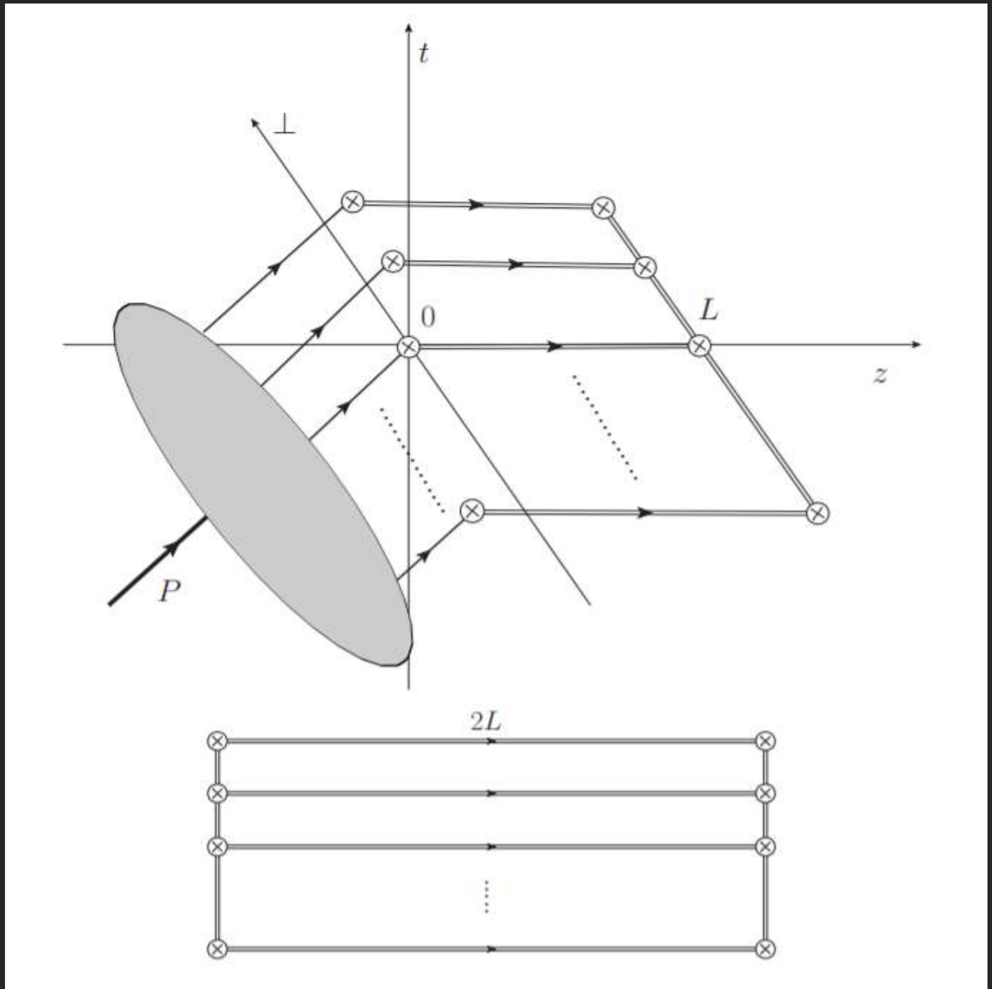
(2106.05310)

- Similar to the TMDPDFs, we define the quasi-LFWF amplitudes.

$$\widetilde{\psi}_N^{\pm}(x_i, b_{i\perp}, \mu, \zeta_{z,i}) = \lim_{L\to\infty} \frac{\int \prod d\lambda_i e^{-i\lambda_i x_i} \langle 0|P_N \prod \Phi_i^{\pm}(\lambda_i n_z + b_{i\perp}; L)|P\rangle}{\sqrt{Z_E(2L, b_{i\perp}, \mu)}}.$$

1. $\Phi_i^{\pm}(\xi; L) = \mathcal{P} \exp\left[ig \int_0^{\mp L - \zeta^z} A^z(\xi + \lambda n_z) d\lambda\right] \phi_i(\xi)$ with gauge link along space –like direction $n_z = (0,0,0,1)$.
2. Z_E: expectation value of $N+1$ Wilson-lines. Removes large L divergence.
3. Rapidity scales $\zeta_{z,i} = 4x_i^2 P_z^2$ generated.

