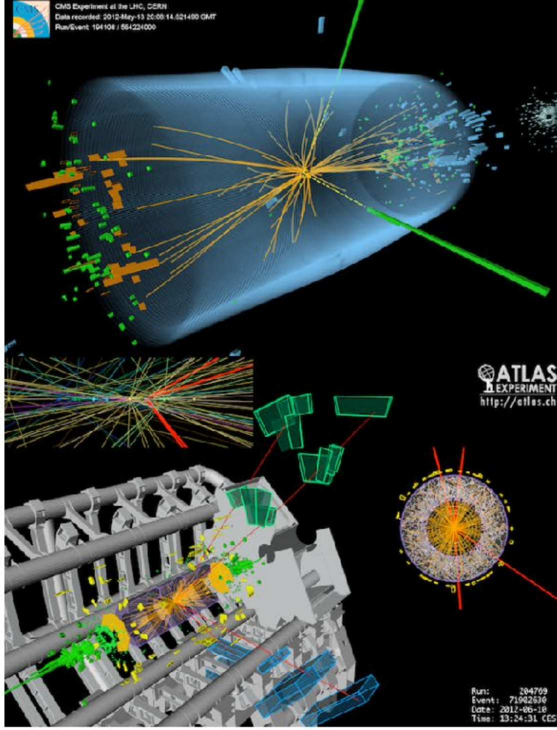
Hadrons, Type II Superconductors, and Cosmological Constant

PRD104,076010 (2021)
[arXiv:2103.15768]

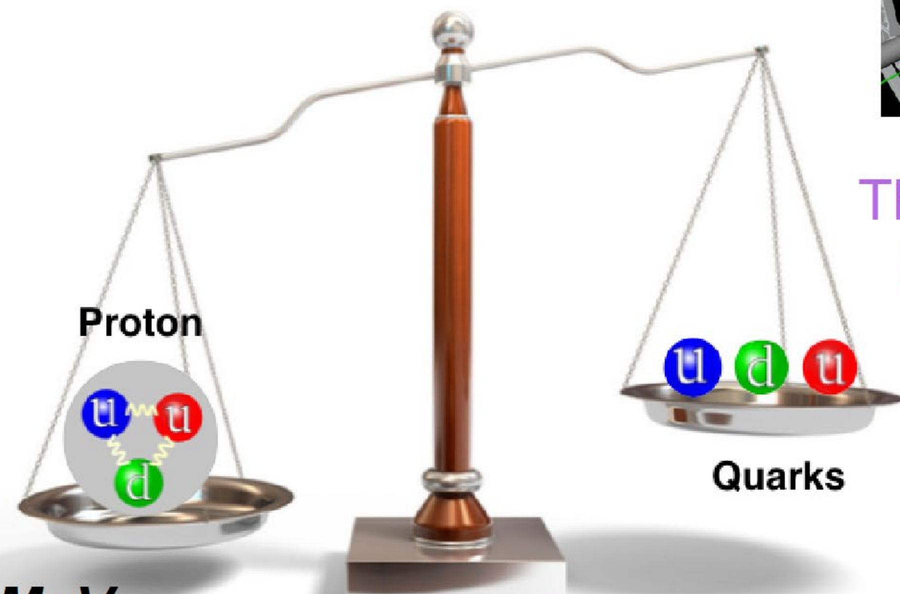
- Mass decomposition of hadrons from Hamiltonian and gravitational form factors
- Pressure balance of hadrons and role of trace anomaly
- Cosmological constant in Einstein's equation
- Type II superconductors
- Puzzle of pion mass and trace anomaly

June 17, 2022 INT

Motivation



Where does the proton mass come from, and how ?



But the mass of the proton is
938.272046(21) MeV.
~100 times of the sum of the quark masses!

The Higgs boson make the u/d quark having masses (2GeV MS-bar):

$m_u = 2.08(9)\ MeV$
$m_d = 4.73(12)\ MeV$

Laiho, Lunghi, & Van de Water, Phys.Rev.D81:034503,2010

Mass (Rest Energy) from Hamiltonian

- Energy momentum tensor (EMT)

$$T_{\mu\nu} = \frac{1}{4}\bar{\psi}\gamma_{(\mu}\overleftrightarrow{D}_{\nu)}\psi + G_{\mu\alpha}G_{\nu\alpha} - \frac{1}{4}\delta_{\mu\nu}G^2$$

- Separate the EMT into traceless part and trace part

$$T_R^{\mu\nu} = \overline{T}_R^{\mu\nu} + \frac{1}{4}\eta^{\mu\nu}(T_\rho^\rho)_R$$

X. Ji (1995)

- Hamiltonian -- $H = \int d^3\vec{x}\, T^{00}(x)$

- With equation of motion (scale dependent)

$$\begin{aligned} H_m &= \int d^3\vec{x} \sum_f m_f \bar{\psi}_f \psi_f , \\ H_E(\mu) &= \int d^3\vec{x} \sum_f (\psi_f^\dagger i\vec{\alpha}\cdot\vec{D}\psi_f)_M , \\ H_g(\mu) &= \int d^3\vec{x}\, \frac{1}{2}(B^2 + E^2)_M , \\ H_{tr} &= \int d^3\vec{x}\, \frac{1}{4}(T_\mu^\mu)_R . \end{aligned}$$

