Hadronic Tensor with Lattice QCD
- Neutrino-Nucleon Scattering

- Neutrino-nucleon scattering via Euclidean hadronic tensor

- Inverse problem algorithms

- Elastic scattering and form factors
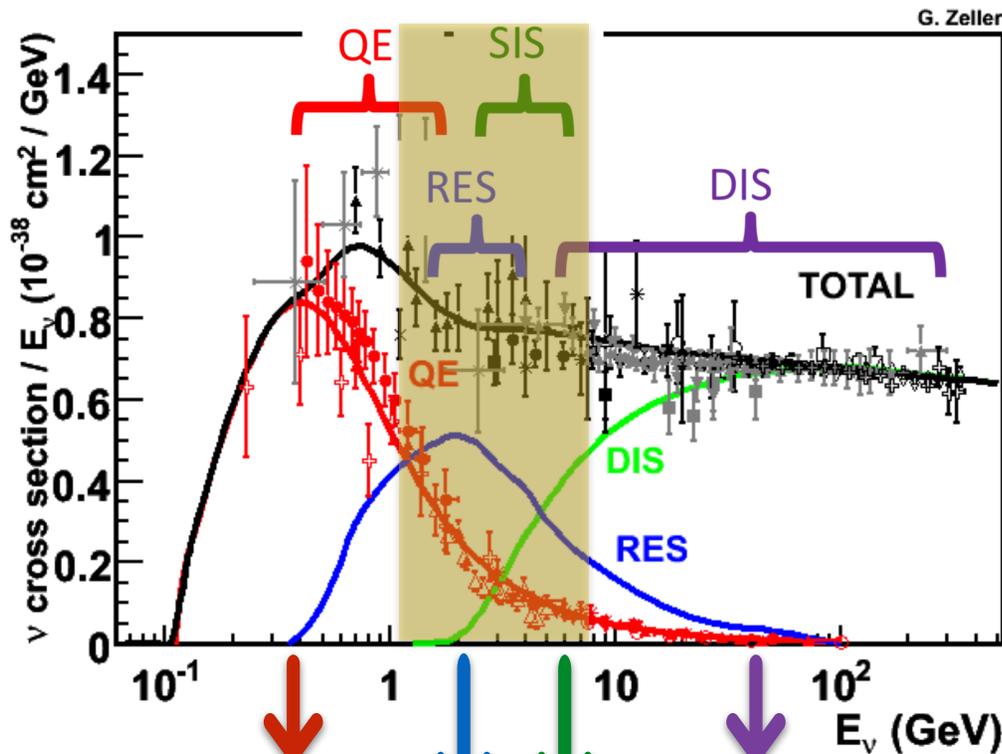
- Resonances and DIS

Parton Workshop, INT, Sept. 12, 2022

Hadronic tensor and neutrino-nucleus scattering

✦ New long-baseline neutrino experiments are in preparation: T2K, NOvA, PINGU, ORCA, Hyper-Kamiokande, DUNE...

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Omar Benhar

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J.A. Formaggio and G.P. Zeller, RMP84, 1307 (2012)

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Whitepapers on Neutrino-Nucleus Scattering

- USQCD – A. Kronfeld et al. (Kronfeld:2019nfb)

- Snowmass – L.A. Ruso et al. (Ruso:2022qes)

Hadronic Tensor in Euclidean Path-Integral Formalism

- Lepton Nucleon scattering
  In Minkowski space

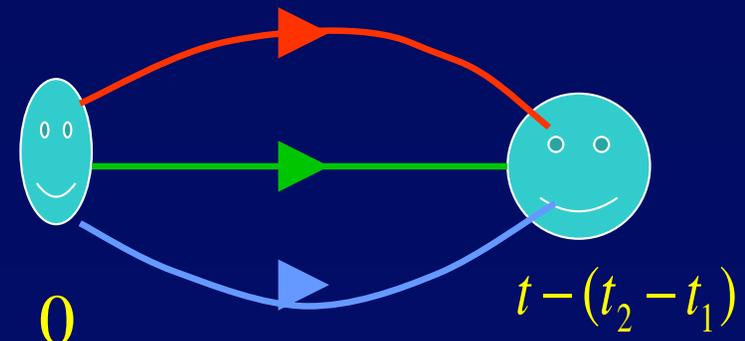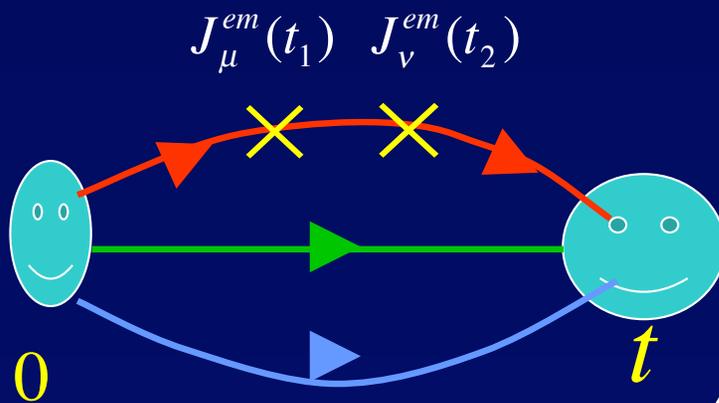$$\frac{d^2\sigma}{dE'd\Omega} = \frac{\alpha^2}{q^4}(\frac{E'}{E})l^{\mu\nu}W_{\mu\nu}$$

$$W_{\mu\nu}(\vec{q},\vec{p},\nu) = \frac{1}{\pi}\text{Im}\,T_{\mu\nu} = \left\langle N(\vec{p})\Big|\int\frac{d^4x}{4\pi}\,e^{iq\cdot x}J_\mu(x)J_\nu(0)\Big|N(\vec{p})\right\rangle_{\text{spin avg}}$$

$$= \frac{1}{2}\sum_n\int\prod_{i=1}^{n}\left[\frac{d^3p_i}{(2\pi)^3 2E_{pi}}\right](2\pi)^3\delta^4(p_n-p-q)<N(\vec{p})|J_\mu|n><n|J_\nu|N(\vec{p})>_{\text{spin avg}}$$

$$W_{\mu\nu}(p,q) = -W_1(q^2,\nu)(g_{\mu\nu}-\frac{q_\mu q_\nu}{q^2})+W_2(q^2,\nu)(p_\mu-\frac{p\cdot q}{q^2}q_\mu)(p_\nu-\frac{p\cdot q}{q^2}q_\nu)$$

- Euclidean path-integral

KFL and S.J. Dong, PRL 72, 1790 (1994)
KFL, PRD 62, 074501 (2000)

