Lattice calculations of the heavy quark diffusion coefficient in quenched QCD

Viljami Leino
Helmholtz Institute Mainz, JGU Mainz

Based on:
Nora Brambilla, V.L., Peter Petreczky, Antonio Vairo: PRD. 102 (2020) [hep-lat/2007.10078]
Nora Brambilla, V.L., Julian Mayer-Steudte, Peter Petreczky: hep-lat/2206.02861

Heavy Flavor Production in Heavy-Ion and Elementary Collisions
INT, Seattle
24.10.2022

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See talk by Julian tomorrow

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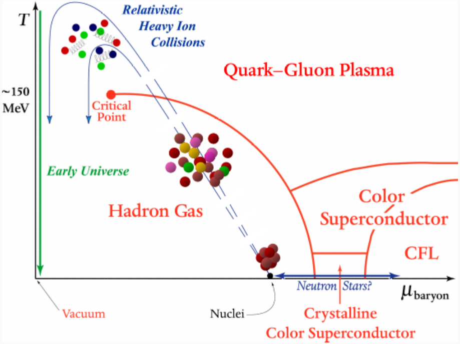
Outline

- Introduction
- Diffusion in strongly coupled plasma
    - Heavy Quarks
    - Heavy Quarkonium
- Lattice basics
- Measurements
- Results
- Conclusions

Introduction: QCD and QGP

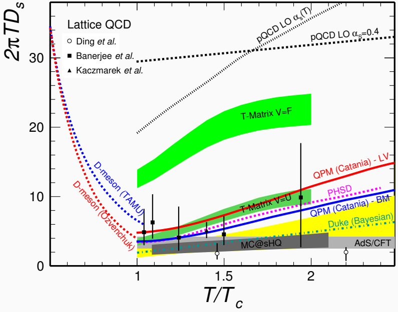
- We aim to understand the strongly coupled Quark Gluon Plasma (QGP)
- QGP generated at particle accelerators such as LHC/RHIC

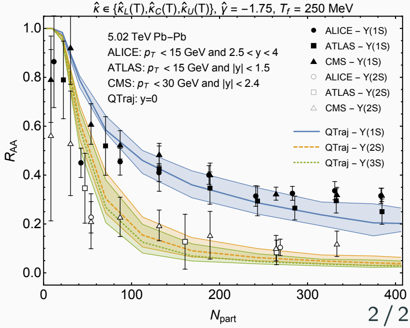


- The QGP can be described in terms of transport coefficients
- In this talk we focus on the heavy quark momentum diffusion coefficient κ

Motivation: diffusion coefficient

- Nuclear modification factor R_{AA} and elliptic flow v_2 described by spatial diffusion coefficient D_x
- Multiple theoretical models predicting wide range of values
- Non-perturbative lattice simulations needed
- κ (γ) dominant source of variation in R_{AA} in some models





UP: X. Dong CIPANP (2018)
DOWN: N. Brambilla, M. Escobedo, M. Strickland, A. Vairo, P. Vander Griend and J. Weber, JHEP 05 (2021) 136

Heavy Quark in medium

- Heavy quark energy changes only little when colliding with medium
$$E_k \sim T\,, \quad p \sim \sqrt{MT} \gg T$$
- HQ momentum is changed by random kicks from the medium
  $\rightarrow$ Brownian motion; Follows Langevin dynamics
$$\frac{dp_i}{dt} = -\frac{\kappa}{2MT} p_i + \xi_i(t)\,, \quad \langle \xi(t)\xi(t') \rangle = \kappa\delta(t-t')$$
- Heavy quark momentum diffusion coefficient κ related also to:
  Spatial diffusion coefficient $D_\mathrm{s} = 2T^2/\kappa$,
  Drag coefficient $\eta_\mathrm{D} = \kappa/(2MT)$,
  Heavy quark relaxation time $\tau_\mathrm{Q} = \eta_\mathrm{D}^{-1}$
- Considering full Lorentz force:
$$F(t) = \dot{p} = q\,(E + v \times B))\,(t)$$
- $\langle v^2 \rangle \sim \mathcal{O}(\frac{T}{M})$ correction to HQ momentum diffusion
$$\kappa_\mathrm{tot} \simeq \kappa_\mathrm{E} + \frac{2}{3}\langle \mathrm{v}^2 \rangle \kappa_\mathrm{B}$$

Moore and Teaney PRC71 (2005), Caron-Huot and Moore JHEP02 (2008)
A. Bouttefeux and M. Laine JHEP 12 (2020) 150, M. Laine JHEP 06 (2021) 139

