# Factorized Approach to QED Radiations in PVDIS

Parity-Violation and other EW Physics at JLab 12 GeV and Beyond
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Tianbo Liu (刘天博)

*Key Laboratory of Particle Physics and Particle Irradiation (MOE)*
*Institute of Frontier and Interdisciplinary Science, Shandong University*

**In collaboration with: W. Melnitchouk, J.W. Qiu, N. Sato**

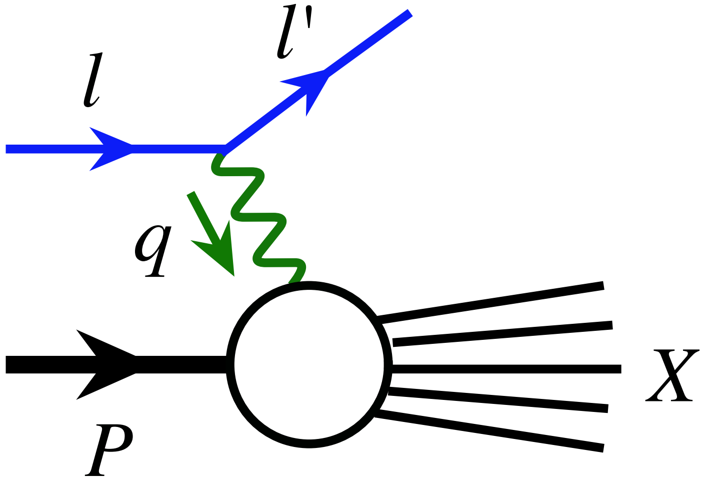
山东大学(青岛)
SHANDONG UNIVERSITY, QINGDAO

粒子物理与粒子辐照教育部重点实验室
Key Laboratory of Particle Physics and Particle Irradiation (MOE)
粒子科学技术研究中心
Research Center for Particle Science and Technology

# Lepton-Hadron Deep Inelastic Scattering

## Inclusive DIS at a large momentum transfer $Q \gg \Lambda_{\rm QCD}$

- dominated by the scattering of the lepton off an active quark/parton
- not sensitive to the dynamics at a hadronic scale ~ 1/fm
- collinear factorization: $\sigma \propto H(Q) \otimes \phi_{a/P}(x, \mu^2)$
- overall corrections suppressed by $1/Q^n$

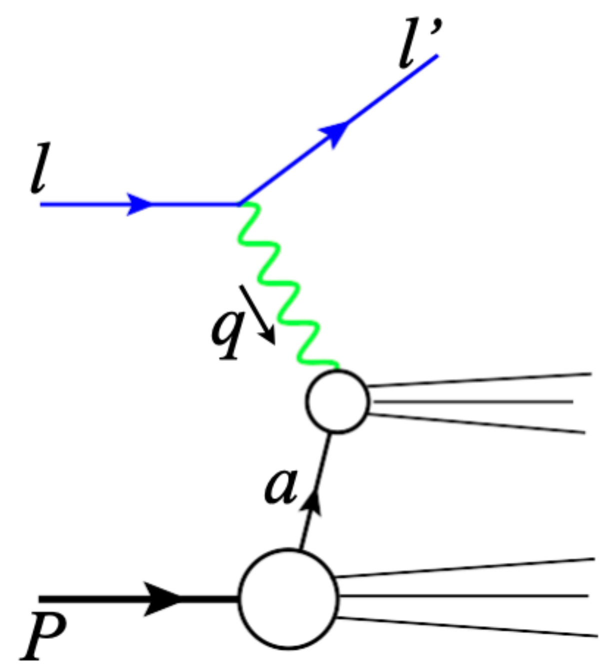



## QCD factorization

- provides the probe to "see" quarks, gluons and their dynamics indirectly
- predictive power relies on

— precision of the probe

— universality of $\phi_{a/P}(x, \mu^2)$



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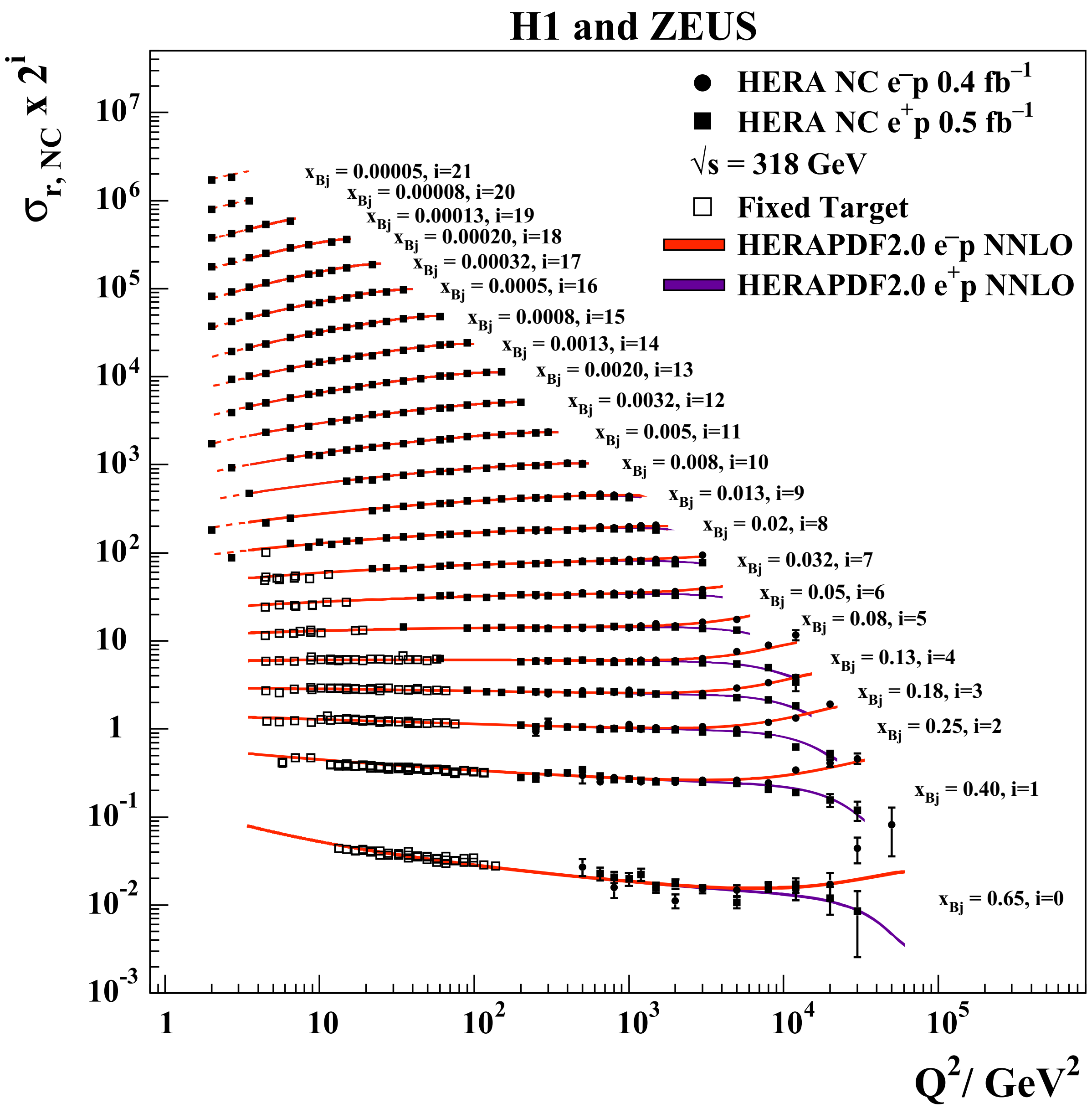
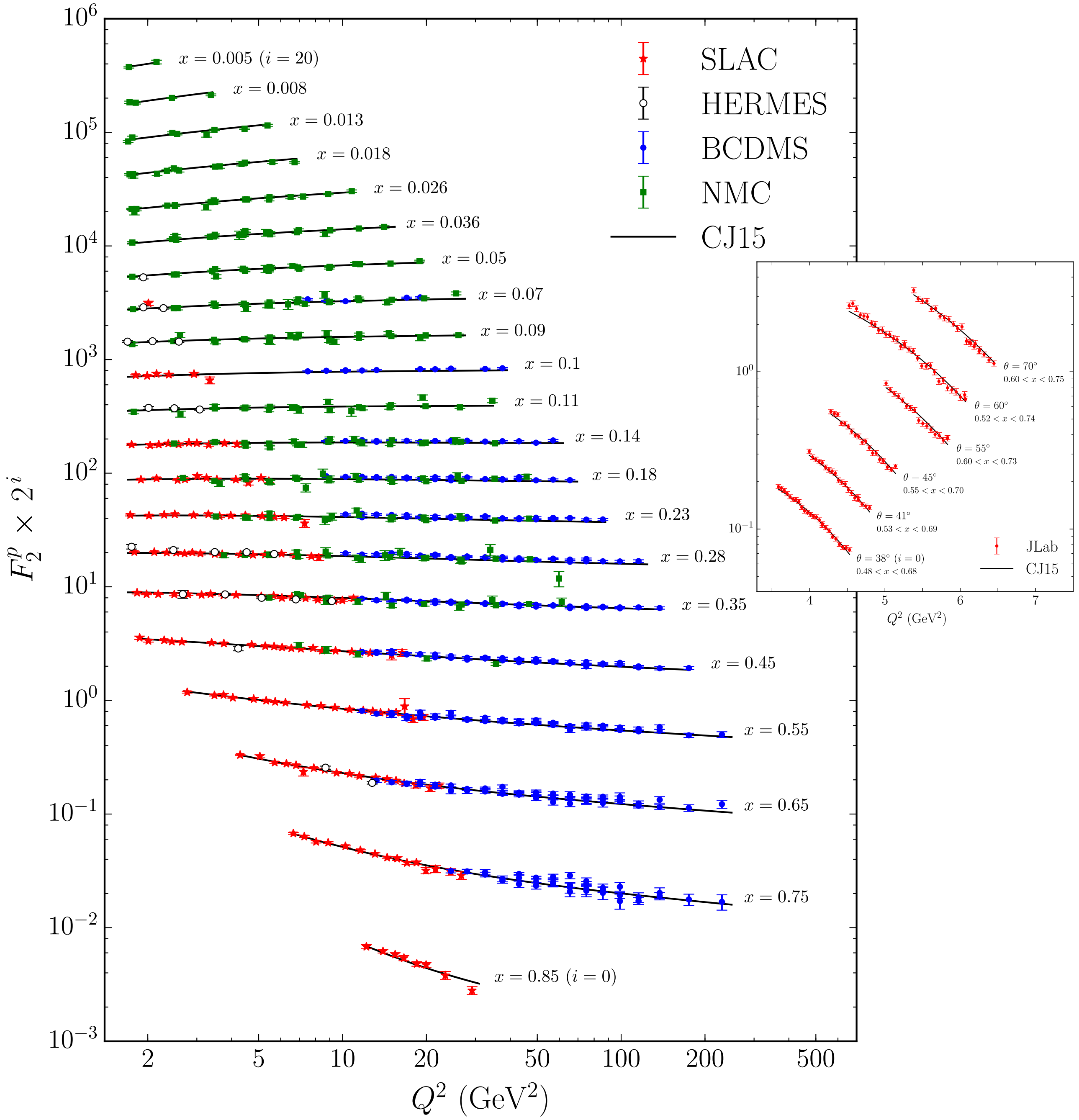
# Lepton-Hadron Deep Inelastic Scattering