$J_\mu^{em}(t_1)$ $J_\nu^{em}(t_2)$

0 t

0 $t-(t_2-t_1)$

W$_{\mu\nu}$ in Euclidean Space

$$\tilde{W}_{\mu\nu}(\vec{q},\vec{p},\tau = t_2 - t_1) = \frac{\frac{E_P}{M_N}\text{Tr} < \Gamma_e \chi_N(\vec{p},t)\sum_{\vec{x}}\frac{1}{4\pi}\ e^{-i\vec{q}\cdot\vec{x}}J_\mu(\vec{x},t_2)J_\nu(0,t_1)\chi_N^\dagger(\vec{p},0) >}{\text{Tr} < \Gamma_e \chi_N(\vec{p},t)\chi_N^\dagger(\vec{p},0) >}$$

$$\xrightarrow{t-t_2 \gg 1/\Delta E_P,\ t_1 \gg 1/\Delta E_P}$$

$$= \frac{1}{4\pi}\sum_n (\frac{2m_N}{2E_n})\ \delta_{\vec{p}_n - \vec{p} - \vec{q}}\ < N(\vec{p})|J_\mu|n><n|J_\nu|N(\vec{p}) >_{\text{spin avg}}\ e^{-(E_n-E_P)\tau}$$

$$=\ < N(\vec{p})|\sum_{\vec{x}}\frac{e^{-i\vec{q}\cdot\vec{x}}}{4\pi}J_\mu(\vec{x},\tau)J_\nu(0,0)|N(\vec{p}) >_{\text{spin avg}}$$

Inverse Laplace transform – formally correct but not practical

$$W_{\mu\nu}(\vec{q},\vec{p},\nu) = \frac{1}{i}\int_{c-i\infty}^{c+i\infty} d\tau\ e^{\nu\tau}\ \tilde{W}_{\mu\nu}(\vec{q},\vec{p},\tau)$$
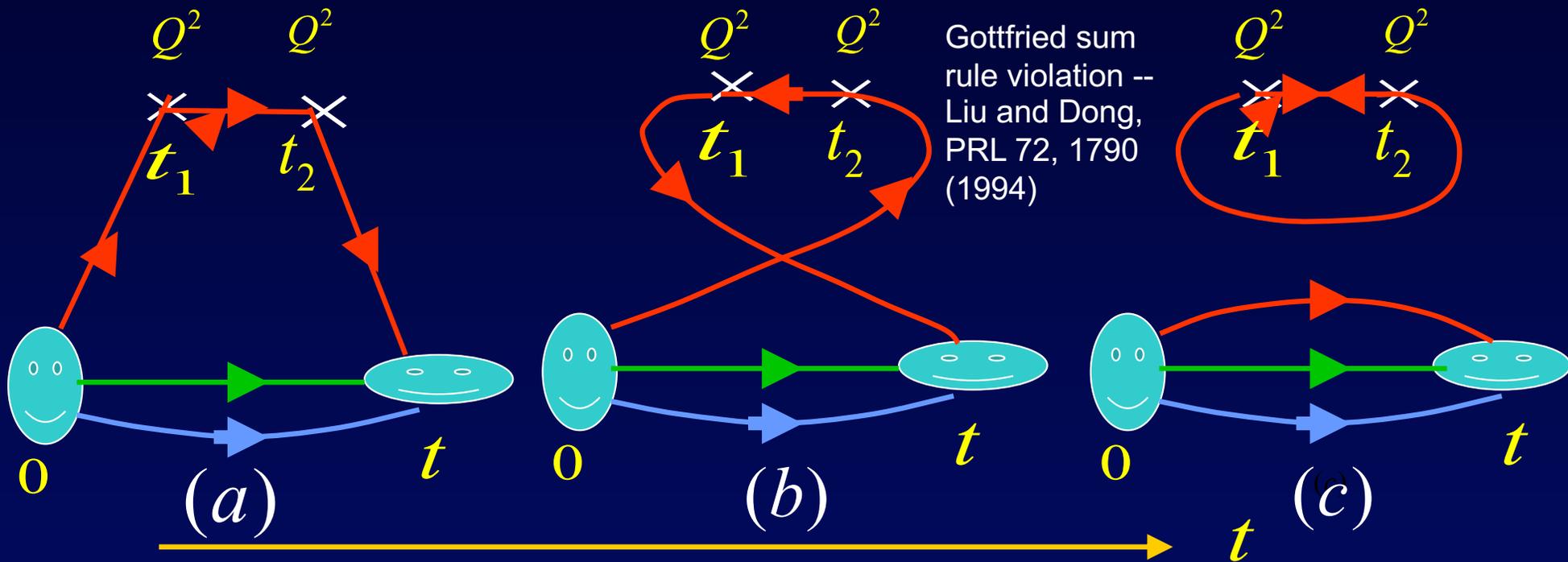
Laplace transform

$$\tilde{W}_{\mu\nu}(p,\vec{q},\tau) = \int d\nu W_{\mu\nu}(p,\vec{q},\nu)e^{-\nu\tau} \implies \text{inverse problem}$$
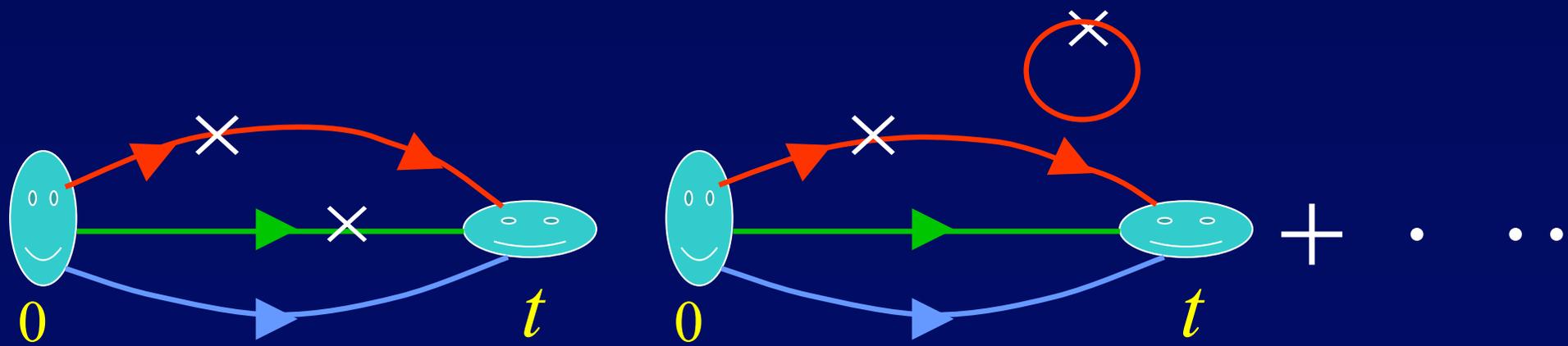
Hadronic Tensor
$$q = q_V + q_{CS}$$

Connected sea $\overline{q}_{CS}$

Disconnected sea $q_{DS} = (\neq ?) \, \overline{q}_{DS}$

Gottfried sum rule violation -- Liu and Dong, PRL 72, 1790 (1994)

