Nuclear matter, Neutron Stars
&
Correlations

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INT
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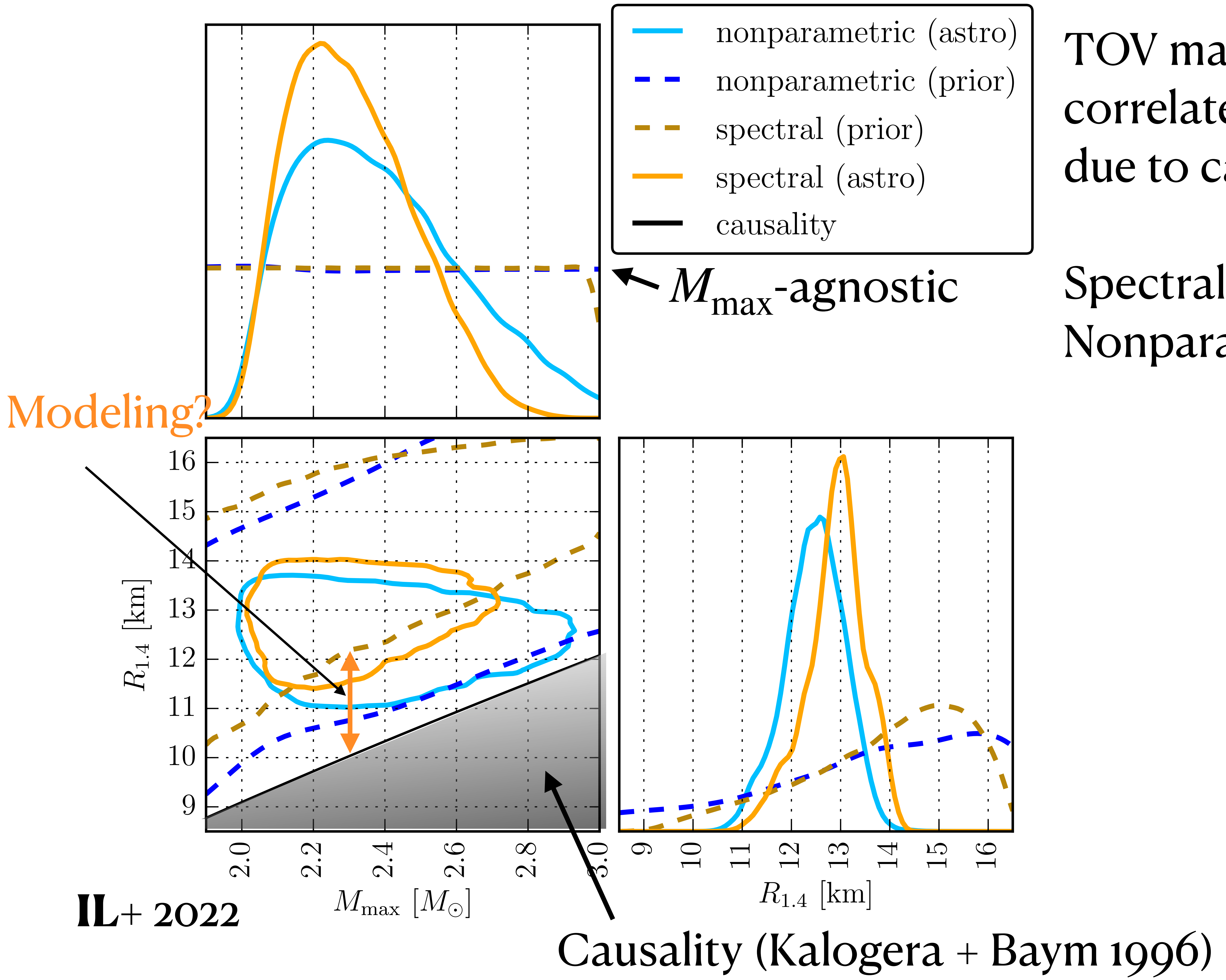
10.1103/PhysRevD.105.043016
https://arxiv.org/abs/2201.06791

Different Priors, Different Results



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TOV maximum mass and radius of a 1.4 solar mass NS are correlated among equation of state candidates due to causality

Spectral model sees a "tighter correlation" than the Nonparametric model — not likely due to causality!

Implicit Correlations - Mock Data

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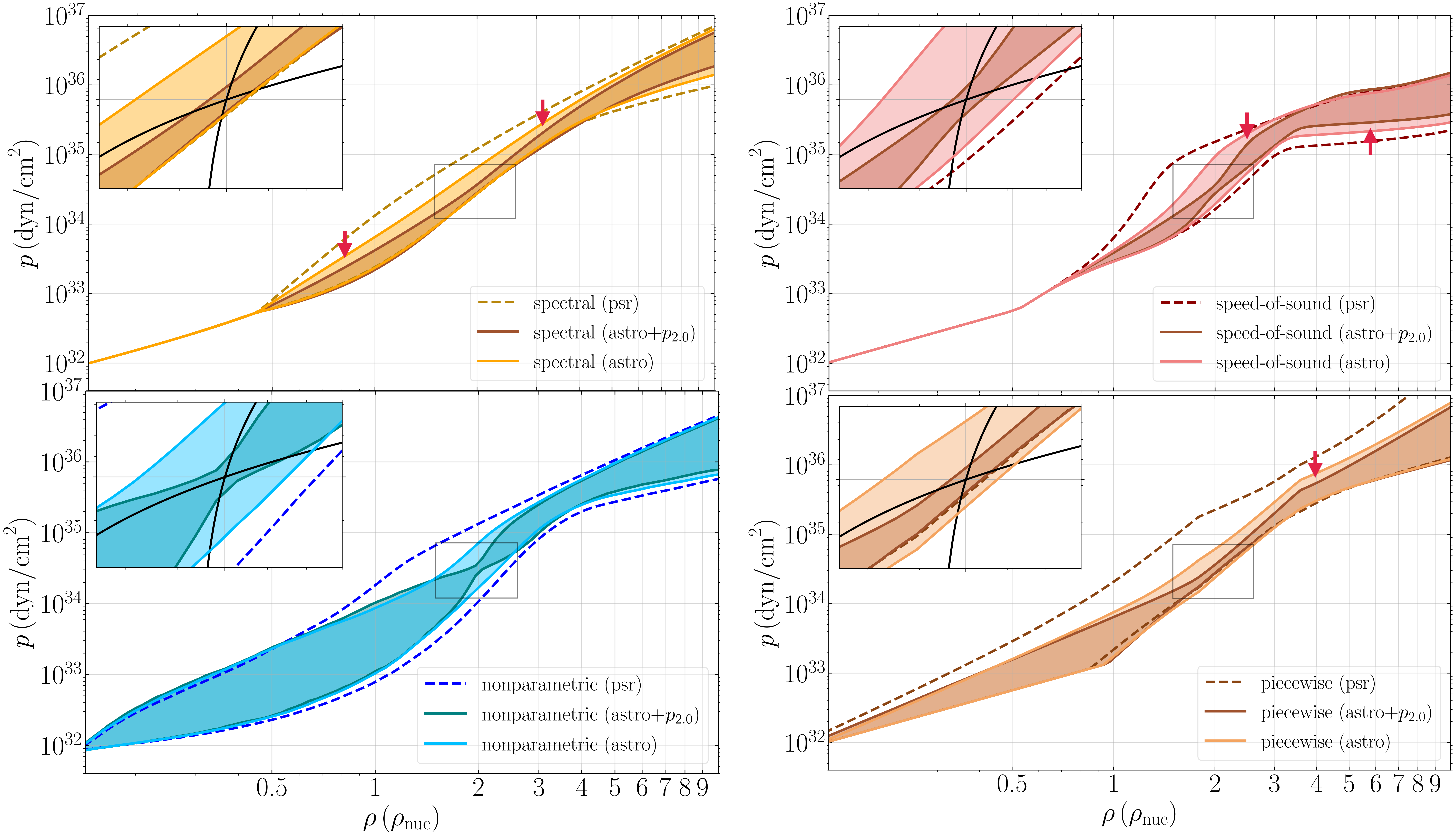
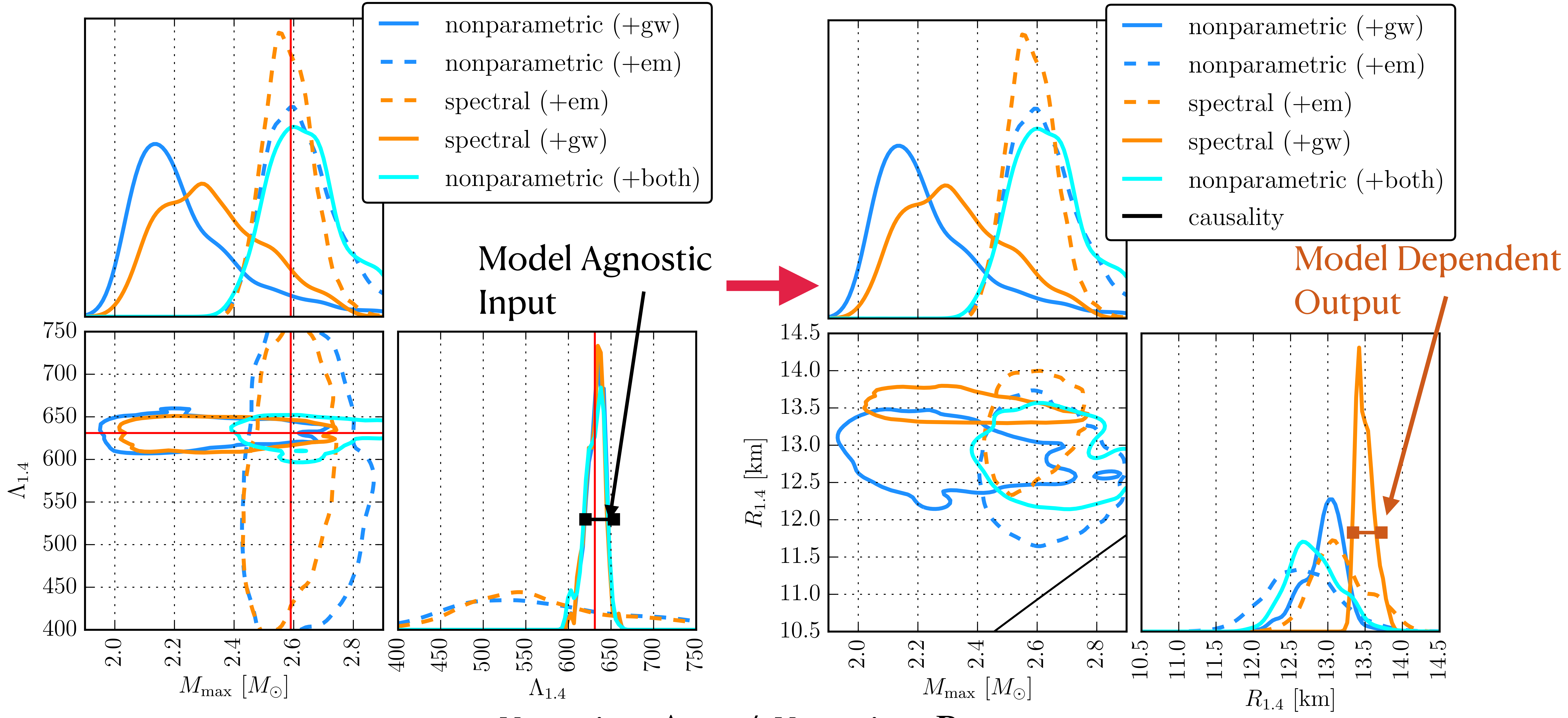


So What?

"Preliminary"



Knowing $\Lambda_{1.4} \neq$ Knowing $R_{1.4}$

Conclusions

- Phenomenological models of the nuclear equation of state can build in (often hidden) correlations due to the functional form of the EoS

- Nonparametric models (such as the Gaussian Process model), can provide more flexibility in inference of the EoS (but do not guarantee it)

- Need to be very careful when talking about translating constraints between variables, both micro and macroscopically

Inferring the EoS — In practice

- Want to establish a probability distribution on candidate equations of state given observed astrophysical data

Equation of state candidate

$$P(\varepsilon_i \,|\, d) = P(\varepsilon \,|\, d_1, d_2, \ldots) \propto P(d_1, d_2, \ldots \,|\, \varepsilon_i) \times \boxed{P(\varepsilon_i)}$$

Prior

Astrophysical data

Inferring the EoS — In practice

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Equation of state candidate

$$P(\varepsilon_i \,|\, d) = P(\varepsilon \,|\, d_1, d_2, \ldots) \propto P(d_1, d_2, \ldots \,|\, \varepsilon_i) \times P(\varepsilon_i)$$

Astrophysical data

Phenomenological

Parametrize a *functional form* (i.e. Spectral, Piecewise-polytrope)

Nonparametric methods, i.e. Gaussian process (GP)

Tabulated models from nuclear theory

Nonparametric: Gaussian Process

Gaussian Process Regression (Landry and Essick 2018)

Tabulate a draw $\phi(p_i) = \ln(1/c_s^2(p_i) - 1)$ @ Pressures p_i from a multivariate Gaussian distribution

Parameters for the covariance kernel are chosen to Control "shape" of EoS distribution

Model-Agnostic Prior (broadest range of models)

