πN states in Nucleon two-, Three-, and Four-point Functions

- importance of correct prior

- Multiple hadron states and volume dependence
- πN states in $1/2^-$ (S_{11}) channel and volume enhancement
- πN states in nucleon two-point function – Bayesian Reconstruction
- πN state contaminations and contributions in nucleon form factors and neutrino-nucleon scattering.

July 10, 2024
INT - Seattle

Spectral Distribution as an Inverse problem

- Multi-exponential fit

$$C_2(t) = \sum_i W_i e^{-E_i t}, \quad W_i > 0$$

  - Initial values
  - Dependence on priors
  - No definite evidence of πN states in nucleon correlators
  - Generalized eigenvalue problem (GEVP)
- Barkus-Gilbert, Maximum Entropy, Bayesian Reconstruction (BR)
  - BG good for broad distributions
  - BR good for discrete states
- BR -- Y. Burnier and A. Rothkopf, PRL 111, 182003 (2013)
  - Prior for asymptotic value
  - No priors for the excited states positions and spectral weight

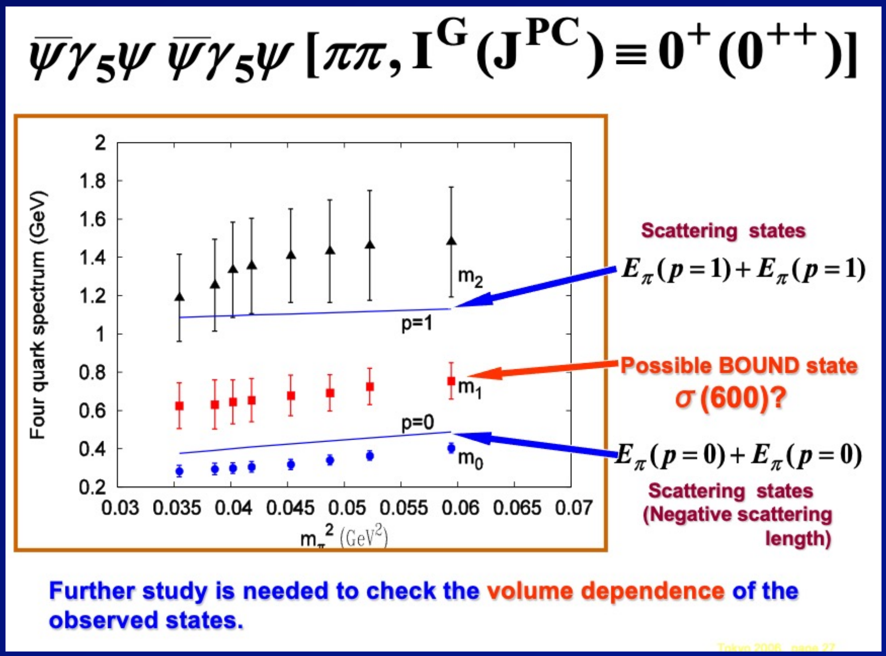
Multi-hadron States and Volume Dependence

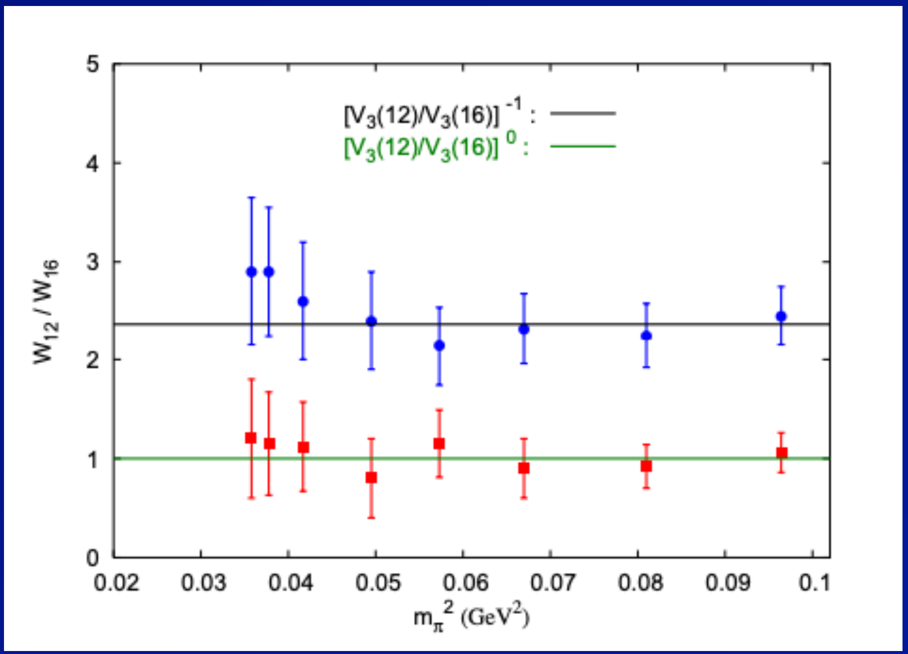
$$C_2(t) = \sum_i W_i e^{-E_i t}, \;\; W_i > 0$$

- Main motivation is to assess the excited state contamination to control the systematic errors of the nucleon matrix elements and the πN contribution in neutrino-nucleon scattering.
- Each hadron state in the finite volume introduces a $1/V_3$ dependence from normalization.
- Volume dependence has been used to discern the multi-hadron nature in hadron correlators.

Muti-hadon States from Volume Dependence

- σ meson from 4-quark interpolation field to differentiate one-particle from two-particle states (i.e. $\pi\pi$) – N. Mathur et. al., hep-ph/0607110





- Similar volume study was carried out for the $uudd\bar{s}$ pentaquark candidate to show it is a KN state, not a pentaquark state -- N. Mathur, et al., hep-ph/0406196.

πN States in the Nucleon Two-point Function?

- Being a two-hadron state, πN state has a volume suppression as compared to the nucleon state.
- Assuming $\langle 0|\chi(0)|N(\vec{p})\pi(-\vec{p})\rangle \approx \langle 0|\chi(0)|N(\vec{p}=0)\rangle / f_\pi$

$$\frac{C_{\pi N}(t)}{C_N(t)} \approx \frac{3}{8f_\pi^2 m_\pi L^3}\sum_{\vec{p}} \frac{m_\pi}{E_\pi}\frac{E_N - M_N}{2E_N}\left[1 - g_A \frac{E_{\rm tot}+M_N}{E_{\rm tot}-M_N}\right]^2 e^{-(E_{\rm tot}-M_N)t} = \mathrm{R}e^{-(E_{\rm tot}-M_N)t}$$

O. Bär – 1503.03649

When m_π = 139 MeV, L = 5.5 fm ($m_\pi L \sim 4$), R = 1.6% for $|\vec{p}| = 1$

- Lattice variational calculation with q^3 and πN interpolators at a small volume, the calculated R is 22(1)%, while the above formula gives 21%.

