Developments in Determinations of the Neutron Star Equation of State from Observations

J. M. Lattimer

Finite-Temperature Effects in Multi-Messenger Astrophysics
INT-26-96W, Seattle, WA
15-19 June 2026

Acknowledgements

Funding Support:

DOE - Nuclear Physics

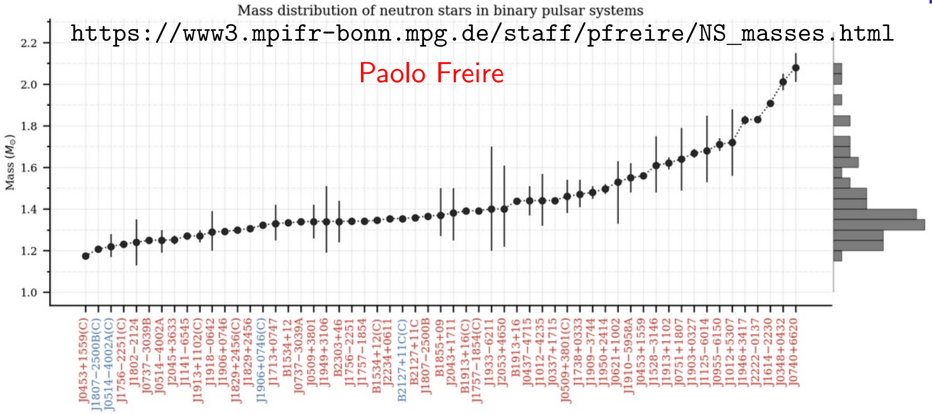
NASA - Neutron Star Interior Composition ExploreR (NICER)

NSF - Neutrinos, Nuclear Astrophysics and Symmetries (PFC - N3AS)

Recent Collaborators:

Boyang Sun (Stony Brook), Tianqi Zhao (Berkeley), Sophia Han (TDLI), Probit Kalita and Bharat Kumar (Rourkela, NIT), Tuhin Malik(Coimbra U.)

Masses of Pulsars in Binaries from Pulsar Timing



https://www3.mpifr-bonn.mpg.de/staff/pfreire/NS_masses.html

Paolo Freire

Largest: 2.08 ± 0.07 $M_\odot$
Smallest: 1.174 ± 0.004 $M_\odot$

Several other NS masses have been measured by other means, including some estimated to be more than $2M_\odot$ (e.g., black widow pulsars) and smaller than $1M_\odot$ (HESS J1731-347), but their mass uncertainties are generally large and even disputed.

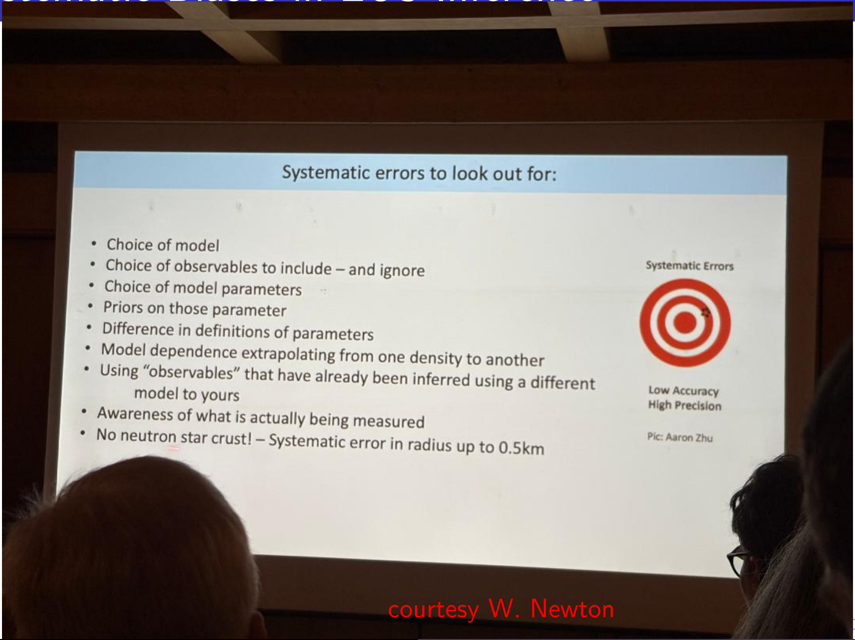
What is the Maximum Mass?