LaMET formulation of LFWFs



The quasi-LFWF amplitude $\widetilde{\psi}_N^-$ (upper) and Z_E (lower).

Rapidity evolution of quasi-LFWFs

- One can show ** that $\widetilde{\psi_N^{\pm}}(x_i, b_{i\perp}, \mu, \zeta_{z,i})$ satisfies the rapidity evolution equation:
$$\frac{d}{d \ln P^z} \ln \widetilde{\psi_N^{\pm}}\left(x_i, b_{i\perp}, \mu, \zeta_{z,i}\right) = K_N(b_{i\perp}, \mu) + \sum_i G(\zeta_{z,i}, \mu) \,.$$

1. Non-perturbative part: Collins-Soper kernel $K_N(b_{i\perp}, \mu)$.
2. Perturbative part: $G(\zeta_{z,i}, \mu)$. $K_N + \sum_i G$ is RG invariant.
3. Collins-Soper kernel can be extracted from ratios.***

**Ji, Sun, Xiong, and Yuan, arXiv:1405.7640.
***Ebert, Stewart, and Zhao, arXiv:1811.00026.
***Shanahan,Wagman, and Zhao,arXiv:2003.0606.

Factorization of quasi-LFWF amplitudes

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- At large P^z, the quasi-LFWF factorizes

$$\tilde{\psi}_N^{\pm}(x_i, b_{i\perp}, \mu, \zeta_{z,i})\sqrt{S_{rN}\,(b_{i\perp}, \mu)} = e^{\ln\frac{\mp\zeta_{z,i}-i0}{\zeta,i}K_N(b_{i\perp},\mu)}\, H_N^{\pm}\left(\frac{\zeta_{z,i}}{\mu^2}\right)\psi_N^{\pm}(x_i, b_{i\perp}, \mu, \zeta_i)$$

A. The $e^{\ln\frac{\mp\zeta_{z,i}-i0}{\zeta,i}K_N(b_{i\perp},\mu)}$ is due to K_N term of rapidity evolution of $\tilde{\psi}_N$.

B. The perturbative kernel $H_N^{\pm}\left(\frac{\zeta_{z,i}}{\mu^2}\right)$ is due to G term of rapidity evolution .

C. The $\psi_N^{\pm}$ is the targeting LFWFs.

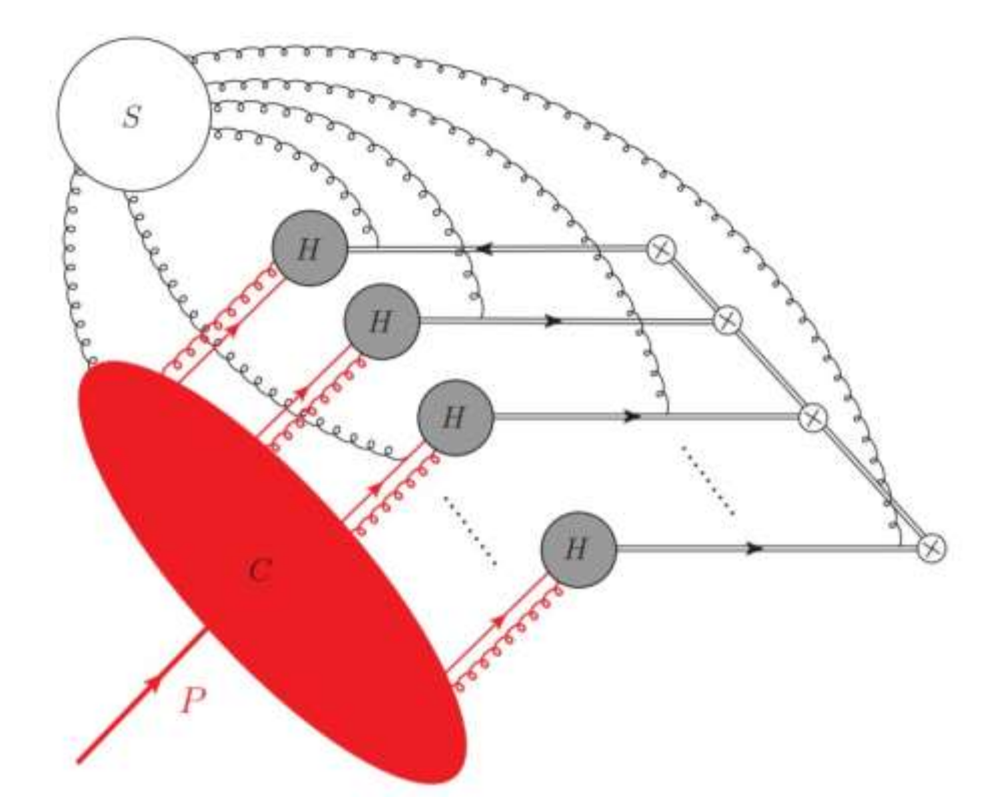
D. The factor $S_{rN}\,(b_{i\perp}, \mu)$ is the generalized reduced soft function.