(a) (b) (c)

Cat's ears diagrams are suppressed by O(1/Q²). $\quad q_i^- = q_i^{v+cs} - \overline{q}_i^{cs} + q_i^{ds} - \overline{q}_i^{ds} \equiv q_i^v + q_i^{ds} - \overline{q}_i^{ds}$

Kinematics

$$\tilde{W}_{\mu\nu}(p,\vec{q},\tau) = \frac{E_p}{m_N}\frac{\text{Tr}[\Gamma_e C_4]}{\text{Tr}[\Gamma_e C_2]} \rightarrow \sum_{\vec{x_2}\vec{x_1}} e^{-i\vec{q}\cdot(\vec{x_2}-\vec{x_1})}\langle p,s\,|\,J_\mu(\vec{x_2},t_2)J_\nu(\vec{x_1},t_1)\,|\,p,s\rangle$$

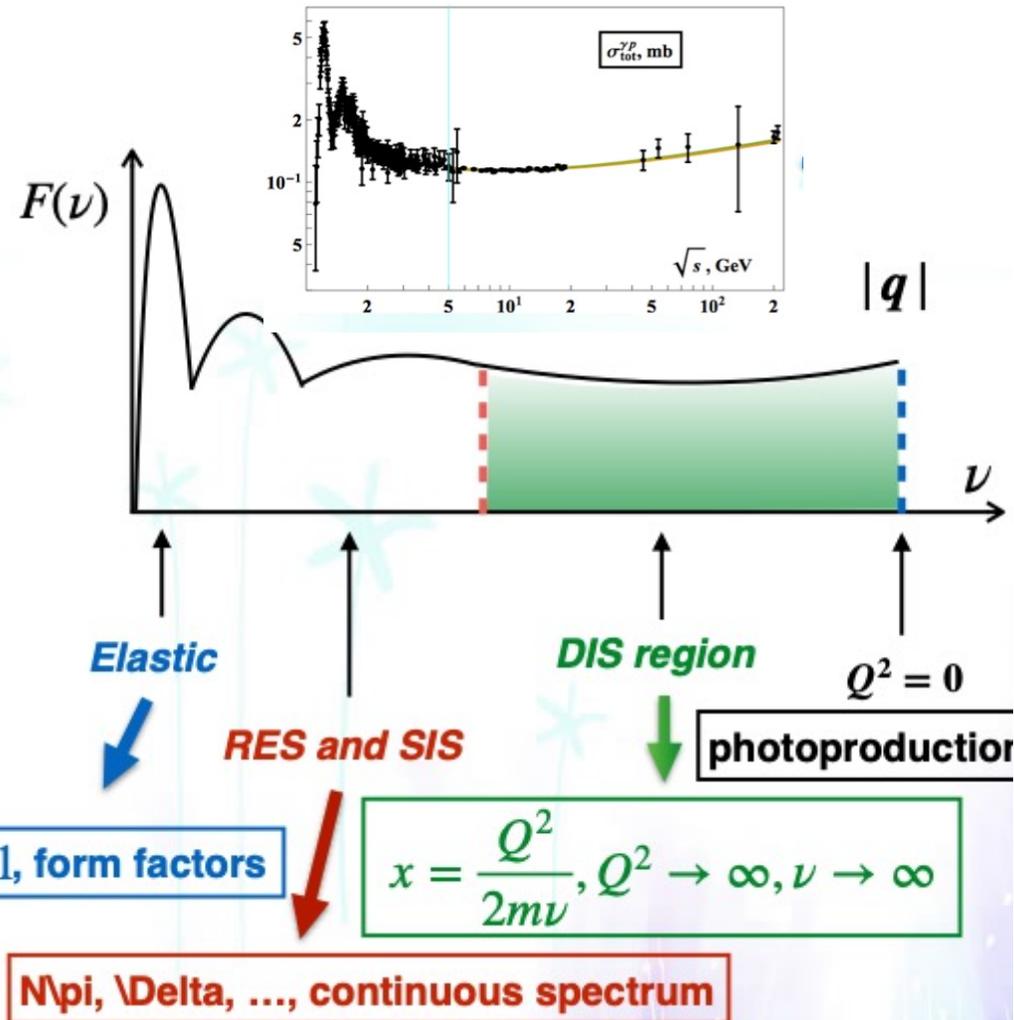$$\tilde{W}_{\mu\nu}\left(p,\vec{q},\tau\right) = \int d\nu\, W_{\mu\nu}\left(p,\vec{q},\nu\right)e^{-\nu\tau}$$

$$Q^2 = -q^2 = |\vec{q}|^2 - \nu^2$$

$$x = \frac{Q^2}{2m\nu}$$

$$W^2 = m^2 + 2m\nu - Q^2$$

1. Both Q^2 and ν need to be large (difficulty?)

2. Will have a range of ν (feature?)

$x = 1$, **form factors**

$x = \dfrac{Q^2}{2m\nu}, Q^2 \rightarrow \infty, \nu \rightarrow \infty$

N\pi, \Delta, …, continuous spectrum

Elastic   *RES and SIS*   *DIS region*   photoproduction

$Q^2 = 0$

Inverse problems are ubiquitous

✦ Extracting spectral functions from lattice data: $c_2(t) = \int d\omega\, e^{-\omega t} \boldsymbol{\rho(\omega)}$

Y. Burnier and A. Rothkopf, PRL 111, 182003 (2013)

✦ Global fittings of PDFs: $F_i = \sum_a C_i^a \otimes \boldsymbol{f_a}$

✦ Lattice calculation of Quasi-PDFs: $\tilde{q}(x, P_3) = \dfrac{2P_3}{4\pi} \displaystyle\sum_{z=-z_{\max}}^{z_{\max}} e^{-ixP_3 z}\, h_\Gamma(P_3, z)$

J. Karpie et. al., JHEP04, 057 (2019)

$$\tilde{q}(x, \mu^2, P^z) = \int_0^1 \frac{dy}{y} Z\left(\frac{x}{y}, \frac{\mu}{P^z}\right) q(y, \mu^2) + \mathcal{O}\left(\Lambda^2/(P^z)^2, M^2/(P^z)^2\right)$$