Minimum Maximum Mass

- PSR B1516+02B (Freire et al. 2008) $M = 2.08 \pm 0.19\ M_\odot$
- PSR J1614+2230 (Arzoumanian et al. 2018) $M = 1.908 \pm 0.016 M_\odot$
- PSR J1748-2021B (Clifford 2019) $M = 2.53^{+0.05}_{-0.08}\ M_\odot$ (total mass is $2.675 \pm 0.022 M_\odot$, but inclination is unknown)
- PSR J0348+0432 (Saffer et al. 2025) $M = 1.806 \pm 0.037\ M_\odot$
- PSR 1957+20 (van Kerkwijk 2010) $M = 2.4 \pm 0.3\ M_\odot$; black widow pulsar (BWP)
- PSR J1311-3430 (Romani et al. 2012) $M = 2.22 \pm 0.10 M_\odot$ BWP
- PSR J1544+4937 (Tang et al. 2014) $M = 2.06 \pm 0.56! M_\odot$ BWP
- PSR 2FGL J1653.6-0159 (Romani et al. 2014) $M > 1.96\ M_\odot$
- PSR J1227-4859 (de Martino et al. 2014) $M = 2.2 \pm 0.8\ M_\odot$ redback pulsar.
- PSR J0740+6620 (Fonseca et al. 2021) $M = 2.08 \pm 0.07\ M_\odot$
- PSR J0952-0607 (Romani et al. 2022) $M = 2.35 \pm 0.17 M_\odot$ BWP
- PSR J0514-4002E(C) (Barr et al. 2024) $M = 2.40 \pm 0.31\ M_\odot$ companion to $1.49 \pm 0.31\ M_\odot$ pulsar

Maximum Maximum Mass from GW170817: $\simeq$ 2.2-2.3 $M_\odot$?

Systematic Biases in EOS Inference



courtesy W. Newton

Extremal Properties of Neutron Stars

Rhoades & Ruffini (1974) examined the problem of determining the maximum neutron star mass from the TOV equations using control theory. The star is divided into two regions:
1) a low-density outer region with $\mathcal{E} < \mathcal{E}_{low}$ having a mass M_{low}, which includes the crust and the region close to the nuclear saturation density where the EOS is well-known;
2) the neutron star core with mass $M - M_{low}$.

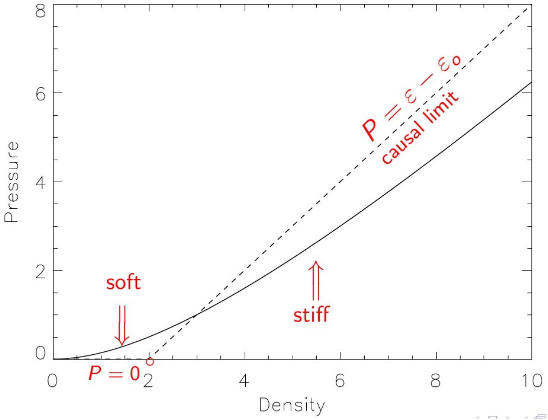
They chose $\mathcal{E}$ as the independent variable, and identified an EOS control variable $u = c_s^2/c^2 \leq 1$ to satisfy causality. The structure equations are linear in u:

$$\frac{dm}{d\mathcal{E}} = 4\pi r^2 \mathcal{E} u, \quad \frac{dr}{d\mathcal{E}} = -\frac{c^2}{G}\frac{r(rc^2 - 2GM)}{(\mathcal{E} + P)(mc^2 + 4\pi r^3 P)}u, \quad \frac{dP}{d\mathcal{E}} = u.$$

Pontryagin's Principle states a necessary condition to maximize M for a given $\mathcal{E}_c$ is that u can only be 0 or 1 for $\mathcal{E} \geq \mathcal{E}_{low}$.

We recently proved the sufficient condition requires $u = 1$ for $\varepsilon \geq \varepsilon_{low}$, and that this also maximizes R for a given $\mathcal{E}_c$, or M.

The theoretical maximum occurs when $P = 0$ for $\varepsilon \leq \varepsilon_0$, producing a self-bound star.



