In-medium form factors and spin polarization

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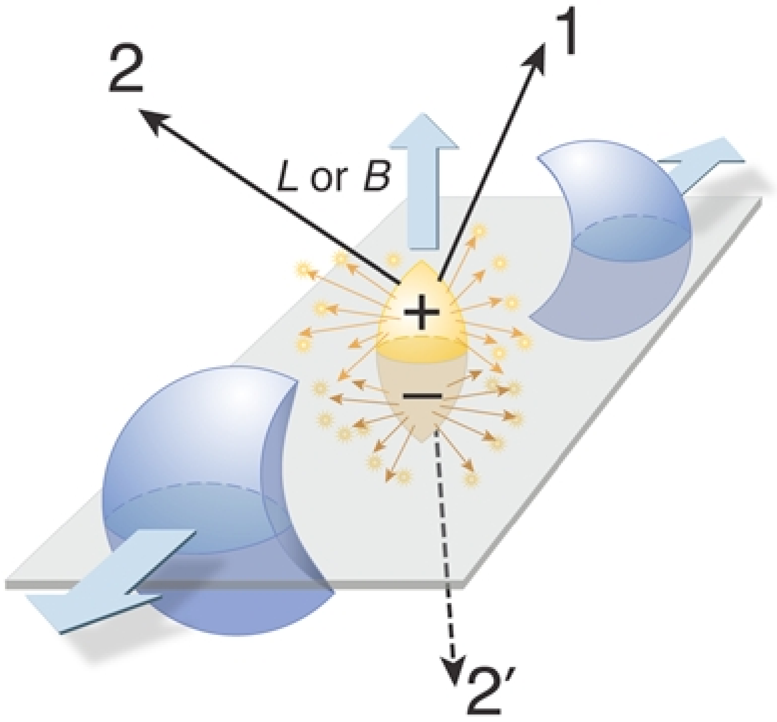
Chirality and Criticality in Heavy Ion Collisions, INT, Aug 21-25, 2023

Outline

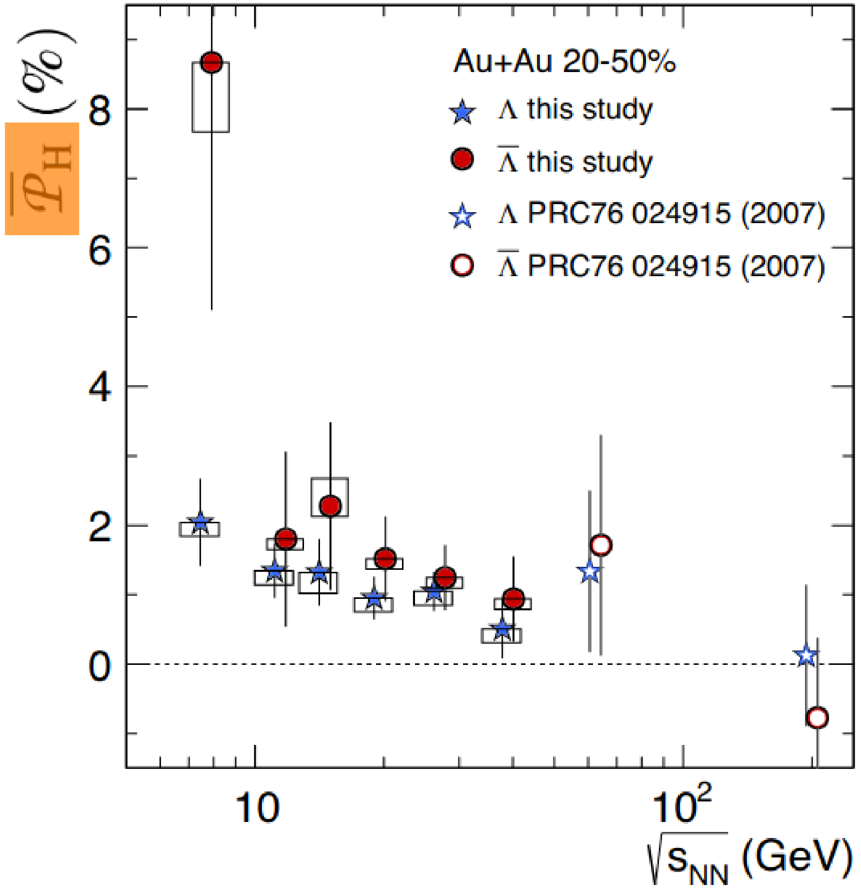
- Spin polarizations in heavy ion collisions
- Success and limitation of quantum kinetic theory
- Spin polarization from correlation function & form factors
- Electromagnetic form factors in vacuum and in medium
- Gravitational form factors in vacuum and in medium
- Summary and outlook