Factorization of quasi-LFWF amplitudes

- The reduced diagram.

1. Hard (H), collinear (C) and soft (S) sub-diagrams.
2. Hard cores are disconnected with each-other: no convolution.

- Can also be derived in soft-collinear effective theory.



The reduced soft function

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- The reduced soft function can be defined in two ways.

1. In on-light-cone schemes, it is defined as a ratio
$$S_{rN}(b_{i\perp},\mu) = \lim_{\delta^+,\delta^-\to 0} \frac{S_N^-(b_{i,\perp},\mu,\delta^+,\delta^-)}{S_N^-(b_{i\perp},\mu,\delta^+,n_z)S_N^-(b_{i\perp},\mu,\delta^-,n_z)}$$
2. In off-light-cone scheme, generalized soft function at large rapidities
$$S_N^{\pm}(b_{i\perp},\mu,Y,Y') = \exp\left[K_N(b_{i\perp},\mu)\ln[\mp e^{Y+Y'} - i0] + \mathcal{D}_N(b_\perp,\mu)\right].$$
3. $S_{rN}(b_{i\perp},\mu) = \exp\left[-\mathcal{D}_N(b_\perp,\mu)\right]$ is the rapidity independent part.
4. Can be simulated through form-factors.

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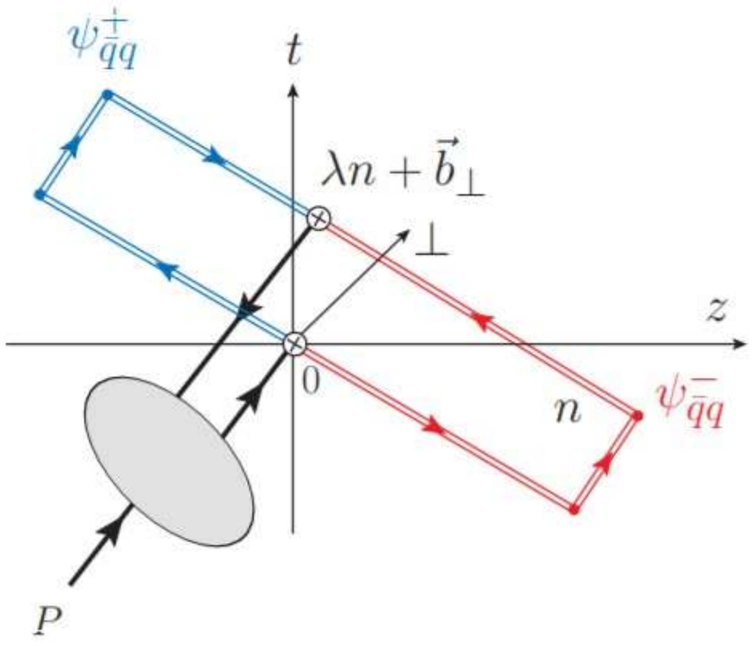
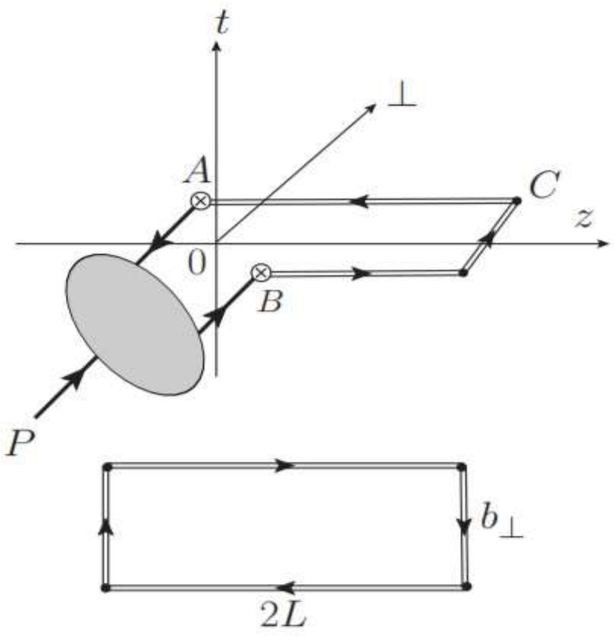
Example: leading LFWF for a pseudo-scalar meson