ε_o is the only EOS parameter

The TOV solutions scale with ε_o

$w = \varepsilon/\varepsilon_o$
$y = P/\varepsilon_o = w - 1$
$x = r\sqrt{G\varepsilon_o}/c^2$
$z = m\sqrt{G^3\varepsilon_o}/c^2$

Prakash & Lattimer (2011) showed the maximum mass occurs in the self-bound case ($P_o = 0$) when $\mathcal{E}_c = 3.034\mathcal{E}_o$ or $P_c = 2.034\mathcal{E}_o$.

Maximum pssible compactness occurs when $\beta = \dfrac{GM}{Rc^2} = \dfrac{1}{2.824}$.

Maximum baryon chemical potential $\mu_c = \ln(5.068)\mu_o \simeq 2.09$ GeV.

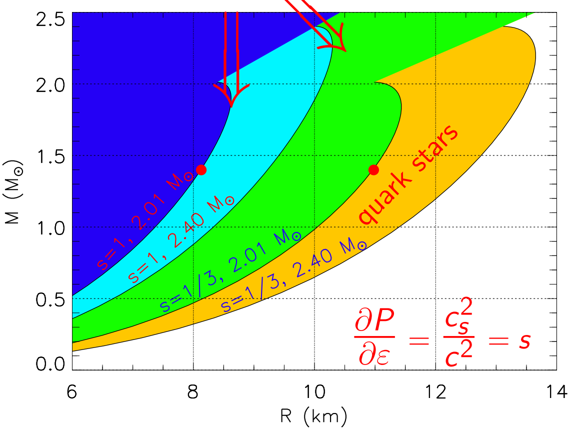
Causality + GR Limits and the Maximum Mass

A lower limit to the maximum mass sets a lower limit to the radius for a given mass.

Similiarly, a precise (M, R) measurement sets an upper limit to the maximum mass.

$1.4M_\odot$ stars must have $R > 8.15M_\odot$.

$M - R$ curves for maximally compact EOS



Self-bound case

No crust ($P = 0$ for $\mathcal{E} \leq \mathcal{E}_o$)

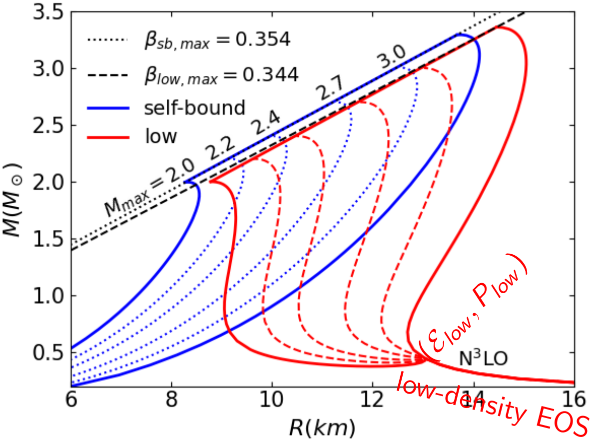
$M - R$ Limits; Neutron Star Crust and Neutron Matter

A different *bang-bang* solution minimizes R for a given ε_c or M.

In this case, $u = 0$ from ε_{low} to ε_t, and $u = 1$ for higher densities.

Each value of ε_t has an associated M_{max} (intermediate red dashed curves).

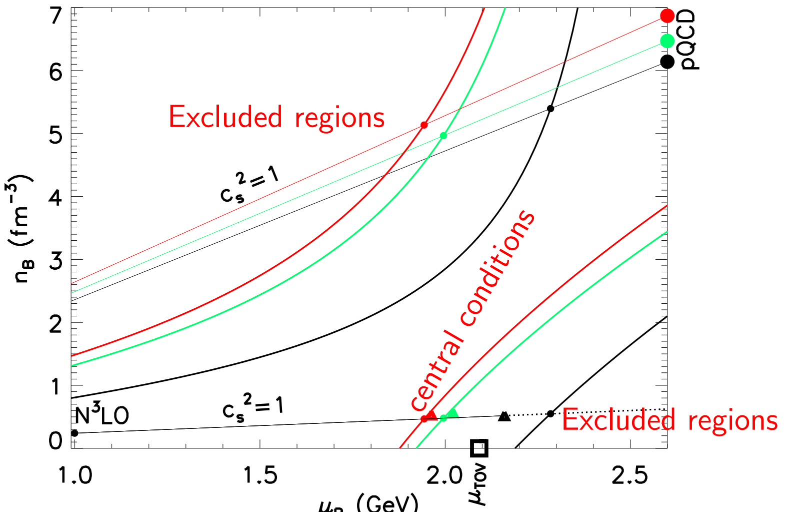
The true minimum radius for a given mass obtains when M_{max} is set to M_{min} $\simeq 2M_\odot$ (left-most red bound).