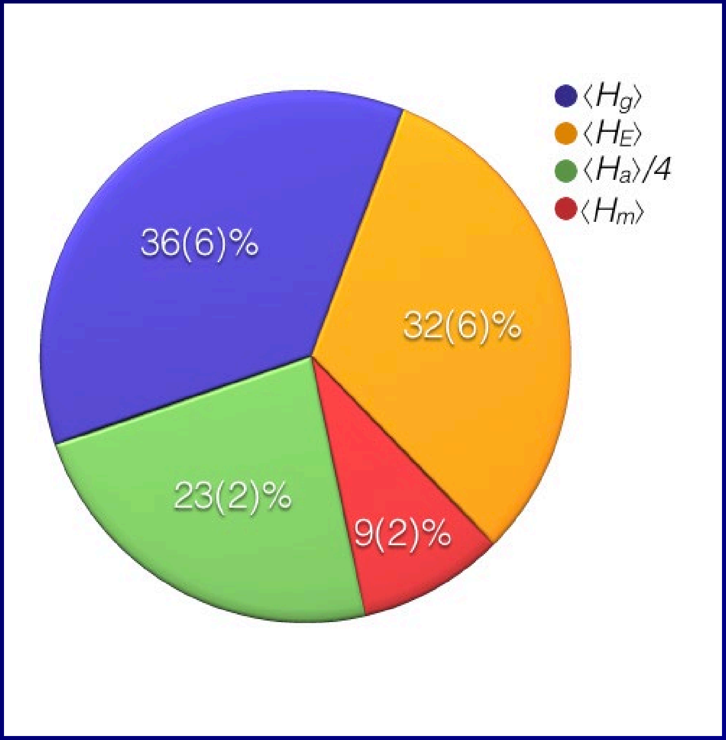
Quark kinetic and potential energy

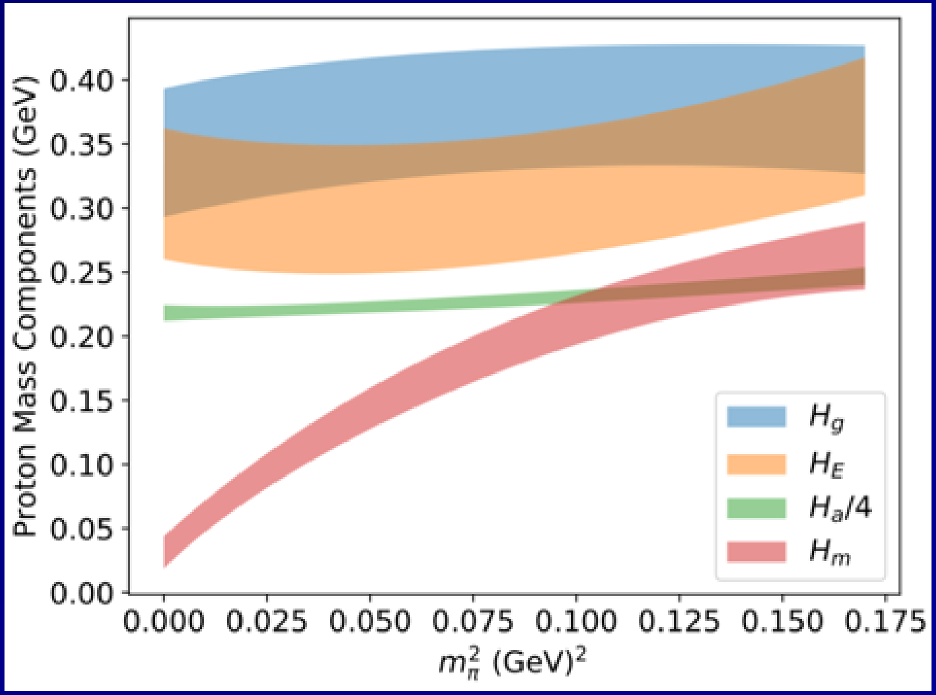
Glue field energy

$$E_0 = M = \langle H_M \rangle + \langle H_E(\mu)\rangle + \langle H_g(\mu)\rangle + \langle H_{tr}\rangle ,$$

Proton Mass Decomposition

Lattice calculation with systematics (physical pion mass, continuum, infinite volume extrapolations, renormalization)





Y.B. Yang et al (χQCD), PRL 121, 212001 (2018)
Physic 11, 118 (2018); ScienceNews, Nov. 16 (2018)

Experimentally Measurable Decomposition

- Separate the EMT into traceless part and trace part

$$T_R^{\mu\nu} = \overline{T}_R^{\mu\nu} + \frac{1}{4}\eta^{\mu\nu}(T_\rho^\rho)_R$$

- Hamiltonian -- $H = \int d^3\vec{x}\, T^{00}(x)$

$$H_q(\mu) = \int d^3\vec{x}\, (\frac{i}{4}\sum_f \bar{\psi}_f \gamma^{\{0} \overleftrightarrow{D}^{0\}} \psi_f - \frac{1}{4} T^\mu_{q\,\mu})_M ,$$

Quark momentum fraction (scale dependent)

$$H_g(\mu) = \int d^3\vec{x}\, \frac{1}{2}(B^2 + E^2)_M ,$$

Glue momentum fraction (scale dependent)

$$H_{tr} = \int d^3\vec{x}\, \frac{1}{4}(T^\mu_\mu)_R . \quad T^\mu_\mu = \sum_f m_f \bar{\psi}_f \psi_f + \left[\sum_f m_f \gamma_m(g) \bar{\psi}_f \psi_f + \frac{\beta(g)}{2g} F^{\alpha\beta} F_{\alpha\beta}\right]$$

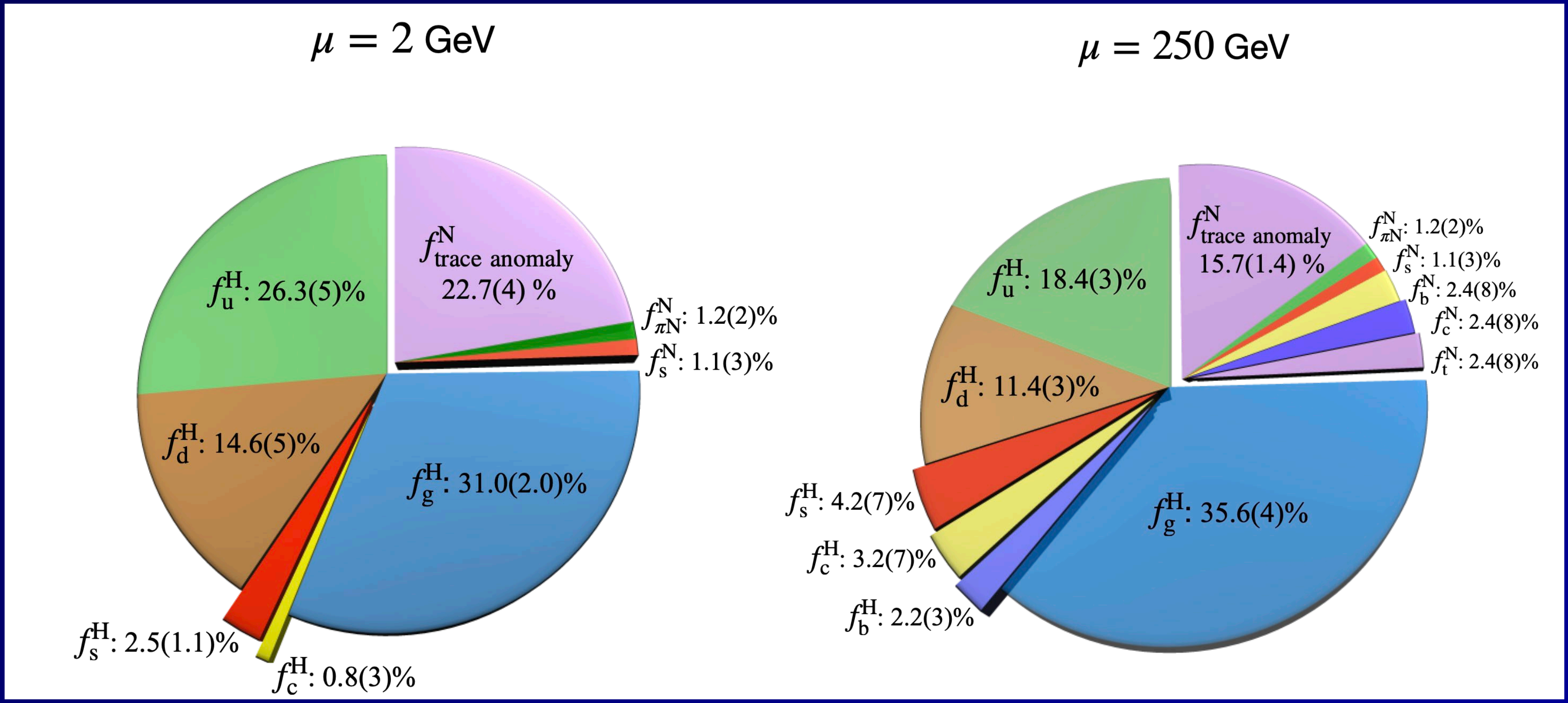
- Rest energy -- $E_0 = M = \langle H_{q_f}(\mu)\rangle + \langle H_g(\mu)\rangle + \langle H_{tr}\rangle ,$

