Constraints on the EOS and nuclear symmetry energy from experiments and observations

J. M. Lattimer

Department of Physics & Astronomy

STONY BROOK UNIVERSITY

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The r-process and the nuclear EOS
after LIGO-Virgo's third observing run
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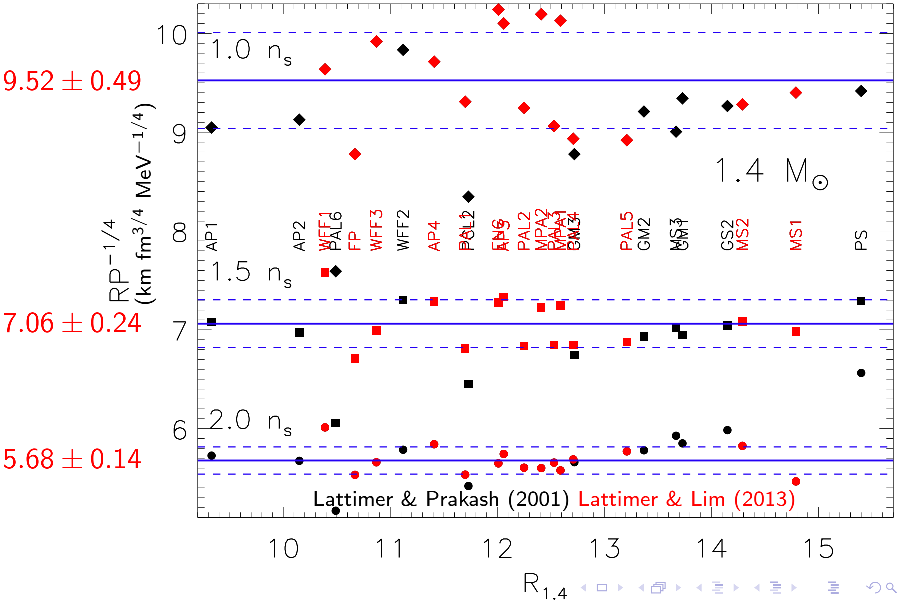
Recent Collaborators:

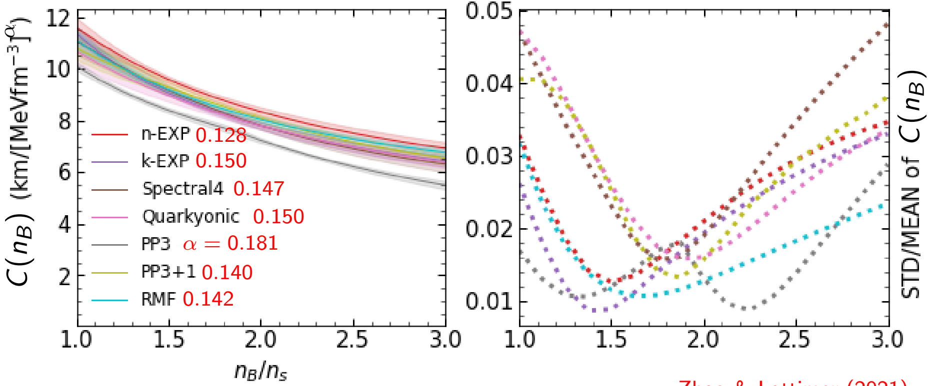
Duncan Brown & Soumi De (Syracuse), Christian Drischler (Berkeley), Sophia Han & Sanjay Reddy (INT), Madappa Prakash & Tianqi Zhao (Ohio), Evgeni Kolomeitsev (Matej Bei, Slovakia), Akira Ohnishi (YITP, Kyoto), Achim Schwenk (Darmstadt), Andrew Steiner (Tennessee) & Ingo Tews (LANL)

The Radius – Pressure Correlation



$R_{1.4} - P_{\rm NSM}$ Correlation with Parameterized EOSs

Optimize $R_{1.4} = C(n_B) P_{\rm NSM}(n_B)^\alpha$ for $n_B/n_s \in [1, 3]$



Zhao & Lattimer (2021)

Nuclear Symmetry Energy and the Pressure

The symmetry energy is the difference between the energies of pure neutron matter ($x=0$) and symmetric ($x=1/2$) nuclear matter:

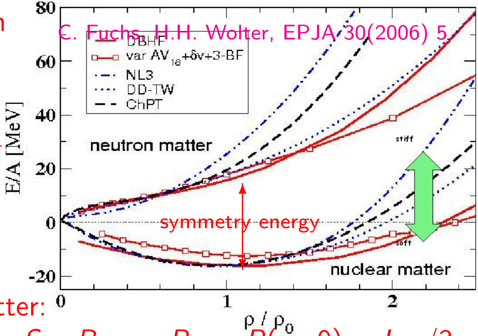
$$S(n) = E(n, x=0) - E(n, x=1/2)$$

Usually approximated as an expansion around the saturation density (n_s) and isopin symmetry ($x=1/2$):

$$E(n,x) = E(n,1/2) + (1-2x)^2 S_2(n) +$$

$$S_2(n) = \mathbf{S_v} + \frac{\mathbf{L}}{3}\frac{n-n_s}{n_s} + \ldots$$

$$\mathbf{S_v} \simeq 31 \text{ MeV}, \quad \mathbf{L} \simeq 50 \text{ MeV}$$



Extrapolated to pure neutron matter:

$$E_N = E(n_s, 0) \approx S_v + E(n_s, 1/2) \equiv S_v - B, \qquad P_N = P(n_s, 0) = L n_s/3$$

