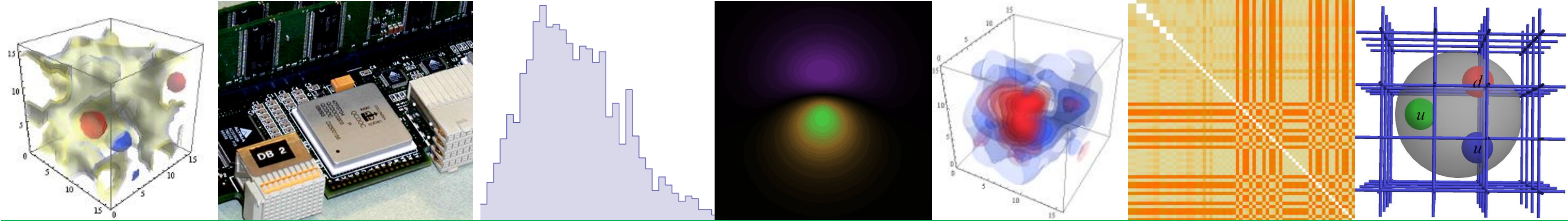


Advancing Nucleon and Pion Parton Distributions with Lattice QCD: Insights and Impact on Global Analyses

HUEY-WEN LIN

This work of HL is supported by the NSF under grant PHY 2209424 & 1653405, DOE under DE-SC0024053 and the Research Corporation for Science Advancement through the Cottrell Scholar Award

@LinQCD

NSF

U.S. DEPARTMENT OF ENERGY

RESEARCH CORPORATION for SCIENCE ADVANCEMENT

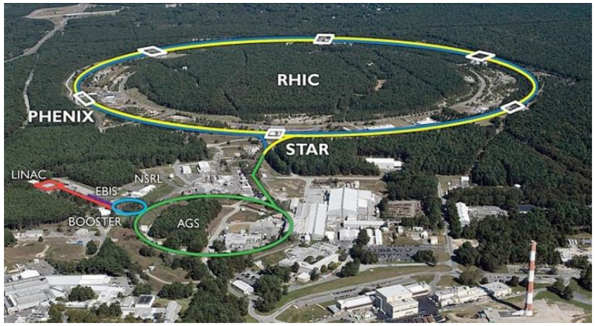
Outline

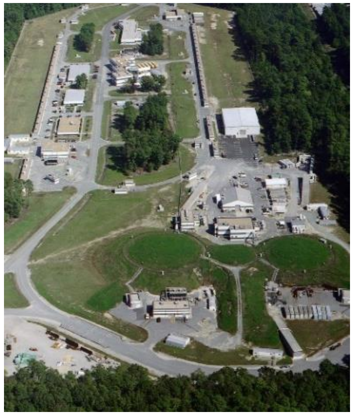
- Lattice QCD and Parton Distribution Functions
- Selected Nucleon and Meson *x*-Dependent Parton Distributions
- Impact of Lattice-QCD PDFs on Global Fits

Parton Distribution Functions

§ PDFs are universal quark/gluon distributions of nucleon

- Many ongoing/planned experiments
  (BNL, JLab, J-PARC, COMPASS, GSI, EIC, EIcC, LHeC, ...)

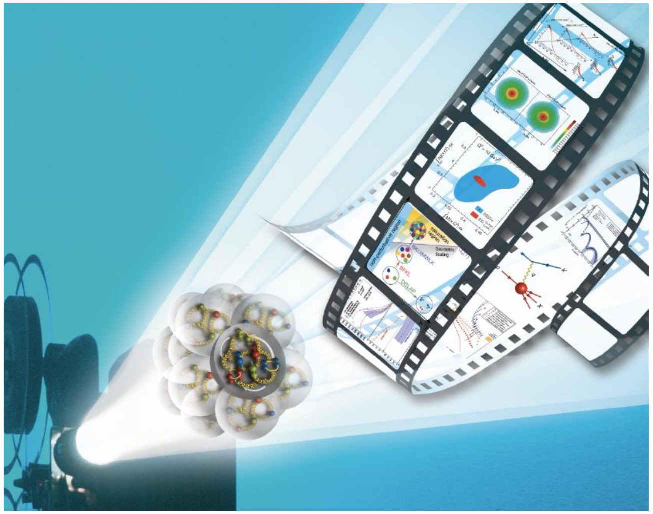




Electron Ion Collider: The Next QCD Frontier

Imaging of the proton

How are the sea quarks and gluons, and their spins, distributed in space and momentum inside the nucleon?



EIC White Paper, 1212.1701; [The Present and Future of QCD](#) (2303.02579)

Global Analysis

§ Experiments cover diverse kinematics of parton variables

- Global analysis takes advantage of all data sets

Theory Input → **Global Analysis of PDFs** ← **Exp't Input**

§ Some choices made for the analysis

- Choice of data sets and kinematic cuts
- Strong coupling constant $\alpha_s(M_Z)$
- How to parametrize the distribution

$$xf(x,\mu_0) = a_0 x^{a_1}(1-x)^{a_2} P(x)$$

- Assumptions imposed

  SU(3) flavor symmetry, charge symmetry, strange and sea distributions

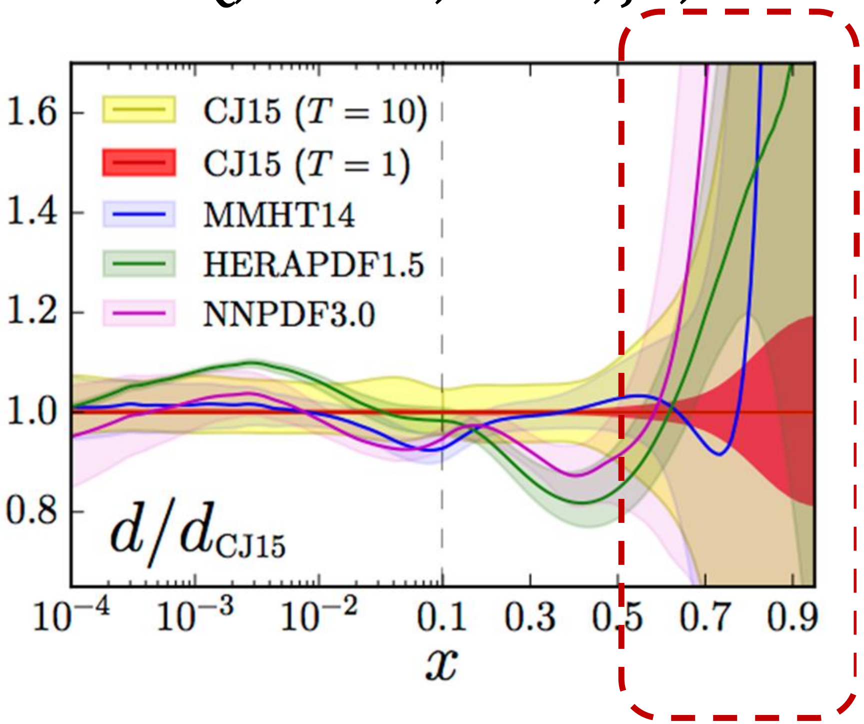
$$s = \bar{s} = \kappa(\bar{u} + \bar{d})$$

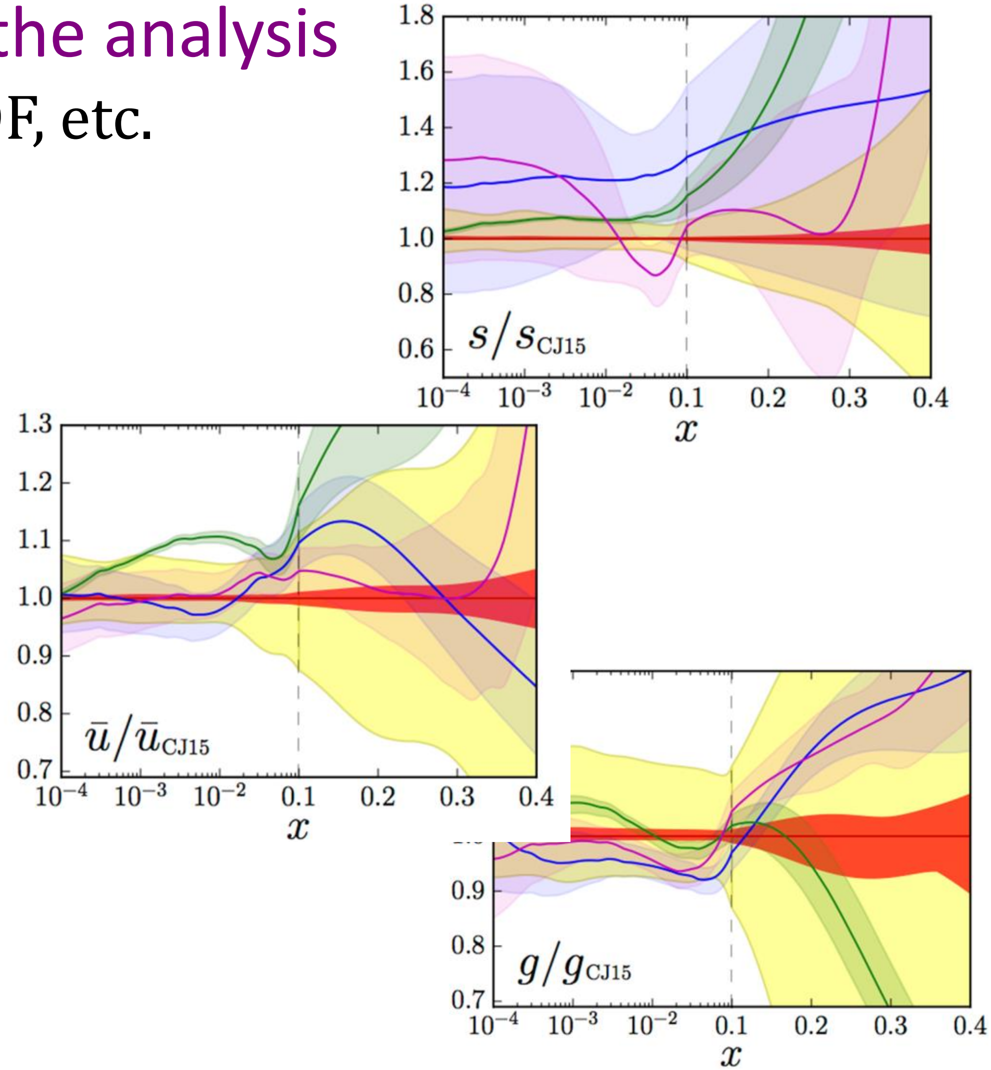
Global Analysis

§ Discrepancies appear when data is scarce

§ Many groups have tackled the analysis

- CTEQ, MSTW, ABM, JR, NNPDF, etc.





CTEQ-JLAB
https://www.jlab.org/theory/cj/

Lattice QCD 101

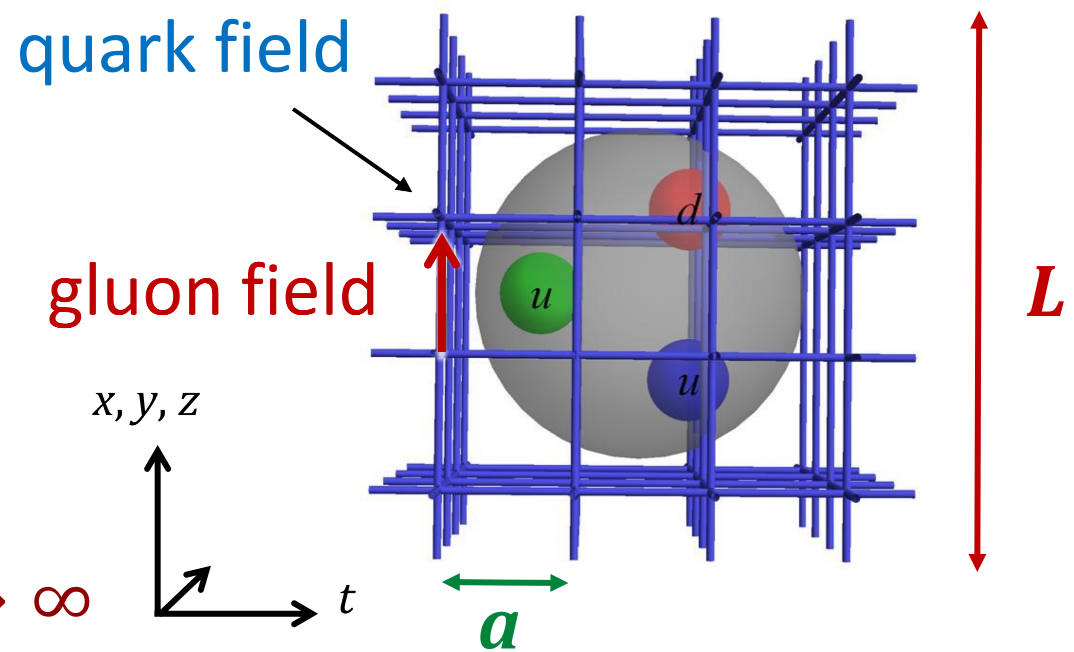
§ Lattice QCD is an ideal theoretical tool for investigating the strong-coupling regime of quantum field theories

§ Physical observables are calculated from the path integral

$$\langle 0|O(\bar{\psi},\psi,A)|0\rangle = \frac{1}{Z}\int \mathcal{D}A\,\mathcal{D}\bar{\psi}\,\mathcal{D}\psi\, e^{iS(\bar{\psi},\psi,A)}O\,(\bar{\psi},\psi,A)$$

in **Euclidean** space

- Quark mass parameter (described by m_π)
- Impose a UV cutoff
  discretize spacetime
- Impose an infrared cutoff
  finite volume



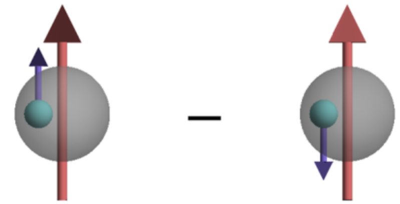
§ Recover physical limit

$m_\pi \to m_\pi^{\text{phys}}, \boldsymbol{a} \to \mathbf{0}, \boldsymbol{L} \to \infty$

Moments of PDFs

- PDG-like rating system or average
- LatticePDF Workshop
  - Lattice representatives came together and devised a rating system
- Lattice QCD/global fit status

$$\langle x^{n-1}\rangle_{\delta q} = \int_{-1}^{1} dx\, x^{n-1}\delta q(x)$$



LatticePDF Report, 1711.07916, 2006.08636

Moment	Collaboration	Reference	N_f	DE	CE	FV	RE	ES		Value	Global Fit
g_T	ETMC 19	(Alexandrou *et al.*, 2019b)	2+1+1	■	★	○	★	★	**	0.926(32)	0.10 — 1.1
	PNDME 18	(Gupta *et al.*, 2018)	2+1+1	★	★	★	★	★	*	0.989(32)(10)	
	χQCD 20	(Horkel *et al.*, 2020)	2+1	■	★	○	★	★	†	1.096(30)	
	LHPC 19	(Hasan *et al.*, 2019)	2+1	○	★	○	★	★	*	0.972(41)	
	Mainz 19	(Harris *et al.*, 2019)	2+1	★	○	★	★	★		$0.965(38)(^{+13}_{-41})$	
	JLQCD 18	(Yamanaka *et al.*, 2018)	2+1	■	○	○	★	★		1.08(3)(3)(9)	
	ETMC 19	(Alexandrou *et al.*, 2019b)	2	■	★	○	★	★	**	0.974(33)	
	ETMC 17	(Alexandrou *et al.*, 2017d)	2	■	★	■	★	★		1.004(21)(02)(19)	
	RQCD 14	(Bali *et al.*, 2015)	2	○	★	★	★	■		1.005(17)(29)	
$\langle 1\rangle_{\delta u^-}$	ETMC 19	(Alexandrou *et al.*, 2019b)	2+1+1	■	★	○	★	★	**	0.716(28)	-0.14 — 0.91
	PNDME 18	(Gupta *et al.*, 2018)	2+1+1	★	★	★	★	★	*	0.784(28)(10)	
	JLQCD 18	(Yamanaka *et al.*, 2018)	2+1	■	○	○	★	★		0.85(3)(2)(7)	
	ETMC 17	(Alexandrou *et al.*, 2017d)	2	■	★	■	★	★		0.782(16)(2)(13)	
$\langle 1\rangle_{\delta d^-}$	ETMC 19	(Alexandrou *et al.*, 2019b)	2+1+1	■	★	○	★	★	**	-0.210(11)	-0.97 — 0.47
	PNDME 18	(Gupta *et al.*, 2018)	2+1+1	★	★	★	★	★	*	-0.204(11)(10)	
	JLQCD 18	(Yamanaka *et al.*, 2018)	2+1	■	○	○	★	★		-0.24(2)(0)(2)	
	ETMC 17	(Alexandrou *et al.*, 2017d)	2	■	★	■	★	★		-0.219(10)(2)(13)	
$\langle 1\rangle_{\delta s^-}$	ETMC 19	(Alexandrou *et al.*, 2019b)	2+1+1	■	★	○	★	★	**	-0.0027(58)	N/A
	PNDME 18	(Gupta *et al.*, 2018)	2+1+1	★	★	★	★	★	*	-0.0027(16)	
	JLQCD 18	(Yamanaka *et al.*, 2018)	2+1	■	○	○	★	★		-0.012(16)(8)	
	ETMC 17	(Alexandrou *et al.*, 2017d)	2	■	★	■	★	★		-0.00319(69)(2)(22)	

Moments of PDFs

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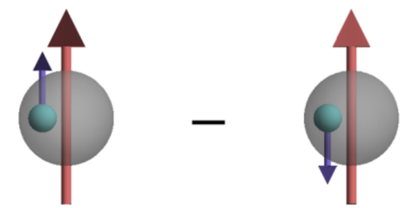
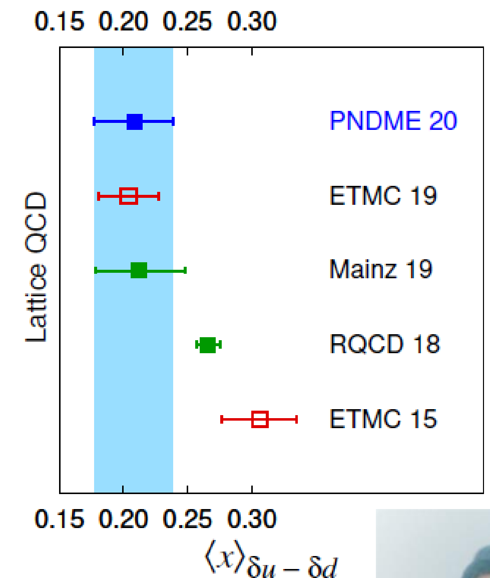
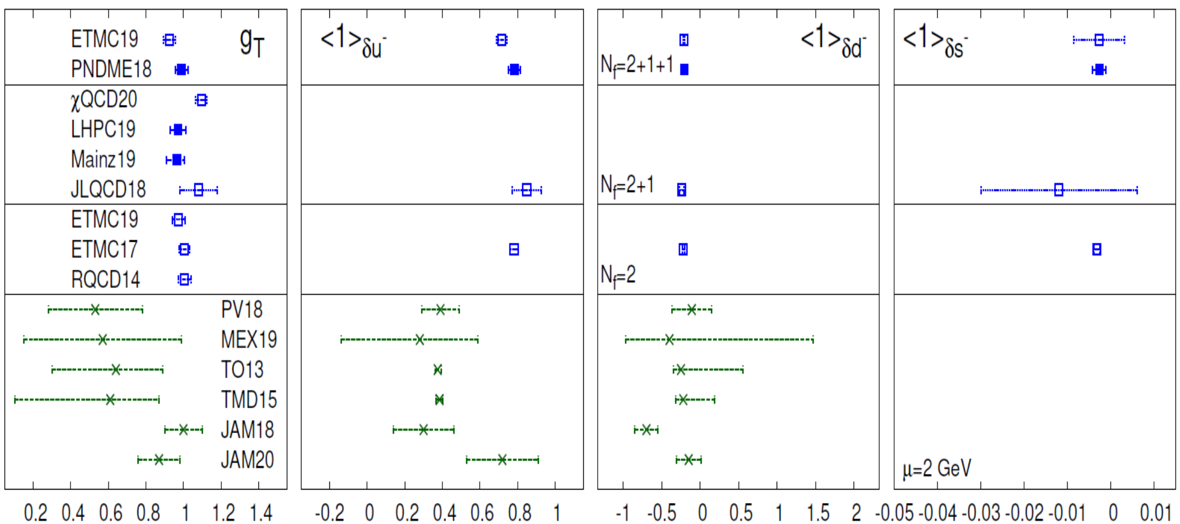
- LatticePDF Workshop
  - Lattice representatives came together and devised a rating system
- Recent lattice QCD/global fit status

$$\langle x^{n-1}\rangle_{\delta q} = \int_{-1}^{1} dx\, x^{n-1}\delta q(x)$$





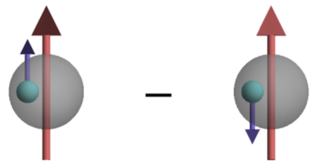
LatticePDF Reports, 1711.07916, 2006.08636