$$\langle H_{q_f}(\mu)\rangle = \frac{3}{4}\sum_f \langle x\rangle_f(\mu) M, \qquad \langle H_g(\mu)\rangle = \frac{3}{4}\langle x\rangle_g(\mu) M,$$

$$\langle H_{tr}\rangle = \frac{1}{4} M.$$

<x> - momentum fraction

Mass Decomposition from Hamiltonian



$$f_f^H = \langle H_q \rangle / M = \frac{3}{4}\langle x \rangle_f(\mu), \qquad f_g^H = \langle H_g \rangle / M = \frac{3}{4}\langle x \rangle_g(\mu),$$

$$f^N_{\pi N} = \frac{1}{4}\frac{\sigma_{\pi N}}{M}, \qquad f^N_s = \frac{1}{4}\frac{\sigma_s}{M}, \qquad f^N_{\text{trace anomaly}} = \frac{1}{4}\frac{\langle H_{\text{ta}} \rangle}{M}$$

Y.B. Yang et al (χ QCD) [arXiv: 1511.15089]

Momentum fractions from CT18 (T.J. Hou et al, PRD, arXiv:1912.10053) at $\mu = 2$ GeV and 250 GeV.

Mass from Gravitational FF

- Gravitational Form factors from the EMT matrix elements

$$\langle P'|(T^{\mu\nu}_{q,g})_R(\mu)|P\rangle/2M_N = \bar{u}(P')[T_{1_{q,g}}(q^2,\mu)\gamma^{(\mu}\bar{P}^{\nu)} + T_{2_{q,g}}(q^2,\mu)\frac{\bar{P}^{(\mu}i\sigma^{\nu)\alpha}q_\alpha}{2M_N} + D_{q,g}(q^2,\mu)\frac{q^\mu q^\nu - g^{\mu\nu}q^2}{M_N} + \bar{C}_{q,g}(q^2,\mu)M_N\eta^{\mu\nu}]u(P)$$

  - T_1 and T_2 : momentum and angular momentum [Ji]
  - D term: deformation of space = elastic property - [Polyakov]
  - C-bar term: pressure-volume work - [Lorce, Liu]

$$M_N(q,g) = \langle P|(T^{00}_{q,g})_{\rm M}(\mu)|P\rangle|_{\vec{P}=0}/2M_N = \langle x\rangle_{q,g}(\mu)M_N + \bar{C}_{q,g}(0,\mu)M_N$$

$$\langle P|(T^{ii}_{q,g})_{\rm M}(\mu)|P\rangle|_{\vec{P}=0}/2M_N = -3\bar{C}_{q,g}(0,\mu)M_N$$

$$\bar{C}_q + \bar{C}_g = \frac{1}{4}(\sum_f f_f^N + f_a^N) - \frac{1}{4}(\langle x\rangle_q + \langle x\rangle_q) = 0 \qquad \partial_\nu T^{\mu\nu} = 0$$

$$M_N = \frac{3}{4}(\langle x\rangle_q(\mu) + \langle x\rangle_g(\mu))M_N + \frac{1}{4}(\sum_f f_f^N + f_a^N)M_N$$

Same as from Hamiltonian

Mass and Pressure from Gravitational FF

- Mass $M_N = M_N(q) + M_N(g)$

$$M_N(q,g) = \langle P|(T^{00}_{q,g})_{\rm M}(\mu)|P\rangle|_{\vec{P}=0}/2M_N = \langle x\rangle_{q,g}(\mu)M_N + \bar{C}_{q,g}(0,\mu)M_N$$

Note: Being scale dependent, separate quark and glue T^{00} are renormalized and mixed.

- What are $\bar{C}_q$ and $\bar{C}_g$?

$$\langle P|(T^{ii}_{q,g})_{\rm M}(\mu)|P\rangle|_{\vec{P}=0}/2M_N = -3\bar{C}_{q,g}(0,\mu)M_N$$

$$3\bar{C}_{q,g}(0,\mu)M_N = [\langle P|\eta_{\mu\nu}(T^{\mu\nu}_{q,g})_{RM}|P\rangle - \langle P|(T^{00}_{q,g})_{RM}(\mu)|P\rangle]/2M_N$$

$$\bar{C}_q(0,\mu) = \tfrac{1}{4}\sum_f(f_f^N - \langle x\rangle_f(\mu)),\ \ \bar{C}_g(0,\mu) = \tfrac{1}{4}(f_a^N - \langle x\rangle_g(\mu))$$

- Therefore,

$$M_N = \tfrac{3}{4}(\langle x\rangle_q(\mu) + \langle x\rangle_g(\mu))M_N + \tfrac{1}{4}(\textstyle\sum_f f_f^N + f_a^N)M_N$$

the same as from the Hamiltonian.

Role of Trace Anomaly --String tension in charmonium

- Heavy quarkonium is confined by a linear potential.
- Constant vacuum energy density and flux tube

$$V(r) = |\epsilon_{vac}|\, A\, r = \sigma\, r$$

- Infinitely heavy quark with Wilson loop

$$V(r) + r\frac{\partial V(r)}{\partial r} = \frac{\langle \frac{\beta}{2g}(\int d^3\vec{x}\, F^2)\, W_L(r,T)\rangle}{\langle W_L(r,T)\rangle}.$$

Dosch, Nachtmann, Rueter - 9503386; Rothe - 9504102

- For charmonium

$$2\sigma\langle r\rangle = \langle H_\beta\rangle_{\bar{c}c} = \frac{\langle \bar{c}c|\frac{\beta}{2g}\int d^3\vec{x}\, F^2|\bar{c}c\rangle}{\langle \bar{c}c|\bar{c}c\rangle}$$

$$\langle H_\beta\rangle_{\bar{c}c} = M_{\bar{c}c} - (1+\gamma_m)\langle H_m\rangle_{\bar{c}c}.$$

- Lattice calculation of charmonium (W. Sun et al.,2012.06228)

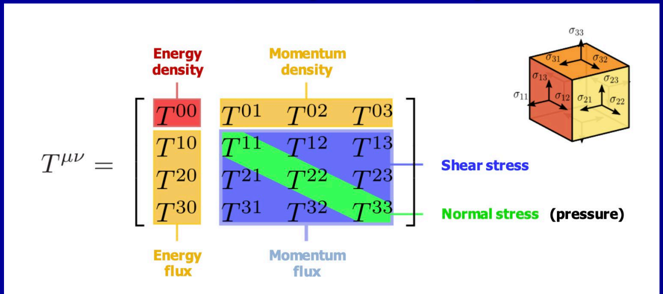
$$\langle H_\beta\rangle_{\bar{c}c} = 199\,\mathrm{MeV}$$ → σ = 0.153 GeV^2

- Cornell potential fit of charmonium → σ = 0.164(11) GeV^2

Mateu:2018zym

Trace Anomaly and Gluon Condensate

- What is trace anomaly? What dynamical role does it play, if any?
  - Note $\langle P|(T^{ii}_{q,g})_{\mathrm{M}}(\mu)|P\rangle|_{\vec{P}=0}/2M_N = -3\bar{C}_{q,g}(0,\mu)M_N$
  - $\frac{1}{3}\langle P|(T^{ii}_{q,g})|P\rangle|_{\vec{P}=0}$ is pressure-volume work
  - The pressure balance equation