Self-bound case ($P_o = 0$)

With low density EOS $0 < \mathcal{E} < \mathcal{E}_{low}$

Could pQCD Impose Additional Constraints?

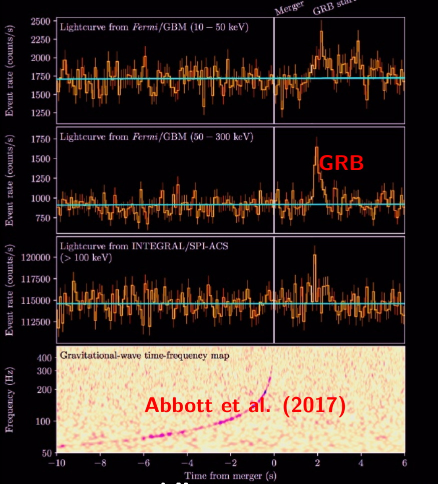


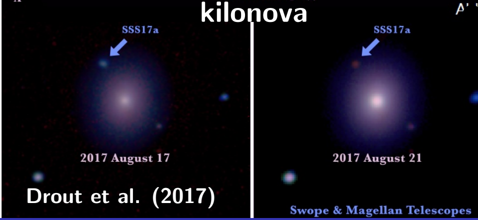
How Can a Neutron Star's Radius Be Measured?

- X-ray observations of thermal emission from quiescent and bursting neutron stars
- X-ray phase-resolved spectroscopy of millisecond pulsars (NICER, NewAthena)
- Gravitational wave observations of merging neutron stars
- Pulsar timing of relativistic binary systems with spin-orbit coupling
- Gamma-ray bursters showing quasi-periodic oscillations
- Quasi-periodic oscillations seen in some accreting neutron star binaries