global spin polarization in heavy ion collisions



$$L_{ini} \sim 10^5 \hbar \rightarrow S_{final}$$

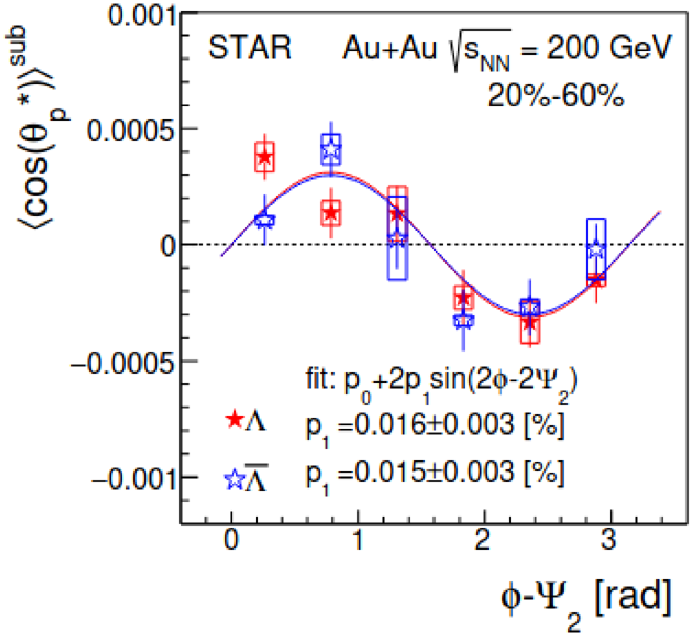
Liang, Wang, PRL 2005, PLB 2005



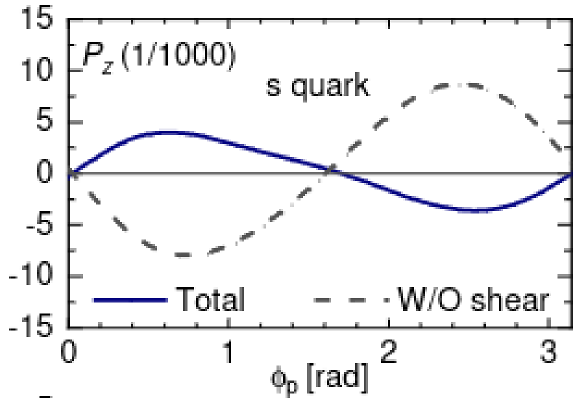
STAR collaboration, Nature 2017

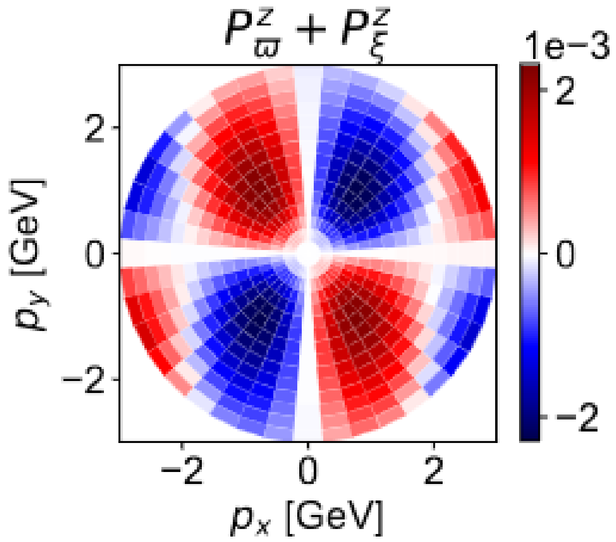
$$e^{-\beta(H_0 - \mathbf{S}\cdot\omega)}$$

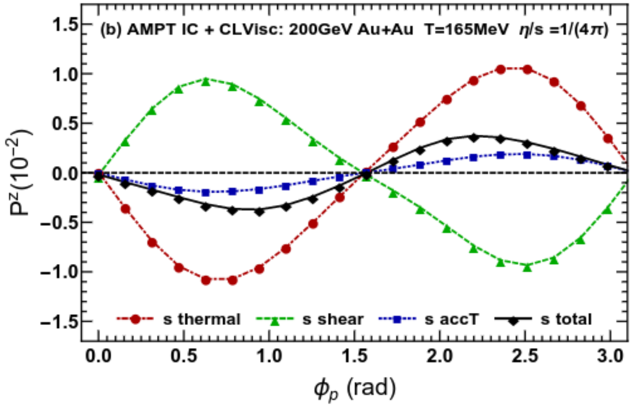
local spin polarization in heavy ion collisions



STAR collaboration, PRL 2019







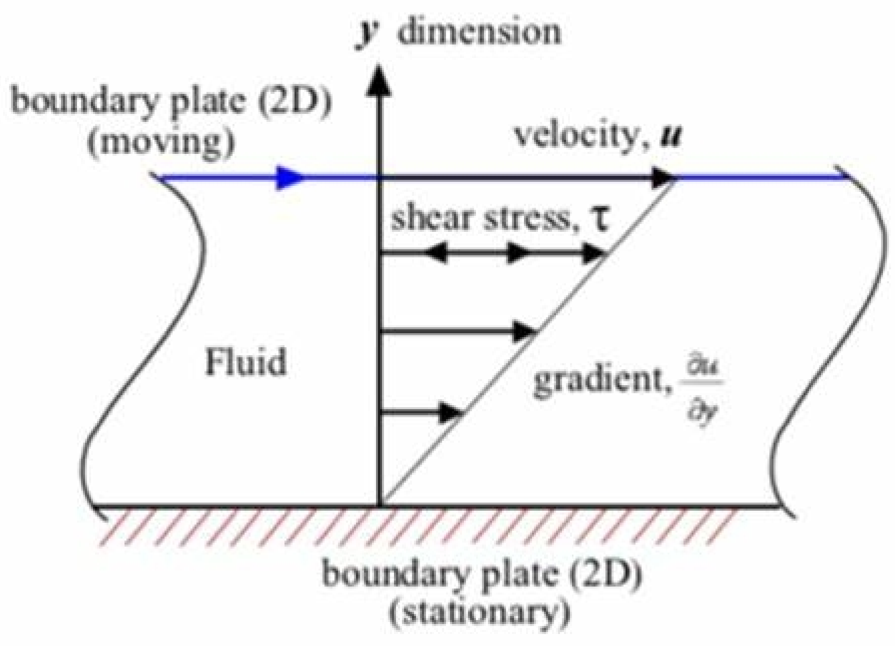
vorticity $\mathcal{P}^i \sim \omega^i$

shear $\mathcal{P}^i \sim \epsilon^{ijk}\hat{p}_j\hat{p}_l\sigma_{kl}$

Fu, Liu, Pang, Song, Yin, PRL 2021
Becattini, et al, PRL 2021
Yi, Pu, Yang, PRC 2021

talks by Yin, Buzzegoli

QKT description of shear induced spin polarizarion



$\mathcal{P}^i \sim \epsilon^{ijk}\hat{p}_j\hat{p}_l\sigma_{kl}$ + state change

steady state: collision nonvanishing

Spin polarization $$\mathcal{A}^\mu = -2\pi\hbar\left[a^\mu f_A + \frac{\epsilon^{\mu\nu\rho\sigma}P_\rho u_\sigma \mathcal{D}_\nu f}{2(P\cdot u + m)}\right]\delta(P^2 - m^2)$$