Neutron star matter (beta equilibrium) is nearly neutron matter:

$$\frac{\partial(E+E_e)}{\partial x} = 0, \quad P(n_s, x_\beta) \simeq \frac{L n_s}{3}\left[1 - \left(\frac{4S_v}{\hbar c}\right)^3 \frac{4-3S_v/L}{3\pi^2 n_s}\right]$$

Energy Expansions

$$S(u) = E_N(u) - E_{1/2}(u), \qquad u = n/n_s$$

$$E_{1/2}(u) = -B + \frac{K_{1/2}}{18}(u-1)^2 + \frac{Q_{1/2}}{162}(u-1)^3 + \cdots$$

$$E_N(u) = S_V - B + \frac{L}{3}(u-1) + \frac{K_N}{18}(u-1)^2 + \frac{Q_N}{162}(u-1)^3 + \cdots$$

$$S(u) = S_V + \frac{L}{3}(u-1) + \frac{K_{\rm sym}}{18}(u-1)^2 + \frac{Q_{\rm sym}}{162}(u-1)^3 + \cdots$$

Empirical saturation properties:

$n_s = 0.155 \pm 0.005 \text{ fm}^{-3}$, $B = 16 \pm 1$ MeV, $K_{1/2} = 230 \pm 20$ MeV

261 nuclear interactions fit to nuclei yield these correlations:

$$K_{\rm sym} = 3.501L - 305.67 \pm 24.26 \text{ MeV}$$

$$Q_{\rm sym} = -6.443L + 708.74 \pm 118.14 \text{ MeV}$$

$$Q_{1/2} = -0.870L - 354.71 \pm 178.04 \text{ MeV}$$

Symmetry Parameter Correlation from Masses

Liquid drop model approximately valid

$E_{\rm sym}(N,Z) = (S_V A - S_s A^{2/3}) I^2$

$\chi^2 = \frac{1}{\mathcal{N}\sigma_D^2}\sum_{i=1}^{\mathcal{N}}(E_{ex,i} - E_{sym,i})^2$

$\chi_{vv} = \frac{2}{\mathcal{N}\sigma_D^2}\sum_{i=1}^{\mathcal{N}} I_i^4 A_i^2 \simeq 61.6\,\sigma_D^{-2}$

$\chi_{ss} = \frac{2}{\mathcal{N}\sigma_D^2}\sum_{i=1}^{\mathcal{N}} I_i^4 A_i^{4/3} \simeq 1.87\,\sigma_D^{-2}$

$\chi_{vs} = -\frac{2}{\mathcal{N}\sigma_D^2}\sum_{i=1}^{\mathcal{N}} I_i^4 A_i^{5/3} \simeq -10.7\,\sigma_D^{-2}$

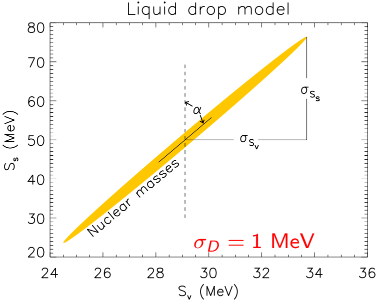
$\sigma_{S_V} = \sqrt{\frac{2\chi_{ss}}{\chi_{vv}\chi_{ss}-\chi_{sv}^2}} \simeq 2.3\,\sigma_D$

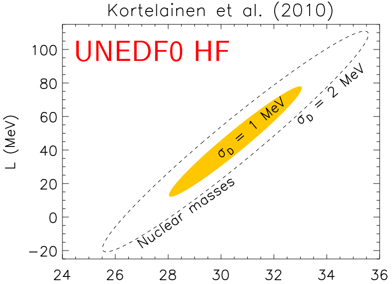
$\sigma_{S_s} = \sqrt{\frac{2\chi_{vv}}{\chi_{vv}\chi_{ss}-\chi_{sv}^2}} \simeq 13.2\,\sigma_D$

$\alpha = \frac{1}{2}\tan^{-1}\frac{2\chi_{vs}}{\chi_{vv}-\chi_{ss}} \simeq 9^\circ.8$