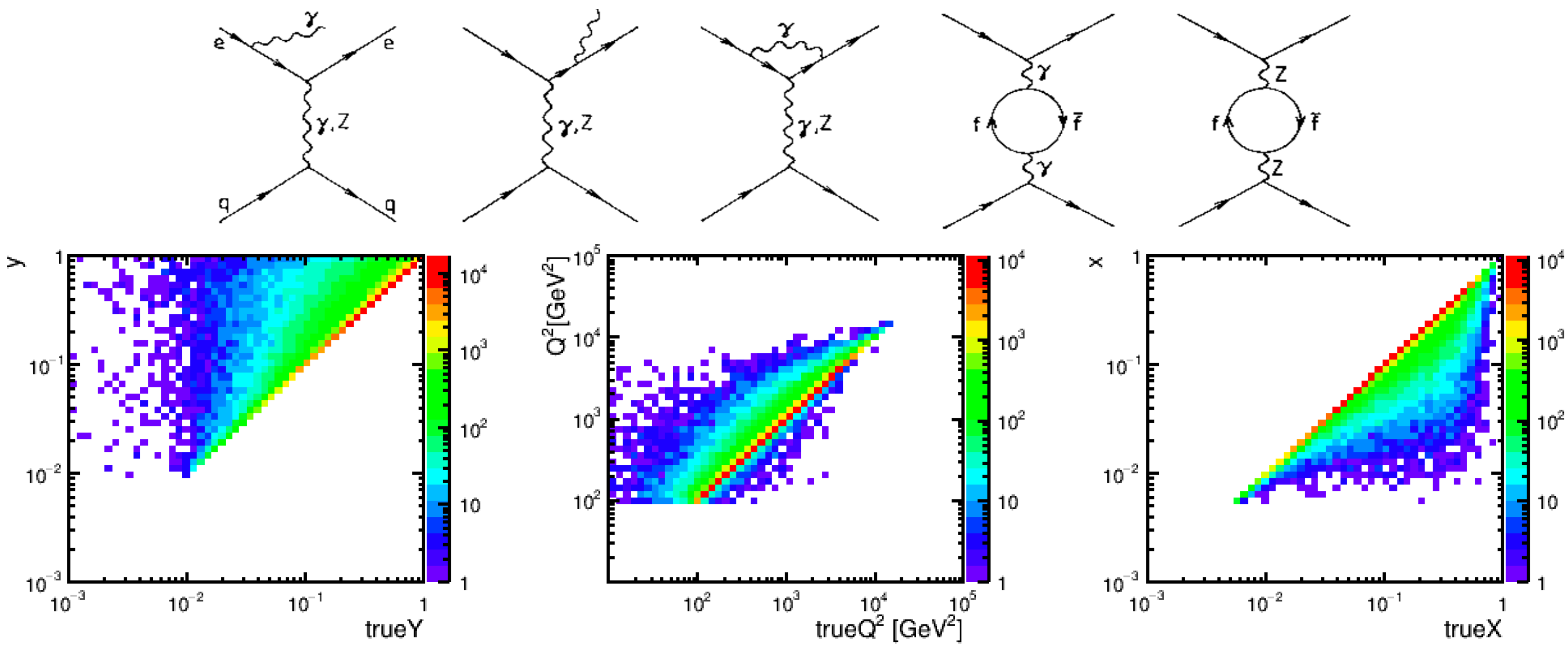



H. Abramowicz *et al.*, EPJC 78, 580 (2015).

A. Accardi *et al.*, PRD 93, 114017 (2016).

# Kinematics with Radiative Effects




[Figures from X. Chu at 2nd EIC YR workshop]

*Kinematic experience by the parton* $\neq$ *Kinematic reconstructed from observed momenta*

*QED radiation will have significant impact due to kinematic shift, although α is small.*

# Traditional Method to Handle QED Radiation

Radiative correction (RC) to Born kinematics:

$$\sigma_{\text{measured}} = \sigma_{\text{No QED radiation}} \otimes \eta_{\text{RC}}$$

RC factor

*"In many nuclear physics experiments, radiative corrections quickly become a dominant source of systematics. In fact, the uncertainty on the corrections might be the dominant source for high-statistics experiment"*

—— *EIC Yellow Report*

Problems or challenges:

The determination of RC factor relies on Monte Carlo simulation.

Usually depends on the physics we want to extract, hence introducing bias.

Also depends on experimental acceptance.

*increasingly difficult for reactions beyond inclusive DIS, e.g. SIDIS …*

Multidimensional kinematic shift, challenge to decouple 18 structure functions.

Almost impossible to determine the virtual photon event by event, and thus the *true photon-hadron frame*.

Problematic to define $P_{hT}$ and azimuthal angles, essential for TMD physics.

# Basic Ideas of Our Approach

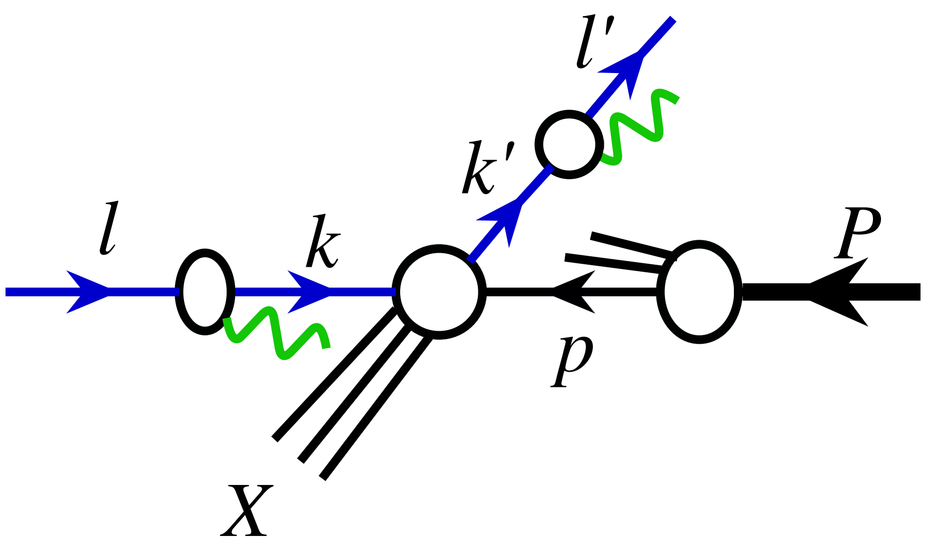
- Do not try to invent any scheme to treat QED radiation to match Born kinematics. — No radiative correction!