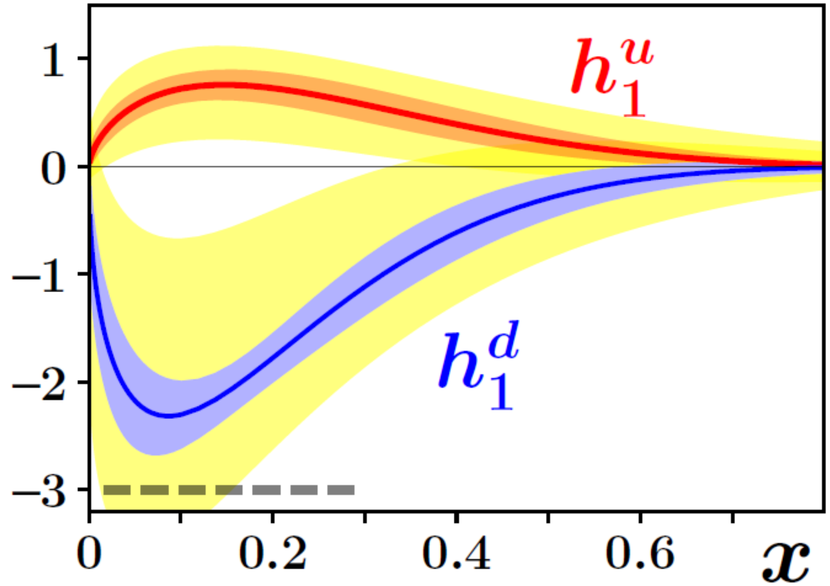
S. Mondal et al 2005.13779

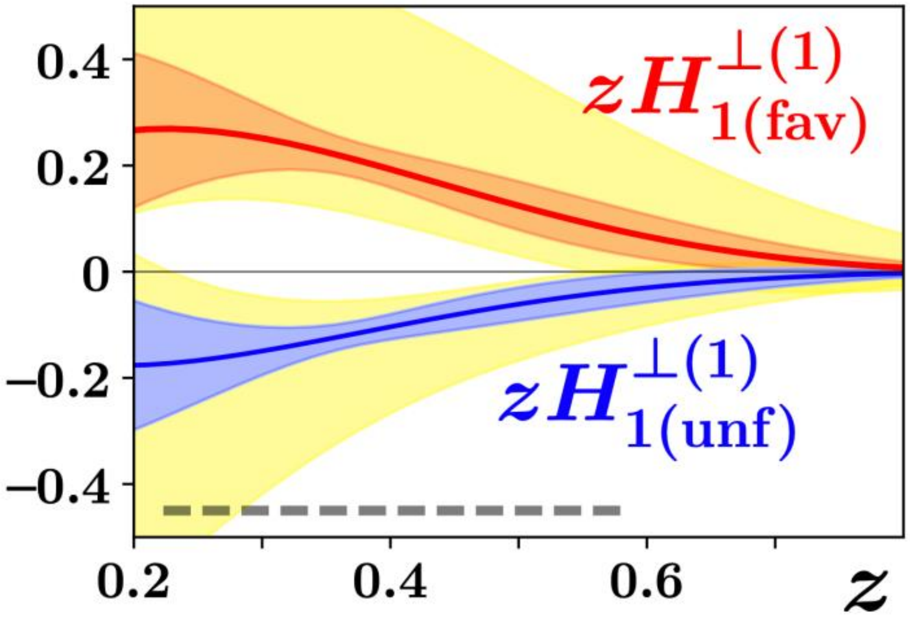
From Charges to PDFs

§ Improved transversity distribution with LQCD tensor charge, $g_T = \int_{-1}^{1} dx\, (\, h_1^u(x) - h_1^d(x)\,)$



- Global analysis with 12 extrapolation forms: $g_T = 1.006(58)$
- Use to constrain the global analysis fits to SIDIS $\pi^{\pm}$ production data from proton and deuteron targets





Lin, Melnitchouk, Prokudin, Sato, 1710.09858, Phys. Rev. Lett. 120, 152502 (2018)

Outline

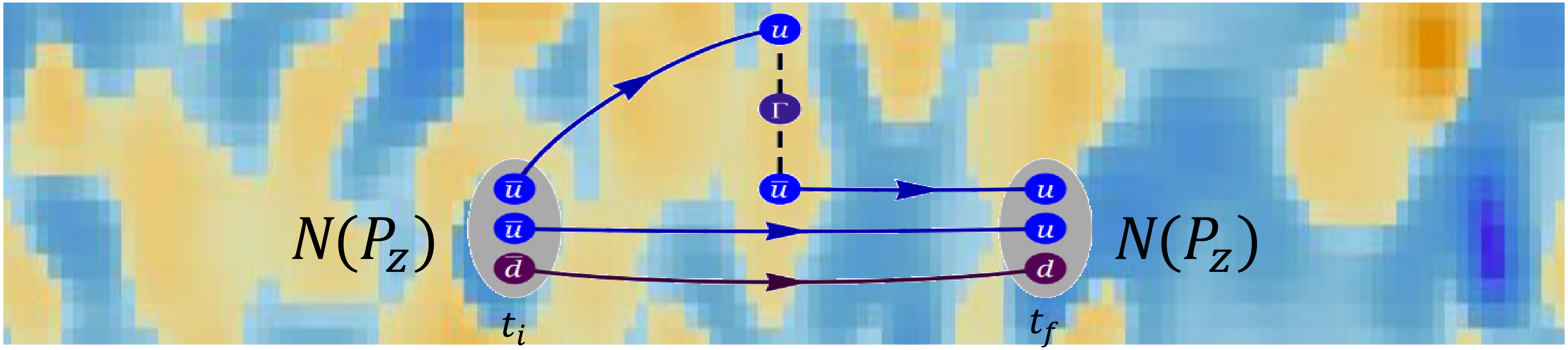
§ Lattice QCD and Parton Distribution Functions

§ **Selected Nucleon and Meson *x*-Dependent Parton Distributions**

§ Impact of Lattice-QCD PDFs on Global Fits

Lattice Parton Method

§ Large-momentum effective theory (LaMET)/quasi-PDF
(X. Ji, 2013; See 2004.03543 for review)



§ Compute quasi-distribution via

$$\tilde{q}(x,\mu,P_z) = \int \frac{dz}{4\pi} e^{-izk_z} \left\langle P \left| \bar{\psi}(z)\Gamma \exp\left(-ig\int_0^z dz' A_z(z')\right)\psi(0) \right| P \right\rangle$$

§ Recover true distribution (take $P_z \to \infty$ limit)

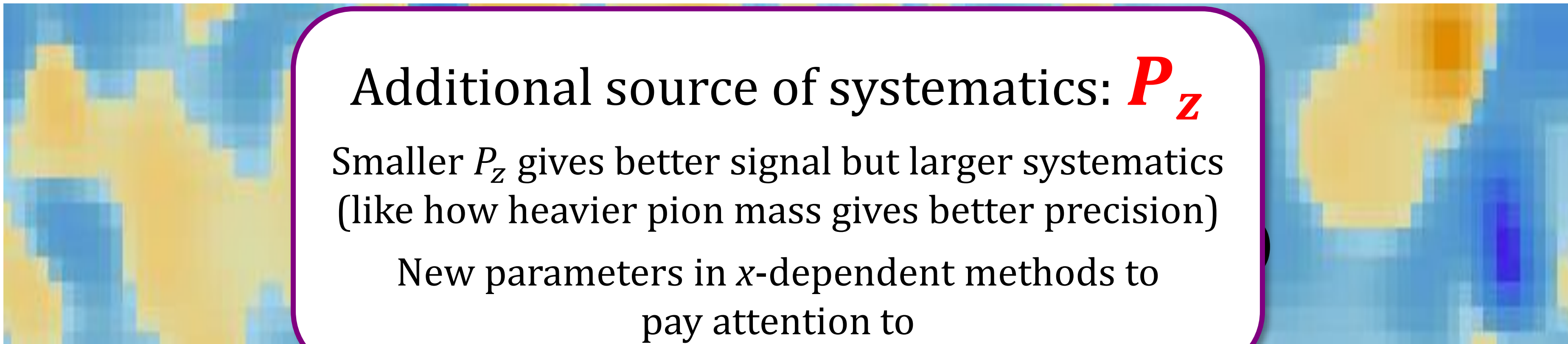
$$\tilde{q}(x,\mu,P_z) = \int_{-\infty}^{\infty} \frac{dy}{|y|} C\left(\frac{x}{y},\frac{\mu}{P_z}\right) \mathbf{q(y,\mu)} + \mathcal{O}\left(\frac{M_N^2}{\mathbf{P_z^2}}, \frac{\Lambda_{\mathrm{QCD}}^2}{(x\mathbf{P_z})^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{((1-x)\mathbf{P_z})^2}\right)$$

X. Xiong e.a., 1310.7471; J.-W. Chen e.a., 1603.06664

Lattice Parton Method

§ Large-momentum effective theory (LaMET)/quasi-PDF

(X. Ji, 2013; *See* 2004.03543 for review)



Additional source of systematics: $\boldsymbol{P_z}$

Smaller P_z gives better signal but larger systematics (like how heavier pion mass gives better precision)

New parameters in x-dependent methods to pay attention to

§ Compute quasi-distribution via

$$\tilde{q}(x,\mu,P_z) = \int \frac{dz}{4\pi} e^{-izk_z} \left\langle P \left| \bar{\psi}(z)\Gamma \exp\left(-ig\int_0^z dz' A_z(z')\right)\psi(0)\right| P \right\rangle$$

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X. Xiong e.a., 1310.7471; J.-W. Chen e.a., 1603.06664

Direct x-Dependent Structure

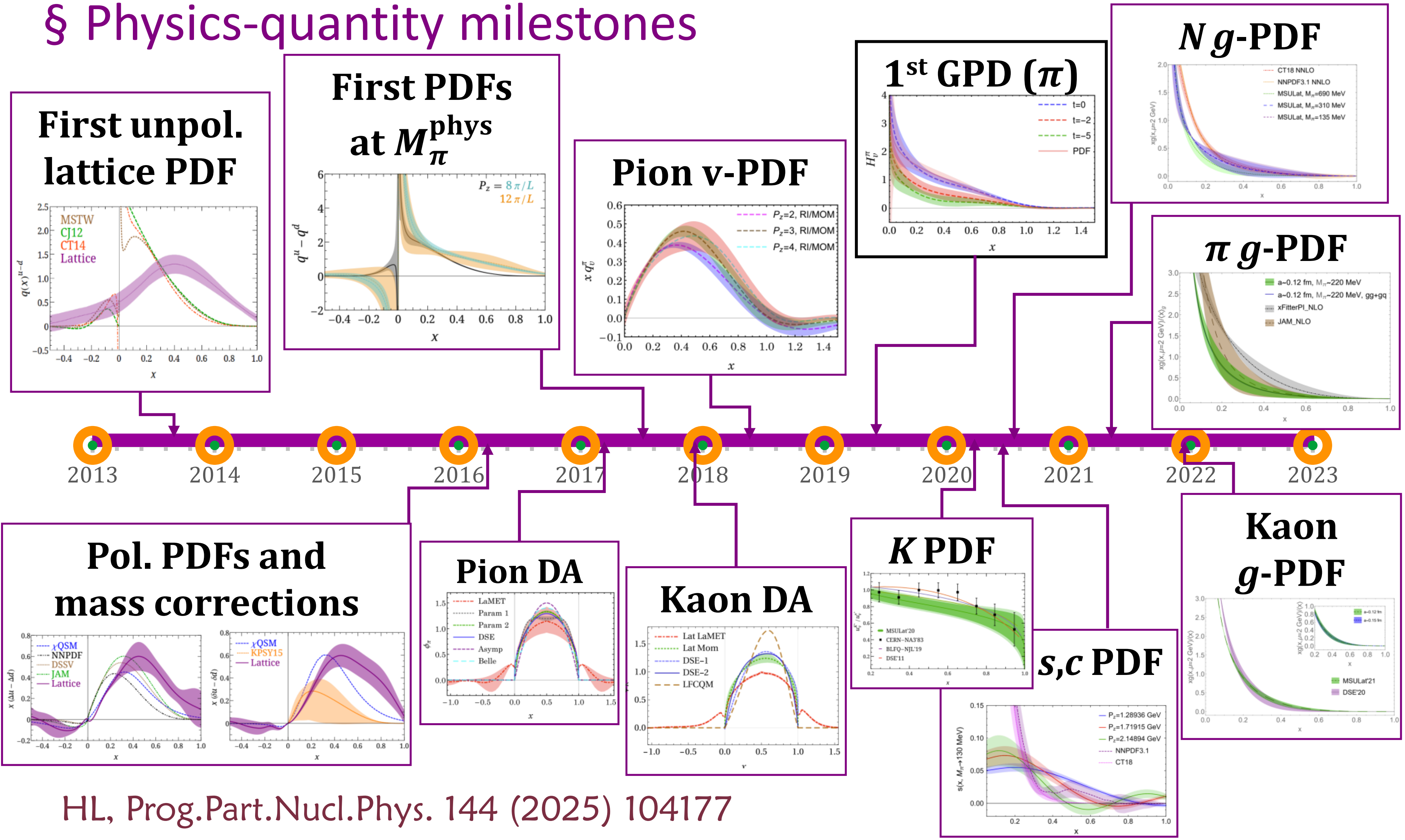
§ Longstanding obstacle to lattice calculations!

Quantities that can be calculated on the lattice today = $\sum$ Wanted PDFs, GPDs, etc. $\times$ pQCD-calculated kernel

- **Quasi-PDF**/large-momentum effective theory (LaMET) (X. Ji, 2013; See 2004.03543 for review)
- **Pseudo-PDF** method: differs in FT (A. Radyushkin, 2017)
- Lattice cross-section method (**LCS**) (Y Ma and J. Qiu, 2014, 2017)
- Compton amplitude method (A.J. Chambers et al., 1703.01153)
- Hadronic tensor currents (Liu et al., hep-ph/9806491, … 1603.07352)
- Euclidean correlation functions (RQCD, 1709.04325)

Lattice Parton Calculations

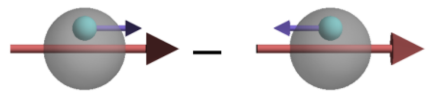
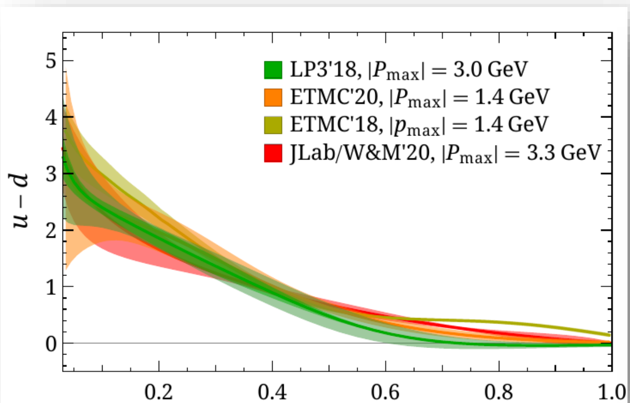
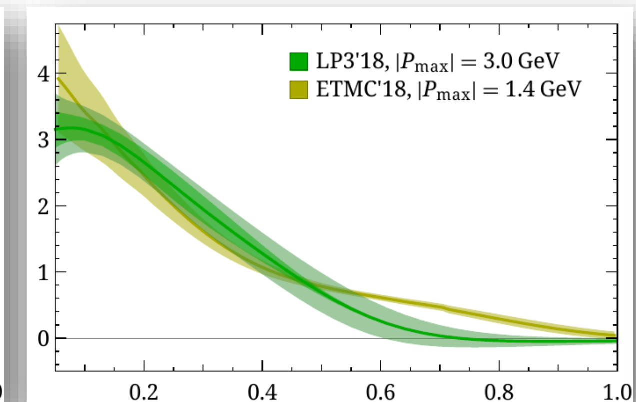
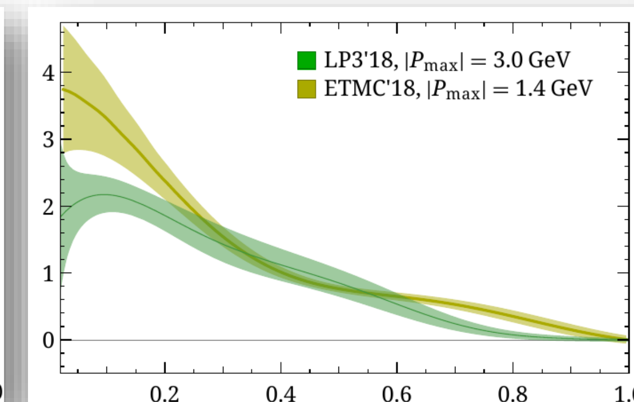
§ Physics-quantity milestones

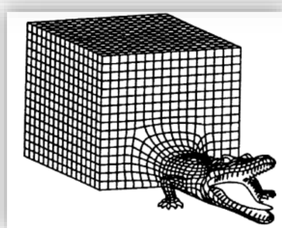


HL, Prog.Part.Nucl.Phys. 144 (2025) 104177

Lattice Example Results

§ Summary of PDF results at physical pion mass

unpolarized	longitudinally polarized	transversely polarized
		
$u(x) - d(x)$	$\Delta u(x) - \Delta d(x)$	$\delta u(x) - \delta d(x)$
		



Finite volume,
Discretization,
...

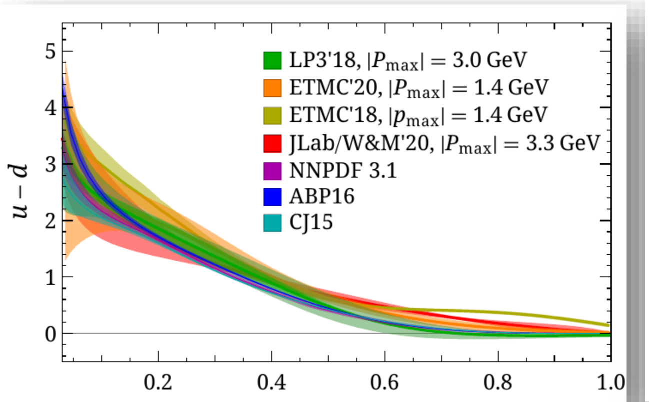
2006.08636 (PDFLattice2019)

Lattice Example Results

§ Summary of PDF results at physical pion mass

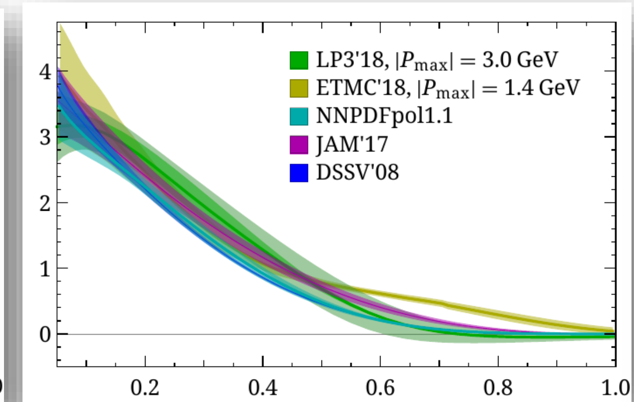
unpolarized

$u(x) - d(x)$



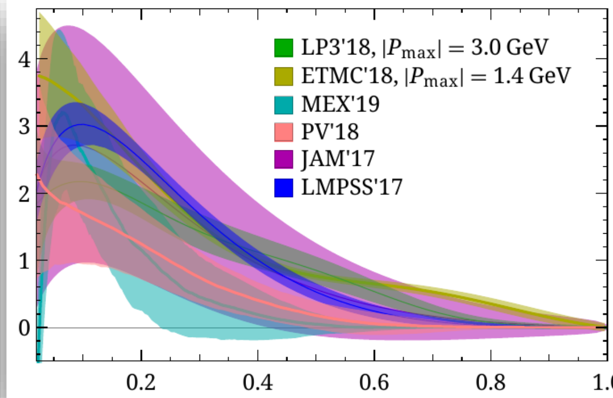
longitudinally polarized

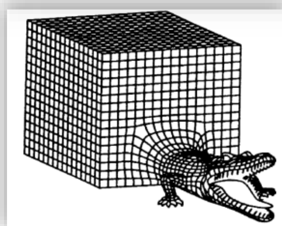
$\Delta u(x) - \Delta d(x)$



transversely polarized

$\delta u(x) - \delta d(x)$





Finite volume,
Discretization,
...