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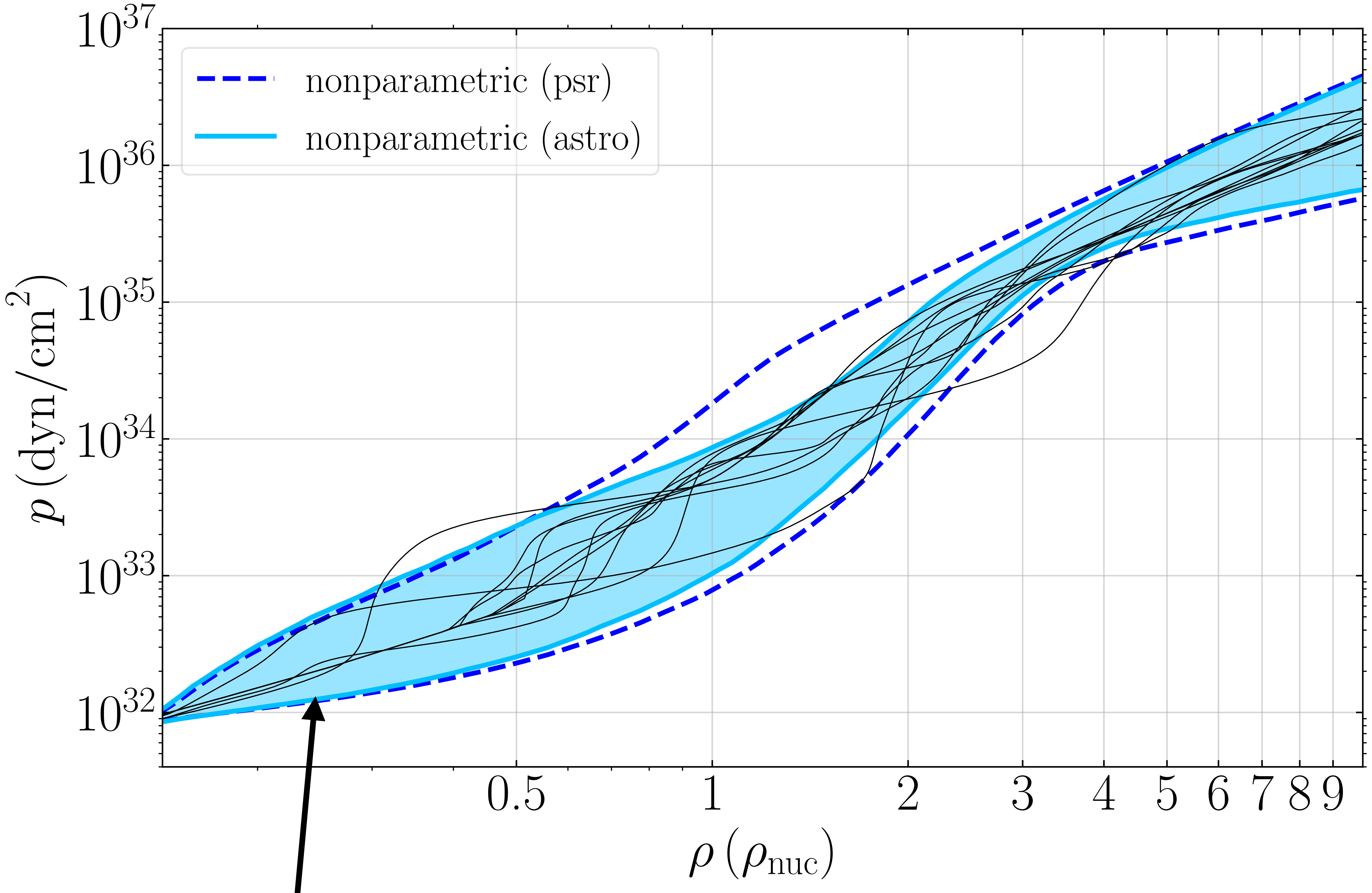


90% credible interval for $p(\rho)$
psr -> just heavy pulsar mass measurements
astro -> Heaviest pulsar, 2 NICER x-ray, 2 GWs

Parametric

Spectral (Lindblom 2010)

Parametrize the adiabatic index

$$p(\rho) = \rho^{\Gamma(x)} \qquad \Gamma(x) = \sum_{i=0}^{n} \gamma_i \left(\log(x)\right)^i$$

Piecewise-polytrope (Read 2008)

A polytrope with multiple segments

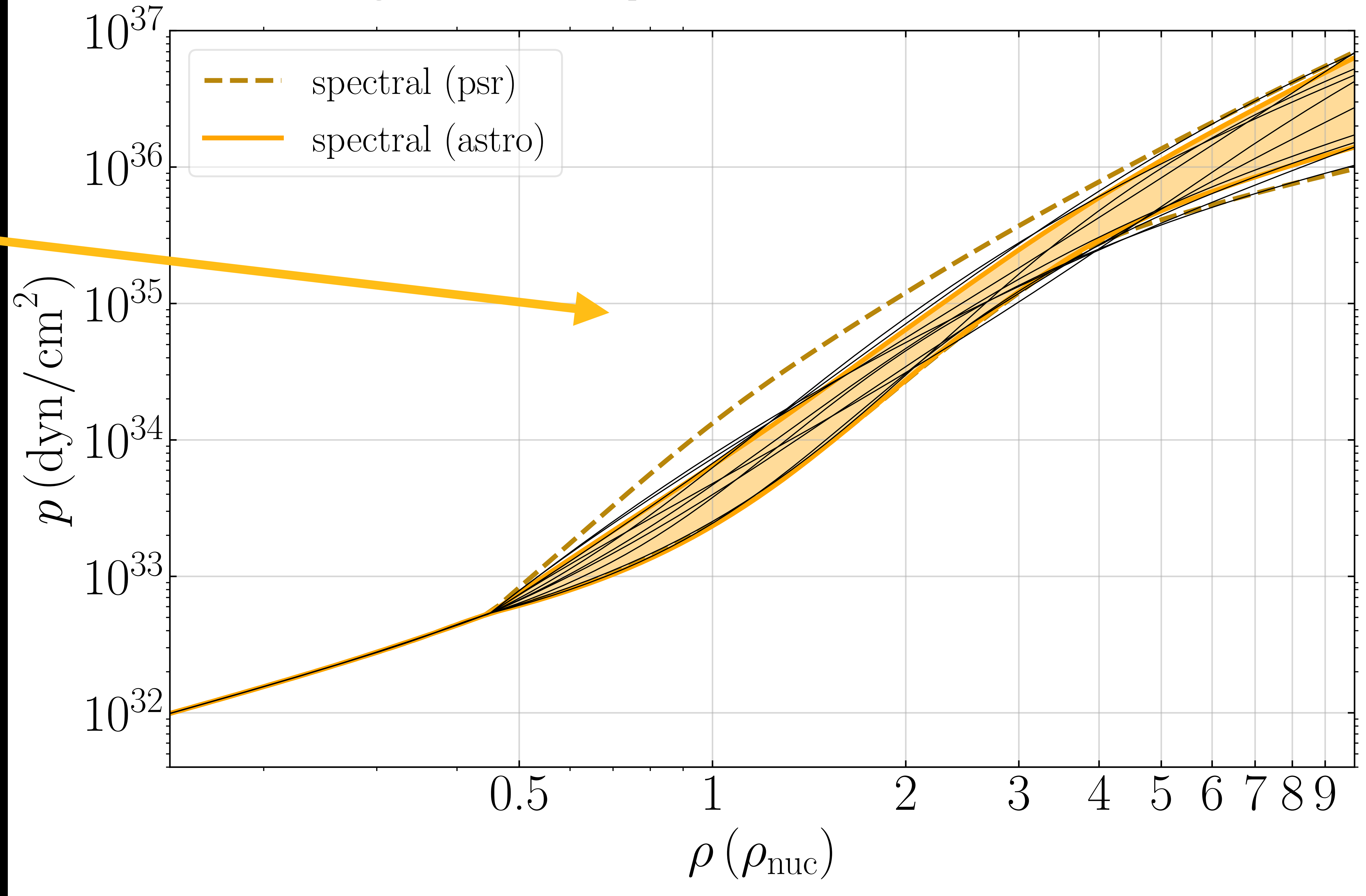
$$p(\rho) = \begin{cases} K_1 \rho^{\Gamma_1} & : \rho < \rho_1 \\ K_2 \rho^{\Gamma_2} & : \rho_1 < \rho < \rho_2 \\ K_3 \rho^{\Gamma_3} & : \rho_2 < \rho \end{cases}$$

Direct **speed-of-sound** (Greif 2018)

A bump in the speed of sound before asymptotic behavior

$$\frac{c_s^2(z)}{c^2} = a_1 e^{-\frac{1}{2}(z-a_2)^2/a_3^2} + a_6 + \frac{\frac{1}{3} - a_6}{1 + e^{-a_5(z-a_4)}}$$