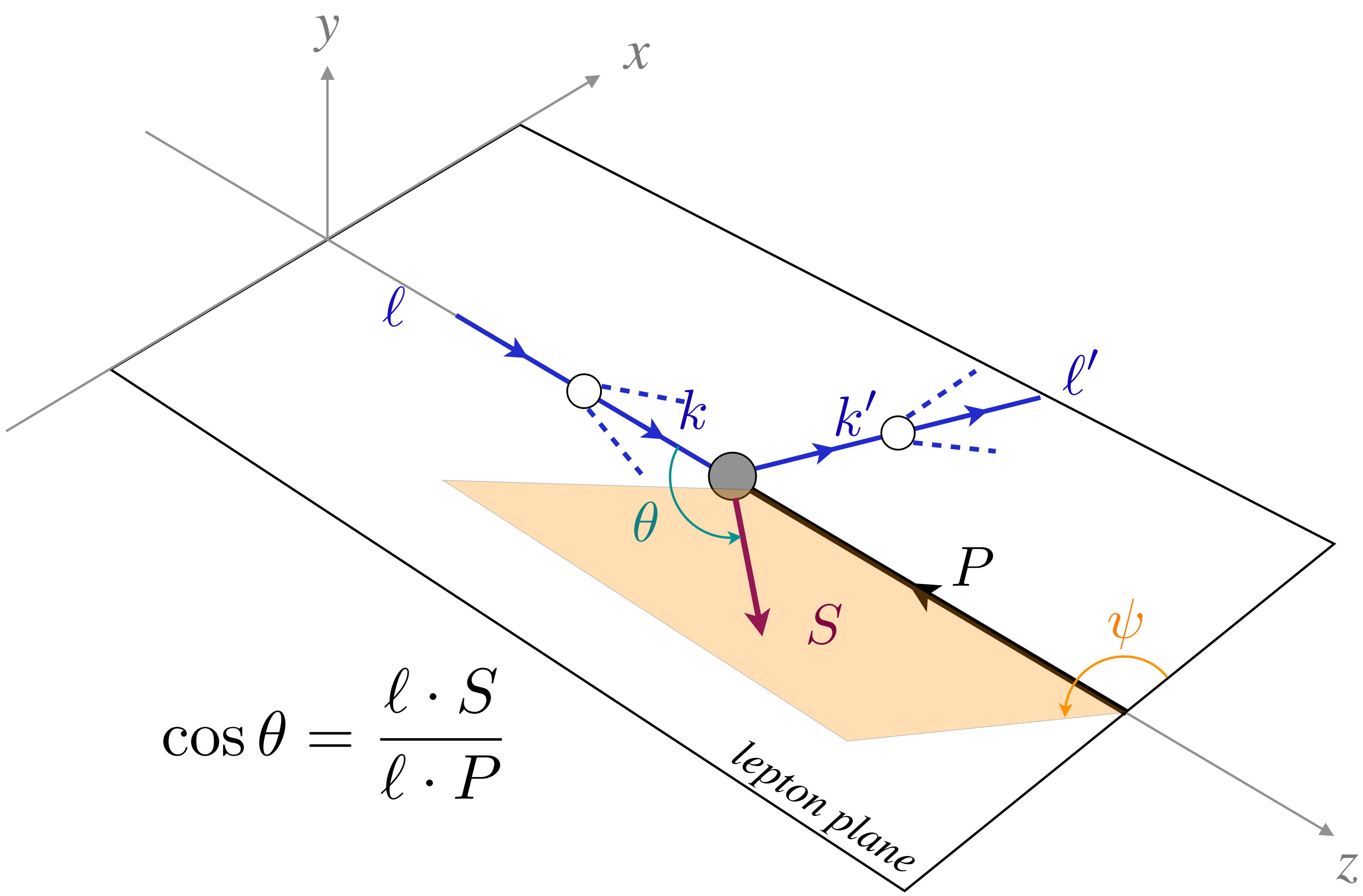
- Generalize the QCD factorization to include Electroweak theory, resum the logarithmic enhanced QED contributions.
  — QED radiation is part of the production cross sections.
  — treat QED radiation in the same way as QCD radiation is treated.

- Same systematically improvable treatment of QED contributions for both inclusive DIS and SIDIS.

T. Liu, W. Melnitchouk, J.W. Qiu, N. Sato,
Phys. Rev. D 104, 094033 (2021), J. High Energy Phys. 11 (2021) 157.

# Inclusive DIS with QED




$$\cos\theta = \frac{\ell \cdot S}{\ell \cdot P}$$

Define inclusive DIS as inclusive lepton scattering with large $\ell'_T$

*in lepton-hadron frame*

# Factorized Approach to inclusive DIS

Unpolarized inclusive DIS cross section:

lepton fragmentation function (LFF)

lepton distribution function (LDF)

$$E'\frac{\mathrm{d}\sigma_{\ell P\to\ell' X}}{\mathrm{d}^3\ell'} = \frac{1}{2s}\sum_{i,j,a}\int_{\zeta_{\min}}^{1}\frac{\mathrm{d}\zeta}{\zeta^2}\int_{\xi_{\min}}^{1}\frac{\mathrm{d}\xi}{\xi} D_{e/j}\left(\zeta,\mu^2\right) f_{i/e}\left(\xi,\mu^2\right)$$

$$\times\int_{x_{\min}}^{1}\frac{\mathrm{d}x}{x} f_{a/N}\left(x,\mu^2\right)\widehat{H}_{ia\to jX}\left(\xi\ell, xP, \ell'/\zeta,\mu^2\right)+\cdots$$

$$\zeta_{\min} = -\frac{t+u}{s},\quad \xi_{\min} = -\frac{u}{\zeta s+t},\quad x_{\min} = -\frac{\xi t}{\zeta\xi s+u}$$

LO (no RC):

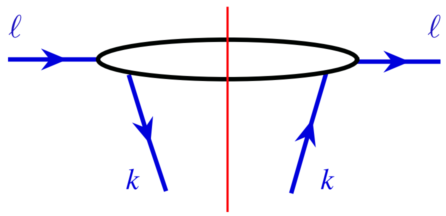
$$\sigma_{eq}^{(2,0)} = D_{e/e}^{(0)}\otimes f_{e/e}^{(0)}\otimes f_{q/q}^{(0)}\otimes\widehat{H}_{eq\to eX}^{(2,0)} = \widehat{H}_{eq\to eX}^{(2,0)}$$

$$f_{i/e}^{(0)}(\xi) = \delta_{ie}\delta(1-\xi)\qquad D_{e/j}^{(0)}(\zeta) = \delta_{ej}\delta(1-\zeta)$$

$$\widehat{H}_{eq\to eX}^{(2,0)} = \frac{4\alpha^2 e_q^2}{\zeta}\left[\frac{(\zeta\xi xs)^2+(xu)^2}{(\xi t)^2}\right]\delta(\zeta\xi xs+xu+\xi t)$$ (parity conserved part)

# LDF and LFF

## Lepton distribution function:




$$f_{i/e}(\xi) = \int \frac{dz^-}{4\pi} e^{i\xi\ell^+ z^-} \langle e| \overline{\psi}_i(0)\gamma^+ \Phi_{[0,z^-]}\, \psi_i(z^-)|e\rangle$$

LO: $f^{(0)}_{i/e}(\xi) = \delta_{ie}\delta(1-\xi)$ NLO($\overline{\text{MS}}$): $f^{(1)}_{e/e}(\xi,\mu^2) = \frac{\alpha}{2\pi}\left[\frac{1+\xi^2}{1-\xi}\ln\frac{\mu^2}{(1-\xi)^2 m_e^2}\right]_+$

## Lepton fragmentation function:

$$D_{e/j}(\zeta) = \frac{\zeta}{2}\sum_X \int \frac{dz^-}{4\pi} e^{i\ell'^+ z^-/\zeta}\, \text{Tr}\left[\gamma^+ \langle 0|\overline{\psi}_j(0)\Phi_{[0,\infty]}\,|e,X\rangle\langle e,X|\psi_j(z^-)\Phi_{[z^-,\infty]}|0\rangle\right]$$

