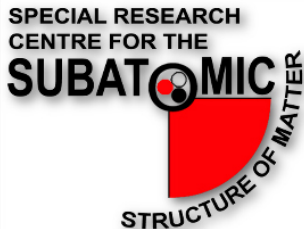


# Centre Vortices in the Foundation of Matter



Derek Leinweber

In collaboration with:

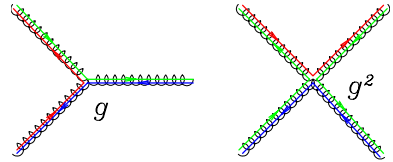
James Biddle, Waseem Kamleh,  
Amalie Trewartha, & Adam Virgili



THE UNIVERSITY  
*of* ADELAIDE

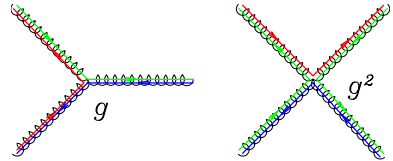
# Introduction

- The self interactions of gluons in QCD make the empty vacuum unstable to the formation of quark and gluon condensates.



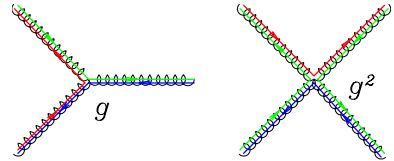
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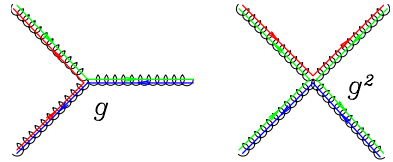


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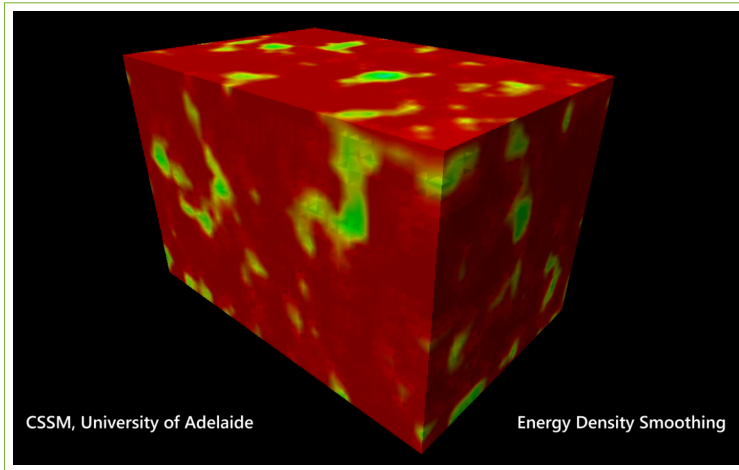
- The self interactions of gluons in QCD make the empty vacuum unstable to the formation of quark and gluon condensates.
- These ground-state QCD-vacuum fields permeate spacetime and form the foundation of matter.
- Quarks carrying the quantum numbers of hadrons are embedded in these fields.
- Commence exploring the ground-state field structure by examining the energy density

$$\sum_{\alpha=1}^8 \left( E_{\alpha}^2(\vec{x}, t) + B_{\alpha}^2(\vec{x}, t) \right) .$$

of the chromo-electric and chromo-magnetic fields.



## Introduction: CSSM Visualisations on YouTube



# Introduction

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- Is there something more fundamental that captures the salient features of QCD?
  - Confinement.
  - Dynamical generation of mass via Chiral Symmetry breaking.
- What is the key feature or mechanism that gives rise to the nucleon mass?
- Can we gain a deeper understanding on how the mass of a hadron emerges?

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# Centre-Vortices in the Ground-State QCD-Vacuum Fields

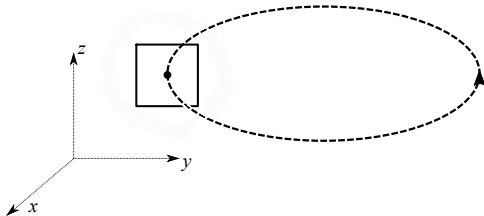
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  - What is the impact of dynamical fermions on centre-vortex structure?
- What is the origin of dynamical mass generation and confinement in QCD?

# Vortex Structure in the Colour Fields of the QCD Vacuum



## What Are Centre Vortices?

- Centre vortices in 3D are tube-like topological defects present in the QCD vacuum.
- We locate thin vortex lines on the lattice.
- The vortex line can be thought of as the 'axis of rotation' of the vortex.



**Figure:** A centre vortex (dashed line) intersecting a lattice plaquette (solid square).

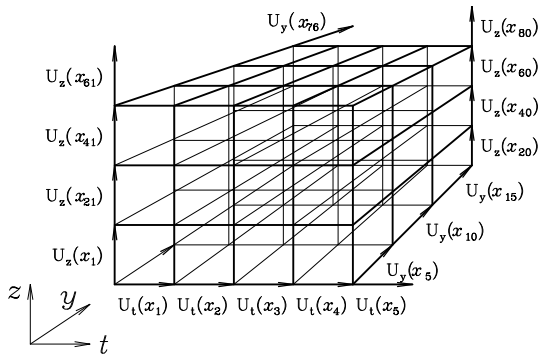
How do you find centre vortices?

## Lattice Links

- On the lattice, the **gluon-field** is encoded in terms of the **link variable**

$$U_{\mu}^{ab}(x) \simeq \exp \left( i a g A_{\mu}^{ab}(x) \right),$$

a  $3 \times 3$  complex special-unitary matrix.



# 1. Maximal Centre Gauge

- Gauge transformations bring the links close to an element of the group centre.
- The centre elements of  $SU(3)$  are the three cube roots of 1, namely

$$Z(3) = \exp\left(\frac{2\pi i}{3} m\right) I, \quad m \in \{-1, 0, +1\}$$

- This is done by maximising the functional

$$R = \sum_x \sum_{\mu} |\text{tr}[U_{\mu}(x)]|^2$$

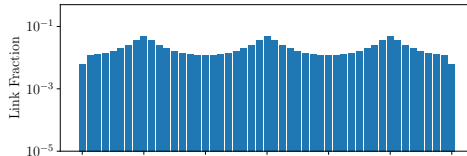
- This is called **Maximal Centre Gauge**

# 1. Maximal Centre Gauge

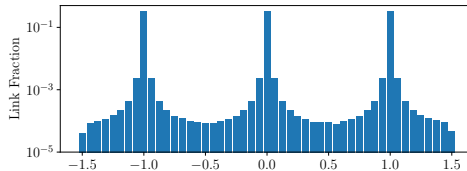
- Distribution of link phases following gauge fixing to Maximal Centre Gauge.

$$\text{tr } U_{\mu}^{\text{MCG}}(x) = \underbrace{r_{\mu}(x)}_{\text{real}} \exp \left( \underbrace{\left( \frac{2\pi i}{3} \phi_{\mu}(x) \right)}_{-\pi < \text{phase} \leq \pi} \right), \quad -\frac{3}{2} < \phi_{\mu}(x) \leq \frac{3}{2}.$$

- $\phi_{\mu}(x)$  before gauge fixing.



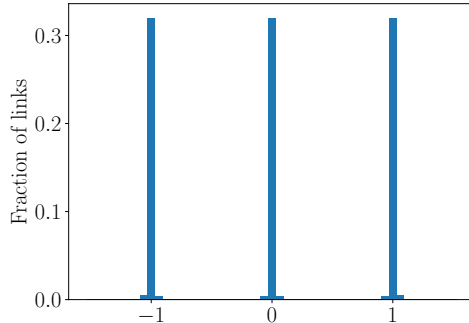
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## 2. Centre Projection

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- Project onto  $Z(3)$

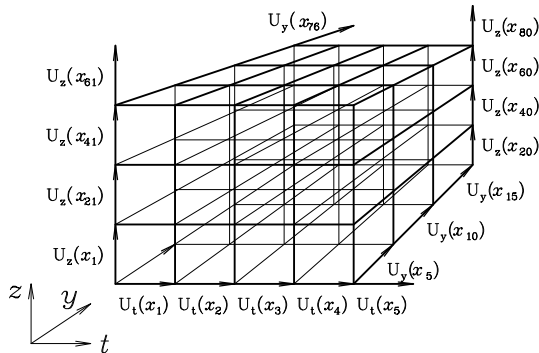
$$U_{\mu}^{\text{MCG}}(x) \rightarrow Z_{\mu}(x) = \exp\left(\frac{2\pi i}{3} m_{\mu}(x)\right) I, \quad m_{\mu}(x) \in \{-1, 0, +1\},$$

*i.e.* a cube-root of 1 times the identity matrix.

- Eight degrees of freedom are replaced by one of the three cube-roots of 1.

### 3. Identifying Vortices

- Examine the product of  $Z_\mu(x)$  around each elementary square (plaquette).
- Each plaquette takes a value from  $Z(3)$ .



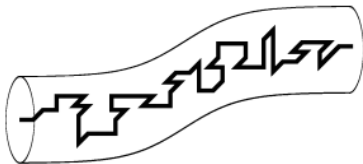
### 3. Identifying Vortices

- Non-trivial plaquettes with values

$$\exp\left(\frac{2\pi i}{3} m\right) \neq 1, \quad \text{i.e. } m \in \{-1, +1\},$$

identify our thin vortices.