2006.08636 (PDFLattice2019)

Isovector PDFs Update

§ Nucleon isovector PDF calculated directly at **physical pion mass**

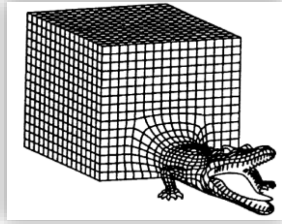
- NNLO matching & treat leading-renormalon effects
  - Leading-renormalon resummation (LRR)
  - Renormalization-group resummation (RGR)

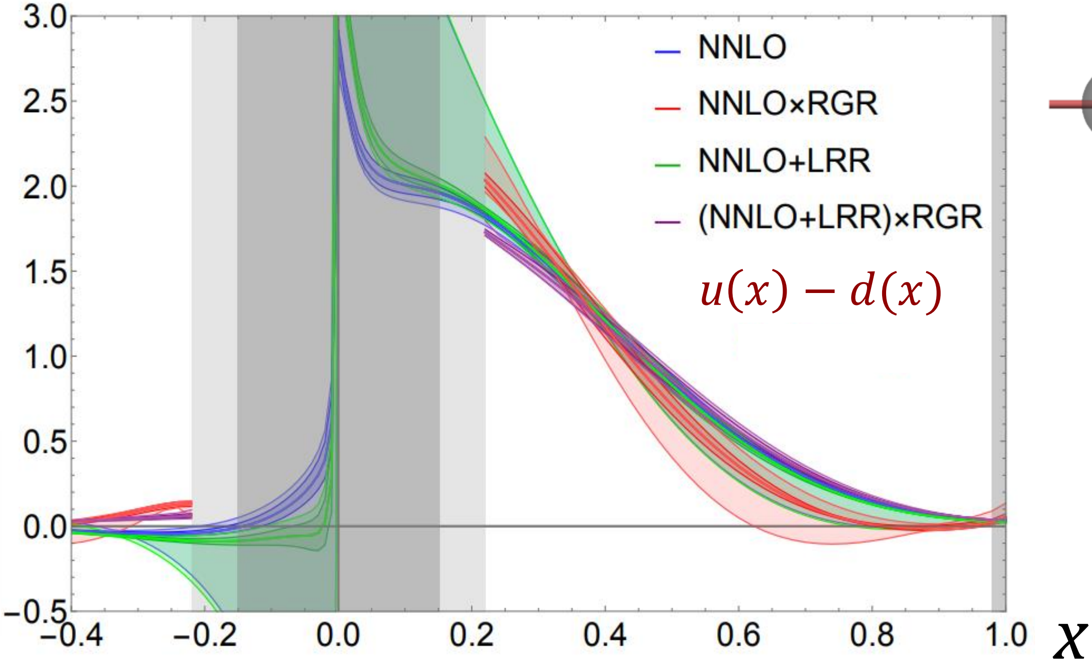
R. Zhang, et. al.
PLB 844, 138081 (2023)

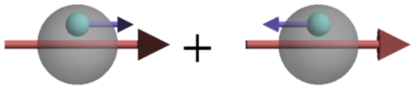
- $N_f = 2+1+1$ clover/HISQ, $a \approx 0.09$ fm, $P_z \approx 2$ GeV

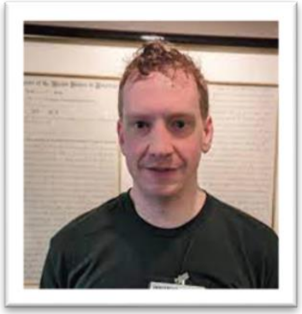
J. Holligan, HL (MSULat), 2312.10829 [hep-lat]

Wanted PDFs, GPDs, etc…









P: Jack Holligan

Isovector PDFs Update

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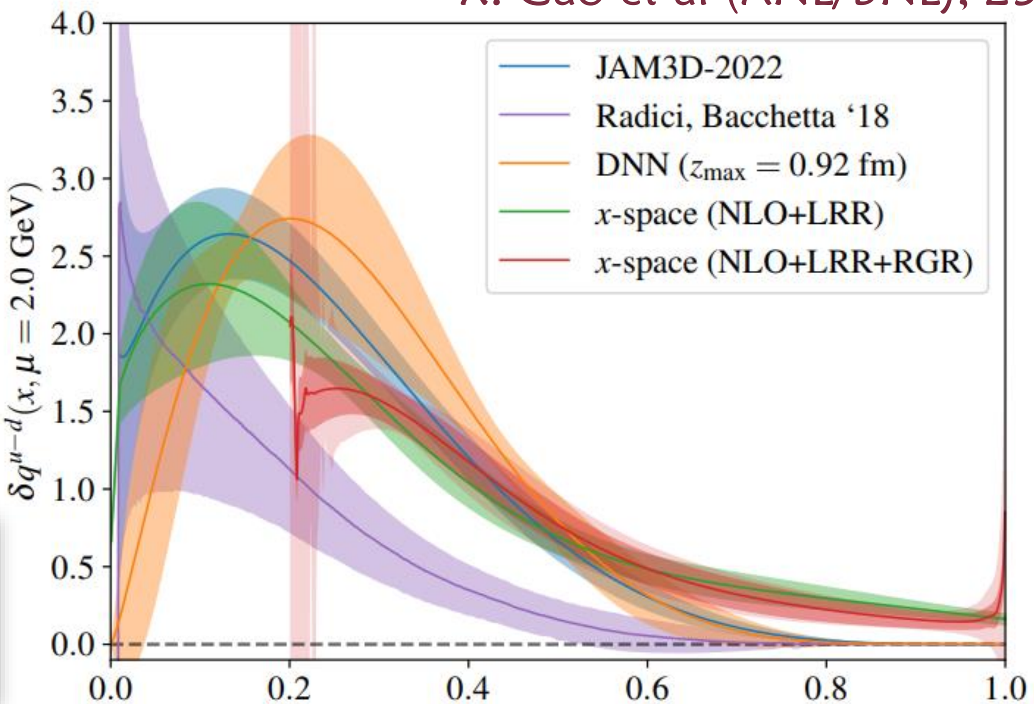
  - Leading-renormalon resummation (LRR)
  - Renormalization-group resummation (RGR)

R. Zhang, et. al.
PLB 844, 138081 (2023)

- $N_f = 2{+}1$ clover/HISQ, $a \approx 0.076$ fm, $P_z \approx 1.5$ GeV

X. Gao et al (ANL/BNL), 2310.19047 [hep-lat]

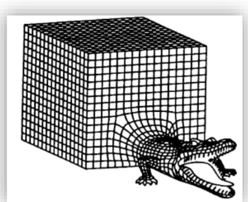
Wanted PDFs, GPDs, etc…



x

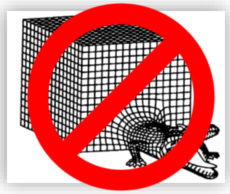
Transversity

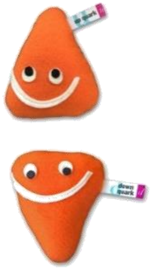
$\delta u(x) - \delta d(x)$



Continuum PDF

§ Nucleon PDFs using quasi-PDFs in the continuum limit

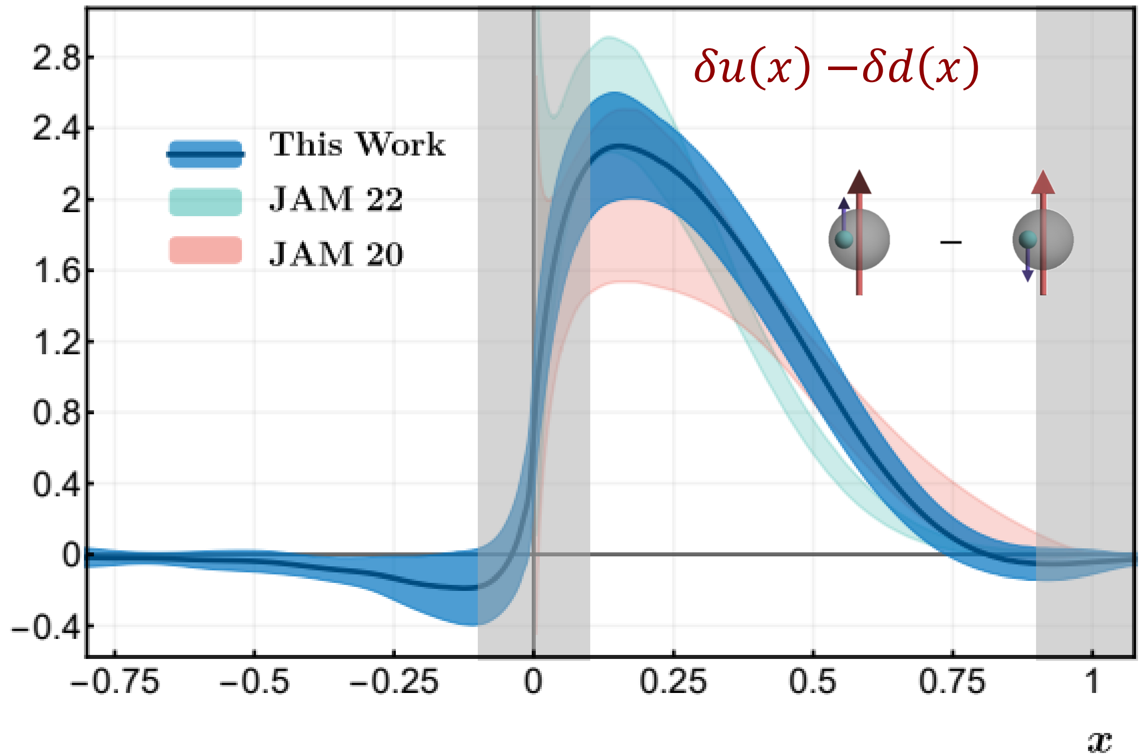




- Lattice details: clover/2+1 clover (LPC)
  - $a \approx \{0.49, 0.64, 0.85, 0.98\}$ fm,
  - $M_\pi \in [222, 354]$-MeV pion,
  - $M_\pi L \in [3.9, 8.1]$
  - $P_z \in [1.8, 2.8]$

F. Yao et al (LPC), 2208.08008

Wanted PDFs, GPDs, etc…

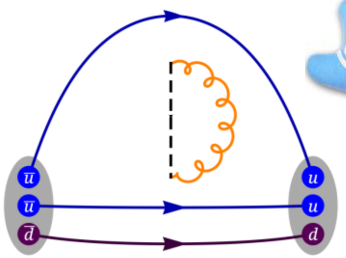


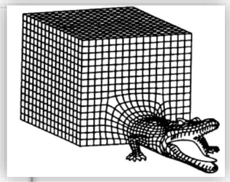
Nucleon Gluon PDF (2020)

§ Gluon PDF using pseudo-PDF

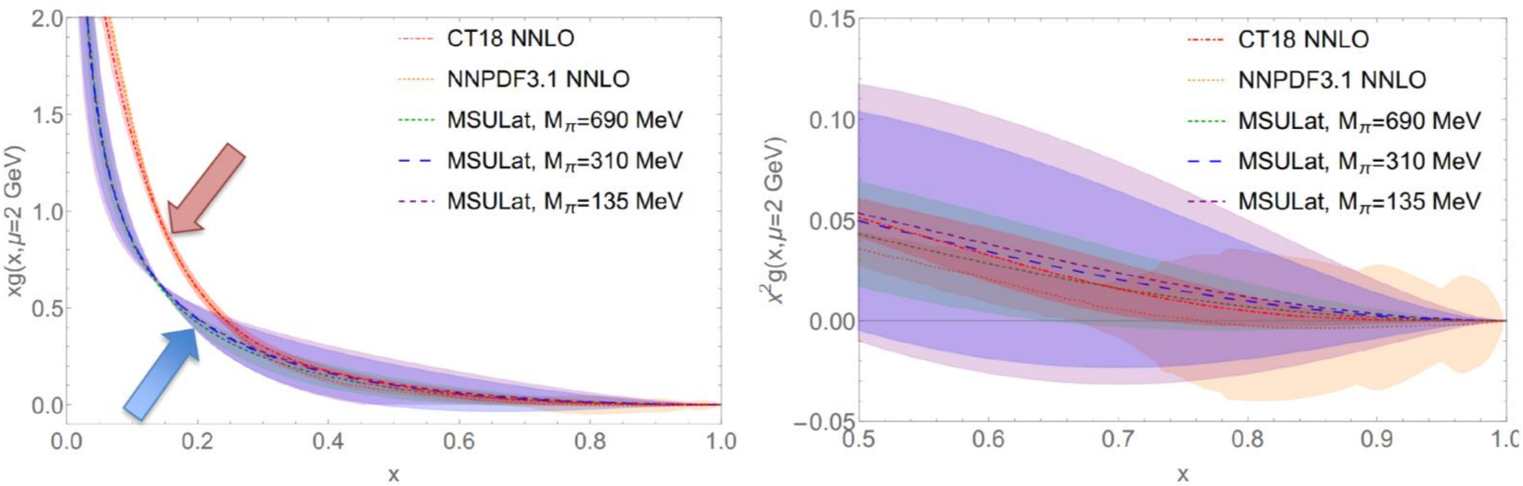
- Lattice details: clover/2+1+1 HISQ 0.12 fm, 310-MeV sea pion
- Study strange/light-quark

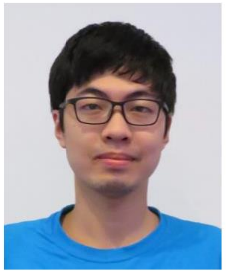
Z. Fan. et al (MSULat), 2007.16113





The comparison of the reconstructed unpolarized gluon PDF from the function form with CT18 NNLO and NNPDF3.1 NNLO gluon unpolarized PDF at $\mu = 2\ GeV$ in the $\overline{\text{MS}}$ scheme.





Slide by Zhouyou Fan@DNP2020

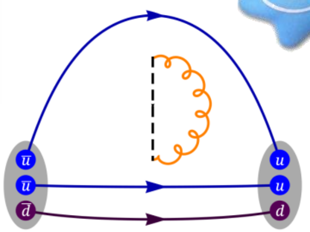
FG: Zhouyou Fan

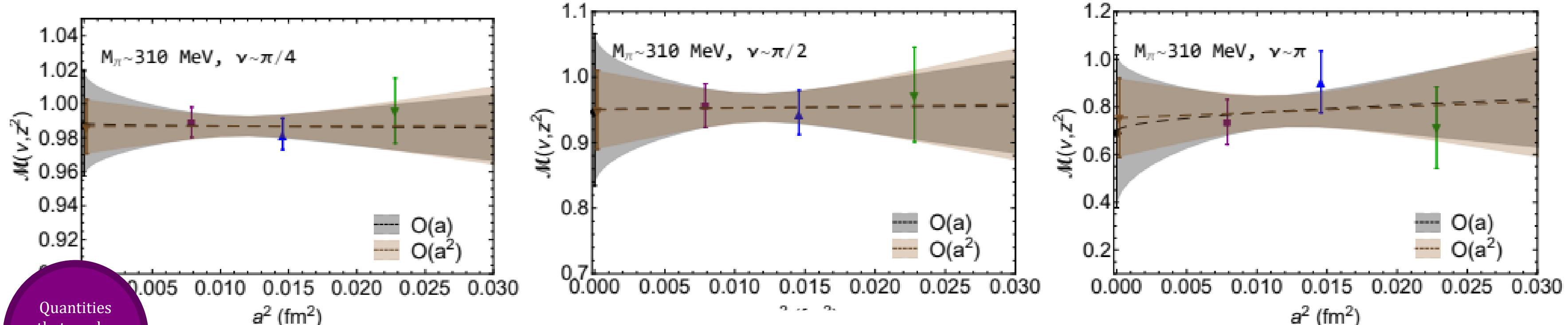
Gluon PDF in Nucleon

§ Continuum Gluon PDF w/ pseudo-PDF

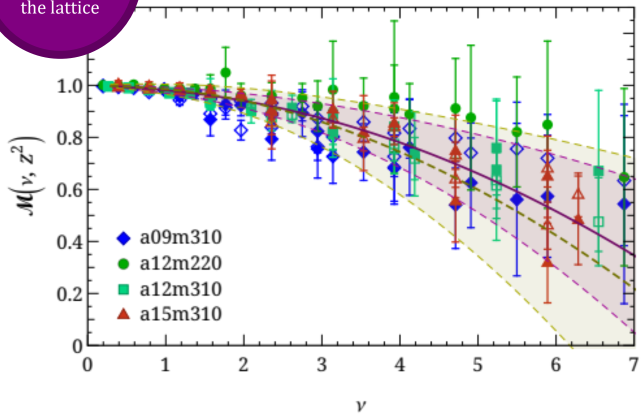
- 2+1+1 HISQ {0.09, 0.12, 0.15} fm
  [220, 310, 700]-MeV pion, 10^5–10^6 statistics