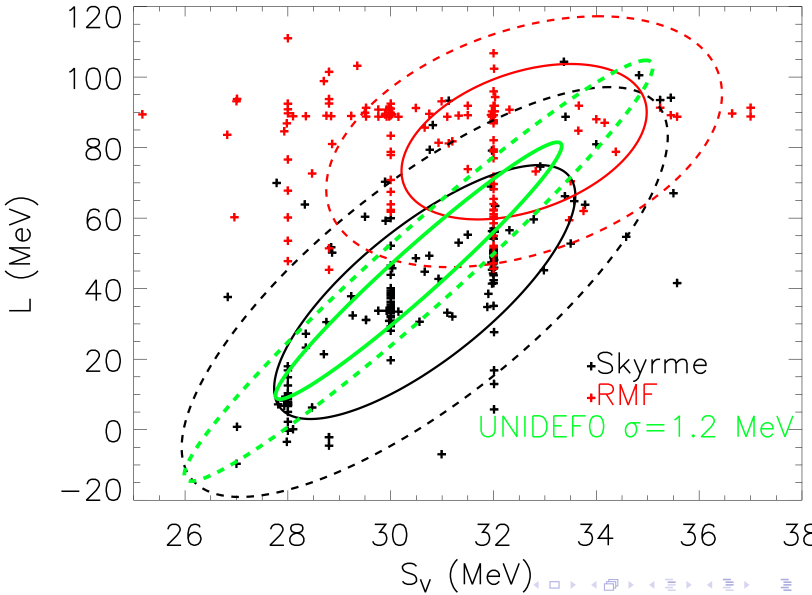
$r_{vs} = -\frac{\chi_{vs}}{\sqrt{\chi_{vv}\chi_{ss}}} \simeq 0.997$

S_s is highly correlated with L and S_V.



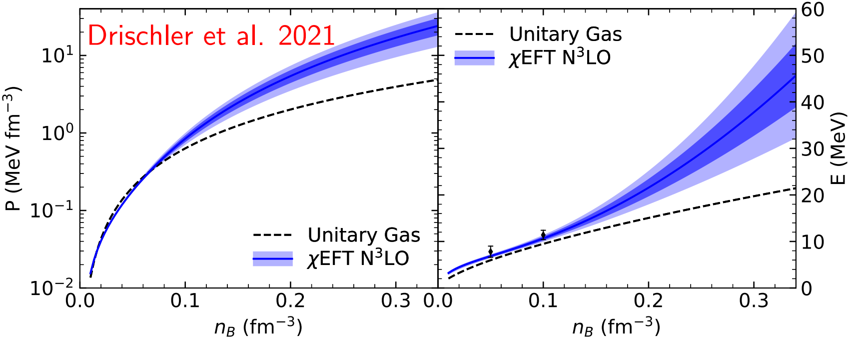


Correlations from Fitting Nuclei



Theoretical Neutron Matter Studies

Recently developed chiral effective field theory allows a systematic expansion of nuclear forces at low energies based on the symmetries of quantum chromodynamics. It exploits the gap between the pion mass (the pseudo-Goldstone boson of chiral symmetry-breaking) and the energy scale of short-range nuclear interactions established from experimental phase shifts. It provides the only known consistent framework for estimating energy uncertainties.



Symmetry Parameters From Neutron Matter

Pure neutron matter calculations are more reliable than symmetric matter calculations.

Symmetric matter emerges from a delicate cancellation sensitive to short- and intermediate-range three-body interactions at N^2LO that are Pauli-blocked in pure neutron matter.

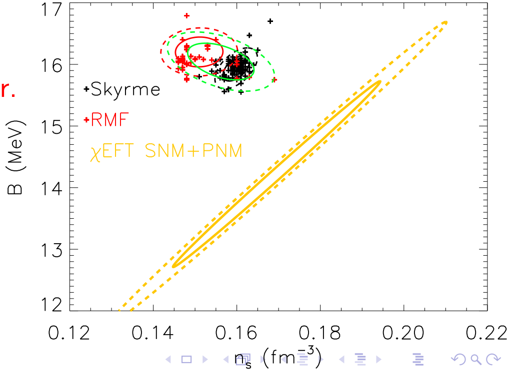
N^3LO symmetric matter calculations don't saturate within empirical ranges for n_s and B, and introduce spurious correlations in symmetric matter.