Quarkonium in medium

- Quarkonium in QGP (environment energy scale πT)

$$M \gg \frac{1}{a_0} \gg \pi T \gg E \,, \quad \tau_R \gg \tau_E \sim 1/\pi$$

- HQ mass M, Bohr radius a_0, binding energy E, correlation time τ_E
- Quarkonium in fireball can be described by Limbland equation

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -i[H,\rho] + \sum_{n,m} h_{nm} \left(L_i^n \rho L_i^{m\dagger} - \frac{1}{2}\{L_i^{m\dagger} L_i^n, \rho\} \right)$$

- All terms depend on two free parameters κ and γ
- Described by adjoint correlator
- κ turns out to be the heavy quark diffusion coefficient and related to thermal width $\Gamma(1S) = 3a_0^2\kappa$
- γ is correction to the heavy quark-antiquark potential and related to mass shift $\delta M(1S) = 2a_0^2\gamma/3$
- Unquenched lattice measurements of $\Gamma(1S)$ and $\delta M(1S)$ available

Brambilla *et.al.*PRD96 (2017), Brambilla *et.al.*PRD97 (2018)

κ from perturbation theory

- From kinetic theory we can derive

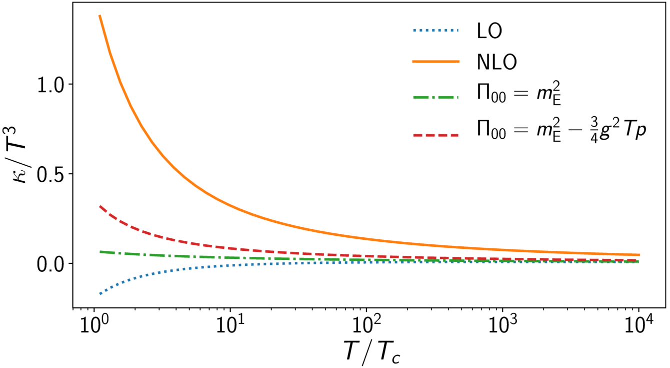
$$\kappa^{\mathrm{LO}} = \frac{g^4 C_{\mathrm{F}}}{12\pi^3} \int_0^{\infty} q^2 \mathrm{d}q \int_0^{2q} \frac{p^3 \mathrm{d}p}{(p^2 + \Pi_{00}^2)^2} \times N_{\mathrm{c}} n_{\mathrm{B}}(q)(1 + n_{\mathrm{B}}(q)) \left(2 - \frac{p^2}{q^2} + \frac{p^4}{4q^4}\right)$$

- Solution depends on assumptions
  - Is $m_{\mathrm{E}} \ll T$
  - How to expand the temporal gluon self-energy $\Pi_{00} \simeq m_{\mathrm{E}}$
- Alternatively from Kubo formula:

$$\frac{\kappa}{T^3} = \frac{g^4 C_f N_c}{18\pi} \left[\left(\ln \frac{2T}{m_{\mathrm{E}}} + \xi \right) + \frac{m_{\mathrm{E}}}{T} C \right], \quad \xi \simeq -0.64718$$

- Truncated LO: $C = 0$, NLO: $C \simeq 2.3302$
- $N_f = 0$ assumed here

κ from perturbation theory



- Clearly $m_{\rm E} \ll T$ is too strict assumption on small T
- Huge perturbative variation

  $\Rightarrow$ needs non-perturbative measurements
- Also huge scale dependence trough $m_{\rm E} = g(\mu)T$
- Here we have scale from NLO EQCD $\mu \sim 2\pi T$

Moore and Teaney PRC71 (2005), Caron-Huot and Moore JHEP02 (2008)

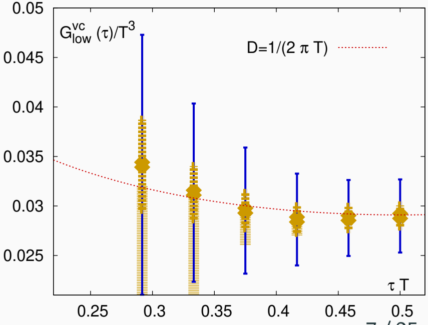
Current-Current correlator

$$\rho_V^{\mu\nu}(\omega) = \int_{-\infty}^{\infty} \mathrm{d}t e^{i\omega t} \int \mathrm{d}^3 x \left\langle \frac{1}{2} [\mathcal{J}^\mu(t,x), \mathcal{J}^\nu(0,0)] \right\rangle$$