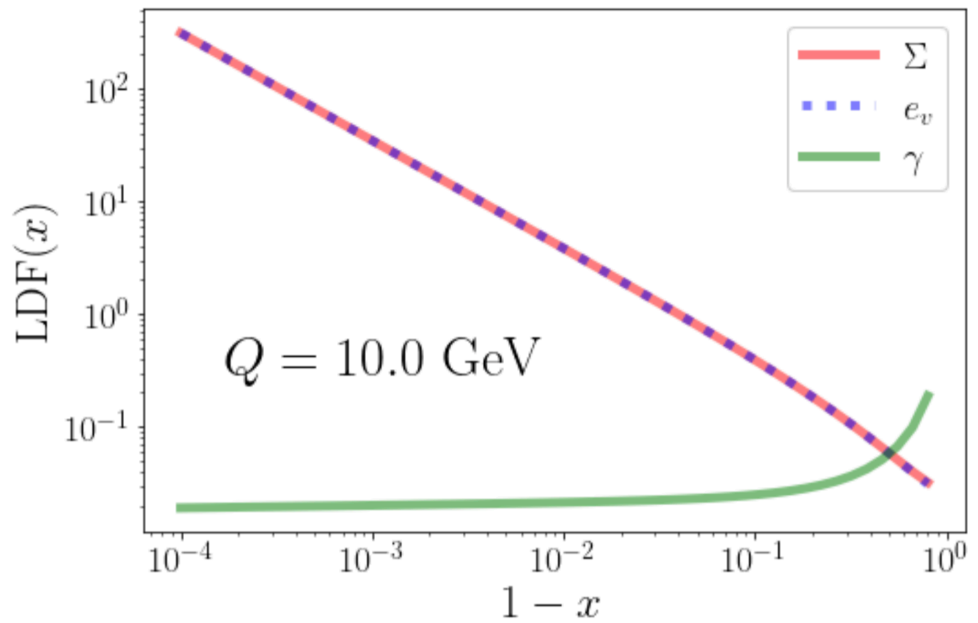
LO: $D^{(0)}_{e/j}(\zeta) = \delta_{ej}\delta(1-\zeta)$ NLO($\overline{\text{MS}}$): $D^{(1)}_{e/e}(\zeta,\mu) = \frac{\alpha}{2\pi}\left[\frac{1+\zeta^2}{1-\zeta}\ln\frac{\zeta^2\mu^2}{(1-\zeta)^2 m_e^2}\right]_+$

## Resum:

$$\frac{\partial}{\partial \ln\mu^2}\begin{pmatrix} f_+ \\ f_\gamma \end{pmatrix} = \begin{pmatrix} P_{ee} & P_{e\gamma} \\ P_{\gamma e} & P_{\gamma\gamma} \end{pmatrix} \otimes \begin{pmatrix} f_+ \\ f_\gamma \end{pmatrix}$$

QED DGLAP evolution

Similar for LFF



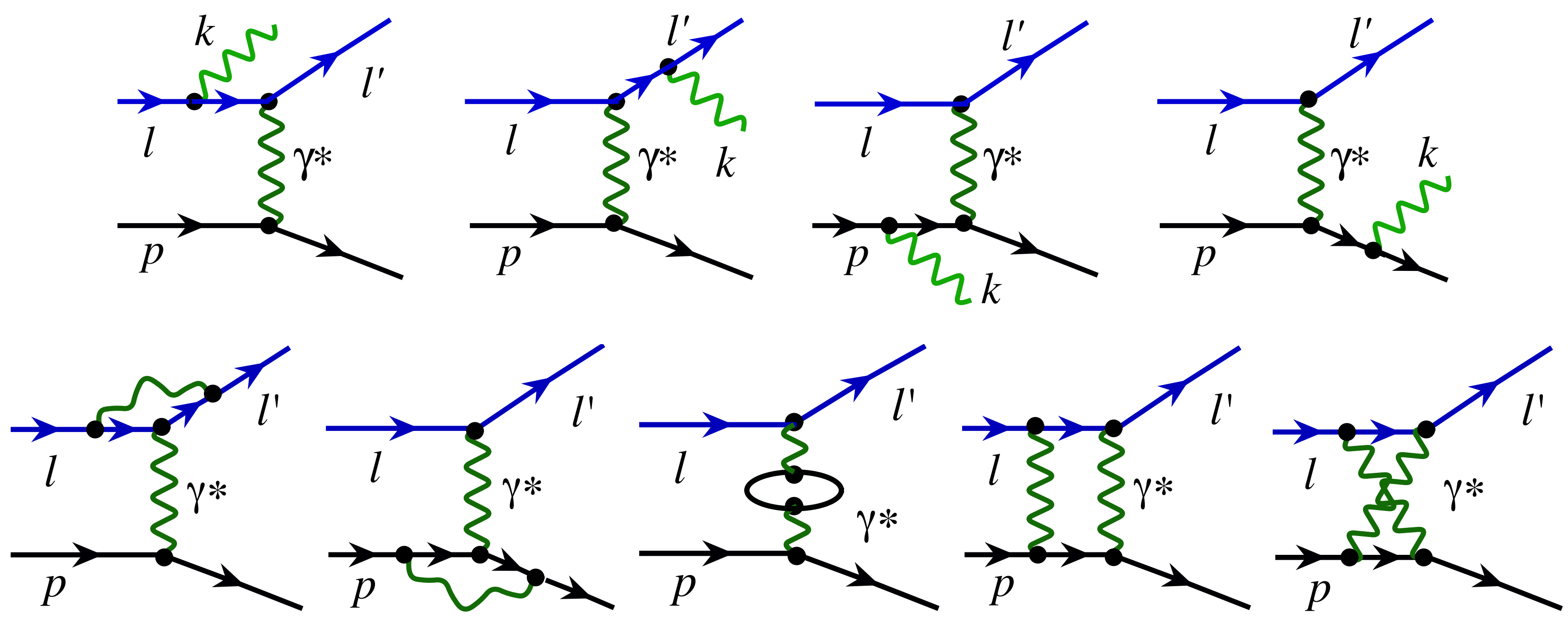
# Hard Part of Inclusive DIS

LO:

$$\sigma_{eq}^{(2,0)} = D_{e/e}^{(0)} \otimes f_{e/e}^{(0)} \otimes f_{q/q}^{(0)} \otimes \widehat{H}_{eq\to eX}^{(2,0)} = \widehat{H}_{eq\to eX}^{(2,0)}$$

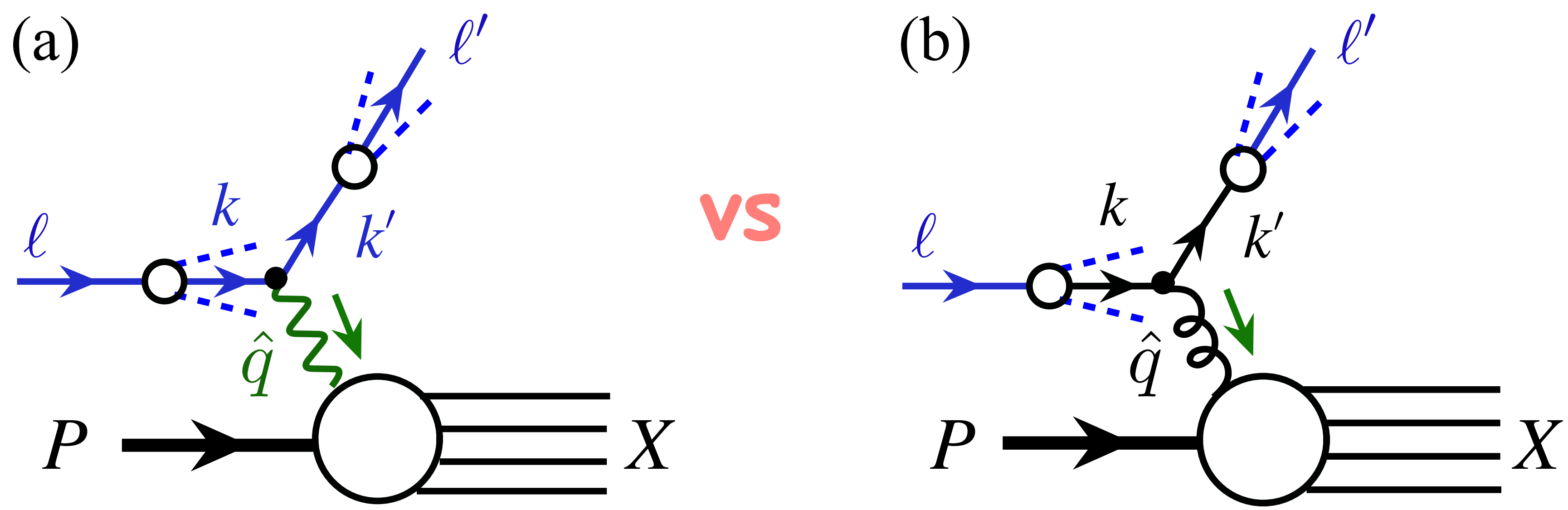
$$\widehat{H}_{eq\to eX}^{(2,0)} = \frac{4\alpha^2 e_q^2}{\zeta}\left[\frac{(\zeta\xi x s)^2 + (xu)^2}{(\xi t)^2}\right]\delta(\zeta\xi x s + xu + \xi t)$$

NLO:




$$\widehat{H}_{eq\to eX}^{(3,0)} = \sigma_{eq}^{(3,0)} - D_{e/e}^{(1)} \otimes \widehat{H}_{eq\to eX}^{(2,0)} - f_{e/e}^{(1)} \otimes \widehat{H}_{eq\to eX}^{(2,0)} - f_{q/q}^{(1)} \otimes \widehat{H}_{eq\to eX}^{(2,0)}$$

# One Boson Exchange Approximation




At higher order one can find quark/gluon distribution in LDF and LFF.