$$\mathcal{D}_\nu = \partial_\nu - \Sigma_\nu^> - \Sigma_\nu^< \frac{1-f}{f}$$

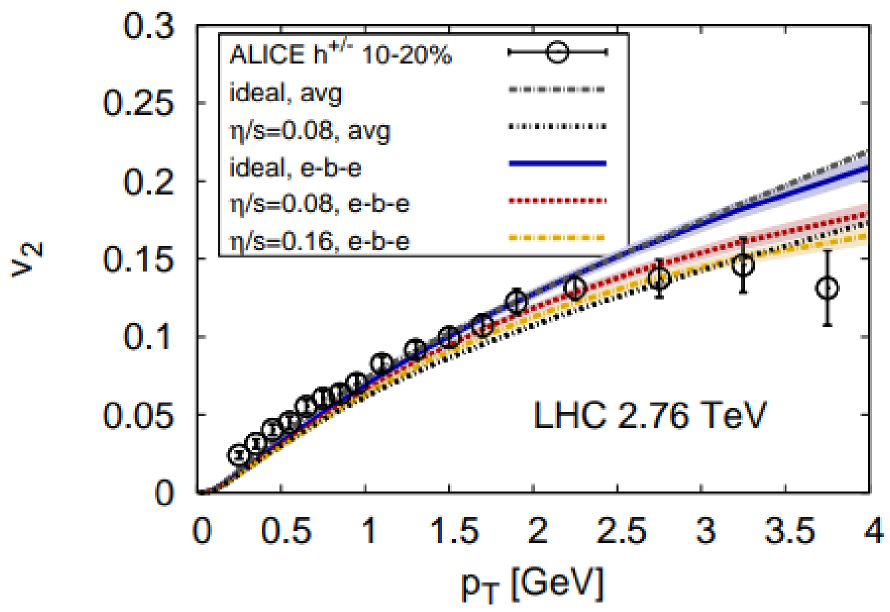
SL, Z.y. Wang, JHEP 2022

Quantum kinetic theory (QKT): pro and cons

QKT = Boltzmann + spin

Hidaka, Pu, Wang, Yang, PPNP 2022

Pro: systematic treatment of spin transport
Cons: assumes medium weakly coupled, not well-justified for QGP



kinetic theory: $\frac{\eta}{s} \sim O\left(\frac{1}{g^4}\right) \sim 1.7$ for $\alpha_s = 0.3$

Arnold, Moore, Yaffe 2003

phenomenology $\frac{\eta}{s} \simeq 0.08$

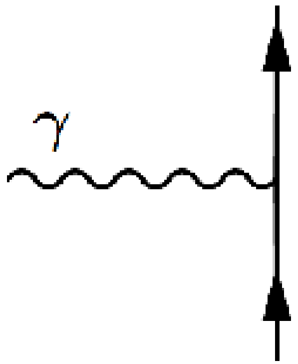
QGP medium might not be weakly coupled

Spin polarization in heavy ion collisions

for $S = \frac{1}{2}$ particle (consider high temperature limit, quark massless)

$S_i \sim B_i$

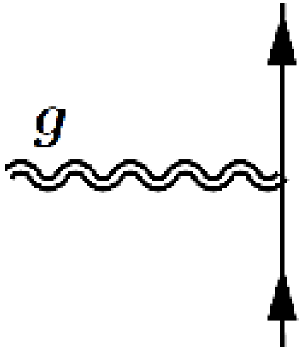
$S_i \sim \epsilon^{ijk} \hat{p}_j E_k$



polarization in external EM fields

$S_i \sim \omega_i$

$S_i \sim \epsilon^{ijk} \hat{p}_j \hat{p}_l \sigma_{kl}$



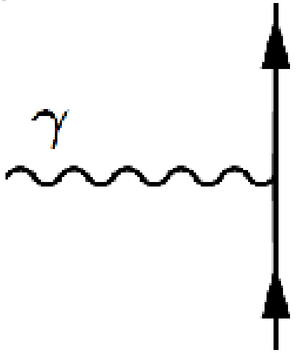
polarization in off-equilibrium state: hydro gradient (mimicked by metric fields)

Spin polarization from correlation functions

Wigner function

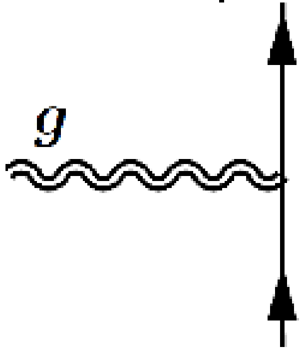
$$S^{<}_{\alpha\beta}(X=\frac{x+y}{2},P)=\int d^4(x-y)e^{iP\cdot(x-y)/\hbar}\left(-\langle\bar{\psi}_\beta(y)\psi_\alpha(x)\rangle\right)$$

- polarization in external EM fields



$$\langle S^{<}(X,P)\rangle_{\mathrm{eq},A_\mu}$$

- polarization in off-equilbrium state



$$\langle S^{<}(X,P)\rangle_{\mathrm{eq},h_{\mu\nu}}$$

$\langle\,\rangle$ taken in medium, not necessarily weakly coupled

Spin polarization from QKT(CKT)

right-handed fermion in EM fields

$$S^<(X=\frac{x+y}{2},P)=\int d^4(x-y)e^{iP\cdot(x-y)/\hbar}U(y,x)\left(-\langle\psi^\dagger(y)\psi(x)\rangle\right)$$

gauge link $U(y,x)=\mathcal{P}\exp[-i\int_x^y dz^\mu A_\mu(z)]$

$$S^<\equiv\bar{\sigma}_\mu S^{<\mu}$$

$$\partial_X \ll P$$

$$S^{<0}=-2\pi\left[\delta(P^2)p_0 f(p_0)+\mathbf{p}\cdot\mathbf{B}\delta'(P^2)f(p_0)\right]$$

$$S^{<i}=-2\pi\left[\delta(P^2)p_i f(p_0)+\left(p_0 B_i-\epsilon^{ijk}P_jE_k\right)\delta'(P^2)f(p_0)\right]$$

absorb e in E&B fields

O(∂^0)

E, B ~ O(∂)

spin-magnetic coupling

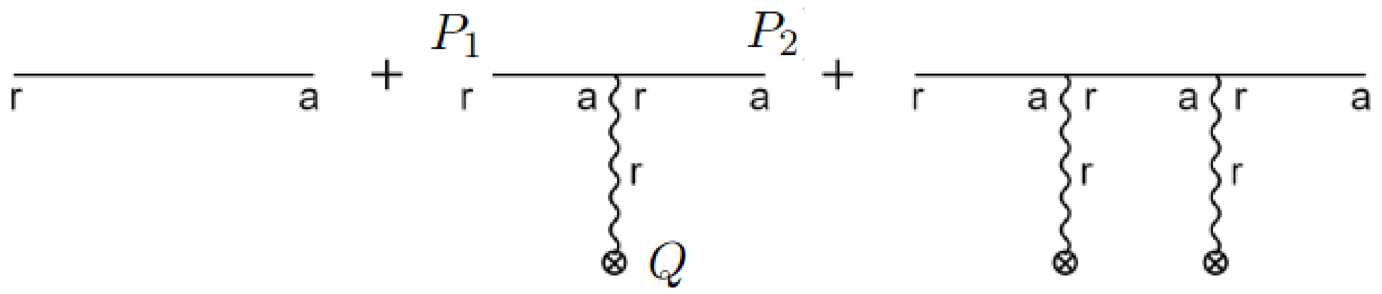
spin Hall effect

Stephanov, Yin, PRL 2012
Son, Yamamoto, PRL 2012
Hidaka, Pu, Yang, PRD 2018
Gao, Liang, Wang, PRD 2019