C.B. Lang, S. Prelovsek et. al. – 1610.01422

- Hard to discern with a q^3 interpolation operator for the nucleon, but should appear in $1/2^-$ channel where S-wave πN is the ground state ($M = m_\pi + m_N$)

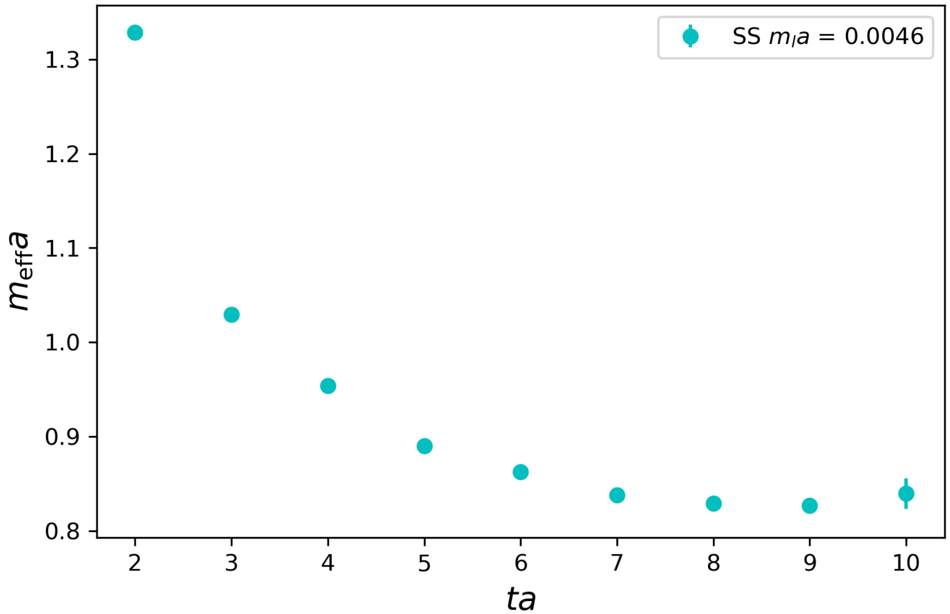
Effective Mass of ½⁻ (S_{11}) Channel
Smeared and Wall Sources