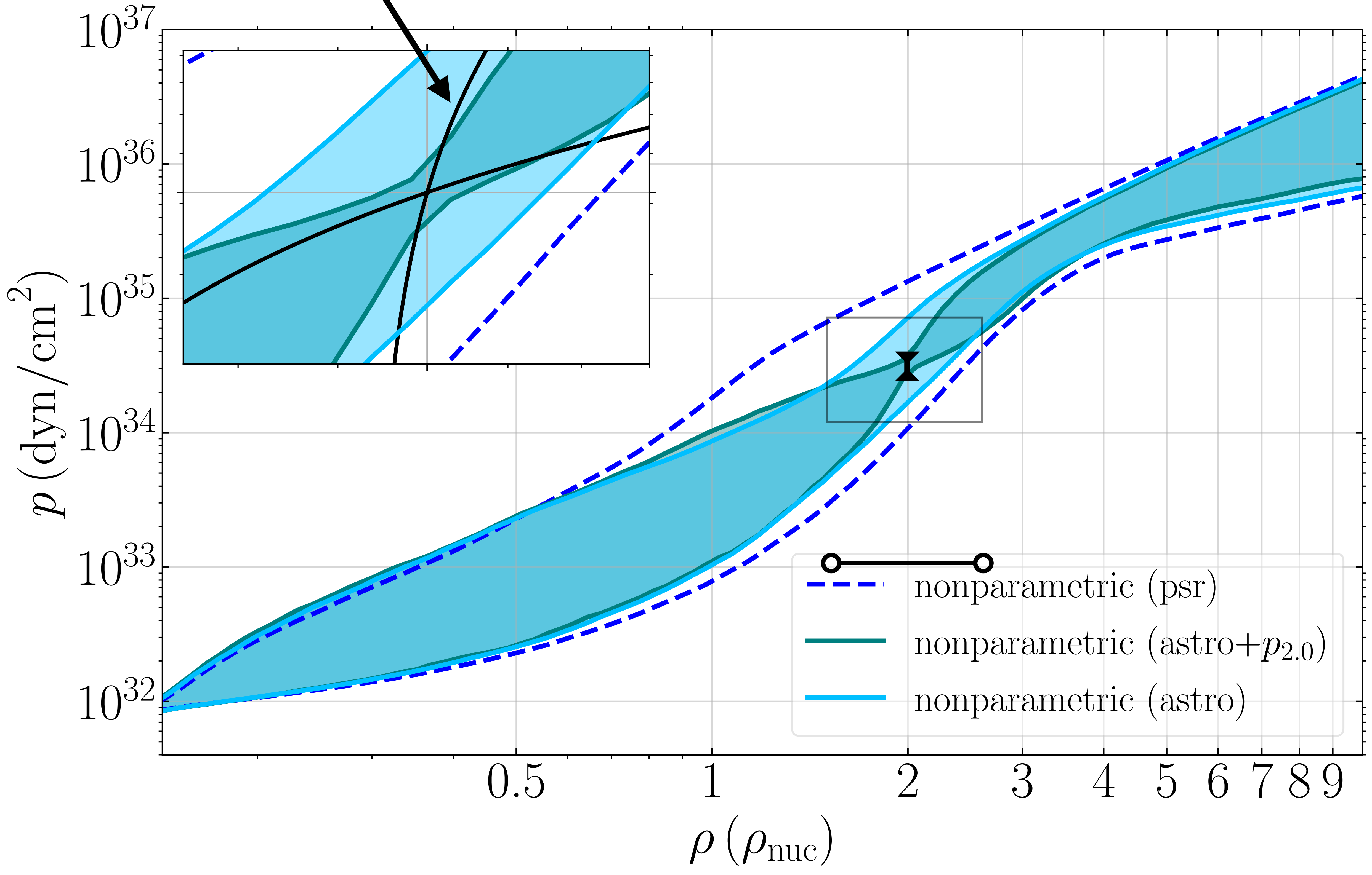
E.g. for the Spectral Parametrization



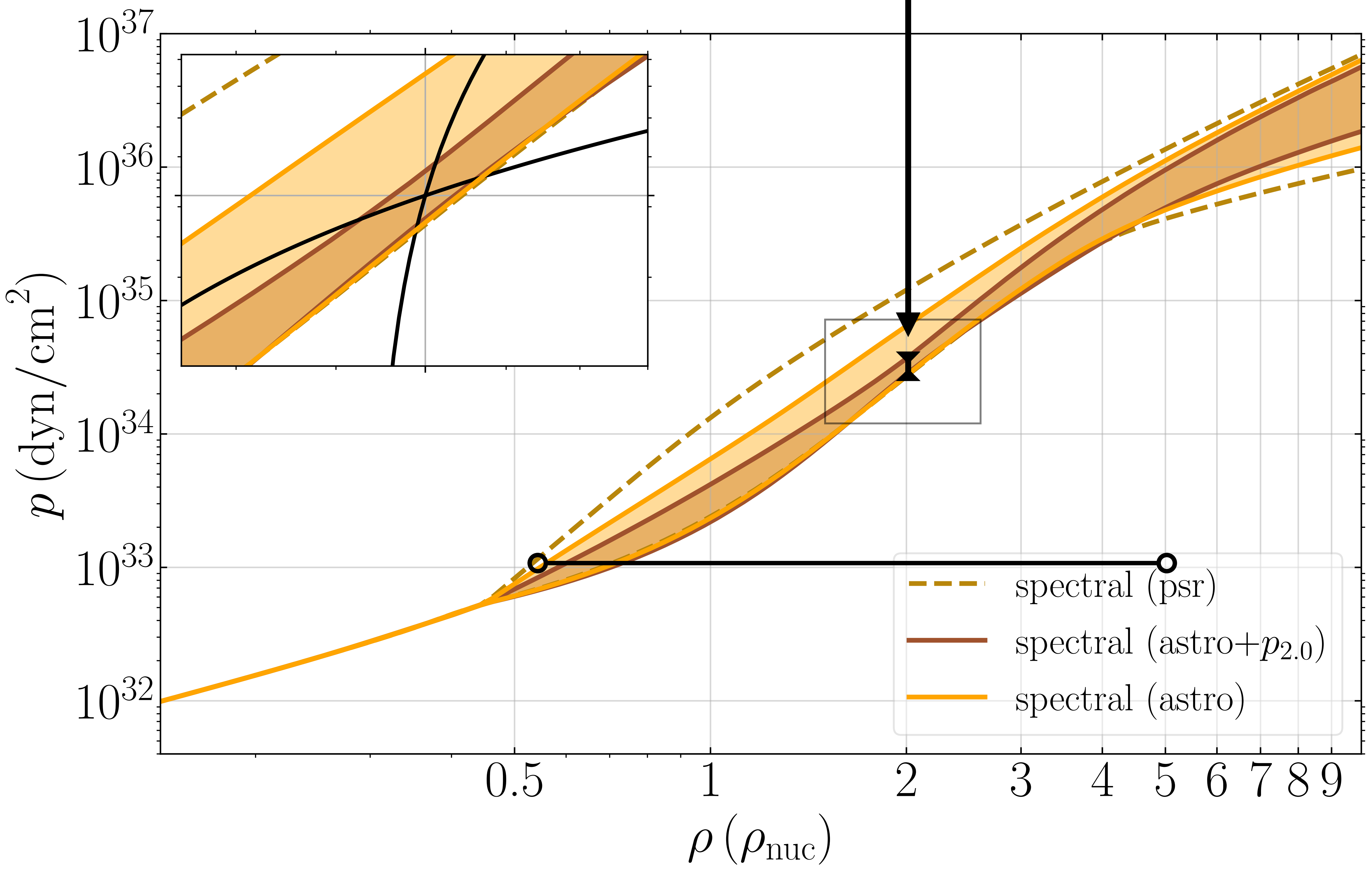
Correlations

Correlations between astro observables <=> Correlations between density scales

Approximate causality and stability limits



Tight constraint a $2\rho_{\mathrm{nuc}}$



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Implicit Correlations

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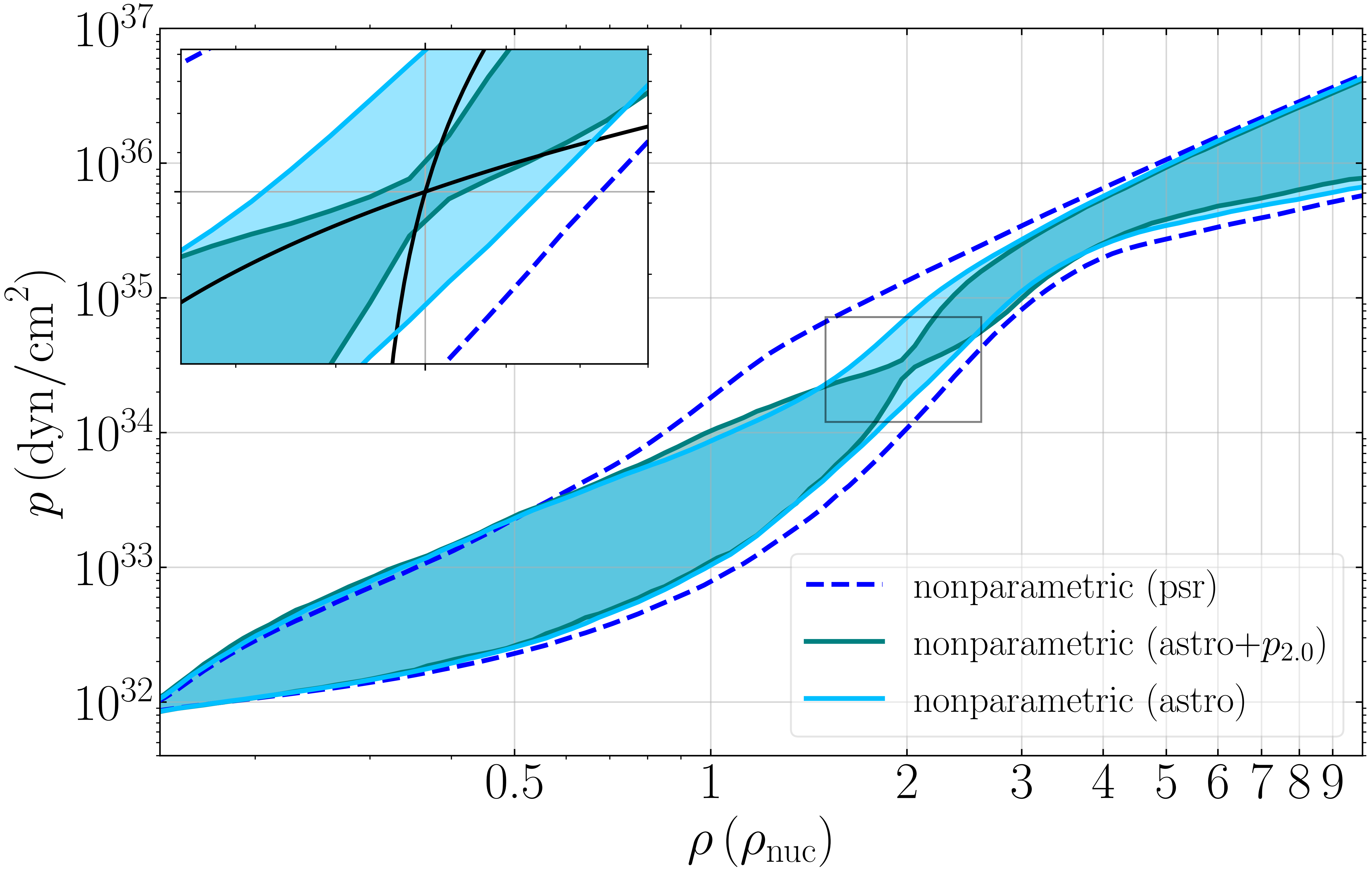

Quantifying correlations — Mutual Information

How much information is gained about other density Scales by knowing the EoS at some fixed density

$$I(p_a, p_b) \equiv \int dp_a dp_b P(p_a, p_b) \ln\left(\frac{P(p_a, p_b)}{P(p_a)P(p_b)}\right)$$

Implicit Correlations

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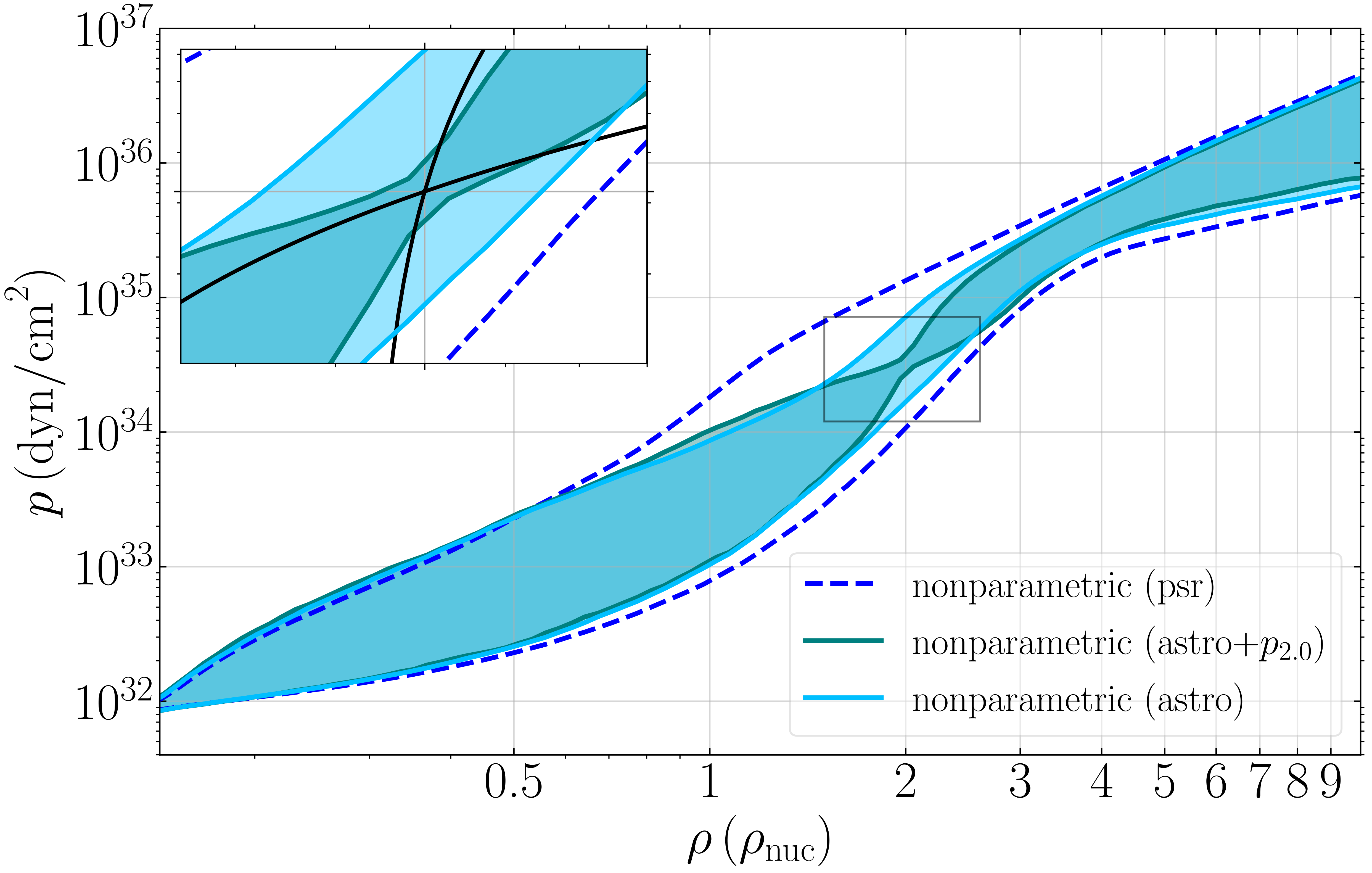

Quantifying correlations — Mutual Information

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Also a K-L divergence!

$$I(p_a, p_b) = \int dp_a P(p_a) \int dp_b P(p_b \mid p_a) \ln\left(\frac{P(p_b \mid p_a)}{P(p_b)}\right)$$

Difference in knowledge about p_b after learning p_a

Changing this analogous to adding a tight Pressure “mock-measurement”

Implicit Correlations

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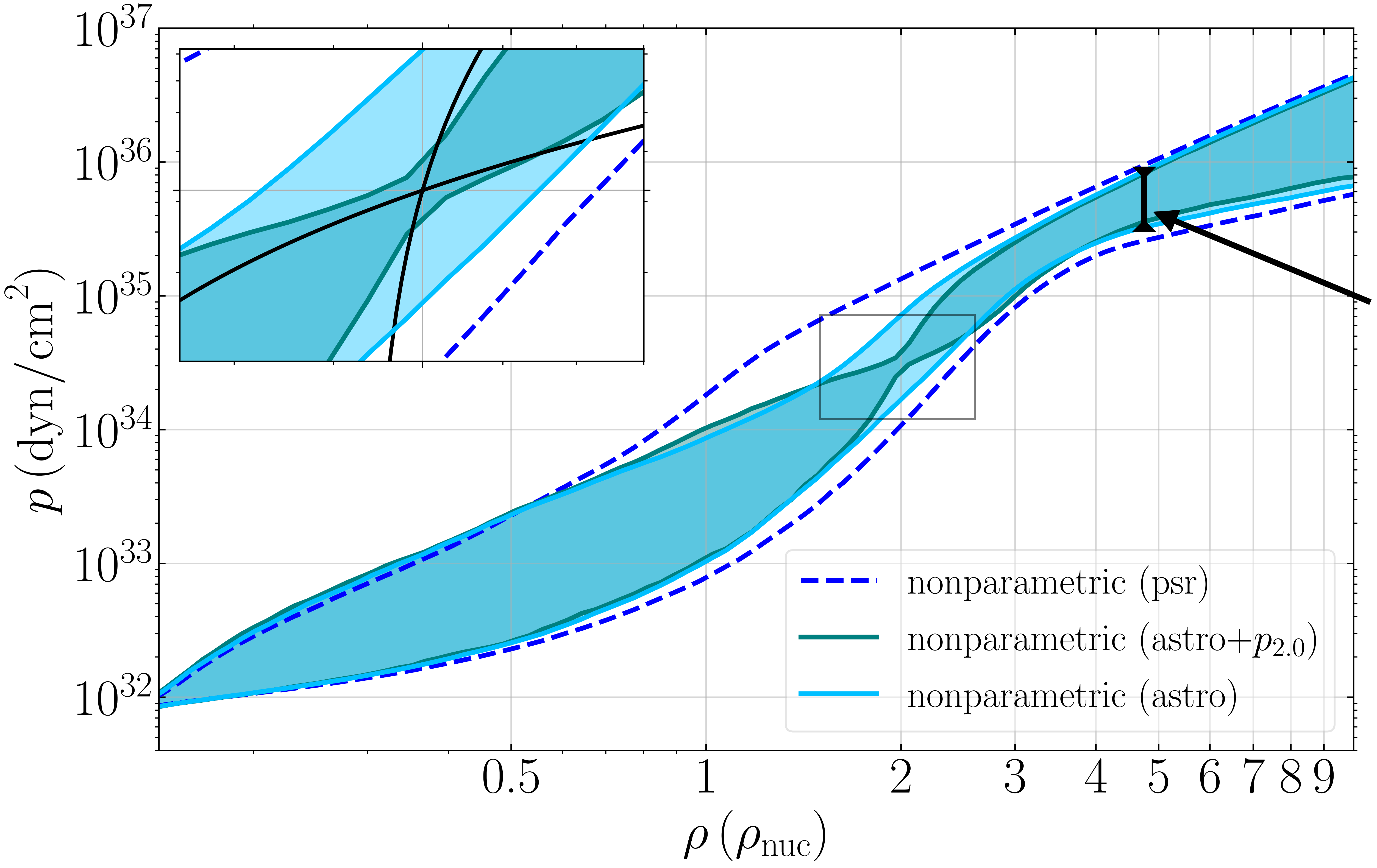


$$I(p_a, p_b) = \int dp_a P(p_a) \int dp_b P(p_b \,|\, p_a) \ln\left(\frac{P(p_b \,|\, p_a)}{P(p_b)}\right)$$

