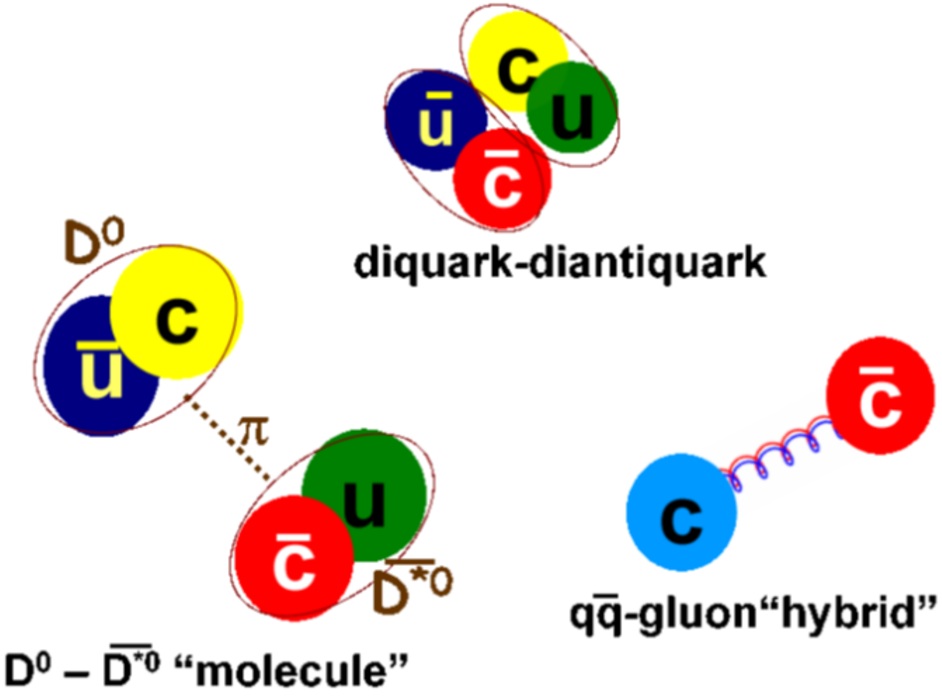
Unifying Diquark and Molecular Models of Exotics



Richard Lebed

Arizona State University

Accessing and Understanding the QCD Spectra

Institute for Nuclear Theory

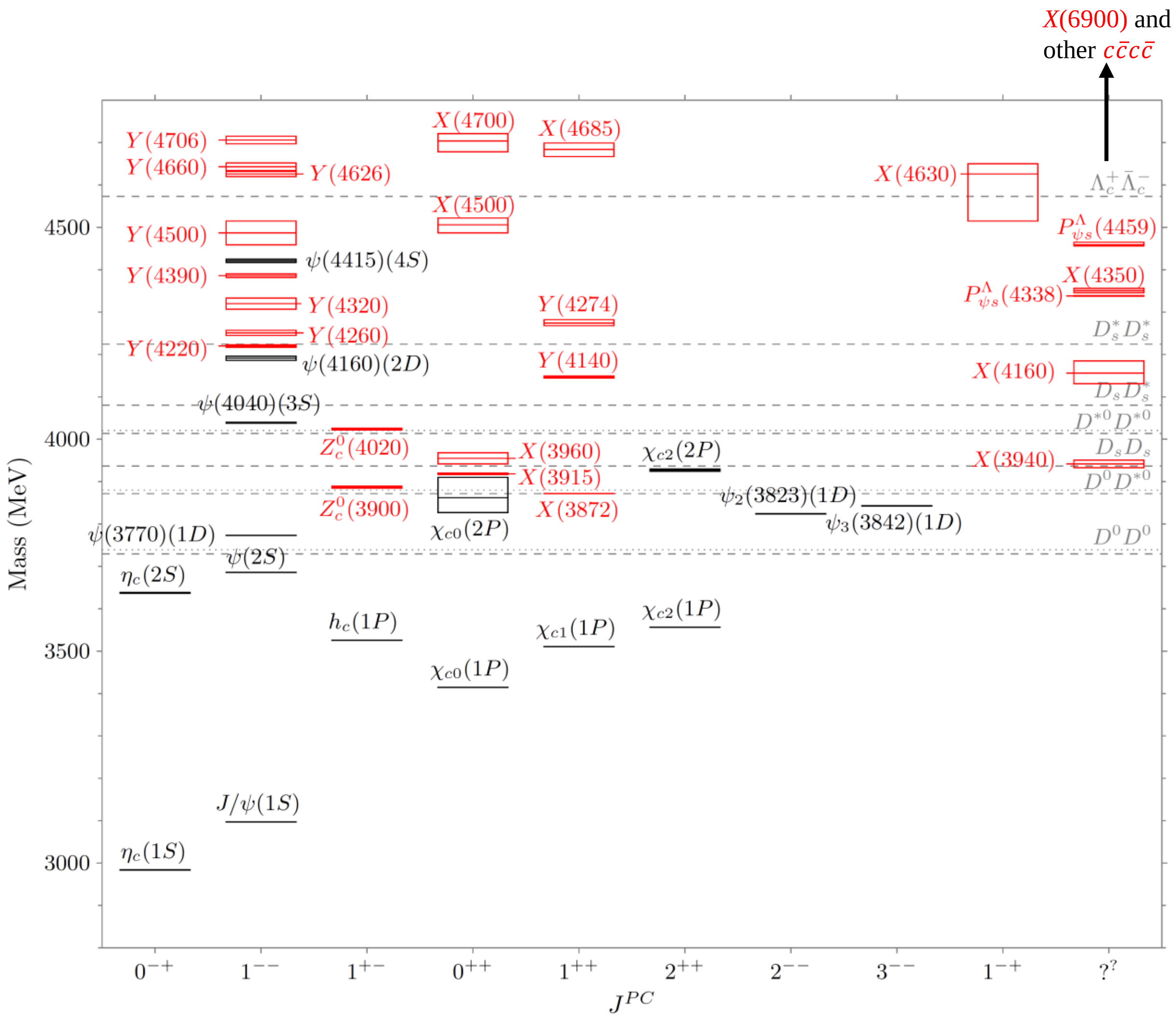
March, 2023

The Hadron Renaissance: 2023 Anniversaries

- **25 years**:
  First confirmed J^{PC}-exotic hybrid candidate $\pi_1(1600)$
  [E852 Collaboration, Brookhaven: PRL **81** (1998) 5760]

- **20 years**:
  First confirmed tetraquark candidate $X(3872)$
  [Belle Collaboration, KEK: PRL **91** (2003) 262001]

Neutral hidden-charm system, March 2023



Several of the states are quite close to di-hadron thresholds

Most prominent example:

$$m_{X(3872)} - m_{D^0} - m_{D^{*0}}$$
$$= -40 \pm 90 \text{ keV}$$

cf. the deuteron:

$$m_d - m_p - m_n$$
$$= -2.2452(2) \text{ MeV}$$

But many are *not* close to thresholds!
e.g., the 1^{--} Y states

Heavy-quark exotics census: March 2023

- **64** observed heavy-quark exotics, both tetraquarks and pentaquarks
  - 49 in the charmonium sector (neutral & charged, including open-strange)
  - 5 in the (much less explored) bottomonium sector
  - 4 with a single c quark (and an s, a u, and a d)
  - 1 with a single b quark (and an s, a u, and a d)
  - 4 with all c and $\bar{c}$ quarks
  - 1 with two c quarks
- A naïve count estimates well over 100 more exotics await discovery

Not all exotic candidates have heavy quarks

- $\pi_1(1600)$ (discovered 1998) is believed to be a hybrid meson because its $J^{PC} = 1^{-+}$ is not accessible to $q\bar{q}$ states

- $f_0(1710)$ is believed to have a sizeable glueball component because the quark model predicts one fewer 0^{++} states than are seen, and of them $f_0(1710)$ shows up most prominently in J/ψ decays (a glue-rich environment)

- $\phi(2170)$ has a peculiar decay pattern and may be an $s\bar{s}g$ hybrid or the $s\bar{s}q\bar{q}$ tetraquark analogue to the $c\bar{c}q\bar{q}$ state $Y(4230)$

…*and others*

The plan:

1) Develop a model that predicts a full spectrum for the expected exotics

2) Determine both their mass spectrum and decay patterns

3) Check whether yet-unobserved states are absent for some good reason

4) Seek to understand why some lie quite close to di-hadron thresholds

- Let us here use the **diquark** quasiparticle δ as the core constituent

Why diquarks?

- Short-distance QCD $\mathbf{3} \otimes \mathbf{3} \to \overline{\mathbf{3}}$ *attractive diquark coupling* is *fully half as strong* as $\mathbf{3} \otimes \overline{\mathbf{3}} \to \mathbf{1}$ (*confining attraction*)

- The $SU(2)$ analogue: Just as one computes a spin-spin coupling,
$$\vec{s}_1 \cdot \vec{s}_2 = \frac{1}{2}\left[(\vec{s}_1 + \vec{s}_2)^2 - \vec{s}_1{}^2 - \vec{s}_2{}^2\right],$$
particles in representations 1 & 2 combined into representation 1+2:
If s_1, s_2 = spin $\frac{1}{2}$, and $\vec{s}_1 + \vec{s}_2$ = $\boxed{\text{spin } 0: -\frac{\mathbf{3}}{\mathbf{4}}}$; $\boxed{\text{spin } 1: +\frac{\mathbf{1}}{\mathbf{4}}}$

- This scaling holds at leading-order QCD, but survives up to 3rd order [Anzal, Kiyo, Sumino, NPB **838**, 28 (2010) & **890**, 569 (2015)]