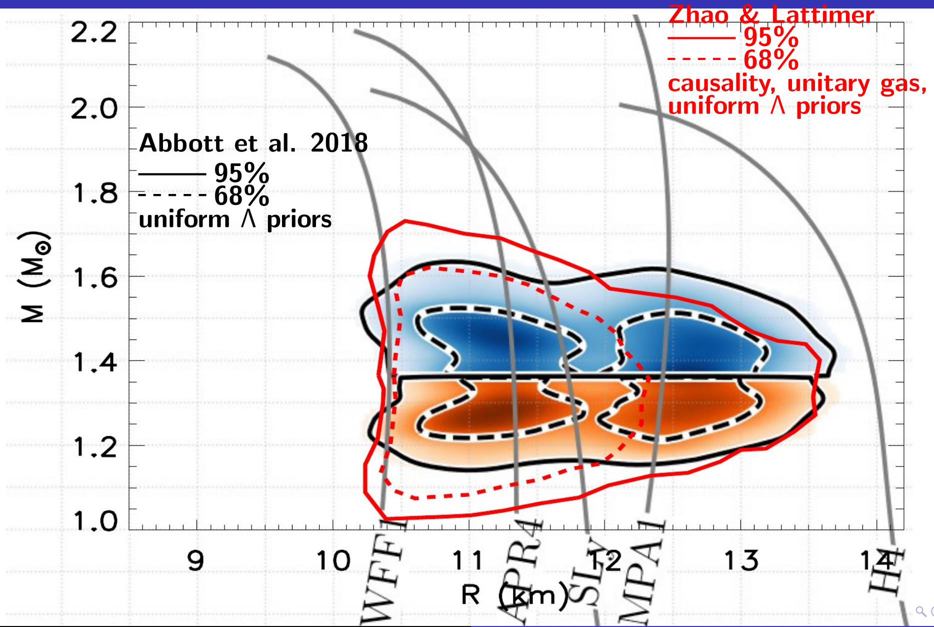
GW170817

- LVC detected a signal consistent with a BNS merger, followed 1.7 s later by a weak gamma-ray burst.
- $\sim$ 10100 orbits observed over 317 s.
- $\mathcal{M} = 1.186 \pm 0.001\ M_\odot$
- $M_\mathrm{T} = M_A + M_B \gtrsim 2^{6/5}\mathcal{M} = 2.725 M_\odot$
- $E_\mathrm{GW} > 0.025 M_\odot c^2$
- $D_L = 40^{+8}_{-14}$ Mpc
- $75 < \tilde{\Lambda} < 560$ (10.9 km$< \bar{R} < 13.3$ km)
- $M_\mathrm{ejecta} \sim 0.06 \pm 0.02\ M_\odot$
- Blue ejecta: $\sim 0.01 M_\odot$
- Red ejecta: $\sim 0.05 M_\odot$
- Highly opaque ejecta implies substantial r-process production
- M_T+Ejecta+GRB: $M_{max} \lesssim 2.22 M_\odot$

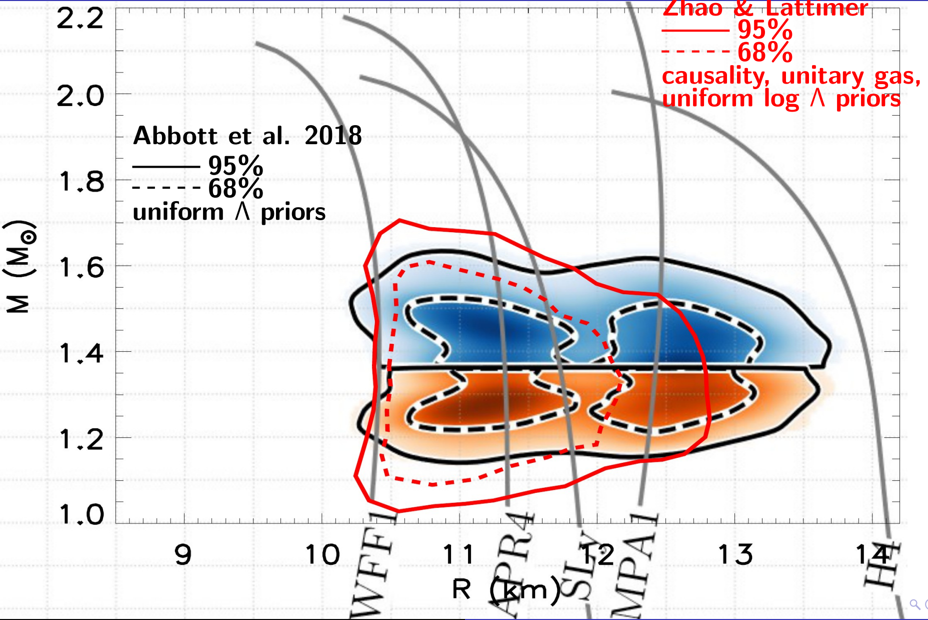




Systematic Biases in Deformability Extractions



Systematic Biases in Deformability Extractions



Summary of NICER Observations

PSR J0030+0451

$M = 1.34^{+0.15}_{-0.16} M_\odot$, $R = 12.71^{+1.14}_{-1.79}$ km [Riley et al. 2019]

$M = 1.44^{+0.13}_{-0.14} M_\odot$, $R = 13.02^{+1.24}_{-1.06}$ km [Miller et al. 2019]

$M = 1.40^{+0.13}_{-0.12} M_\odot$, $R = 11.71^{+0.88}_{-0.83}$ km [Vinciguerra et al. 2024]

PSR J0740+6620 $M = (2.07 \pm 0.07) M_\odot$ [Fonseca et al. 2021]

$R = 12.39^{+1.30}_{-0.98}$ km [Riley et al. 2021]

$R = 13.7^{+2.6}_{-1.5}$ km [Miller et al. 2021]

$R = 12.76^{+1.49}_{-1.02}$ km [Salmi et al. 2022]

PSR J0437-7415 $M = (1.42 \pm 0.04) M_\odot$ [Reardon et al. 2024]

$R = 11.36^{+0.95}_{-0.63}$ km [Choudhury et al. 2024]

$R = 13.25^{+0.32}_{-0.31}$ km [Qi et al. 2026]