- Above spectral function with heavy quark current correlator $\mathcal{J} = \bar{Q}\gamma^i Q$ has a pole at $\omega = -iD_x k^2$. Solving the pole position leads to Kubo formula with susceptibility χ:

$$D_x = \frac{1}{3\chi^{00}} \lim_{\omega\to\infty} \sum_{i=1}^{3} \frac{\rho_V^{ii}(\omega)}{\omega}, \ \chi^{00} = \beta \int \mathrm{d}x^3 \left\langle \frac{1}{2} \left[\mathcal{J}^0(t,0), \mathcal{J}^0(0,0)\right] \right\rangle$$

- Very narrow transport peak at zero
- Heavier quark $\Rightarrow$ narrower peak
- Very weak signal insensitive to D
  (Petreczky and Teaney PRD 73 (2006), Petreczky EPJC62 (2008))
- Better approach:
  Integrate out heavy quarks



Heavy Quark limit

- In the $M \to \infty$ limit we can define:

$$\kappa = \frac{1}{3T\chi} \sum_{i=1}^{3} \lim_{\omega \to 0} \left[\lim_{M \to \infty} \int_{-\infty}^{\infty} \mathrm{d}t \; e^{i\omega(t-t')} \int \mathrm{d}^3 x \left\langle \frac{1}{2} \left\{ \mathcal{F}^i(t,x), \mathcal{F}^i(0,0) \right\} \right\rangle \right]$$

- Where $\mathcal{F} = M \frac{\hat{\mathcal{J}}^i(0,0)}{\mathrm{d}t}$ and $\hat{\mathcal{J}}^i(x) = \bar{\psi}(x)\gamma^\mu \psi(x)$ is the heavy quark current
- The heavy quark force in static limit:

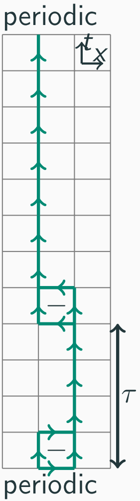
$$M \frac{\mathrm{d}\hat{\mathcal{J}}^i}{\mathrm{d}t} = \left\{ \phi^\dagger E^i \phi - \theta^\dagger E^i \theta \right\} + \phi^\dagger \left\{ \frac{[D^i, D^2 + \sigma \cdot B]}{2M} + \cdots \right\} \phi$$

- Where ϕ , θ are HQ and $\mathrm{H\bar{Q}}$ operators, E^i chromoelectric field
- Now the euclidean correlator is defined as:

$$\kappa = \sum_{i=1}^{3} \lim_{M \to \infty} \frac{\beta}{3\chi} \int \mathrm{d}t \mathrm{d}x^3 \left\langle \frac{1}{2} \left\{ \left[\phi^\dagger g E^i \phi - \theta^\dagger g E^i \theta \right] (t,x), \left[\phi^\dagger g E^i \phi - \theta^\dagger g E^i \theta \right] (0,0) \right\} \right\rangle$$

Casalderrey-Solana and Teaney PRD 74 (2006),
Caron-Huot *et.al.*JHEP04 (2009), Bouttefeux and Laine JHEP12 (2020)

Euclidean correlator



- Switch to Euclidean electric fields $G_{\mathrm{E}} =$

$$-\frac{1}{3T\chi}\sum_{i=1}^{3}\lim_{M\to\infty}\int \mathrm{d}^3x \left\langle \left[\phi^\dagger gE_i\phi - \theta^\dagger gE_i\theta\right](\tau,x)\left[\phi^\dagger gE_i\phi - \theta^\dagger gE_i\theta\right](0,0)\right\rangle$$

- After simplifying the propagators of ϕ and θ in $M\to\infty$:

$$G_{\mathrm{E}}(\tau) = -\frac{1}{3}\sum_{i=1}^{3}\frac{\langle \mathrm{Re}\,\mathrm{Tr}\,[U(\beta,\tau)gE_i(\tau,0)U(\tau,0)gE_i(0,0)]\rangle}{\langle \mathrm{Re}\,\mathrm{Tr}\,[U(\beta,0)]\rangle}$$

$$G_{\mathrm{B}}(\tau) = \sum_{i=1}^{3}\frac{\langle \mathrm{Re}\,\mathrm{Tr}\,[U(1/T,\tau)B_i(\tau,0)U(\tau,0)B_i(0,0)]\rangle}{3\,\langle \mathrm{Re}\,\mathrm{Tr}\,U(1/T,0)\rangle}$$

- Free of transport peak at zero
- Field strength tensor components need discretization
- Choose corner discretization for this study
- On lattice there is a self-energy contribution that generates a multiplicative renormalization

Following Caron-Huot *et.al.*JHEP04 (2009),

Connecting G_{E} to κ

$$G_{\mathrm{E}}(\tau) = \int_0^\infty \frac{\mathrm{d}\omega}{\pi}\rho(\omega)\frac{\cosh\left(\frac{\omega}{T}[\tau T - \frac{1}{2}]\right)}{\sinh\frac{\omega}{2T}} \qquad \kappa = \lim_{\omega\to 0}\frac{2T}{\omega}\rho(\omega)$$