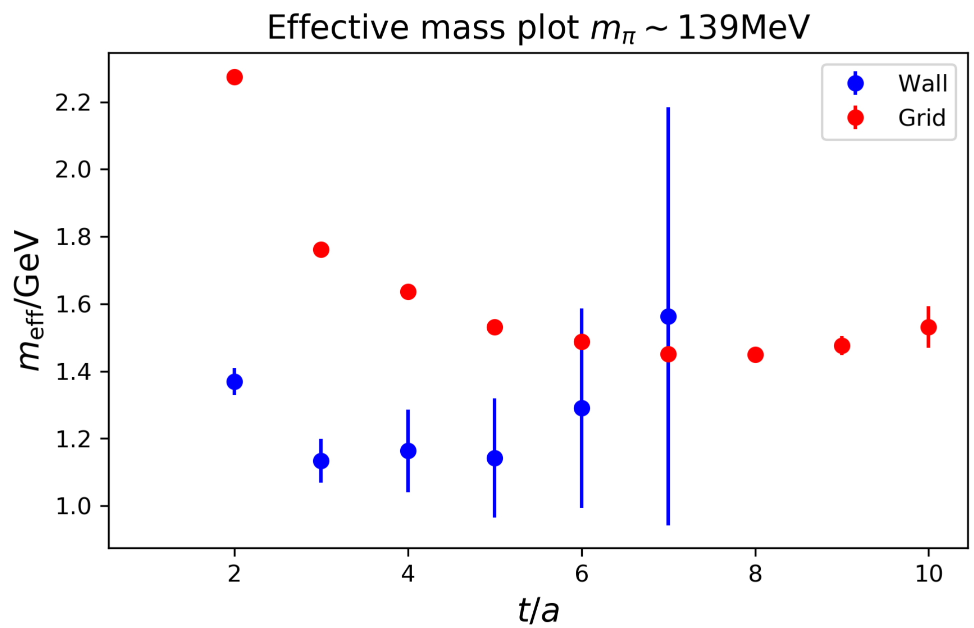
- Wall source has a V enhancement

  m_π = 139 MeV

$$C_2(t) = \sum_i W_i e^{-E_i t}$$

$$m_{\text{eff}} = \log(C_2(t)/C_2(t-1))$$



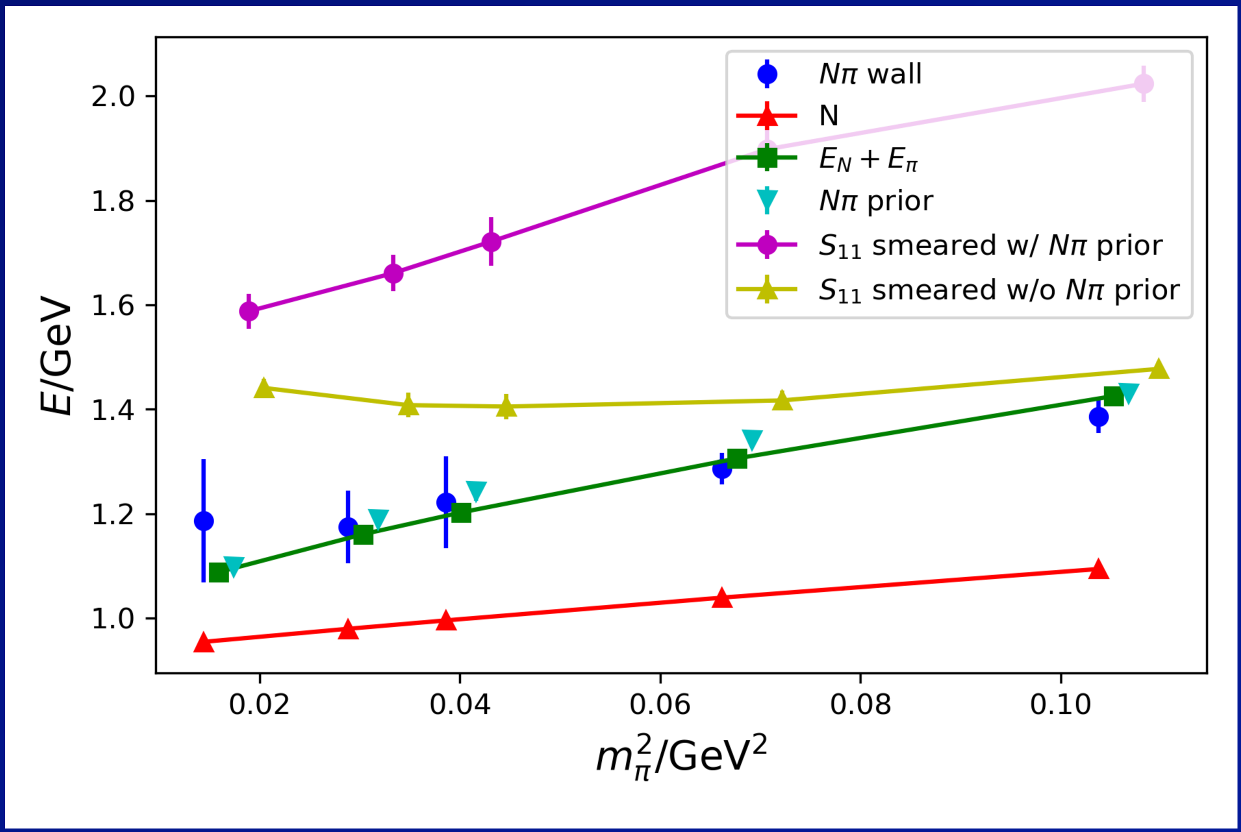


Smeared source

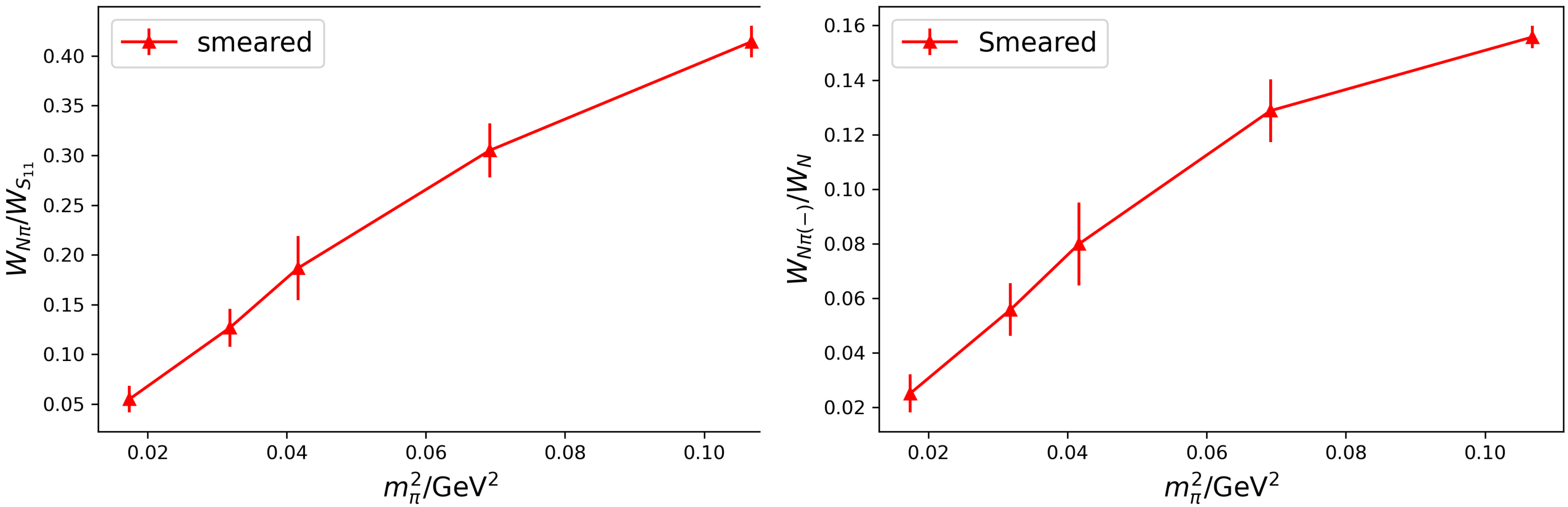
Smeared vs wall sources

BR for ½⁻ (S_{11}) Channel

- πN is in an S-wave with E ≈ $m_\pi + m_{N.}$
- Smeared source vs wall source
- Wall source has a V enhancement
- Prior of πN in the smeared source fit



Ratio of Spectral Weights



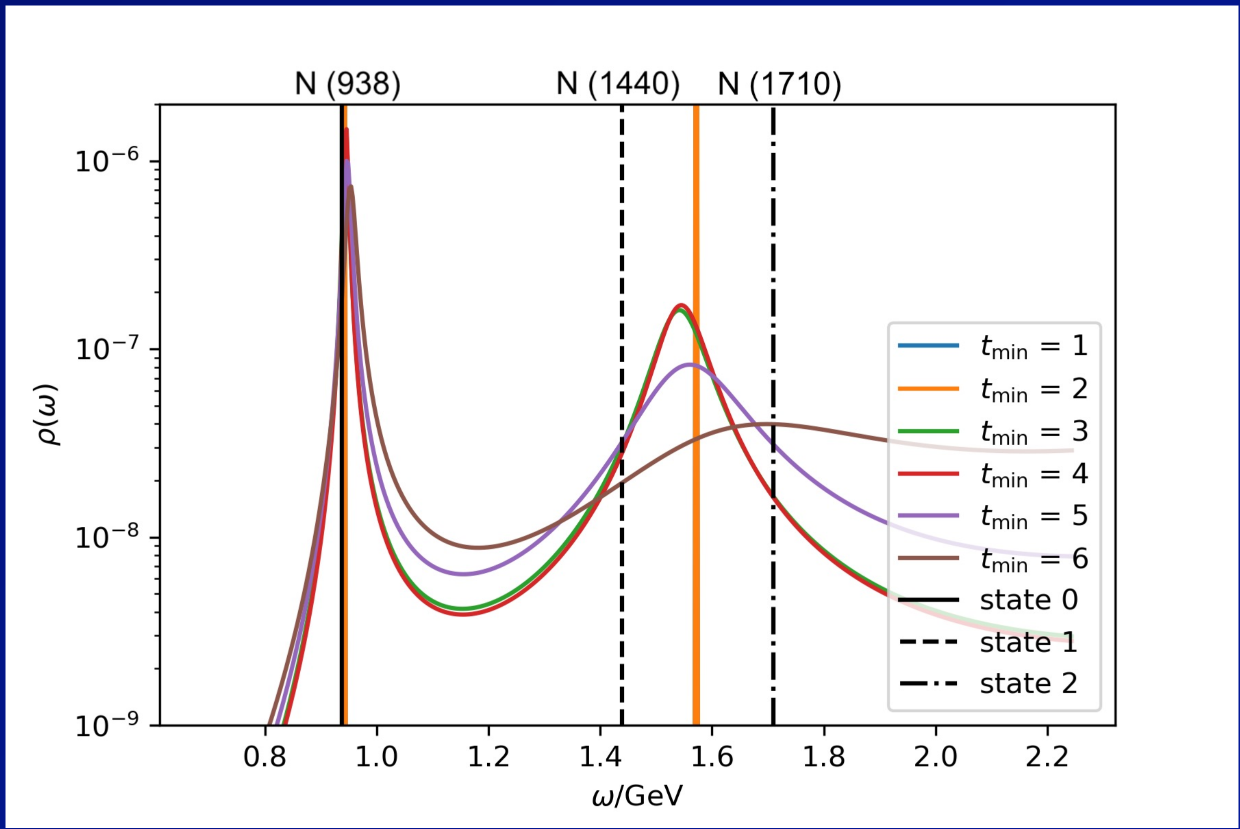
$$\frac{C_{\pi N}(t)(1/2^-)}{C_N(t)(1/2^+)} \approx \frac{3}{8 f_\pi^2 m_\pi L^3} \sum_{\vec{p}} \frac{m_\pi}{E_\pi} \frac{E_N + M_N}{2E_N} \left[1 - g_A \frac{E_{\rm tot} - M_N}{E_{\rm tot} + M_N}\right]^2 e^{-(E_{\rm tot} - M_N)t} = \mathrm{R}e^{-(E_{\rm tot} - M_N)t}$$

O. Bär – 1503.03649

When m_π = 139 MeV, L = 5.5 fm ($m_\pi L \sim 4$), R = 1.6%

Bayesain Restrcution for Nucleon Two-Point Function

- $48^3 \times 96$ lattice, overlap on DWF, a = 0.114 fm, m_π = 139 MeV
- m_N = 949.1 (1.5) MeV



Nucleon Three-Point Function

- Peculiar features
  - $g_A(A_0) \approx 0.8\, g_A(A_i)$ J. Liang et al., 1612.04388
  - Goldberger-Treiman relation for axial form factors from PCAC

$$2m_N\, g_A(q^2) + q^2 h_A(q^2) = \frac{2m_\pi^2\, f_\pi^2\, g_{\pi NN}(q^2)}{m_\pi^2 - q^2}$$ K.F. Liu et al, hep-lat/9406007

does not hold for light pion mass.