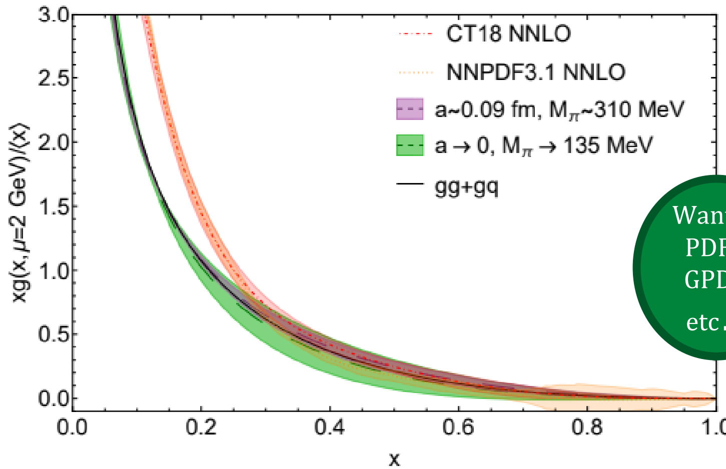
Z. Fan et al (MSULat), 2210.09985





Quantities that can be calculated on the lattice



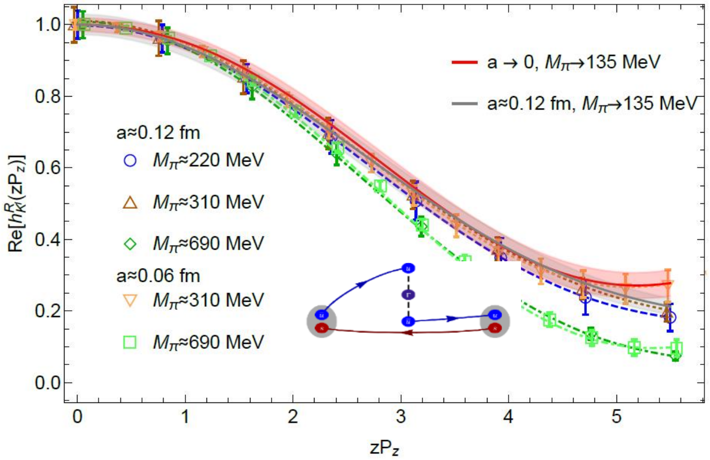


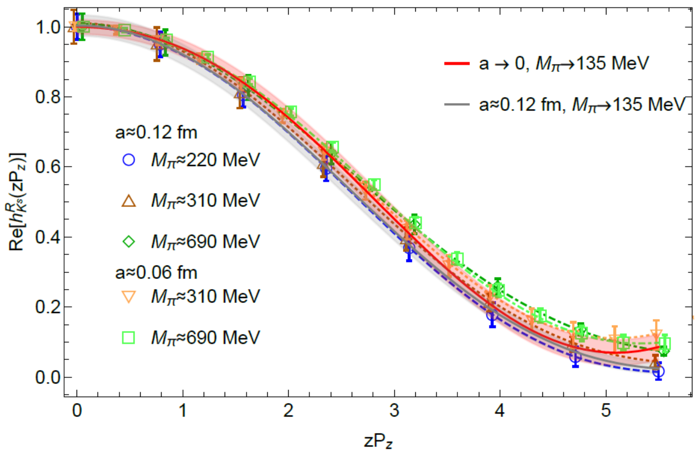
Wanted PDFs, GPDs, etc…

G: Bill Good

Meson Valence-quark PDFs

§ Pion/Kaon PDFs using quasi-PDF in the continuum limit

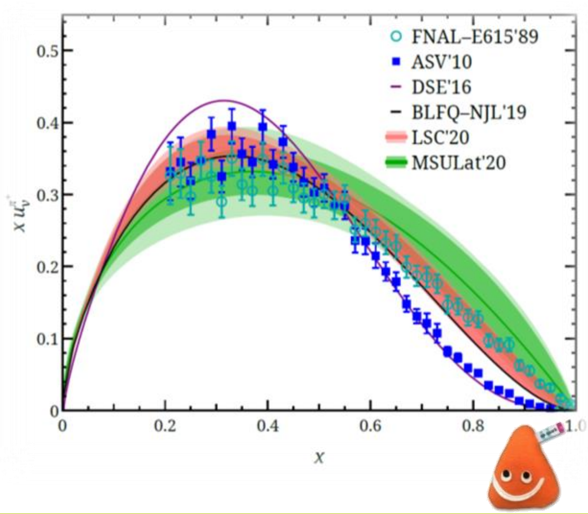


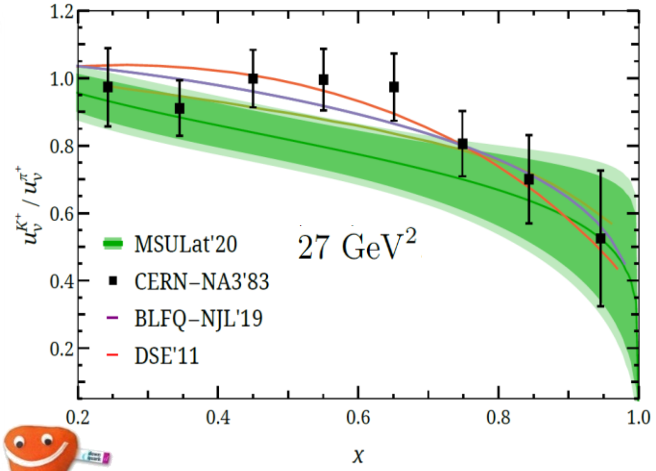


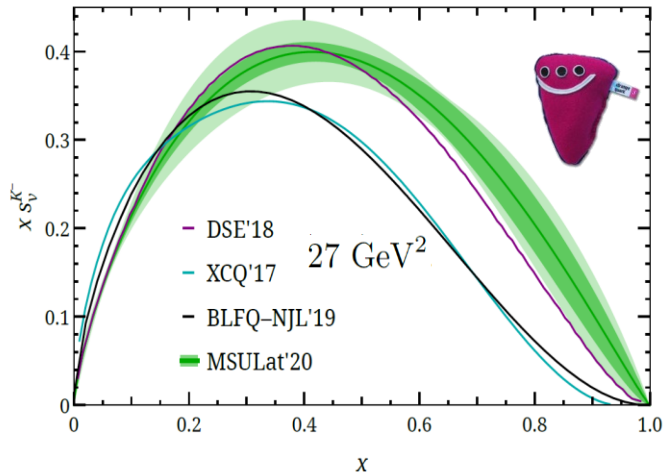
Quantities that can be calculated on the lattice

MSULat, 2003.14128

Wanted PDFs, GPDs, etc…







Valence-quark PDFs Update

§ Pion PDFs calculated directly at physical pion mass

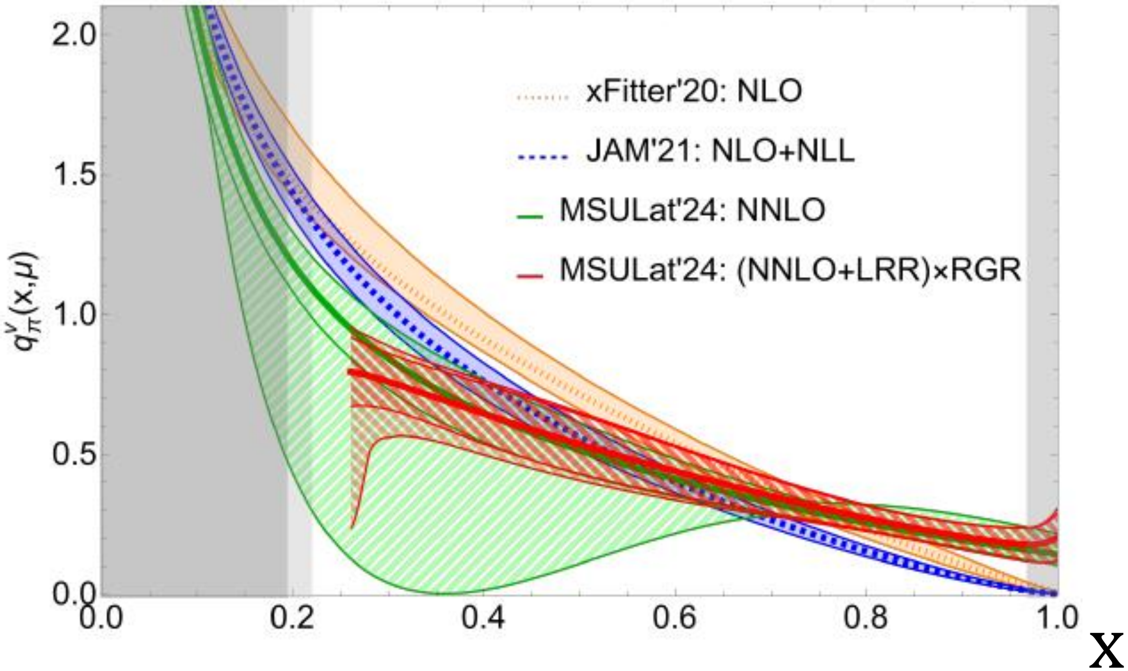
- NNLO matching & treat leading-renormalon effects
  - Leading-renormalon resummation (LRR)
  - Renormalization-group resummation (RGR)

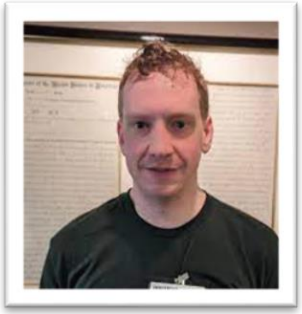
R. Zhang, et. al. PLB 844, 138081 (2023)

- N_f=2+1+1 clover/HISQ, a~0.09 fm

J. Holligan, HL (MSULat), 10.1088/1361-6471/ad3162

Wanted PDFs, GPDs, etc…

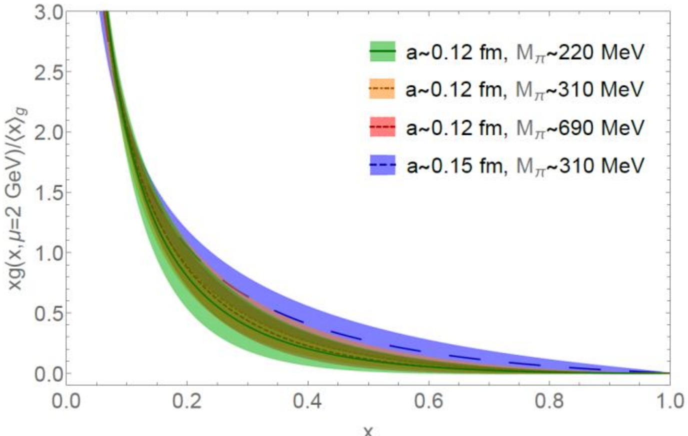


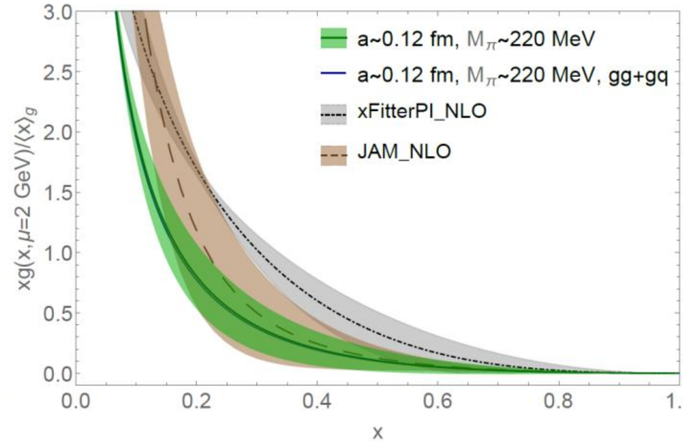


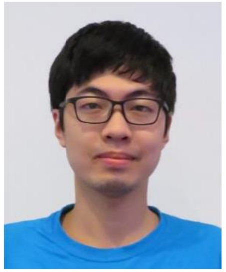
P: Jack Holligan

Meson Gluon PDFs

§ First pion and kaon gluon PDFs $g(x)/\langle x \rangle$ using pseudo-PDF



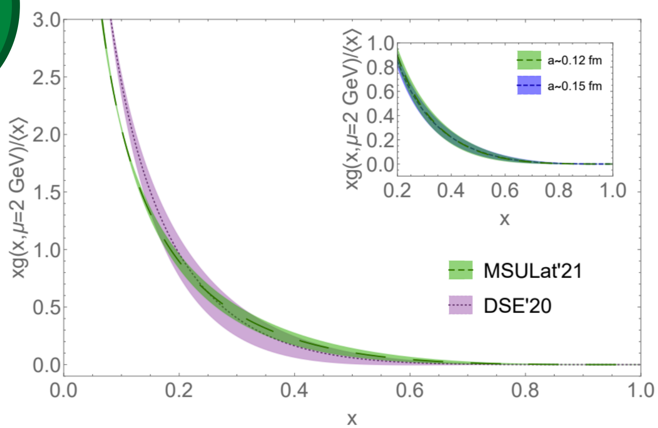




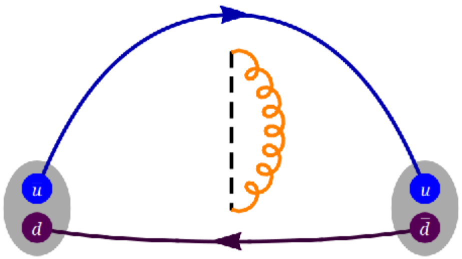
G: Zhouyou Fan

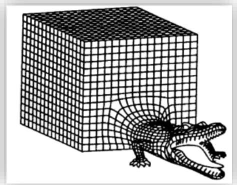
Wanted PDFs, GPDs, etc…

2104.06372, Fan et al. (MSULat); 2112.03124, Salas-Chavira et al. (MSULat)



G: Alejandro Salas-Chavira





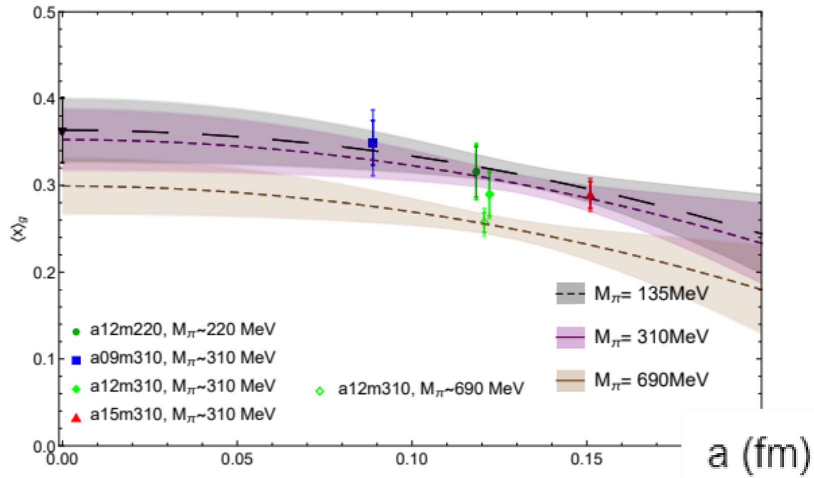
finite-volume, discretization, heavy quark mass, …

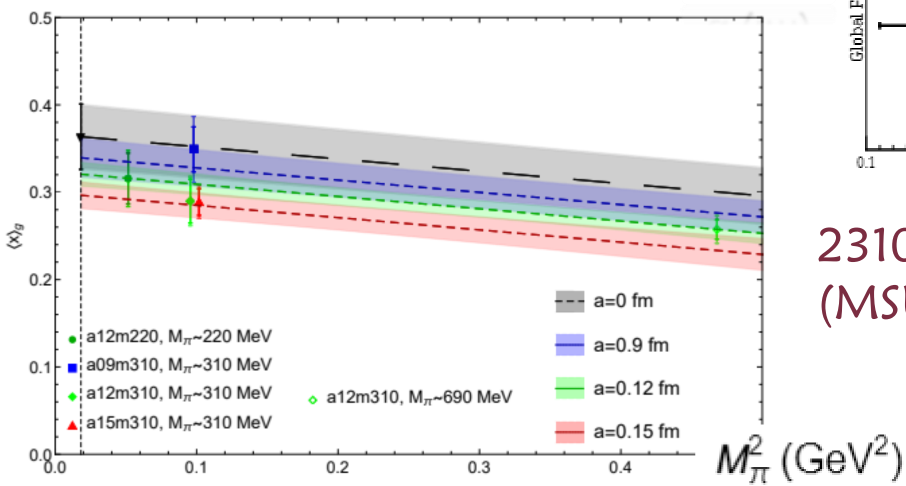
§ What does lattice QCD say about $g(x)$?

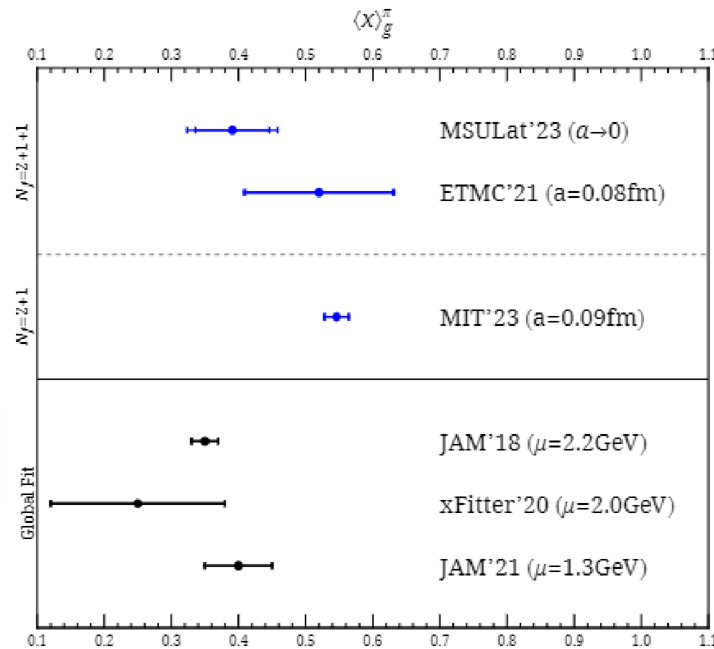
Pion Gluon PDF Update

§ Study discretization systematic in $\langle x \rangle_{\{\pi,g\}}$

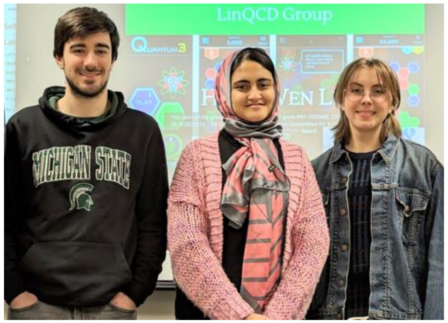
- Lattice details: clover/HISQ, HISQ, a ~{0,15, 0.12, 0.09} fm







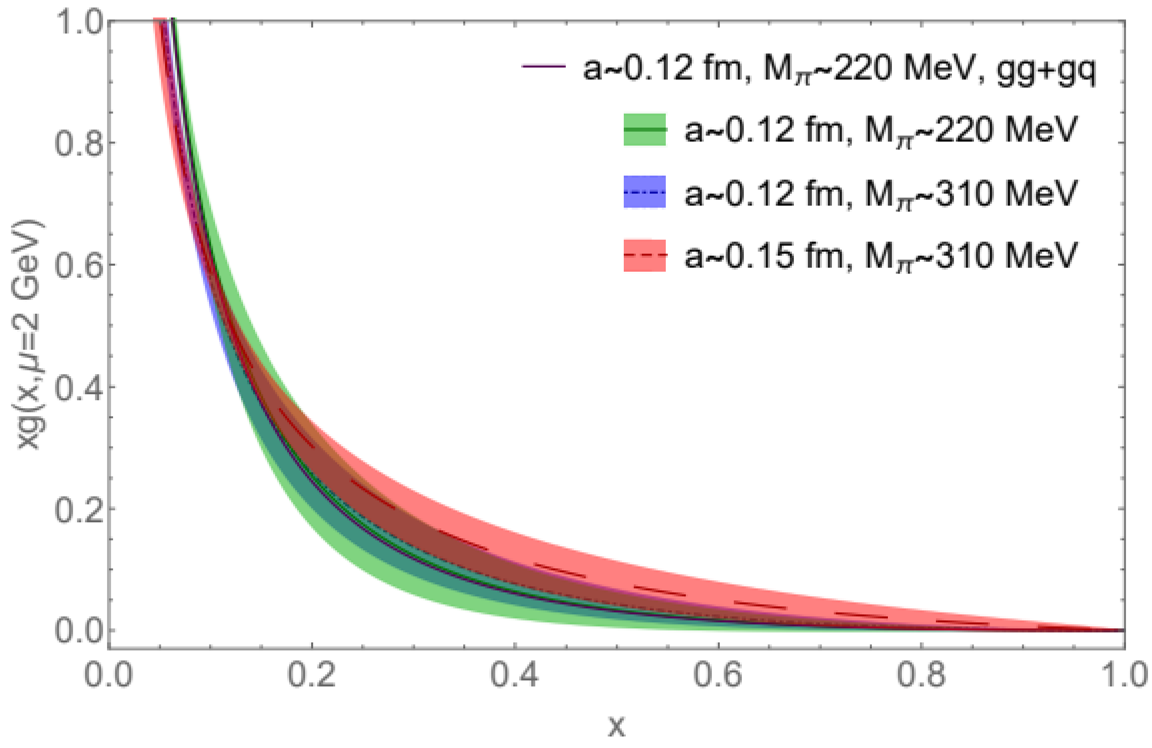
2310.12034, Good et al. (MSULat)



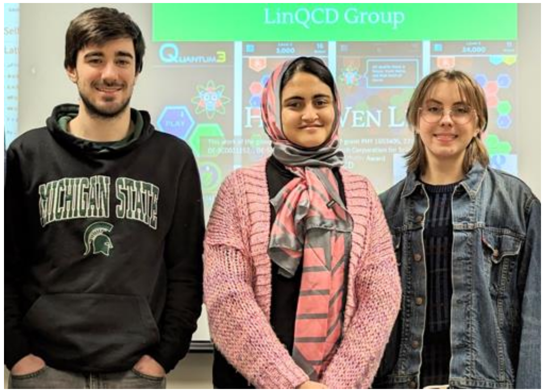
Pion Gluon PDF Update

§ Back to Pion gluon PDF $g(x)$

- Update previous calculated $g(x)/\langle x \rangle$ in 2021



2310.12034, Good et al. (MSULat)