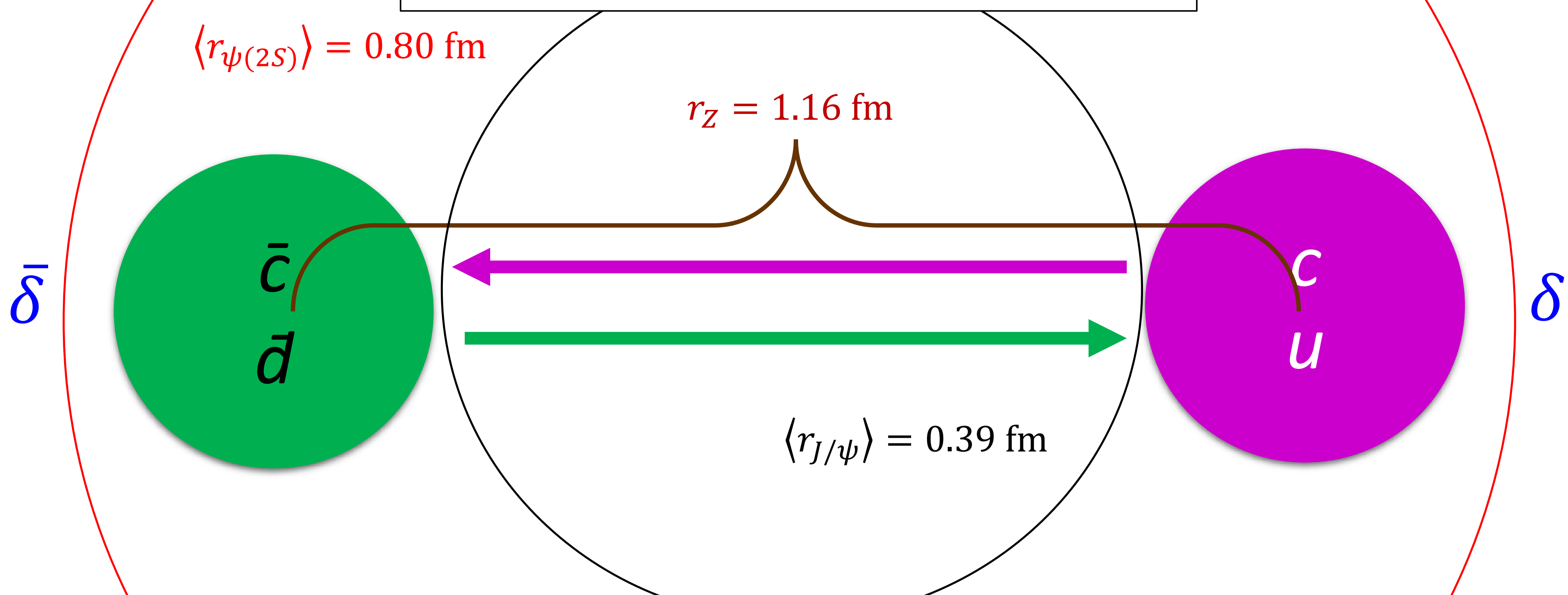
- The existence of this attraction out to distances as large as ~ 1 fm is supported by lattice calculations [Bali, PRD **62**, 114503 (2000)]

Diquarks allow large, but still strongly bound, states

Belle [K. Chilikin *et al.*, PRD **90**, 112009 (2014)] finds:

$$\frac{\mathrm{B.R.}\,[Z_c^-\,(4430) \to \psi(2S)\pi^-]}{\mathrm{B.R.}\,[Z_c^-\,(4430) \to J/\psi\pi^-]} > \mathbf{10}$$

and LHCb [R. Aaij *et al.*, PRL **112**, 222002 (2014)]
has not yet reported seeing the J/ψ $(1S)$ mode



The dynamical diquark *picture*:

Brodsky, Hwang, RFL [PRL **113**, 112001 (2014)]

- Heavy quarks provide nucleation points for diquark formation
- Spatial separation of heavy quarks during production process (in heavy-hadron decays or in high-energy collisions) leaves diquarks as identifiable constituent components of multiquark hadrons
- Diquark-antidiquark pair remain strongly connected by color flux tube → tetraquark $(Qq)_{\bar{\mathbf{3}}}(\bar{Q}\bar{q})_{\mathbf{3}} = \delta\bar{\delta}$
- Same color-triplet mechanism supports pentaquark formation, using a triquark: $[Q_{\mathbf{3}}(\bar{q}_1\bar{q}_2)_{\mathbf{3}}]_{\bar{\mathbf{3}}}(\bar{Q}\bar{q})_{\mathbf{3}}$ {RFL [PLB **749**, 454 (2015)]}

The dynamical diquark *model*:

RFL [PRD **96**, 116003 (2017)]

- Exotic eigenstate: configuration in which kinetic energy of the heavy di-(tri-)quarks is converted into potential energy of the color flux tube
- Two heavy, slow sources connected by light degrees of freedom (d.o.f.)? That's the adiabatic approximation → ordinary Schrödinger equation
- At energies where only one interaction potential function is important (*i.e.*, away from level crossings), can use the single-channel approximation
- Together, these form the Born-Oppenheimer (BO) approximation
- BO potentials created by light d.o.f. are same ones in lattice simulations of heavy-quark hybrids, labeled by axial quantum numbers such as in the potentials Σ_g^+, Π_u^-, *etc*.

Base states of the model (to combine w/BO potentials)

$$J^{PC}=0^{++}: \quad X_0 \equiv |0_\delta, 0_{\bar\delta}\rangle_0 \,, \quad X_0' \equiv |1_\delta, 1_{\bar\delta}\rangle_0 \,,$$

$$J^{PC}=1^{++}: \quad X_1 \equiv \frac{1}{\sqrt{2}}\left(|1_\delta, 0_{\bar\delta}\rangle_1 + |0_\delta, 1_{\bar\delta}\rangle_1\right) ,$$

$$J^{PC}=1^{+-}: \quad Z \equiv \frac{1}{\sqrt{2}}\left(|1_\delta, 0_{\bar\delta}\rangle_1 - |0_\delta, 1_{\bar\delta}\rangle_1\right) ,$$

$$Z' \equiv |1_\delta, 1_{\bar\delta}\rangle_1 \,,$$

$$J^{PC}=2^{++}: \quad X_2 \equiv |1_\delta, 1_{\bar\delta}\rangle_2 \,,$$

Diquark spin basis

Notation: Maiani *et al.*, PRD **89** (2014) 114010

Light-quark/ heavy-quark spin basis

$$0^{++} \begin{cases} \tilde X_0 \equiv |0_{q\bar q}, 0_{Q\bar Q}\rangle_0 = +\frac{1}{2}X_0 + \frac{\sqrt{3}}{2}X_0' \,, \\ \tilde X_0' \equiv |1_{q\bar q}, 1_{Q\bar Q}\rangle_0 = +\frac{\sqrt{3}}{2}X_0 - \frac{1}{2}X_0' \,, \end{cases}$$

$$X_1 \equiv |1_{q\bar q}, 1_{Q\bar Q}\rangle_1 ,$$

$$1^{+-} \begin{cases} \tilde Z \equiv |1_{q\bar q}, 0_{Q\bar Q}\rangle_1 = \frac{1}{\sqrt{2}}(Z' + Z) \,, \\ \tilde Z' \equiv |0_{q\bar q}, 1_{Q\bar Q}\rangle_1 = \frac{1}{\sqrt{2}}(Z' - Z) \,. \end{cases}$$

$$X_2 \equiv |1_{q\bar q}, 1_{Q\bar Q}\rangle_2 ,$$

and analogous definitions, $P_{1/2}$, *etc.* for pentaquarks

Exotics spectroscopy using BO potentials: Tetraquarks

[with analogous results for pentaquarks]

Boldface = quantum numbers that are exotic for $q\bar{q}$

BO potential	State notation State J^{PC}		
$\Sigma_g^+(1S)$	$\tilde{X}_{0\,S}^{(0)++}$ 0^{++}	$\tilde{Z}_S^{(1)++}$, $\tilde{Z}_S'^{(1)++}$ $2\times 1^{+-}$	$\tilde{X}_{0\,S}'^{(0)++}$, $X_{1\,S}^{(1)++}$, $X_{2\,S}^{(2)++}$ $[0,1,2]^{++}$
$\Sigma_g^+(1P)$	$\tilde{X}_{0\,P}^{(1)++}$ 1^{--}	$[\tilde{Z}_P^{(0),\mathbf{(1)},(2)}]^{++}$, $[\tilde{Z}_P'^{(0),\mathbf{(1)},(2)}]^{++}$ $2\times(0,\mathbf{1},2)^{-+}$	$\tilde{X}_{0\,P}'^{(1)++}$, $[X_{1\,P}^{\mathbf{(0)},(1),(2)}]^{++}$, $[X_{2\,P}^{(1),(2),(3)}]^{++}$ $[1,\ (\mathbf{0},1,2),\ (1,2,3)]^{--}$
$\Sigma_g^+(1D)$	$\tilde{X}_{0\,D}^{(2)++}$ 2^{++}	$[\tilde{Z}_D^{(1),\mathbf{(2)},(3)}]^{++}$, $[\tilde{Z}_D'^{(1),\mathbf{(2)},(3)}]^{++}$ $2\times(1,\mathbf{2},3)^{+-}$	$\tilde{X}_{0\,D}'^{(2)++}$, $[X_{1\,D}^{(1),(2),(3)}]^{++}$, $[X_{2\,D}^{(0),(1),(2),(3),(4)}]^{++}$ $[2,\ (1,2,3),\ (0,1,2,3,4)]^{++}$
$\Pi_u^+(1P)$ & $\Sigma_u^-(1P)$	$\tilde{X}_{0\,P}^{(1)-+}$ 1^{+-}	$[\tilde{Z}_P^{(0),(1),(2)}]^{-+}$, $[\tilde{Z}_P'^{(0),(1),(2)}]^{-+}$ $2\times(0,1,2)^{++}$	$\tilde{X}_{0\,P}'^{(1)-+}$, $[X_{1\,P}^{\mathbf{(0)},(1),\mathbf{(2)}}]^{-+}$, $[X_{2\,P}^{(1),\mathbf{(2)},(3)}]^{-+}$ $[1,\ (\mathbf{0},1,\mathbf{2}),\ (1,\mathbf{2},3)]^{+-}$
$\Pi_u^-(1P)$	$\tilde{X}_{0\,P}^{\mathbf{(1)}+-}$ $\mathbf{1}^{-+}$	$[\tilde{Z}_P^{\mathbf{(0)},(1),(2)}]^{+-}$, $[\tilde{Z}_P'^{\mathbf{(0)},(1),(2)}]^{+-}$ $2\times(\mathbf{0},1,2)^{--}$	$\tilde{X}_{0\,P}'^{\mathbf{(1)}+-}$, $[X_{1\,P}^{(0),\mathbf{(1)},(2)}]^{+-}$, $[X_{2\,P}^{\mathbf{(1)},(2),\mathbf{(3)}}]^{+-}$ $[\mathbf{1},\ (0,\mathbf{1},2),\ (\mathbf{1},2,\mathbf{3})]^{-+}$
$\Sigma_u^-(1S)$	$\tilde{X}_{0\,S}^{(0)-+}$ 0^{-+}	$\tilde{Z}_S^{(1)-+}$, $\tilde{Z}_S'^{(1)-+}$ $2\times 1^{--}$	$\tilde{X}_{0\,S}'^{(0)-+}$, $X_{1\,S}^{\mathbf{(1)}-+}$, $X_{2\,S}^{(2)-+}$ $[0,\mathbf{1},2]^{-+}$
$\Pi_u^+(1D)$	$\tilde{X}_{0\,D}^{(2)-+}$ 2^{-+}	$[\tilde{Z}_D^{(1),(2),(3)}]^{-+}$, $[\tilde{Z}_D'^{(1),(2),(3)}]^{-+}$ $2\times(1,2,3)^{--}$	$\tilde{X}_{0\,D}'^{(2)-+}$, $[X_{1\,D}^{\mathbf{(1)},(2),\mathbf{(3)}}]^{-+}$, $[X_{2\,D}^{(0),\mathbf{(1)},(2),\mathbf{(3)},(4)}]^{-+}$ $[2,\ (\mathbf{1},2,\mathbf{3}),\ (0,\mathbf{1},2,\mathbf{3},4)]^{-+}$

All states in $S, D,$ *etc.* multiplets are $P = +$
All states in $P, F,$ *etc.* multiplets are $P = -$