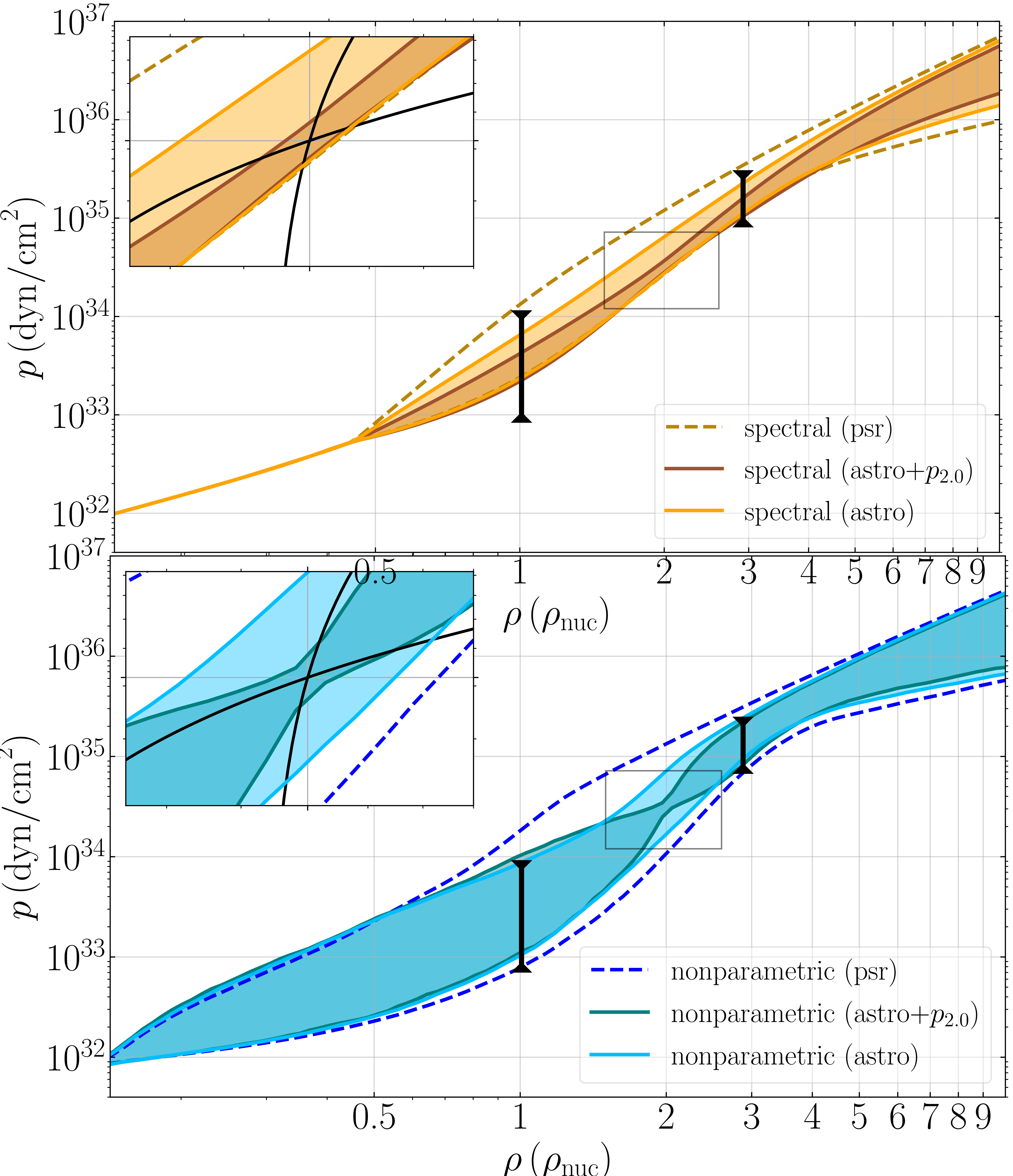
Caveats!

Scales with overall uncertainty of marginal distributions

Want to keep I small even with large entropy in Marginal distributions $P(p_a)$, …

Implicit Correlations

IL+ 2022



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Caveats!

Scales with overall uncertainty of marginal distributions

Want to keep I small even with large entropy in Marginal distributions $P(p_a)$, …

$$I\left(\ln(p_{1.0}), \ln(p_{1.5}), \ln(p_{2.0}), \ln(p_{3.0}), \ln(p_{4.0})\right)$$

	PSR	Astro	Astro+$p_{2.0}$
Nonparametric	3.7	3.1	2.9
Spectral	6.6	5.5	4.7
Polytrope	5.7	4.6	3.8
Speed of sound	5.0	4.7	4.3

Simulated Astrophysical Data

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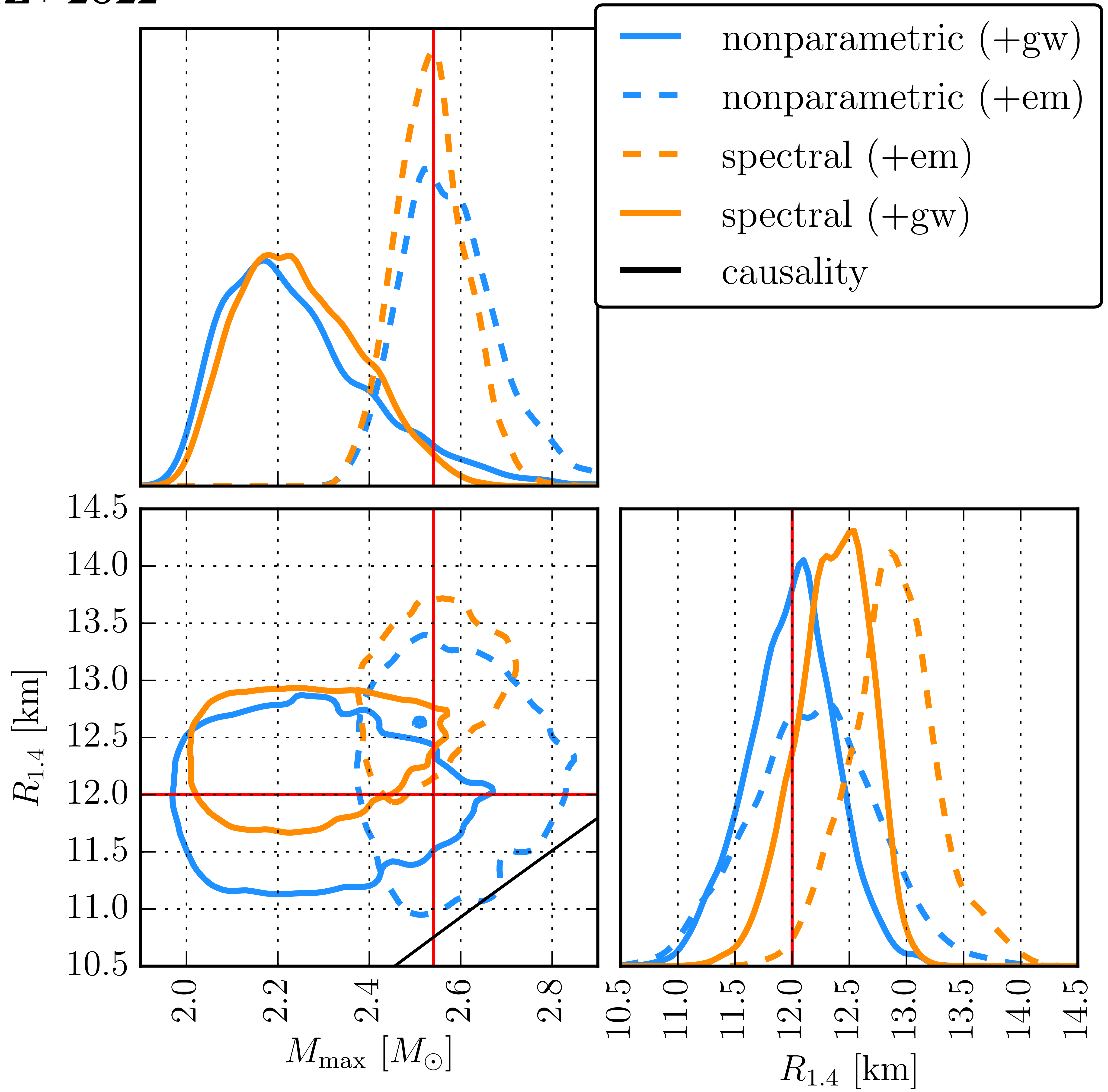
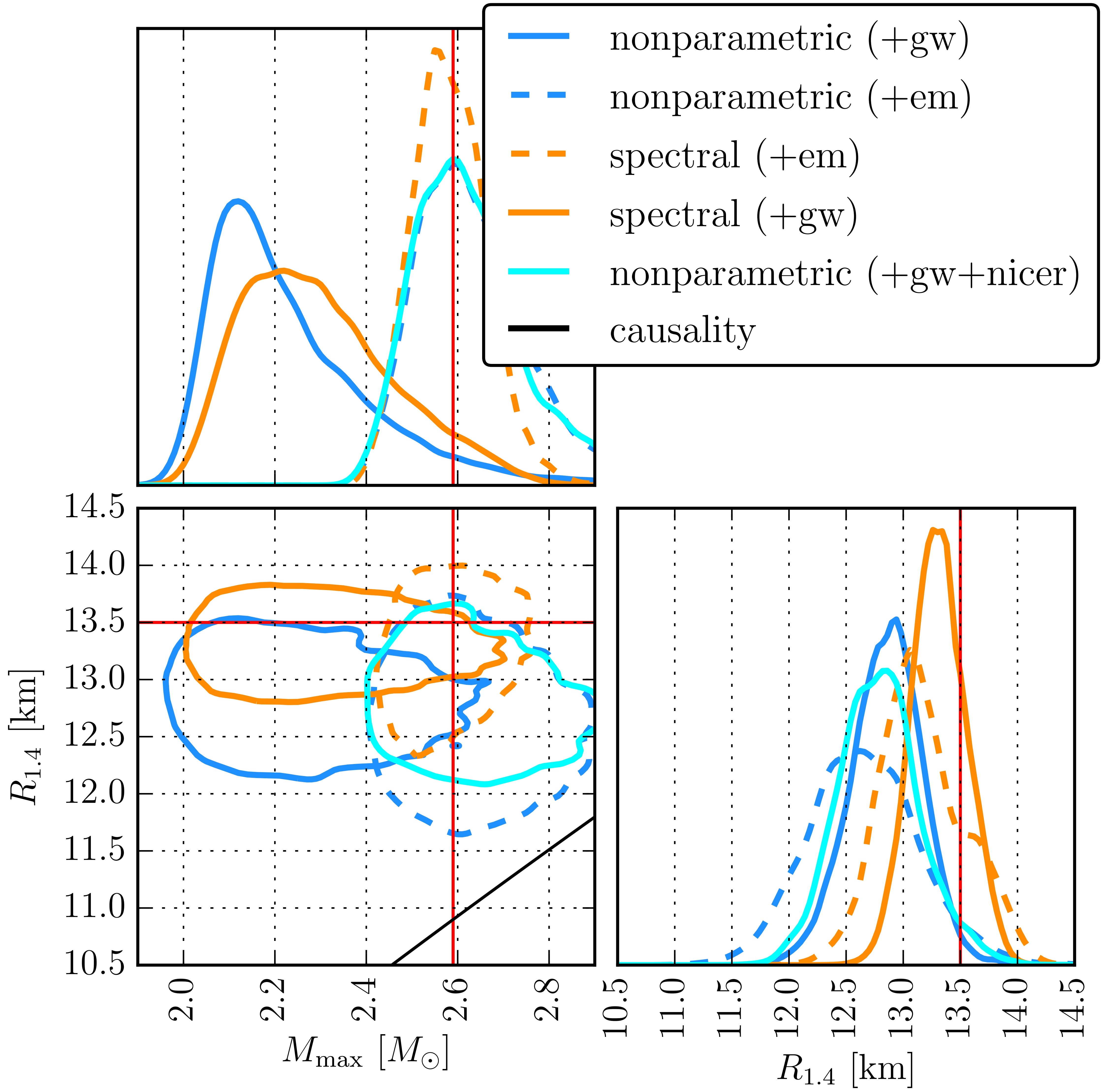


We inject gravitational-wave (gw) and x-ray-radio (em) observations on top of existing constraints

We intentionally choose an EoS that we expect the Spectral model to fail to recover

Gives a sense of tension that may arise from combining constraints using models with unphysical correlations

Simulated Astrophysical Data



We inject gravitational-wave (gw) and x-ray-radio (em) observations on top of existing constraints