Spin polarization from field theory

$$S^{<\mu} = -(S^{\mu}_{ra} - S^{\mu}_{ar}) f(P_0)$$

$$S_{ra} =$$



$$Q = P_2 - P_1$$

$$P = \frac{1}{2}(P_1 + P_2)$$

resummation to all order in A, and up to O(Q) reproduces the CKT results

Two lessons:

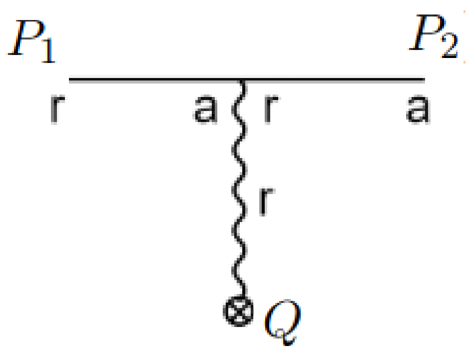
- CKT equivalent to the tree-level vertex (Lorentz invariant)
- Electric field can't do work to fermion (implicit in CKT)

n^{μ} frame vector $\quad E^{\mu} = F^{\mu\nu} n_{\nu}$ $\quad Q \cdot n = 0$ static

no-work condition $\quad E \cdot P = 0$ → $P \cdot Q = 0$ orthogonal

SL, Tian, 2306.14811

Correlation function for scattering process



on-shell condition $P^2 = 0$

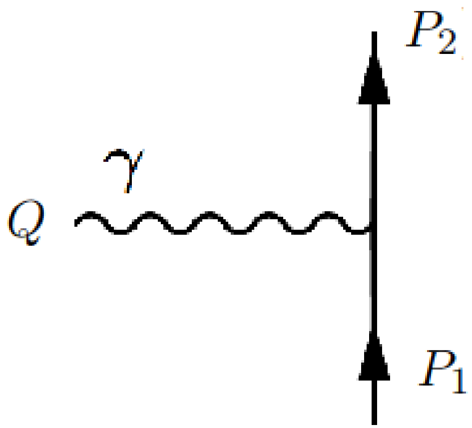
no-work condition $P \cdot Q = 0$

$\rightarrow$ $P_{1,2}^2 \sim O(Q^2)$

interpretation: scattering of fermions on EM fields

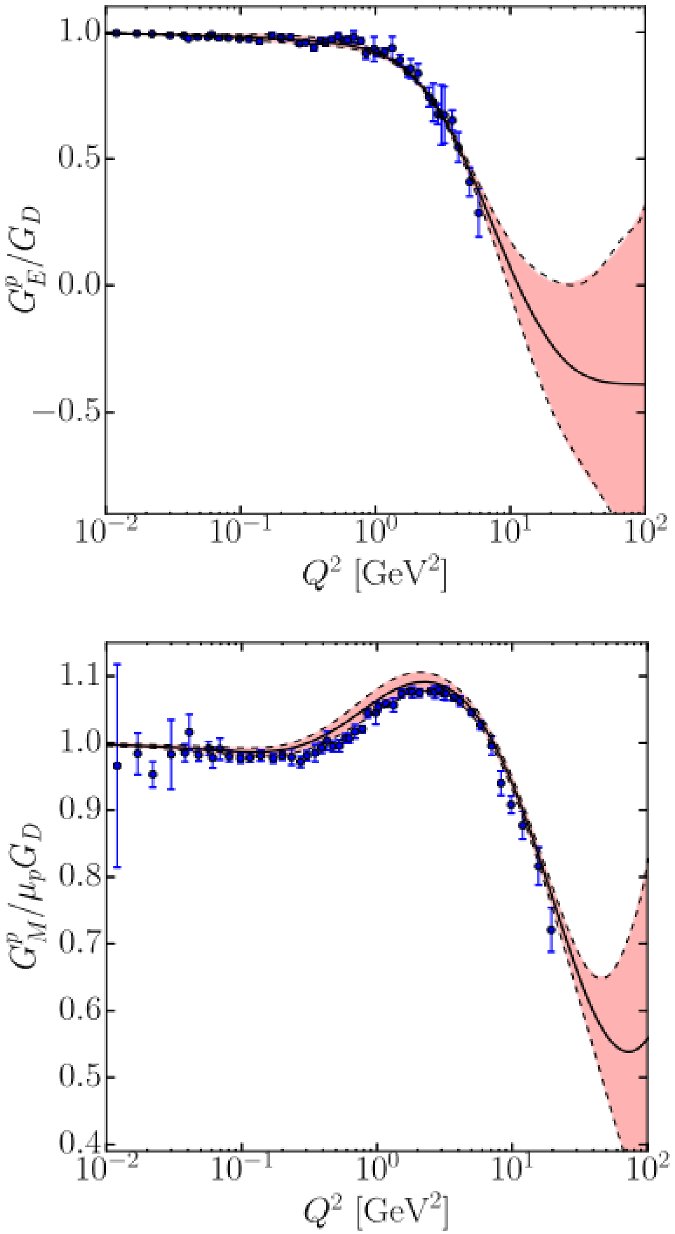
can be described more generally by form factors!