Pion Gluon PDF Update

§ Back to Pion gluon PDF $g(x)$

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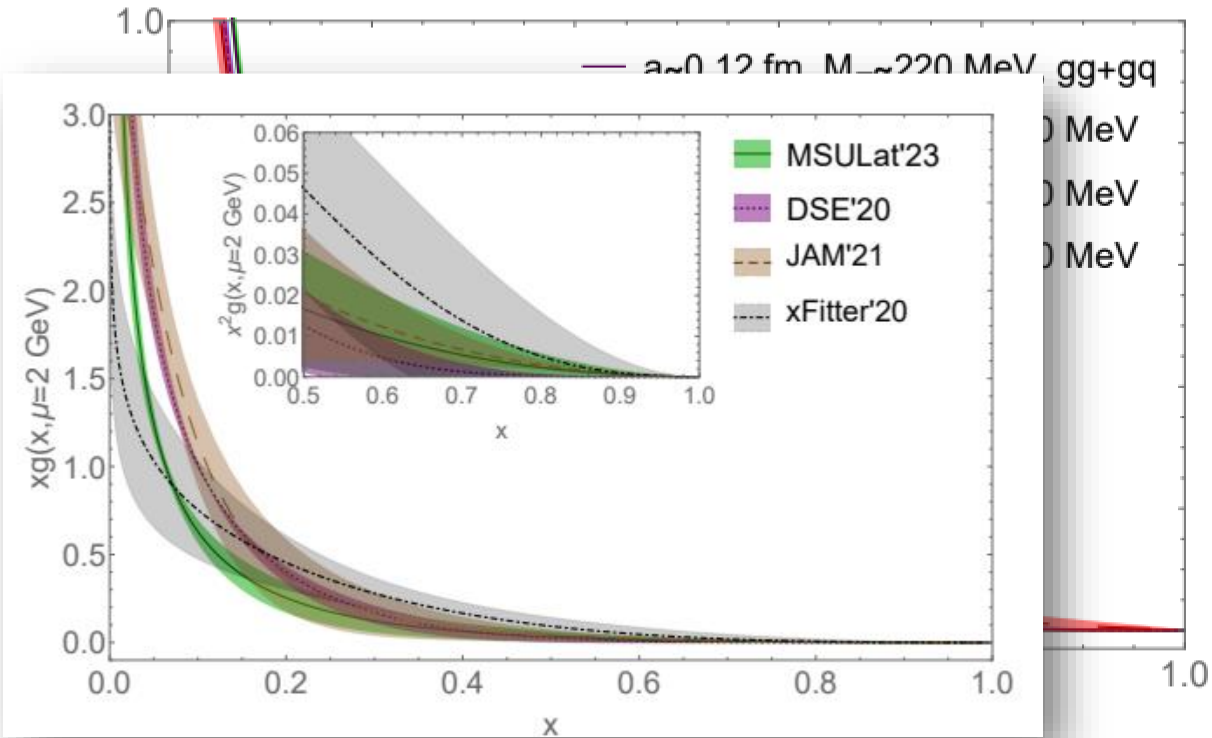
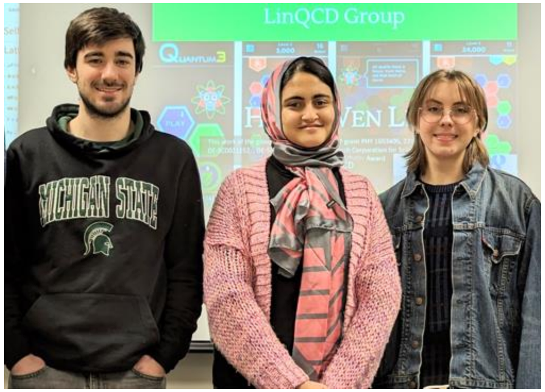


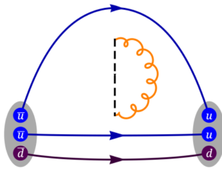
2310.12034, Good et al. (MSULat)



First LaMET Gluon PDF

§ Gluon PDF w/ quasi-PDF (no parameterization)

- 2+1+1 clover/HISQ 0.12 fm, 690-MeV pion, 10^6 statistics
  Good et al (MSULat), 2505.13321
- PQCD calculated Wilson coefficient and matching kernel

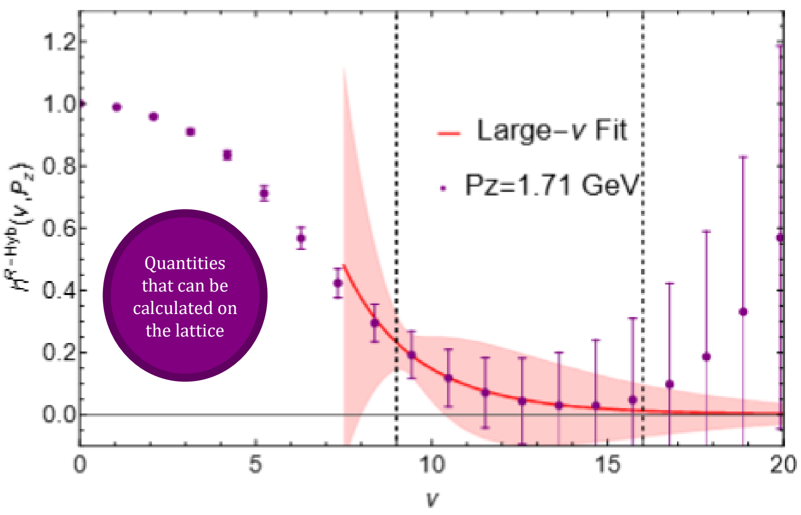


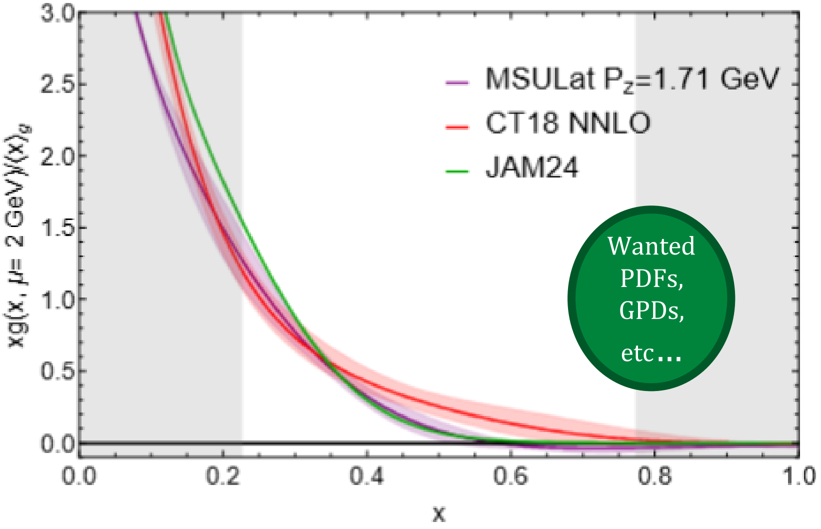
$$C_0 = 1 + \frac{\alpha_s C_A}{2\pi}\left(\frac{5}{6}\ln\left(\frac{z^2\mu^2 e^{2\gamma_E}}{4}\right) + \frac{3}{2}\right)$$

$$C^{\text{hyb.}}\left(\xi, \frac{\mu}{p^z}\right) = C^{\text{ratio}}\left(\xi, \frac{\mu}{p^z}\right) + \frac{\alpha_s C_A}{2\pi}\frac{5}{6}\left[-\frac{1}{|1-\xi|} + \frac{2\text{Si}((1-\xi)z_s p^z)}{\pi(1-\xi)}\right]_+$$

Formula by Fei Yao (BNL)

Plots by Bill Good (MSU)





Hadron Tomography

§ Lots of progress on tomography by many collaborations

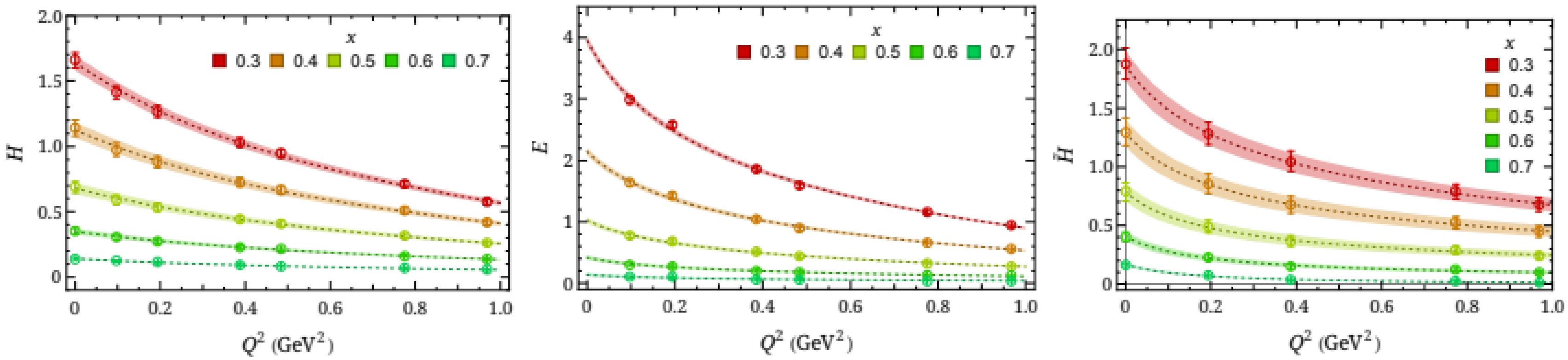
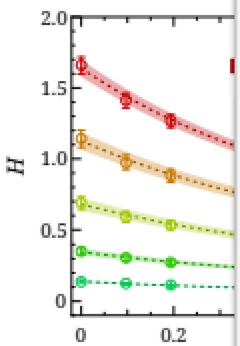


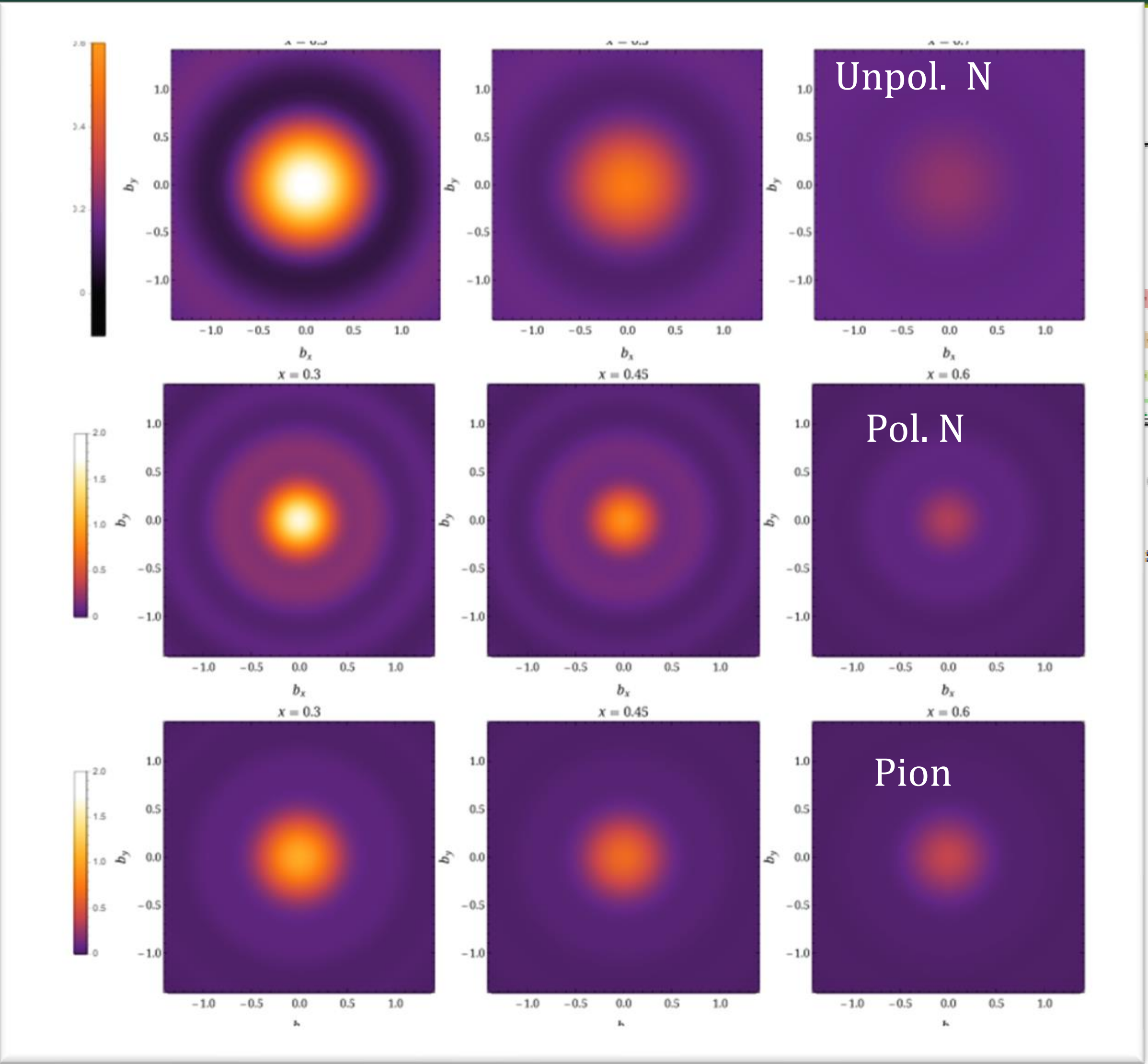
Figure 4.1: Nucleon isovector H (left), E (middle) and $\tilde{H}$ (right) GPDs at $\xi = 0$ with z-expansion to Q^2 at selected x values.

HL, Prog.Part.Nucl.Phys. 144 (2025) 104177

Hadron Tomography

§ Lots of ... borations





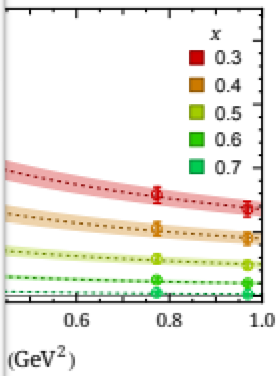


Figure 4.1: ... selected x values.

HL, Prog.Part.Nucl.Phys. 144 (2025) 104177

Impact of Lattice-QCD PDFs on Global Fits

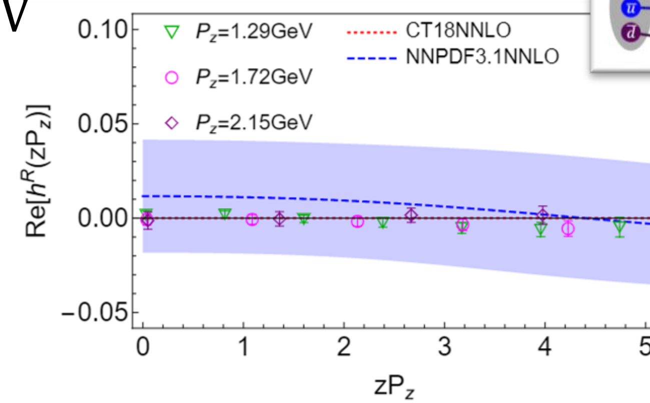
First Lattice Strange PDF

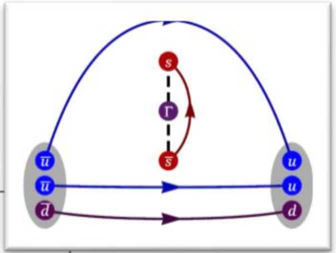
§ Results by MSULat/quasi-PDF method

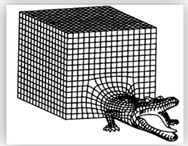
- Clover on 2+1+1 HISQ, 0.12-fm 310-MeV QCD vacuum
- Extrapolated to $M_\pi \approx 140$ MeV

R. Zhang et al (MSULat), 2005.01124

$$\mathrm{Re}[h(z)] \propto \int dx\,(s(x) - \bar{s}(x))\cos(xzP_z)$$







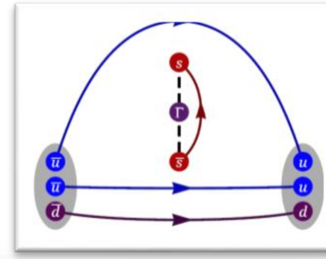
Quantities that can be calculated on the lattice

FG: Rui Zhang

Lattice Strangeness Asymmetry Impact

§ Results by MSULat/quasi-PDF method

- Clover on 2+1+1 HISQ, 0.12-fm 310-MeV QCD vacuum
- Extrapolated to $M_\pi \approx 140$ MeV, $P_z \approx 1.7$ GeV



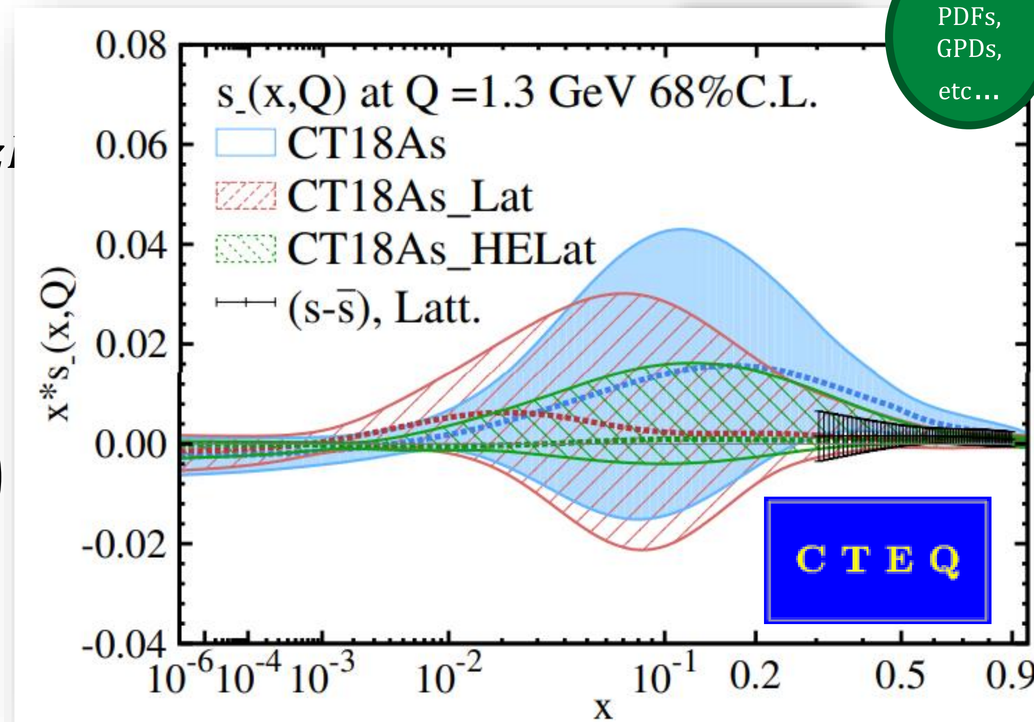
R. Zhang et al (MSULat), 2005.01124

$$\mathrm{Re}[h(z)] \propto \int dx\, \left(s(x) - \bar{s}(x)\right) \cos(xz$$

§ From quasi-PDF to PDF

$$\tilde{f}_q(x, P_z) = \int_{-1}^{1} \frac{dy}{|y|} f_q(y) C_{q/q}(x, y, P_z, \mu) + O\left(\frac{\Lambda_{\mathrm{QCD}}^2}{x^2 P_z^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{(1-x)^2 P_z^2}\right)$$