NUCLEON ISOVECTOR COUPLINGS FROM $N_f = 2$ …

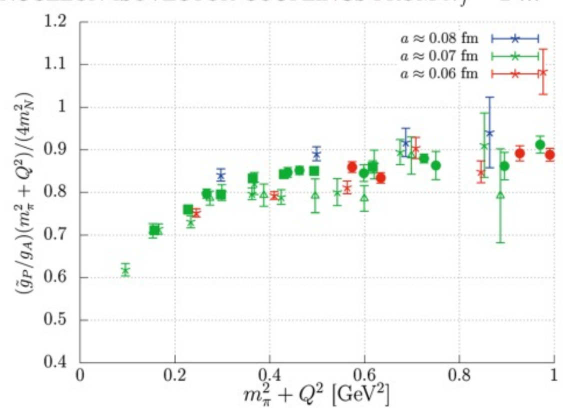


FIG. 27 (color online). The ratio of form factors $\tilde{g}_P(Q^2)/g_A(Q^2)$, normalized with respect to the single pole dominance expectation, as a function of the virtuality Q^2. Data from all 11 ensembles are plotted on top of each other. Symbols are as in Fig. 1. Deviations from unity quantify violations of the pole dominance model.

G. Bali et al.,1412.7336

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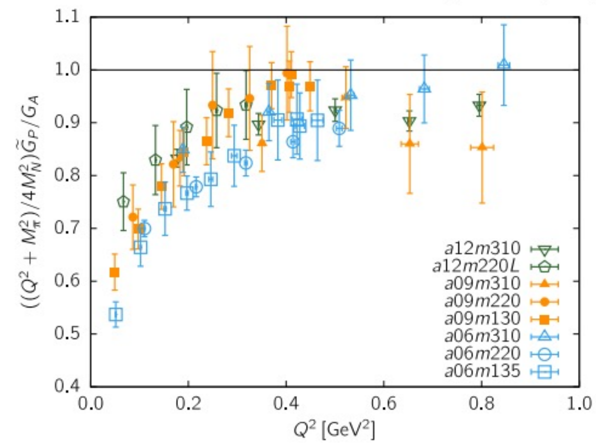
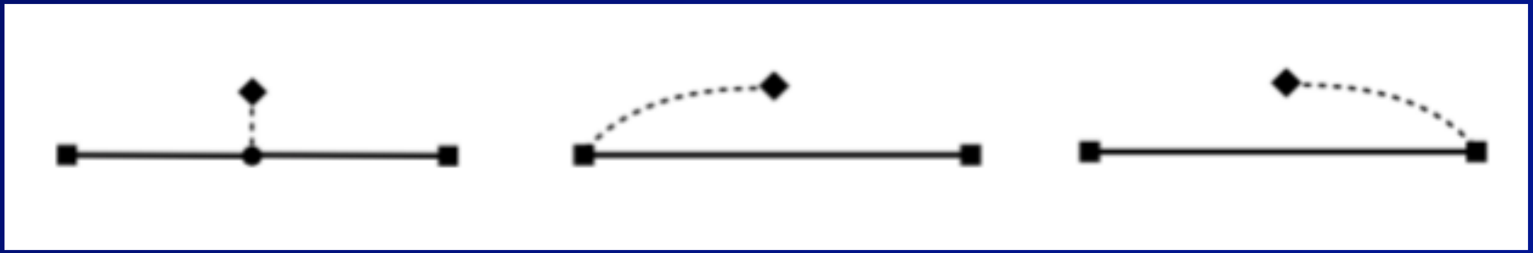


FIG. 13. Plot of the ratio $(Q^2 + M_\pi^2)\tilde{G}_P(Q^2)/(4M_p^2 G_A(Q^2))$ versus Q^2 for the eight ensembles. Validity of the pion pole-dominance hypothesis, given in Eq. (11), requires that this ratio is unity for all Q^2. Our data show significant deviations, especially for $Q^2 \lesssim 0.2$ GeV^2.

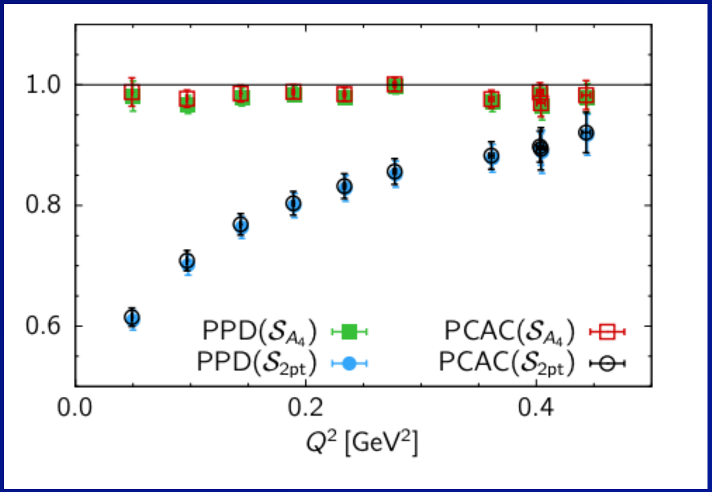
R. Gupta et al., 1705.06834

Current Induced πN Contamination

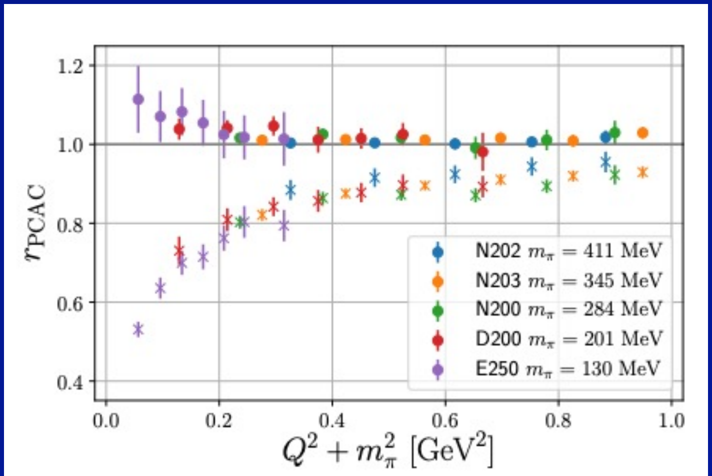
- O. Bär -- 1812.09191



- This is particularly important for pseudoscalar current which couples to the pion and has a low excitation energy.