Dynamical diquark model, first numerical results

Giron, RFL, Peterson [JHEP **05**, 061 (2019)]

- When the heavy sources coincide, some BO potentials become degenerate (called *parity doubling* in atomic physics)
  $\rightarrow$ requires development of a coupled Schrödinger equation solver
- Our detailed simulations showed that all known exotics fit into the ground-state Σ_g^+ BO potential, and in their $1S$, $1P$, $2S$, $2P$, $1D$ orbitals
- Using the lattice-simulated BO potentials, the result for the $(cq)(\bar{c}\bar{q})$ $\Sigma_g^+(1S)$ multiplet-average mass naturally matches $m_{X(3872)}$, and those for $\Sigma_g^+(1P)$, $\Sigma_g^+(2S)$ beautifully match $m_{Y(4220)}$, $m_{Z_c(4430)}$, respectively
- But these are multiplet-average masses—Need to include fine structure

First numerical results of the model

[Giron, RFL, and Peterson, JHEP **05** (2019) 061]

Fixed to $X(3872)$

Right atop $Z_c(4430)$

Right atop $Y(4220)$

Right atop $Y(4660)$

Right atop $X(4500)$

BO states	Potential	M (GeV)	m_δ	$\langle 1/r\rangle^{-1}$ (fm)	$\langle r\rangle$
$\Sigma_g^+(1S)$	JKM	**3.8711**	1.8747	0.27202	0.36485
	CPRRW	**3.8721**	1.8535	0.27519	0.36904
	BGS	**3.8718**	1.9402	0.21347	0.30268
$\Sigma_g^+(2S)$	JKM	4.4430	1.8747	0.42698	0.69081
	CPRRW	4.4410	1.8535	0.43057	0.69640
	BGS	4.4674	1.9402	0.42621	0.69756
$\Sigma_g^+(1P)$	JKM	4.2457	1.8747	0.48968	0.56601
	CPRRW	4.2435	1.8535	0.49379	0.57067
	BGS	4.3471	1.9402	0.48361	0.56787
$\Sigma_g^+(2P)$	JKM	4.7128	1.8747	0.62445	0.84285
	CPRRW	4.7092	1.8535	0.62911	0.84913
	BGS	4.7416	1.9402	0.65333	0.89663
$\Sigma_g^+(1D)$	JKM	4.5318	1.8747	0.66414	0.73132
	CPRRW	4.5282	1.8535	0.66921	0.73668
	BGS	4.6151	1.9402	0.69780	0.77323
$\Sigma_g^+(2D)$	JKM	4.9476	1.8747	0.78634	0.98022
	CPRRW	4.9431	1.8535	0.79199	0.98697
	BGS	4.9486	1.9402	0.84597	1.0645
$\Pi_u^+(1P)$ &	JKM	4.9156	1.8747	0.44931	0.56950
$\Sigma_u^-(1P)$	CPRRW	4.8786	1.8535	0.44614	0.56438

Dynamical diquark model, fine structure & isospin

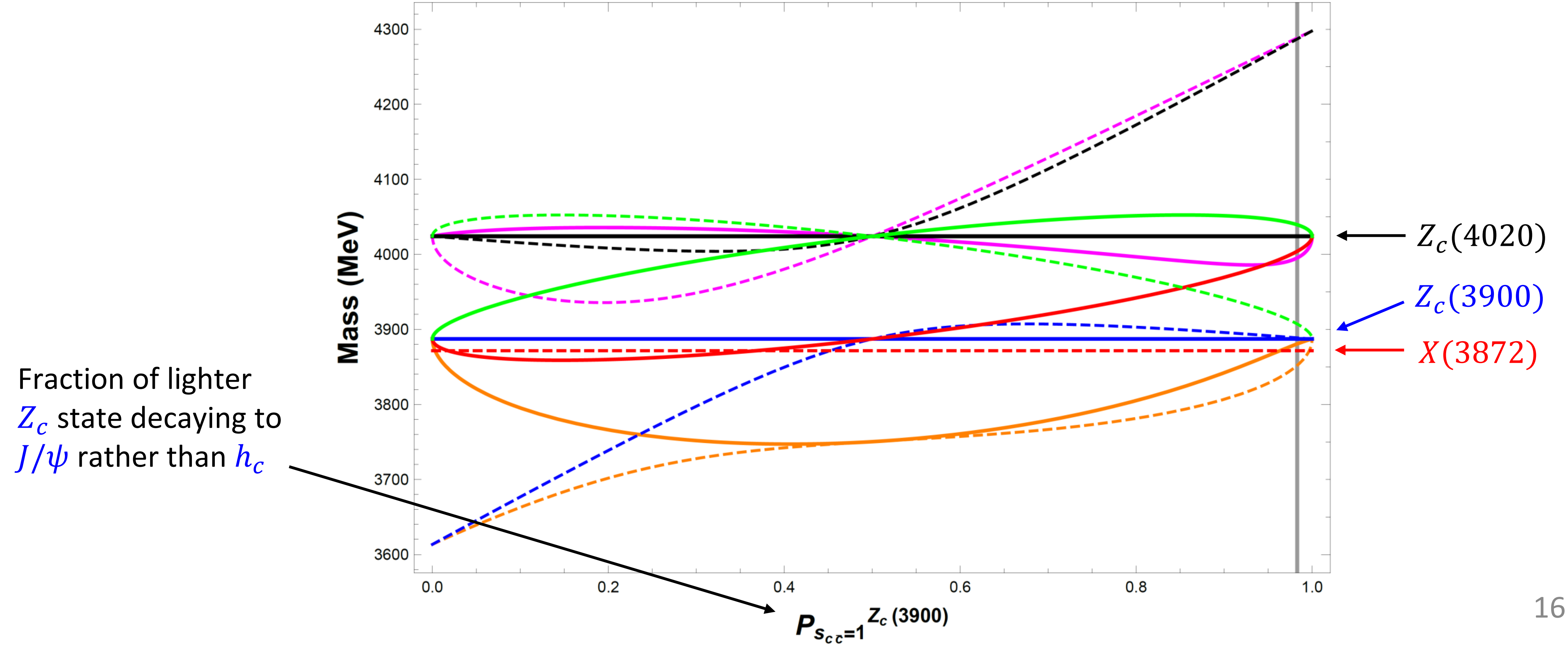
Giron, RFL, Peterson [JHEP **01**, 124 (2020)]

- With only a few known exotics in each multiplet, need to identify the most physically important perturbation Hamiltonian operators
- *e.g.*, the multiplet $(cq)(\bar{c}\bar{q})\ \Sigma_g^+(1S)$ contains $6\ I=0$ and $6\ I=1$ states, and we know only $X(3872)$ [$I=0$], $Z_c(3900)$, $Z_c(4020)$ [$I=1$]
- Fixes 2 operators, taken to be:
  (1) quark spin-spin coupling *within* each diquark, and
  (2) isospin/spin exchange *between* diquarks (analogous to π exchange)
- Naturally predicts $X(3872)$ to be lightest narrow state in multiplet
- Naturally predicts $Z_c(3900)$ to decay preferentially to J/ψ ($s_{c\bar{c}}=1$) and $Z_c(4020)$ to h_c ($s_{c\bar{c}}=0$), as is experimentally observed

Dynamical diquark model: $(cq)(\bar{c}\bar{q})$ ground states

Figure from J. Giron, PhD dissertation (2021)

- The model also predicts masses for the other 9 states in $\Sigma_g^+(1S)$:



If the model is any good, it must also apply to other flavor sectors

Using the same Hamiltonian operators (plus spin-orbit & tensor for orbitally excited states), apply to:

- the $(cq)(\bar{c}\bar{q})$ negative-parity states {Giron, RFL [PRD **101**, 074032 (2020)]}
- the $(bq)(\bar{b}\bar{q})$ sector {Giron, RFL [PRD **102**, 014036 (2020)]}
- the $(cs)(\bar{c}\bar{s})$ sector {Giron, RFL [PRD **102**, 014036 (2020)]}
- the $(cc)(\bar{c}\bar{c})$ sector {Giron, RFL [PRD **102**, 074003 (2020)]}
- the $(cq)(\bar{c}\bar{s})$ sector {Giron, RFL, Martinez [PRD **104**, 054001 (2021)]}
- the $(cu)(\bar{c}ud)$ & $(cs)(\bar{c}ud)$ **pentaquarks** {Giron, RFL [PRD **104**, 114028 (2021)]}

But what about the closeness of some exotics to di-hadron thresholds?

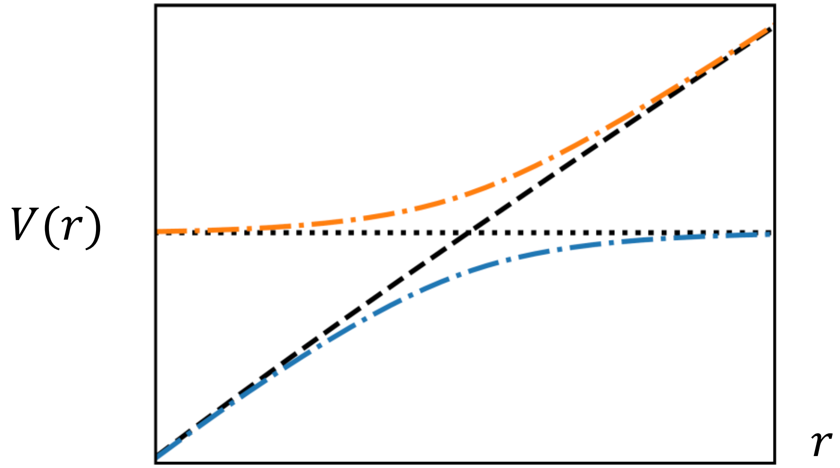
- Since the constituents are the same, *e.g.*, $(cq)(\bar{c}\bar{q})$ *vs.* $(c\bar{q})(\bar{c}q)$, *some* exotics may lie naturally close (~ 10 MeV) to thresholds

- But $m_{X(3872)} - m_{D^0} - m_{D^{*0}} = -40 \pm 90$ keV *cannot* be an accident!
- This binding energy is much smaller than expected for "conventional" hadron molecule—more likely a threshold rescattering effect [coupling to near-on-shell particles leads to an enhanced amplitude]

- A great deal of theory work has been performed to explain some exotics as purely threshold effects, but not every threshold seems to have a prominent associated state

Diabatic corrections

- But what if both types of interaction potentials are present (diquark-antidiquark <u>and</u> di-hadron threshold)?
- This is a well-known problem in atomic physics: One must perform a coupled-channel calculation to find mixed-configuration eigenstates near level crossing, where the adiabatic approximation fails
- Rigorous method to incorporate these effects: **diabatic approach**