- The LFWF is defined with the same operator as TMDPDFs.
- The quasi-LFWF is defined with the same operators as quasi-TMDPDFs.
- Same anomalous dimensions, rapidity evolutions as TMDPDFs.



LFWF (left).

Quasi LFWF and Z_E (right).



Example: leading LFWF for a pseudo-scalar meson

- The matching formula reads

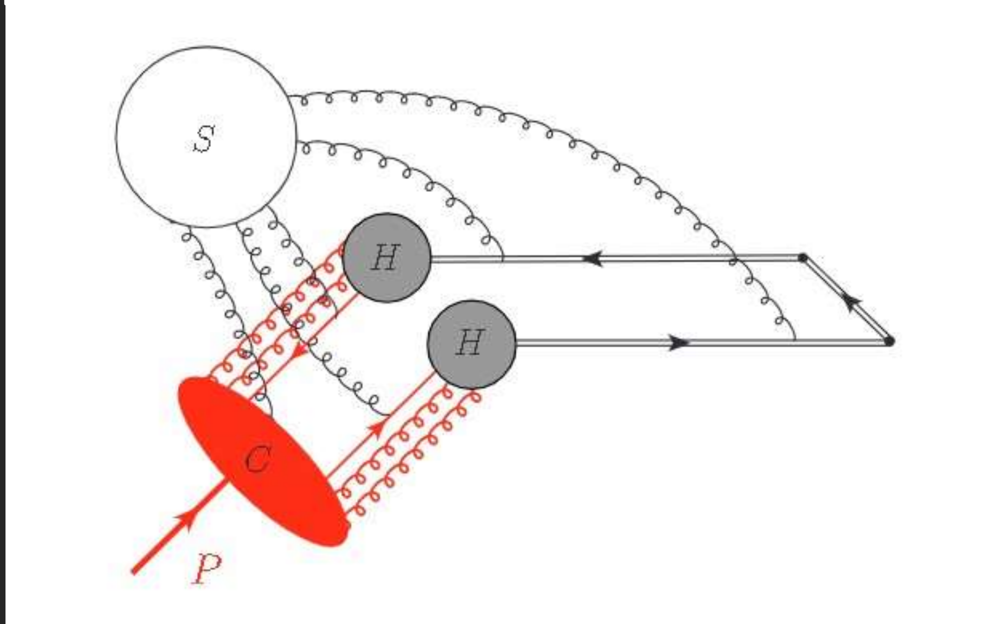
$$\tilde{\psi}^{\pm}_{\bar{q}q}(x, b_\perp, \mu, \zeta_{z,i})\sqrt{S_{r1}(b_\perp, \mu)} = e^{\ln\frac{\mp\zeta_{z,i}-i0}{\zeta,i}K_1(b_\perp,\mu)} \prod H_1^{\pm}\left(\frac{\zeta_{z,i}}{\mu^2}\right)\psi^{\pm}_{\bar{q}q}(x, b_\perp, \mu, \zeta_i)$$