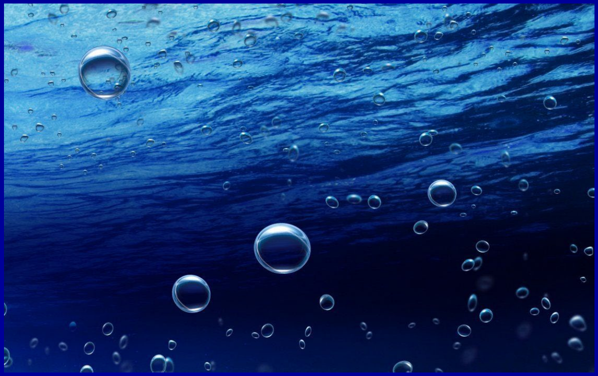


$$PV = -\frac{dE}{dV}V = -(\bar{C}_q + \bar{C}_g) = \frac{1}{4}(\langle x\rangle_q + \langle x\rangle_q) - \frac{1}{4}(\textstyle\sum_f f_f^N + f_a^N) = 0$$

- Nucleon is a bubble in the sea of gluon condensate, where

$$\langle H_a\rangle = -\epsilon_{vac}V,$$

$$\epsilon_{vac} = \frac{\beta(g)}{2g}\langle 0|F^{\alpha\beta}F_{\alpha\beta}|0\rangle < 0$$



$$E_0 = E_T + E_S,$$

$$E_S = \frac{1}{4}[\langle H_m\rangle + \langle H_a\rangle] \propto V,$$

$$E_T = \langle H_{q_f}(\mu)\rangle + \langle H_g(\mu)\rangle \propto V^{-1/3}$$

$$P_{\text{total}} = -\frac{dE_0}{dV} = -\frac{E_S}{V} + \frac{1}{3}\frac{E_T}{V} = 0$$

$$\downarrow$$

$$-\epsilon_{\text{vac}}$$

- (MIT bag model, E_0 = BV+ $\Sigma_{q,g}$/R)

Mass-Pressure Correspondence

- Mass

$$M_N = \tfrac{3}{4}(\langle x\rangle_q(\mu) + \langle x\rangle_g(\mu))M_N + \tfrac{1}{4}(\textstyle\sum_f f_f^N + f_a^N)M_N$$

- Pressure-volume work

$$PV = \tfrac{1}{4}(\langle x\rangle_q(\mu) + \langle x\rangle_g(\mu))M_N - \tfrac{1}{4}(\textstyle\sum_f f_f^N + f_a^N)M_N$$

- Virial theorem
  - D dimension
  - Mass

$$M = E_T + E_S, \quad E_T = \langle \overline{T}^{00}\rangle = \frac{D-1}{D}M, \; E_S = \frac{1}{D}\langle T^\mu_\mu\rangle = \frac{1}{D}M$$

  - Pressure

$$PV = \frac{1}{D-1}\langle T^{ii}\rangle = -\frac{1}{D-1}[\langle T^\mu_\mu\rangle - \langle T^{00}\rangle]$$

$$= -\frac{1}{D-1}[DE_s - E_T - E_s] = -E_s + \frac{1}{D-1}E_T = 0$$

Mass-Pressure Correspondence

- Other mass decomposition formulae
  - Trace $$\frac{\langle P|\int d^3\vec{x}\, T^\mu_\mu(x)|P\rangle}{\langle P|P\rangle}\Big|_{\vec{P}=0} = M_N$$

$$T^\mu_\mu = \sum_f m_f \bar{\psi}_f \psi_f + \left[\sum_f m_f \gamma_m(g) \bar{\psi}_f \psi_f + \frac{\beta(g)}{2g} F^{\alpha\beta} F_{\alpha\beta}\right]$$

No kinetic energy, no connection to pressure

  - Gravitational FF without $\bar{C}$

$$M_N = (\langle x\rangle_q(\mu) + \langle x\rangle_g(\mu)) M_N$$

No potential energy, no relation to pressure

- Virial Theorem
  - Coulomb potential: H = T + V, 2<T> = - <V>
  - E = - ½ <T>, E = ½ <V> are not good physics explanation of the decomposition of the binding energy.

Generic bound state energy and pressure correspondence

- Hadronic model mass – pressure relation

  – MIT bag model (E_0 = BV+ $\Sigma_{q,g}$/R),
  pressure: $\partial E_0 / \partial R = 0$

  – Skyrmion: Derrick's theorem ($r \rightarrow \lambda r$)

$$\mathcal{L}_2 = f_\pi^2\, Tr\left[\partial_\mu U^\dagger \partial^\mu U\right] \longrightarrow \lambda$$

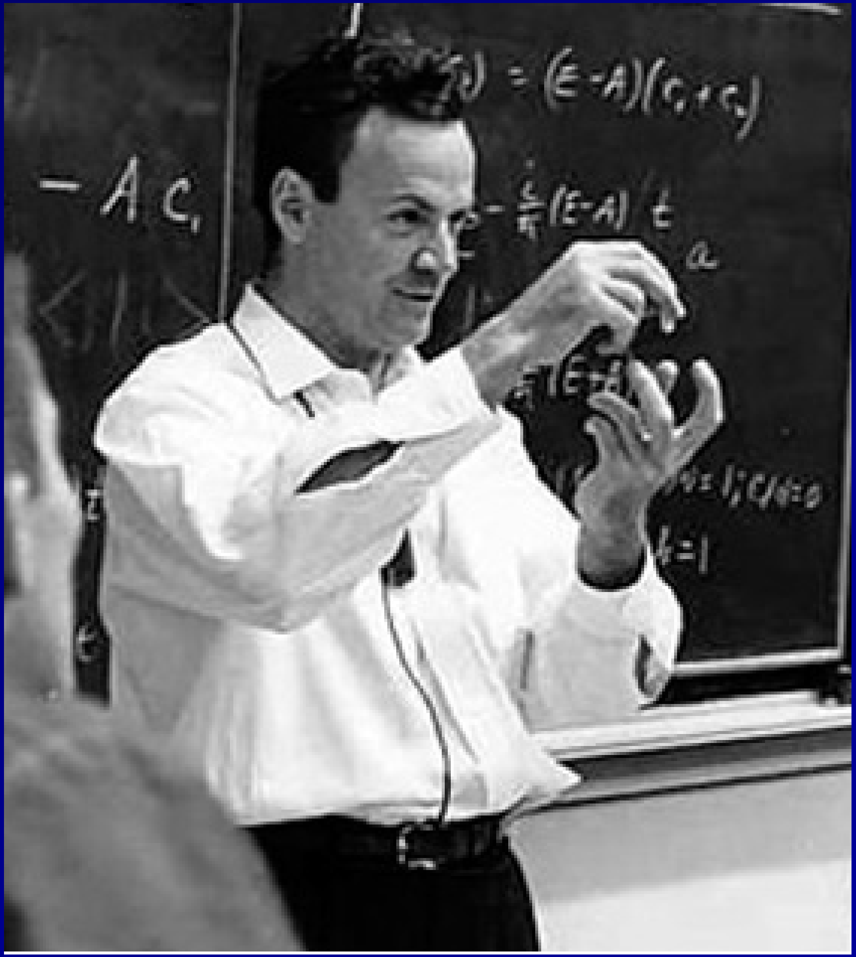
$$\mathcal{L}_4 = \frac{1}{e^2}\, Tr\left[U^\dagger \partial_\mu U, U^\dagger \partial_\nu U\right]^2 \longrightarrow 1/\lambda$$

  – Potential models: kinetic energy ↘ r, potential energy ↗ r.