--- $= V_{\text{diquark}}(r)$

... $= V_{\text{thresh}}(r)$

Diabatic corrections

- Choose a separation $\boldsymbol{r}_0$ of heavy sources at which mixing is small
- Solve Schrödinger equation for eigenstates $|\xi_i(\boldsymbol{r}_0)\rangle$, where i labels unmixed diquark/di-hadron components
- Given interaction Hamiltonian for light degrees of freedom H_{light}, compute diabatic potential matrix $V_{ji}(\boldsymbol{r}, \boldsymbol{r}_0) \equiv \langle \xi_j(\boldsymbol{r}_0) | H_{\text{light}} | \xi_i(\boldsymbol{r}_0) \rangle$
- Rest of the Hamiltonian is the heavy-source kinetic-energy operator, $\mathrm{K} = \mathrm{diag}\{-\hbar^2\nabla^2/2\mu_i\}$
- Solve the coupled Schrödinger equation $[\mathrm{K} + \mathrm{V}(\boldsymbol{r})]\Psi(\boldsymbol{r}) = E\Psi(\boldsymbol{r})$ for mass eigenstates $|\Psi\rangle$, expressed as linear combinations of $|\xi_i\rangle$

Diabatic corrections

- Only missing ingredient: What is the mixing potential of H_{light} (gives off-diagonal elements V_{ji} of diabatic potential matrix)?
- Lattice simulations will be able to calculate these (*e.g.*, string-breaking potential static energies), but in the meantime can model them as Gaussians that rapidly transition at the level crossing
- Diabatic approach was first applied to mixing of hadron thresholds with conventional quarkonium: Bruschini & Gonzalez [PRD **102**, 074002 (2020)]
- We use the same techniques, but instead mixing is with diquark states
  The coupled-channel Schrödinger solver from prior work comes in very handy!
  {RFL, Martinez [PRD **106** (2022) 7, 074007]}

Diabatic framework first results

RFL & Martinez [PRD **106** (2022) 7, 074007]

- It is not at all unnatural for a diquark $(\delta\bar{\delta})$ state near a threshold to acquire a very large di-hadron component, while others do not:

J^{PC}	E (MeV)	$\delta\bar{\delta}$	$D\bar{D}^*$	$D_s\bar{D}_s$	$D^*\bar{D}^*$	$D_s^*\bar{D}_s^*$	$\langle r\rangle$(fm)	$\langle r^2\rangle^{1/2}$(fm)
0^{++}	3905.4	63.0%		27.4%	8.4%	1.2%	0.596	0.605
1^{++}	3871.5	8.6%	91.4%				4.974	5.459
2^{++}	3922.3	83.1%		1.5%	13.9%	1.5%	0.443	0.497

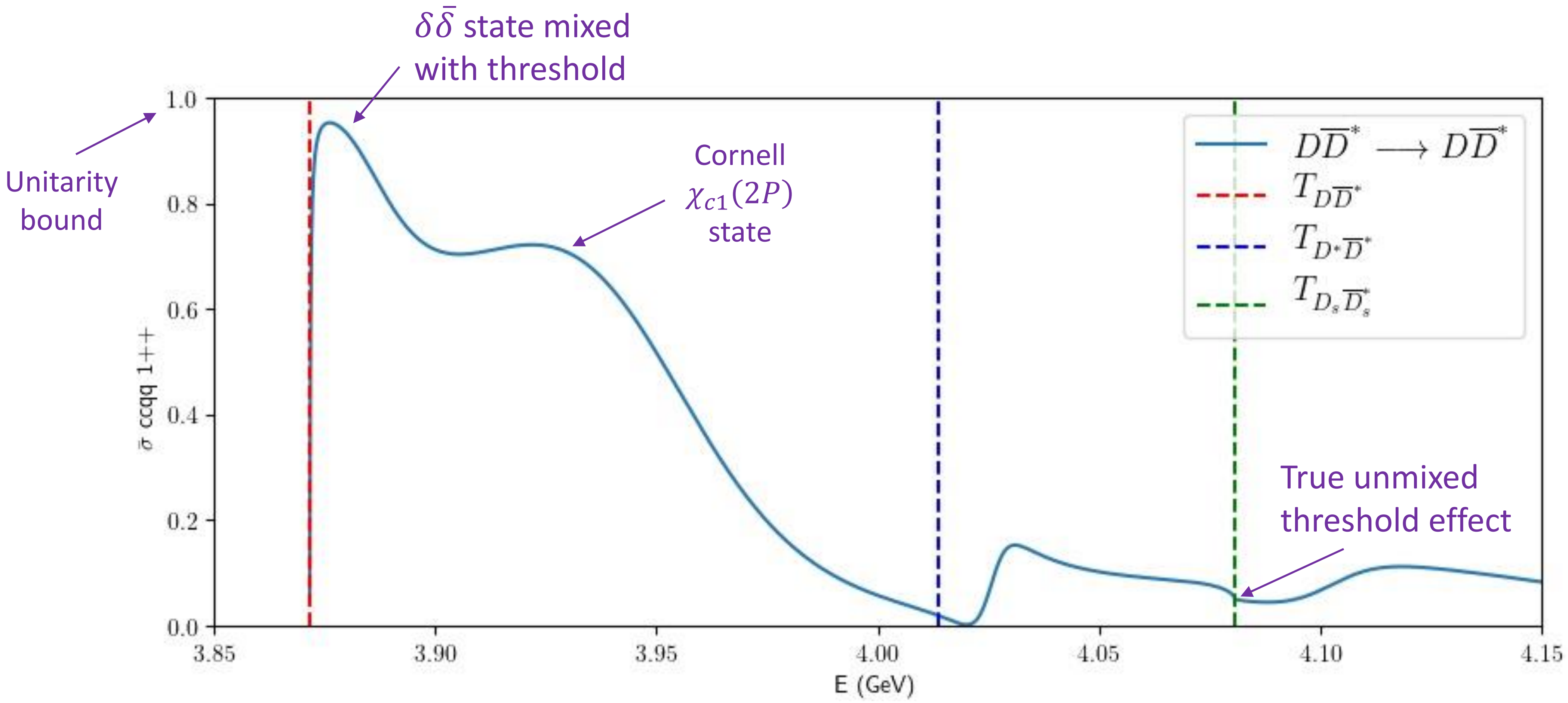
- Knowing explicitly the diquark (short-distance) as well as di-hadron (long-distance) components allows one to probe effects sensitive to short-distance structure, such as radiative decays: *e.g.*, B.R.$[X(3872) \longrightarrow \gamma\psi(2S)] = (4.5 \pm 2.0)\%$

Diabatic framework: The future

- Is this a true unification of diquark and molecular models?
  Here, the threshold potential is just treated as a free di-hadron state, but changing it to a binding potential would be trivial (*future work*)
- Current calculations treat all pre-diabatic $\Sigma_g^+(1S)$ states as degenerate, but fine structure is easy to incorporate into H_{light} (*future work*)
- Computing mass eigenvalues this way is rigorous only for states below or slightly above thresholds. States substantially above thresholds are broad resonances—Should be treated as poles in scattering amplitudes
  Here again, Bruschini & Gonzalez [PRD **104**, 074025 (2021)]
  provides a relevant diabatic framework (*future work*)

Preliminary result: $D\bar{D}^*$ scattering amplitude

RFL & Martinez, in preparation



Summary & Conclusions

1) So many heavy-quark exotics have now been observed that
a theory to predict their complete spectrum has become **imperative**
2) Molecules alone are not enough: Many exotics lie far from constituent thresholds
3) Models based upon diquarks hold promise:
Fully predicted spectrum, whole state bound by strong QCD forces,
many phenomenological successes (especially in the dynamical diquark model)
4) But several exotics *are* very close to thresholds⟶the adiabatic nature of the
Born-Oppenheimer approximation can be generalized to the **diabatic approach**
when di-hadron thresholds are nearby, unifying diquark & molecular pictures
5) First calculations of dynamical diquark model using diabatic framework complete,
research in multiple future directions now underway

Backup Slides

For decades, hadronic spectroscopy was the core of high-energy physics

- 1947: Discovery of $\pi^{\pm}$, $K^{\pm}$, K^0
- 1950 ~ 1965: The hadron zoo; strangeness; the Eightfold Way; the quark model; color charge
- 1974: Charmonium; evidence for asymptotic freedom & QCD
- 1977: Bottomonium; 3rd generation of quarks needed for CP violation
- 1983: First full reconstruction of B meson decays
- 1983: W & Z bosons. Look for top quark! Look for Higgs! Look for BSM!!
- 1983– Hadron spectroscopy: Fill out the quark-model multiplets 

Previously, on "Rich's Talks"…

- "***The XYZ Affair***"
- Defining a hadron as *any* $\mathrm{SU(3)_{color}}$-neutral (and hence, unconfined) compound of quarks and gluons, then…
- every single hadron* discovered from the proton [Rutherford, 1919] until **2003** has been either a $q\bar{q}$ meson or a qqq baryon
- Even Gell-Mann and Zweig, in their foundational quark-model papers (1964), anticipated other possibilities
- *A state $\pi_1(1600)$ with quantum numbers $J^{PC} = 1^{-+}$ (not allowed for a $q\bar{q}$ meson) was first observed by E852 at Brookhaven in 1998 but not confirmed elsewhere until 2010

Classes of possible exotic hadrons

- The rule for forming color-neutral singlets is simple:
  (# of q) – (# of $\bar{q}$) $= 0 \bmod 3$, & any number of g except one by itself
- gg, ggg, … (*glueball*)
- $q\bar{q}g$, $q\bar{q}gg$, … (*hybrid meson*)
- $q\bar{q}q\bar{q}$, $q\bar{q}q\bar{q}q\bar{q}$, … (*tetraquark*, *hexaquark*, …)
- $qqqq\bar{q}$, $qqqqqq q\bar{q}$, … (*pentaquark*, *octoquark*, …)
- $qqqqqq$, … (*dibaryon*, …)

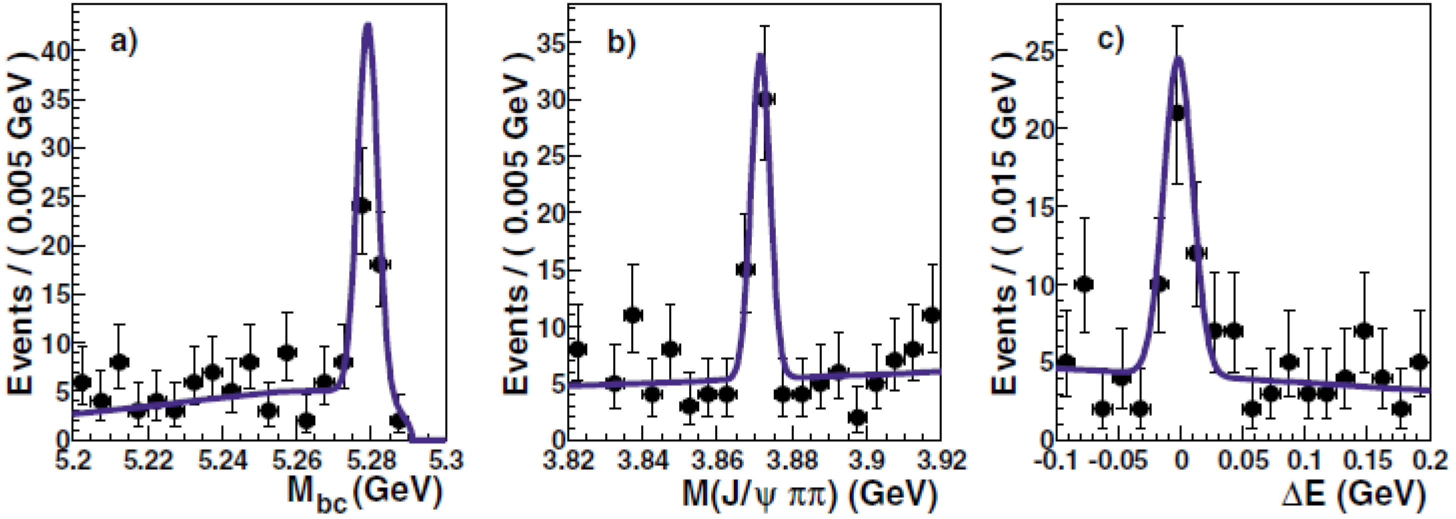
And then, in 2003…

The **Belle Collaboration** at KEK found evidence for a narrow new particle at 3872 MeV