We infer symmetry parameters from $E_N(n_s)$ and $P_N(n_s)$ using

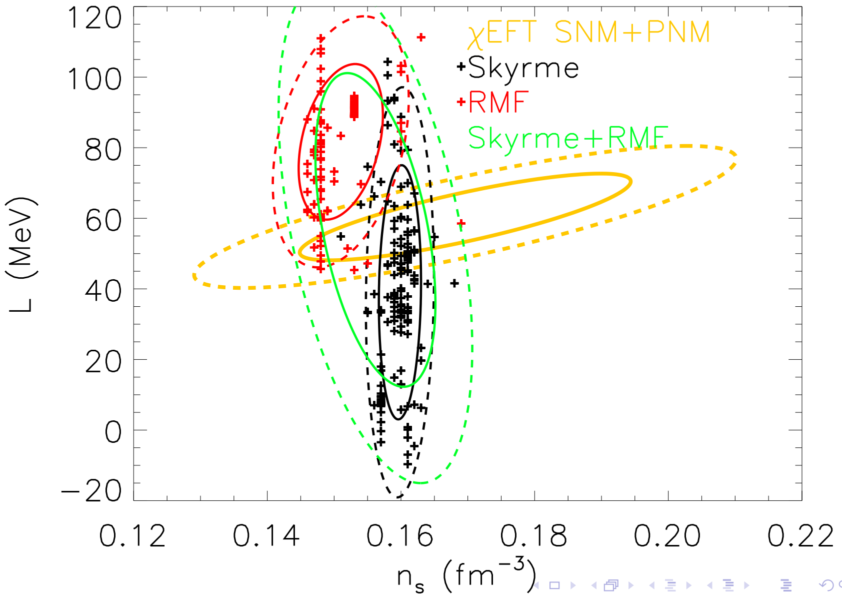
$$S_V = E_N(n_s) + B$$

$$L = 3P_N(n_s)/n_s$$

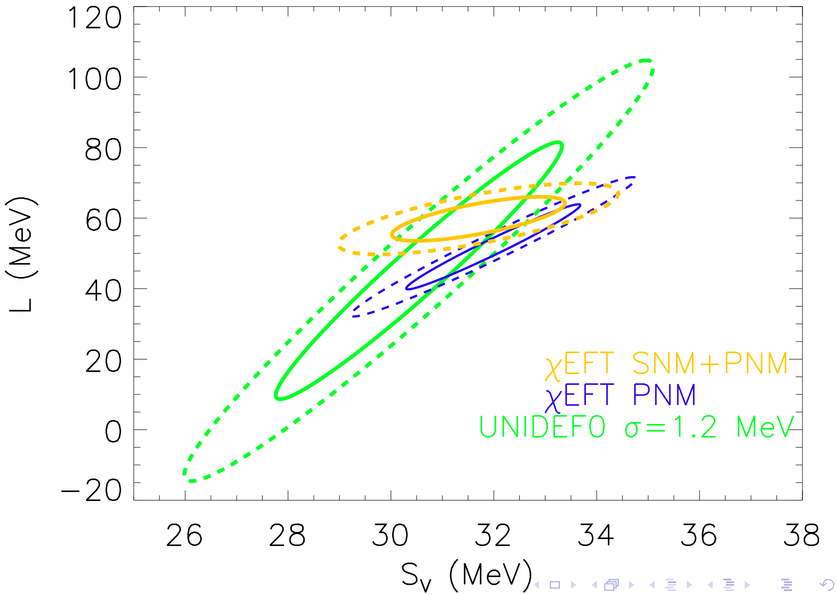
and include uncertainties in E_N, P_N, n_s and B.



Correlations from Chiral EFT



Correlations from Chiral EFT



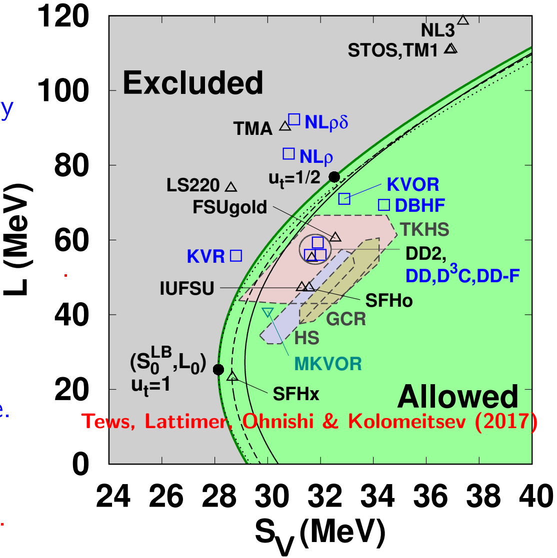
Bounds From The Unitary Gas Conjecture

The Conjecture:

Neutron matter energy is larger than that of the unitary gas $E_{UG} = \xi_0 (3/5) E_F$, or

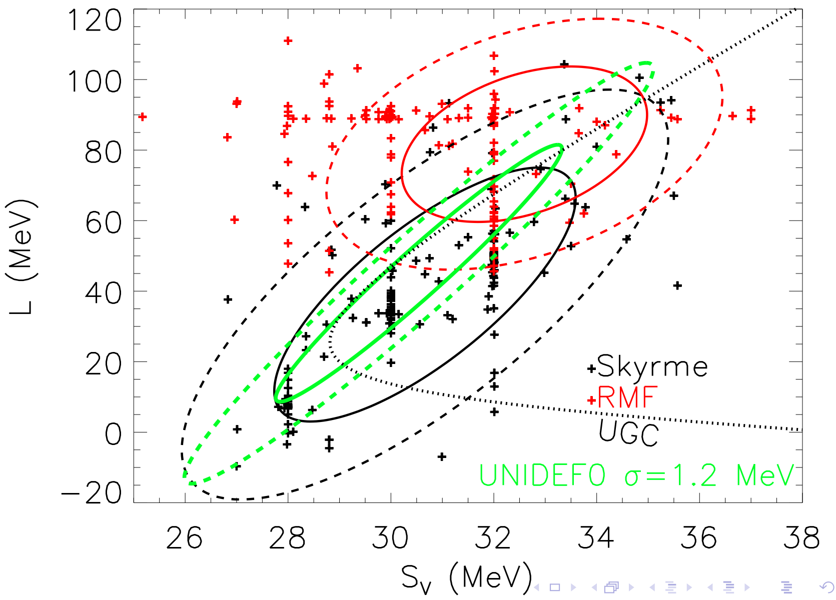
$$E_{UG} \simeq 12.6 \left(\frac{n}{n_s}\right)^{2/3} \mathrm{MeV}$$

The unitary gas consists of fermions interacting via a pairwise short-range s-wave interaction with infinite scattering length and zero range. Cold atom experiments show a universal behavior with the Bertsch parameter $\xi_0 \simeq 0.37$.