X. Xiong et. al., PRD90:014051 (2014)

✦ Lattice cross sections:

$$\sigma_n(\omega, \xi^2, P^2) = \sum_a \int_{-1}^1 \frac{dx}{x} f_a(x, \mu^2) \times K_n^a(x\omega, \xi^2, x^2 P^2, \mu^2) + O(\xi^2 \Lambda_{\mathrm{QCD}}^2)$$
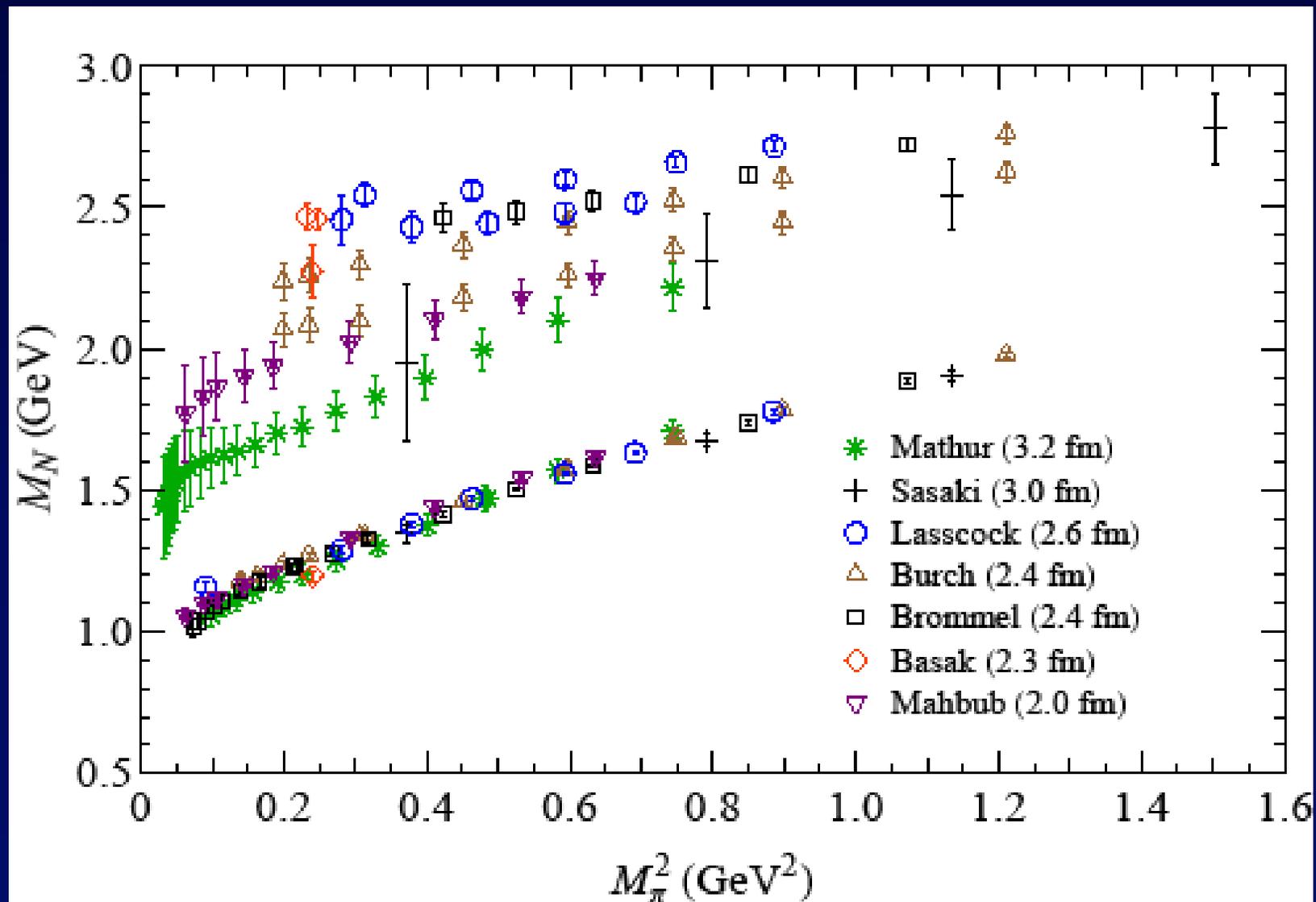
Y.-Q. Ma and J.-W. Qiu, PRL120, 022003 (2018)

✦ Lattice calculation of Pseudo-PDFs: $\mathfrak{M}_R(\nu, \mu^2) \equiv \int_0^1 dx\, \cos(\nu x)\, q_v(x, \mu^2)$
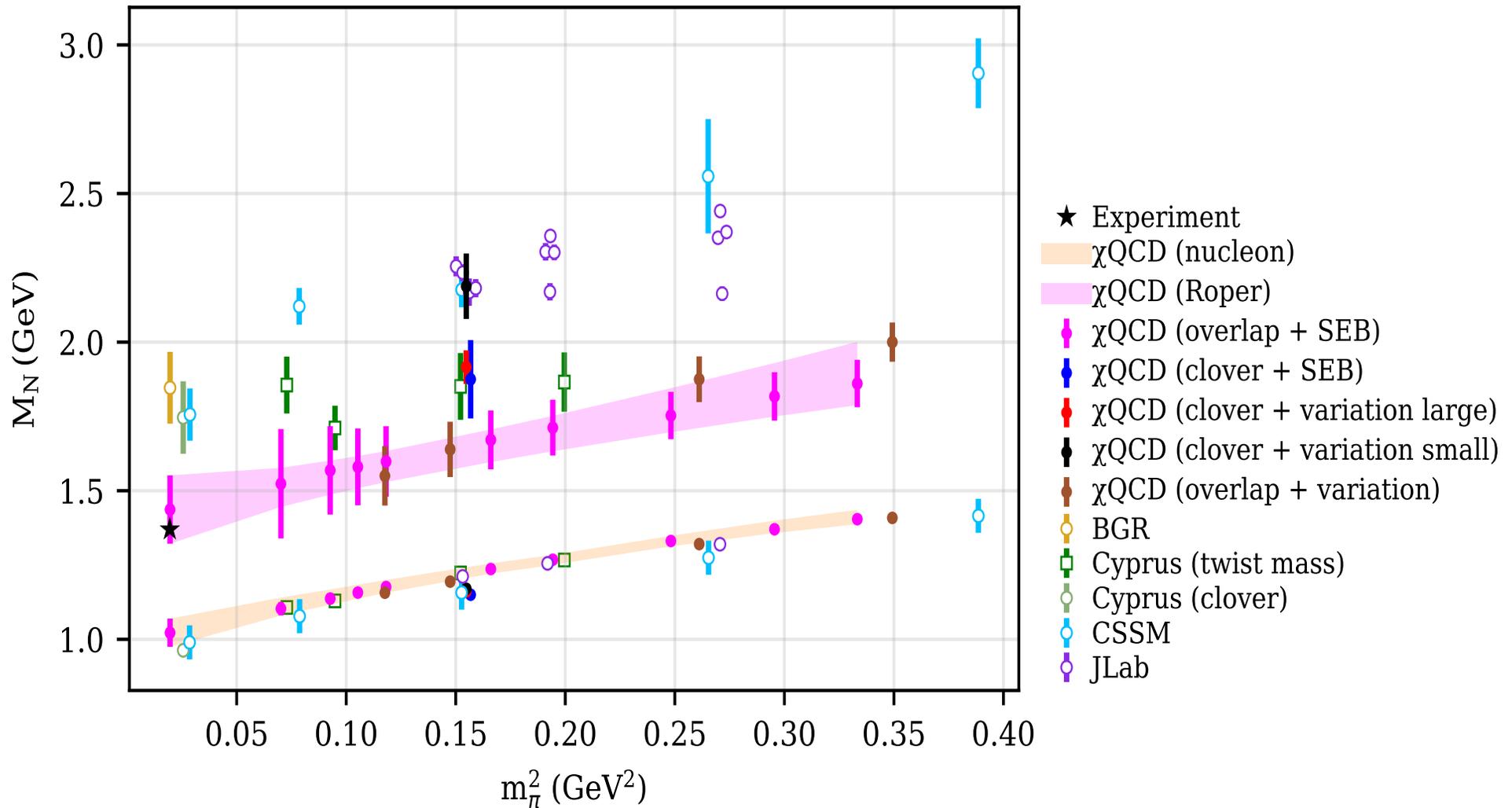
K. Orginos et al., PRD96, 094503 (2017)