T. Hou, HL, M. Yan, C. Yuan, 2211.11064



Wanted PDFs, GPDs, etc…

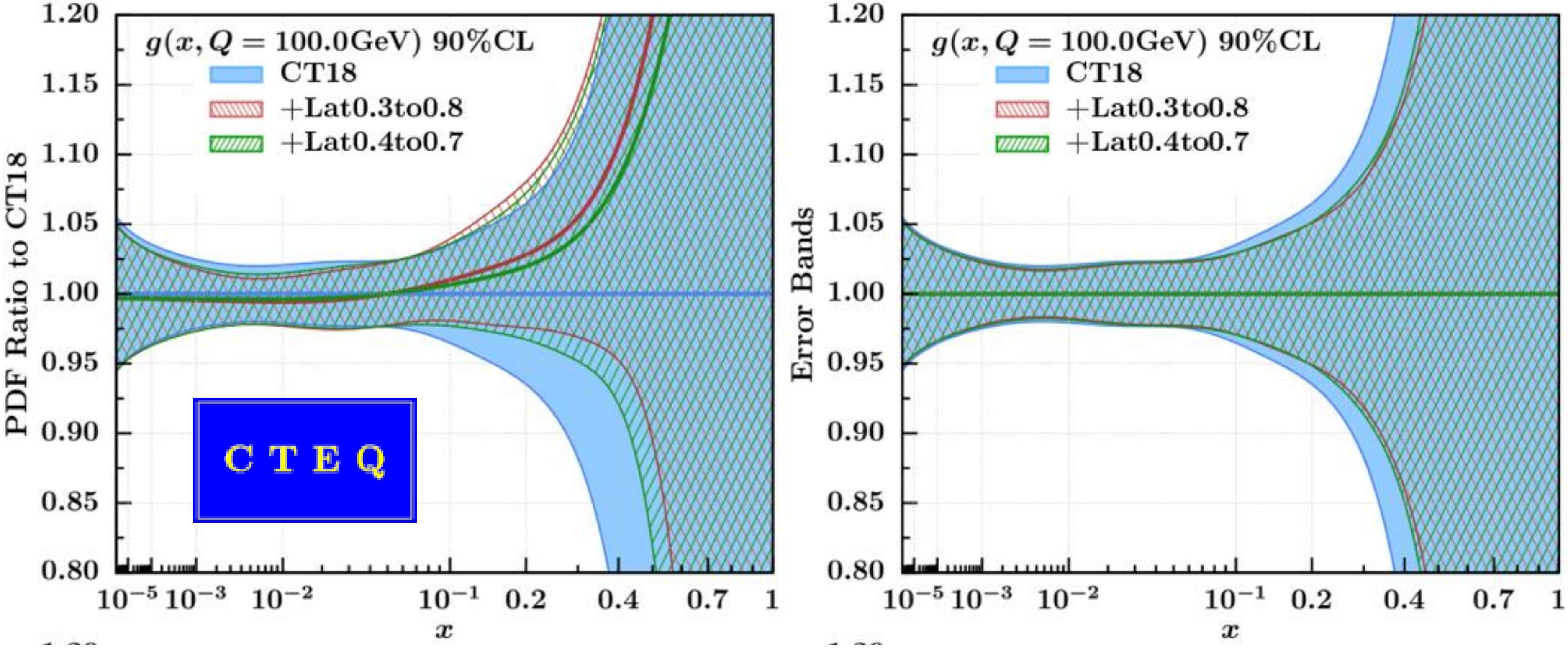
§ The strangeness asymmetry $s(x,Q) - \bar{s}(x,Q)$ at $x > 0.2$ is difficult to measure, but can be predicted in lattice QCD

Nucleon Gluon PDF Impact

§ Impact study with CTEQ-TEA analysis

- Take lattice inputs in the region where no strong experimental data constraints, $x \in [0.3, 0.8]$

Plots by Alim Ablat (Xinjiang U.); 2502.10630



Nucleon Gluon PDF Impact

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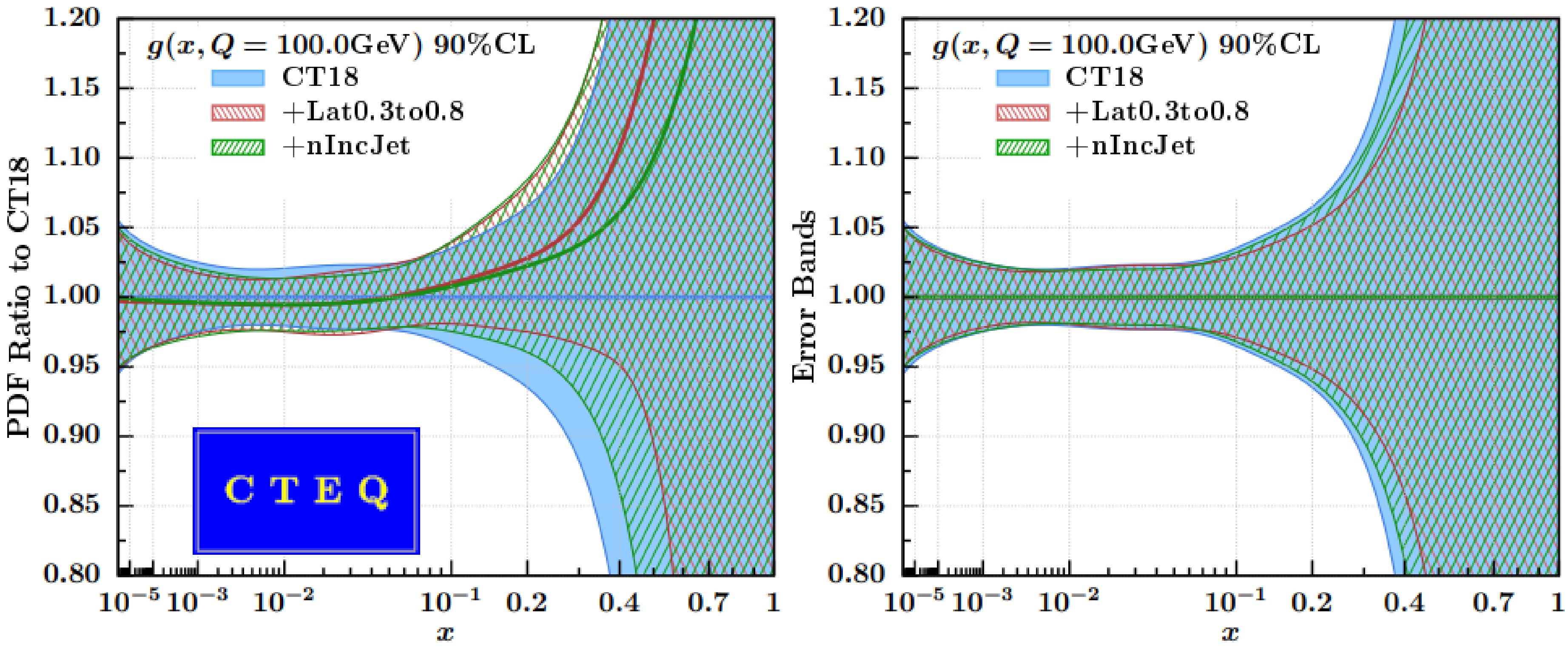

Nucleon Gluon PDF Impact

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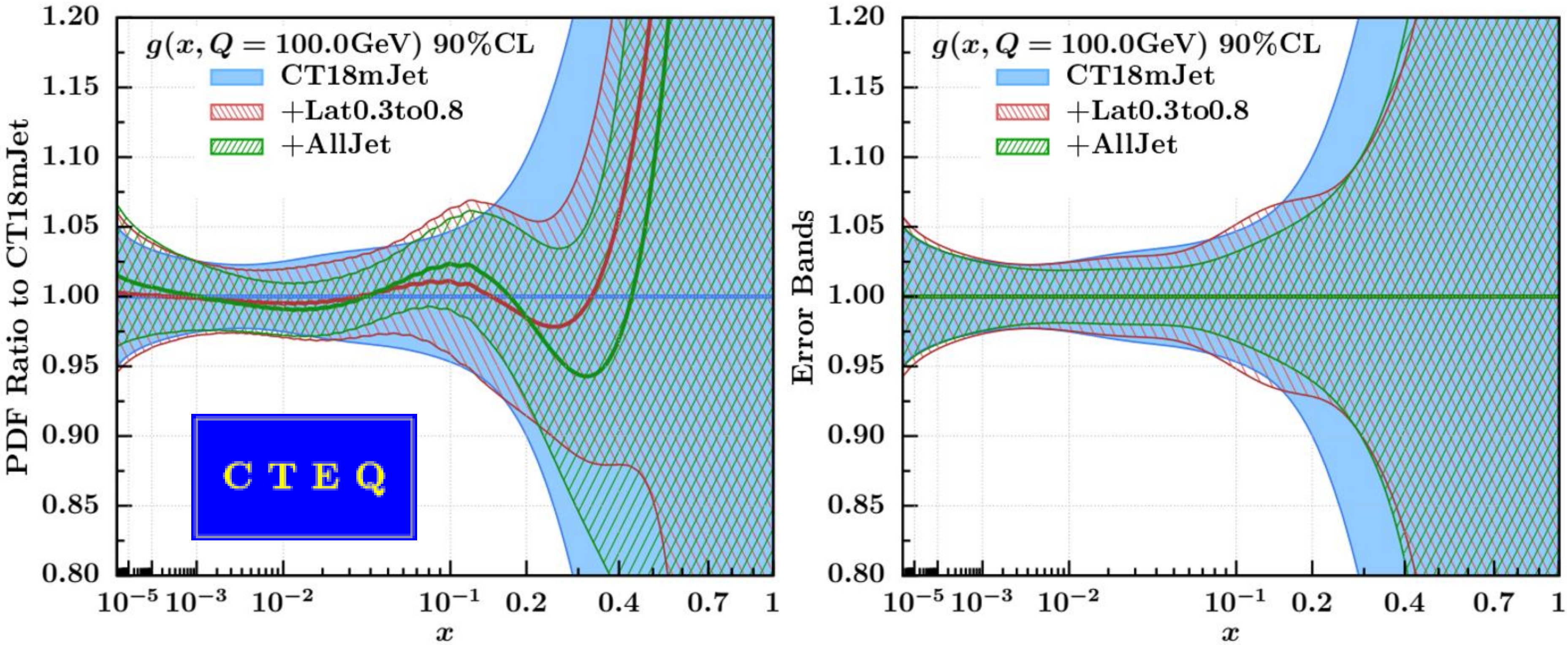
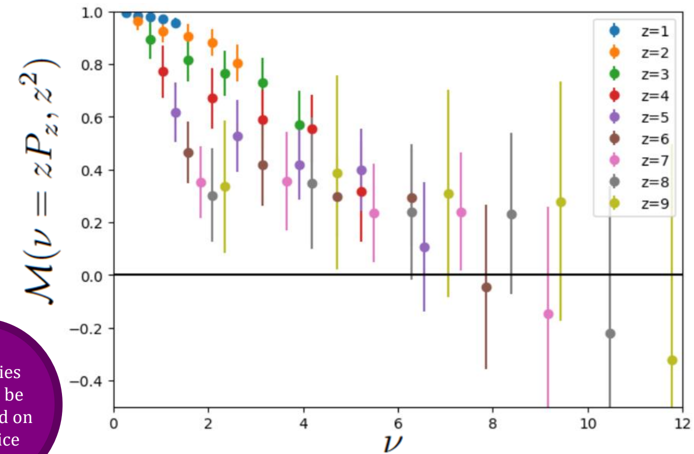


Pion Gluon PDF Impact

§ Preliminary study with JAM analysis

- Treat lattice reduced pseudo-Ioffe time distribution (RpITD) as "cross-section" inputs in the JAM global fit

Plots by Bill Good (MSU)



W. Good et al, 2409.02750 [hep-lat]

Quantities that can be calculated on the lattice today = Σ Wanted PDFs, GPDs, etc. × pQCD-calculated kernel

Quantities that can be calculated on the lattice

$$\mathcal{M}(\nu, z^2) = \int_0^1 dx \frac{xg(x)}{\langle x_g \rangle} \left[R_{gg}(x\nu, z^2\mu^2) + R_r(x\nu, z^2\mu^2) \frac{\langle x_S \rangle}{\langle x_g \rangle} \right] + \int_0^1 dx \frac{xf_S(x)}{\langle x_g \rangle} R_{gq}(x\nu, z^2\mu^2) + \mathcal{O}(z^2 m^2, z^2 \Lambda_{\rm QCD})$$

Lattice data

Non-perturbative PDFs and moments

Perturbative matching kernels

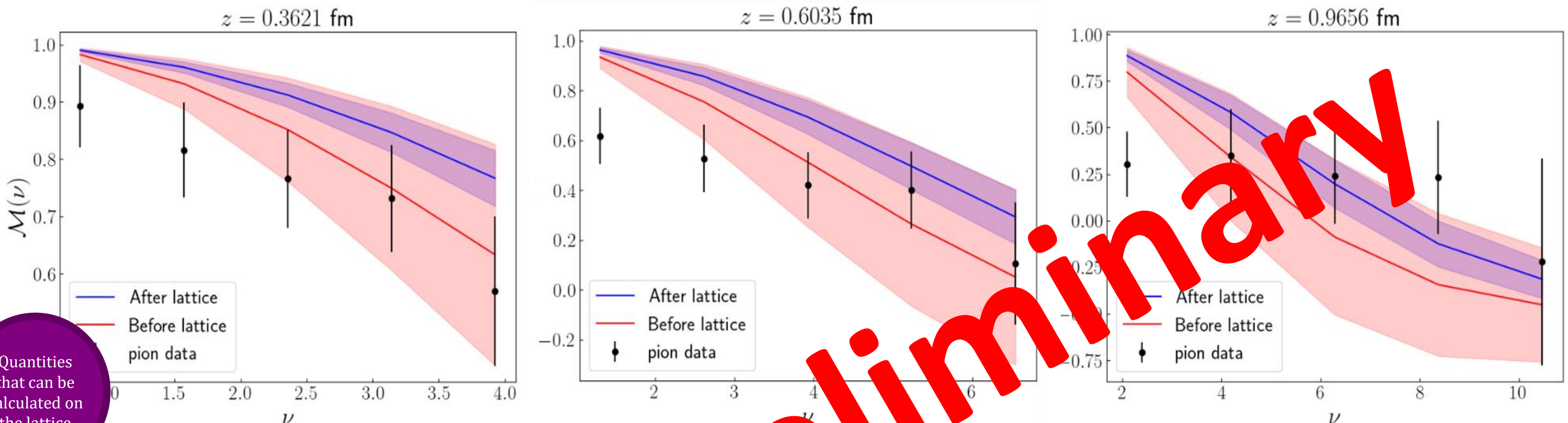
Residual systematic error

A. V. Radyushkin, PRD96, 034025 (2017);
Balitsky, et al., PLB808, 135621 (2020)

Pion Gluon PDF Impact

§ Preliminary study with JAM analysis

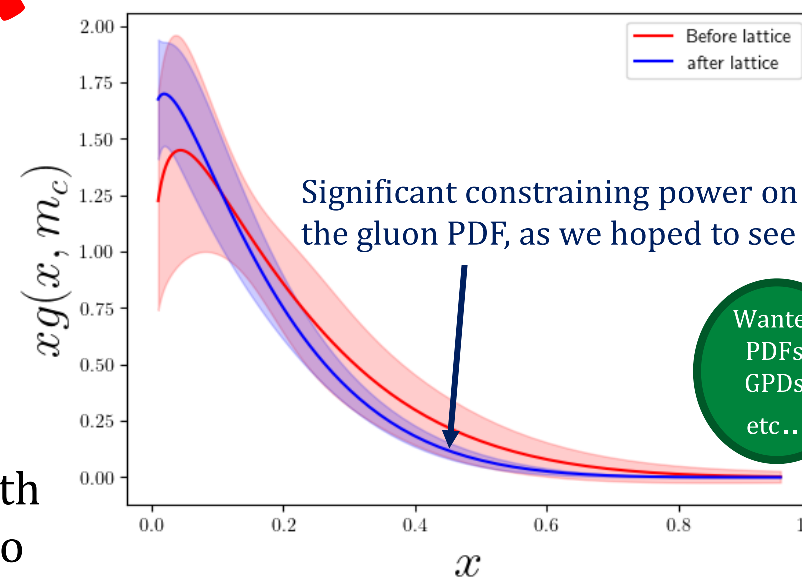
Plots by Bill Good (MSU)



Quantities that can be calculated on the lattice

$$\mu^2 = \max\left[m\ldots, \frac{4}{\ldots_E z^2}\right]$$

also in Collaboration with Wally and Nobuo



Wanted PDFs, GPDs, etc…

Lattice Progress & Challenges

§ Beyond the standard twist-2 collinear PDFs

- Generalized parton distributions (GPDs) for the pion and unpolarized/polarized nucleon
- Transverse-momentum– dependent distributions (TMDs)
  - Collins-Soper kernel, soft function and wavefunctions
- Twist-3 PDFs and GPDs

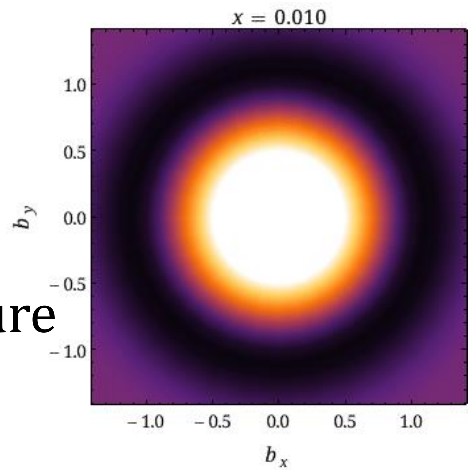
For more details and references, refer to 2202.07193

§ Challenges ahead for precision PDFs

- Large momentum is essential
  - With sufficient statistics nucleons may reach 5 GeV
- Methods for signal-to-noise improvement
  - Gluonic observables, new ideas for large momentum
- Access small-x physics; some methods have inverse problem in PDF extraction, more computational resources, etc.