Electromagnetic form factors in vacuum



$$Q = P_2 - P_1$$

$$P = \frac{1}{2}(P_1 + P_2)$$



PLB 2018, Ye et al

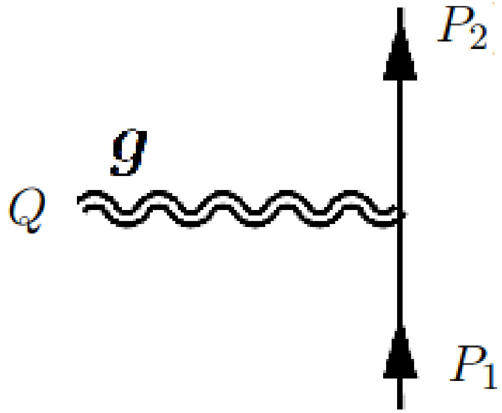
$$\langle P_2 | J^\mu(Q) | P_1 \rangle = \bar{u}(P_2)\Big[\gamma^\mu F_1(Q^2) + \frac{i\sigma^{\mu\nu}Q_\nu}{2m} F_2(Q^2)\Big] u(P_1)$$

$$= \bar{u}(P_2)\Big[\frac{P^\mu}{m} G_E(Q^2) + \frac{i\epsilon^{\mu\nu\rho\sigma}Q_\nu P_\rho \gamma_\sigma \gamma^5}{2m^2} G_M(Q^2)\Big] u(P_1)$$

Form factors parameterize interaction based on symmetry

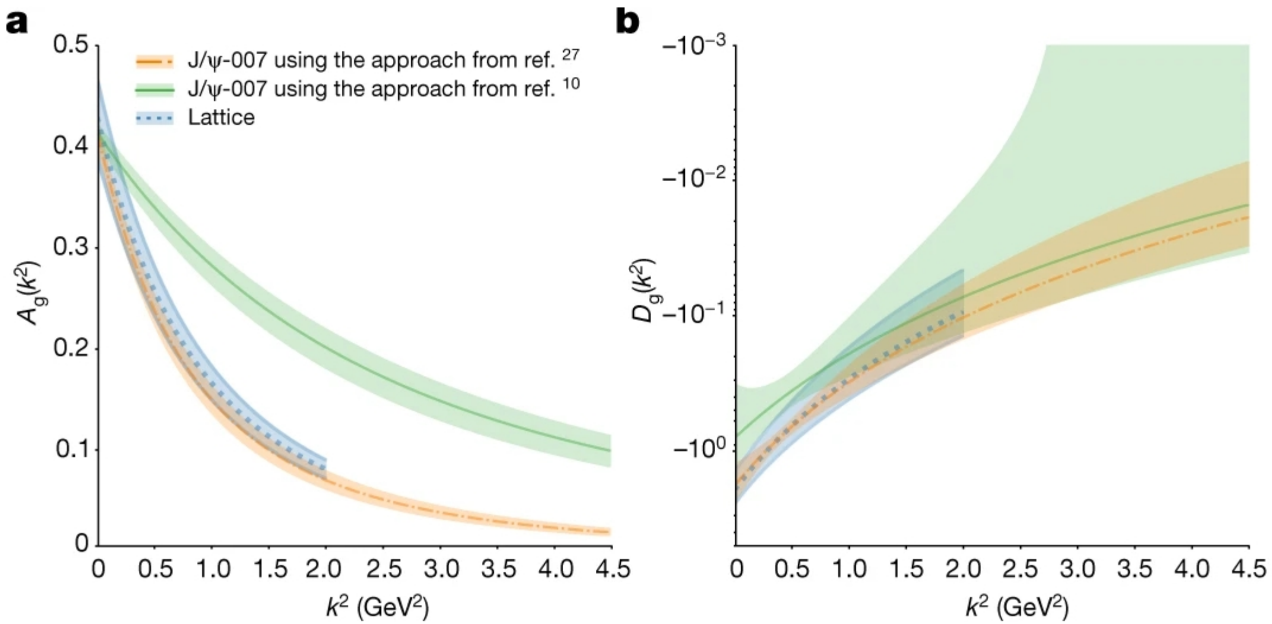
- accessible experimentally
- charge distribution, magentic moment

Gravitational form factors in vacuum



$$Q = P_2 - P_1$$

$$P = \frac{1}{2}(P_1 + P_2)$$



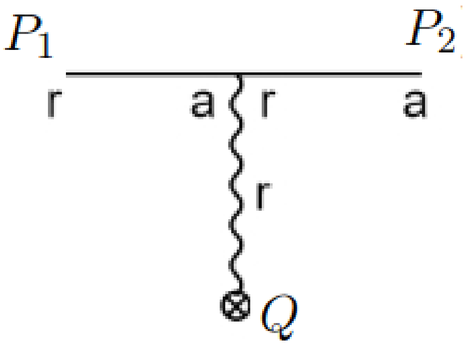
Nature 2021, Meziani et al

$$\langle P_2 | T^{\mu\nu}(Q) | P_1 \rangle = \bar{u}(P_2)\left[\frac{P^\mu P^\nu}{m} A(Q^2) + \frac{iP^{\{\mu}\sigma^{\nu\}\rho}Q_\rho}{2m} J(Q^2) + \frac{Q^\mu Q^\nu - \eta^{\mu\nu}Q^2}{4m} D(Q^2)\right] u(P_1)$$

Form factors parameterize interaction based on symmetry

- ➢ accessible experimentally
- ➢ mass distribution, gravitomagnetic moment, internal structures

EM form factors in vacuum



$P_i \cdot \bar{\sigma} = u(P_i)u(P_i)^\dagger$ from propagators

$$u(P_2)u^\dagger(P_2)i\sigma^\mu u(P_1)u^\dagger(P_1)A_\mu$$

$$u^\dagger(P_2)i\left(n^\mu + \hat{p}^\mu + \frac{i\epsilon^{\mu\nu\rho\sigma}n_\nu P_\rho Q_\sigma}{2(P\cdot n)}\right)u(P_1)$$

n^μ frame vector

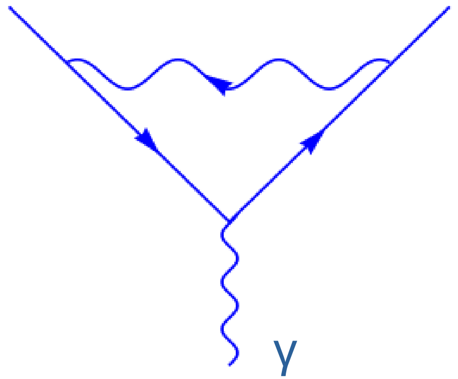
no-work condition

$Q \cdot n = 0$ Ward identity satisfied by each structure,

$P \cdot Q = 0$ three form factors (FF) degenerate

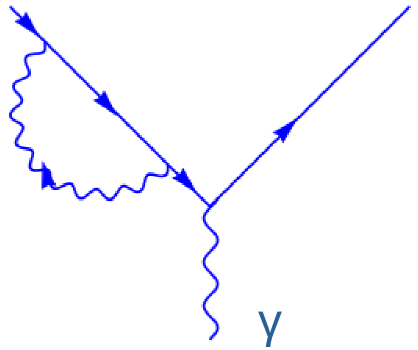
SL, Tian, 2306.14811

What to expect for FF in medium?