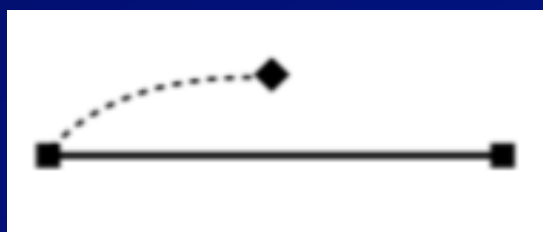
R. Gupta et al., 1905.06470

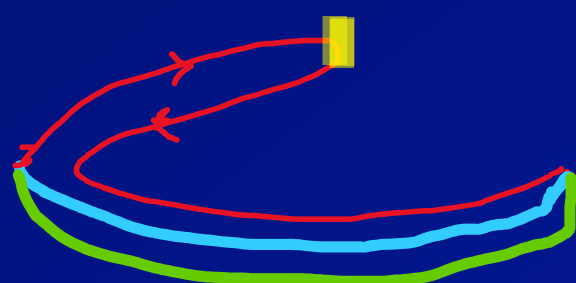


G. Bali et al., 1911.13150

Current Induced πN Contamination

- Boomerang diagram



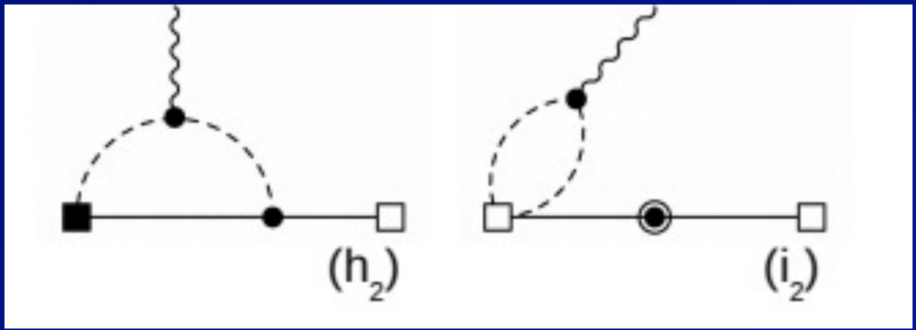


- The current position is summed over, leading to a V enhancement to overcome the 1/v suppression of πN state.

$\pi\pi N$ Contamination

- There are πN and $\pi\pi$N contamination in the pion-nucleon sigma term calculation due to the coupling of the scalar current to $\pi\pi$ -- R. Gupta et al., 2105.12095 which leads to an enhancement of the πN sigma term.

- ChPT



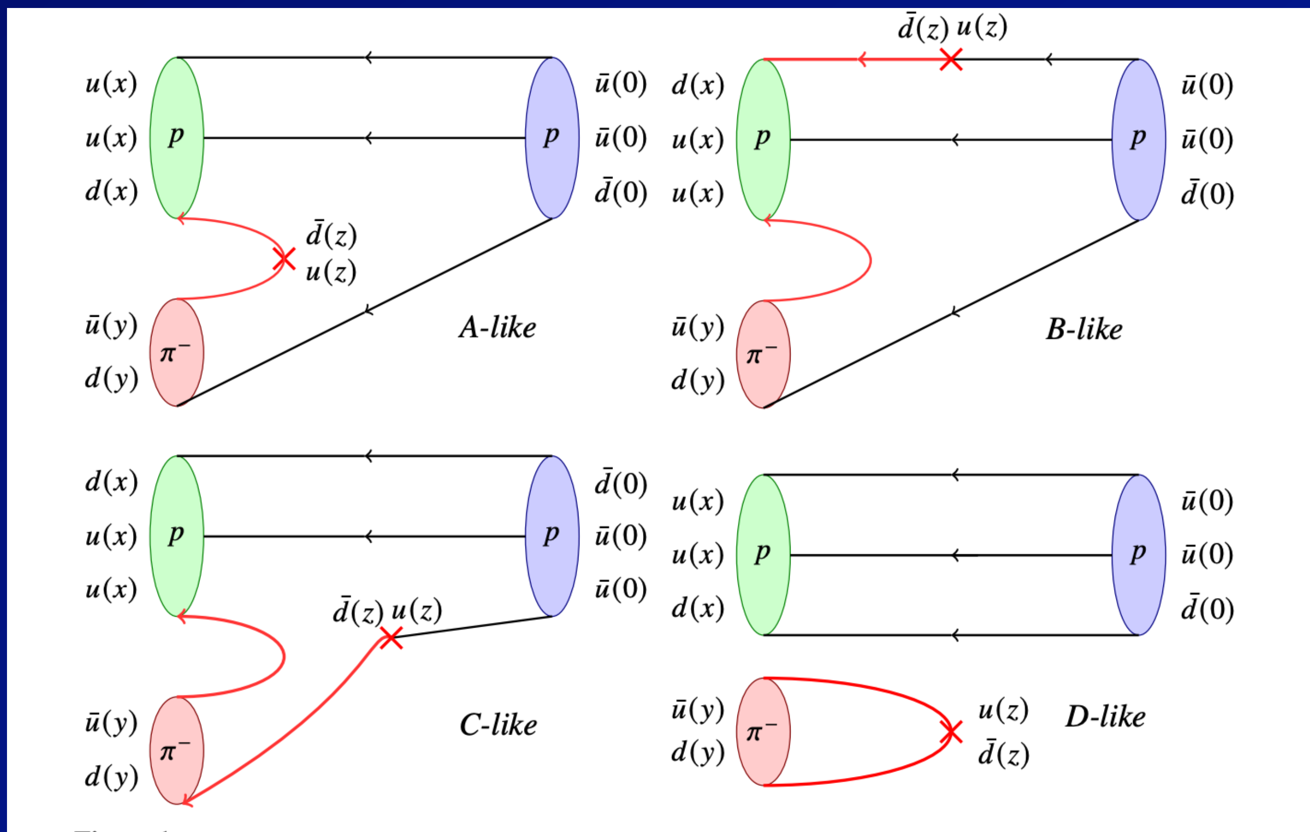
Both are 1/V suppressed.

- Question – Is large $g_{\sigma\pi\pi}$ coupling enough to overcome the 1/V suppression at certain V?

- Question on nEDM ? Vector current coupling to $\pi\pi$ is much weaker than that of scalar current (cf. muon g-2 calculation).