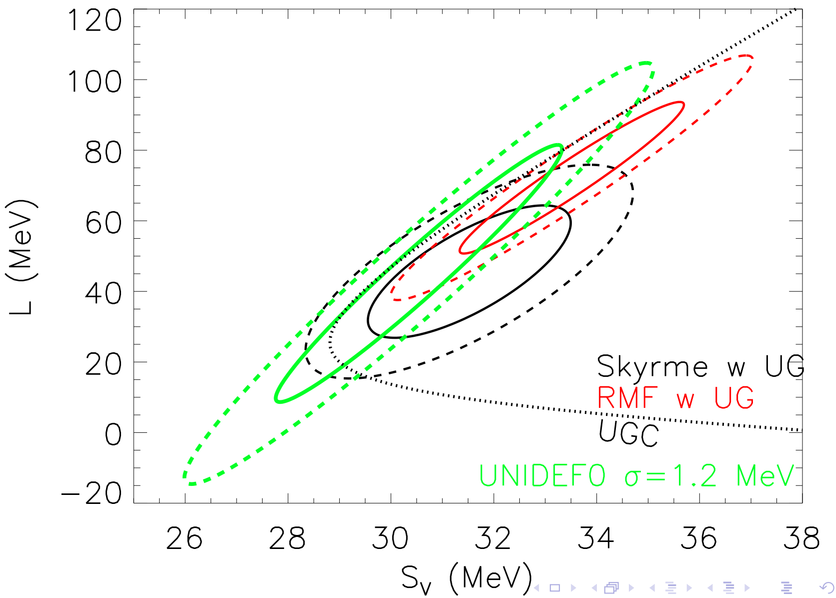


$S_v \geq 28.6$ MeV; $L \geq 25.3$ MeV; $P_N(n_s) \geq 1.35$ MeV fm^{-3}; $R_{1.4} \geq 9.7$ km