In the broad mass range of charmonium, but it behaves *very unlike* a pure $c\bar{c}$ state

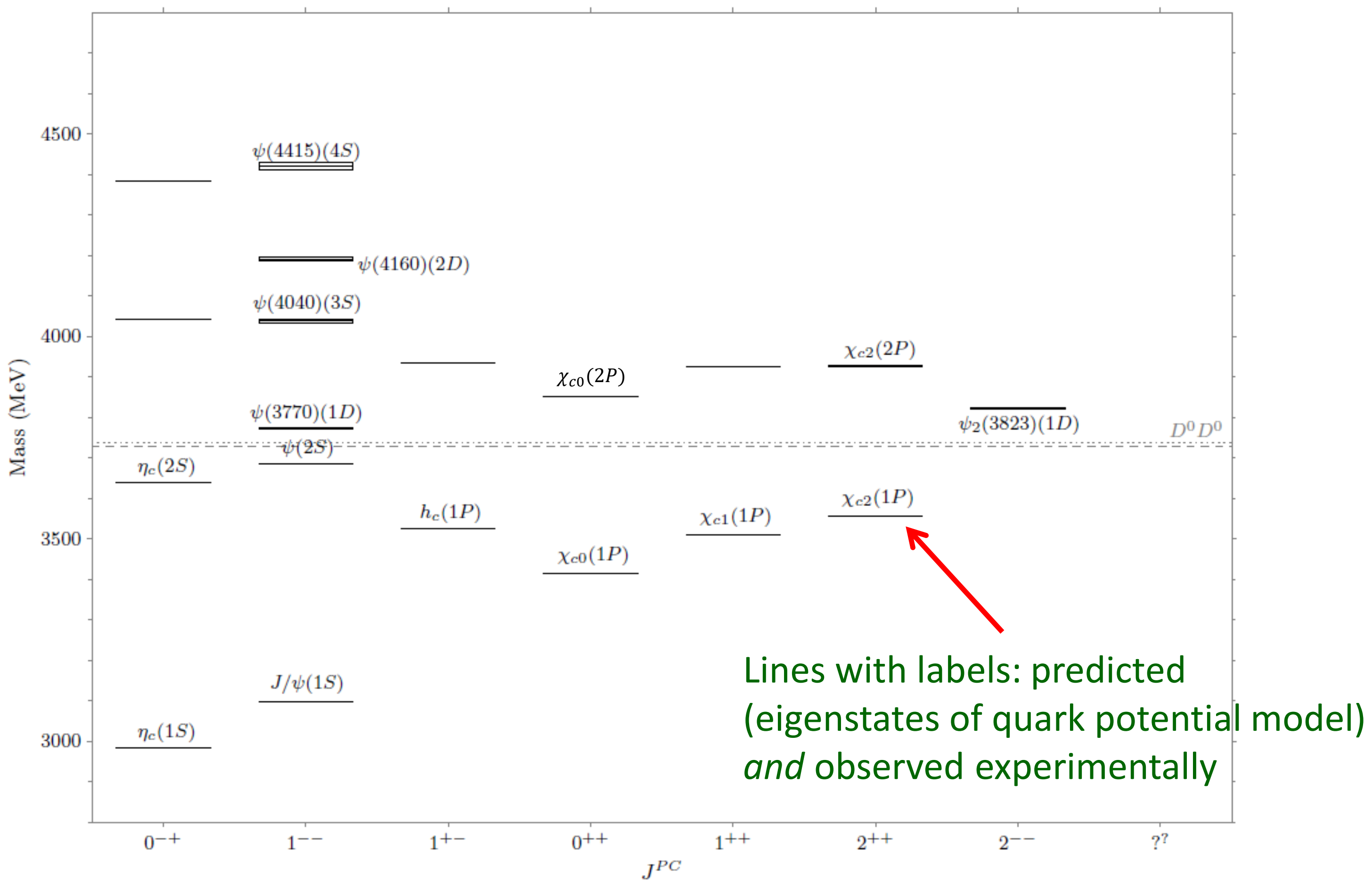
Almost certainly a hadron of valence quark content $c\bar{c}q\bar{q}$, has $J^{PC} = 1^{++}$

S.K. Choi *et al.*, Phys. Rev. Lett. **91** (2003) 262001

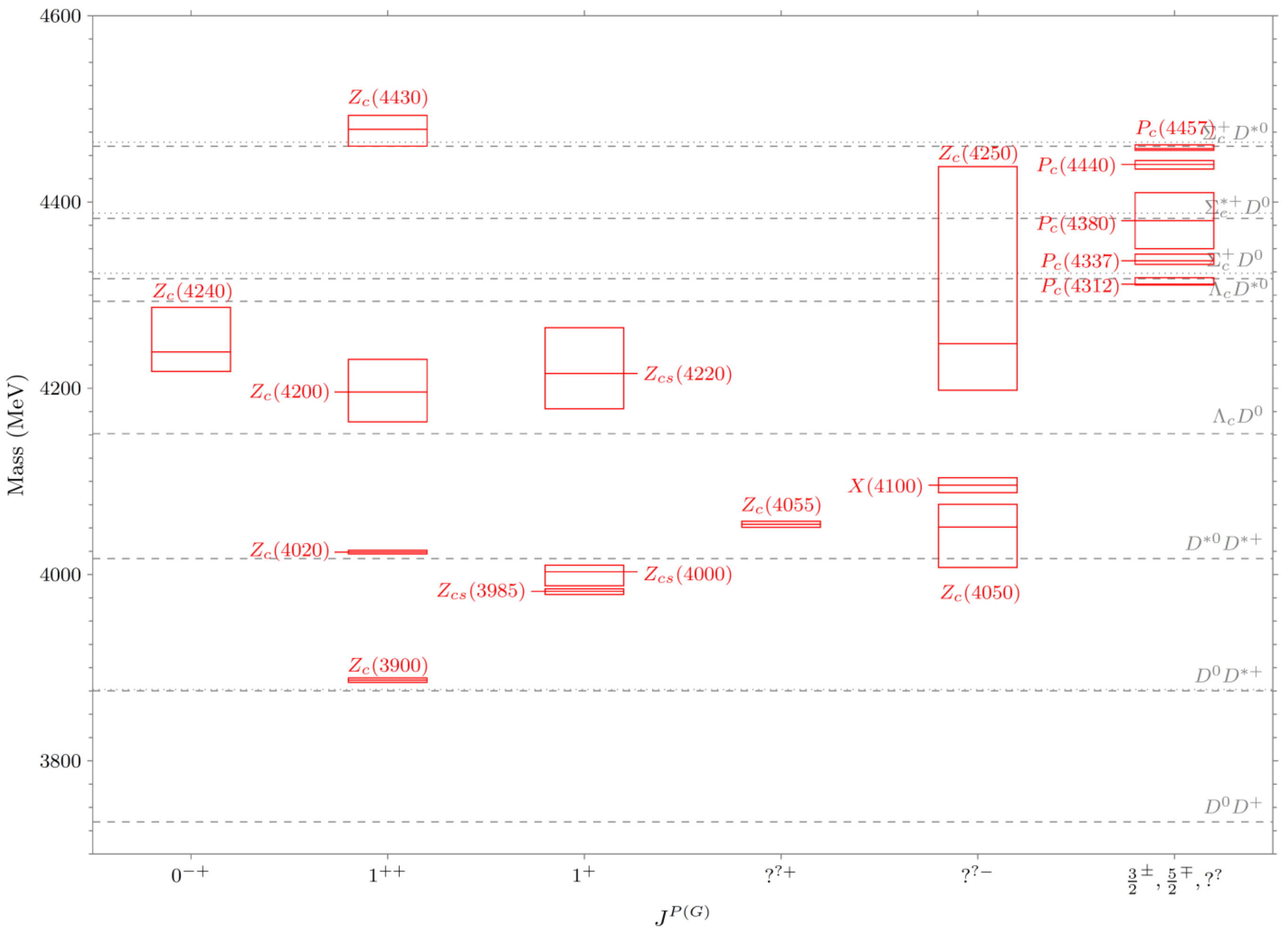


Reminder: The primary goal of Belle was the search for *CP* violation in the *B* system

What the charmonium system should look like



Charged hidden-charm sector: September 2022



Even the Naming Scheme Has to Be Updated

LHCb Collaboration, 2206.15233

- "I found a tetraquark hadron with valence-quark content c, s, u, d"

- "Which one? There are three kinds, not counting antiparticles:"
- $c\bar{s}u\bar{d}$: $T^{\mathrm{X}}_{c\bar{s}\,J}(\mathrm{mass})^{++}$
- $c\bar{s}\bar{u}d$: $T^{\mathrm{X}}_{c\bar{s}\,J}(\mathrm{mass})^{0}$
- $cs\bar{u}\bar{d}$: $T^{\mathrm{X}}_{cs\,J}(\mathrm{mass})^{0}$

J = total spin,
X = symbol for parity & isospin
(*e.g.*, $\mathrm{X} \to a$ for $P = +$, $I = 1$)

- Examples of all three of these types have already been observed!