Direct calculation of <N|J|Nπ>

L. Barca et al., 2405.20875



Part of $$C_3(t_f,t) \rightarrow \langle 0|\chi_{3q}|N\pi\rangle e^{-E_{N\pi}(t_f-t)}\langle N\pi|J|N\rangle e^{-E_N t}\langle N|\chi_{3q}^\dagger|0\rangle$$

Lepton-nucleon Scattering – Hadronic Tensor $W_{\mu\nu}$ in Euclidean Space

$$\tilde{W}_{\mu\nu}(\vec{q},\vec{p},\tau=t_2-t_1)=\frac{\frac{E_P}{M_N}\mathrm{Tr}<\Gamma_e\chi_N(\vec{p},t)\sum_{\vec{x}}\frac{1}{4\pi}\; e^{-i\vec{q}\cdot\vec{x}}J_\mu(\vec{x},t_2)J_\nu(0,t_1)\chi_N^\dagger(\vec{p},0)>}{\mathrm{Tr}<\Gamma_e\chi_N(\vec{p},t)\chi_N^\dagger(\vec{p},0)>}$$

$$\xrightarrow[t-t_2>>1/\Delta E_P,\; t_1>>1/\Delta E_P]{}$$

$$=\frac{1}{4\pi}\sum_n\left(\frac{2m_N}{2E_n}\right)\,\delta_{\vec{p}_n-\vec{p}-\vec{q}}\;\;<N(\vec{p})\,|\,J_\mu\,|\,n><n\,|\,J_\nu\,|\,N(\vec{p})>_{\text{spin avg}}\;\;e^{-(E_n-E_P)\tau}$$

$$=\;<N(\vec{p})\,|\sum_{\vec{x}}\frac{e^{-i\vec{q}\cdot\vec{x}}}{4\pi}J_\mu(\vec{x},\tau)J_\nu(0,0)\,|\,N(\vec{p})>_{\text{spin avg}}$$

KFL and S.J. Dong, PRL 72, 1790 (1994)
KFL, PRD 62, 074501 (2000)

Inverse Laplace transform – formally correct but not practical

$$W_{\mu\nu}(\vec{q},\vec{p},\nu)=\frac{1}{i}\int_{c-i\infty}^{c+i\infty}d\tau\;e^{\nu\tau}\;\tilde{W}_{\mu\nu}(\vec{q},\vec{p},\tau)$$

Laplace transform

$$\tilde{W}_{\mu\nu}(p,\vec{q},\tau)=\int d\nu W_{\mu\nu}(p,\vec{q},\nu)e^{-\nu\tau}$$

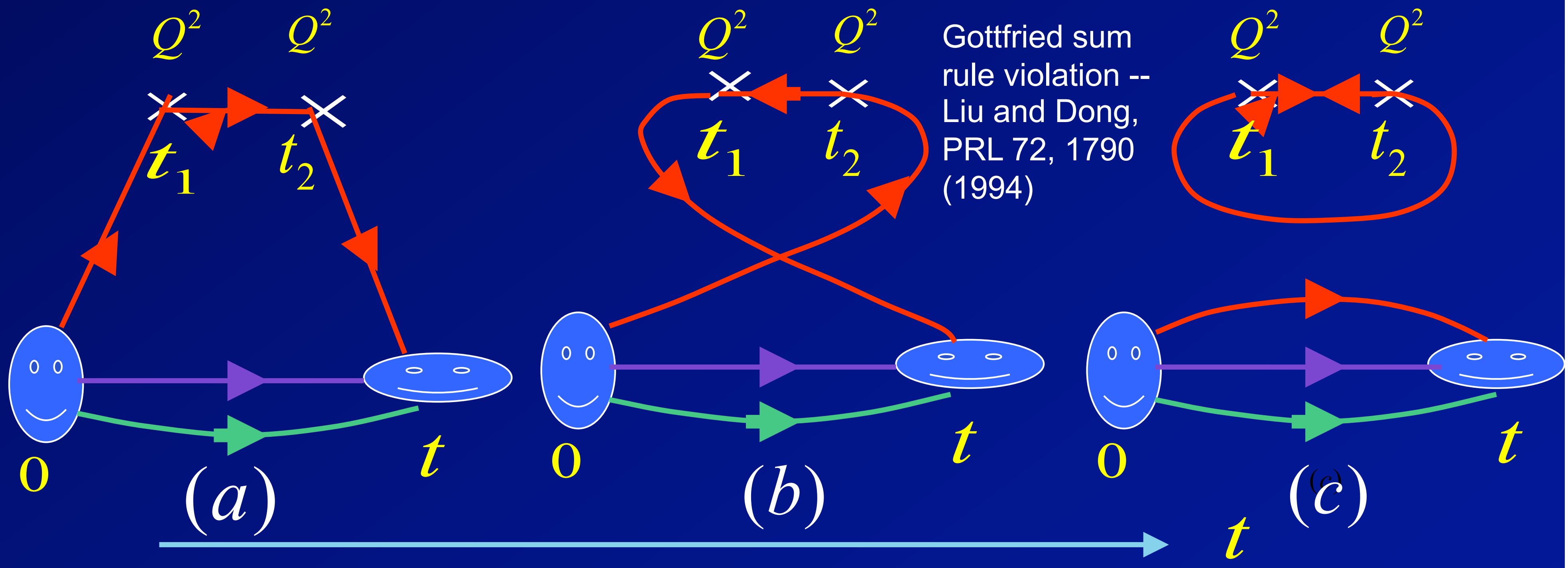
→ inverse problem

Hadronic Tensor

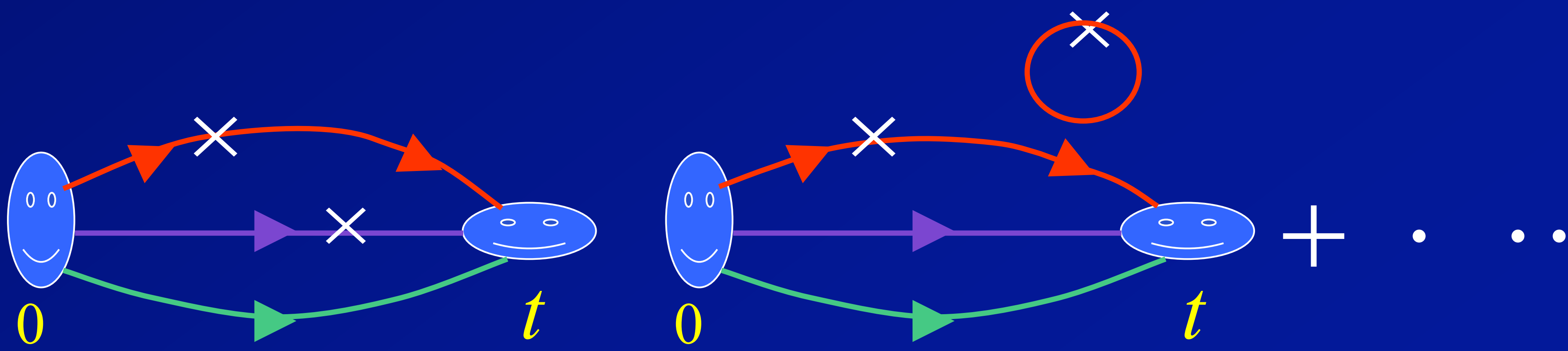
$$q = q_V + q_{CS}$$

Connected sea $\bar{q}_{CS}$

Disconnecrted sea $q_{DS} = (\neq ?)\ \bar{q}_{DS}$



Gottfried sum rule violation -- Liu and Dong, PRL 72, 1790 (1994)



Cat's ears diagrams are suppressed by O(1/Q²). $q_i^- = q_i^{v+cs} - \bar{q}_i^{cs} + q_i^{ds} - \bar{q}_i^{ds} \equiv q_i^v + q_i^{ds} - \bar{q}_i^{ds}$