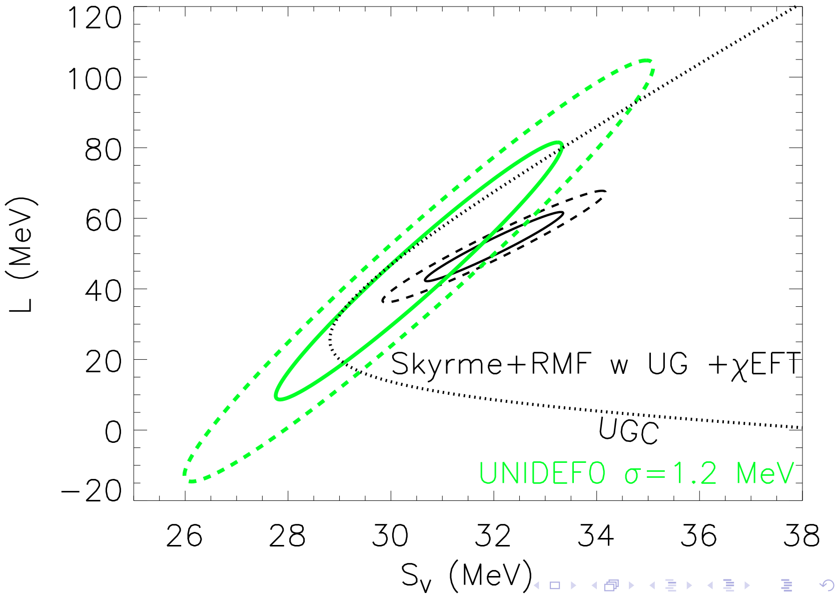
Before Applying the UG Constraint



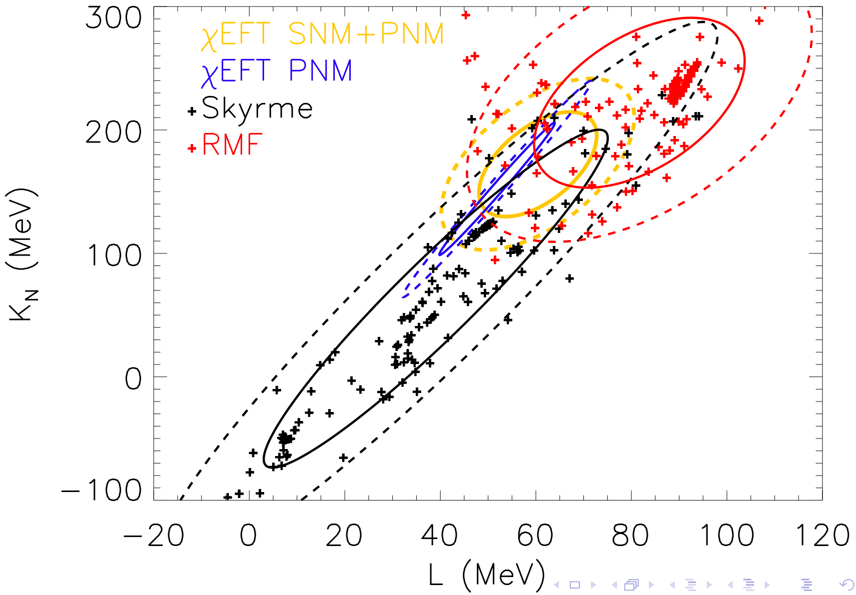
After Applying the UG Constraint



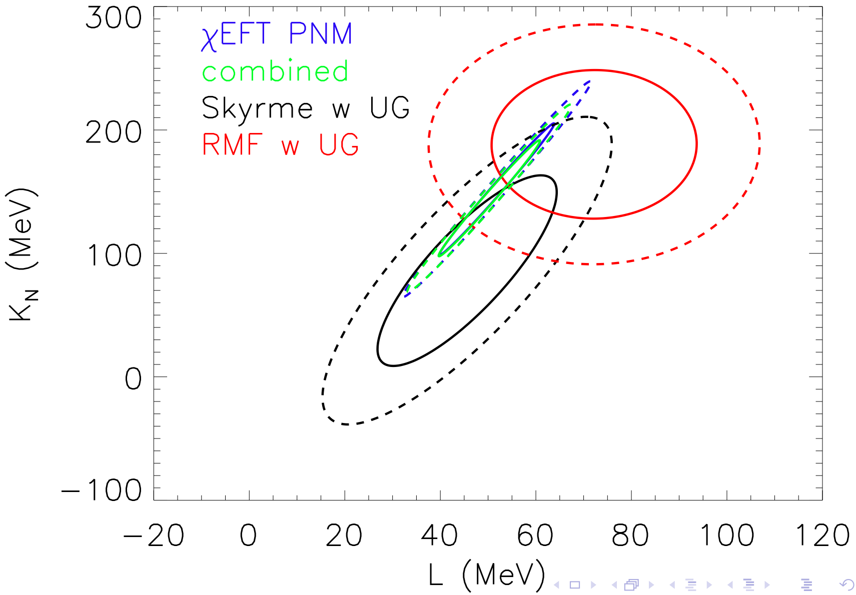
Combining Mass Fits With Chiral EFT



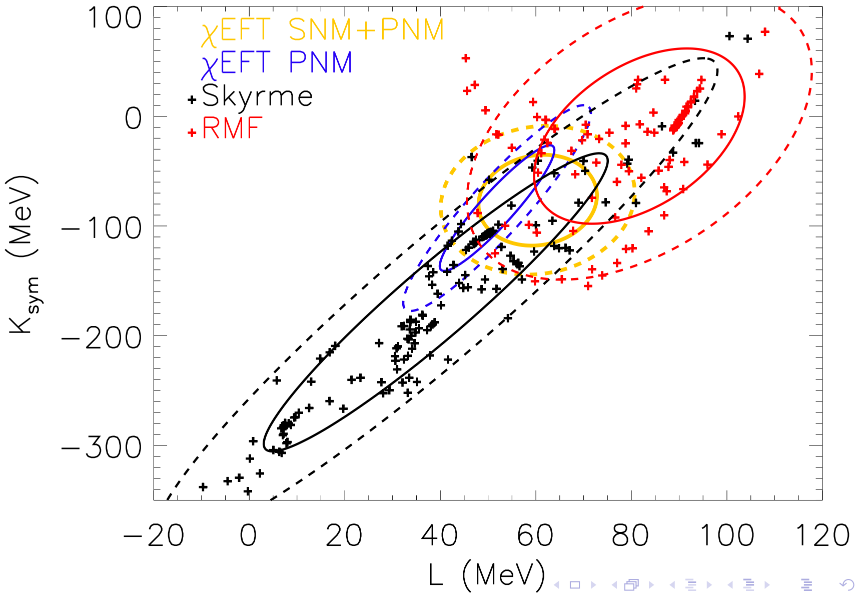
$K_N - L$ Correlations



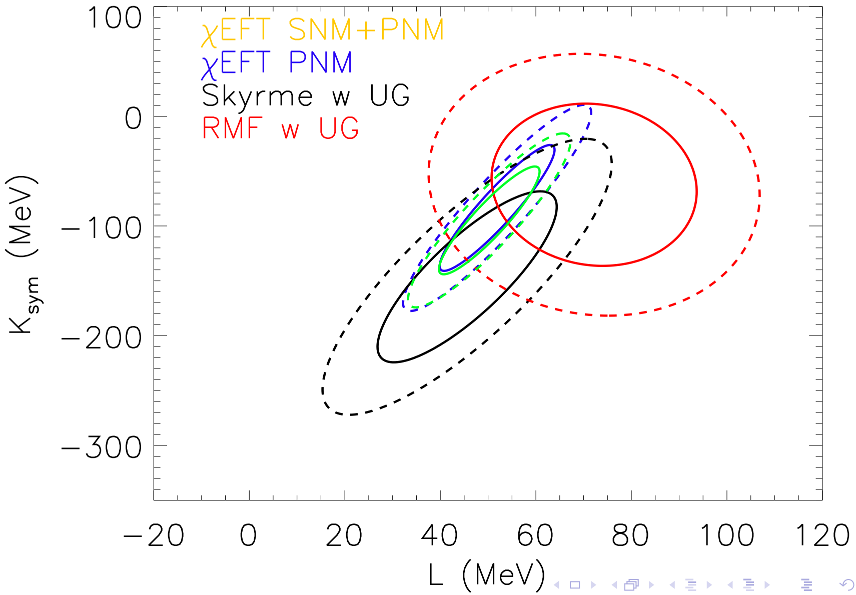
$K_N - L$ Correlations



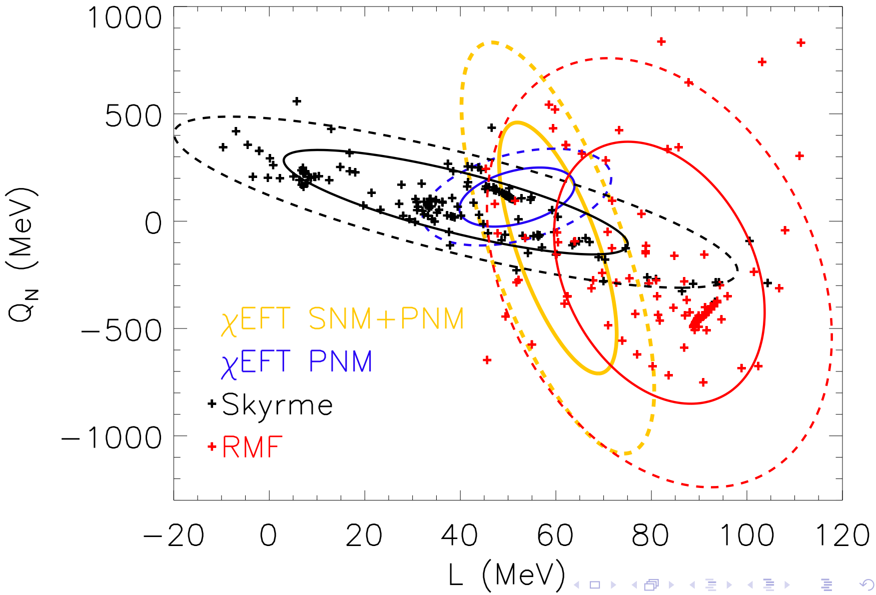
$K_{sym} - L$ Correlations