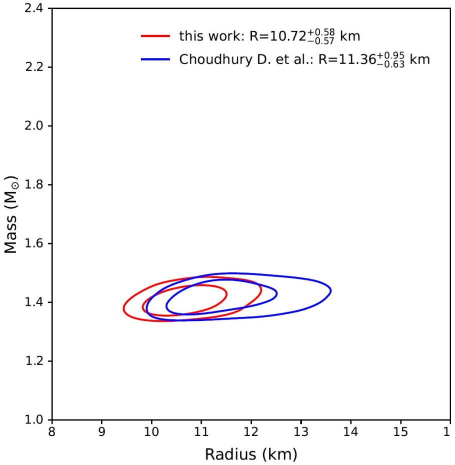
PSR J1231-1411

$M = 1.04^{+0.05}_{-0.03}$, $R = 12.6^{+0.31}_{-0.33}$ km [Salmi et al. 2024]

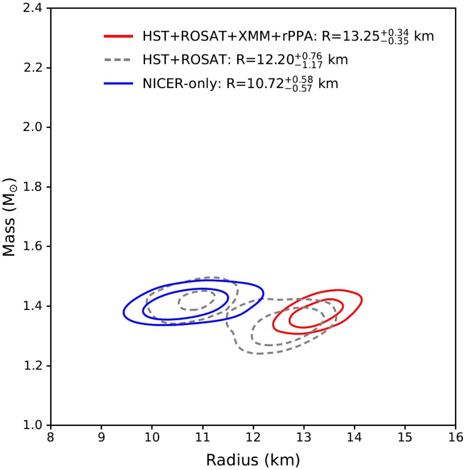
PSR J0614-3329 $M = (1.44 \pm 0.07) M_\odot$ [Miles et al. 2025]

$R = 10.29^{+1.01}_{-0.86}$ km [Mauviard et al. 2025]

Qi et al. 2025

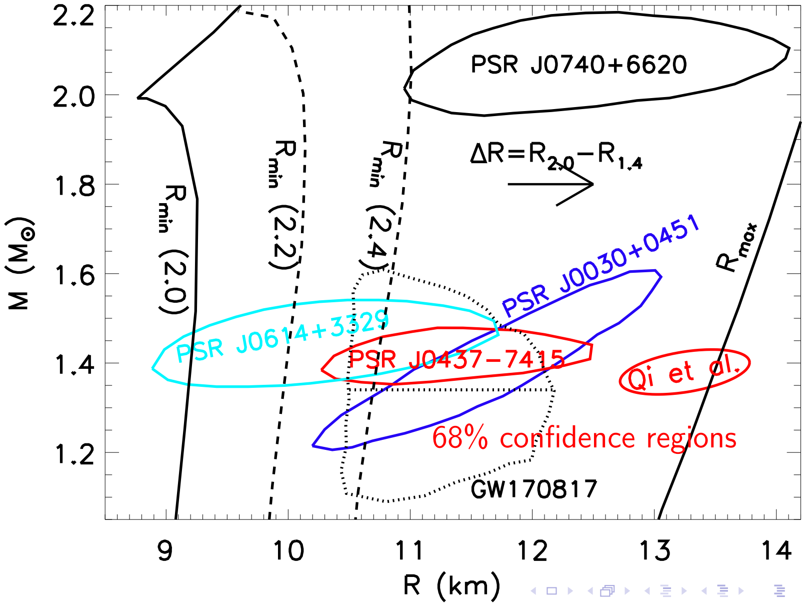


this work: R=10.72$^{+0.58}_{-0.57}$ km
Choudhury D. et al.: R=11.36$^{+0.95}_{-0.63}$ km
Mass (M$_\odot$)
Radius (km)

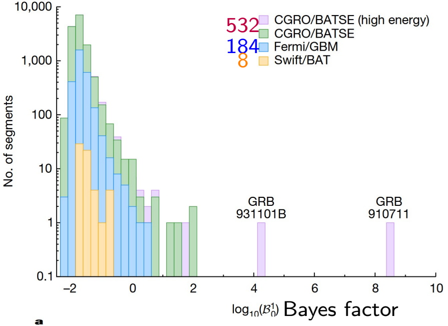


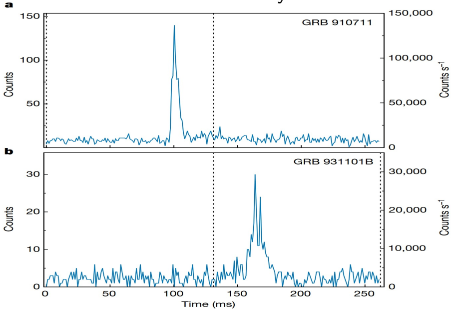
HST+ROSAT+XMM+rPPA: R=13.25$^{+0.34}_{-0.35}$ km
HST+ROSAT: R=12.20$^{+0.76}_{-1.17}$ km
NICER-only: R=10.72$^{+0.58}_{-0.57}$ km
Mass (M$_\odot$)
Radius (km)