Kinematics

$$\tilde{W}_{\mu\nu}(p,\vec{q},\tau) = \frac{E_p}{m_N}\frac{\mathrm{Tr}[\Gamma_e C_4]}{\mathrm{Tr}[\Gamma_e C_2]} \to \sum_{\vec{x}_2\vec{x}_1} e^{-i\vec{q}\cdot(\vec{x}_2-\vec{x}_1)}\langle p,s\,|\,J_\mu(\vec{x}_2,t_2)J_\nu(\vec{x}_1,t_1)\,|\,p,s\rangle$$

$$\tilde{W}_{\mu\nu}\left(p,\vec{q},\tau\right) = \int d\nu W_{\mu\nu}\left(p,\vec{q},\nu\right)e^{-\nu\tau}$$

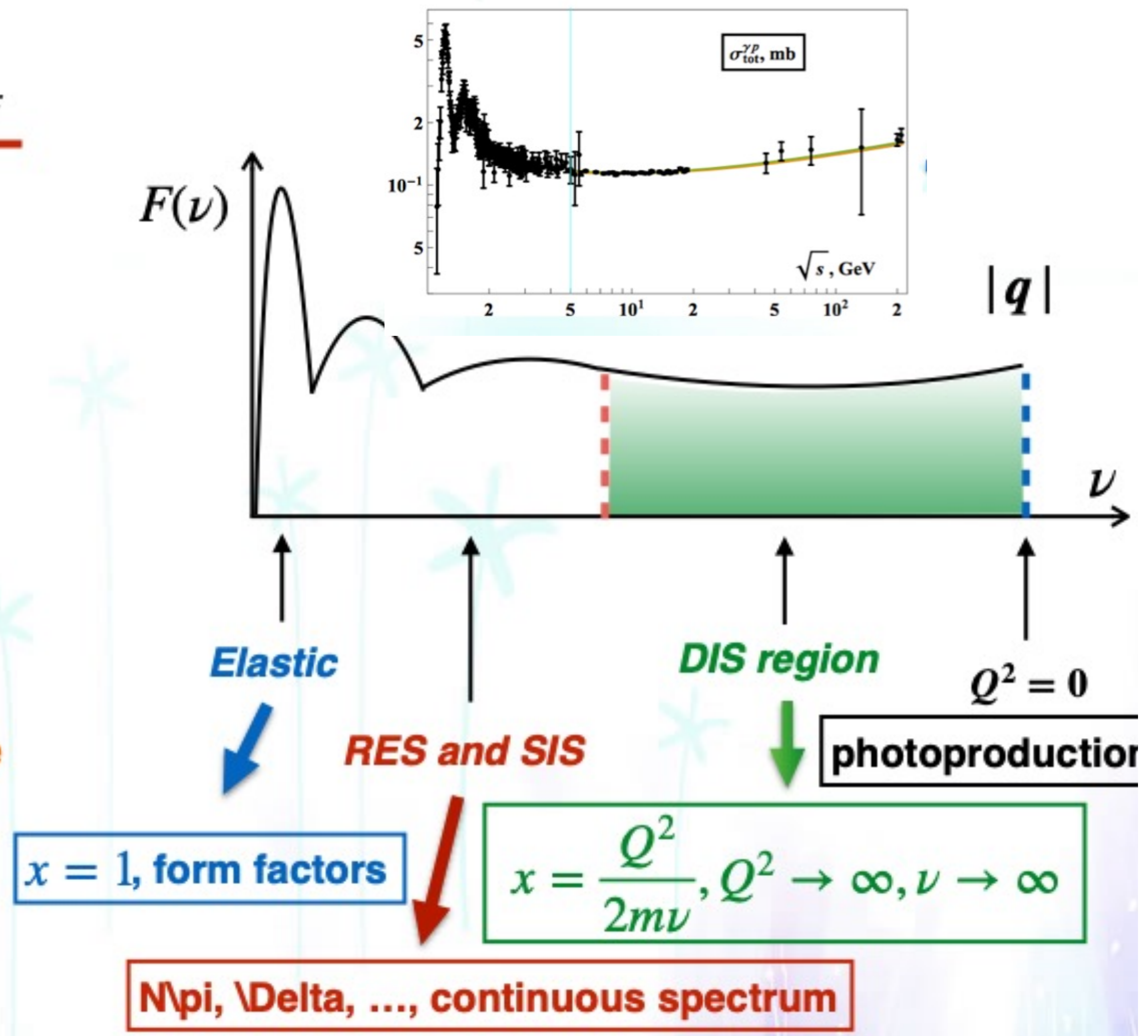
$$Q^2 = -q^2 = |\vec{q}|^2 - \nu^2$$

$$x = \frac{Q^2}{2m\nu}$$

$$W^2 = m^2 + 2m\nu - Q^2$$

1. Both Q^2 and ν need to be large (difficulty?)

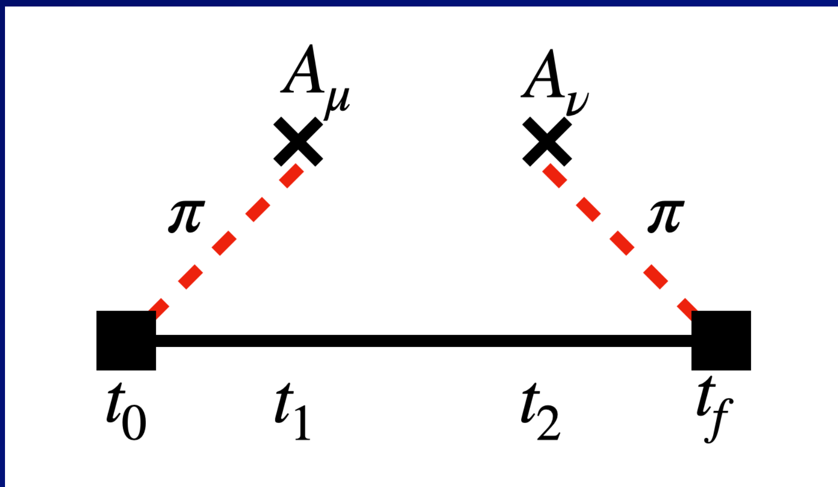
2. Will have a range of ν (feature?)