X

Roper Puzzle
Quenched Lattice Calculations

Dynamical Fermions

M. Sun et al.
1911.02635

Visualize 2-point Functions

◆ **Backus-Gilbert (BG)**　　*G. Backus and F. Gilbert, Geophysical Journal International 16, 169 (1968)*

◆ **Maximum Entropy (ME)**　　*E Rietsch et. al., JOURNAL OF GEOPHYSICS, 42:489 (1977)*
　　　　　　　　　　　　　　　M. Asakawa et. al., Prog. Part. Nucl. Phys. 46, 459 (2001)

◆ **Bayesian Reconstruction (BR)**

　　　　　　　　　　　Y. Burnier and A. Rothkopf, PRL 111, 182003 (2013)

$$P[\omega \,|\, D, \alpha, m] \propto e^{Q'(\omega)} \qquad Q' = \alpha S - L - \gamma(L - N_\tau)^2$$

$$S = \sum_\nu \left[1 - \frac{\omega(\nu)}{m(\nu)} + \log\left(\frac{\omega(\nu)}{m(\nu)}\right) \right] \Delta\nu$$

$$P[\omega \,|\, D, m] = \frac{P[D \,|\, \omega, I]}{P[D \,|\, m]} \int d\alpha P[\alpha \,|\, D, m]$$

Hyper parameter alpha is integrated over

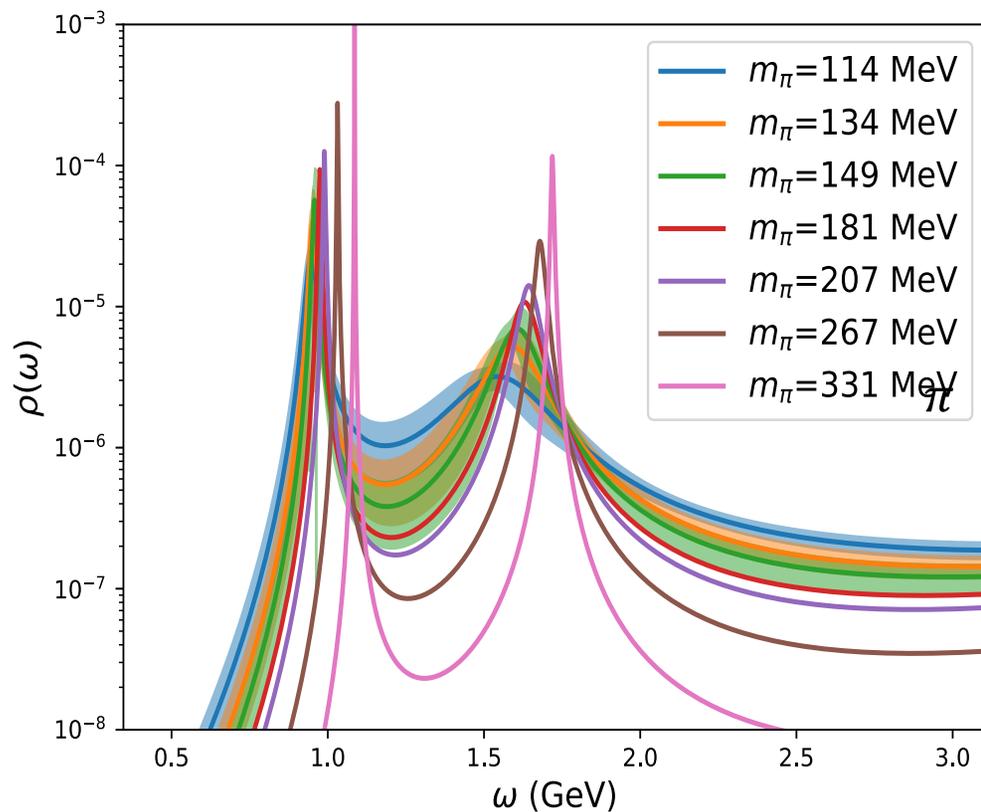Maximum search is in the entire parameter space (O(10³))

High precision architecture is needed (e.g., 512-bit floating point number).

Overlap fermion on 2+1 flavor DWF, $48^3 \times 96$, m_π = 139 MeV, L = 5.5 fm
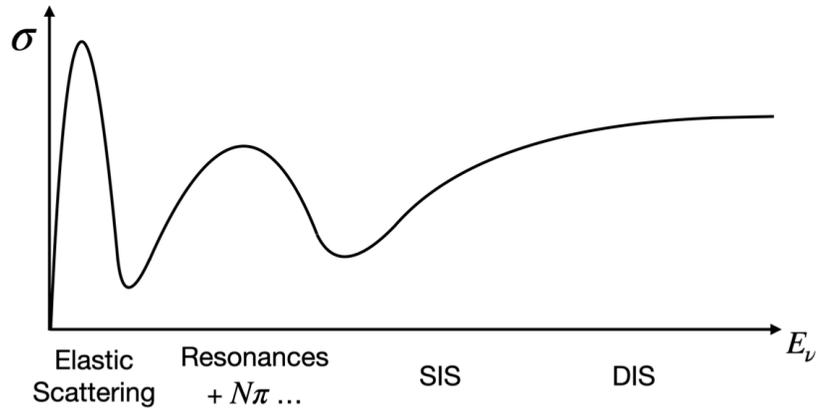