Feynman’s quote

From the very beginning of his first-ever lecture comes this timeless gem (mentioned in Daniel Bor’s excellent *The Ravenous Brain*) that set the tone for both Feynman’s academic contribution and his broader cultural legacy:

If, in some cataclysm, all of scientific knowledge were to be destroyed, and only one sentence passed on to the next generation of creatures, what statement would contain the most information in the fewest words?



I believe it is the atomic hypothesis that all things are made of atoms — little particles that move around in perpetual motion, attracting each other when they are a little distance apart, but repelling upon being squeezed into one another. In that one sentence, you will see, there is an enormous amount of information about the world, if just a little imagination and thinking are applied.

Trace Anomaly and Cosmological Constant

- Vacuum energy density is indeed a constant which is analogous to the cosmological constant in the $g^{\mu\nu}$ term as Einstein introduced for a static universe.

$$R_{\mu\nu} + \frac{1}{2} R\, g_{\mu\nu} - \Lambda\, g_{\mu\nu} = 8\pi G\, T_{\mu\nu} \qquad \Lambda = 4\pi G \rho$$

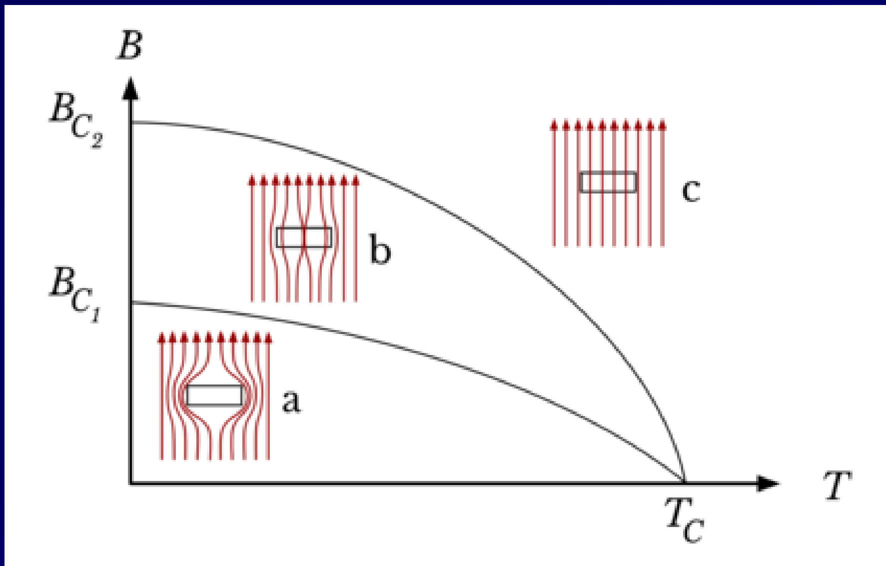
- Friedman equation for the accelerating expansion of the universe

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) + \frac{\Lambda}{3}$$

$$\rho_{\rm vac} = \frac{\Lambda}{8\pi G}$$

$$P_{\rm vac} = -\frac{\Lambda}{8\pi G}$$

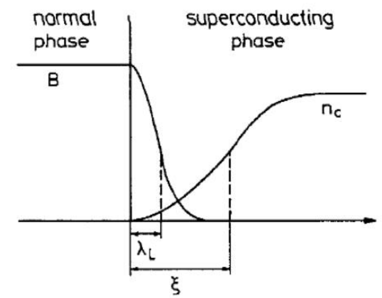
Type II Superconductor



Physics of type I and II superconductors

- "London Penetration Depth" λ_L
  is the e-fold decay length of the magnetic field from the superconductor skin due to the Meissner effect (in the range of 10 to 10^3 nm)

- "Coherence Length" ξ
  the average size of Cooper in the superconductor (in the range of 10 to 100 nm, I.e. much larger than the inter-atomic distance typically of 0.1 to 0.3 nm.



Ginzburg-Landau Parameter κ

$$\kappa = \lambda_L / \xi \Rightarrow \begin{cases} k < 1/\sqrt{2} \Leftrightarrow \text{type I} \\ k > 1/\sqrt{2} \Leftrightarrow \text{type II} \end{cases}$$

material	In	Pb	Sn	Nb
λ_L [nm]	24	32	≈ 30	32
ξ [nm]	360	510	≈ 170	39

Ginzburg–Landau equations

$$\alpha\psi + \beta|\psi|^2\psi + \frac{1}{2m}(-i\hbar\nabla - 2e\mathbf{A})^2\psi = 0$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j} \;\; ; \;\; \mathbf{j} = \frac{2e}{m}\mathrm{Re}\{\psi^*(-i\hbar\nabla - 2e\mathbf{A})\psi\}$$

$$|\psi|^2 = n_s$$

London penetration depth

$$\lambda_L = \sqrt{\frac{m}{4\mu_0 e^2 \psi_0^2}}$$

Coherent length ξ

$$\kappa = \lambda_L/\xi$$

Energetics and Pressure

Normal Phase

Superconducting Phase

- Type II superconductor

$$F = F_s + F_B + F_{sc}$$

$$F_s = \text{ cost of condensation energy}$$

$$F_B = \int dv\, B^2/2\mu_0 \ \text{(magnetic energy)}$$

$$F_{sc} = 1/2 \int dv\, \lambda_L^2 J_s \cdot J_s \ \text{(supercurrent kinetic energy)}$$

- Variational model (J.R. Clem, Jour. Low Temp. Phys. 18, 5/6 (1975))

$$\frac{|\psi|^2}{n_0} = \frac{n_s}{n_0} = \frac{\rho^2}{\rho^2 + R^2} \underset{\rho \to \infty}{\longrightarrow} 1$$

$$\frac{1}{\sqrt{2}H_c}\frac{E}{l} = \phi_0 H_c'/4\pi \ \text{ where } \phi_0 = hc/2e, \ \sqrt{2}H_c = \kappa\phi_0/2\pi\lambda_L^2$$

$$H_c' = \underbrace{\kappa R'/8}_{F_s} + \underbrace{1/8\kappa + K_0(R')/2\kappa R'\, K_1(R')}_{F_B + F_{sc}}, \ \text{ where } \ R' = R/\lambda_L$$

SC, Hadrons, Cosmos

- Type II Superconductor

$$P_s = -\frac{\partial F_s}{\partial V} < 0, \quad P_{B+sc} = -\frac{\partial F_B + F_{sc}}{\partial V} > 0$$

- Hadrons

$$P_{tr} = -\frac{\partial E_S}{\partial V} < 0, \quad P_{q+g} = -\frac{\partial E_T}{\partial V} > 0$$

- Cosmos $$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) + \frac{\Lambda}{3}, \quad \Lambda > 0 \longrightarrow \frac{\ddot{a}}{a} > 0$$

- The common theme is the existence of a condensate.
- Hadrons: condensates from breaking of conformal and chiral symmetries. SC: Cooper pair condensate from gauge symmetry breaking. Cosmos: ?