1. Two rapidity scales $\zeta_z = 4x^2(P^z)^2$ and $\bar{\zeta}_z = 4\bar{x}^2(P^z)^2$. $x + \bar{x} = 1$.
2. The hard kernel relates to TMDPDF cases through: $H_{TMD}(\frac{\zeta_z}{\mu^2}) = \left|H_1^{\pm}\left(\frac{\zeta_z}{\mu^2}\right)\right|^2$.
3. The $S_{r1}(b_\perp, \mu)$ the same as the case of TMDPDFs.
4. Imaginary part non-vanishing. Required by analyticity. Scale-invariant at 1-loop.

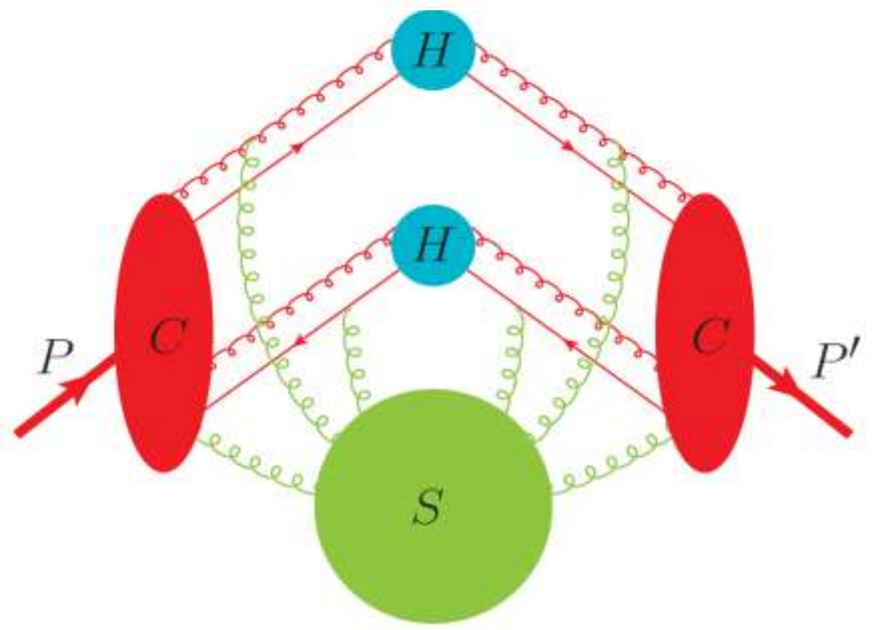
Example: leading LFWF for a pseudo-scalar meson

- The reduced soft function can be obtained through a space-like form-factor.
- Preliminary lattice results of S_{r1} exists.

The reduced diagram of the quasi LFWV $\widetilde{\psi_{\bar{q}q}}(x, b_\perp, \mu, P)$.



The reduced diagram of the form factor $\langle P' | \bar{\psi}\Gamma\psi(b_\perp)\bar{\psi}\Gamma\psi(0) | P \rangle$.



The light-meson formalism of S_{r1}

Ji, Y. Liu, and Y.-S. Liu, arXiv:1910.11415.
Ji, Liu, Liu, Zhang, and Zhao, arXiv:2004.035