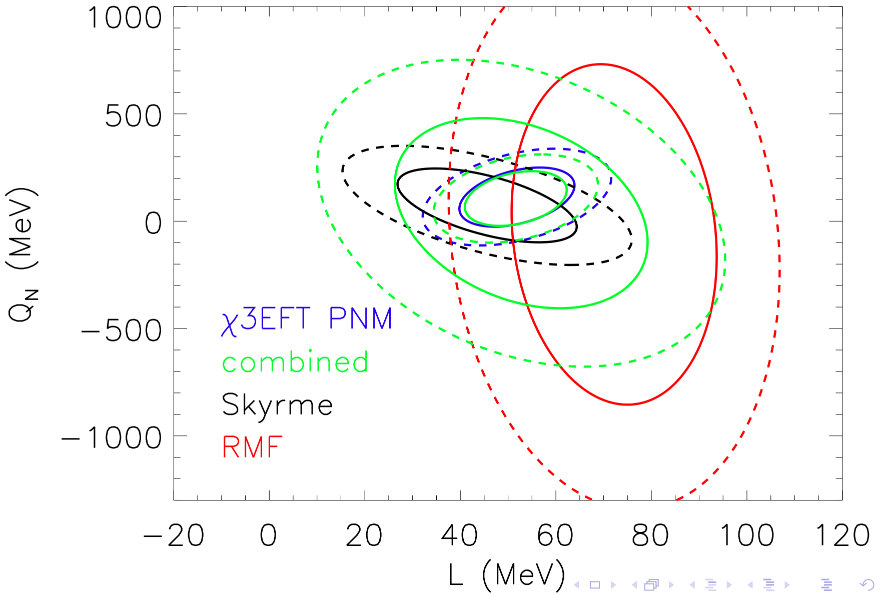
$K_{sym} - L$ Correlations



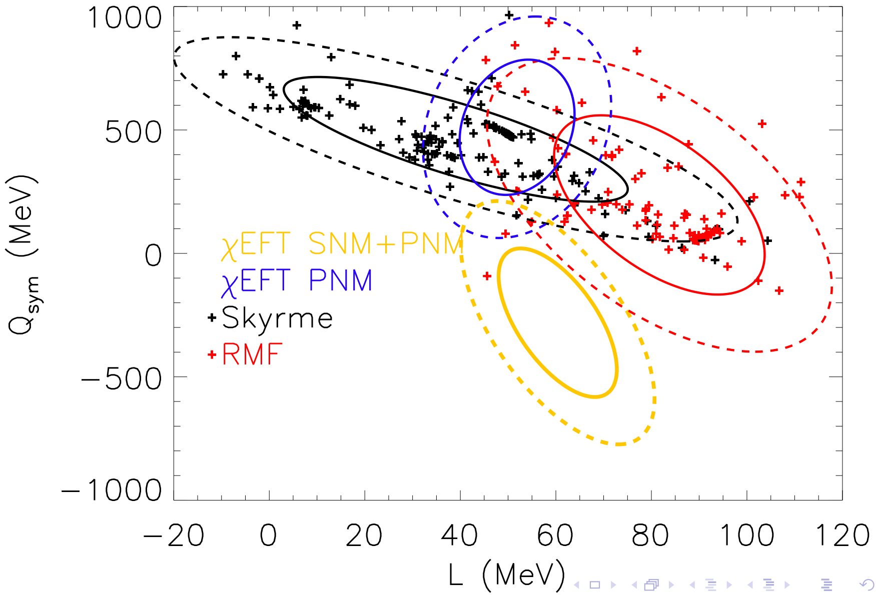
$Q_N - L$ Correlations



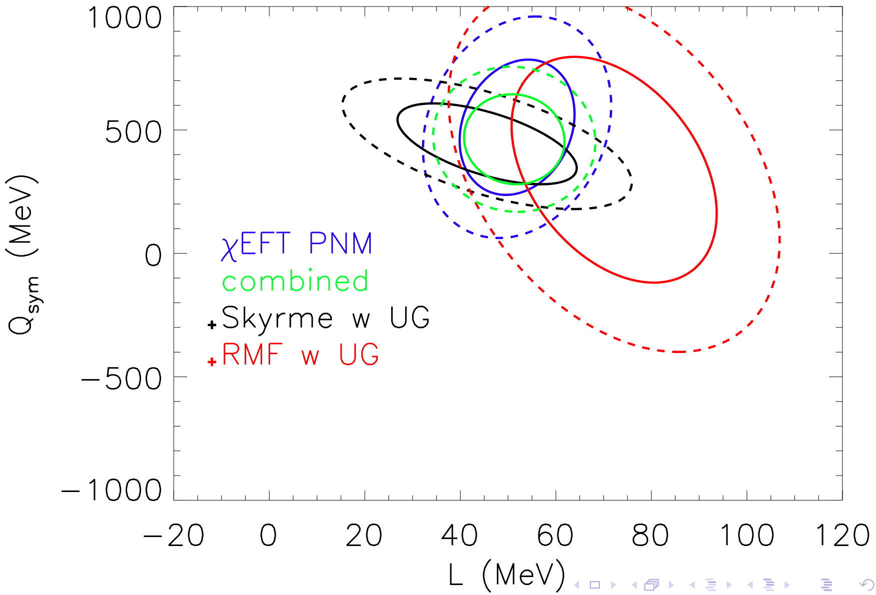
$Q_N - L$ Correlations



$Q_{sym} - L$ Correlations



$Q_{sym} - L$ Correlations



Neutron Skin Thickness

The difference between the mean neutron and proton radii in the liquid droplet model is $t_{np} = R_n - R_p$

$$t_{np} = \frac{2r_o I}{3}\frac{S_s}{S_V}\left[1 + S_s A^{-1/3}/S_V\right]^{-1} \qquad r_{np} = \sqrt{<R_n>^2 - <R_p>^2}$$