*RBC/UKQCD 48I, physical pion mass, a~0.11 fm*

Visualize 2-point Functions
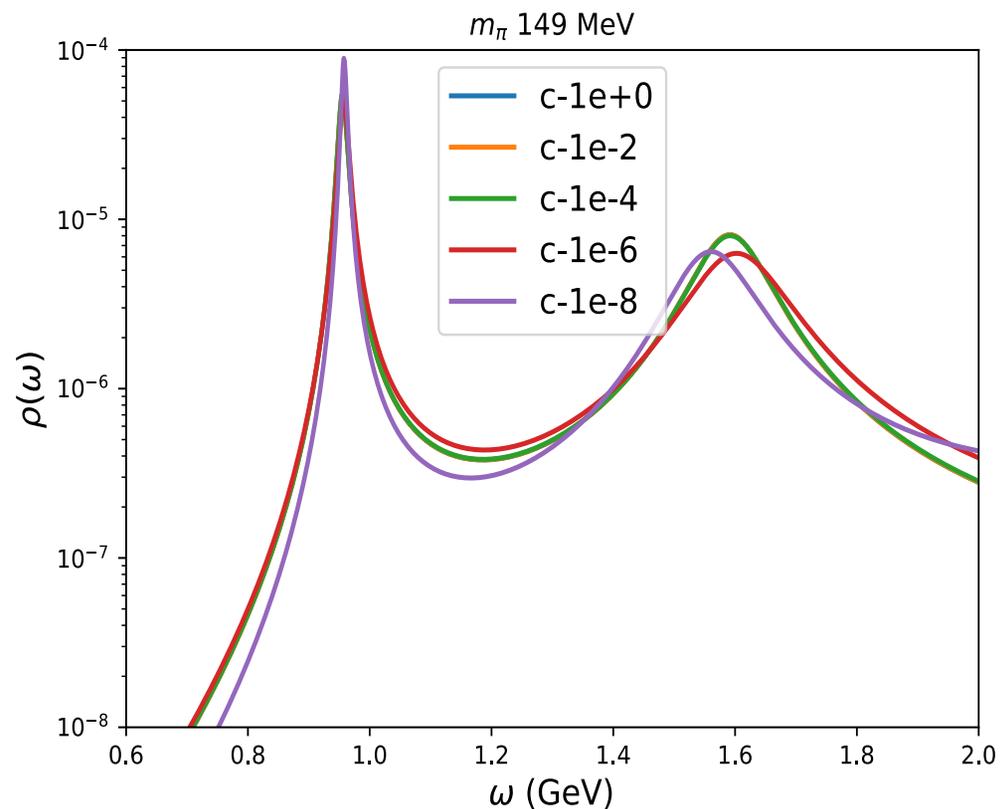
Overlap fermion on 2+1 flavor DWF,
$48^3 \times 96$, $m_\pi = 139$ MeV, $L = 5.5$ fm