The internal structure of exotics is unresolved

Mesons depicted here, but each model has a baryonic analogue

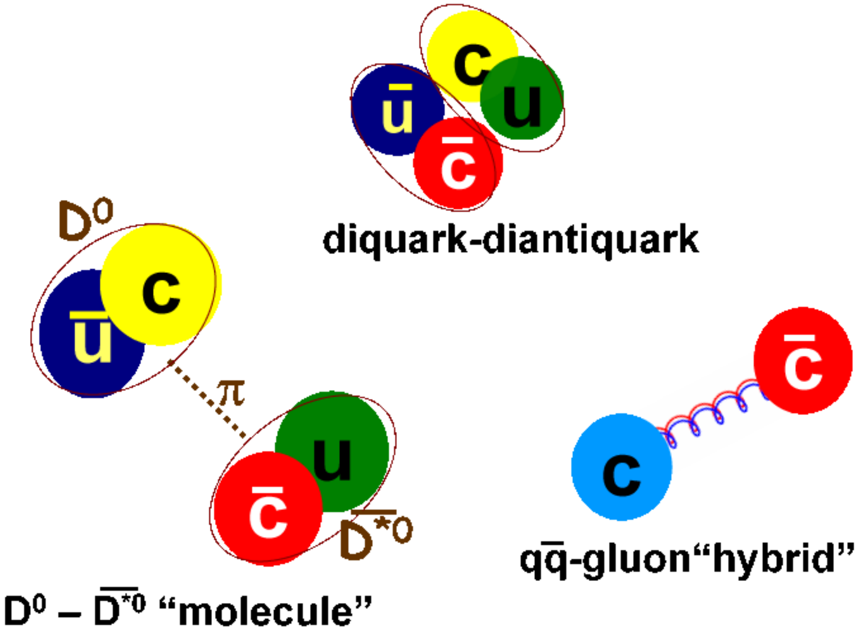
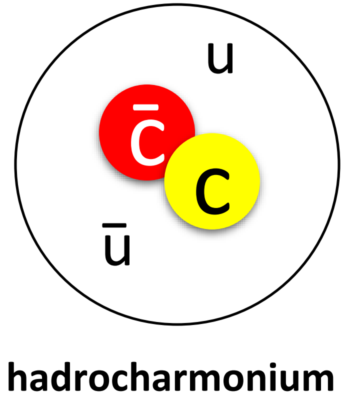
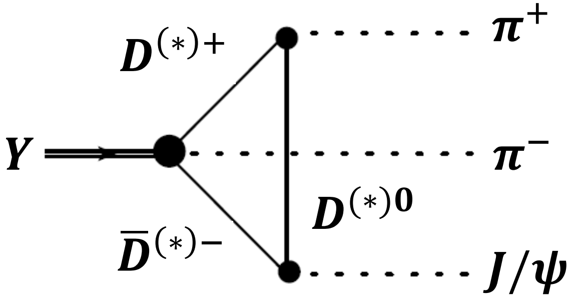


Image adapted from Godfrey & Olsen, Ann. Rev. Nucl. Part. Sci. **58** (2008) 51





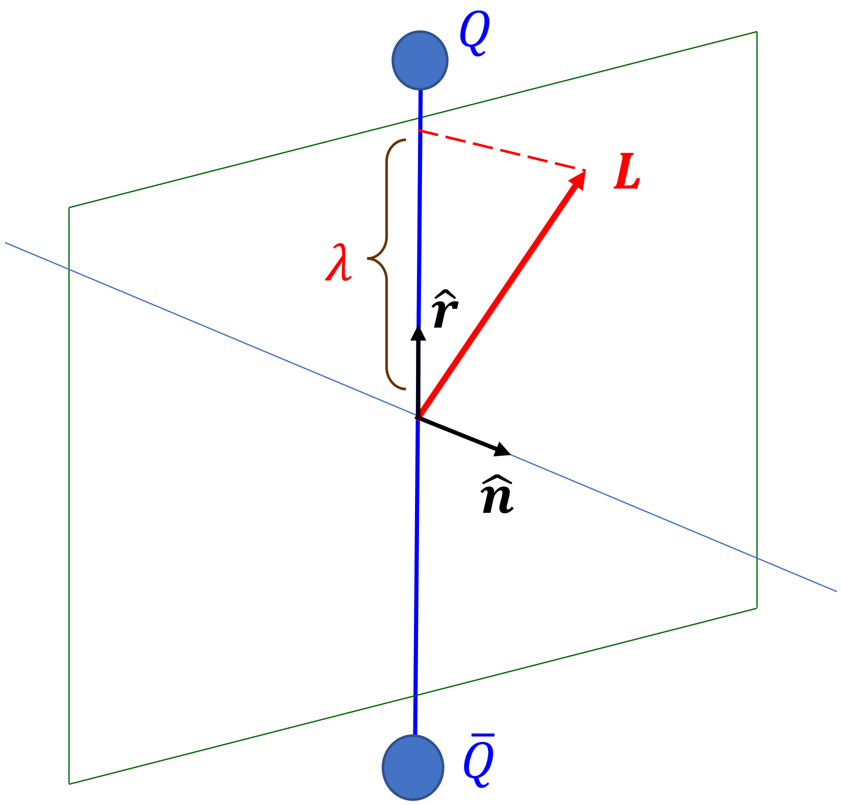
threshold/ rescattering/ cusp effect

“Each of the interpretations provides a natural explanation of parts of the data, but neither explains all of the data. It is quite possible that both kinds of structures appear in Nature. It may also be the case that certain states are superpositions of the compact and molecular configurations.”
—Karliner, Santopinto, *et al*. [Snowmass, “Substructure of Multiquark Hadrons”, 2203.16583]

What constitutes a diquark?

- Most general terms: A **diquark** is *any correlation* between a quark pair (color, flavor, spin, spatial—*any* entanglement)
- Typically treated as a *quasiparticle*
  (a bound state that would be free, if not for color confinement)
  that is a compact subcomponent of a hadron
- [Note: Particles can be correlated without being spatially compact!
  e.g., e^- Cooper pairs in superconductors need not be close together]
- Dynamics due to closely correlated quarks should be apparent in the
  interiors of baryons, but also in exotic hadrons,
  high-energy collisions, quark-gluon plasma, and neutron stars

BO quantum numbers for the "Homonuclear Diatomic" $Q\bar{Q}$ system



- Symmetry group is that of a cylinder, $D_{\infty h}$:
- Rotations about the axis $\hat{\boldsymbol{r}}$ (eigenvalues $\boxed{\lambda \equiv \hat{\boldsymbol{r}} \cdot \boldsymbol{L}}$)
- Reflection (R_{light}) through any plane containing the axis $\hat{\boldsymbol{r}}$ (eigenvalues $\boxed{\epsilon = \pm 1}$)
- Reflection through the origin (P_{light}) is *not* a symmetry since Q, $\bar{Q}$ not equivalent, but $(CP)_{\text{light}}$ *is* a symmetry (eigenvalues $\boxed{\eta = \pm 1}$, called g and u, respectively)

BO quantum numbers for the "Homonuclear Diatomic" $Q\bar{Q}$ system

- $\lambda \equiv \hat{\boldsymbol{r}} \cdot \boldsymbol{L}$ is a pseudoscalar:
  Invariant under rotations, odd under reflections
  Reflection R_{light} gives physically equivalent system, but $\lambda \to -\lambda$
  $\Longrightarrow$ Energy eigenvalue depends only upon $\Lambda \equiv |\lambda|$
- The BO potentials are thus labeled by Λ_η^ϵ $(\equiv \Gamma)$
  - $\Lambda = 0,1,2,\cdots$ are labeled, respectively, by the letters $\Sigma, \Pi, \Delta, \cdots$ (analogous to $S, P, D, \cdots$)
  - If heavy sources not truly "homonuclear" (*e.g.*, $b\bar{c}g$ or $c\bar{c}qqq$), then $(CP)_{\mathrm{light}}$ eigenvalue η is lost (only Λ^ϵ remains); symmetry group is that of a cone, $C_{\infty v}$
- Dynamical diquark model: Heavy sources interact via $V_\Gamma(r)$

Ordering of the BO potentials

- How do we know what are lowest, next lowest, *etc.* BO potentials? That's nonperturbative QCD!
- Calculation of hybrids with pure-glue configurations: the result of numerous lattice QCD simulations
- State-of-the-art results:
  Hadron Spectrum Collaboration, JHEP **1207** (2012) 126; **1612** (2016) 089
- But it has a very long history:
  Griffiths, Michael, Rakow: PLB **129B** (1983) 351
  Juge, Kuti, Morningstar: Nucl. Phys. Proc. Suppl. **63** (1998) 326;
  PRL **82**, (1999) 4400; PRL **90** (2003) 161601
  Bali *et al*.: PRD **62** (2000) 054503
  Bali, Pineda: PRD **69** (2004) 094001
  Foster *et al*.: PRD **59** (1999) 094509
  Marsh, Lewis: PRD **89** (2014) 014502
  Capitani *et al*.: PRD **99** (2019) 034502

Ordering of the BO potentials

- But all pure-glue simulations agree:
  - Ground-state potential: Σ_g^+
  - 1st excited potential: Π_u; 2nd excited potential: Σ_u^-
- For small heavy-source separations, some potentials become degenerate gluelumps and mix, *e.g.*, $\Pi_u^+(1P)$ and $\Sigma_u^-(1P)$
  [Λ doubling: Berwein *et al.*, PRD **92** (2015) 114019]
- **Strategy** [Jesse F. Giron, RFL, Curtis T. Peterson JHEP **05** (2019) 061]:
  - Choose a particular diquark mass m_δ and a BO potential Γ whose interaction potential $V_\Gamma(r)$ is calculated on the lattice
  - Feed into Schrödinger equation solver (possibly coupled equations), obtain energy eigenvalues for levels such as $\Sigma_g^+(2D)$, $\Pi_u^+(1P)$, *etc.*
  - Adjust diquark mass to match some observed state [*e.g.*, $X(3872)$] to some eigenvalue, and see if predicted spectrum matches other states

Exotics spectroscopy using BO potentials

RFL, Phys. Rev. D **96** (2017), 116003

- Given quantum numbers of the light d.o.f., combine with the heavy quantum numbers to find the full spectrum of states
- Most efficient: Re-express diquark spins $(s_\delta, s_{\bar{\delta}})$ in terms of spin carried by light quarks $(s_{q\bar{q}})$, by heavy quarks $(s_{Q\bar{Q}})$, and by total spin carried by all quarks (S) [using $9j$ symbols]
- Tetraquark discrete symmetry quantum numbers:
$$P = \epsilon(-1)^{\Lambda+L}, \qquad C = \eta\epsilon(-1)^{\Lambda+L+s_{q\bar{q}}+s_{Q\bar{Q}}}$$
- Pentaquark discrete symmetry quantum numbers:
$$P = \epsilon(-1)^{\Lambda+L+1}, \qquad C \text{ no longer good}$$
- Build multiplets based on BO potentials (including radial n and orbital L quantum numbers) and states classified using $s_{q\bar{q}}, s_{Q\bar{Q}}, S$

Exotics spectroscopy using BO potentials:

Pentaquarks

BO potential	State notation State J^P	
$\Sigma^+(1S)$	$\tilde{P}^{(\frac{1}{2})+}_{\frac{1}{2}S}$, $\tilde{P}'^{(\frac{1}{2})+}_{\frac{1}{2}S}$ $2\times\frac{1}{2}^-$	$P^{(\frac{3}{2})+}_{\frac{3}{2}S}$ $\frac{3}{2}^-$
$\Sigma^+(1P)$	$\left[\tilde{P}^{(\frac{1}{2}),(\frac{3}{2})}_{\frac{1}{2}P}\right]^+$, $\left[\tilde{P}'^{(\frac{1}{2}),(\frac{3}{2})}_{\frac{1}{2}P}\right]^+$ $2\times\left(\frac{1}{2},\frac{3}{2}\right)^+$	$\left[P^{(\frac{1}{2}),(\frac{3}{2}),(\frac{5}{2})}_{\frac{3}{2}P}\right]^+$ $\left(\frac{1}{2},\frac{3}{2},\frac{5}{2}\right)^+$
$\Sigma^+(1D)$	$\left[\tilde{P}^{(\frac{3}{2}),(\frac{5}{2})}_{\frac{1}{2}D}\right]^+$, $\left[\tilde{P}'^{(\frac{3}{2}),(\frac{5}{2})}_{\frac{1}{2}D}\right]^+$ $2\times\left(\frac{3}{2},\frac{5}{2}\right)^-$	$\left[P^{(\frac{1}{2}),(\frac{3}{2}),(\frac{5}{2}),(\frac{7}{2})}_{\frac{3}{2}D}\right]^+$ $\left(\frac{1}{2},\frac{3}{2},\frac{5}{2},\frac{7}{2}\right)^-$
$\Pi^+(1P)$ & $\Sigma^-(1P)$	$\left[\tilde{P}^{(\frac{1}{2}),(\frac{3}{2})}_{\frac{1}{2}P}\right]^-$, $\left[\tilde{P}'^{(\frac{1}{2}),(\frac{3}{2})}_{\frac{1}{2}P}\right]^-$ $2\times\left(\frac{1}{2},\frac{3}{2}\right)^-$	$\left[P^{(\frac{1}{2}),(\frac{3}{2}),(\frac{5}{2})}_{\frac{3}{2}P}\right]^-$ $\left(\frac{1}{2},\frac{3}{2},\frac{5}{2}\right)^-$
$\Pi^-(1P)$	Same as $\Sigma^+(1P)$	
$\Sigma^-(1S)$	$\tilde{P}^{(\frac{1}{2})-}_{\frac{1}{2}S}$, $\tilde{P}'^{(\frac{1}{2})-}_{\frac{1}{2}S}$ $2\times\frac{1}{2}^+$	$P^{(\frac{3}{2})-}_{\frac{3}{2}S}$ $\frac{3}{2}^+$
$\Pi^+(1D)$	$\left[\tilde{P}^{(\frac{3}{2}),(\frac{5}{2})}_{\frac{1}{2}D}\right]^-$, $\left[\tilde{P}'^{(\frac{3}{2}),(\frac{5}{2})}_{\frac{1}{2}D}\right]^-$ $2\times\left(\frac{3}{2},\frac{5}{2}\right)^+$	$\left[P^{(\frac{1}{2}),(\frac{3}{2}),(\frac{5}{2}),(\frac{7}{2})}_{\frac{3}{2}D}\right]^-$ $\left(\frac{1}{2},\frac{3}{2},\frac{5}{2},\frac{7}{2}\right)^+$

e.g.,

$$\tilde{P}_{\frac{1}{2}} \equiv \left|\frac{1}{2}_{s_{qqq}}, 0_{s_{Q\bar{Q}}}\right\rangle_{S=\frac{1}{2}}$$

Phenomenological modeling

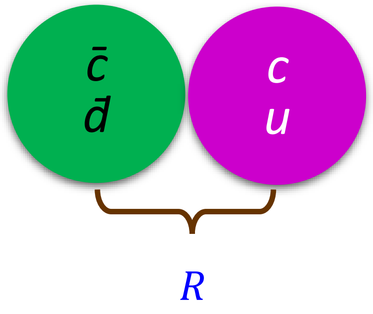
[Snowmass white paper, 2203.16583]

- The internal structure of heavy-quark exotics is not yet resolved, so multiple approaches must continue to be developed
- Heavy-quark ($m_Q \gg \Lambda_{\mathrm{QCD}}$) hadrons (especially multiquark exotics) admit features not available for light-quark ones:
- Usually fewer decay modes (hence narrower); anomalous decay modes (*e.g.*, $X(3872) \to J/\psi\,\rho$); small KE for m_Q hence heavy-quarks nucleates quark clusters (Hadronic molecules? Diquark compounds?)
- Large m_Q allows for scale separation from lighter d.o.f.: effective field theory, Born-Oppenheimer approximation

Does finite diquark size matter?

[Jesse F. Giron, RFL, Curtis T. Peterson JHEP **01** (2020) 124]

- Crude calculation:
  Model the diquarks to have no wave function overlap when their centers exceed some distance R apart
  (*i.e.*, hard spheres of radius $R/2$)



- Pointlike assumption is $R = 0$
- For $r < R$, use same potential $V_\Gamma(r)$ as in original calculation to act only on the heavy quarks, while light valence quarks become inert contribution to state mass
- Result: Spacings between mass multiplets only change by $30 - 40\ \mathrm{MeV}$ until $R{\sim}0.8\ \mathrm{fm}$
  $\Longrightarrow$ Diquark radii up to $0.4\ \mathrm{fm}$ lead to relatively small changes

Fine structure of the multiplets: The model

- All that is known about the states in the $\Sigma_g^+(1S)$ multiplet can be incorporated using a 3-parameter Hamiltonian:

$$H = M_0 + 2\kappa_{qQ}(\mathbf{s}_q \cdot \mathbf{s}_Q + \mathbf{s}_{\bar{q}} \cdot \mathbf{s}_{\bar{Q}}) + V_0\, \boldsymbol{\tau}_q \cdot \boldsymbol{\tau}_{\bar{q}}\, \boldsymbol{\sigma}_q \cdot \boldsymbol{\sigma}_{\bar{q}}$$

Common multiplet mass (as computed above)

Internal diquark spin-spin coupling

Isospin-spin coupling of light quarks (same form as in πNN exchange)

- In addition, pairs of states with degenerate J^{PC} can mix:

$$0^{++} \quad \begin{pmatrix} \bar{X}_0 \\ \bar{X}_0' \end{pmatrix} = \begin{pmatrix} \cos\theta_X & \sin\theta_X \\ -\sin\theta_X & \cos\theta_X \end{pmatrix} \begin{pmatrix} X_0 \\ X_0' \end{pmatrix}$$

$$1^{+-} \quad \begin{pmatrix} \bar{Z} \\ \bar{Z}' \end{pmatrix} = \begin{pmatrix} \cos\theta_Z & \sin\theta_Z \\ -\sin\theta_Z & \cos\theta_Z \end{pmatrix} \begin{pmatrix} Z \\ Z' \end{pmatrix}$$

Fine structure in the $P = -$ states

[Giron & RFL, PRD **101** (2020) 074032]

- Hamiltonian for $L > 0$ states like $\Sigma_g^+(1P)$ requires 2 additional operators, spin-orbit and tensor:

$$\Delta H_{LS} = V_{LS}\, \mathbf{L} \cdot \mathbf{S} \quad , \qquad \Delta H_T = V_T\, \boldsymbol{\tau}_q \cdot \boldsymbol{\tau}_{\bar{q}}\, S_{12}^{(q\bar{q})},$$

$$S_{12} \equiv 3\, \boldsymbol{\sigma}_1 \cdot \boldsymbol{r}\, \boldsymbol{\sigma}_2 \cdot \boldsymbol{r}/r^2 - \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$$

- Since only 5 Hamiltonian parameters for 28 states (as well as mixing angles, *e.g.*, 3 for the 4 Y states), the system is already almost completely constrained by data
- Example: The fits with lowest χ^2 predict the sole $\Sigma_g^+(1P)$ 0^{--}, $I = 1$ state to match the candidate $Z_c(4240)$ [seen in LHCb's $Z_c(4430)$ paper PRL **112** (2014) 222002]

Fine structure in $c\bar{c}s\bar{s}$ and $b\bar{b}q\bar{q}$ states

[Giron & RFL, PRD **102** (2020) 014036]

- Several exotic candidates [$Y(4140), Y(4274), X(4350), X(4500), Y(4626), X(4700)$] so far seen only to decay to $J/\psi\ \phi$ or to $D_s\bar{D}_{sJ}$,
  hence are natural $c\bar{c}s\bar{s}$ candidates
- Furthermore, $X(3915)$ (likely 0^{++}) is a weird state:
  Does not fit well as $c\bar{c}$ or $c\bar{c}q\bar{q}$ (no open-charm decays)
  Proposed lowest $c\bar{c}s\bar{s}$ state [RFL & Polosa, PRD **93** (2016) 094024]
- 0 isospin$\Longrightarrow$ $\Sigma_g^+(1S)$: only 2 Hamiltonian parameters, 6 states
- $X(3915), Y(4140), X(4350)$ fit well in $c\bar{c}s\bar{s}$ $\Sigma_g^+(1S)$ multiplet
- But $Y(4274)$ does not! Fits well as conventional $\chi_{c1}(3P)$

Fine structure in $c\bar{c}s\bar{s}$ and $b\bar{b}q\bar{q}$ states

[Giron & RFL, PRD **102** (2020) 014036]

- For $b\bar{b}q\bar{q}$, only known $\Sigma_g^+(1S)$ ($P = +$) candidates are $Z_b(10610)$ & $Z_b(10650)$, both $J^{PC} = 1^{+-}$, $I = 1$
- But here one has an important extra piece of information: Both Z_b's decay to $\Upsilon(1S), \Upsilon(2S), \Upsilon(3S)$ and to $h_b(1P), h_b(2P)$
- And $Z_b(10610)$ prefers Υ ($s_{b\bar{b}} = 1$), while $Z_b(10650)$ prefers h_b ($s_{b\bar{b}} = 0$), by roughly a $3:1$ ratio $\Rightarrow$ 2 masses and $P_{s_{b\bar{b}}=1}\big(Z_b(10610)\big)$ are enough to fix all 3 Hamiltonian parameters and all 12 $\Sigma_g^+(1S)$ masses:

J^{PC}	$I=0$		$I=1$	
0^{++}	10569.7	10695.7	10575.4	10644.9
1^{++}	10598.9		10621.4	
1^{+-}	10613.2	10691.1	**10607.2**	**10652.2**
2^{++}	10637.9		10660.4	

(MeV)