Inverse Problem : Spectral Eos -> NP analysis

Slow convergence, but no bias

Modified parametric priors

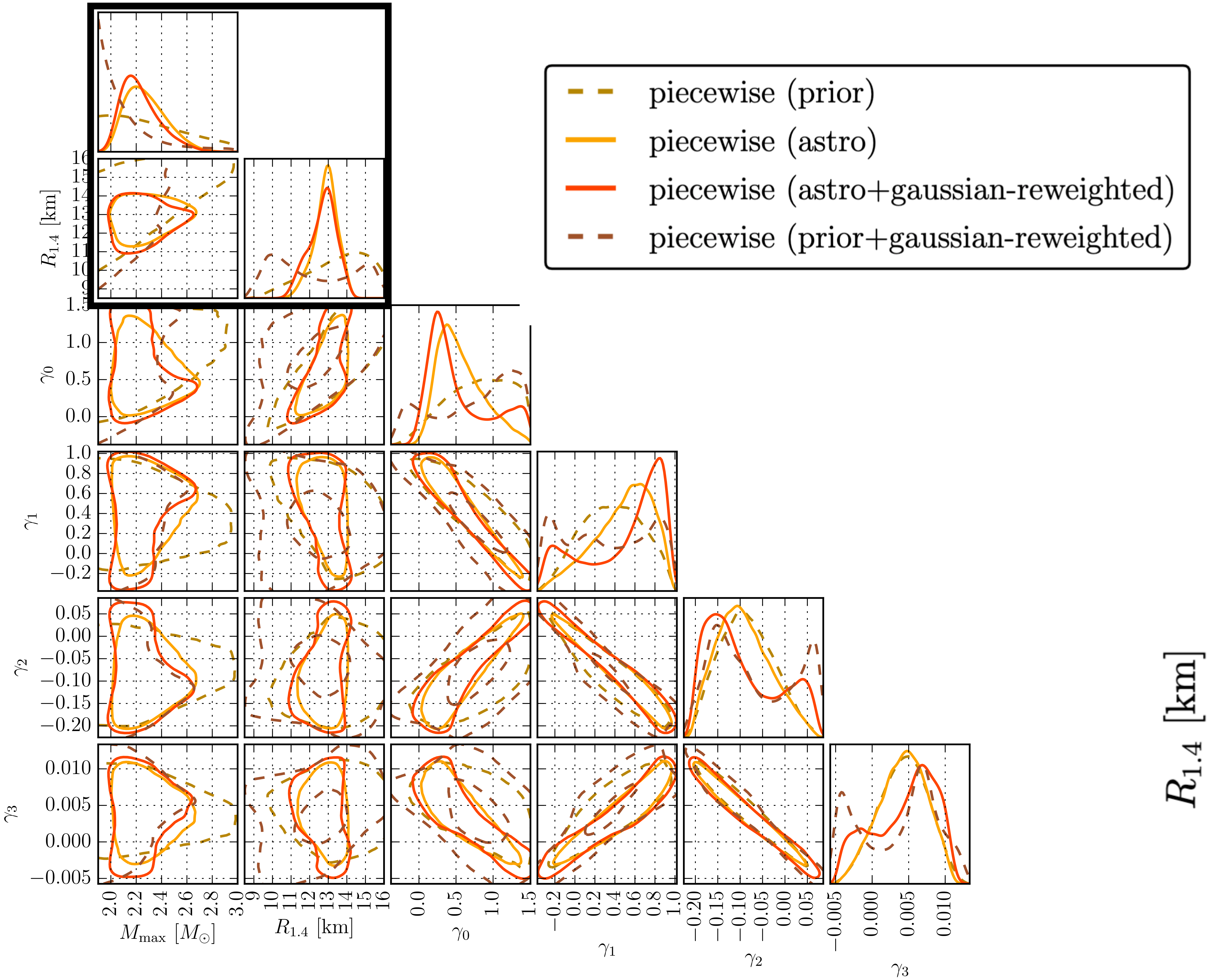
Why not just modify the parametric models to get more flexibility?

E.g.

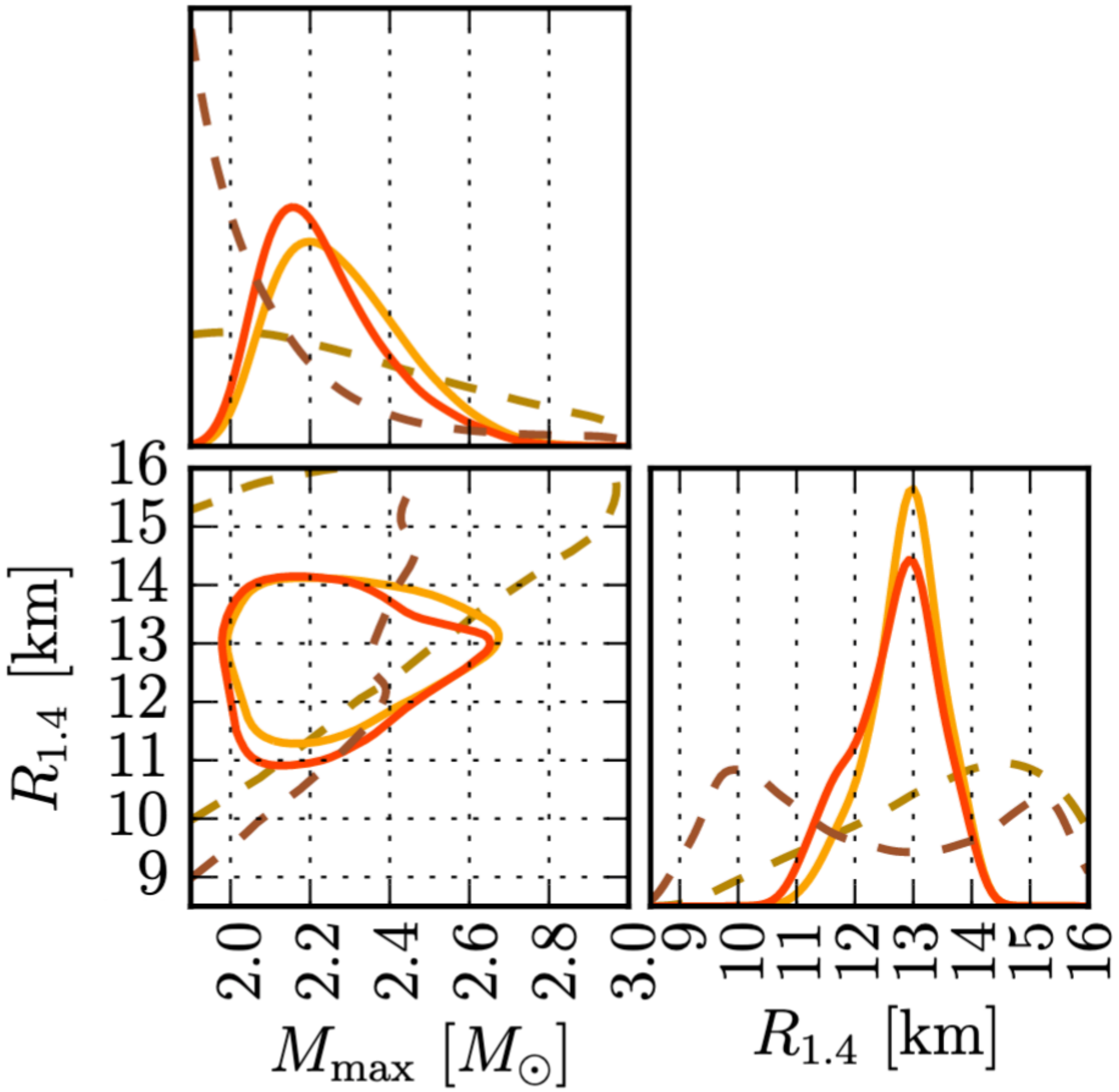
$$p(\rho) = \rho^{\Gamma}; \quad \Gamma(p) = \sum_{i=0}^{3} \gamma_i \log(p/p_0)^i + \text{more terms}$$

IL+ 2022

Modified parametric priors



We find changing the
Prior on parameters doesn't
Remove the correlations



Modified parametric priors

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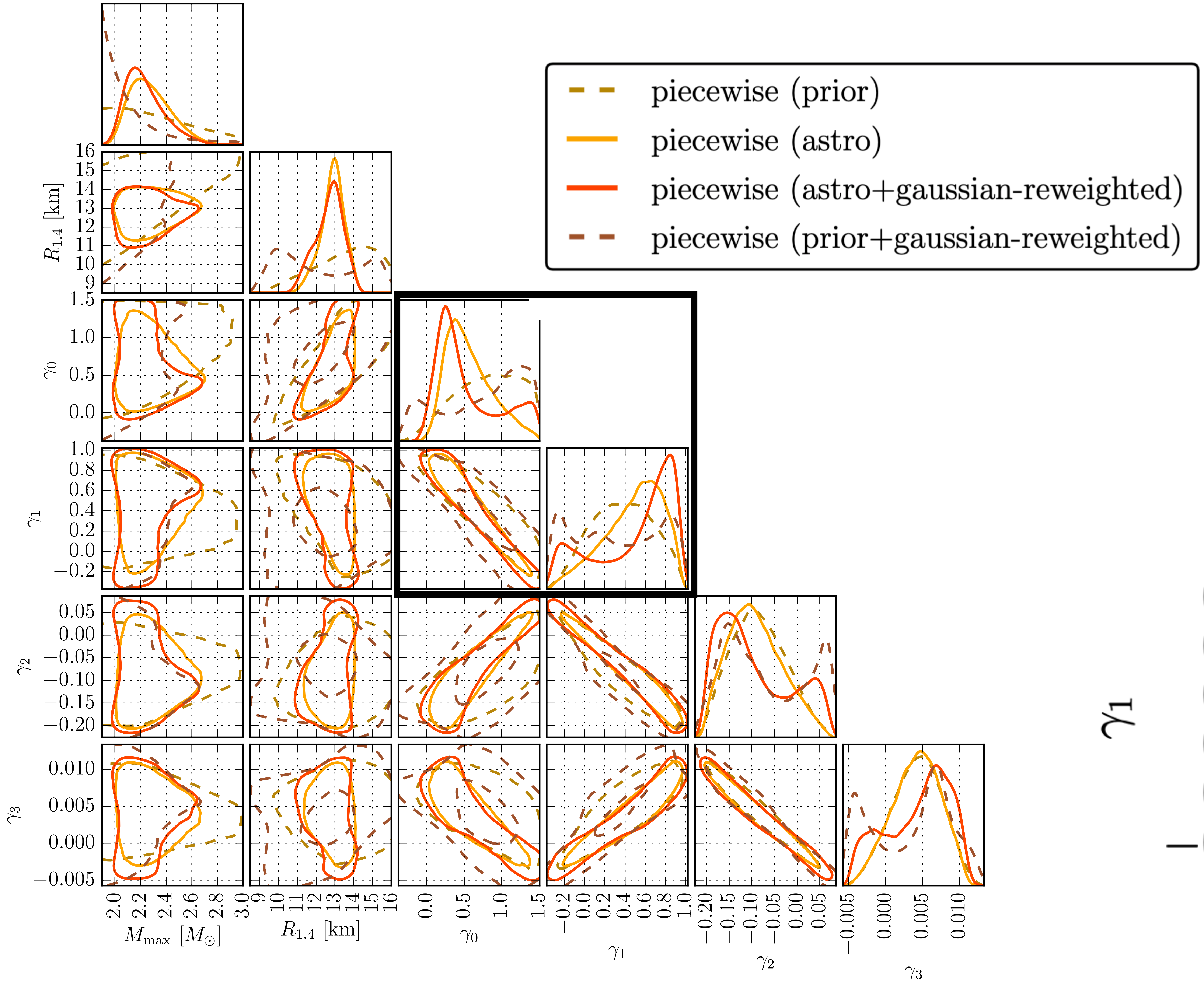
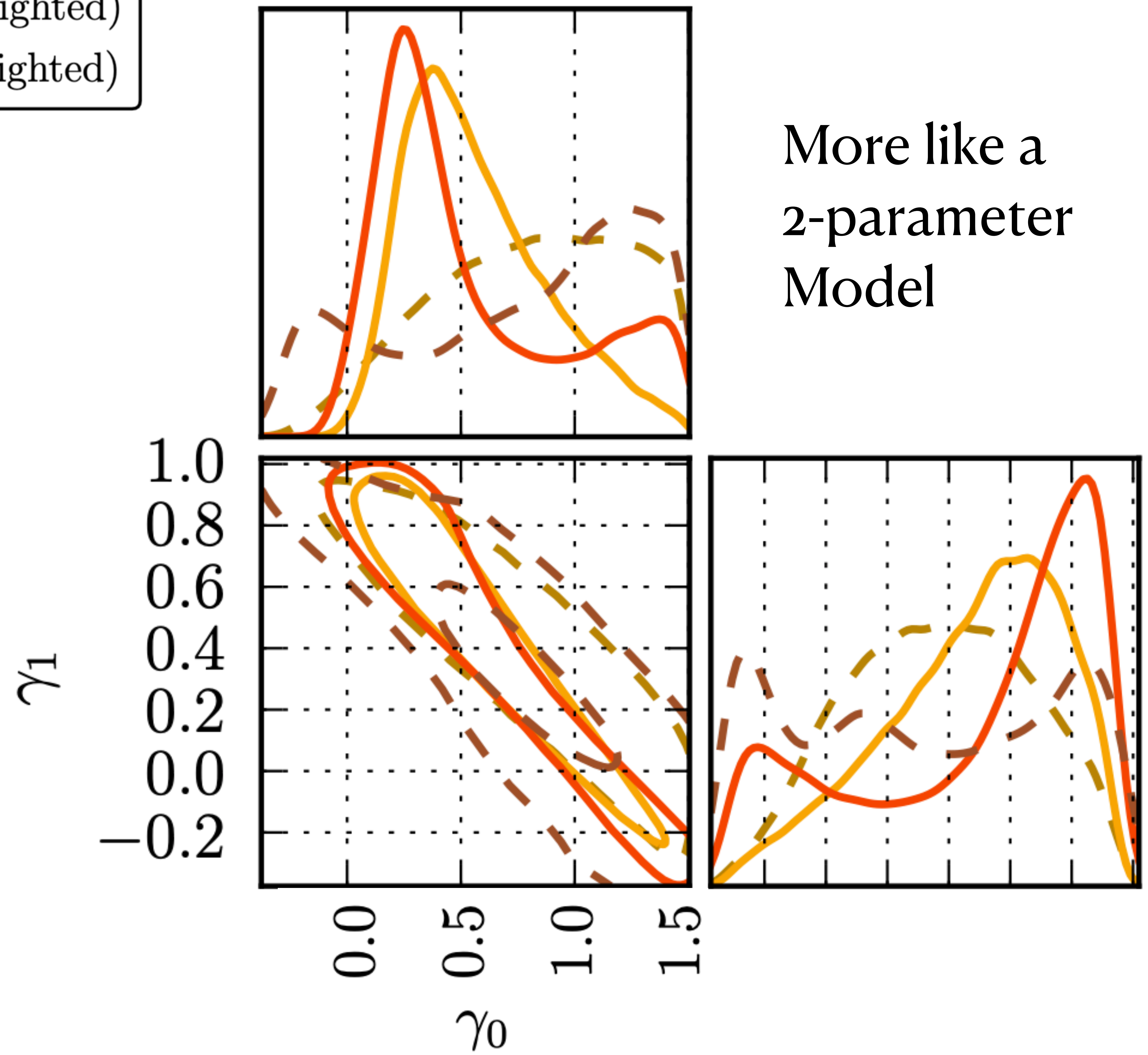


With the current data we only Really only infer γ_0 & γ_1



More like a 2-parameter Model

Modified parametric priors

Why not just modify the parametric models to get more flexibility?