- Thin vortices locate the centre of the physical thick vortices



**Figure:** An example of a vortex path embedded within a thick vortex.  
M. Engelhardt, H. Reinhardt, Nuclear Physics B **585** (2000) 597

# Configurations

- This projection allows us to define 3 sets of configurations:
  - Untouched -  $U_\mu(x)$
  - Vortex Only -  $Z_\mu(x)$
  - Vortex Removed -  $R_\mu(x) = Z_\mu^\dagger(x) U_\mu(x)$

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  - Untouched -  $U_\mu(x)$
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  - Vortex Removed -  $R_\mu(x) = Z_\mu^\dagger(x) U_\mu(x)$
- 4 ensembles
  - $20^3 \times 40$  pure gauge (PG), spacing  $a = 0.125$  fm
  - $32^3 \times 64$  pure gauge (PG), spacing  $a = 0.100$  fm
  - $32^3 \times 64$  dynamical 2 + 1 flavour, spacing  $a = 0.1022$  fm,  $m_\pi = 701$  MeV
  - $32^3 \times 64$  dynamical 2 + 1 flavour, spacing  $a = 0.0933$  fm,  $m_\pi = 156$  MeV
  - S. Aoki, *et al.* (PACS-CS), Phys. Rev. D **79**, 034503.

## Phenomenology of Centre Vortices

# Static Quark Potential

- Measures the potential energy between two massive, static quarks at separation  $r$ .
- Serves as an indicator of confining behaviour in the form of a linear long-range potential.
- Typically described via the Cornell potential

$$V(r) = V_0 - \frac{\alpha}{r} + \sigma r$$

# Centre Vortices and Confinement – Pure Gauge Sector

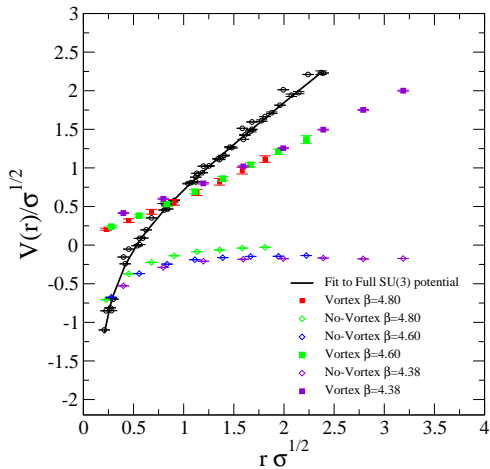


Figure from

Bowman et al, *Phys. Rev. D* **84**, 034501 (2011).

MCG procedure cannot simultaneously identify all SU(3) vortex matter.

O’Cais et al, *Phys. Rev. D* **82**, 114512 (2010).



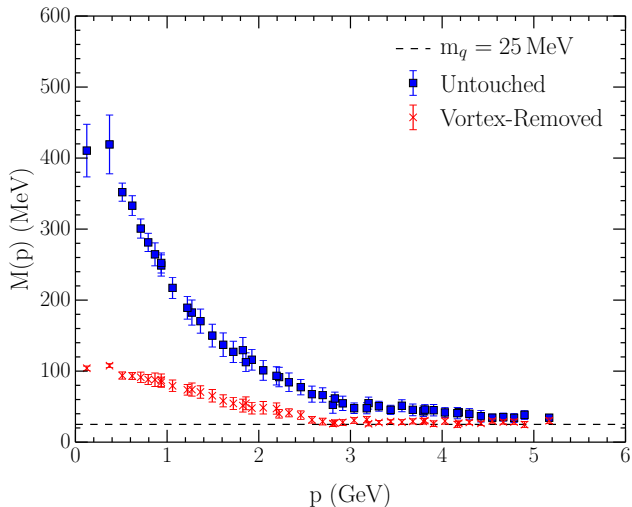
# Centre Vortices and the Landau-Gauge Quark Propagator

- Probe dynamical mass generation using the quark propagator

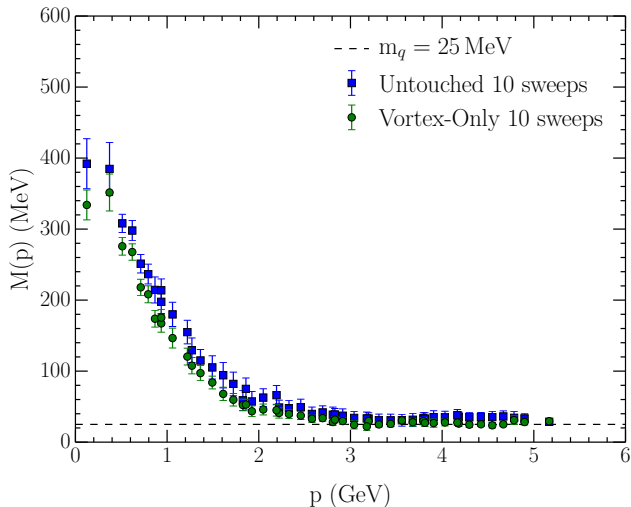
$$S(p) = \frac{Z(p)}{i\not{p} + M(p)},$$

- Enhancement of the mass function,  $M(p)$ , at low momenta indicates dynamical mass generation.
- Renormalisation function,  $Z(p)$ , is typically infrared suppressed.
- Consider the Overlap-Dirac fermion action
  - Provides a lattice implementation of chiral symmetry,
  - No additive mass renormalisation,
  - Sensitive to the topological structure of the gauge fields.

# Quark Mass Function in Pure Gauge Theory



# Vortex-Only Mass function, 10 sweeps of cooling



## Restoration of Chiral Symmetry

- If vortices are responsible for  $D\chi SB$ , then their removal should restore chiral symmetry

$$SU 2_L \times SU 2_R \times U(1)_A$$

- Expect hadrons related by chiral transformations to become degenerate

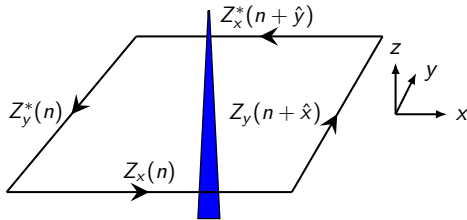
$$\begin{array}{ccc} \pi & \xleftrightarrow{U(1)_A} & a_0 \\ \rho & \xleftrightarrow{SU 2_L \times SU 2_R} & a_1 \\ N & \xleftrightarrow{SU 2_L \times SU 2_R} & \Delta \end{array}$$

- At light quark masses, all symmetries are observed to be restored.
- A. Trewartha, W. Kamleh and DBL, J. Phys. G **44** (2017) 125002 [arXiv:1708.06789 [hep-lat]].

What do centre vortices look like?

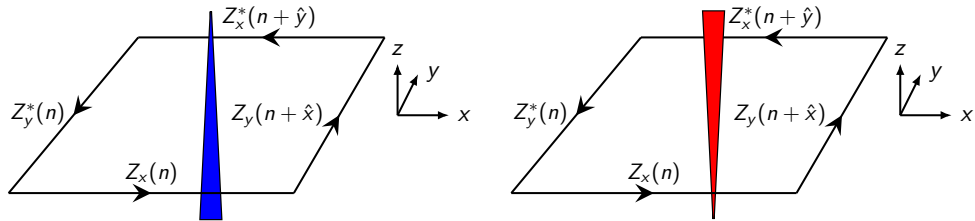
# Rendering Projected Vortices

- Vortex directions are indicated using a right-handed coordinate system.
- For example,
  - An  $m = +1$  vortex in the  $x$ - $y$  plane is plotted in the  $+\hat{z}$  direction as a blue jet.

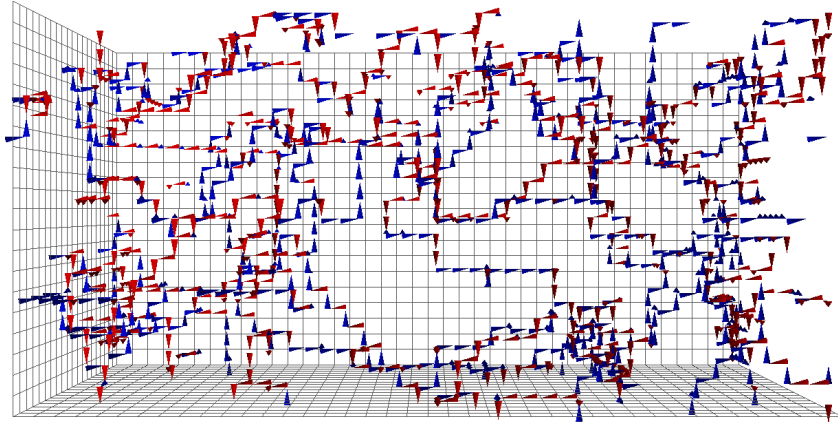


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  - An  $m = -1$  vortex in the  $x$ - $y$  plane is plotted in the  $-\hat{z}$  direction as a red jet.