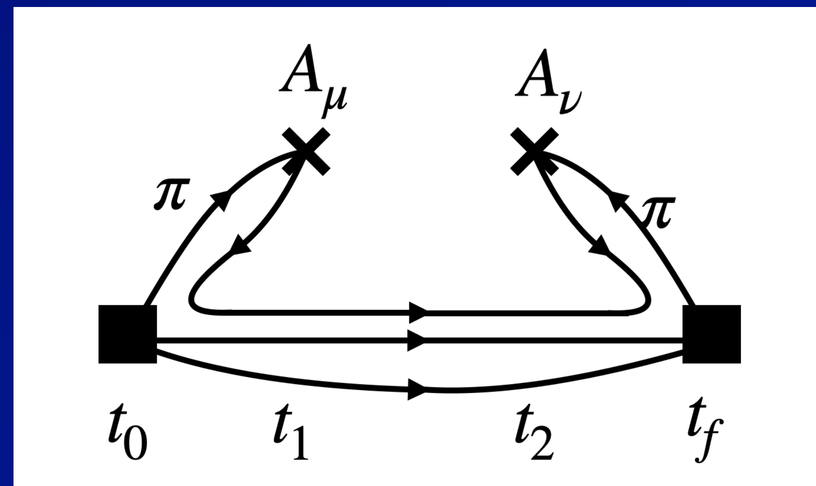


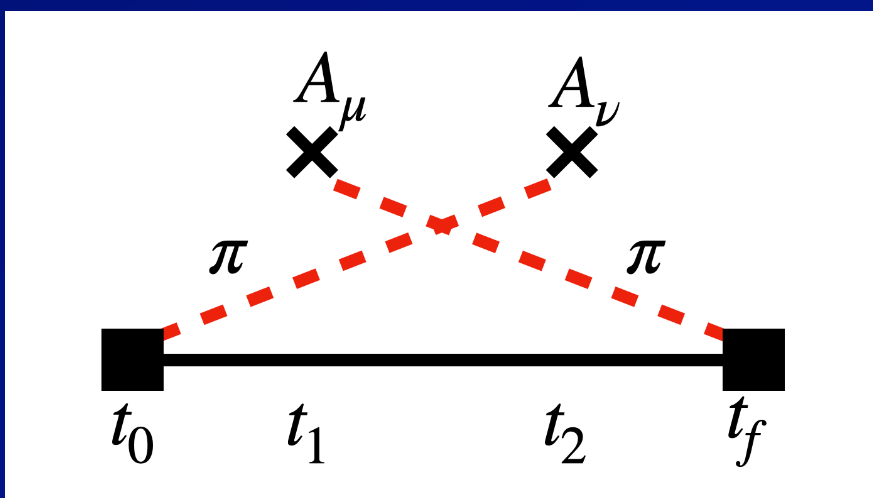
x

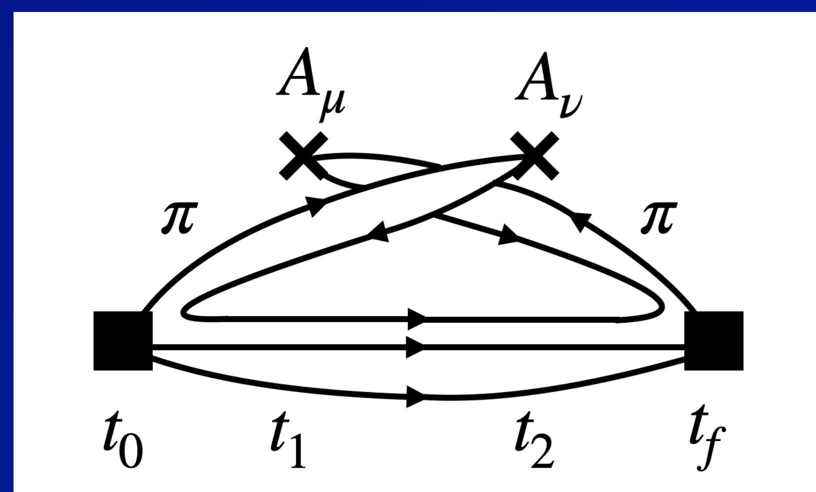
Neutrino-nucleon Scattering

- Excited state contamination



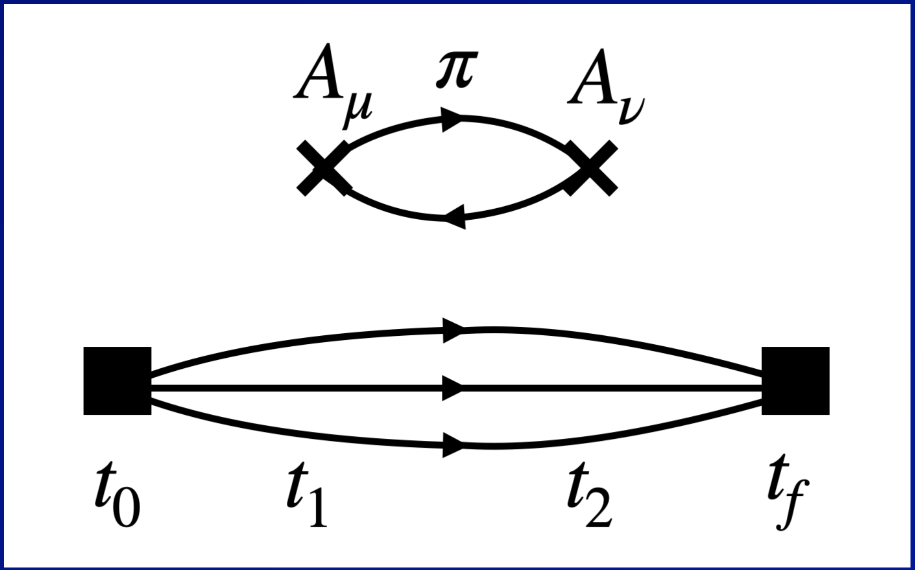




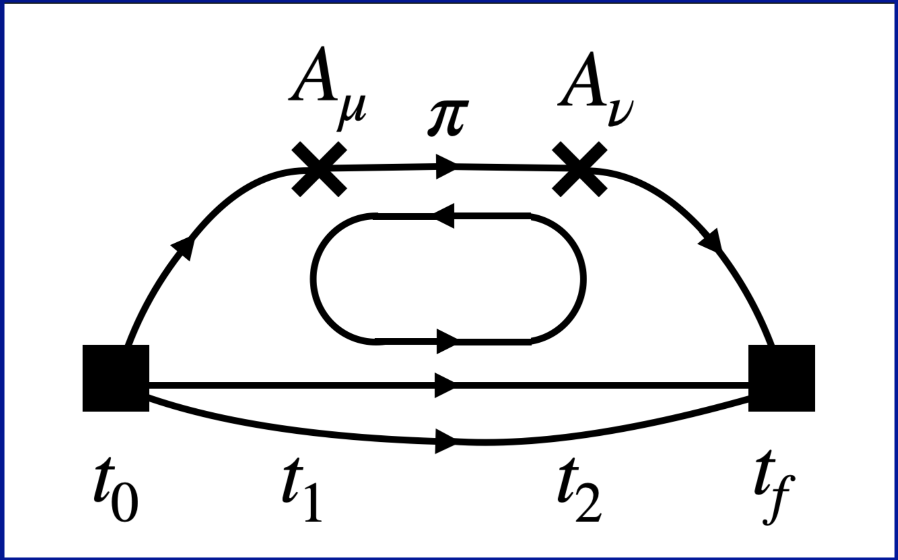


Contribution of <N|J|Nπ> m.e. in neutrino-nucleon scattering cross section

A and D types



B and C types



Summary

- Spectral decomposition of the nucleon two-, three-, and four-point functions are needed for nucleon form factor and neutrino-nucleon scattering calculations in lattice QCD.

- Prior for the πN state is essential for its extraction from the ½$^-$ correlator to obtain the correct S_{11} energy. The existence of the πN state is established from the wall source correlator. This is consistent with Bayesian Reconstruction of the nucleon correlator which sees the Roper without πN state.

- πN contamination due to the induced current is absent in the nucleon two-point function. Including it as a prior leads to the resolution of the current algebra violation.

- Lattice data are not precise enough to discern the small contributon of the πN in two-point function and its current induced volume enhancement. It is essential to obtain relevant priors from theory input and independent lattice data.