Models are either

(1) fine-tuned => extending them without breaking is difficult (spectral + speed of sound)
(2) Need overhaul-type improvements (piecewise-polytrope + speed of sound)

This is already being done!
i.e. Steiner+ 2016 -> better piecewise-polytrope models

But… Extensions are nontrivial.
Best to understand limitations of each model while using it

Not all Correlations are Bad!

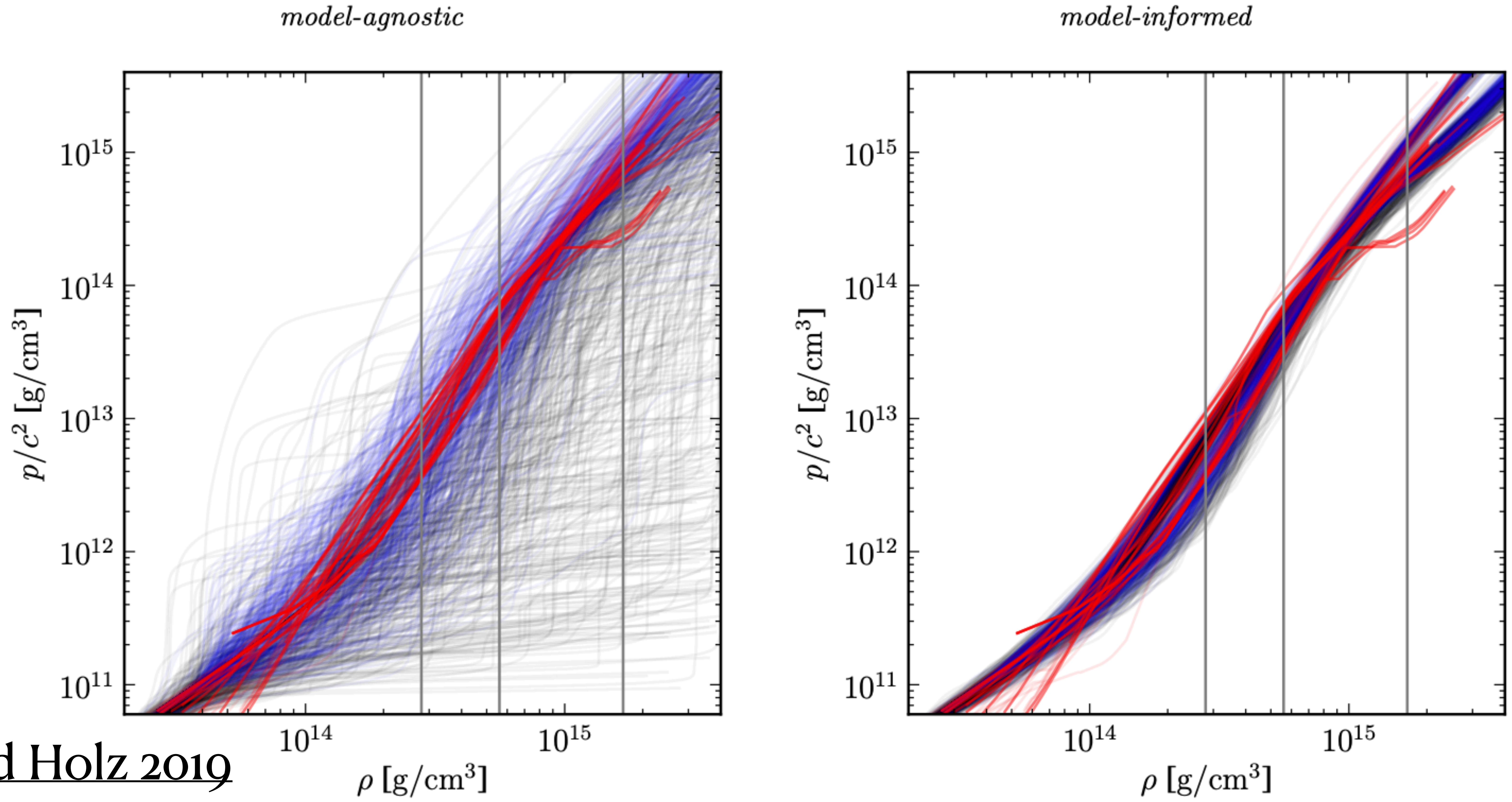
Physical theories have correlations between quantities "F=ma"

Correlated

Goal is to give flexibility in the choice of correlations

See e.g. Miller+2021 : GP with "tight" correlations

Eventually one should *infer* the correlations



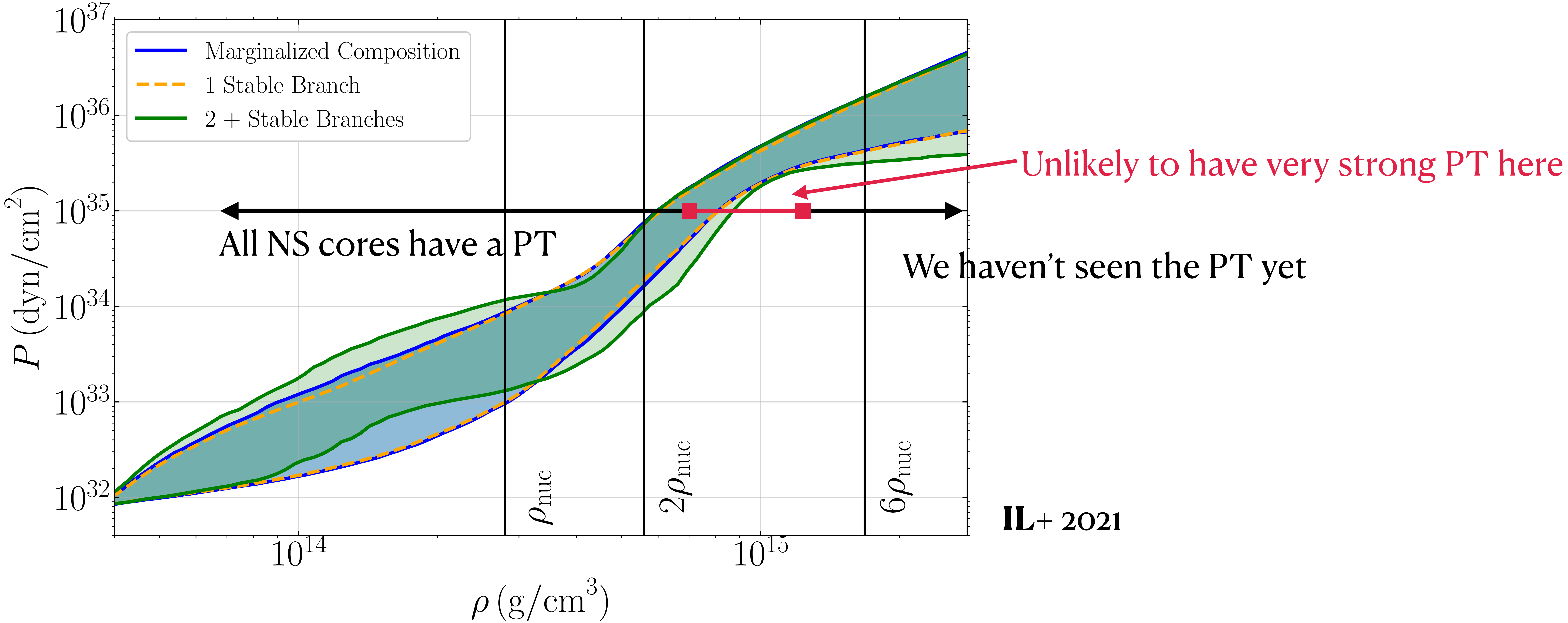
Essick, Landry, and Holz 2019

(Backup) : Strong Phase Transitions

Parametric models struggle to model phase transitions

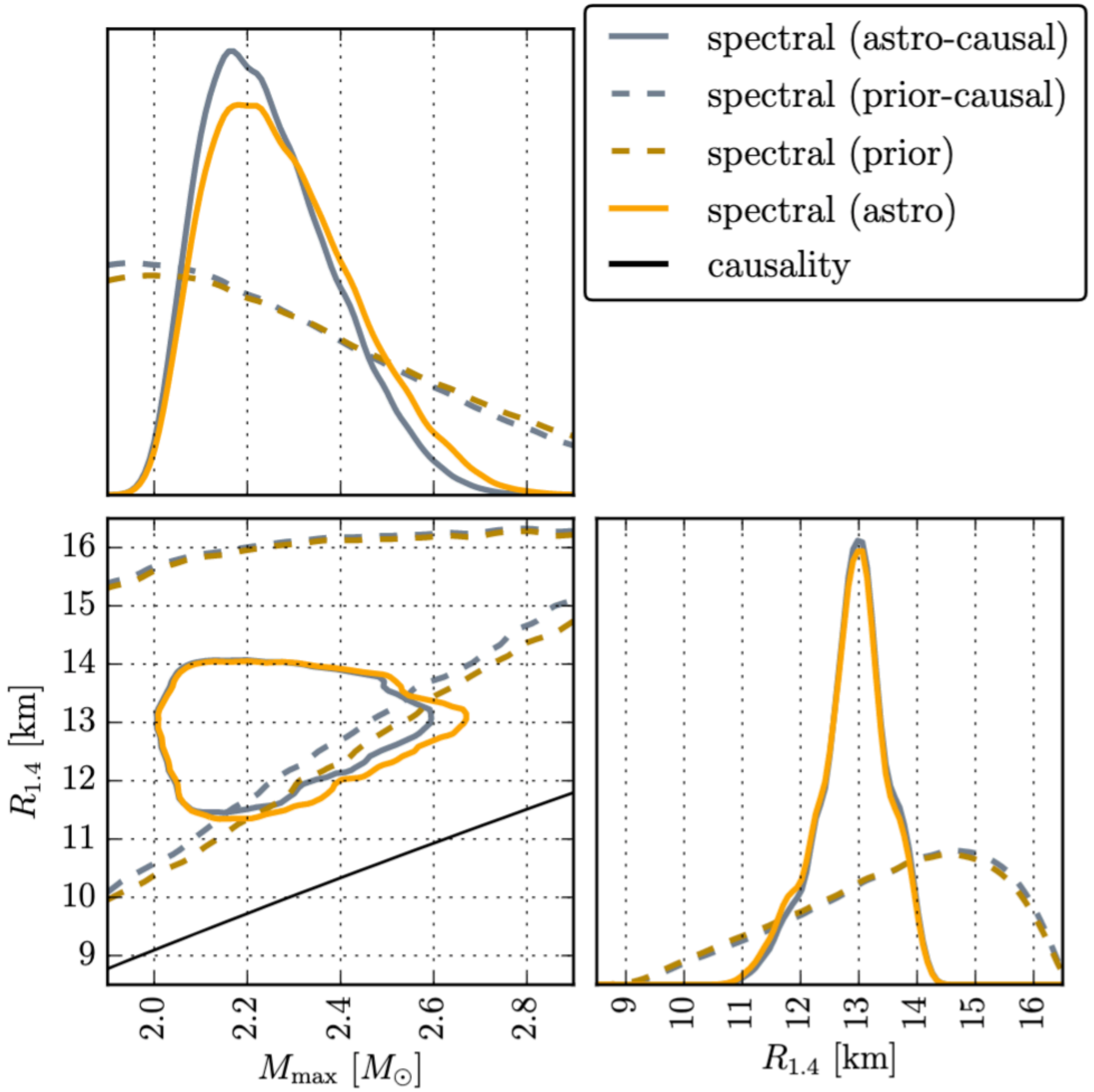
Piecewise-polytope models with variable stitching densities may be able to -> need fine tuning