$t = 1$  J. Biddle, W. Kamleh and DBL, Phys. Rev. D **102**, 034504 [arXiv:1912.09531 [hep-lat]]





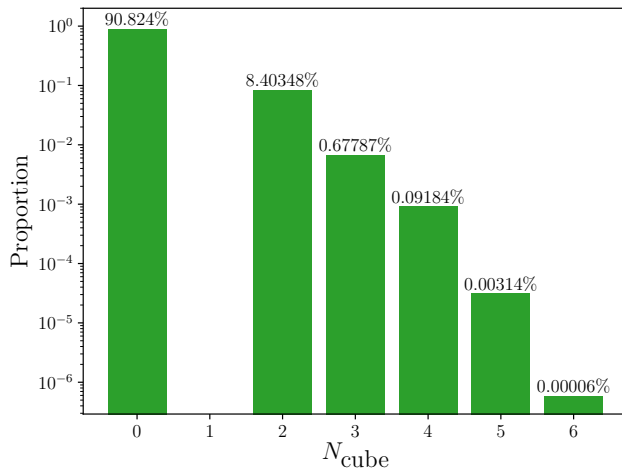
## Visualising Centre Vortices

- Consider the number of vortices entering a 3D cube on the dual lattice.

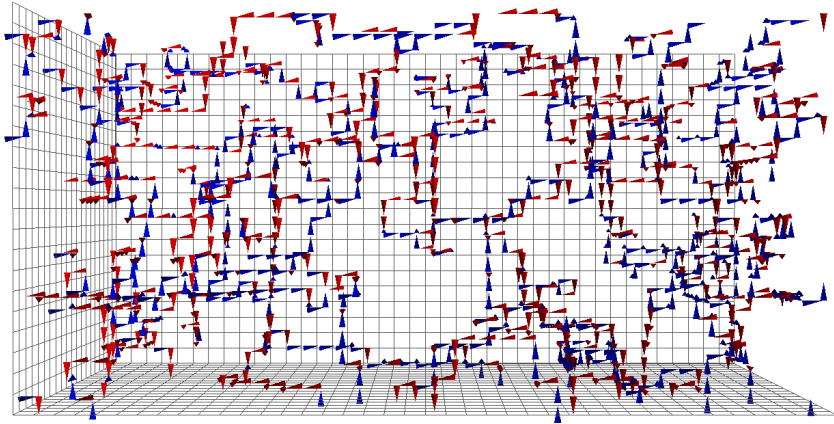
$N_{\text{cube}}(\tilde{x})$	Interpretation
0	No vortices present.
1	Terminating vortex, forbidden by Bianchi*.
2	Vortex line flowing through the cube.
3	Simple three-way vortex monopole.
4	Vortex intersection.
5	Complex five-way monopole path.
6	Vortex intersections or double monopoles.

\*Bianchi identity implies a continuous flow of centre vortex flux through a spatial cube.

# Visualising Centre Vortices

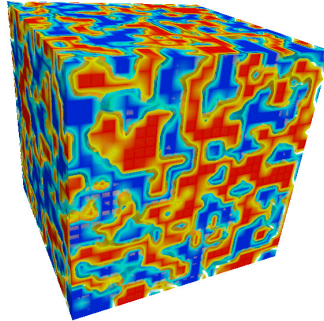


$t = 2$  J. Biddle, W. Kamleh and DBL, Phys. Rev. D **102**, 034504 [arXiv:1912.09531 [hep-lat]]

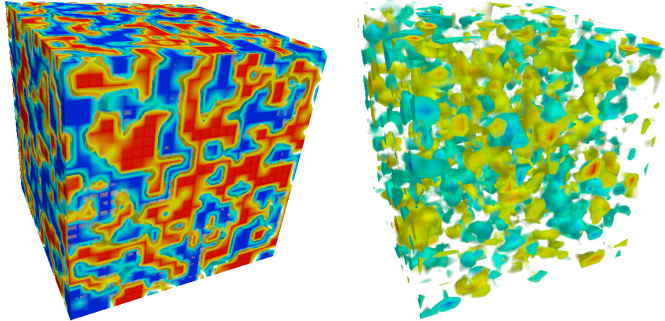


# Impact of Dynamical Fermions on Centre Vortex Structure

# Visualisations of the Topological Charge Density

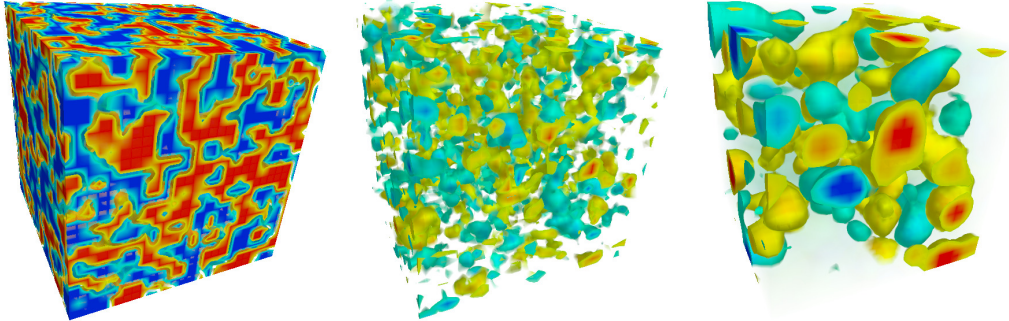


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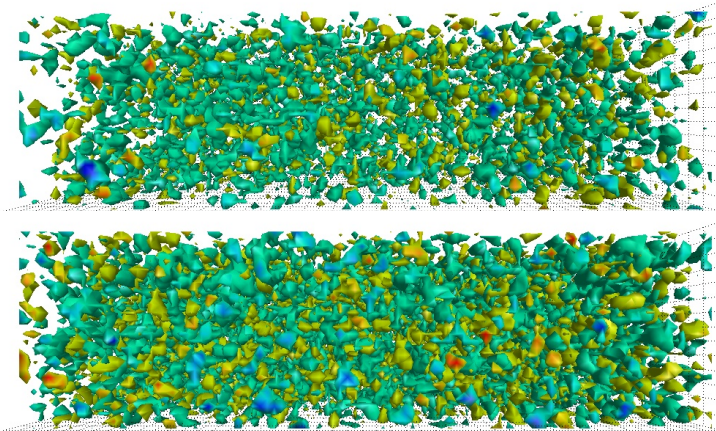
- See “Buried treasure in the sand of the QCD vacuum,” P. J. Moran and DBL, [arXiv:0805.4246 [hep-lat]].

# Visualisations of the Topological Charge Density



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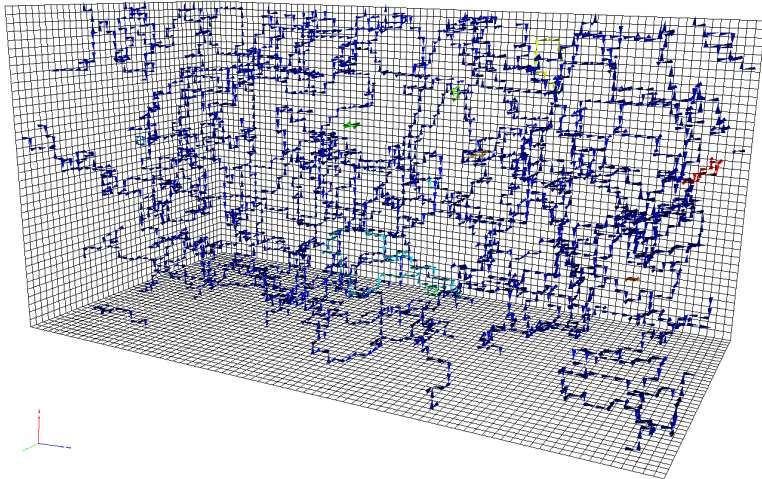
# Pure Gauge versus MILC 2 + 1-Flavour QCD: $m_{u,d} = 27.1$ MeV



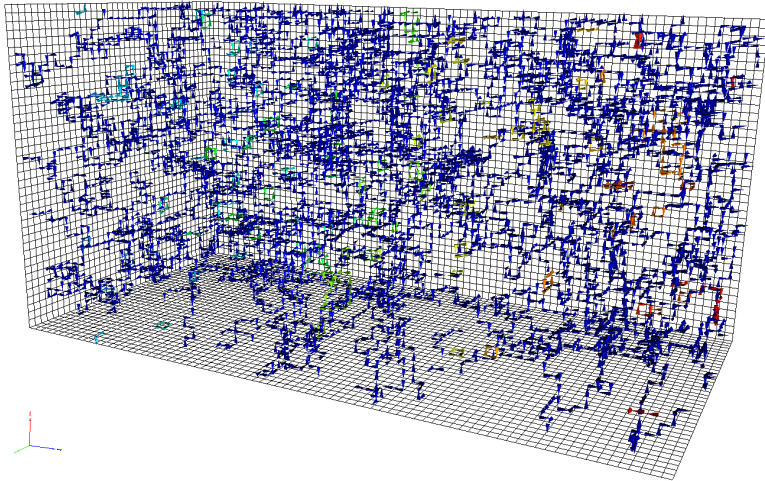
- “Impact of Dynamical Fermions on QCD Vacuum Structure,” P. J. Moran and DBL, Phys. Rev. D **78** (2008) 054506 [arXiv:0801.2016 [hep-lat]].