Pion Mass Puzzle

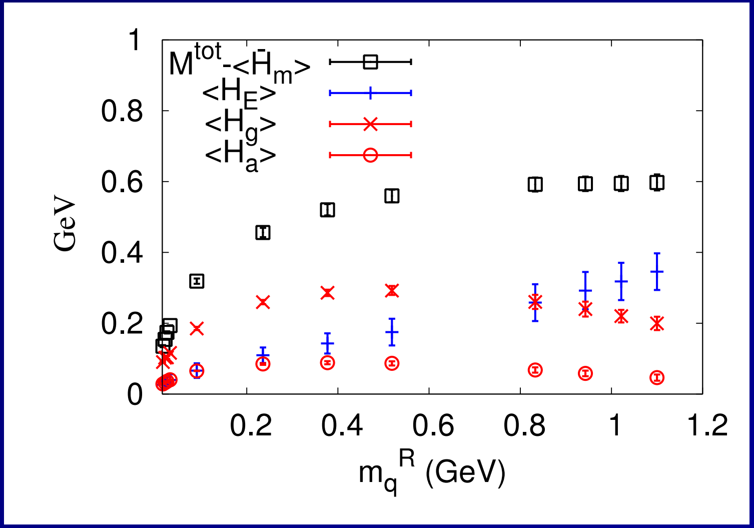
- Pion mass in terms of trace of EMT

$$m_\pi = m\langle\pi|\bar{\psi}\psi|\pi\rangle + \langle\pi|\frac{\beta}{2g}G_{\mu\nu}^2 + m\gamma_m\bar{\psi}\psi|\pi\rangle$$

- Gellmann-Oakes-Renner relation

$$m_\pi^2 = -2m\langle\bar{\psi}\psi\rangle/f_\pi^2, \quad m_\pi^2 \propto m$$

- $\langle\pi|\bar{\psi}\psi|\pi\rangle \propto 1/\sqrt{m}$ But, why should the trace anomaly be proportional to $\sqrt{m}$? V → 0 ?

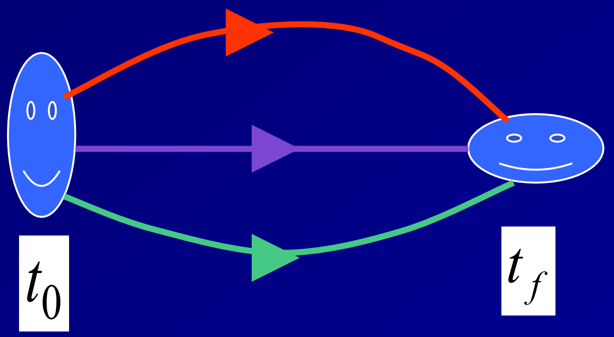


Y.B. Yang et al. (χQCD), PRD (2015); 1405.4440

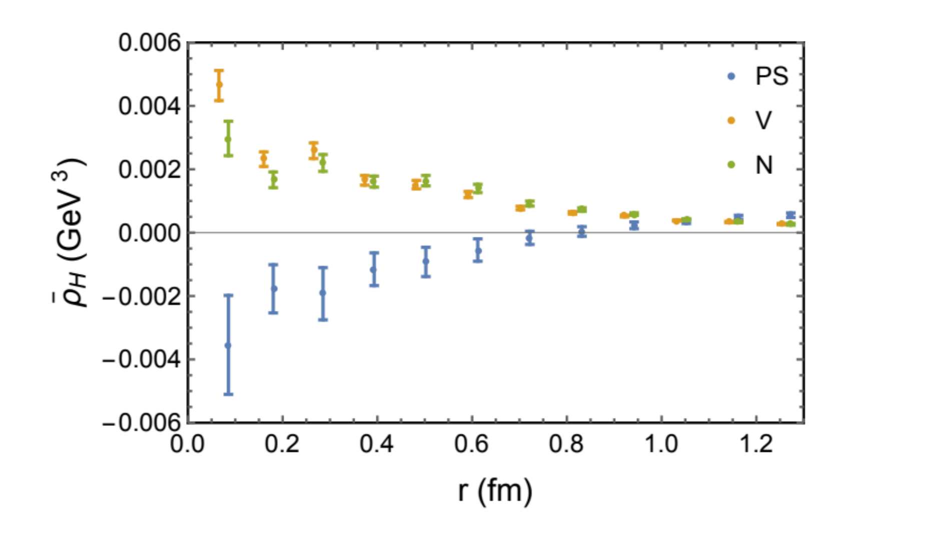
Trace anomaly Distribution

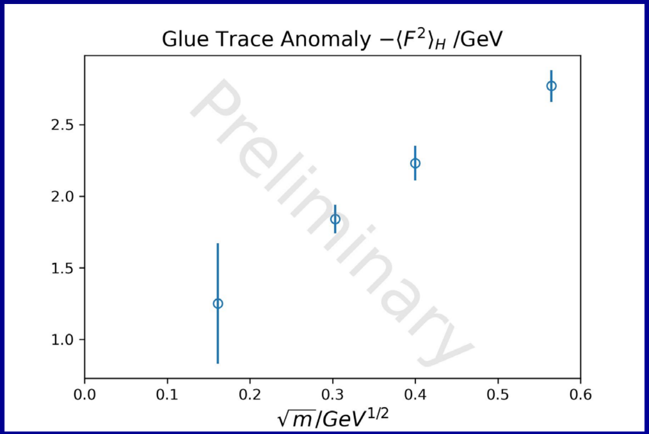
- Distribution as a function of the relative distance between the glue operator and the sink positions.

$$F_{\mu\alpha}F_{\nu\alpha} - \frac{1}{4}\delta_{\mu\nu}F^2$$



- F. He, P. Sun and Y.B. Yang (χQCD) (PRD 2021, 2101.04942)