Mock Data Test

Mock data contain two **isolated states of mass 1.0 and 1.5 GeV,** and **a constant dense spectrum (simulating the continuous spectrum) from 2 GeV to 15 GeV**. The lattice spacing is set to be 0.02 fm and the number of time slices is 100. The signal-to-noise ratio is set to be 100.
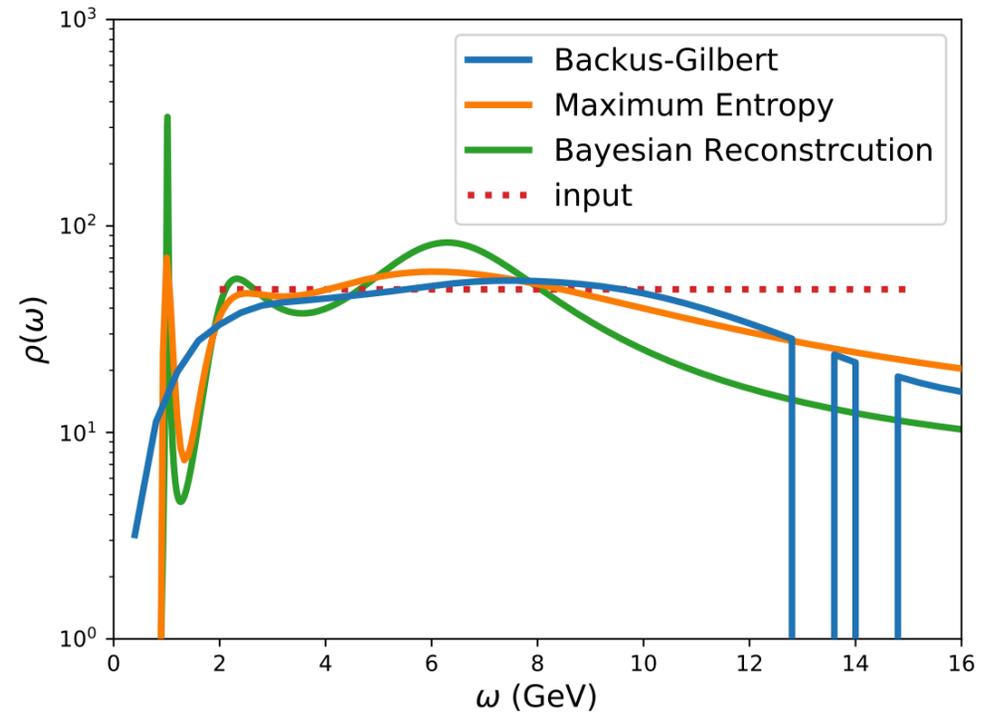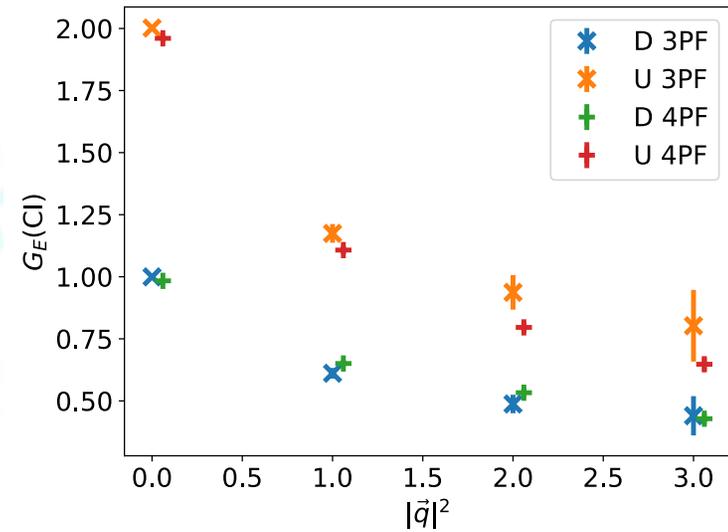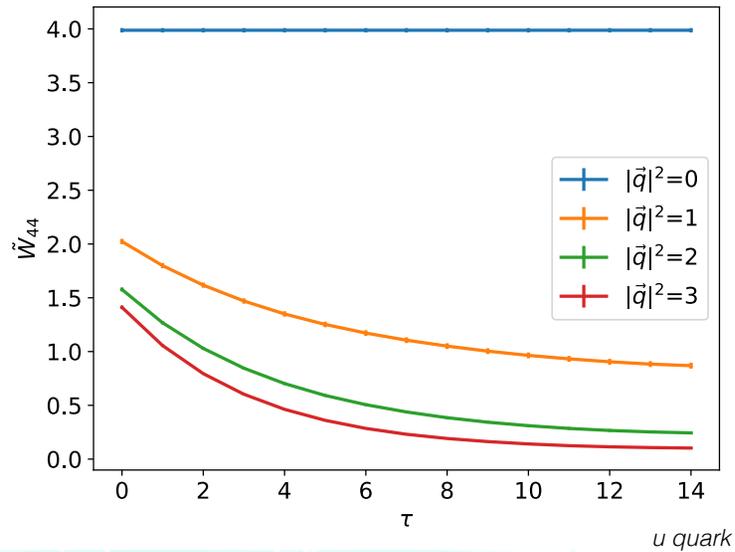
J. Liang et. al., Phys.Rev.D 101 (2020) 11, 114503
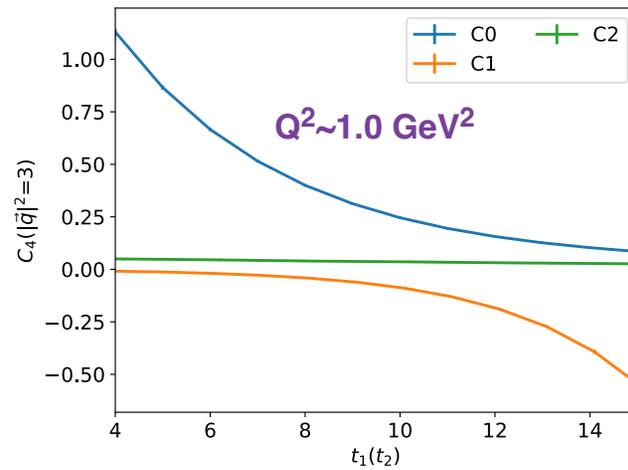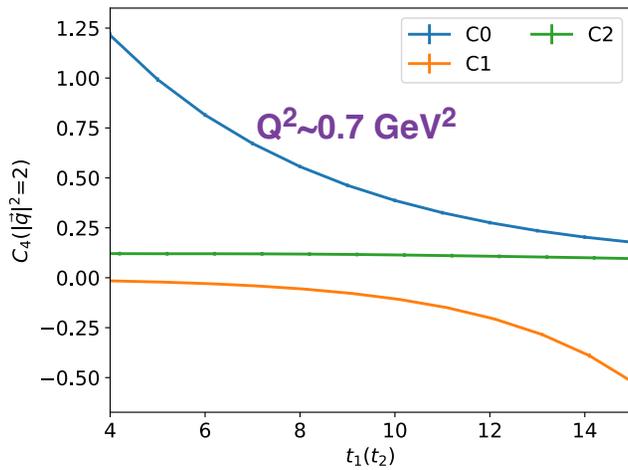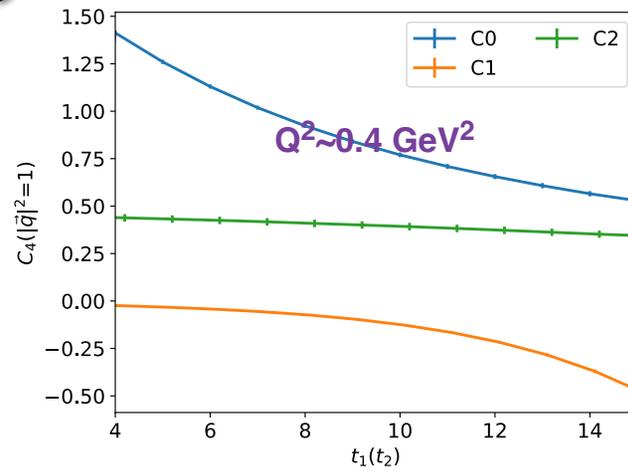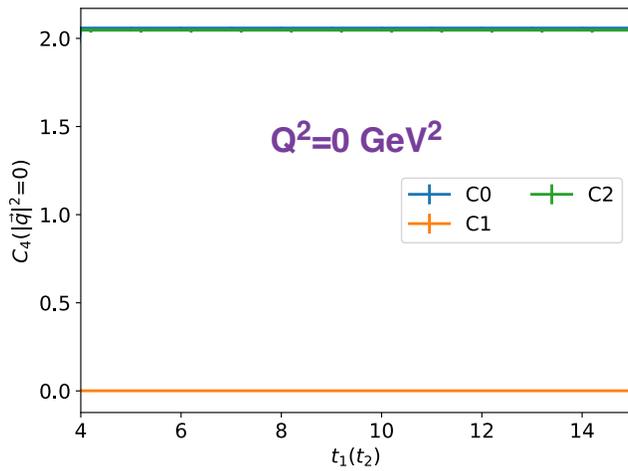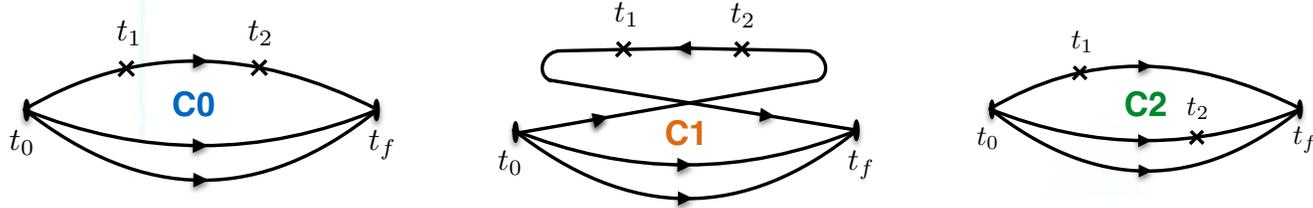
Form Factor Case for Elastic Scattering

$$\widetilde{W}_{44}(\vec{p}, \vec{q}, \tau) = \sum_{\vec{x}_2 \vec{x}_1} e^{-i\vec{q}\cdot(\vec{x}_2 - \vec{x}_1)} \langle p, s \,|\, J_4(\vec{x}_2, t_2) J_4(\vec{x}_1, t_1) \,|\, p, s \rangle = \sum_n A_n e^{-\nu_n \tau}$$

$$A_0 = \langle p, s \,|\, J_4(\vec{q}) \,|\, n = 0 \rangle \langle n = 0 \,|\, J_4(-\vec{q}) \,|\, p, s \rangle = G_E^2(Q^2)$$