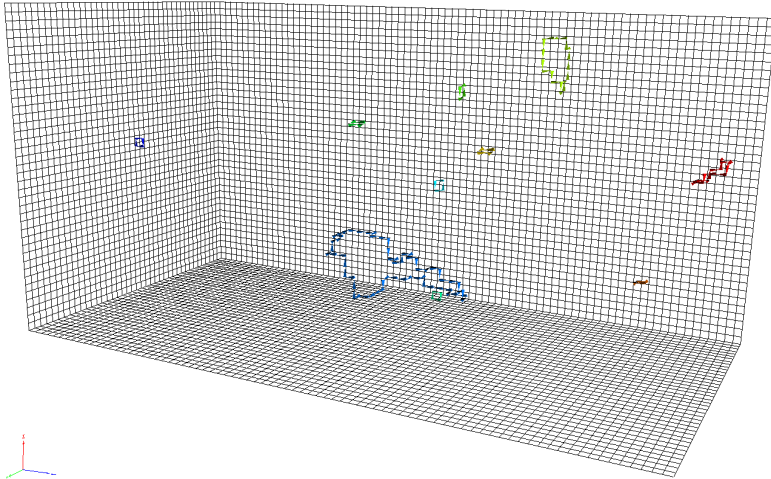
# Vortices on a Pure-Gauge $32^3 \times 64$ Lattice



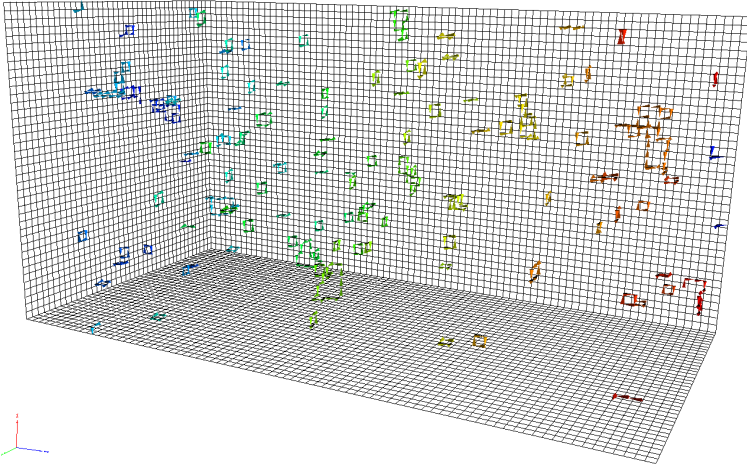
# 2 + 1 Flavour $32^3 \times 64$ Dynamical-Fermion Lattice $m_\pi = 156$ MeV



# Secondary Loops on a Pure-Gauge $32^3 \times 64$ Lattice



# 2 + 1 Flavour $32^3 \times 64$ Dynamical-Fermion Lattice $m_\pi = 156$ MeV



# Impact of Dynamical Fermions on Centre Vortex Structure

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- The number of vortices composing the primary cluster is
  - $3,277 \pm 156$  vortices in the Pure Gauge theory.
  - $5,924 \pm 239$  vortices in Full QCD.

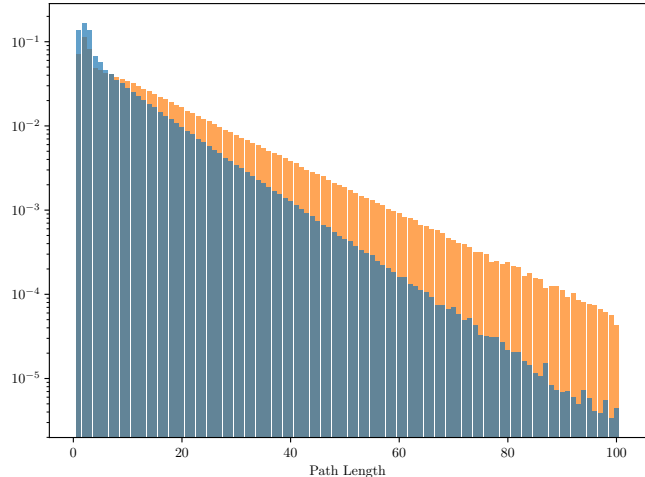
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- The number of vortices composing the primary cluster is
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  - $5,924 \pm 239$  vortices in Full QCD.
- There is further enhancement of the secondary loop structure,
  - Both in number and in complexity via monopoles.



## Impact of Dynamical Fermions: Vortex Path Lengths

- Histogram of vortex path lengths in the percolating cluster.
- **Pure Gauge** and **Dynamical Fermion** ensembles are illustrated.
- Path length is the number of jets from one branching point to the next.
- Distributions are normalised.
- Moderate size loops are exponentially distributed.
  - Fixed probability of branching .
  - Branching is independent of length.



# Impact of Dynamical Fermions on the Centre-Vortex Structure of the Gluon Propagator

## Centre Vortices and the Landau-Gauge Gluon Propagator

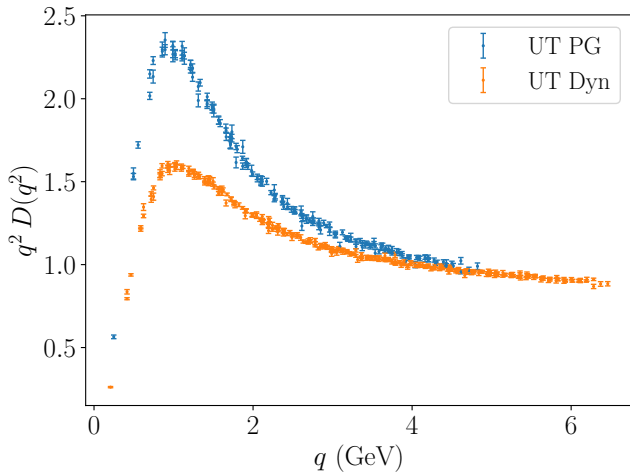
- The nonperturbative scalar gluon propagator in momentum space is

$$D(q^2) \equiv \frac{Z(q^2)}{q^2} \rightarrow \frac{1}{q^2} \text{ at tree level.}$$

- Consider the renormalisation function

$$Z(q^2) = q^2 D(q^2).$$

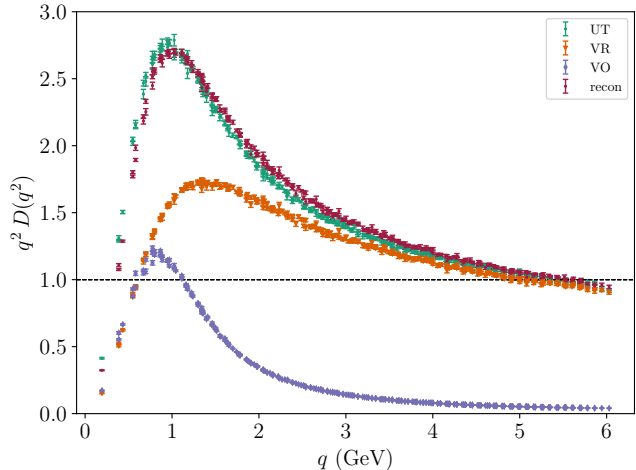
- Renormalise by setting  $Z(q^2) = 1$  at  $q = 5.5$  GeV.
- $32^3 \times 64$  lattices.  $m_\pi = 156$  MeV.



J. C. Biddle, W. Kamleh and DBL, in preparation.

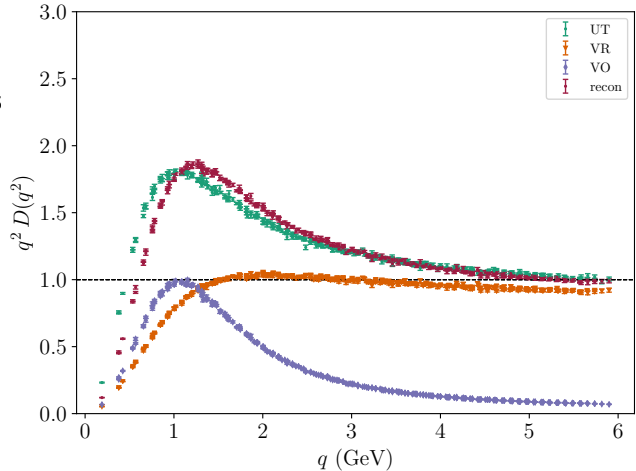
# Gluon Propagator – Pure Gauge Sector

- **Vortex Removal** (VR) suppresses infrared enhancement whilst preserving UV perturbative behaviour.
- **Vortex-Only** (VO) configurations capture the long-distance physics.
- **Reconstruction** of the propagator as a linear combination of the vortex-modified parts recovers full propagator.
- Residual infrared enhancement in the **vortex-removed** result is undesirable.