- Euclidean correlator related to spectral function
- Needs inversion of integral equation
- We use simple procedure of modeling $\rho(\omega)$ and comparing to lattice data
  - $\rho(\omega)$ known at IR and UV
  - Connect IR and UV with i) step ii) line

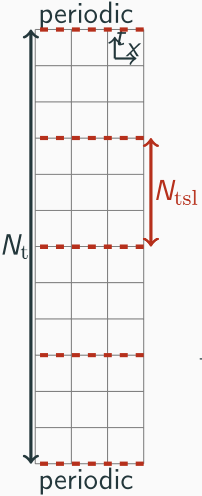
$$\rho^{\mathrm{step}}_{\mathrm{E,B}}(\omega, T) = \rho^{\mathrm{IR}}_{\mathrm{E,B}}(\omega, T)\,\theta(\Lambda-\omega) + \rho^{\mathrm{UV,T=0}}_{\mathrm{E,B}}(\omega, T)\,\theta(\omega-\Lambda)\,,$$

$$\begin{aligned}\rho^{\mathrm{line}}_{\mathrm{E,B}}(\omega, T) = {}& \rho^{\mathrm{IR}}_{\mathrm{E,B}}(\omega, T)\theta(\omega^{\mathrm{IR}}-\omega) + \\ & \left[\frac{\rho^{\mathrm{IR}}_{\mathrm{E,B}}(\omega^{\mathrm{IR}}, T) - \rho^{\mathrm{UV}}_{\mathrm{E,B}}(\omega^{\mathrm{UV}}, T)}{\omega^{\mathrm{IR}}-\omega^{\mathrm{UV}}}\left(\omega-\omega^{\mathrm{IR}}\right) + \rho^{\mathrm{IR}}_{\mathrm{E,B}}(\omega^{\mathrm{IR}}, T)\right] \\ & \times\theta(\omega-\omega^{\mathrm{IR}})\theta(\omega^{\mathrm{UV}}-\omega) + \rho^{\mathrm{UV}}_{\mathrm{E,B}}(\omega, T)\theta(\omega-\omega^{\mathrm{UV}})\,,\end{aligned}$$

- Related: γ not measured yet

$$\gamma = -\frac{1}{3N_c}\int_0^\infty \frac{d\omega}{2\pi}\frac{\rho(\omega)}{\omega} = -\int_0^\infty d\tau\, G_{\mathrm{E}}(\tau)$$

Algorithms



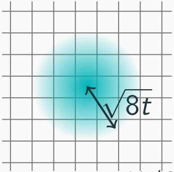
Multilevel

- Algorithm for quenched simulations
- At large N_t observable like Polyakov loop can have poor signal
- Idea: Divide the lattice to temporal slices of size N_{tsl}
- Update each sub-lattice independently keeping boundaries fixed
- Average over different boundary configurations

→ Allows reaching better statistics with less configurations

Gradient flow

- Automatically renormalizing smearing
- Can be generalized to unquenched
- Needs zero flow time limit



Lüscher and Weisz JHEP09 (2001)

Lattice parameters for the multilevel study

T/T_c	$N_t \times N_s^3$	β	$N_{\rm conf}$
1.1	12×48^3	6.407	1350
	16×48^3	6.621	2623
	20×48^3	6.795	2035
	24×48^3	6.940	2535
1.5	12×48^3	6.639	1801
	16×48^3	6.872	2778
	20×48^3	7.044	2081
	24×48^3	7.192	2496
2.2	12×48^3	6.940	1535

T/T_c	$N_t \times N_s^3$	β	$N_{\rm conf}$
3	12×48^3	7.193	1579
	16×48^3	7.432	1553
	20×48^3	7.620	1401
	24×48^3	7.774	1663
6	12×48^3	7.774	1587
	16×48^3	8.019	1556
	20×48^3	8.211	1258
	24×48^3	8.367	1430
2×10^4	12×48^3	14.792	1948

T/T_c	$N_t \times N_s^3$	β	$N_{\rm conf}$
10	12×48^3	8.211	1807
	16×48^3	8.458	2769
	20×48^3	8.651	2073
	24×48^3	8.808	2423
10^4	12×48^3	14.194	1039
	16×48^3	14.443	1157
	20×48^3	14.635	1139
	24×48^3	14.792	1375

- Quenched multilevel simulations

  Code from: Banerjee *et.al.* PRD85 (2012)