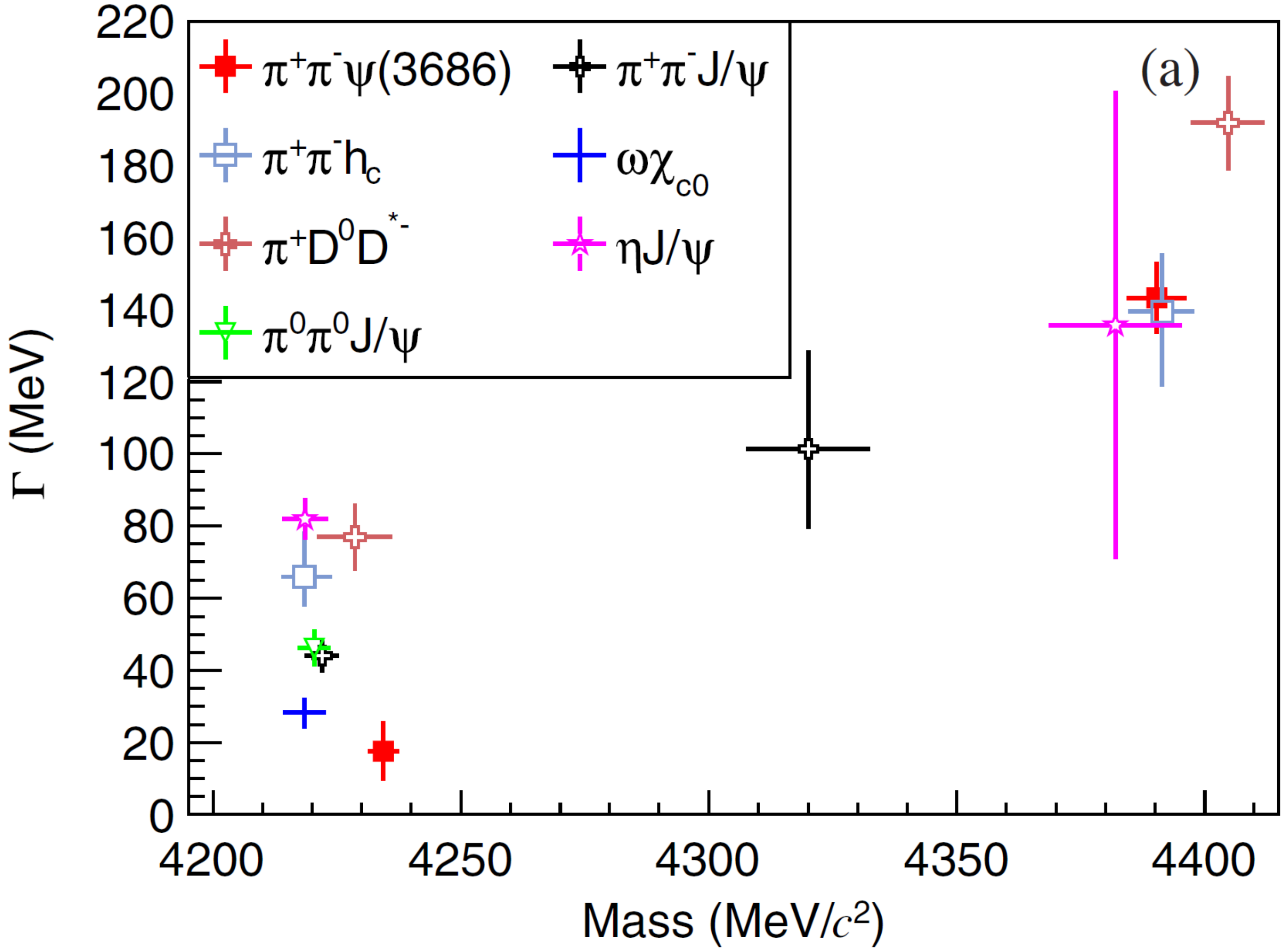
The orbitally excited $\Sigma_g^+(1P)$ multiplet

Giron, RFL [PRD **101**, 074032 (2020)]

- The lightest negative-parity states (like Y) live here:
  14 with $I = 0$, 14 with $I = 1$
- Multiplet $\Sigma_g^+(1P)$ contains precisely $4\ J^{PC} = 1^{--}$, $I = 0$ (Y) states
- Analysis requires 2 more Hamiltonian operators: spin-orbit and tensor
- But which Y states are experimentally confirmed?
  BESIII data is rapidly improving (next slide), but still has ambiguities
- Our analysis predicts full multiplet under several possible assignments:
  e.g., using $Y(4220)$, $Y(4320)$, $Y(4390)$ as inputs predicts
  the $Z_c(4240)$ [$J^{PC} = 0^{--}$, $I = 1$] seen by LHCb [PRL **112**, 22202 (2014)]

BESIII is working to nail down the Y states

PRD **104**, 052012 (2021)



If the model is any good, it must also apply to other flavor sectors

- Using the same Hamiltonian operators, apply to:

- the $(bq)(\bar{b}\bar{q})$ sector {Giron, RFL [PRD **102**, 014036 (2020)]}
  Here, just masses of $Z_b(10610)$, $Z_b(10650)$ ($J^{PC} = 1^{+-}$, $I = 1$) & their B.R.'s to $\Upsilon(nS)$, $h_b(nP)$ are sufficient to predict whole $\Sigma_g^+(1S)$ multiplet
- the $(cs)(\bar{c}\bar{s})$ sector {Giron, RFL [PRD **102**, 014036 (2020)]}
  Here, the 0^{++} $X(3915)$ (peculiar: no open-charm decay) is lowest state, $X(4140)$ ($J^{PC} = 1^{++}$, appears in $J/\psi\,\phi$) is analogue of $X(3872)$, and all other $\Sigma_g^+(1S)$ states are predicted because the Hamiltonian has one less operator (zero isospin!)

If the model is any good, it must also apply to other flavor sectors

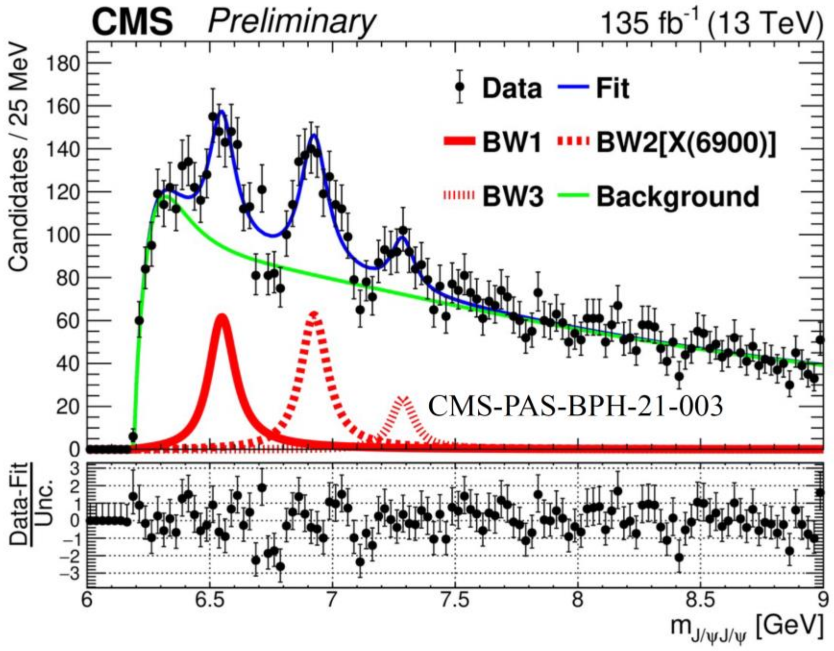
- Using the same Hamiltonian operators, apply to:

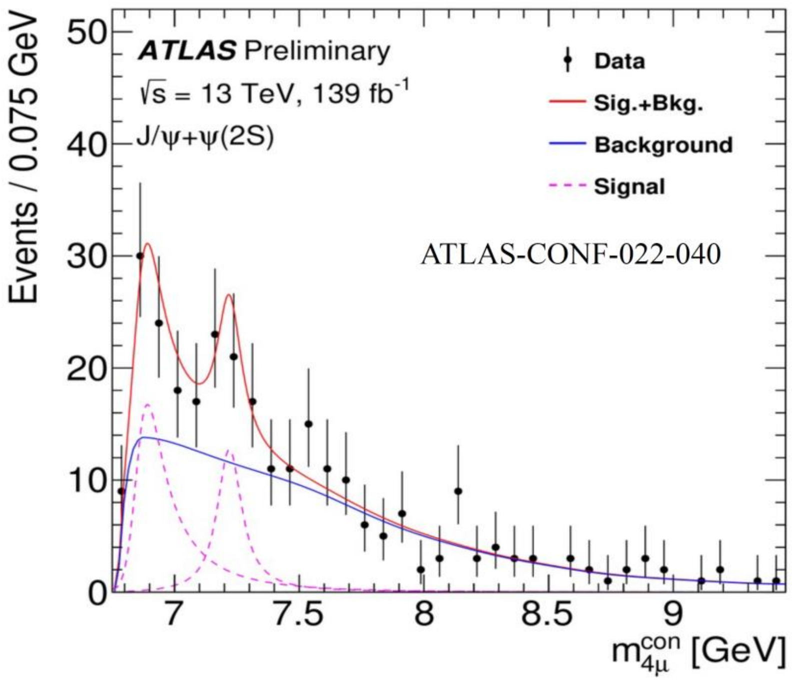
- the $(cq)(\bar{c}\bar{s})$ sector {Giron, Martinez, RFL [PRD **104**, 054001 (2021)]}
  Here, LHCb's recently observed $Z_{cs}(4000)$, $Z_{cs}(4220)$ [PRL **127**, 082001 (2021)] belong to $SU(3)_{\text{flavor}}$ multiplets of $J^{PC} = 1^{++}$ and 1^{+-}, but their strange members can mix, like K_{1A}, K_{1B}
- the $(cu)(\bar{c}ud)$ and $(cs)(\bar{c}ud)$ pentaquarks {Giron, RFL [PRD **104**, 114028 (2021)]}
  Here, all the known nonstrange states: $P_c(4312)$, $P_c(4337)$, $P_c(4450)$, $P_c(4457)$, are easily accommodated

If the model is any good, it must also apply to other flavor sectors

- Using the same Hamiltonian operators, apply to:
- the $(cc)(\bar{c}\bar{c})$ sector {Giron, RFL [PRD **102**, 074003 (2020)]}
  Here, identical particle constraints limit allowed quantum numbers:
  Only 3 $\Sigma_g^+(nS)$ & 7 $\Sigma_g^+(nP)$ states
- Not easy to describe $(cc)(\bar{c}\bar{c})$ states using "conventional" di-hadron molecule picture—diquarks are most plausible explanation
- Our calculations indicate $X(6900) \to J/\psi\, J/\psi$ [Sci. Bull. **65**, 1983 (2020)] observed by LHCb almost certainly in excited orbital, most likely $\Sigma_g^+(2S)$
- So where are the lower $(cc)(\bar{c}\bar{c})$ states?
  Preliminary results from CMS and ATLAS indicate seeing them!

There will be lots of $(cc)(\bar{c}\bar{c})$ hadrons!





from
Matt Durham's
Snowmass talk

- CMS confirms X(6900) and finds other peaks in di-J/ψ mass spectrum
- ATLAS also confirms X(6900) and finds structure in $\psi(2S) + J/\psi$ mass
- **Rich spectrum of $c\bar{c}c\bar{c}$ states accessible with dimuons.**

Diabatic mixed potential

$$\mathrm{V}(r) = \begin{pmatrix} V_{\delta\bar{\delta}}(r) & V_{\mathrm{mix}}(r) \\ V_{\mathrm{mix}}(r) & T_{M_1\overline{M}_2} \end{pmatrix}$$

$$V_{\delta\bar{\delta}}(r) = -\frac{\alpha}{r} + \sigma r + V_0 + m_\delta + m_{\bar{\delta}}$$

$$T_{M_1\overline{M}_2} \equiv m_{M_1} + m_{\overline{M}_2}$$

$$|V_{\mathrm{mix}}^{(i)}(r)| = \frac{\Delta}{2}\exp\left\{-\frac{1}{2}\frac{\left[V_{\delta\bar{\delta}}(r) - T_{M_1\overline{M}_2}^{(i)}\right]^2}{(\rho\sigma)^2}\right\}$$