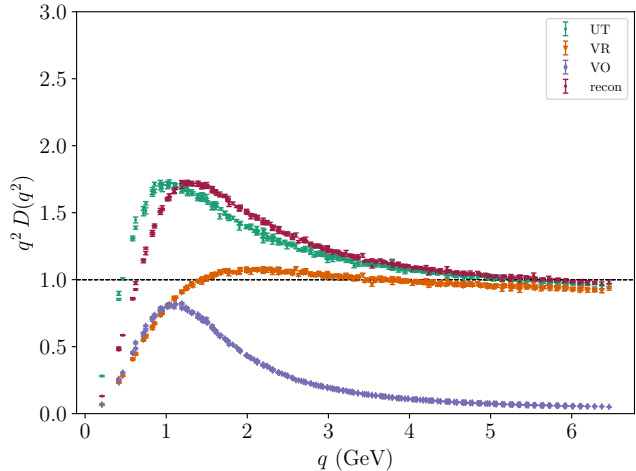
# Gluon Propagator – Dynamical Fermions $m_\pi = 701$ MeV

- **Dynamical fermions** (UT) suppress the overall infrared strength.
- **Vortex Removal** (VR) almost eliminates infrared enhancement.
- **Vortex-Only** (VO) configurations capture the long-distance physics.
- **Reconstruction** is less perfect.

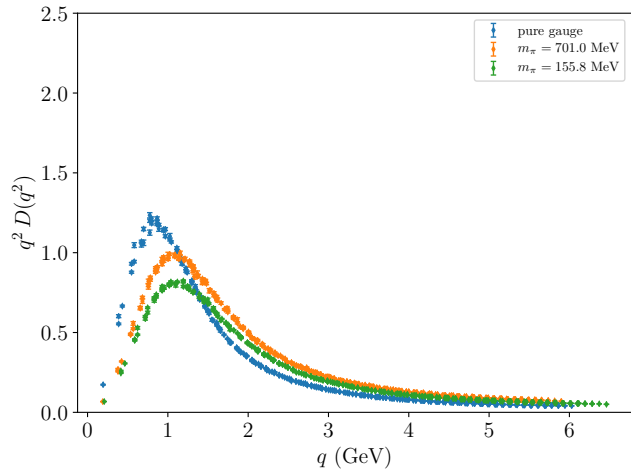


## Gluon Propagator – Dynamical Fermions $m_\pi = 156$ MeV

- Lighter dynamical  $u$  and  $d$  quarks further suppress the infrared enhancement.
- **Centre Vortex** degrees of freedom are able to capture the screening effects of dynamical fermions in QCD.



# Dynamical-Fermion Screening Effects in Vortex-Only Configurations



# The Gluon Propagator and Confinement

- The 1-dimensional Fourier transform of the gluon propagator at zero spatial momentum defines the wall-to-wall correlator

$$C(t) = \int_0^\infty dm \rho(m^2) e^{-mt} .$$

- $C(t)$  is negative in QCD.



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- The 1-dimensional Fourier transform of the gluon propagator at zero spatial momentum defines the wall-to-wall correlator

$$C(t) = \int_0^\infty dm \rho(m^2) e^{-mt} .$$

- $C(t)$  is negative in QCD.
- The spectral density,  $\rho(m^2)$ , cannot be a positive spectral function.
  - A physical state does not have negative norm contributions in its propagator.
  - There is no Källén-Lehmann representation.
  - The corresponding states cannot appear in the physical particle spectrum.

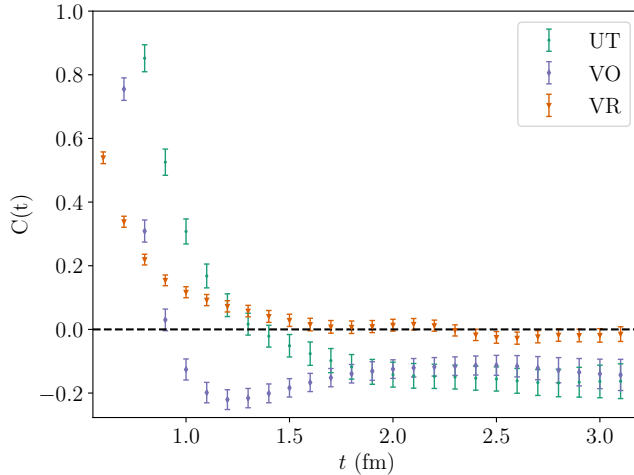
# The Gluon Propagator and Confinement

- The 1-dimensional Fourier transform of the gluon propagator at zero spatial momentum defines the wall-to-wall correlator

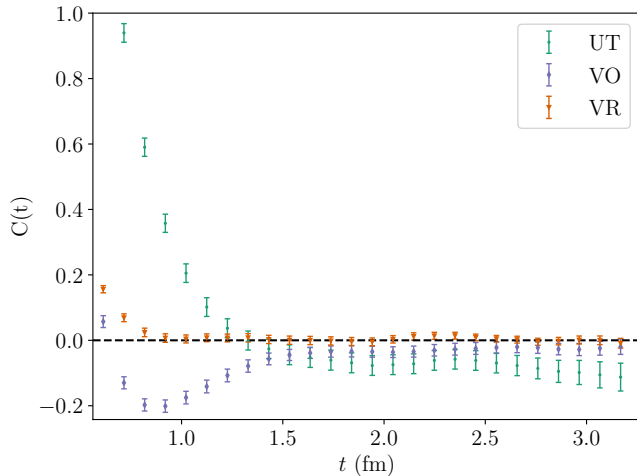
$$C(t) = \int_0^\infty dm \rho(m^2) e^{-mt} .$$

- $C(t)$  is negative in QCD.
- The spectral density,  $\rho(m^2)$ , cannot be a positive spectral function.
  - A physical state does not have negative norm contributions in its propagator.
  - There is no Källén-Lehmann representation.
  - The corresponding states cannot appear in the physical particle spectrum.
- The states are confined from the physical world.
  - J. E. Mandula and M. Ogilvie, Phys. Lett. **B185**, 127 (1987).
  - C. A. Aubin and M. C. Ogilvie, Phys. Lett. **B570**, 59 (2003), hep-lat/0306012.
  - P. O. Bowman, *et al.* Phys. Rev. D **76**, 094505 (2007) [arXiv:hep-lat/0703022 [hep-lat]].

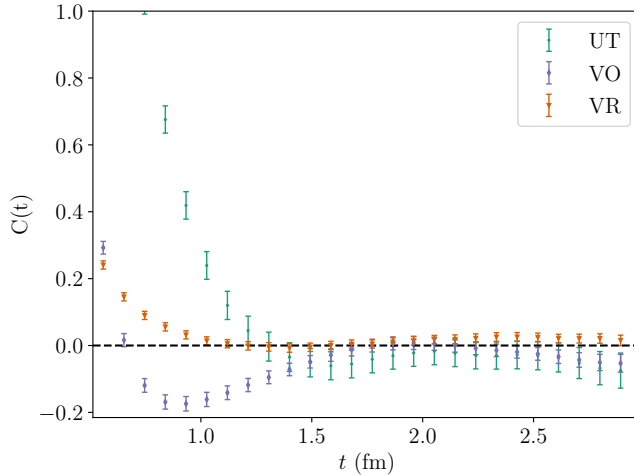
# Positivity Violation in the Pure-Gauge Sector



# Positivity Violation in QCD: $m_\pi = 701$ MeV



# Positivity Violation in QCD: $m_\pi = 156$ MeV



# Impact of Dynamical Fermions on the Centre-Vortex Structure of the Quark Propagator

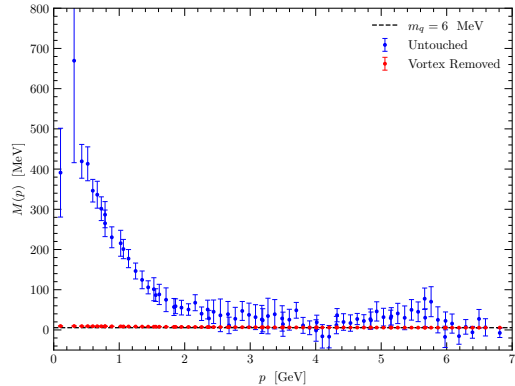
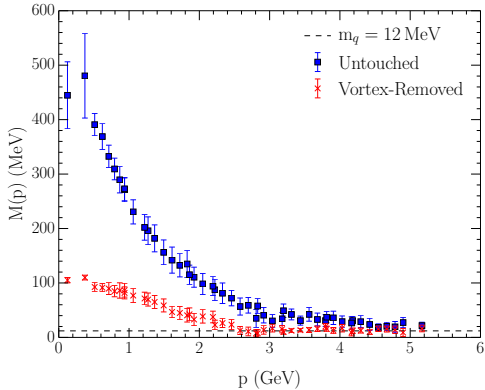
# Centre Vortices and the Landau-Gauge Quark Propagator

- Probe dynamical mass generation using the quark propagator

$$S(p) = \frac{Z(p)}{i\not{p} + M(p)},$$

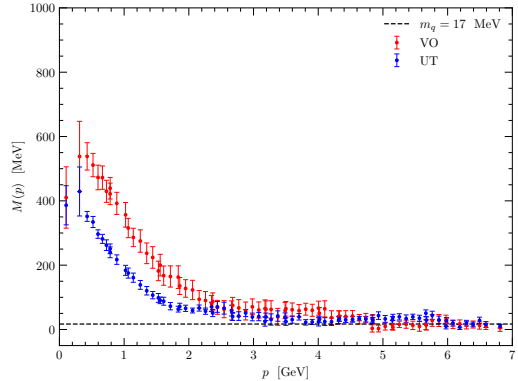
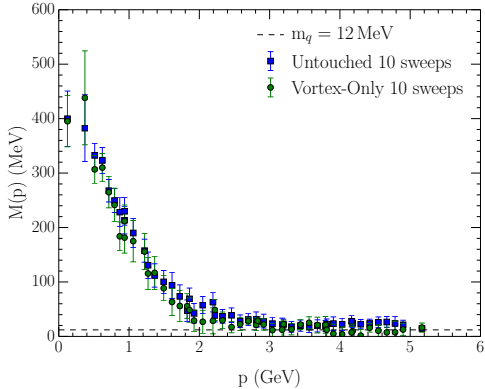
- Enhancement of the mass function,  $M(p)$ , at low momenta indicates dynamical mass generation.
- Renormalisation function,  $Z(p)$ , is typically infrared suppressed.
- Consider the Overlap-Dirac fermion action
  - Provides a lattice implementation of chiral symmetry,
  - No additive mass renormalisation,
  - Sensitive to the topological structure of the gauge fields.