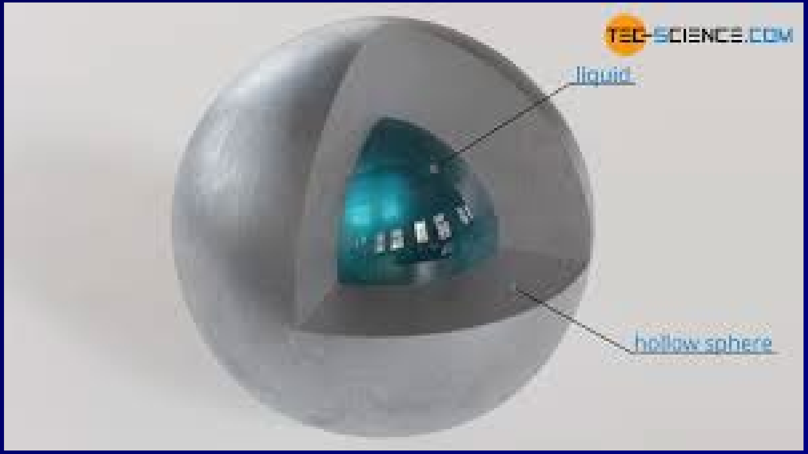


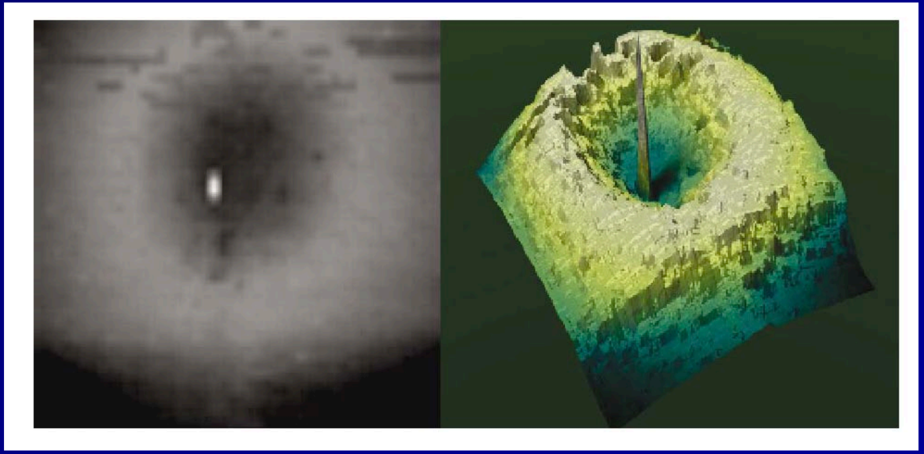


- It changes sign in pion so that the integral approaches zero at the chiral limit.

Pion as a Ring-shaped Type II Superconductor

A. Groeger et al., PRL 90, 237004 (2003)

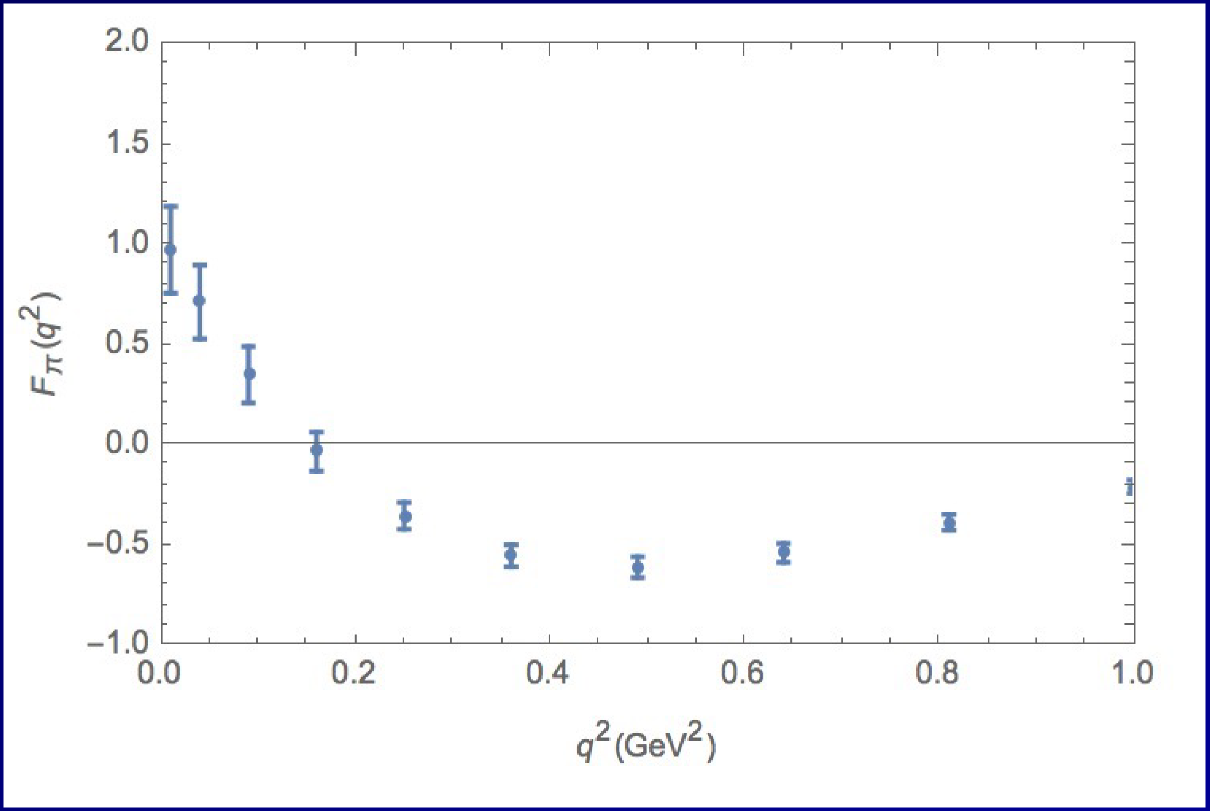




Pion with shell of positive trace anomaly and an inner core with negative trace anomaly.

Niobium, normal conducting vortex ring around a superconducting region,

Pion trace anomaly FF



m_π = 340 MeV

X.B. Tong, J.P. Ma and F. Yuan
arXiv:2203.13493

Summary and Challenges

- From femto-scale to micro-scale to that of the cosmos, Nature seems to choose the same mechanism for confinement or acceleration.
- m_q ← Higgs mechanism
- Quark condensate ← chiral symmetry breaking (restoration at T and μ)
- Trace anomaly (confinement) ← conformal symmetry breaking (conformal phases with multi-flavors and SU(N); finite $T > T_c$)
- Chiral symmetry breaking and conformal symmetry breaking are linked in the case of the pion trace anomaly distribution.
- String theory invented in hadron physic finds its home in quantum gravity.
- Cosmological constant introduced in general relativity is relevant to hadron physics.
- Challenges for EIC and COMPASS is to measure the trace anomaly form factors for the proton and, particularly, the pion.
- Glue condensate is an order parameter for confinement – deconfinement transition.

Virial Theorem

- D dimension
- Mass

$$M = E_T + E_S, \quad E_T = \langle \overline{T}^{00} \rangle = \frac{D-1}{D} M, \; E_S = \frac{1}{D} \langle T^\mu_\mu \rangle = \frac{1}{D} M$$

- Pressure

$$PV = \frac{1}{D-1} \langle T^{ii} \rangle = -\frac{1}{D-1} [\langle T^\mu_\mu \rangle - \langle T^{00} \rangle]$$