(b) is suppressed by selecting events in which the lepton does not have much hadronic energy around it.

One-photon exchange approximation:

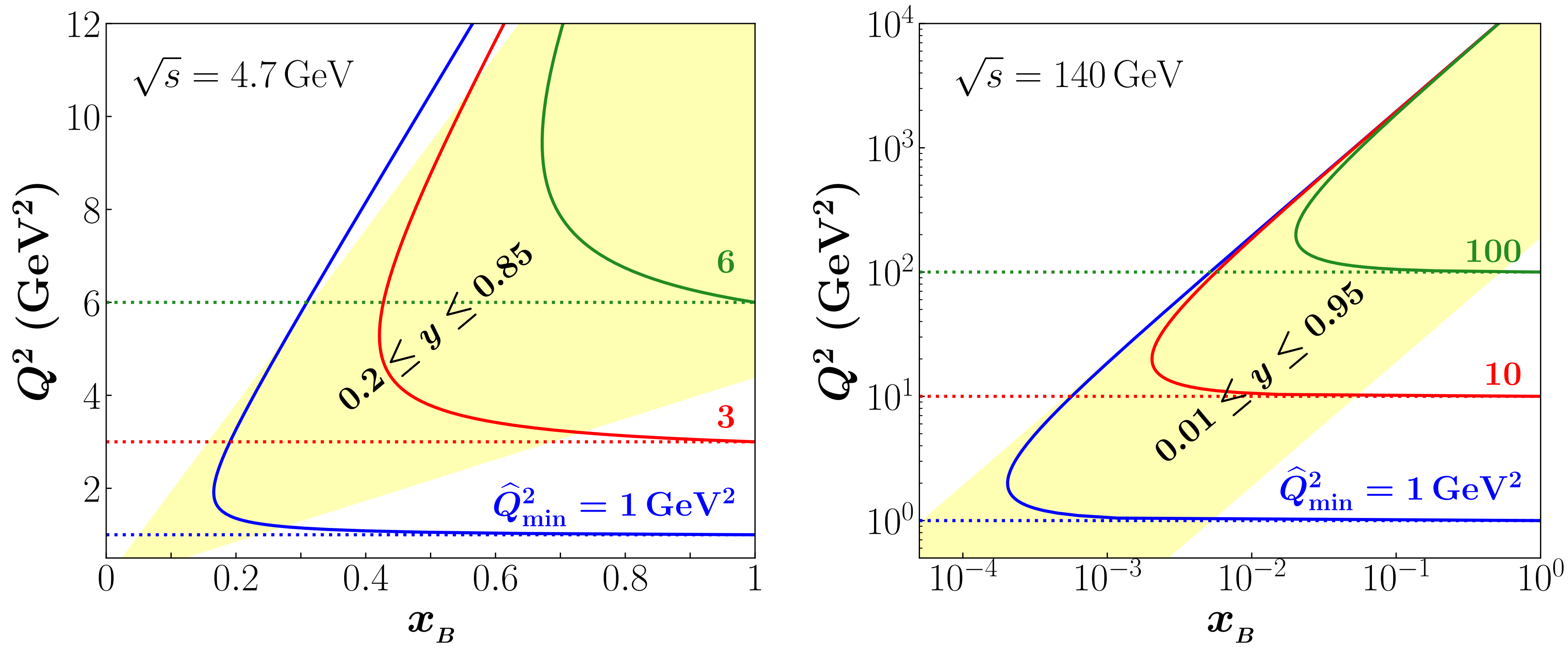
$$\frac{d\sigma_{\ell P\to\ell' X}}{dx_B\, dy} \approx \int_{\zeta_{\min}}^{1} \frac{d\zeta}{\zeta^2} \int_{\xi_{\min}}^{1} d\xi D_{e/e}\left(\zeta,\mu^2\right) f_{e/e}\left(\xi,\mu^2\right)$$

$$\times \frac{4\pi\alpha^2}{\hat{x}_B \hat{y} \widehat{Q}^2}\left[\hat{x}_B \hat{y}^2 F_1\left(\hat{x}_B, \widehat{Q}^2\right) + \left(1-\hat{y}-\frac{1}{4}\hat{y}^2\hat{\gamma}^2\right) F_2\left(\hat{x}_B, \widehat{Q}^2\right)\right]$$

$$\widehat{Q}^2 = -\hat{q}^2 = \frac{\xi}{\zeta} Q^2, \quad \hat{x}_B = \frac{\widehat{Q}^2}{2P\cdot\hat{q}}, \quad \hat{y} = \frac{P\cdot\hat{q}}{P\cdot k}, \quad \hat{\gamma} = \frac{2M\hat{x}_B}{\widehat{Q}}$$

# The Hard Scale

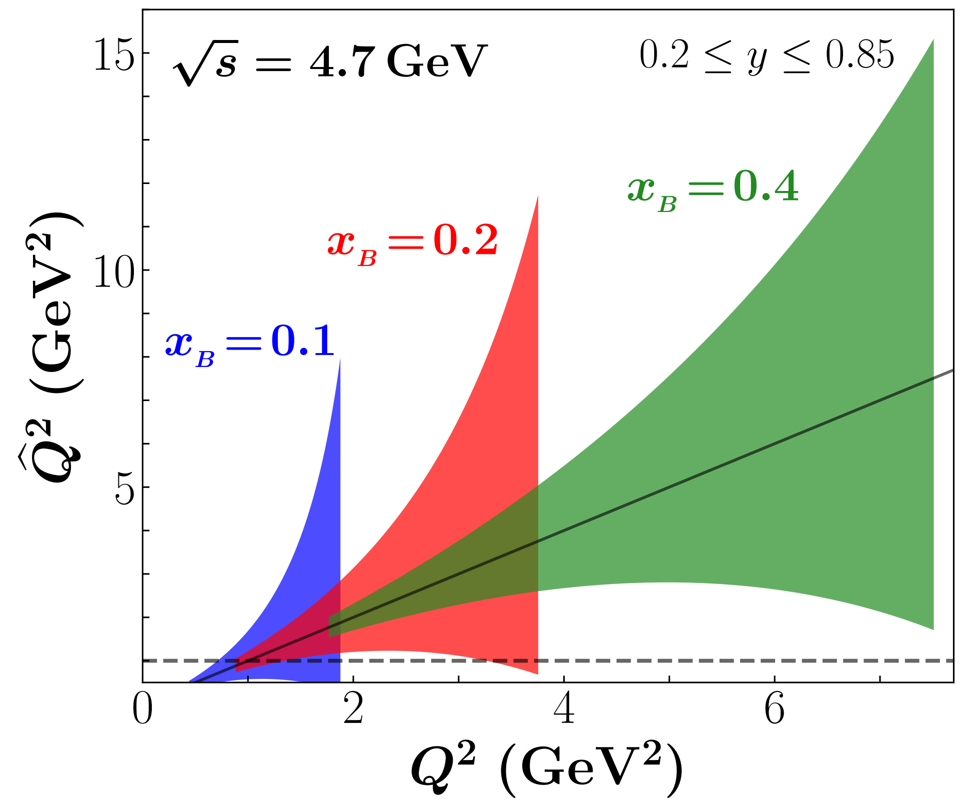
Collision induced QED radiation changes the hard scale from $Q^2$ to $\widehat{Q}^2$

