Inferring Phase transitions from Nonparametric EoSs

Isaac Legred

From Essick, Legred, Chatziioannou, Han, and Landry: 2023 (2305.07411)
Phase Transition is fairly common prediction at high densities

Guenther+2020
But Morphology is uncertain
Big Question:

What are the observable signatures of a PT?

(Is it possible to find a model-agnostic signature?)
Big Question: What are the observable signatures of a PT?

Rapid change in speed of sound?
   How to measure?

Change in tidal deformability?
   Hard to measure

Large change in Radius?

Potentially hadronic / GR effect?
Phase transition may not lead to sharp features/discontinuities in astro observables!
Big Question:
What are the observable signatures of a PT?

Phase transition may not lead to sharp features/discontinuities in astro observables!
Insight:
Look for observable which is universal within a single phase
Insight:
Look for observable which is universal within a single phase
Features in Moment of Inertia:

$$D_M^I \equiv \frac{d \log I / d \log p_c}{d \log M / d \log p_c}$$

Look for a minimum in the derivative of the moment of inertia

Minimum Exists $\Rightarrow$ Phase Transition exists

Minimum Location $\Rightarrow$ End of Phase Transition
Identifying the Onset

Look for running maximum in speed of sound

\[ D^I_M \equiv \frac{d \log I}{d \log p_c} \frac{d \log M}{d \log p_c} \]

\[ \frac{dp}{d\epsilon} \]

\[ c_s^2 = \left( \frac{dp}{d\epsilon} \right) \]
Algorithm

1. Identify a minimum in MOI derivative (end of transition)
2. Identify the global maximum in the speed of sound up to that density
3. Check certain criteria
4. If all criteria are satisfied accept running maximum
Can now “extract” PT parameters

- $p_{\text{onset}}$
- $\rho_{\text{onset}}$
- $M_{\text{onset}}$
- $p_{\text{end}}$
- $\rho_{\text{end}}$
- $M_{\text{end}}$

Quantify the strength of transition

$$\Delta (E/N) \equiv \left( \frac{\varepsilon}{n} \right)_{\text{end}} - \left( \frac{\varepsilon}{n} \right)_{\text{onset}}$$
Can we distinguish EoSs with phase transitions in data?

Most likely - no PT
Most likely - w/ PT

PSR: $\Delta(E/N) \leq 10\,\text{MeV}$
PSR: $\Delta(E/N) \geq 100\,\text{MeV}$

PGX: $\Delta(E/N) \leq 10\,\text{MeV}$
PGX: $\Delta(E/N) \geq 100\,\text{MeV}$
## Inference - Search

### $1.1 - 1.6 M_\odot$

**$\mathcal{D}_M^I$ Features**

<table>
<thead>
<tr>
<th>$\min \Delta(E/N)$ [MeV]</th>
<th>max $\mathcal{L}^{n&gt;0}_{n=0}(\text{PGX})$</th>
<th>$\mathcal{B}^{n&gt;0}_{n=0}(\text{PGX})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.57</td>
<td>1.043 ± 0.020</td>
</tr>
</tbody>
</table>

**Some evidence against strong transitions**

| 50                        | 0.49           | 0.463 ± 0.013  |

| 100                       | 0.26           | 0.152 ± 0.009  |

**No evidence against “weak” transitions**
Inference - Search (Injections)

\[ \Delta(E/N) > 50 \text{ MeV} \quad 1.1 - 1.6M_\odot \]

100 GW Events \rightarrow \text{Injected EoS w/ PT} \Rightarrow \text{Some evidence (not conclusive)}

\[ M_{\text{onset}} \sim 1.5 \]

\text{DBHF}_2507

\text{DBHF}_3504
Inference - Characterization

Feature Extraction

Distribution on EoSs

\[ \Delta(E/N) \text{ [MeV]} \]

- \( M \cap (0.8, 1.1) \)
- \( M \cap (1.1, 1.6) \)
- \( M \cap (1.6, 2.3) \)
- prior(\( M \cap (0.8, 1.1) \))
- prior(\( M \cap (1.1, 1.6) \))
- prior(\( M \cap (1.6, 2.3) \))

Distribution on PT parameters
Characterization (Injections)

Painfully slow convergence!
What can we measure?
Conclusions:

• We have developed a phenomenological scheme to extract PTs from arbitrary EoSs

• We can use this to constrain the parameters of a real PT inside neutron stars

• GWs will be effective probe of tidal parameters at A+ sensitivity

• LIGO GWs likely not informative enough to settle PT question

• Future work includes better prior specification