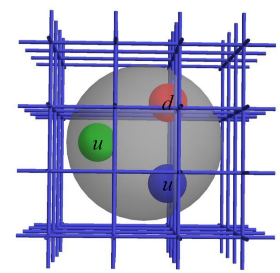
Summary and Outlook

- Exciting era using LQCD to study *x*-dependent parton distributions
  - Bjorken-*x* dependence of parton distributions now widely studied
  - More study of systematics planned for the near future



- Lattice strange and gluon PDFs can have impacts
  - Treat lattice matrix elements as expt inputs in the future
  - Computational resources are needed for precision calculations (community support!)
- Precision and progress are limited by resources

Theory Input → **Global Analysis of PDFs/GPDs** ← **Exp't Input** +



EXCLAIM

Thanks to MILC collaboration for sharing their 2+1+1 HISQ lattices & USQCD/NSF/DOE for computational resources
This work is partially sponsored by grants NSF PHY 1653405 & 1653405, DOE DE-SC0024053 & RCSA Cottrell Scholar Award

Students Wanted

LGT4HEP website: https://lgt4hep.github.io/

LGT4HEP

High Energy Physics Computing Traineeship for Lattice Gauge Theory

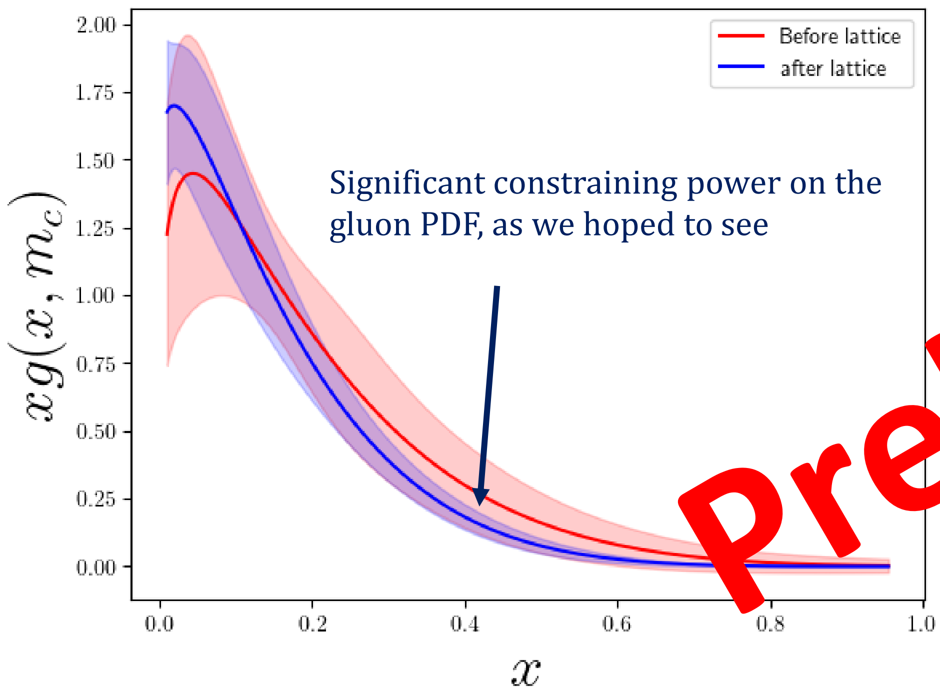
Apply now:

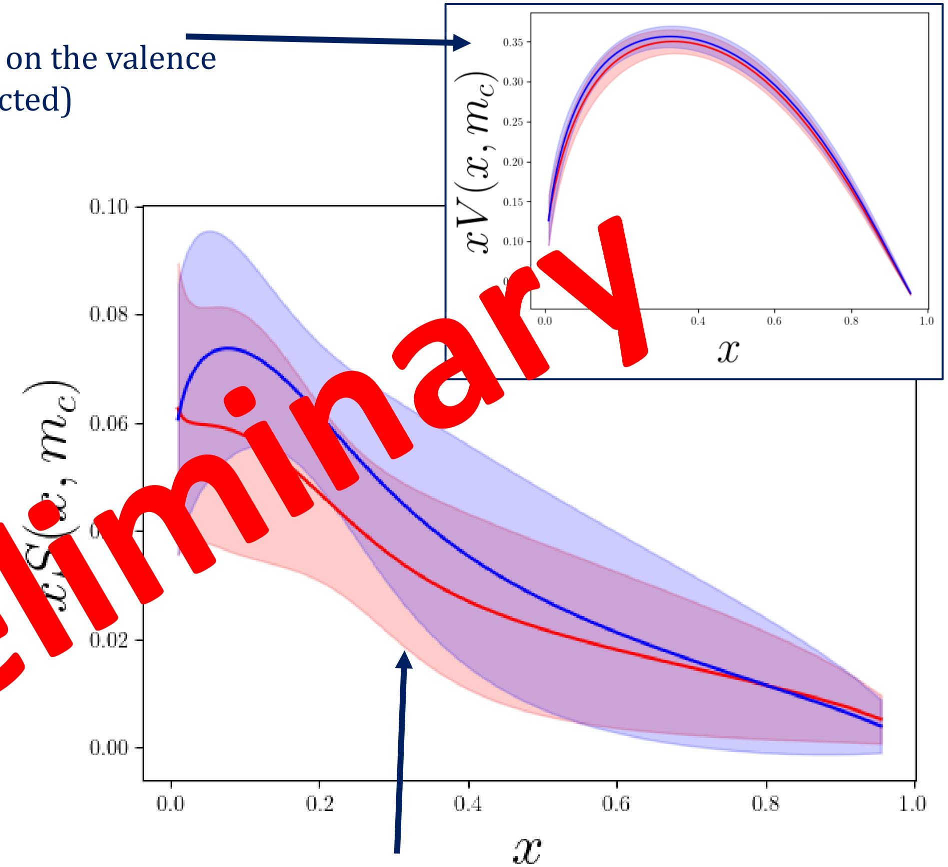
Visit lgt4hep.github.io to learn more and where to apply for the traineeship graduate school program.

Backup Slides

More Pion PDF Updates

Not much effect on the valence quarks (as expected)





Some shape change in the sea quark PDF, need to look at relative errors

Overall compatibility with "before" lattice PDFs

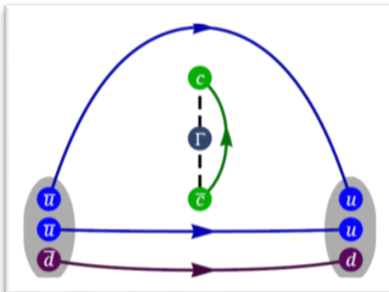
First Lattice Charm PDF

§ Large uncertainties in global PDFs

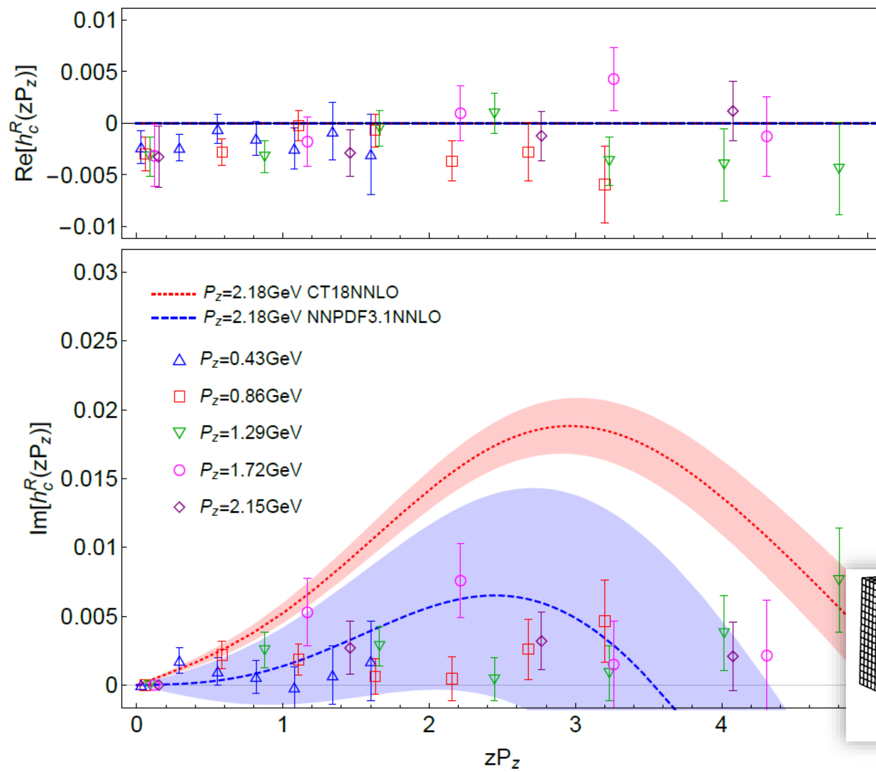
§ Results by MSULat/quasi-PDF method

- Clover on 2+1+1 HISQ 0.12-fm 310-MeV QCD vacuum

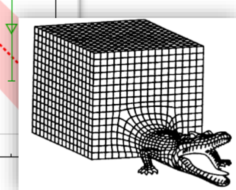
2005.01124, R. Zhang et al (MSULat)



- suggest a symmetric $c - \bar{c}$ distribution
- much smaller than strange PDF



Quantities that can be calculated on the lattice



First Lattice Charm PDF

- Large uncertainties in global PDFs
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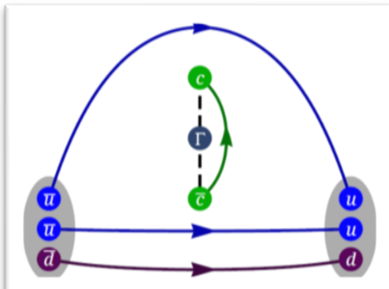
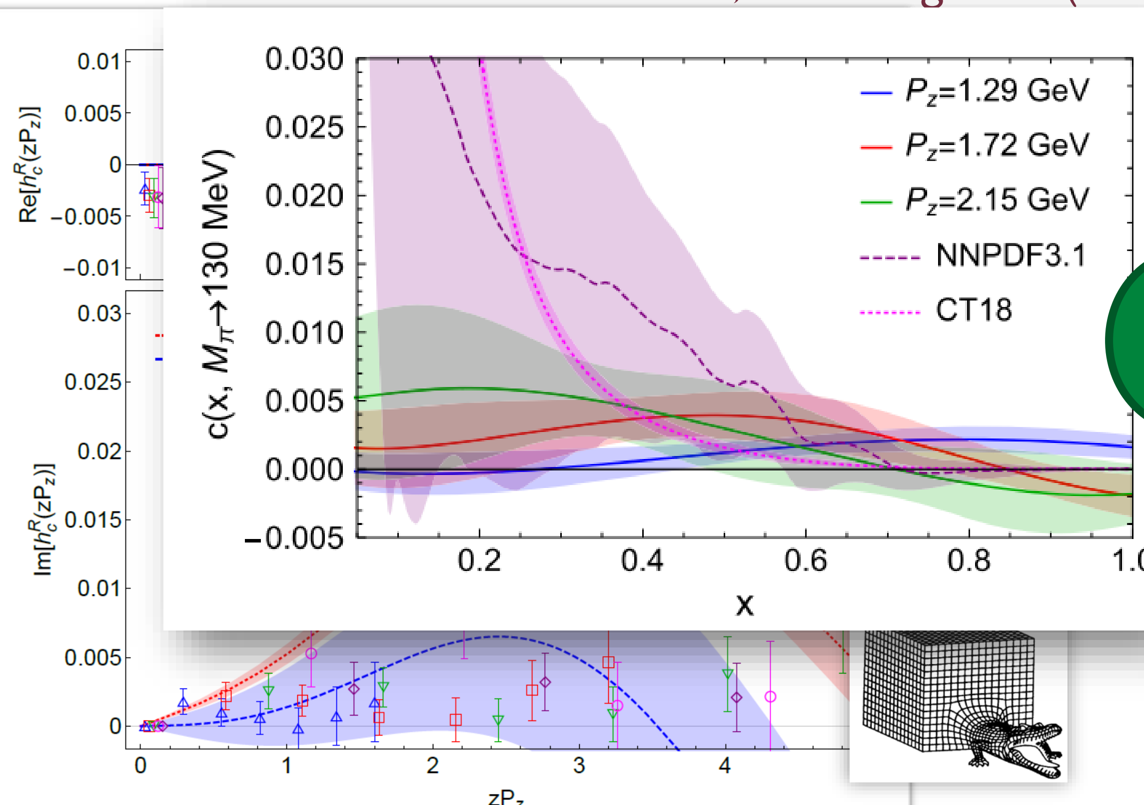
  - Clover on 2+1+1 HISQ 0.12-fm 310-MeV QCD vacuum

2005.01124, R. Zhang et al (MSULat)

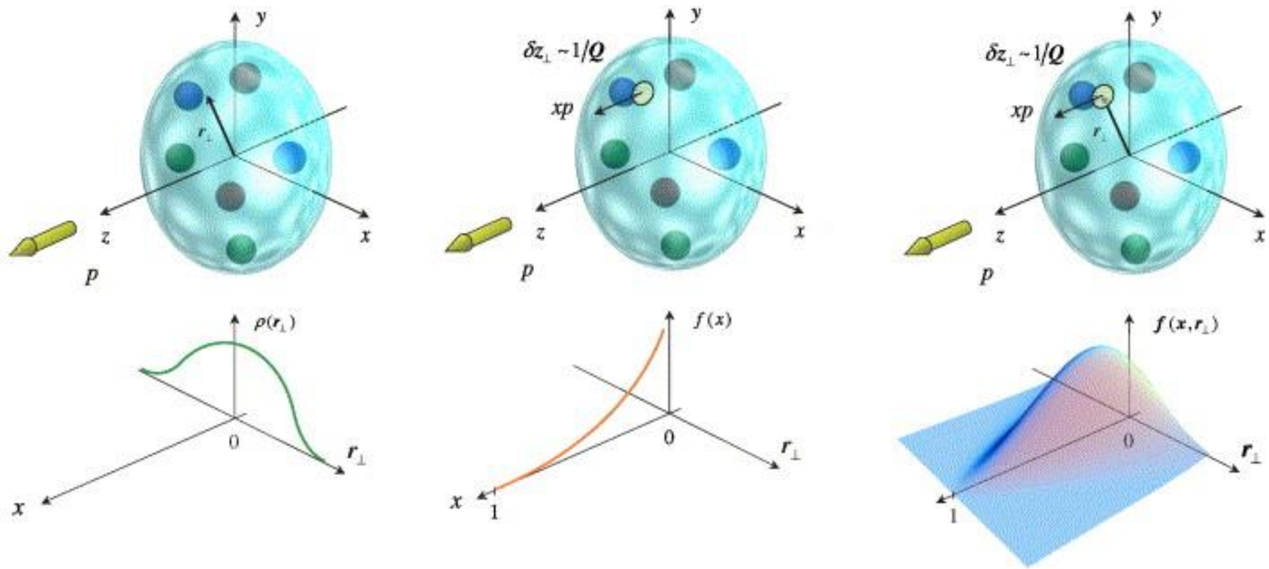


- suggest a symmetric $c-\bar{c}$ distribution
- much smaller than strange PDF



Wanted PDFs, GPDs, etc…

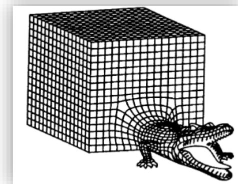
Bjorken-x Dependent GPDs



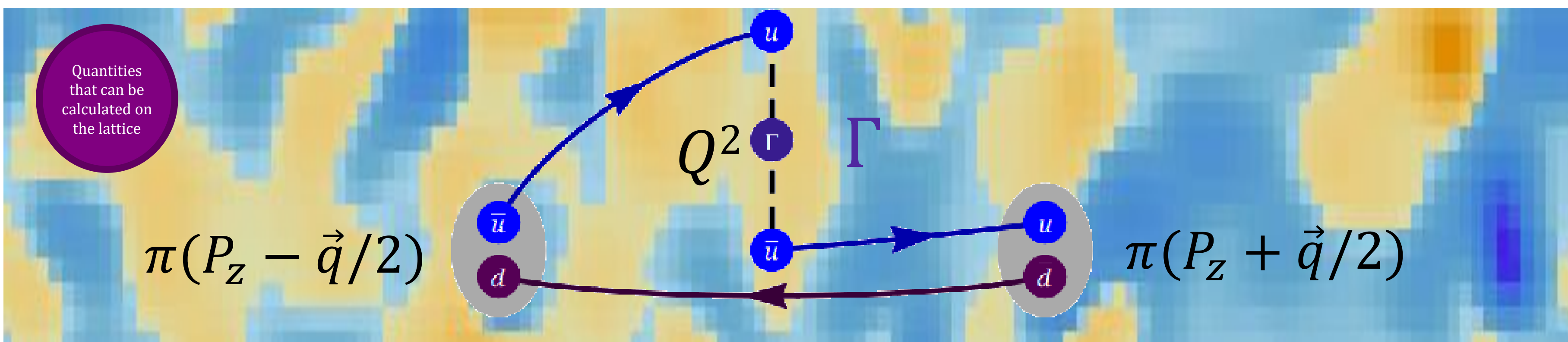
Picture from A. Belitskya and A Radyushkin, Physics Report, 416 (2015)

Generalized Parton Distributions

Single-ensemble result



finite-volume,
discretization,
heavy quark mass,
...



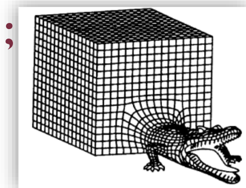
First Lattice GPDs

§ First glimpse into pion GPD using Quasi-PDF/LaMET

- Lattice details: clover/HISQ, **0.12fm**, **310-MeV** pion mass

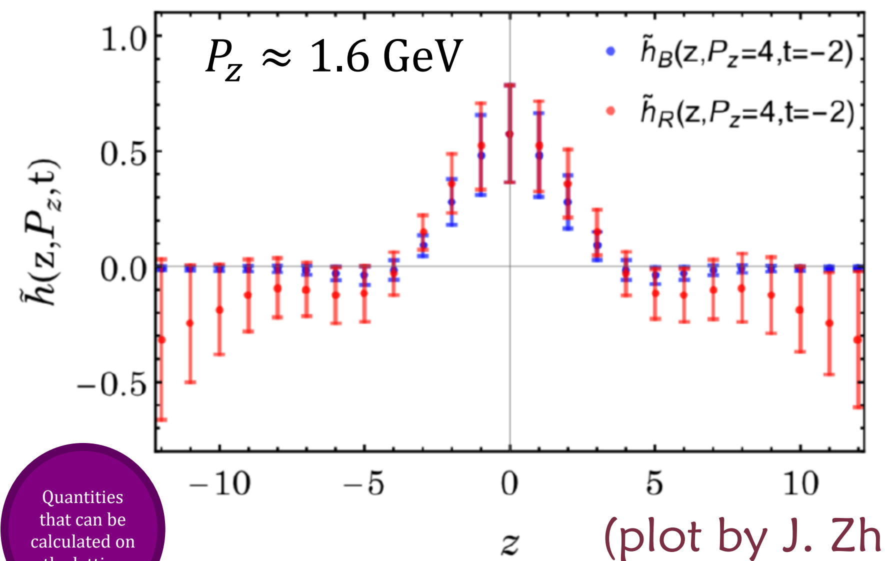
$P_z \approx 1.3, 1.6$ GeV

MILC, Phys. Rev. D, 82 (2010), 074501;
Phys. Rev. D, 87 (2013), 0545056

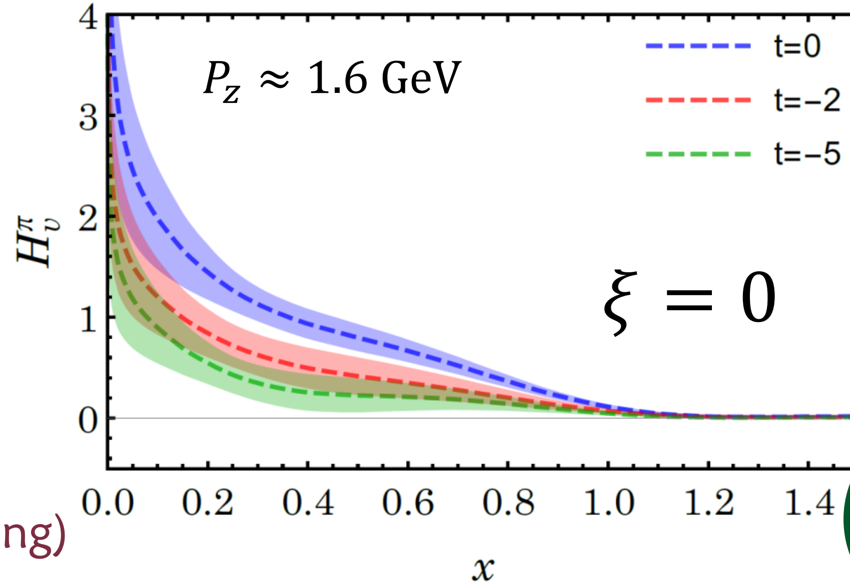


J. Chen, HL, J. Zhang, 1904.1237;

$$H_q^\pi(x,\xi,t,\mu) = \int \frac{d\eta^-}{4\pi} e^{-ix\eta^- P^+} \left\langle \pi(P+\Delta/2) \left| \bar{q}\left(\frac{\eta^-}{2}\right) \gamma^+ \Gamma\left(\frac{\eta^-}{2}, -\frac{\eta^-}{2}\right) q\left(-\frac{\eta^-}{2}\right) \right| \pi(P-\Delta/2) \right\rangle$$



(plot by J. Zhang)



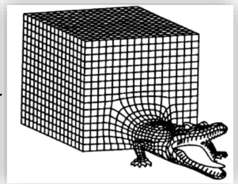
Quantities that can be calculated on the lattice

Wanted PDFs, GPDs, etc…

Valence-Quark Pion GPD

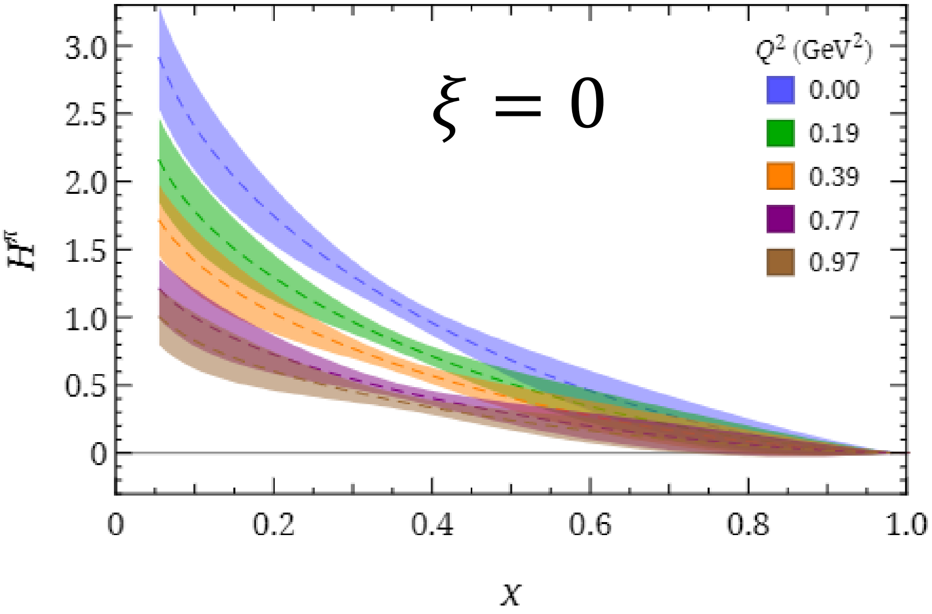
§ Pion GPD (H^π) using quasi-PDFs at physical pion mass