# Pure Gauge vs Dynamical: Vortex-Removed Quark Mass Function





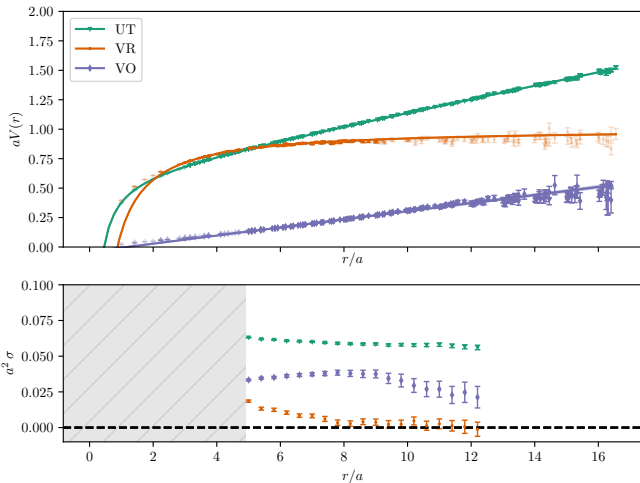
# Pure Gauge vs Dynamical: Vortex-Only Mass Function



# Impact of Dynamical Fermions on the Centre-Vortex Structure of the Static Quark Potential

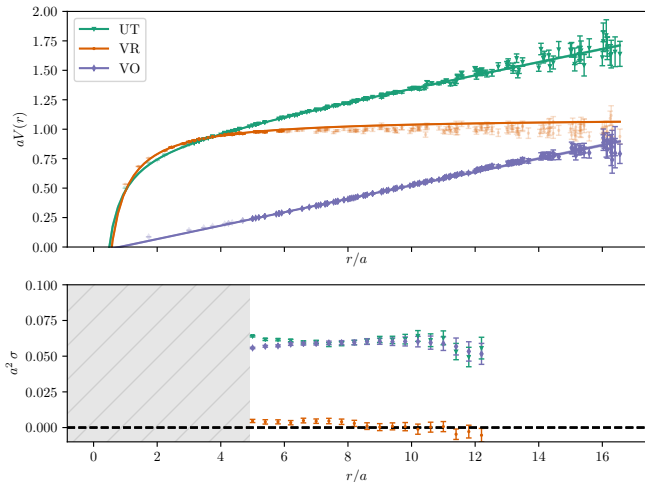
## Static Quark Potential – Pure Gauge Sector

- Lower plot reports the local slope from fits to  $V(r)$  over a forward-looking window from  $r$  to  $r + 4a$ .
- **Vortex removal (VR)** leaves no residual confining potential.
- **Vortex-only (VO)** reproduces only 62% of the **original (UT)** static quark potential.



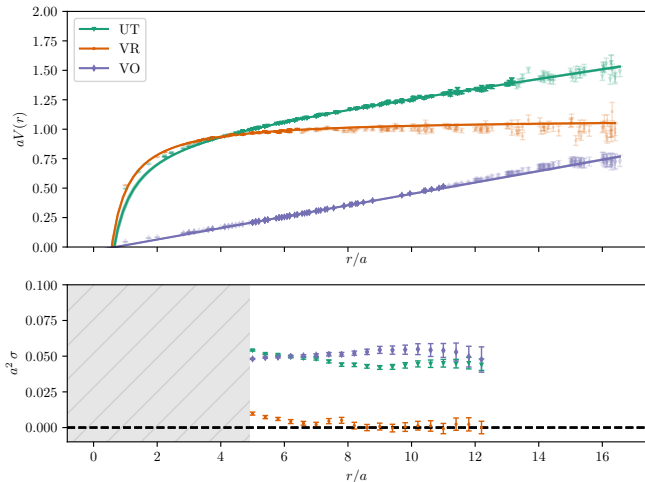
## Introducing Dynamical Fermions ( $m_\pi = 701\text{MeV}$ )

- In the presence of dynamical fermions, **vortices** capture the **full** string tension.
- **Vortex removal** leaves no residual confining potential.
- Centre vortices are the origin of confinement in QCD.



## Lighter dynamical fermions ( $m_\pi = 156\text{MeV}$ )

- Lighter quark masses screen the **confining potential**.
- **Vortices** continue to capture the **full string tension**.
- **Vortex removal** leaves no residual confining potential.
- Centre vortices are the origin of confinement in QCD.



## Visualisation Conclusions

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  - From  $\sim 3,000 \rightarrow 6,000$  vortex links.

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- Enhancement of small vortex paths upon introducing dynamical fermions.
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- Dynamical fermions radically alter the centre-vortex structure of the vacuum fields.

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- In QCD, vortex removal:
  - Suppresses the infrared enhancement of the gluon propagator.
  - Eliminates dynamical mass generation in the mass function of the quark propagator.
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- Conclude that centre vortices provide a common origin for
  - Confinement, and the
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- Centre vortices in the foundation of matter capture the essence of nonperturbative QCD.



## Interactive 3D Visualisation Techniques

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- Rendered in AVS Express Visualisation Edition.  
<http://www.avs.com/solutions/express/>
- Exported in VRML.
- Converted to U3D format via pdf3d ReportGen.  
<https://www.pdf3d.com/products/pdf3d-reportgen/>
- Imported into  $\text{\LaTeX}$  via media9 package.
- Viewed in Adobe acroread (Linux, use 9.4.1 when 3D support was maintained).  
<ftp://ftp.adobe.com/pub/adobe/reader/unix/9.x/9.4.1/>

## Smoothed Vortices vs Instantons

## Quark propagator on vortex only fields

---

- Vortex only configurations consist only of centre elements
- ⇒ very rough
- The Overlap operator has a smoothness condition
- ⇒ 10 sweeps of cooling on vortex only configurations.

## Does 10 sweeps of cooling recreate instantons?

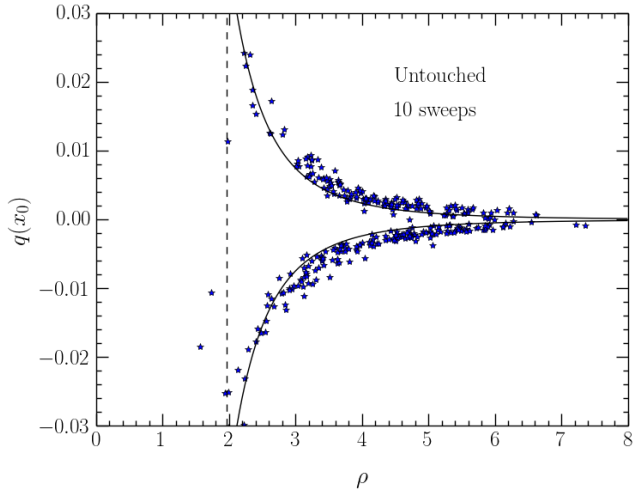
- Scan each configuration for local maxima of the action.
- Fit the instanton profile around them to learn the radius  $\rho$

$$S_0(x) = \xi \frac{6}{\pi^2} \frac{\rho^4}{((x - x_0)^2 + \rho^2)^4}.$$

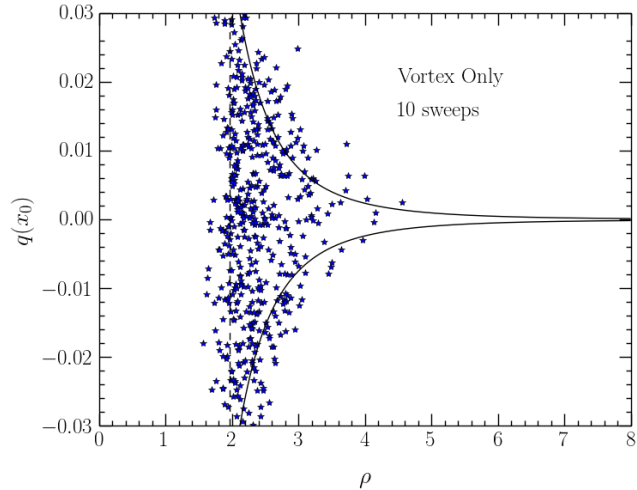
- Compare lattice value for  $q(x_0)$  to the theoretical relationship