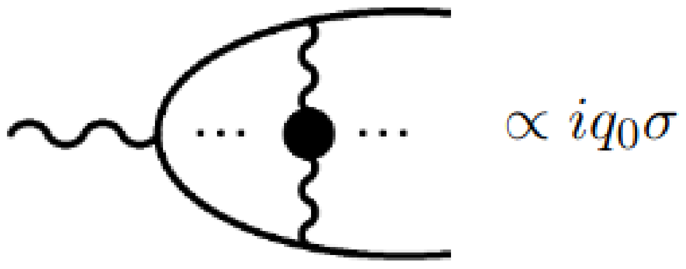
Effect of radiative correction (vacuum)

- both vertex and fermion states corrected by interaction
- Lift degeneracy of FF



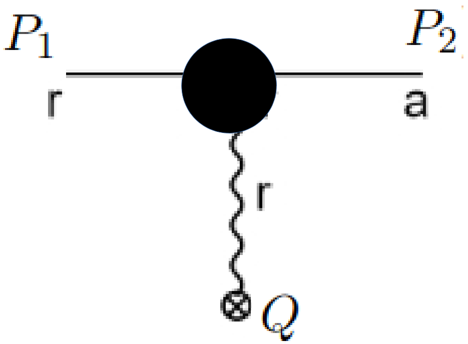
Effect of medium

- enhanced phase space for particles in loop
- breaking of Lorentz invariance, more structures possible
- dissipation effect introduces non-hermiticity, complex form factors



EM form factors in medium

$n^\mu \to u^\mu$ medium frame vector



$$\sigma^\mu \to \Gamma^\mu = F_0 u^\mu + F_1 \hat{p}^\mu + F_2 \frac{i\epsilon^{\mu\nu\rho\sigma} u_\nu P_\rho Q_\sigma}{2(P\cdot u)^2}$$

$$S^{<0} = F_2 \left(\vec{p}\cdot\vec{B}\right) 2\pi\delta'(P^2) f(p_0)$$

$$S^{<i} = \left[F_0 \epsilon^{ijk} E_j p_k + F_1 \left(p_0 B^i - \left(\vec{B}\cdot\vec{p}\right)\hat{p}^i\right) + F_2 \left(\vec{B}\cdot\vec{p}\right)\hat{p}^i\right] 2\pi\delta'(P^2) f(p_0)$$

spin Hall effect

spin-perpendicular magnetic coupling

spin-parallel magnetic coupling

medium interaction can lift the degeneracy of form factors

SL, Tian, 2306.14811

Transformation under time-reversal

$$\Gamma^\mu = F_0 u^\mu + F_1 \hat{p}^\mu + F_2 \frac{i\epsilon^{\mu\nu\rho\sigma} u_\nu P_\rho Q_\sigma}{2(P \cdot u)^2}$$

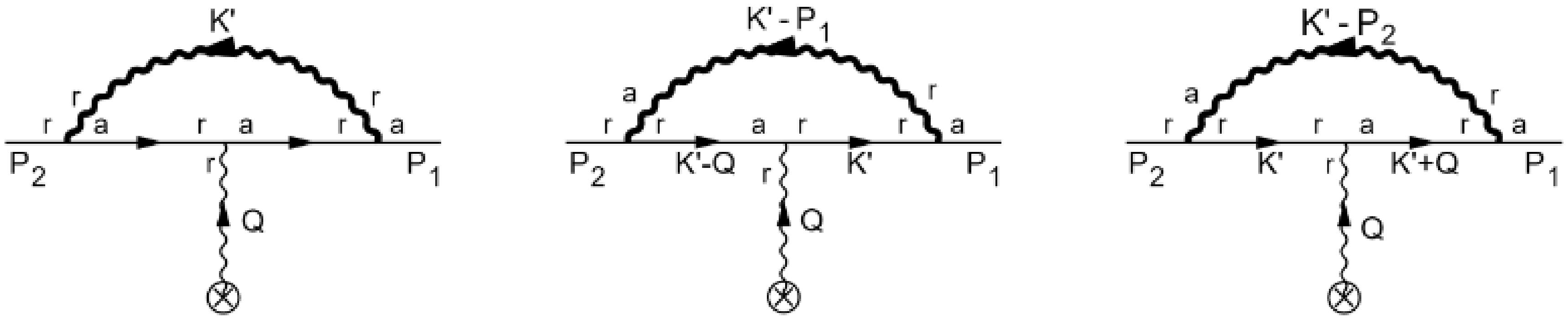
Γ^0 T-even Γ^i T-odd

→ All form factors T-even

no-work condition $Q \cdot n = 0$, $P \cdot Q = 0$ $F_i = F_i(p^2, q^2)$ → All form factors real

SL, Tian, 2306.14811

Example: vertex correction to EMFF



$$-2im_f^2 A_\nu \sigma_\lambda \left(\hat{l}^\lambda \hat{l}^\nu \frac{1}{p^2}\ln\frac{2p}{q} + \hat{p}^\lambda \hat{p}^\nu \frac{1}{p^2} - \hat{p}^\lambda P^\nu \frac{1}{p^3}\ln\frac{2p}{q}\right)$$

$$m_f^2 = \tfrac{1}{8} g^2 T^2 C_F \qquad \hat{l}^i = \tfrac{1}{pq}\epsilon^{ijk} q_j p_k$$

simplificaitons

- medium contribution only (HTL)
- leading contributions as $q \to 0$

SL, Tian, 2306.14811

IR limit and screening

$$-2im_f^2 A_\nu \sigma_\lambda \left(\hat{l}^\lambda \hat{l}^\nu \frac{1}{p^2} \ln\frac{2p}{q} + \hat{p}^\lambda \hat{p}^\nu \frac{1}{p^2} - \hat{p}^\lambda P^\nu \frac{1}{p^3} \ln\frac{2p}{q} \right)$$