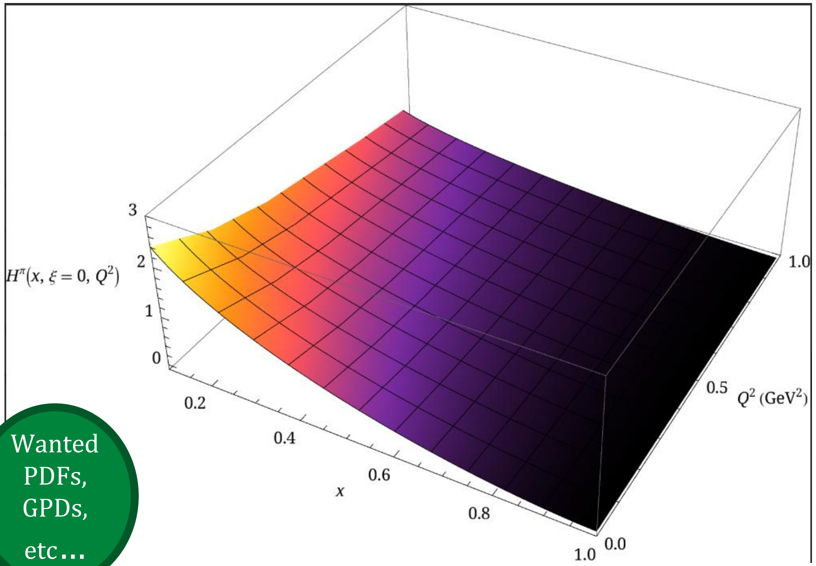
- Lattice details: clover/2+1+1 HISQ
  0.09 fm, 135-MeV pion mass, $P_z \approx 1.7$ GeV
- $\xi = 0$ valence-quark Pion GPD results



finite-volume, discretization,

HL (MSULat), Phys. Lett. B 846 (2023) 138181



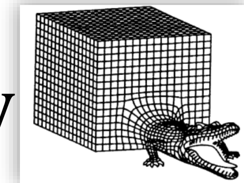


Wanted PDFs, GPDs, etc…

Valence-Quark Pion GPD

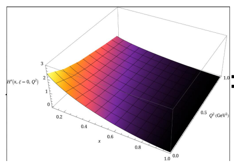
§ Pion GPD (H^π) using quasi-PDFs at physical pion mass

- Lattice details: clover/2+1+1 HISQ
  0.09 fm, 135-MeV pion mass, $P_z \approx 1.7$ GeV
- $\xi = 0$ valence-quark Pion GPD results

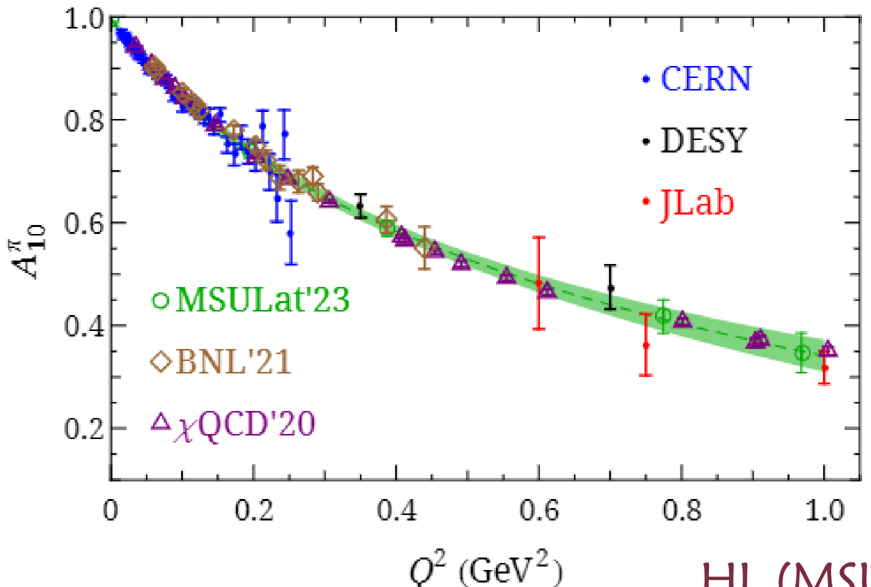


finite-volume, discretization,

$$\int_{-1}^{+1} dx\, x^{n-1}$$



$$= A^{\pi}_{ni}(t)$$

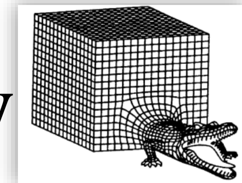


HL (MSULat), Phys. Lett. B 846 (2023) 138181

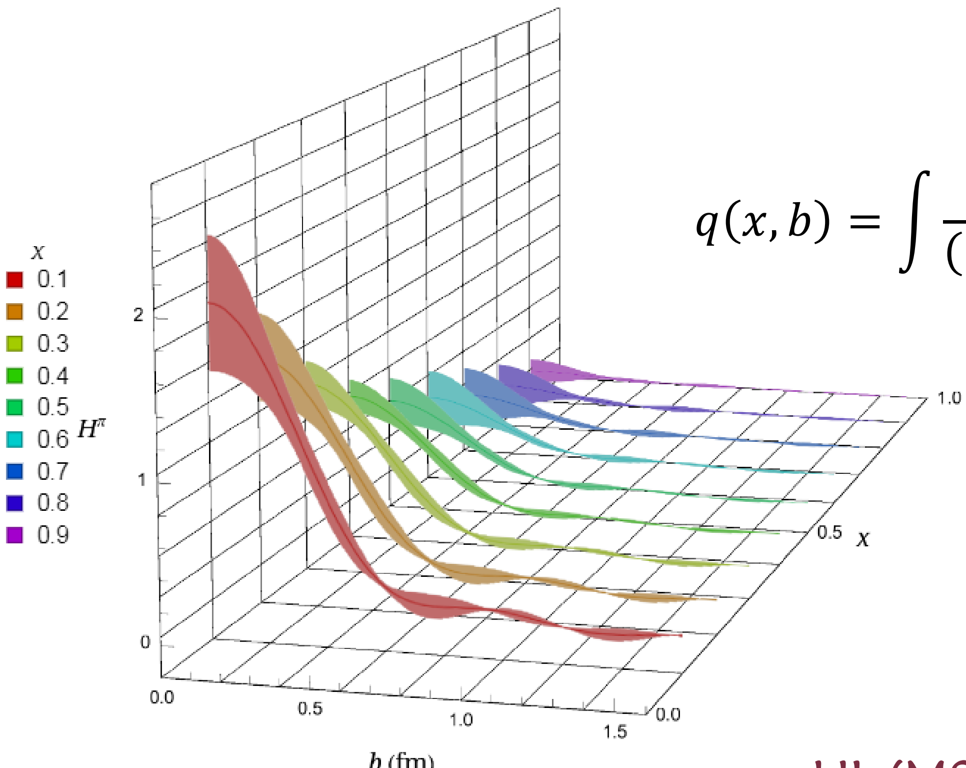
Pion Tomography

§ Nucleon GPD using quasi-PDFs at physical pion mass

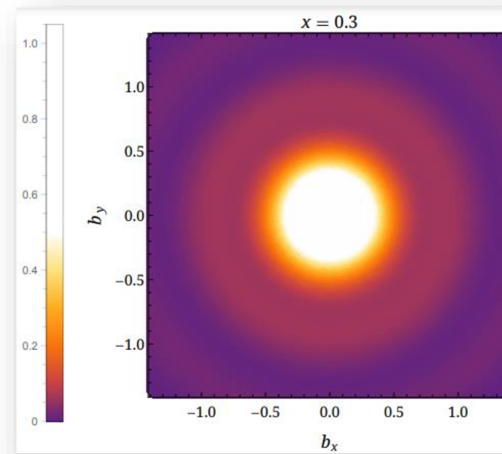
- Lattice details: clover/2+1+1 HISQ
  0.09 fm, 135-MeV pion mass, $P_z \approx 1.7$ GeV
- $\xi = 0$ valence-quark Pion GPD results



finite-volume,
discretization,



$$q(x, b) = \int \frac{d\vec{q}}{(2\pi)^2} H(x, \xi = 0, t = -\vec{q}^2) e^{i\vec{q}\cdot\vec{b}}$$



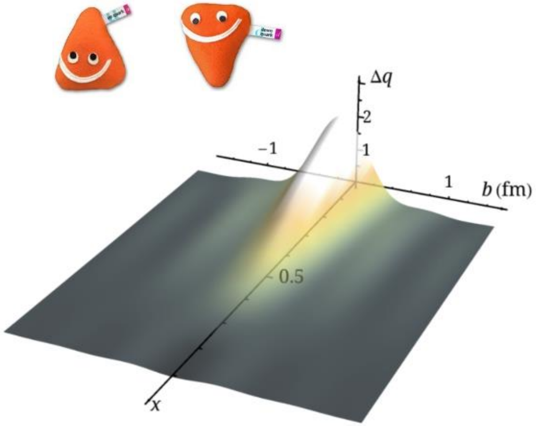
HL (MSULat), Phys. Lett. B 846 (2023) 138181

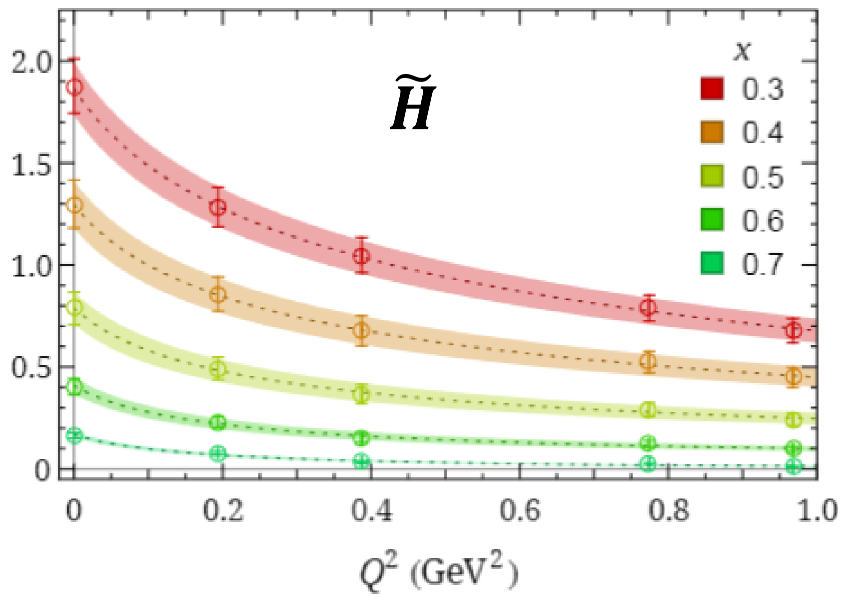
Nucleon Polarized GPDs

§ Helicity GPD ($\widetilde{H}$)using quasi-PDFs at **physical pion mass**

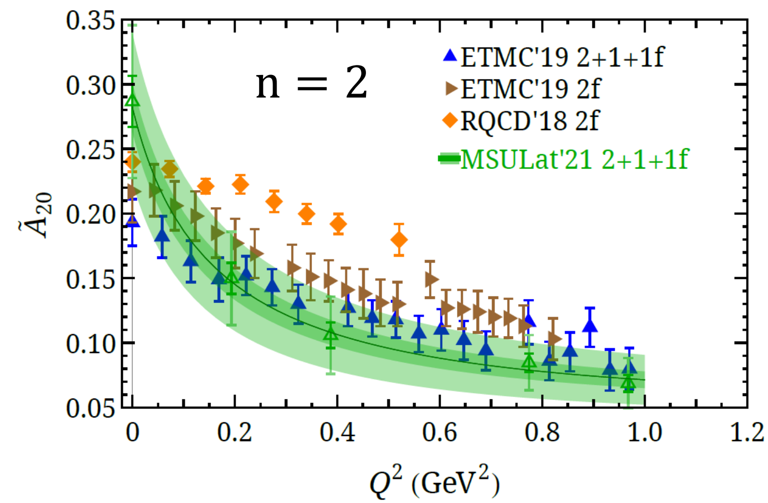
- MSULat: clover/2+1+1 HISQ
  0.09 fm, 135-MeV pion mass, $P_z \approx 2$ GeV
- $\xi = 0$ isovector nucleon (quasi-)GPD results

HL (MSULat), Phys.Lett.B 824 (2022) 136821





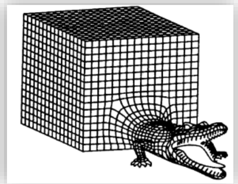
- Take the integral to form moments



Nucleon Tomography

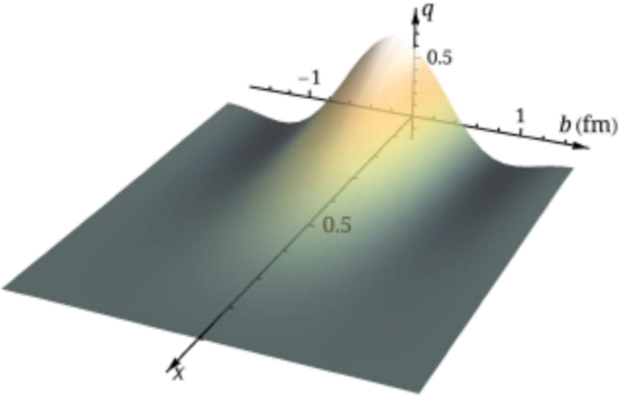
§ Nucleon GPD using quasi-PDFs at physical pion mass

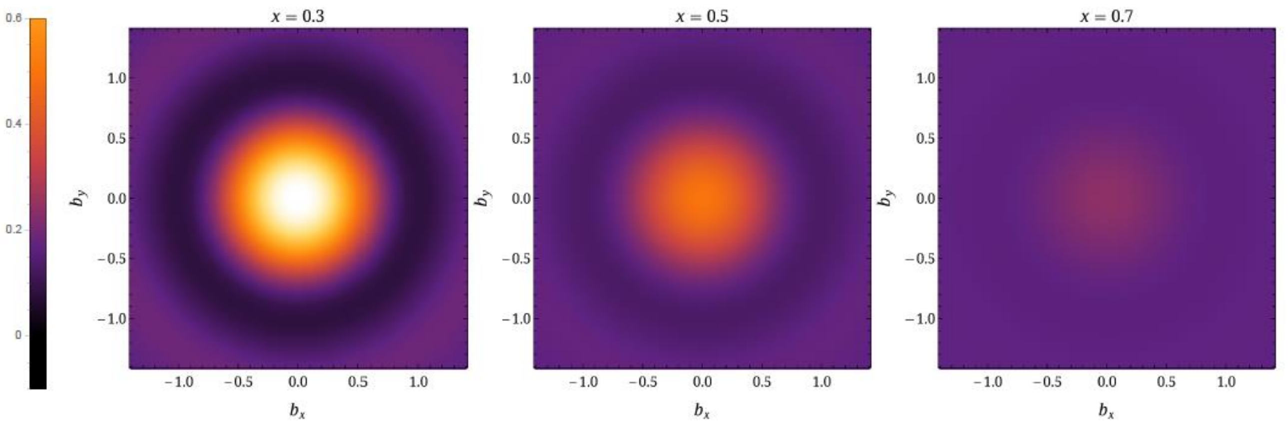
- Lattice details: clover/2+1+1 HISQ
  0.09 fm, 135-MeV pion mass, $P_z \approx 2$ GeV
- $\xi = 0$ isovector nucleon GPD results



finite-volume, discretization,

$$q(x,b) = \int \frac{d\vec{q}}{(2\pi)^2} H(x, \xi = 0, t = -\vec{q}^2) e^{i\vec{q}\cdot\vec{b}}$$





HL, Phys.Rev.Lett. 127 (2021) 18, 182001

Also see work done by ANL/BNL/ETMC, 2209.05373 ,2310.13114

GPD Systematic Update

§ Nucleon isovector GPDs calculated directly at physical pion mass

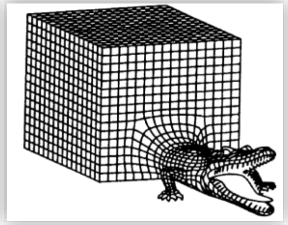
- NNLO matching & treat leading-renormalon effects
  - Leading-renormalon resummation (LRR)
  - Renormalization-group resummation (RGR)

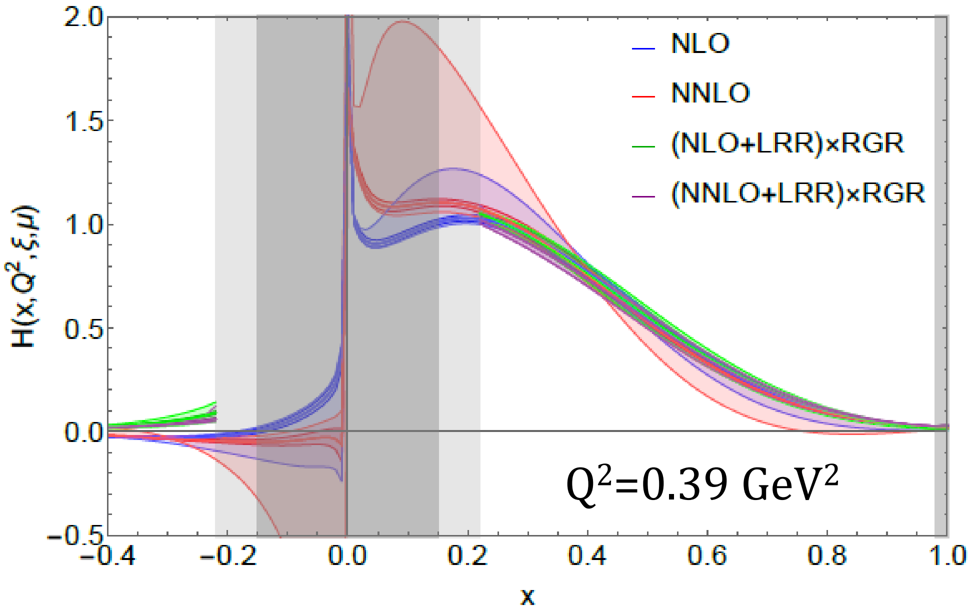
R. Zhang, et. al.
PLB 844, 138081 (2023)

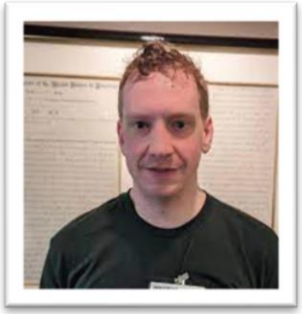
- $N_f = 2+1+1$ clover/HISQ, $a \approx 0.09$ fm, 135-MeV pion, $P_z \approx 2$ GeV

J. Holligan, HL (MSULat), 2312.10829 [hep-lat]

Wanted PDFs, GPDs, etc…







P: Jack Holligan

GPD Systematic Update

§ Nucleon isovector GPDs & pion valence-quark GPDs

- NNLO matching & treat leading-renormalon effects
  - Leading-renormalon resummation (LRR)
  - Renormalization-group resummation (RGR)

R. Zhang, et. al.
PLB 844, 138081 (2023)

- N_f = 2+1+1 clover/HISQ, $a \approx$ 0.09 fm, 135-MeV pion, $P_z \approx$ 2 GeV
  J. Holligan, HL (MSULat), 2312.10829 [hep-lat]
- N_f = 2+1 clover/HISQ, $a \approx$ 0.09 fm, 300-MeV pion, $P_z \approx$ 2 GeV
  H. Ding et al (ANL/BNL/Wuhan), 2407.03516 [hep-lat]

Wanted PDFs, GPDs, etc…