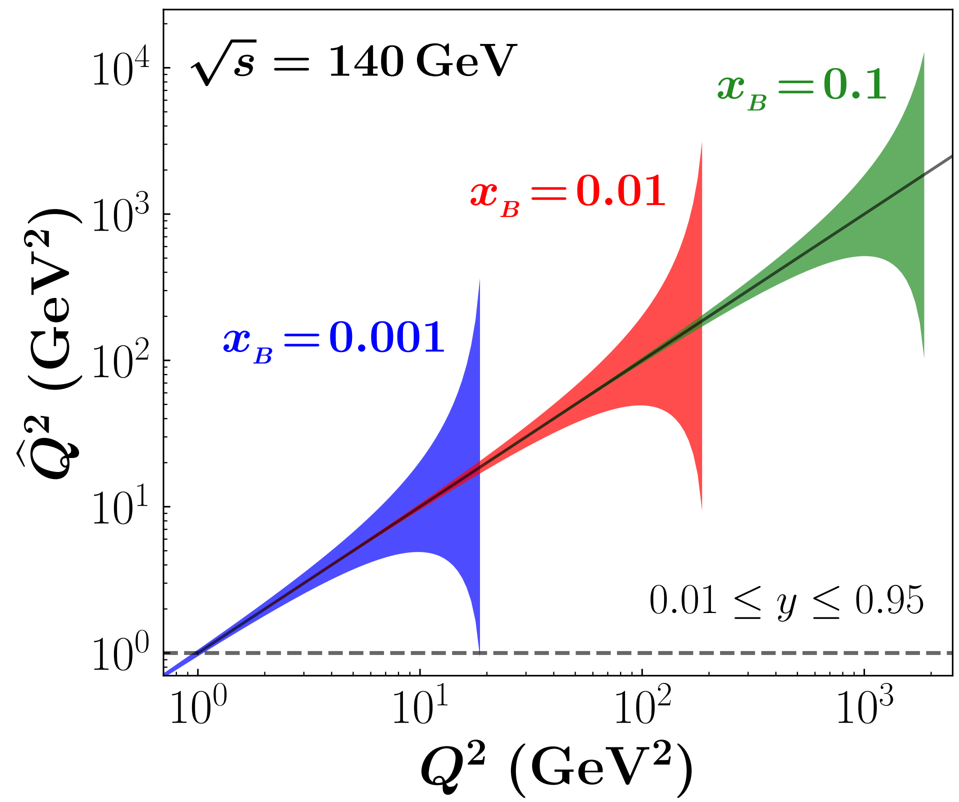



$\widehat{Q}^2$ has a minimum value $\widehat{Q}^2_{\min} = \dfrac{1-y}{1-yx_B} Q^2 < Q^2$

# The Hard Scale

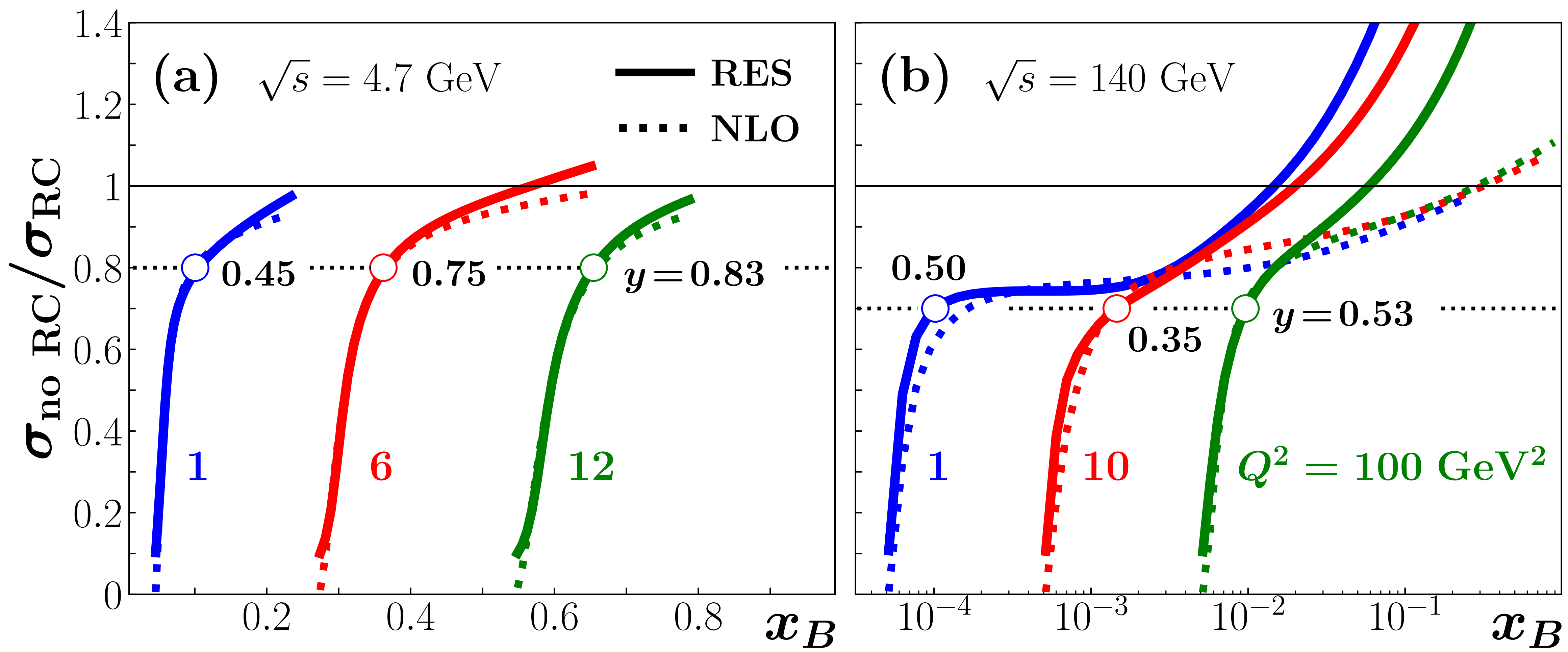
Collision induced QED radiation changes the hard scale from $Q^2$ to $\widehat{Q}^2$



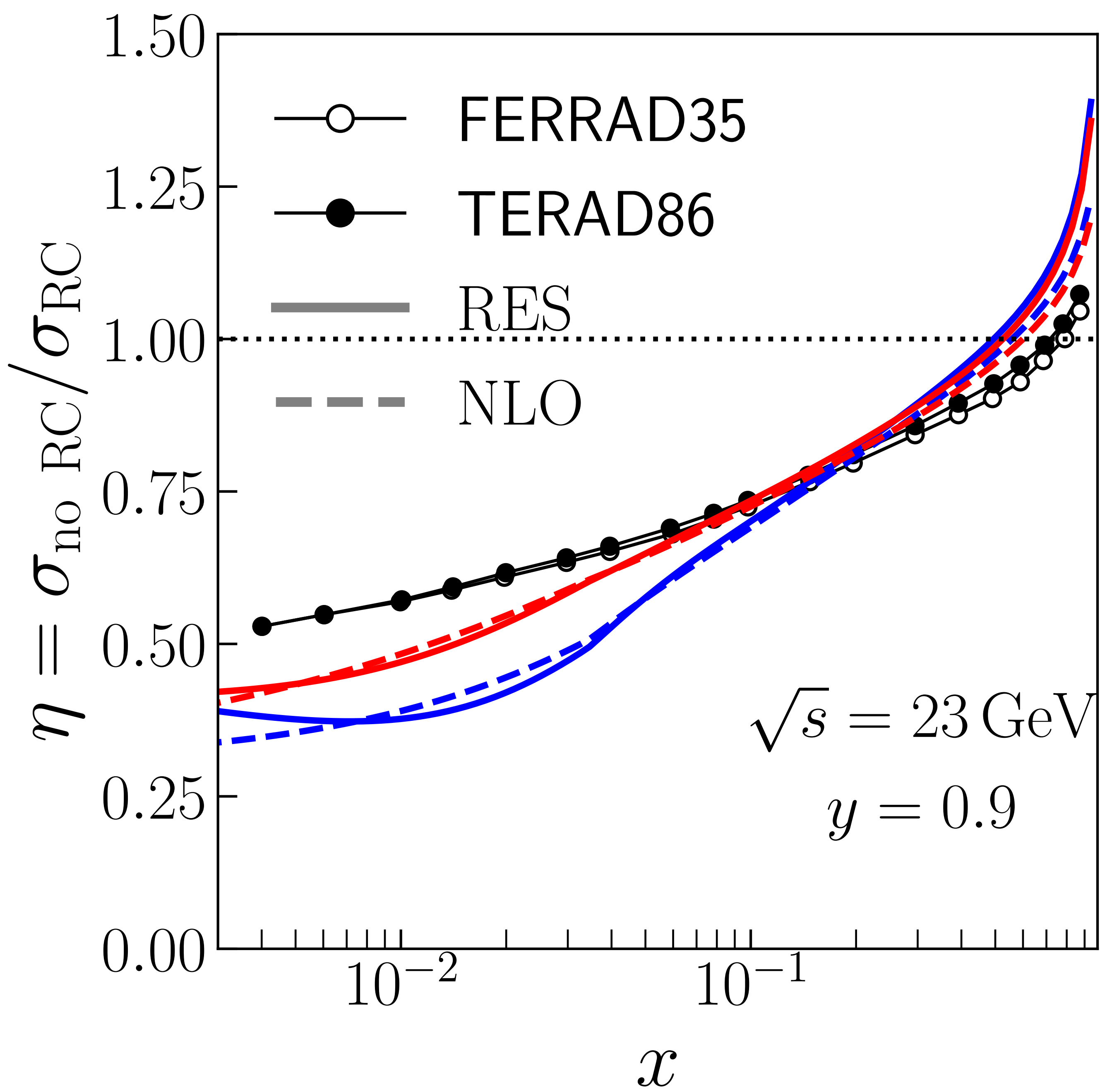




$$\widehat{Q}^2_{\rm min} = Q^2 \frac{(1-y)}{(1-x_B\, y)} \qquad \widehat{Q}^2_{\rm max} = Q^2 \frac{1}{(1-y+x_B\, y)}$$