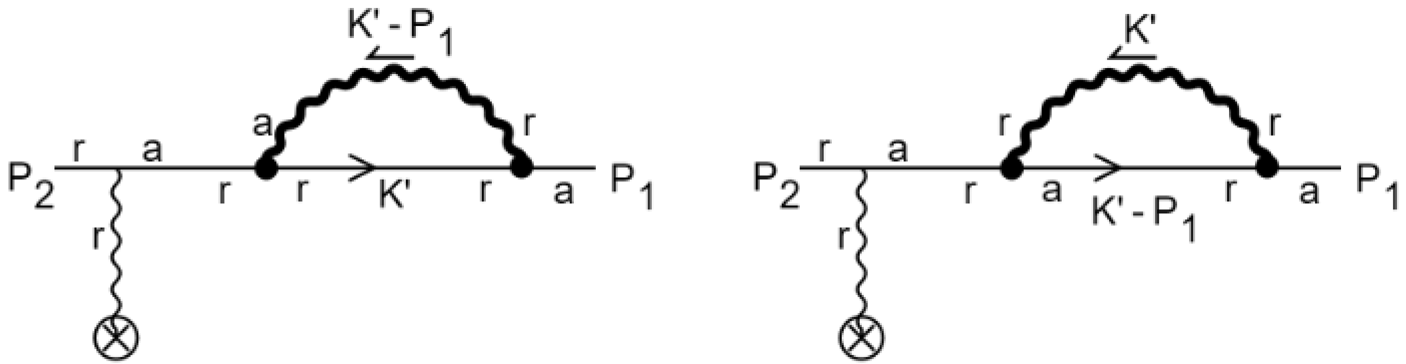
potential IR divergent as $q \to 0$, cutoff by screening effect

$$\delta\Gamma^\nu_{vertex} A_\nu = 2m_f^2 A_\nu \sigma_\lambda \left[\frac{1}{6p^2} \left(2\ln\left(\frac{pT}{m_f^2}\right) + \ln\left(\frac{2pT}{m_g^2}\right) - 36\ln(A) + \ln\left(16\pi^3\right) + 3 \right) \times \left(\hat{l}^\lambda \hat{l}^\nu - \hat{p}^\lambda P^\nu \frac{1}{p} \right) + \hat{p}^\lambda \hat{p}^\nu \frac{1}{p^2} \right],$$

$$m_g^2 = \tfrac{1}{3} g^2 T^2 (C_A + \tfrac{1}{2} N_f) \qquad A \simeq 1.282$$

SL, Tian, 2306.14811

Example: self-energy correction to EMFF



$$S^{ra}(P) = \frac{i}{2}\Delta_+(P)\left(\gamma^0 - \gamma\cdot\hat{p}\right) + \frac{i}{2}\Delta_-(P)\left(\gamma^0 + \gamma\cdot\hat{p}\right)$$

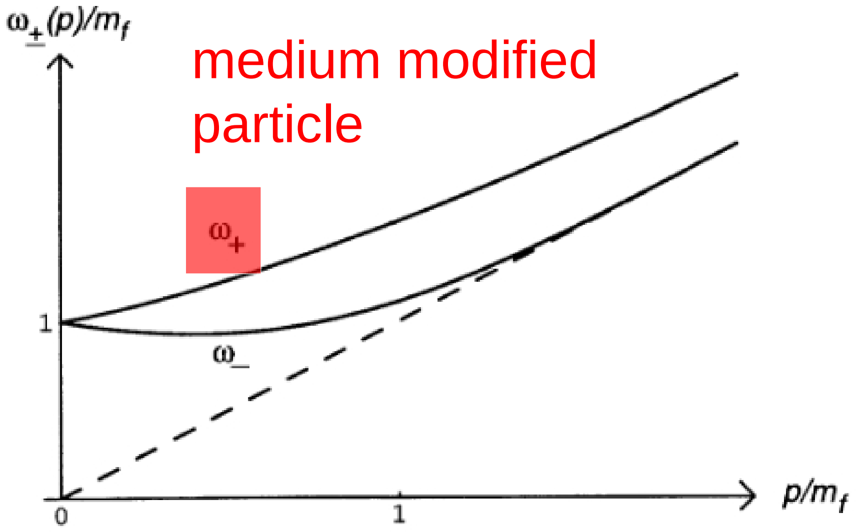
chiral symmetry remains

$$\delta\Gamma^{\mu} = \delta Z_+ \sigma^{\mu}$$

$p \gg m_f$:

$$\delta\Gamma^{\nu}_{self-energy} A_{\nu} = 2\frac{m_f^2}{2p^2}\left(1 - ln\frac{2p^2}{m_f^2}\right)\sigma^{\nu} A_{\nu}.$$

medium modified particle



Le Bellac, thermal field theory

Sum of vertex & self-energy corrections

$$\delta F_0 = \frac{2m_f^2}{p^2} X + \frac{m_f^2}{p^2}\left(1 - ln\frac{2p^2}{m_f^2}\right),$$

spin Hall effect

$$\delta F_1 = \frac{2m_f^2}{p^2}(X - 1) + \frac{m_f^2}{p^2}\left(1 - ln\frac{2p^2}{m_f^2}\right),$$

spin-perpendicular magnetic coupling

$$\delta F_2 = \frac{2m_f^2}{p^2} X + \frac{m_f^2}{p^2}\left(1 - ln\frac{2p^2}{m_f^2}\right),$$

spin-parallel magnetic coupling

$$X = \frac{1}{6}\left(2\ln\left(\frac{pT}{m_f^2}\right) + \ln\left(\frac{2pT}{m_g^2}\right) - 36\ln(A) + \ln\left(16\pi^3\right) + 3\right) \qquad A \simeq 1.282$$

- all form factors real
- partial lift of the degeneracy $\delta F_1 \neq \delta F_2 = \delta F_0$

Gravitational FF in vacuum

FF for massless case $Q \to 0$, ignore D-term

$$\langle P_2|T^{\mu\nu}(Q)|P_1\rangle = \bar{u}(P_2)\left[A(Q^2)\frac{P^\mu P^\nu}{P\cdot n} \pm B(Q^2)\frac{-iP^{\{\mu}\epsilon^{\nu\}\lambda\sigma\rho}\gamma_\lambda n_\sigma Q_\rho}{P\cdot n}\right]u(P_1)$$

compared to massive case

$$\langle P_2|T^{\mu\nu}(Q)|P_1\rangle = \bar{u}(P_2)[\frac{P^\mu P^\nu}{m}A(Q^2) + \frac{iP^{\{\mu}\sigma^{\nu\}\rho}Q_\rho}{2m}J(Q^2) + \frac{Q^\mu Q^\nu - \eta^{\mu\nu}Q^2}{4m}D(Q^2)]u(P_1)$$

tree-level $T^{\mu\nu} = \frac{i}{2}\bar{\psi}\left(\gamma^{\{\mu}\partial^{\nu\}} - \gamma^{\{\mu}\overleftarrow{\partial}^{\nu\}}\right)\psi$.