$$q(x_0) = Q \frac{6}{\pi^2 \rho^4}.$$

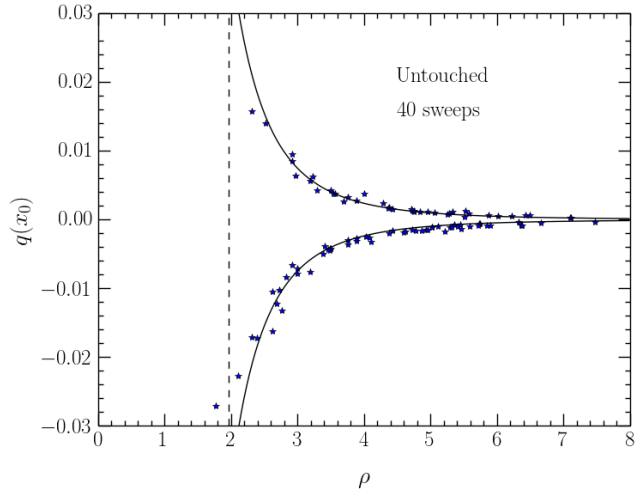
# Instanton $\rho$ vs $q(x_0)$



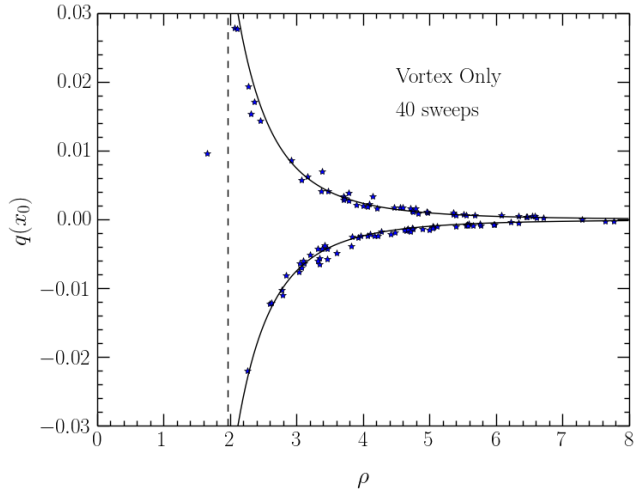
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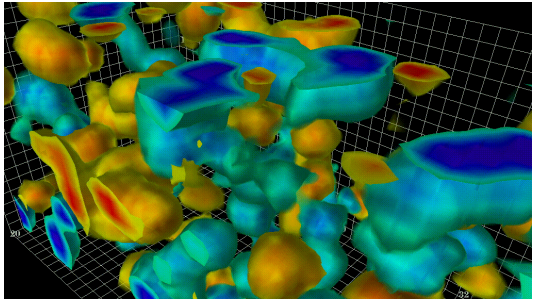
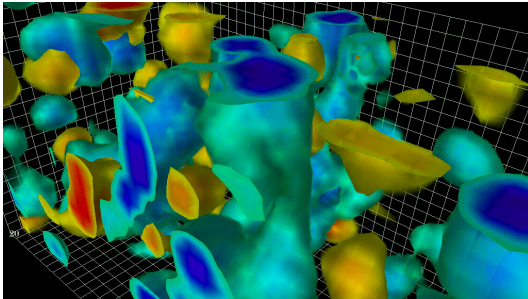


# Instanton $\rho$ vs $q(x_0)$





## Untouched (left) and Vortex-Only comparison (10 sweeps)



## Space-Time Oriented Vortices

# Rendering Space-Time Oriented Projected Vortices

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- Every link in the spatial volume has a forward and backward time-oriented plaquette associated with it.

## Rendering Space-Time Oriented Projected Vortices

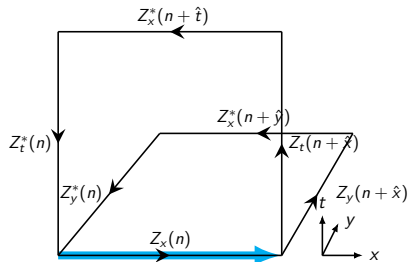
- Every link in the spatial volume has a forward and backward time-oriented plaquette associated with it.
- The three jets associated with the spatial  $x$ - $y$ ,  $y$ - $z$  and  $z$ - $x$  plaquettes, are complemented by
  - Jets in the three forward time  $x$ - $t$ ,  $y$ - $t$  and  $z$ - $t$  plaquettes, and
  - Jets in the three backward time  $x$ - $t$ ,  $y$ - $t$  and  $z$ - $t$  plaquettes.

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  - Jets in the three backward time  $x$ - $t$ ,  $y$ - $t$  and  $z$ - $t$  plaquettes.
- See “Visualization of center vortex structure,” to link vortices to topological charge.  
J. C. Biddle, W. Kamleh and DBL, Phys. Rev. D **102** (2020) 034504 [arXiv:1912.09531 [hep-lat]].

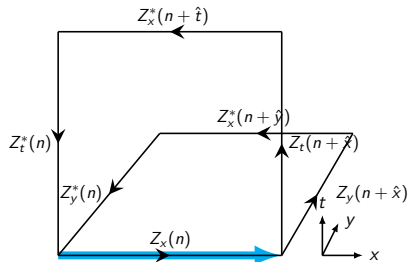
## Rendering Space-Time Oriented Projected Vortices

- If a spatial link belongs to a vortex in a space-time plaquette then:
  - The link is rendered in cyan for an  $m = +1$  vortex.



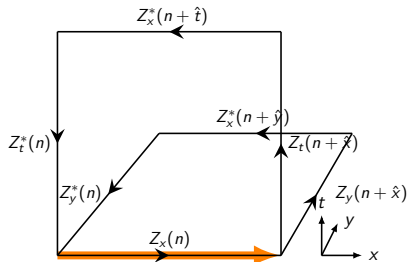
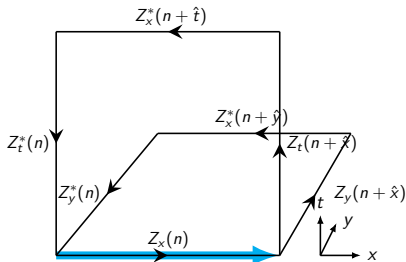
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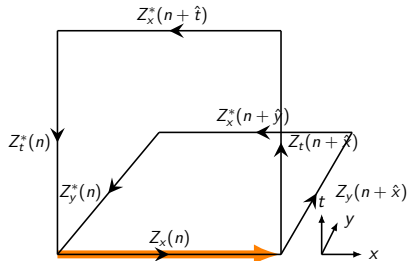
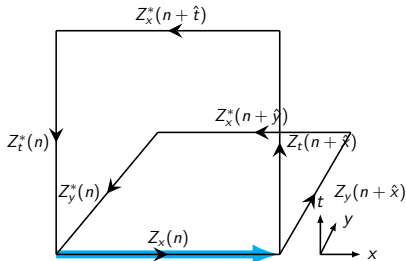
- If a spatial link belongs to a vortex in a space-time plaquette then:
  - The link is rendered in **cyan** for an  $m = +1$  vortex, and in **orange** for  $m = -1$ .
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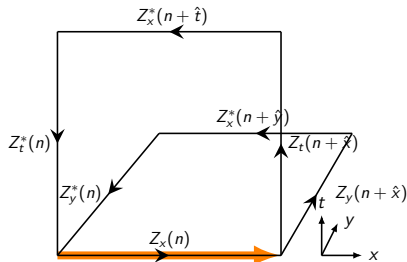
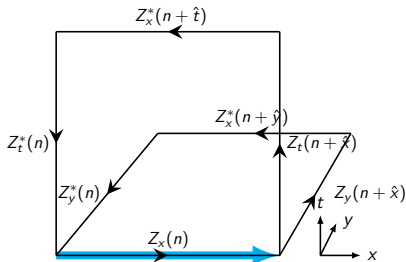
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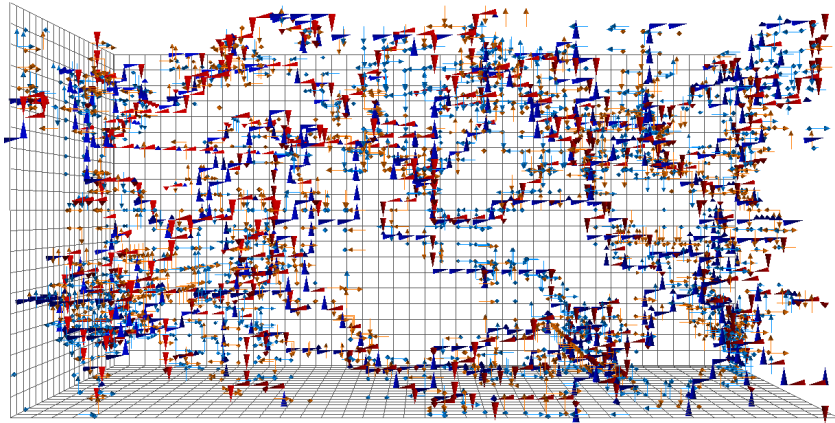


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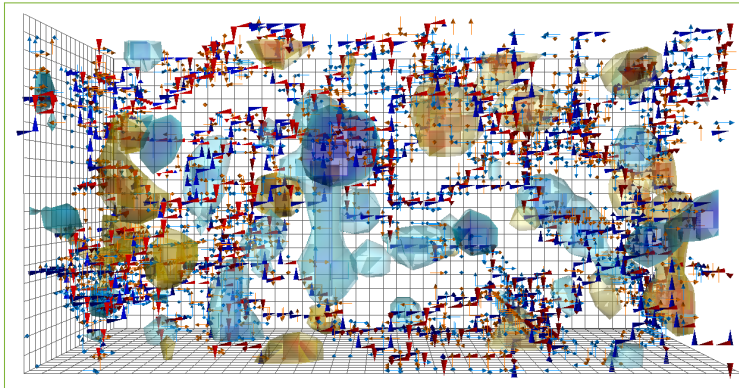
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  - The link is rendered as a positively-directed arrow for forward space-time plaquettes.
  - The link is rendered as a negatively-directed arrow for backward space-time plaquettes.
- As one steps forwards in time, positively-directed links become negatively-directed.