A. Form factor allows form factors factorization into LFWFs.

B. Quasi LFWF factorize into LFWV and S_{r1}.

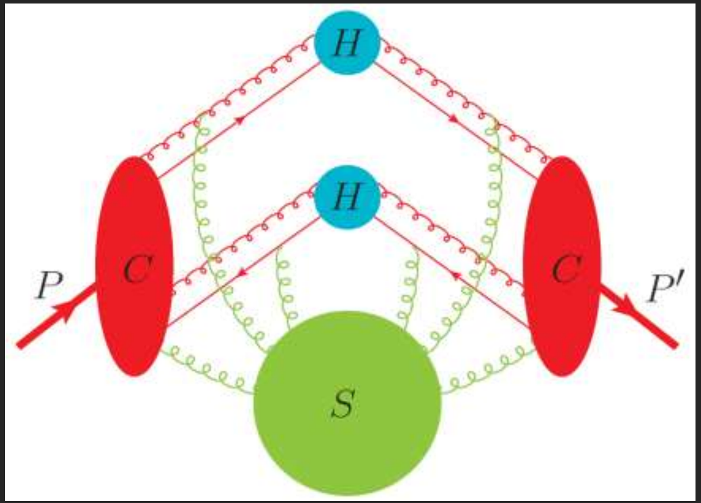
C. Therefore,

Lattice

$$S_r(b_\perp,\mu) = \frac{\langle P'|\bar{\psi}\Gamma\psi(b_\perp)\bar{\psi}\Gamma\psi(0)|P\rangle}{\int dx\, dx' H(x,x')\tilde{\psi}^\dagger(x',b_\perp,\mu,P')\tilde{\psi}(x,b_\perp,\mu,P)}.$$

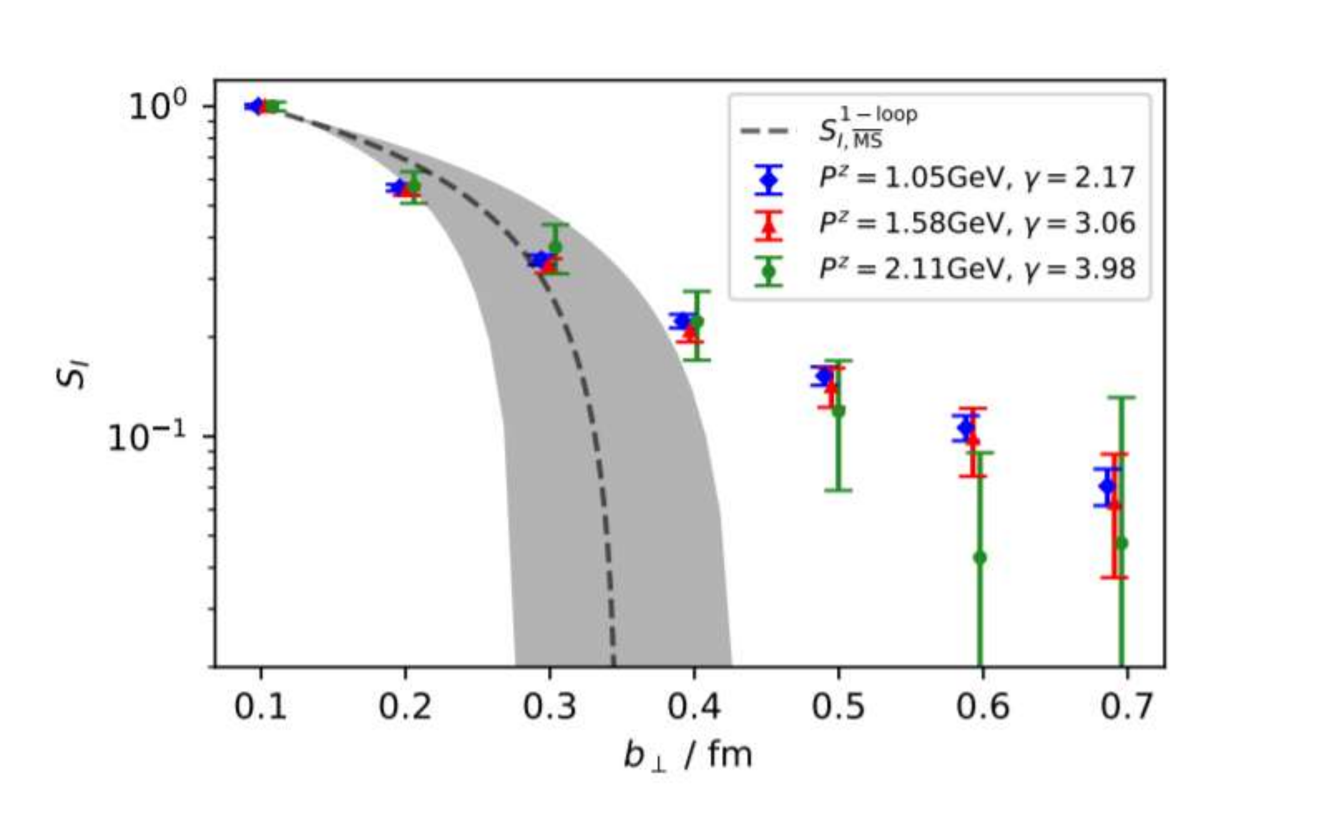
PQCD

The reduced diagram of the form factor $\langle P'|\bar{\psi}\Gamma\psi(b_\perp)\bar{\psi}\Gamma\psi(0)|P\rangle$.