NICER Summary, with *R* Constraints

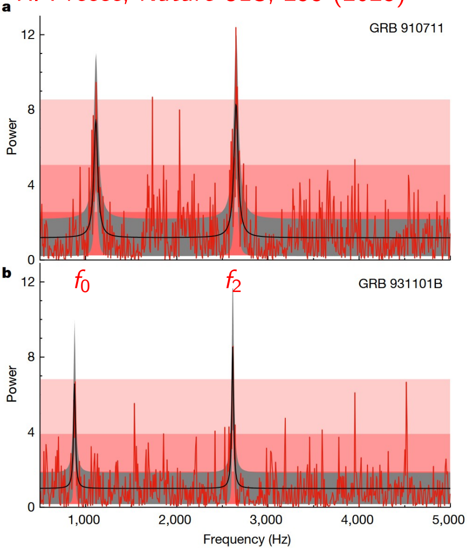


GRB QPOs





C. Chirenti, S. Dichiari, A. Lien, M. C. Mille
R. Preece, *Nature* **613**, 253 (2023)



Neutron Star Constraints from Gamma-Ray Bursts

Quasi-periodic oscillations (QPOs) observed in short GRBs are consistent with radial and quadrupolar vibrations of hypermassive neutron stars (HMNS) formed in the aftermath of BNS mergers.

Numerical relativity simulations show BNS merger remnants radiate gravitational waves with frequency peaks associated with quadrupolar non-radial $\ell = 2$ ($f_2 \sim 1-4$ kHz) and radial $\ell = 0$ ($f_0 \sim 1$ kHz) modes (Bauswein & Janka 2012; Takami et al. 2015; Rezzolla & Takami 2016).

These frequencies appear to obey quasi-universal (little EOS dependence) correlations with the pre-merger chirp mass $\mathcal{M}$ and binary tidal deformability $\tilde{\Lambda}$, both highly correlated with the NS M and R. From the GR simulations of Breschi et al. (2019), Guedes et al. (2025) find for the combinations $x_0 = \mathcal{M} f_0$ and $x_1 = f_2/f_0$

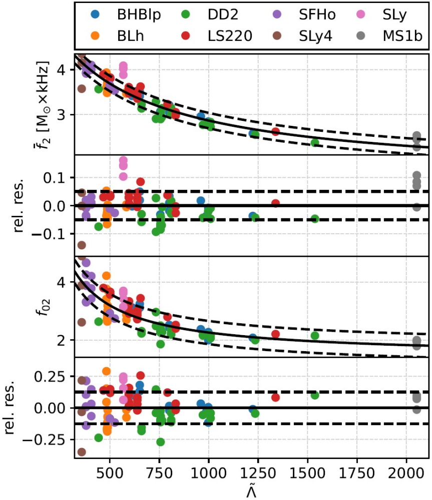
$$x_{0,1} \simeq Q\left(\frac{1 + a_{0,1}\tilde{\Lambda} + b_{0,1}\tilde{\Lambda}^2}{1 + c_{0,1}\tilde{\Lambda} + d_{0,1}\tilde{\Lambda}^2}\right)$$

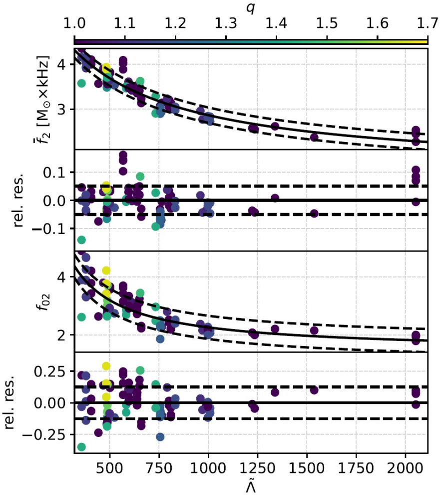
$[Q_0, a_0, b_0, c_0, d_0] = [(1.463 \pm 0.171) M_\odot \mathrm{kHz}, 993.0, 0.7556, 194.8, 0.6973]$,
$[Q_1, a_1, b_1, c_1, d_1] = [2.146 \pm 0.398, 524.8, 10.31, -787.3, 15.57]$.