- 4 sublattices with 2000 updates

- Temperatures between $1.1\,T_c - 10^4\,T_c$

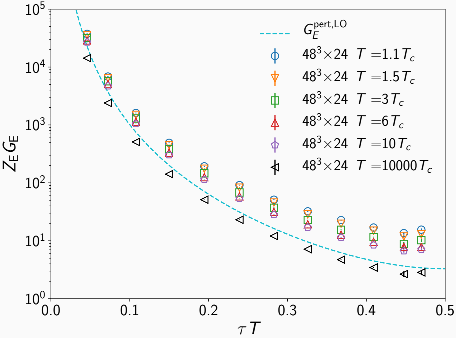
- Scale setting with

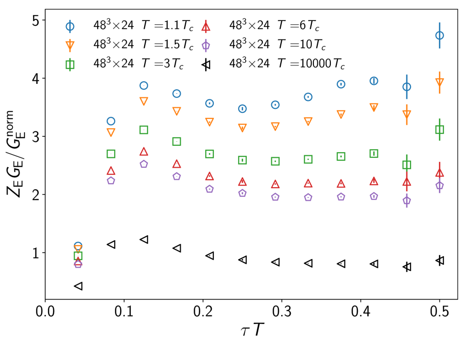
  (Francis *et.al.*PRD91 (2015))

- Other lattice results

  Meyer NJP13 (2011),
  Ding *et.al.*JPG38 (2011),
  Banerjee *et.al.* PRD85 (2012),
  Francis *et.al.* PRD92 (2015)
  Brambilla *et.al.* PRD102 (2020)
  Altenkort *et.al.* PRD103 (2021)
  Banerjee *et.al.* hep-lat/2204.14075
  Banerjee *et.al.* hep-lat/2206.15471

Lattice correlator





$$G_{\mathrm{E}}^{\mathrm{norm}} = \frac{G_{\mathrm{E}}^{\mathrm{pert,LO}}}{g^2 C_f} = \pi^2 T^4 \left[\frac{\cos^2(\pi\tau T)}{\sin^4(\pi\tau T)} + \frac{1}{3\sin^2(\pi\tau T)} \right]$$

- Chromoelectric field E needs discretization
- On Lattice E has non-physical self-energy contribution
  $Z_{\mathrm{E}} = 1 + g_0^2 \times 0.137718569\ldots + \mathcal{O}(g_0^4)$

(Christensen and Laine PLB02 (2016))

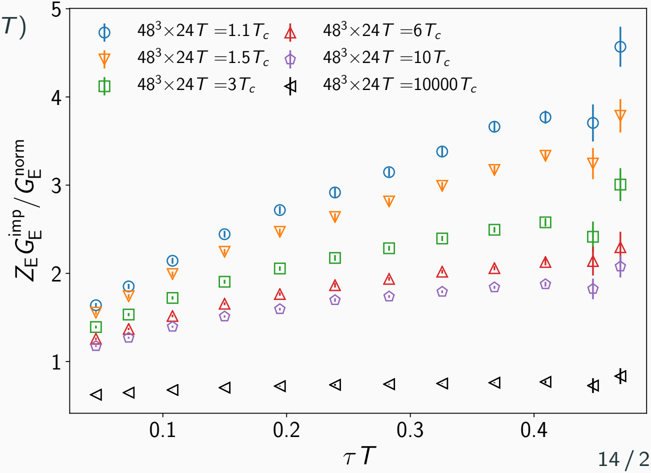
Caron-Huot *et.al.*JHEP04 (2009), Francis *et.al.*PoSLattice (2011)

Tree level improvement

$$G_{E,\,\mathrm{LOlat}} = \frac{1}{3a^4}\int_{-\pi}^{\pi}\frac{\mathrm{d}^3 q}{(2\pi)^3}\,\frac{e^{\bar{q}N_\tau(1-\tau T)}+e^{\bar{q}N_\tau \tau T}}{e^{\bar{q}N_\tau}-1}\,\frac{\tilde{q}^2}{\sinh\bar{q}}\,,$$

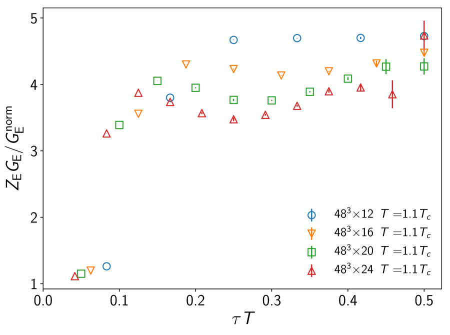
$$\bar{q} = 2\mathrm{arsinh}\left(\frac{\tilde{q}}{2}\right)\,,\quad \tilde{q}^2 = \sum_{i=1}^{3} 4\sin^2\left(\frac{q_i}{2}\right)$$