# Impact on Inclusive DIS



# Comparison with Early Result



# Including Parity-Violating Terms

$$A_{\rm PV} = \frac{\sigma_{\ell(\lambda_\ell=1)P\to\ell' X} - \sigma_{\ell(\lambda_\ell=-1)P\to\ell' X}}{\sigma_{\ell(\lambda_\ell=1)P\to\ell' X} + \sigma_{\ell(\lambda_\ell=-1)P\to\ell' X}} = \frac{\Delta\sigma_{\lambda_\ell}}{\sigma_{\ell P\to\ell' X}}$$

$$\begin{aligned}\frac{\mathrm{d}\Delta\sigma_{\lambda_\ell}}{\mathrm{d}x_B\,\mathrm{d}y} = &\int_{\zeta_{\min}}^{1}\frac{\mathrm{d}\zeta}{\zeta^2}D_{e/e}\left(\zeta,\mu^2\right)\int_{\xi_{\min}}^{1}\mathrm{d}\xi\Delta f_{e/e}\left(\xi,\mu^2\right)\left[\frac{Q^2}{x_B}\frac{\hat{x}_B}{\widehat{Q}^2}\right]\\ &\frac{4\pi\alpha^2}{\hat{x}_B\hat{y}\widehat{Q}^2}\left[-\hat{x}_B\left(\hat{y}-\frac{1}{2}\hat{y}^2\right)F_3^\gamma\left(\hat{x}_B,\widehat{Q}^2\right)\right.\\ &+\eta_{\gamma Z}\left(eg_A^e\hat{x}_B\hat{y}^2F_1^{\gamma Z}\left(\hat{x}_B,\widehat{Q}^2\right)+eg_A^eK_{\hat{y}}F_2^{\gamma Z}\left(\hat{x}_B,\widehat{Q}^2\right)-g_V^e\left(\hat{y}-\frac{1}{2}\hat{y}^2\right)F_3^{\gamma Z}\left(\hat{x}_B,\widehat{Q}^2\right)\right)\\ &+\eta_Z\left(2eg_V^eg_A^e\hat{x}_B\hat{y}^2F_1^Z\left(\hat{x}_B,\widehat{Q}^2\right)+2eg_V^eg_A^eK_{\hat{y}}F_2^Z\left(\hat{x}_B,\widehat{Q}^2\right)\right.\\ &\left.\left.-\left(g_V^e2+g_A^e\right)\hat{x}_B\left(\hat{y}-\frac{1}{2}\hat{y}^2\right)F_3^Z\left(\hat{x}_B,\widehat{Q}^2\right)\right)\right]\end{aligned}$$

$$\begin{aligned}\frac{\mathrm{d}\sigma_{\ell P\to\ell' X}}{\mathrm{d}x_B\,\mathrm{d}y} = &\int_{\zeta_{\min}}^{1}\frac{\mathrm{d}\zeta}{\zeta^2}D_{e/e}\left(\zeta,\mu^2\right)\int_{\xi_{\min}}^{1}\mathrm{d}\xi f_{e/e}\left(\xi,\mu^2\right)\left[\frac{Q^2}{x_B}\frac{\hat{x}_B}{\widehat{Q}^2}\right]\\ &\times\frac{4\pi\alpha^2}{\hat{x}_B\hat{y}\widehat{Q}^2}\left[\hat{x}_B\hat{y}^2F_1^\gamma\left(\hat{x}_B,\widehat{Q}^2\right)+K_{\hat{y}}F_2^\gamma\left(\hat{x}_B,\widehat{Q}^2\right)\right.\\ &+\eta_{\gamma Z}g_V^e\left(\hat{x}_B\hat{y}^2F_1^{\gamma Z}\left(\hat{x}_B,\widehat{Q}^2\right)+K_{\hat{y}}F_2^{\gamma Z}\left(\hat{x}_B,\widehat{Q}^2\right)\right)\\ &\left.+\eta_Z g_V^e 2\left(\hat{x}_B\hat{y}^2F_1^Z\left(\hat{x}_B,\widehat{Q}^2\right)+K_{\hat{y}}F_2^Z\left(\hat{x}_B,\widehat{Q}^2\right)\right)\right]\end{aligned}$$