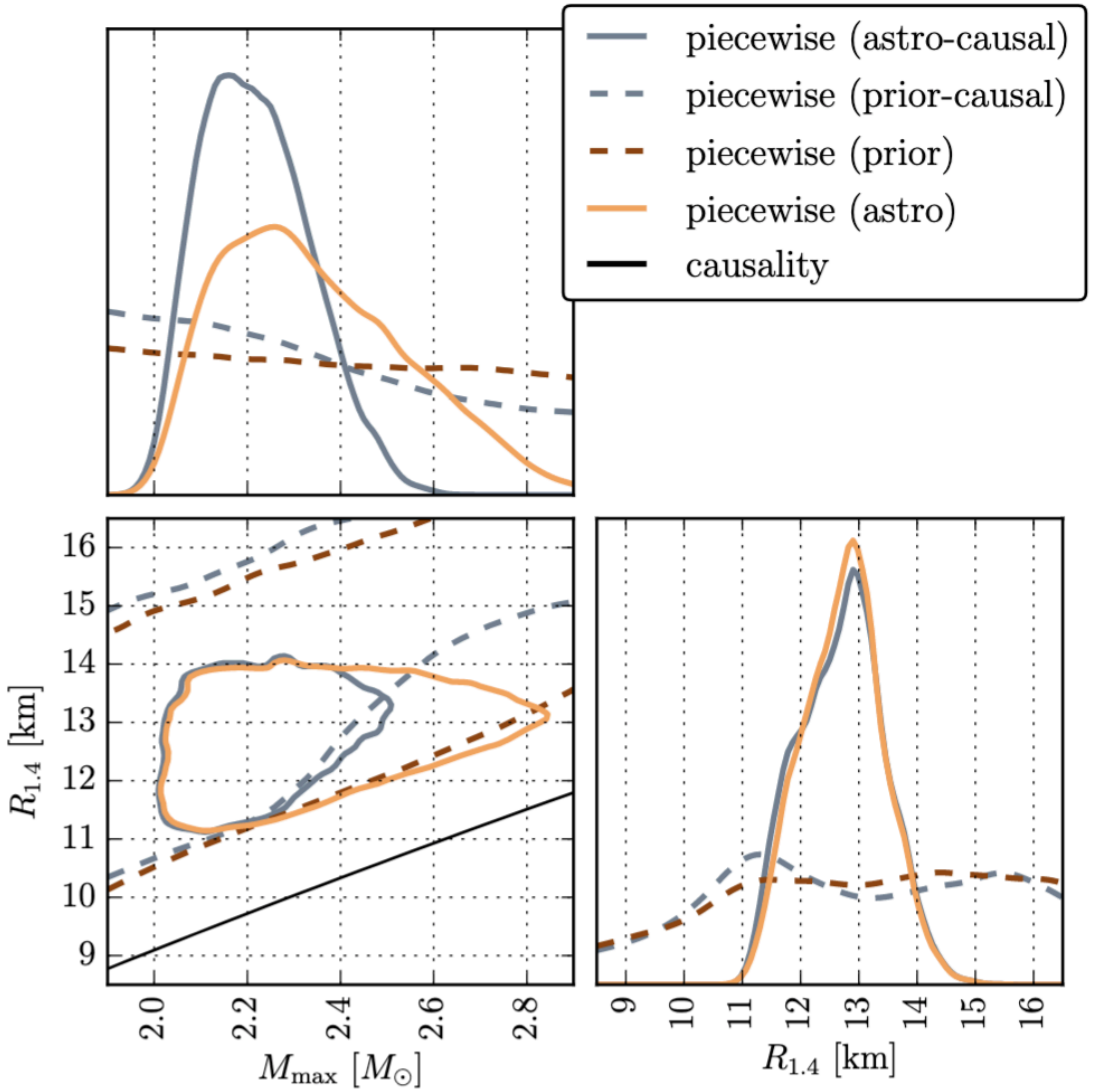
The GP model can produce EoSs mimicking phase transitions generically



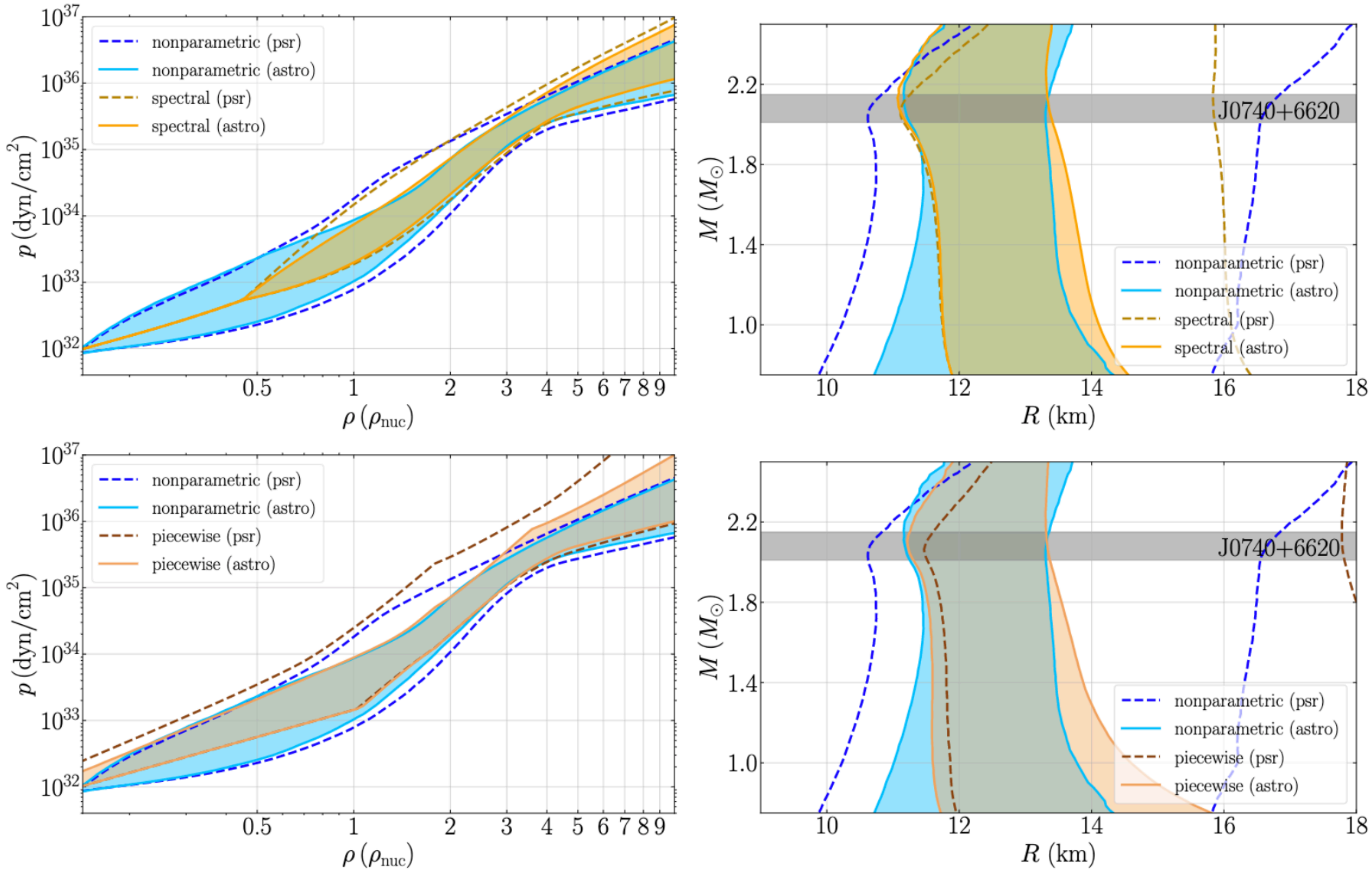
IL+ 2021

(Backup): Causality in Parametric Models



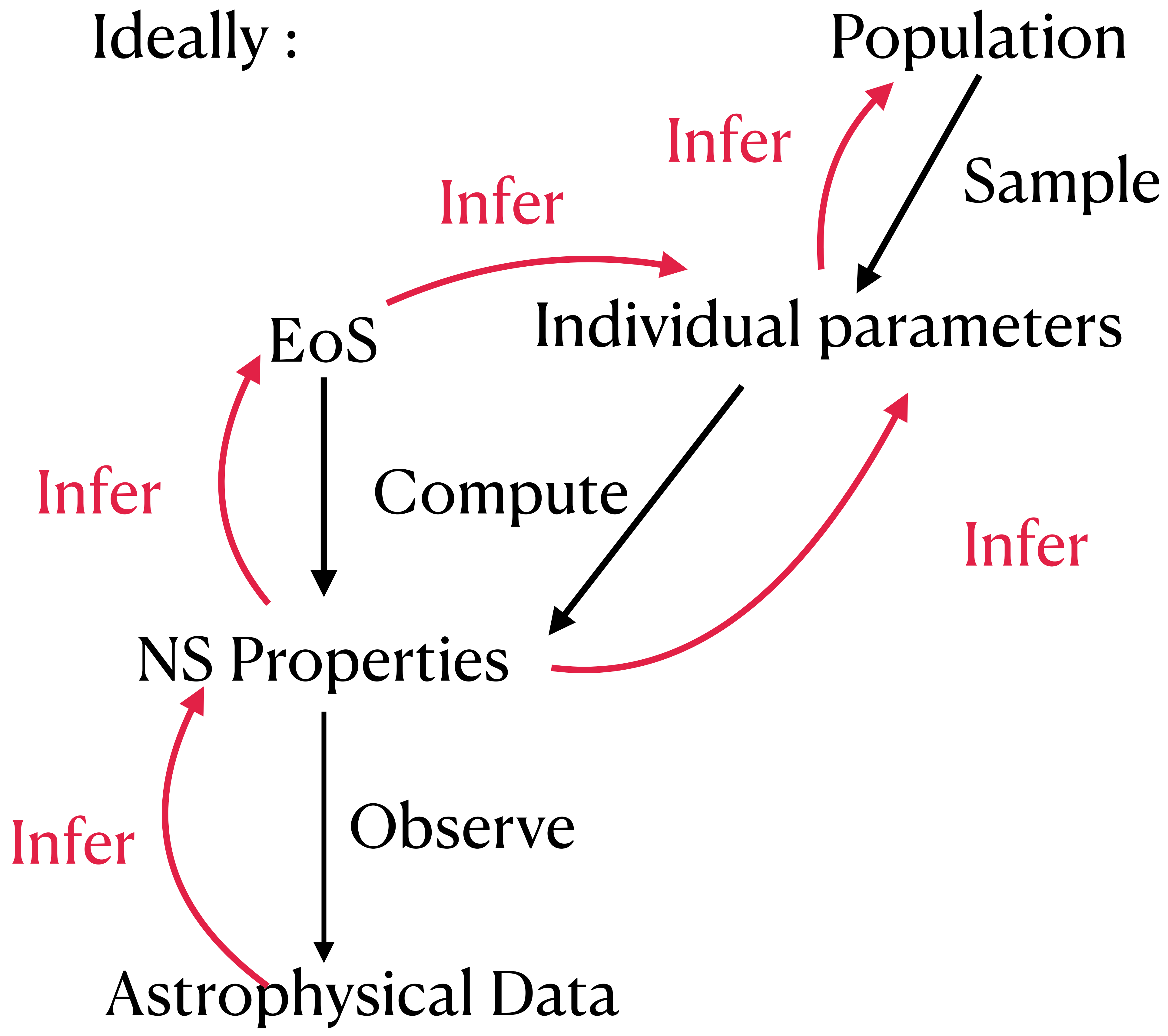


(Backup): More Parametric Results



Astrophysical Data (Brief Aside)

Ideally :



Lots of details

(1) Selection Effects
(2) Interpreting data (GW waveforms, x-ray pulse profiles)
(3) Poorly characterized population of NSs

$$P(\varepsilon_i \,|\, d) = P(\varepsilon \,|\, d_1, d_2, \ldots) \propto \boxed{P(d_1, d_2, \ldots \,|\, \varepsilon_i)} \times P(\varepsilon_i)$$

$$P(d_1, d_2, \ldots \,|\, \varepsilon_i) = P(d_1 \,|\, \varepsilon_i) \times P(d_2 \,|\, \varepsilon_i))\ldots$$

$$P(d_1 \,|\, \varepsilon_i) = \sum_{\text{NS sources}} P(d_1 \,|\, \text{NS source}) \times P(\text{NS Source} \,|\, \varepsilon_i, (\text{Population Model}))$$

Astrophysical Data (Brief Aside)

In practice:

Population

Sample

Individual parameters

EoS

Infer

Compute

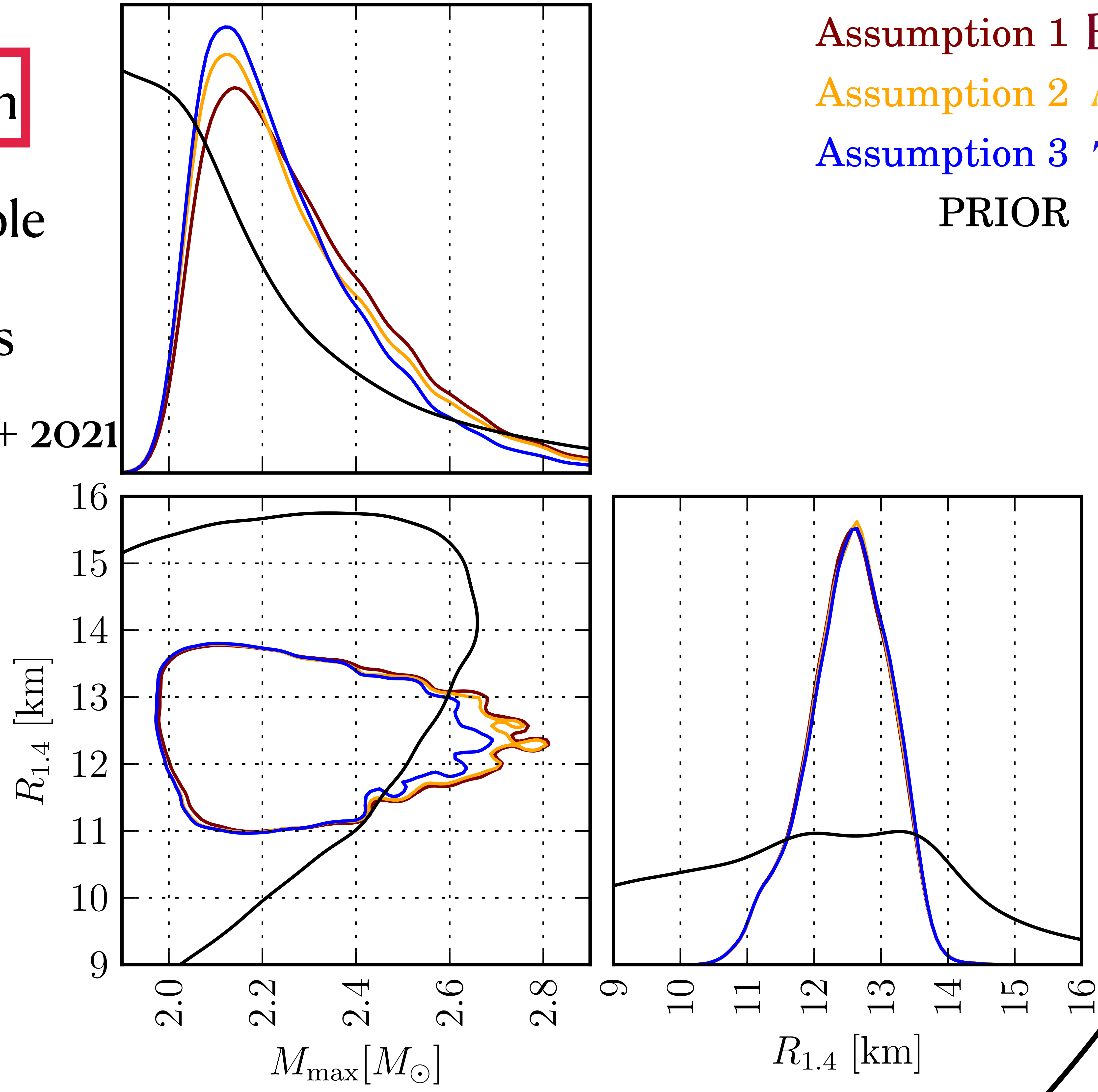
NS Properties

Infer

Observe

Astrophysical Data

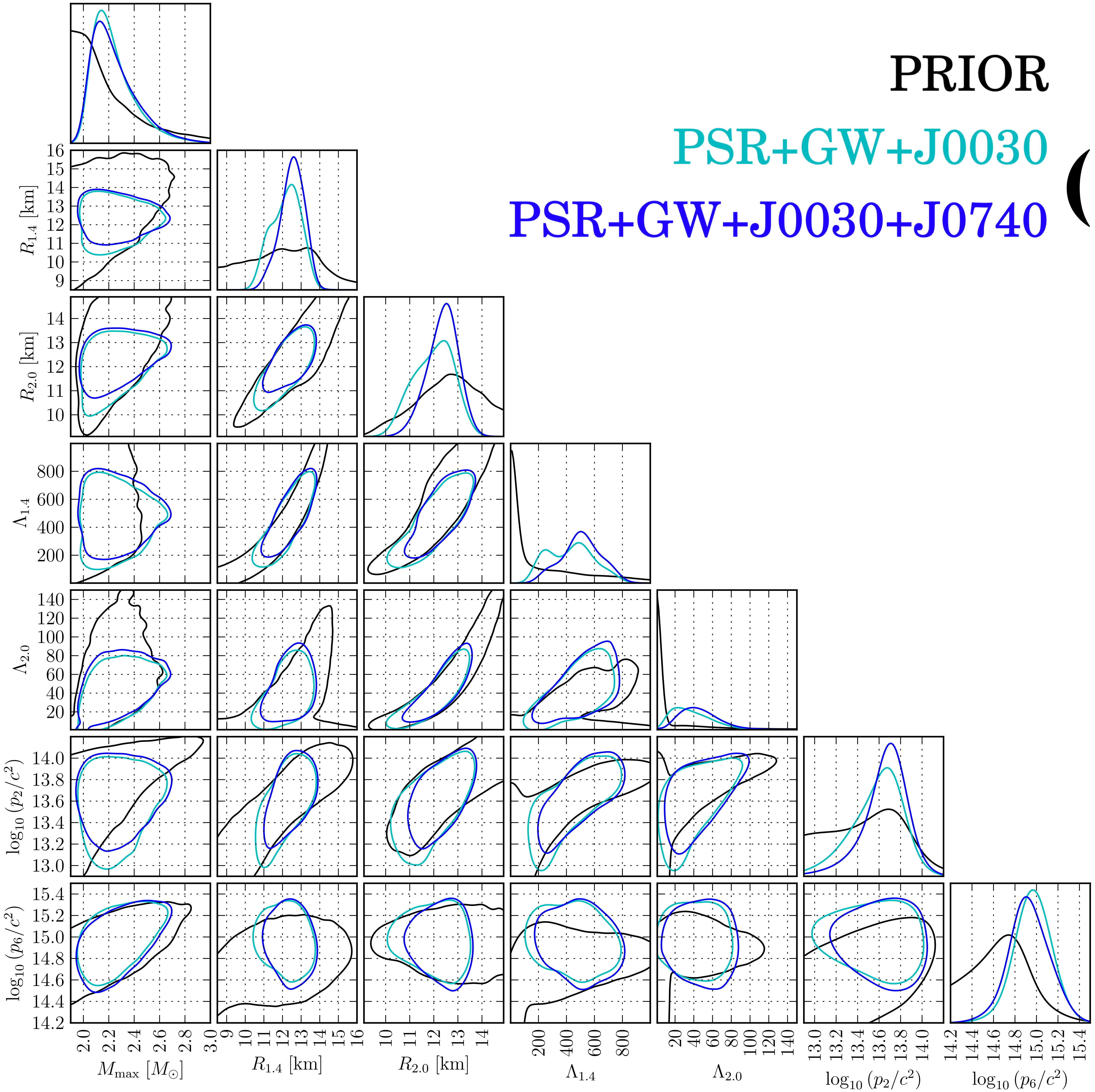
IL+ 2021

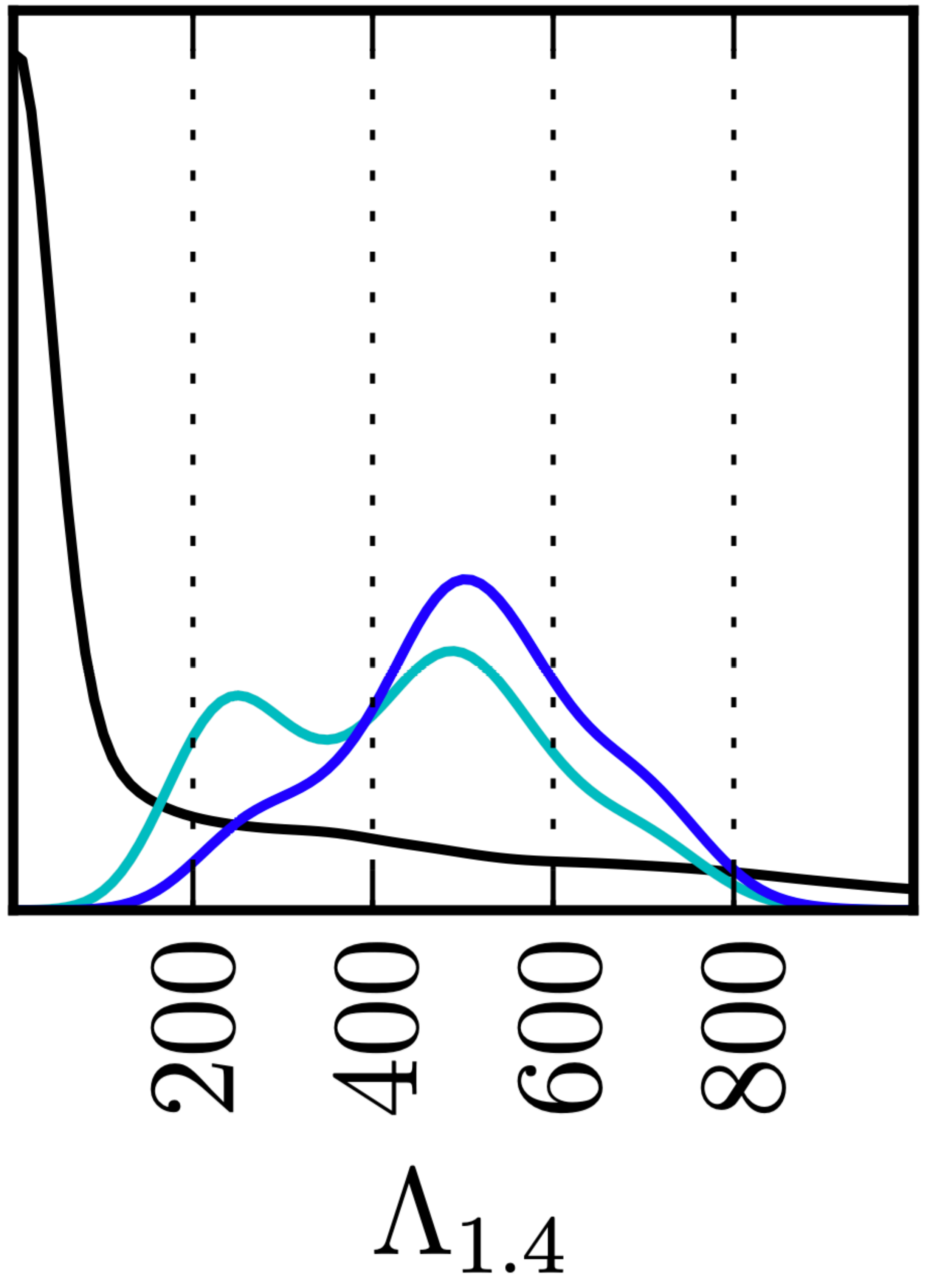


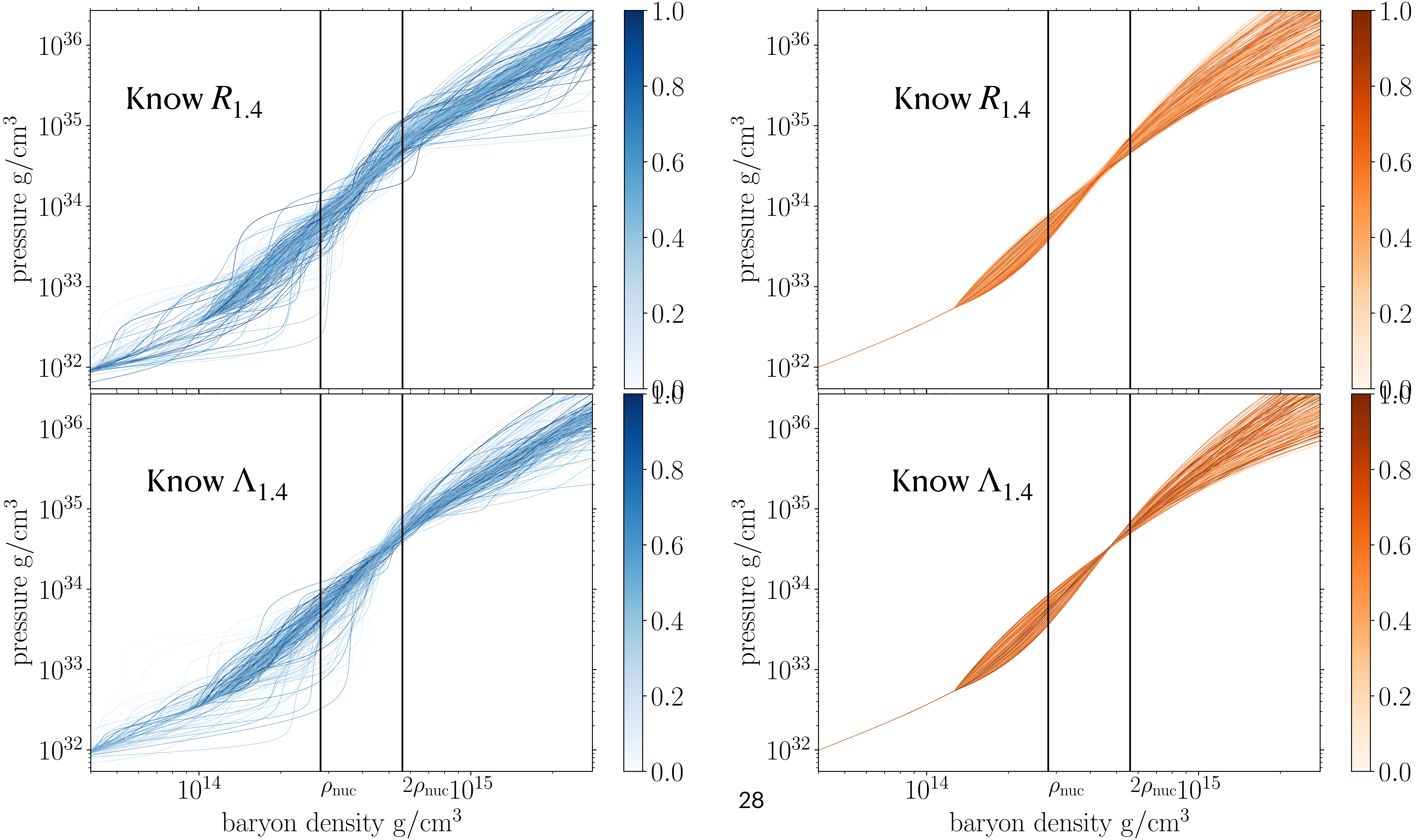
Assumption 1 BHs are just small NSs
Assumption 2 Astrophysically lmtd.
Assumption 3 TOV mass lmtd.
PRIOR

$$P(d_1 \,|\, \varepsilon_i) = \sum_{\text{NS sources}} P(d_1 \,|\, \text{NS source}) \times P(\text{NS Source} \,|\, \varepsilon_i, (\text{Population Model}))$$

(Backup): Corner Plot






Know $R_{1.4}$
Know $\Lambda_{1.4}$
Know $R_{1.4}$
Know $\Lambda_{1.4}$
pressure g/cm^3
baryon density g/cm^3
10^{32}
10^{33}
10^{34}
10^{35}
10^{36}
10^{14}
$\rho_{\rm nuc}$
$2\rho_{\rm nuc}$
10^{15}
0.0
0.2
0.4
0.6
0.8
1.0