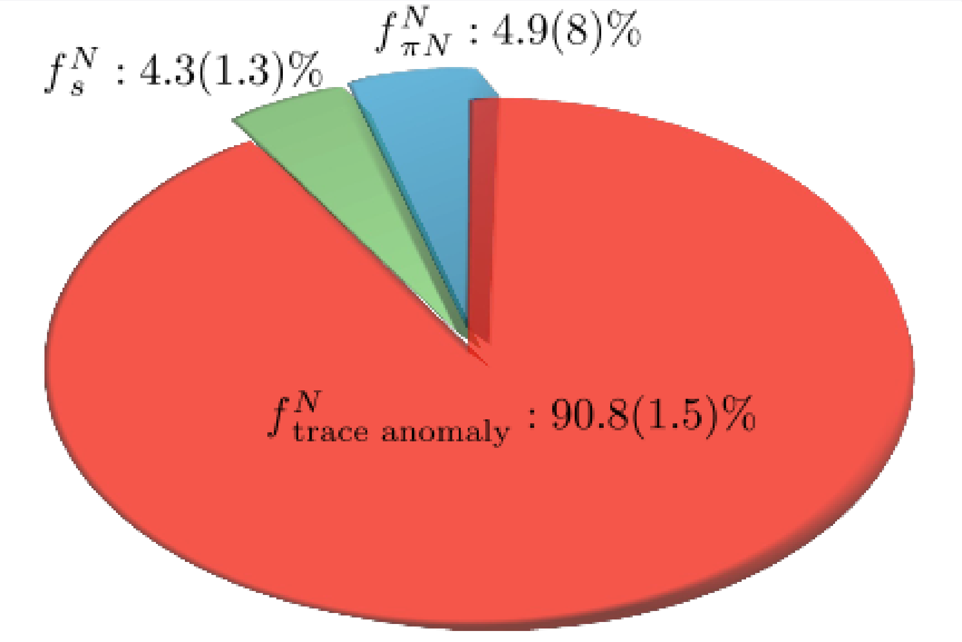
$$= -\frac{1}{D-1} [D E_s - E_T - E_s] = -E_s + \frac{1}{D-1} E_T = 0$$

Trace of EMT (¼ of Hadron Mass)

- Trace of EMT – scalar, frame independent, RG invariant

$$T^{\mu}_{\mu} = \sum_f m_f \bar{\psi}_f \psi_f + \left[\sum_f m_f \gamma_m(g) \bar{\psi}_f \psi_f + \frac{\beta(g)}{2g} F^{\alpha\beta} F_{\alpha\beta} \right]$$

- Lattice calculation of of quark condensate
  - Y.B. Yang et al (χ QCD) [arXiv: 1511.15089]
  - Overlap fermion (Z_m Z_s =1)
  - 3 lattices (one at physical m_π), systematics (volume, continuum)



πN sigma term

$$\sigma_{\pi N} = \frac{m_u + m_d}{2} \langle P | \bar{u}u + \bar{d}d | P \rangle$$

Strangeness sigma term

$$\sigma_s = m_s \langle P | \bar{s}s | P \rangle$$

$$f^N_{\pi N} = \frac{\sigma_{\pi N}}{M_N}, \qquad f^N_s = \frac{\sigma_s}{M_N}$$

Mass and Pressure from Gravitational FF

- Mass $M_N = M_N(q) + M_N(g)$

$$M_N(q,g) = \langle P|(T^{00}_{q,g})_{\rm M}(\mu)|P\rangle|_{\vec{P}=0}/2M_N = \langle x\rangle_{q,g}(\mu)M_N + \bar{C}_{q,g}(0,\mu)M_N$$

Note: Being scale dependent, separate quark and glue T^{00} are renormalized and mixed.

- What are $\bar{C}_q$ and $\bar{C}_g$?

$$\langle P|(T^{ii}_{q,g})_{\rm M}(\mu)|P\rangle|_{\vec{P}=0}/2M_N = -3\bar{C}_{q,g}(0,\mu)M_N$$

$$3\bar{C}_{q,g}(0,\mu)M_N = [\langle P|\eta_{\mu\nu}(T^{\mu\nu}_{q,g})_{RM}|P\rangle - \langle P|(T^{00}_{q,g})_{RM}(\mu)|P\rangle]/2M_N$$

$$\bar{C}_q(0,\mu) = \tfrac{1}{4}\textstyle\sum_f(f^N_f - \langle x\rangle_f(\mu)),\ \ \bar{C}_g(0,\mu) = \tfrac{1}{4}(f^N_a - \langle x\rangle_g(\mu))$$

- Therefore,

$$M_N = \tfrac{3}{4}(\langle x\rangle_q(\mu) + \langle x\rangle_g(\mu))M_N + \tfrac{1}{4}(\textstyle\sum_f f^N_f + f^N_a)M_N$$

the same as from the Hamiltonian.

Trace Anomaly and Cosmological Constant

- Pressure of anomaly:

$$d\langle H_a\rangle = -P_{vac}\,dV\,(dQ = T\,dS = 0), \qquad P_{vac} = -|\epsilon_{vac}| < 0$$

- Quark and glue energy

$$\langle H_E(\mu)\rangle + \langle H_g(\mu)\rangle \propto V^P$$

- Volume dependence of total rest energy

$$\begin{aligned} E_0 &= |\epsilon_{vac}|V + \epsilon_{mat}V^p \\ \frac{dE_0}{dV} &= -P_{vac} - P_k = |\epsilon_{vac}| + p\,\epsilon_{mat}V^{p-1} = 0 \end{aligned}$$

- E_0 = d E_S (d=4) ⇒ p = -1/3 (MIT bag model, $E_0 = BV + \Sigma_{q,g}/R$)
- Rest energy as the sum of scalar trace and tensor traceless parts

$$E_0 = E_T + E_S,$$

$$\begin{aligned} E_T &= \langle H_{q_f}(\mu)\rangle + \langle H_g(\mu)\rangle = \frac{3}{4}[\sum_f \langle x\rangle_f(\mu) + \langle x\rangle_g(\mu)]M, \\ E_S &= \frac{1}{4}[\langle H_m\rangle + \langle H_a\rangle] \end{aligned}$$

Pion Mass Puzzle

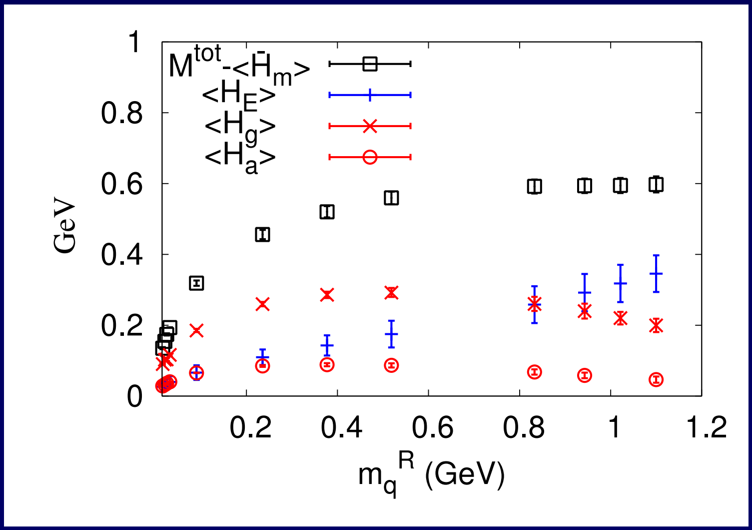
- Pion mass in terms of trace of EMT

$$m_\pi = m\langle \pi|\bar\psi\psi|\pi\rangle + \langle \pi|\frac{\beta}{2g}G^2_{\mu\nu} + m\gamma_m\bar\psi\psi|\pi\rangle$$

- Gellmann-Oakes-Renner relation

$$m_\pi^2 = -2m\langle\bar\psi\psi\rangle/f_\pi^2, \quad m_\pi^2 \propto m$$

- $\langle \pi|\bar\psi\psi|\pi\rangle \propto 1/\sqrt{m}$ But, why should the trace anomaly be proportional to $\sqrt{m}$? V → 0 ?



Y.B. Yang et al. (χQCD), PRD (2015); 1405.4440