→ $A = 1 \quad B = -\frac{1}{2}$

metric perturbation $h_{0i}(t,x) = v_i(t,x)$.

$$i\mathcal{M} \sim \bar{u}(P)\sigma_k u(P) i\epsilon^{ijk} q_j v_i \sim \vec{S}\cdot\vec{\omega}$$

spin-vorticity coupling

SL, Tian, 2302.12450

Gravitational FF in medium

Einstein equivalence principle $B(Q^2 = 0) = -\frac{1}{2}$

spin-vorticity coupling dictated for any $S = \frac{1}{2}$ particle

medium breaks Lorentz invariance,
violating equivalence principle!

Donoghue et al 1984, 1985

Buzzegoli, Kharzeev, PRD 2021

SL, Tian, 2302.12450

$$\Gamma^{\mu\nu} = \gamma \cdot \hat{p} \left(F_0 u^\mu u^\nu + F_1 u^{\{\mu} \hat{p}^{\nu\}} + F_2 \hat{p}^\mu \hat{p}^\nu \right) + \gamma \cdot \hat{l} \left(F_3 \hat{p}^{\{\mu} \hat{l}^{\nu\}} + F_4 u^{\{\mu} \hat{l}^{\nu\}} \right)$$

$$\hat{l}_i = \epsilon^{ijk} \hat{q}_j \hat{p}_k$$

five structures, each satisfies energy-momentum conservation

Example: medium correction to gravitational FF

vertex correction

$$\delta\Gamma^{\mu\nu} = m_f^2\big[-\gamma\cdot\hat{p}P^\mu P^\nu \frac{\ln\frac{2p}{q}}{p^3} - \gamma\cdot\hat{l}P^{\{\mu}\hat{l}^{\nu\}}\frac{\ln\frac{2p}{q}}{p^2} + \gamma\cdot\hat{p}\left(2u^\mu u^\nu + u^{\{\mu}\hat{p}^{\nu\}} + \hat{p}^\mu\hat{p}^\nu\right)\frac{1}{p} + 2\gamma\cdot\hat{l}\hat{l}^{\{\mu}\hat{p}^{\nu\}}\big]$$

self-energy

$$\delta\Gamma^{\mu\nu} = \delta Z_+\gamma^{\{\mu}P^{\nu\}} \qquad \delta Z_+ = \frac{m_f^2}{2p^2}\left(1 - \ln\frac{2p^2}{m_f^2}\right)$$

Application: spin-vorticity coupling receives multiplicative renormalization

e.g. $p = 500\text{MeV}$ $\quad T = 150\text{MeV}$ $\quad \alpha_s = 0.3$

7% suppression of spin-vorticity coupling

SL, Tian, 2302.12450

Summary

- Correlation function description of spin polarization interpreted as scattering problem, leading to form factors description
- In-medium electromagnetic FF lift degeneracy of spin magnetic coupling and spin Hall effect
- In-medium gravitational FF leads to suppression of spin-vorticity coupling

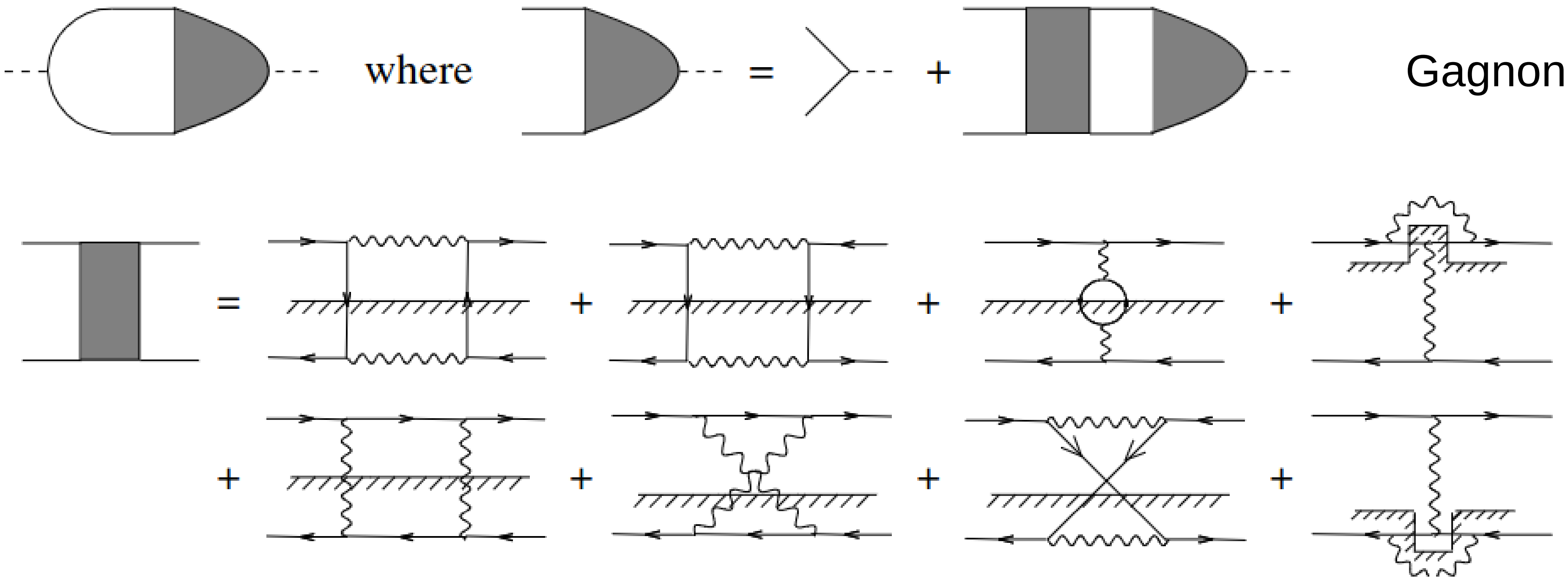
Outlook

- Polarization in strongly coupled medium with holography
- Dissipation effect: complex FF
- Applications to spin polarization in heavy ion collisions

Thank you!

Field theory: collisional (QED)

$$\sigma \;=\; \frac{\beta}{6} \lim_{k^0\to 0,\,\mathbf{k}=0} \int d^4x\; e^{ik\cdot x}\; \langle j_i(t,\mathbf{x}) j^i(0)\rangle_{\rm eq}$$



Gagnon, Jeon, PRD 2007

resummation of two-loop diagrams

$$\tau_{\rm rel} \sim \frac{T}{e^4}$$