The observed frequencies ν are redshifted, $\nu = f(1+z)$.

GR Simulations of Breschi et al. (2019)

Quasi-universal fits insensitive to EOS and q





GRB QPOs (modified from Guedes et al. 2025)

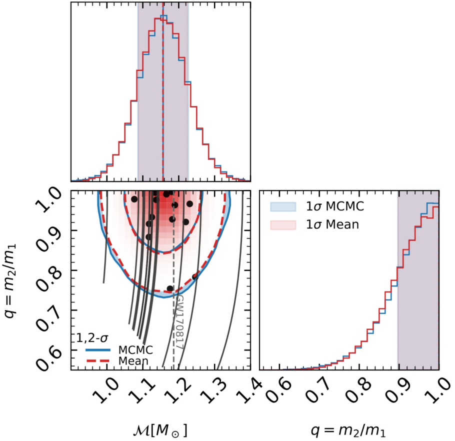
Although observed frequencies are redshifted, their ratio is not. From the x_1 formula, for GRB 910711 (931101B), $\tilde{\Lambda} = 1022 \pm 607$ (595 ± 204).

The x_2 formula gives $\mathcal{M}(1+z) = 1.14 \pm 0.23 M_\odot$ ($1.36 \pm 0.23 M_\odot$).

Both ranges expected from galactic BNS.

Priors for q and z taken from galactic BNS and s-GRBs ($\mathcal{P}(z) \propto z^3$, $\mathcal{P}(q) = \exp(-[q/0.2]^2)$).

$\mathcal{M} \in [1.0, 1.30] M_\odot$ from galactic BNS. Long-lived hypermassive neutron stars can't form if $\mathcal{M}$ is too large.

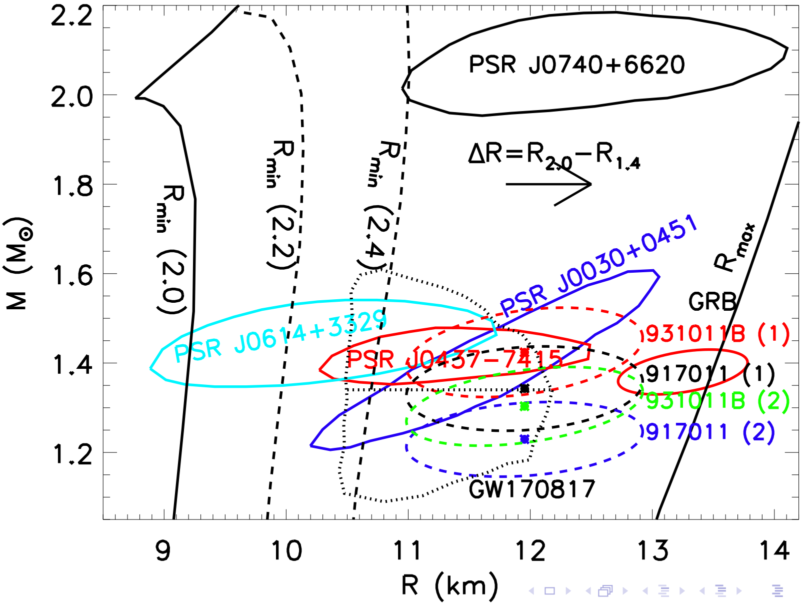


We assume all 4 neutron stars have a common radius R.

A semi-universal relation (Zhao & Lattimer 2018), valid for $1.2 \le M/M_\odot \le 1.6$, is $\Lambda \simeq (0.0088 \pm 0.0008)(Rc^2/GM)^6$.

This implies $R = 11.7 \pm 0.8$ km and masses $1.1 < M/M_\odot < 1.5$.

GRB QPO constraints



Conclusions

M_{max} limits could imply M-R bounds that could alter their observational inferences.

A lower limit to M_{max} imposes a lower limit to radii.

Increasing the upper limit to the density range for which we ‘understand’ the EOS imposes an upper limit to radii.

Systematic biases are important in modeling of observations as well as EOS inferences from them.

Beware of comparing observables inferred from models with differing assumptions.

Could GRBs provide an additional source of M-R information?