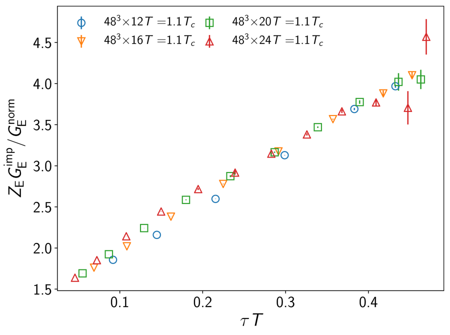
$$G_{E,\,\mathrm{pert}}(\bar{\tau}\bar{T}) = G_{E,\,\mathrm{LOlat}}(\tau T)$$



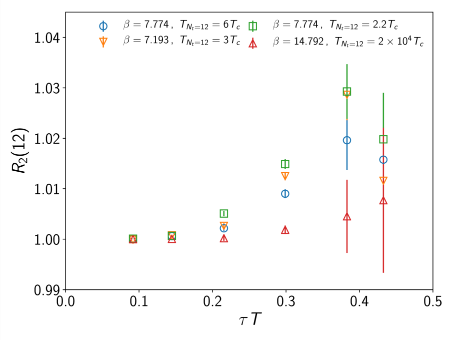
Francis *et.al.*PoSLattice (2011)

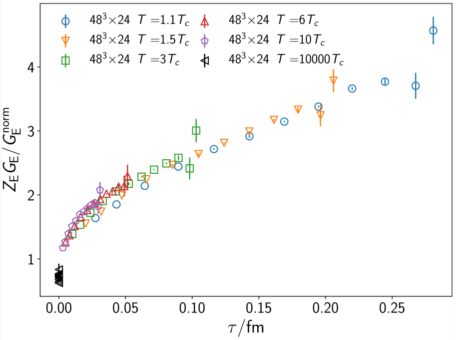
The effect of tree-level improvement





- Greatly decreases N_t dependence
- From now on we only use tree-level improved τT without further indication





$$R_2(N_t) = \frac{G_{\mathrm{E}}(N_t,\beta)}{G_{\mathrm{E}}^{\mathrm{norm}}(N_t)} \Big/ \frac{G_{\mathrm{E}}(2N_t,\beta)}{G_{\mathrm{E}}^{\mathrm{norm}}(2N_t)} \,.$$

- On small physical separation every T shares a scaling (apart from finite size effects)
- Thermal effect nonexistent for $\tau < 0.10$, then grow

Spectral function basics: LO

$$G_{\mathrm{E}}(\tau) = \int_0^\infty \frac{\mathrm{d}\omega}{\pi}\rho(\omega)\frac{\cosh\left(\frac{\omega}{T}[\tau T - \frac{1}{2}]\right)}{\sinh\frac{\omega}{2T}}$$

$$\kappa = \lim_{\omega\to 0}\frac{2T}{\omega}\rho(\omega)\,, \qquad \gamma = -\frac{1}{3N_c}\int_0^\infty \frac{d\omega}{2\pi}\frac{\rho(\omega)}{\omega}$$

- Assume simple behavior on IR ($\omega \ll T$):

$$\rho_{\mathrm{IR}}(\omega) = \frac{\kappa\omega}{2T}$$

- Perturbative behavior in UV in LO ($\omega \gg T$):

$$\rho_{\mathrm{UV}}^{\mathrm{LO}}(\omega) = \frac{g^2(\mu_\omega)C_F\omega^3}{6\pi}\,, \quad \mu_\omega = \max(\omega, \pi T)$$

- Use 5-loop running for the coupling

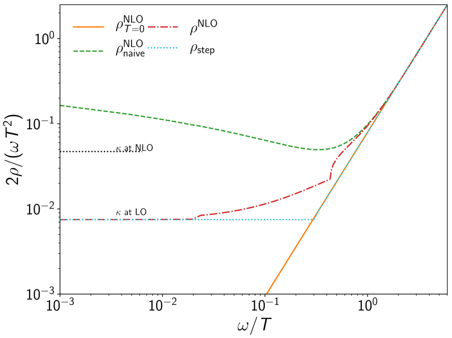
Spectral function: NLO

- NLO $\rho(\omega)$ known from (Burnier *et.al.*JHEP08 (2010))
- Full HTL resummed NLO ρ over corrects and gives negative κ at small T
- Naive QCD NLO $\rho(\omega)$ diverges logarithmically:

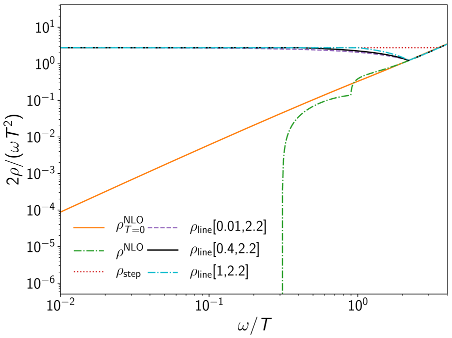
$$\begin{aligned}\rho_{\mathrm{QCD,naive}}(\omega) \quad = \quad & \frac{g^2 C_F \omega^3}{6\pi}\left\{1+\frac{g^2}{(4\pi)^2}\left[N_{\mathrm{c}}\left(\frac{11}{3}\ln\frac{\mu^2}{4\omega^2}+\frac{149}{9}-\frac{8\pi^2}{3}\right)\right]\right\} \\ & + \frac{g^2 C_F}{6\pi}\frac{g^2}{2\pi^2}\left\{N_{\mathrm{c}}\int_0^\infty \mathrm{d}q\, n_{\mathrm{B}}(q)\left[(q^2+2\omega^2)\ln\left|\frac{q+\omega}{q-\omega}\right| + q\omega\left(\ln\frac{|q^2-\omega^2|}{\omega^2}-1\right)\right.\right. \\ & \left.\left. + \frac{\omega^4}{q}\mathbb{P}\left(\frac{1}{q+\omega}\ln\frac{q+\omega}{\omega}+\frac{1}{q-\omega}\ln\frac{\omega}{|q-\omega|}\right)\right]\right\},\end{aligned}$$

- We will use the $T=0$ naive QCD $\rho(\omega)$ as our normalization
- set scale: UV: $\rho^{\mathrm{LO}}_{T=0}=\rho^{\mathrm{NLO}}_{T=0}$, IR: from NLO EQCD

Spectral function behavior



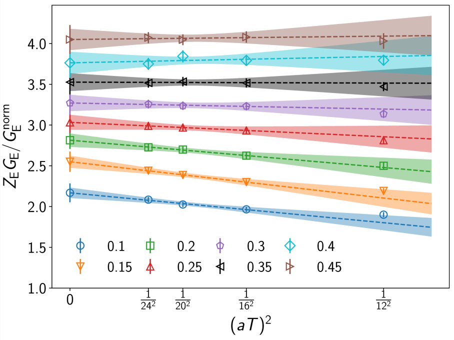
$T = 10\,T_c$

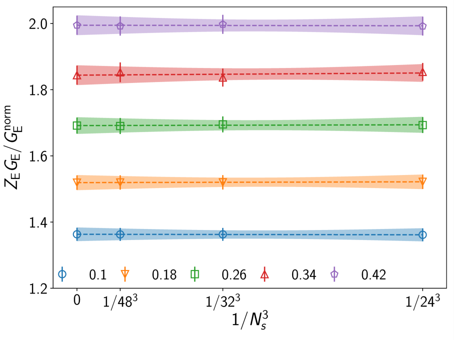


$T = 1.1\,T_c$

- NLO spectral function works only at very high temperatures
- Try different models for $\omega \sim T$ behavior
- Instead of inverting integral equation, compare to ansatz

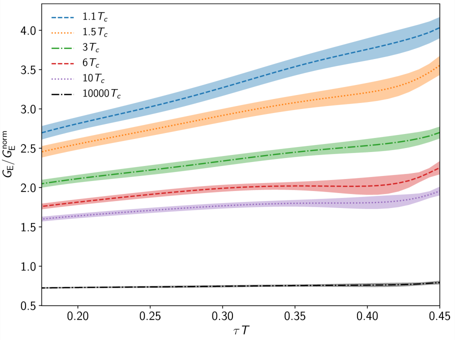
Continuum limit and finite size effects



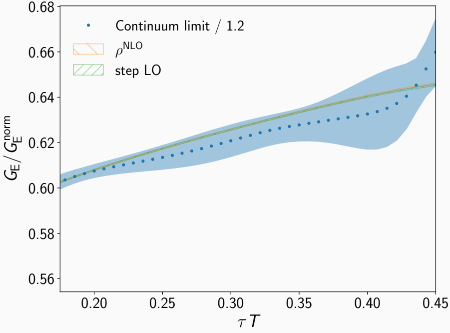


- Use 3 largest lattices for continuum limit
- Check systematics by including the $N_t = 12$ point
- $\chi^2/\mathrm{d.o.f.} < 5$ for $\tau T > 0.20$ when using 3 largest lattices (< 10 with $N_t = 12$)
- Finite size effects are negligible