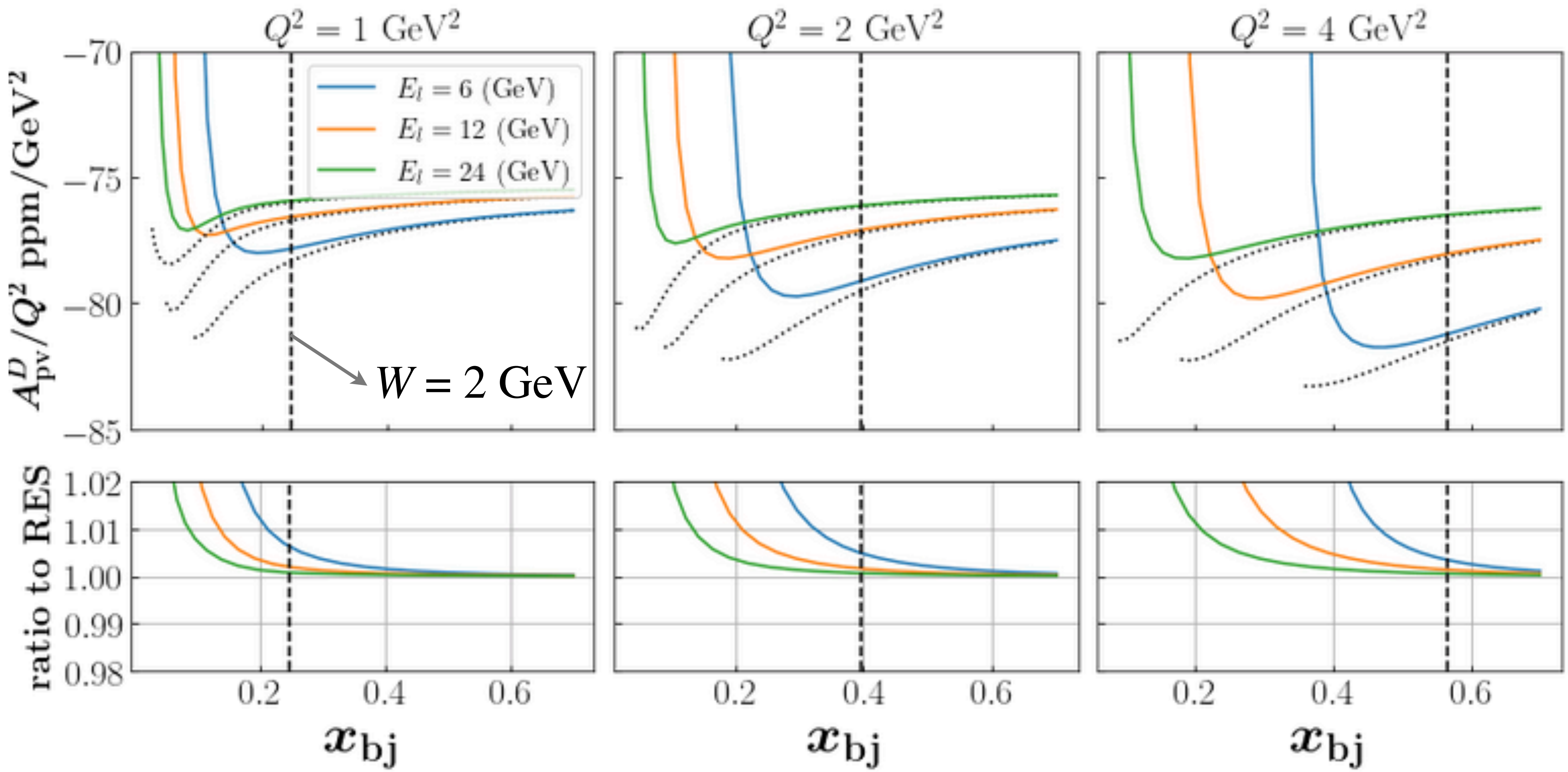
$$\eta_\gamma = 1$$

$$\eta_{\gamma Z} = \frac{G_F M_Z^2}{2\sqrt{2}\pi\alpha}\frac{Q^2}{Q^2+M_Z^2}$$

$$\eta_Z = \eta_{\gamma Z}^2$$

$$K_{\hat{y}} = 1-\hat{y}-\frac{1}{4}\hat{\gamma}^2\hat{y}^2$$

# Impact on $A_{PV}$



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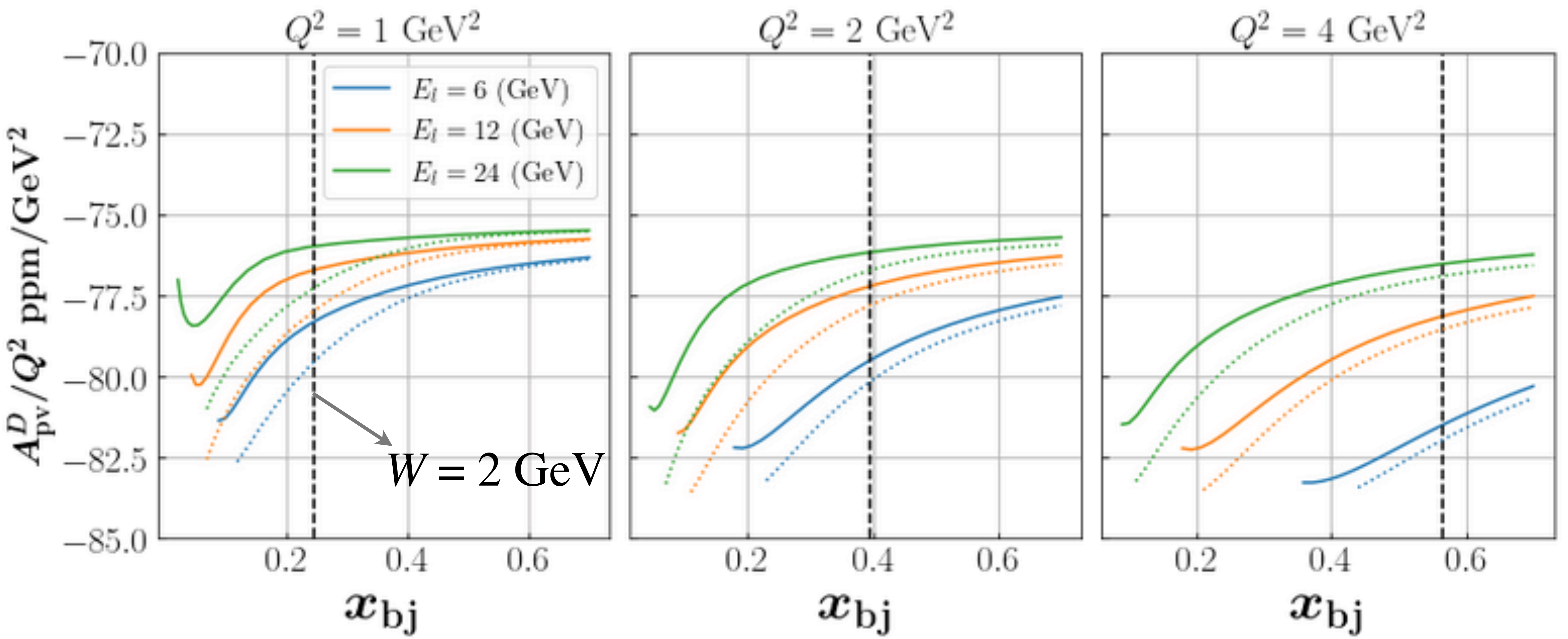
# Comparison with Traditional Method

No QED results: different PDF sets

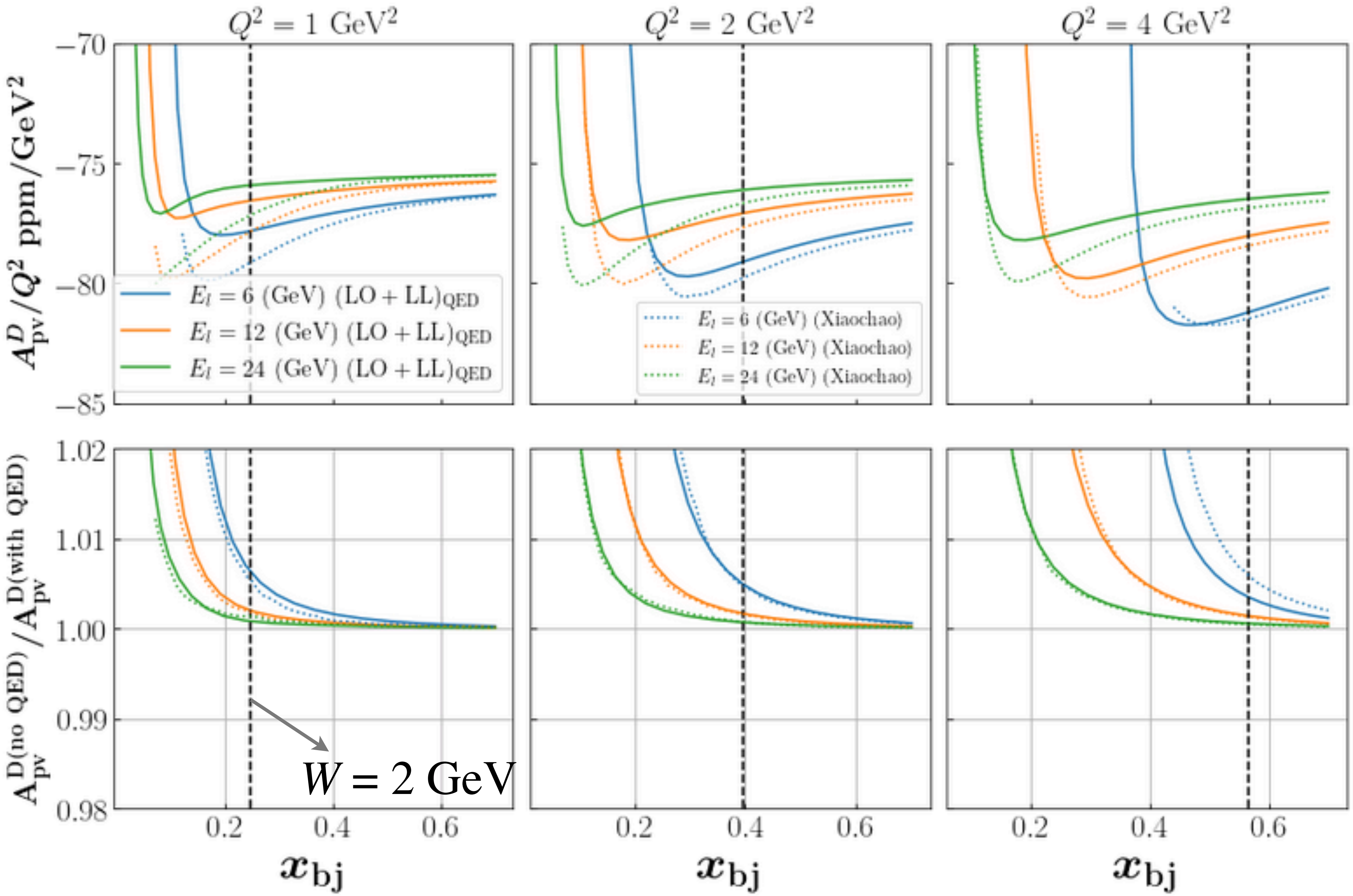



dotted curves from X. Zheng

See Nobuo Sato's talk this afternoon for $A_{\rm pv}$ sensitivity to PDFs

# Comparison with Traditional Method

QED impact on $A_{pv}$




dotted curves from X. Zheng, generated using “Mo&Tsai” approach.

# Summary

- QED radiation effects are important in DIS, and hence precise measurements of nucleon structures and EW physics.
- We propose a factorized approach to treat QED radiations.
  - Treat QED radiation as a part of the production cross section.
  - Generalize QCD factorization to include QED. All perturbatively calculable hard parts are IR safe.
  - Same systematically improvable treatment of QED contributions for inclusive DIS, semi-inclusive DIS …
- QED radiation also has significant effect on $A_{pv}$
  - If calculating the RC factor for $A_{pv}$, our factorized approach gives similar values to that from traditional approach at JLab kinematics.
  - A high energy upgrade of JLab may provide a wider window, where the QED radiation is more controllable for $A_{pv}$.
  - $A_{pv}$ is also a sensitive probe to sea quark PDFs [see Nobuo's talk]

# *Thank you!*