RBC/UKQCD 32IF lattice (a ~ 0.063 fm, pion mass ~ 370 MeV), clover on domain wall
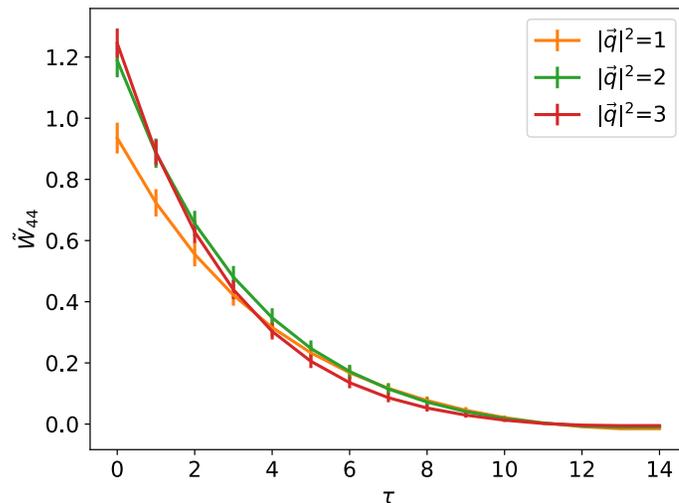

u quark
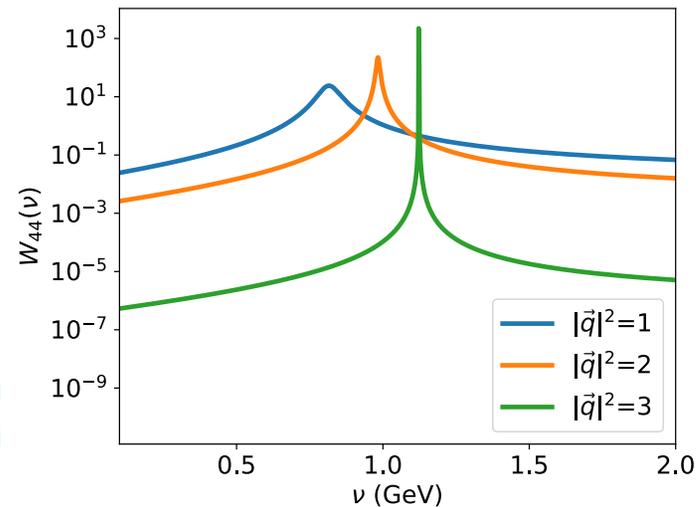
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Higher twist contribution

Beyond the Elastic Contribution

u quark with elastic contribution subtracted

Non-zero inelastic contribution

Need more data with finer lattice spacings and better understanding of the reconstruction
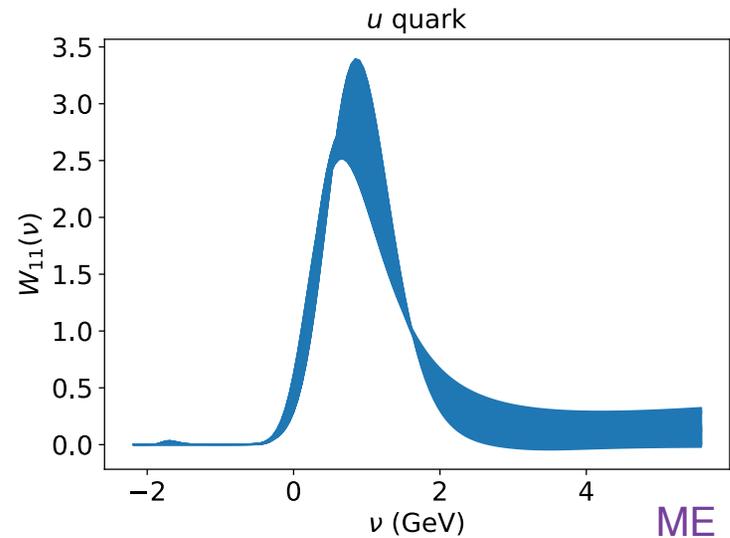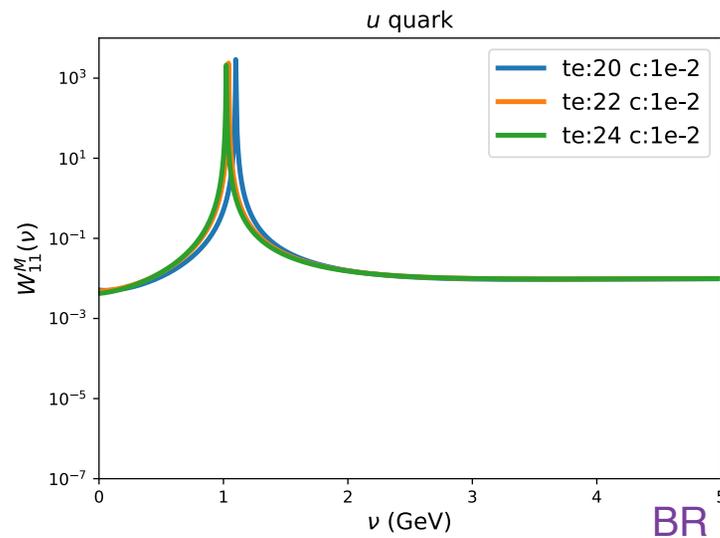
Compare different inverse methods

Phenomenological input

$$W^2 = M_N^2 + 2M_N\nu - Q^2$$

Large Momentum Transfer Case

J. Liang et al.
(Liang:2019frk)

$\mu = \nu = 1$ and $p_1 = q_1 = 0$ $\quad W_{11}(\nu) = F_1(x, Q^2)$

$24^3 \times 128$, $a_s \sim 0.12$ fm, $\xi \sim 3.5$, $m_\pi \sim 380$ MeV, $\dfrac{2\pi}{L} \sim 0.42$ GeV

p	q	E_p	$E_{n=0}$	$|q|$	ν	Q^2	x
(0,3,3)	(0,-6,-6)	2.15	2.15	3.57	[2.96, 3.28]	[4, 2]	[0.16, 0.07]

u quark

BR

u quark

ME

J. Liang et. al., Phys.Rev.D 101 (2020) 11, 114503

Fine lattice spacings!

Better solving the inverse problem!

$$W^2 = (p + q)^2 = M_N^2 - Q^2 + 2E_p\nu - 2\vec{p}\cdot\vec{q}$$

$Q^2 = 11.7$ GeV^2

W = 2.5 GeV

$$|\vec{p} + \vec{q}| = 1.8\,\text{GeV}$$

13

17

Summary and Challenges

- How high an excitation energy can be reached on today's lattice?

$$W^2 = (p + q)^2 = M_N^2 - Q^2 + 2E_p\nu - 2\vec{p}\cdot\vec{q}$$

$$\text{Take } |\tilde{p}| = 1\,\text{GeV}, |\tilde{q}| = 3\,\text{GeV}, \nu = 2.5\,\text{GeV} \longrightarrow Q^2 = 2.75\,\text{GeV}^2, W = 3.35\,\text{GeV}$$

- DIS is perhaps reachable on lattices with a = 0.06 fm or less, but may need to include multi-hadron interpolation field (?)

- Spectral decompositions in lattice 2-, 3-, and 4-point functions of baryons have seen limited success. Need better data.

- Spectral decomposition of 2- and 3-pt functions can be improved with a larger basis of interpolation fields in a variational approach. How about 4-pt functions?

- Can calculate the combination of V_4, V_i, A_4, A_i to isolate different resonance contributions in neutrino-nucleon scattering.