The light-meson formalism of S_{r1}

Lattice Parton Collaboration, arXiv:2005.14572



β	$L^3 \times T$	a (fm)	c_{sw}	κ_l^{sea}	m_π^{sea}(MeV)
3.34	$24^3 \times 48$	0.098	2.06686	0.13675	333
			N_{cfg}	κ_l^v	m_π^v (MeV)
			864	0.13622	547

CLS A654

Preliminary results of S_r by LPC . Only tree-level matching.

The light-meson formalism of S_{r1} Yuan Li and others, arxiv:2106.13027

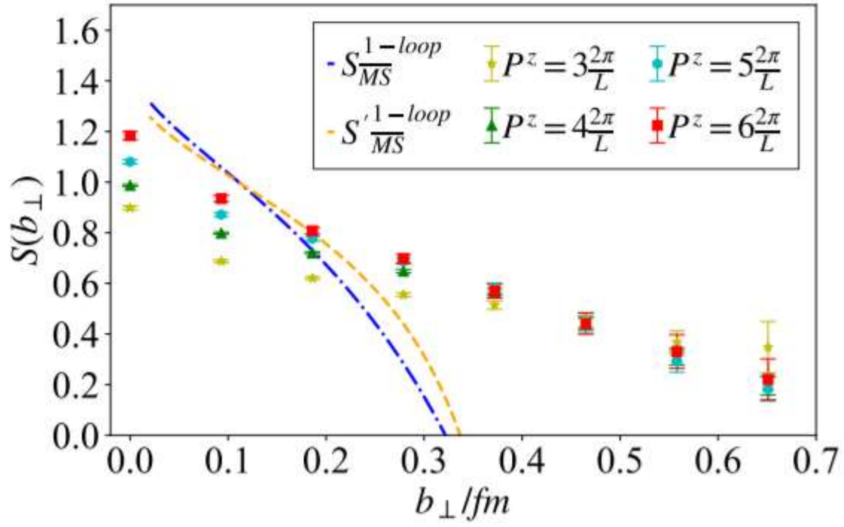


Figure 2. The lattice results of $S(b_\perp)$ for various momenta, together with the one-loop perturbative result $S^{\text{1-loop}}_{\overline{\text{MS}}}$ and its variant $S'^{\text{1-loop}}_{\overline{\text{MS}}}$ with α_s including up to 4 loops. The scale μ in Eq. (17) is set as $\mu = 2$ GeV.

$(L/a)^3 \times T/a$	a (fm)	$a\mu_{sea}$	m^π_{sea}	N_{conf}
$24^3 \times 48$	0.093	0.0053	350	126

$a\mu_{v0}$	m^π_{v0}	$a\mu_{v1}$	m^π_{v1}	$a\mu_{v2}$	m^π_{v2}	$a\mu_{v3}$	m^π_{v3}
0.0053	350	0.013	545	0.018	640	0.03	827

Table I. Parameters of the ensemble used in this work. We list the spatial and temporal extents, L/a and T/a, the lattice spacing a, the sea quark mass μ_{sea}, the pion mass m^π_{sea}, the number of configurations used, N_{conf}, and four valence quark masses μ_{vi} for $i = 0, 1, 2, 3$ together with the associated pion masses m^π_{vi}. All the pion masses are given in units of MeV.

ETMC configuration

Preliminary results of S_r . Only tree-level matching.

Summary and Outlook

- QFT on LF is an effective theory in the infinite rapidity limit.
- LFWF amplitudes can be defined as LF correlators without LF quantization.
- LaMET provides a natural Euclidean formulation of LFWF amplituds.
- Implementation of one-loop matching for lattice calculation.
- Better understanding of LF limit, scheme dependence.