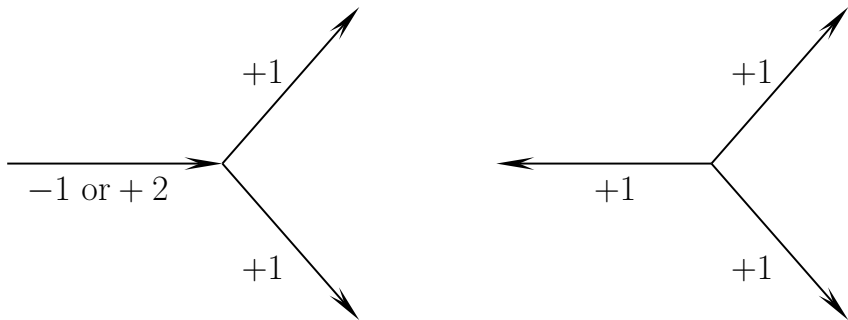
# Time slice $t = 1$



# Animation of Centre Vortex Structure [Google: YouTube CSSM Visualisations](#)



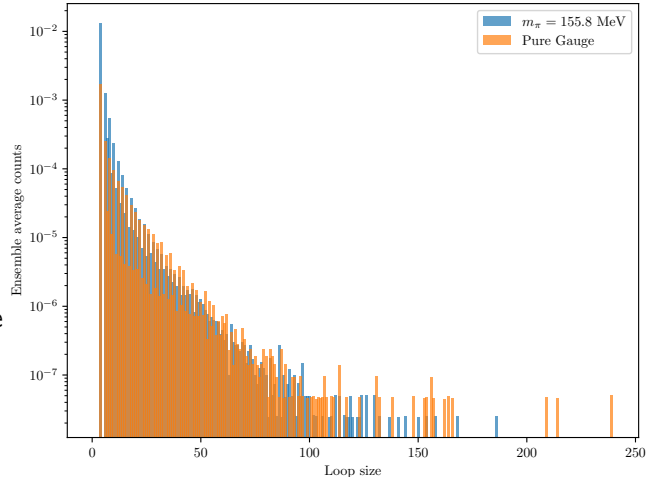
## Visualising Centre Vortices



- A vortex branching point with centre charge  $+2$  flowing into a vertex (left) is equivalent to a vortex monopole with charge  $+1$  flowing out of the vertex (right). Arrows indicate the direction of flow for the labelled charge.
- Our convention illustrates the directed flow of charge  $m = +1$ .

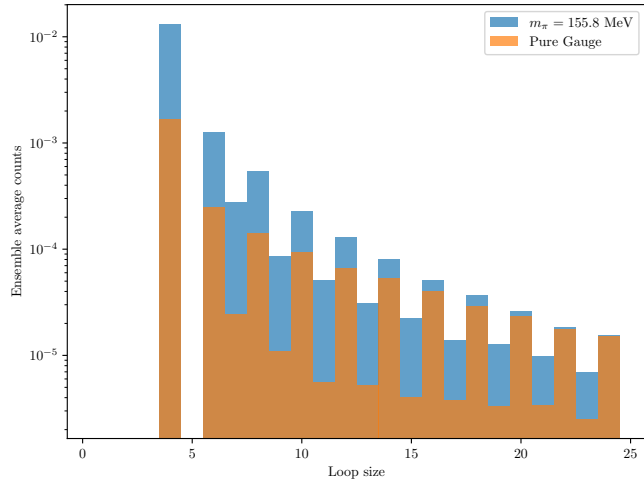
## Impact of Dynamical Fermions: Secondary Loop Lengths

- Histogram of vortex path lengths for each structure within an ensemble.
- The relative proportion of each loop size is indicated.
- Moderate size loops are exponentially distributed.
- There is an order of magnitude increase in the number of small loops in QCD.
- Comb-like structure indicates lower probability of monopoles.



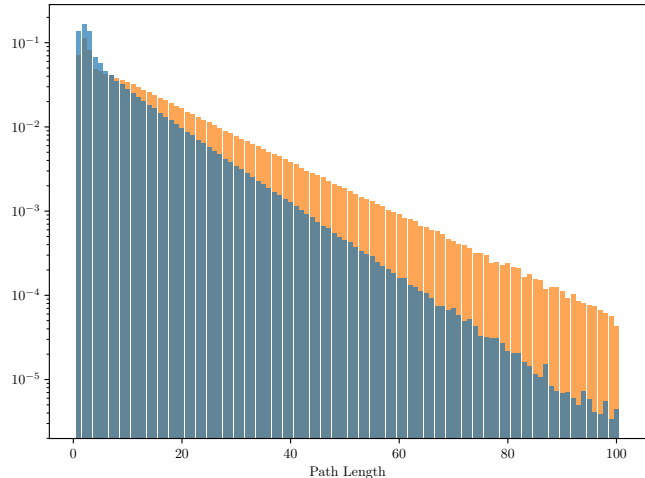
## Impact of Dynamical Fermions: Secondary Loop Lengths

- Path lengths 1, 2, 3 and 5 don't close.
  - Bianchi identity forbidden.
- Odd link-path lengths require a monopole-antimonopole pair.
- Greater relative abundance of monopole-antimonopole pairs in QCD.
  - Tines of the comb are shorter in QCD.



## Impact of Dynamical Fermions: Vortex Path Lengths

- Histogram of vortex path lengths in the percolating cluster.
- **Pure Gauge** and **Dynamical Fermion** ensembles are illustrated.
- Path length is the number of jets from one branching point to the next.
- Distributions are normalised.
- Moderate size loops are exponentially distributed.
  - Fixed probability of branching .
  - Branching is independent of length.





## Potential Models: Lighter dynamical fermions ( $m_\pi = 156\text{MeV}$ )

- Lattice QCD results characterised well by simple models.
- Original (UT):

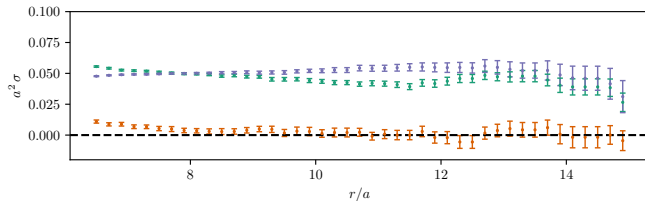
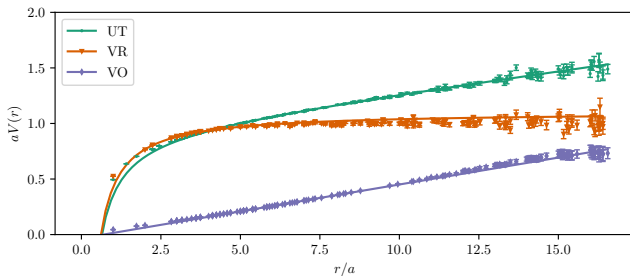
$$V(r) = V_0 - \frac{\alpha}{r} + \sigma r$$

- Vortex-removed (VR):

$$V(r) = V_0 - \frac{\alpha}{r}$$

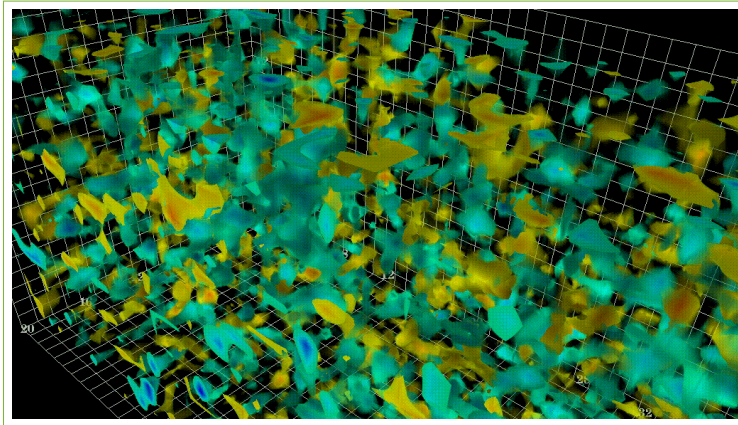
- Vortex-only (VO):

$$V(r) = V_0 + \sigma r$$



## Centre Vortices Versus Instantons

# Untouched Configurations with Cooling



## Vortex Removed Configurations with Cooling