Continuum limit



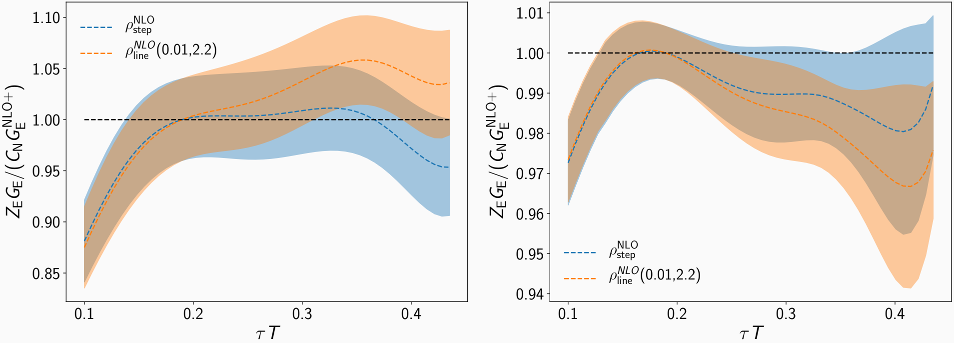
$T = 1.1\, T_c$



$T = 10^4\, T_c$

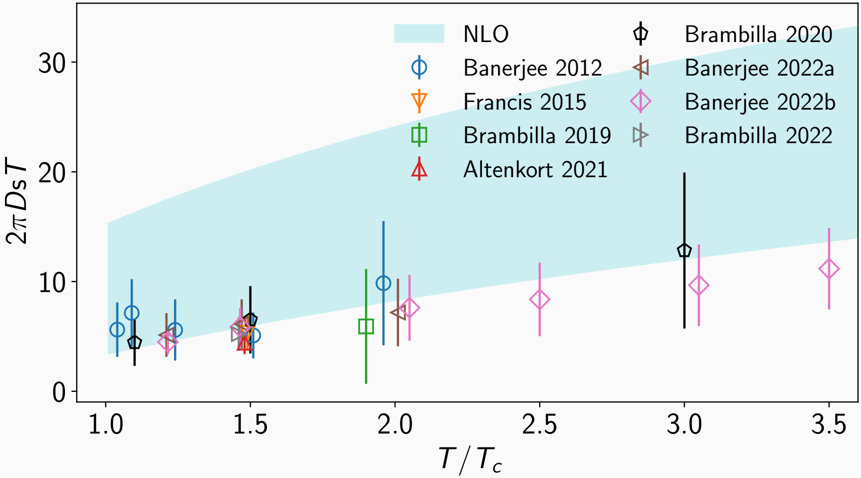
- Data needs additional normalization, do this at $\tau T = 0.19$
- Great agreement to perturbation theory at very high temperatures

κ extraction

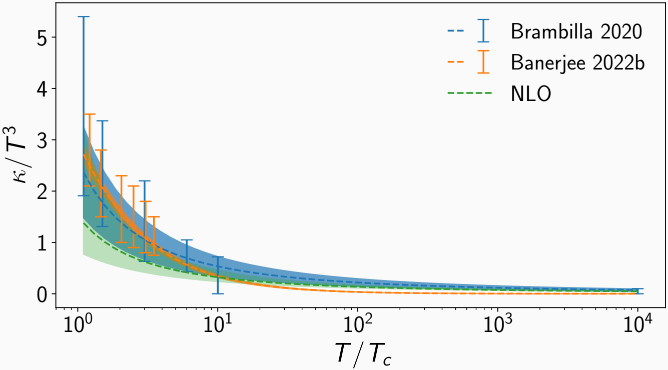


- Take continuum limit of the lattice data
- Normalize with different models for spectral function
- Extract κ as all values that normalize to unity in $0.19 \leq \tau T \leq 0.45$

Multilevel results for D_s



Temperature dependence of κ



- Measure κ in range: $T = 1.1 - 10^4\, T_\mathrm{c}$

$$\frac{\kappa^{\mathrm{NLO}}}{T^3} = \frac{g^4 C_\mathrm{F} N_\mathrm{c}}{18\pi}\left[\ln\frac{2T}{m_\mathrm{E}} + \xi + C\frac{m_\mathrm{E}}{T}\right].$$

- Fit temperature dependence to perturbatively motivated ansatz $C = 3.81(1.33)$
- In Banerjee *et.al.*[2206.15471] different form is used $2\pi T D_\mathrm{s} \sim \alpha + \beta(T/T_\mathrm{c} - 1)$

Brambilla, V.L, Petreczky, Vairo *et.al.*Phys.Rev.D 102 (2020)

Conclusions and Future prospects

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- Fit temperature dependence
- Agreement to perturbation theory at high T
- Agreement to previous results at small T
- Measured $1/M$ corrections (See Julians talk tomorrow)
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  - Measure γ from our data
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Thank you for your attention!