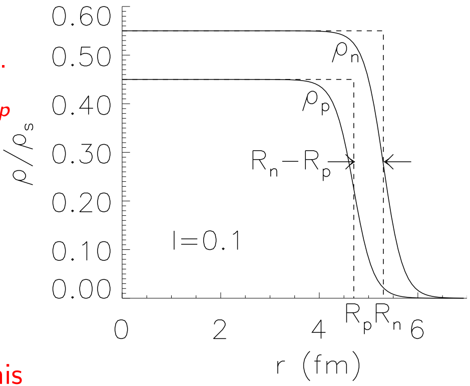
As for masses, this implies an $L - S_V$ correlation for a given t_{np}.

Additionally, Brown found that r_{np} was more highly correlated with dS/dn at $n \sim 2n_s/3$ than with L.

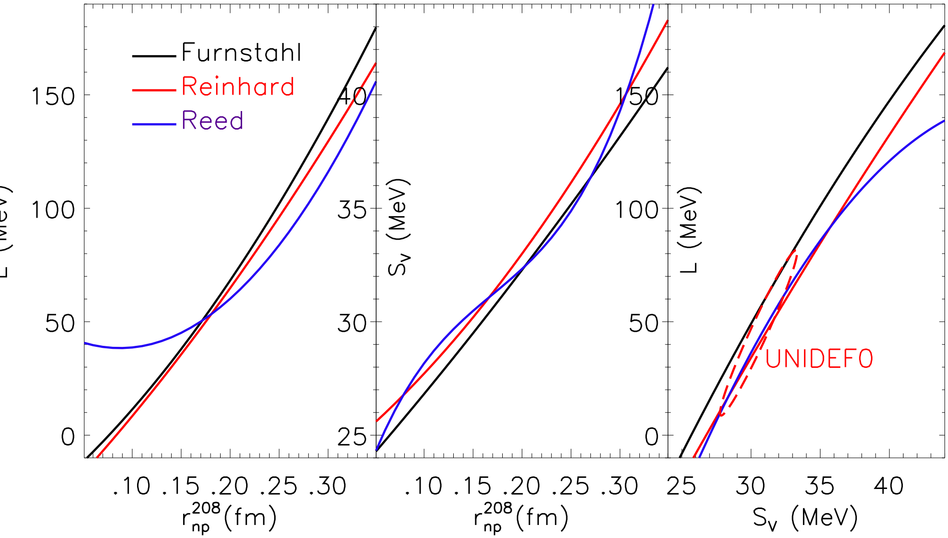
$r_{np} \propto \tilde{L}(2n_s/3)$; $\tilde{L} = 3ndS/dn$.

$$r_{np}^{208} \simeq \frac{(dS/dn)_{0.1}}{(882 \pm 32)\ \mathrm{MeV\ fm}^{-2}}$$

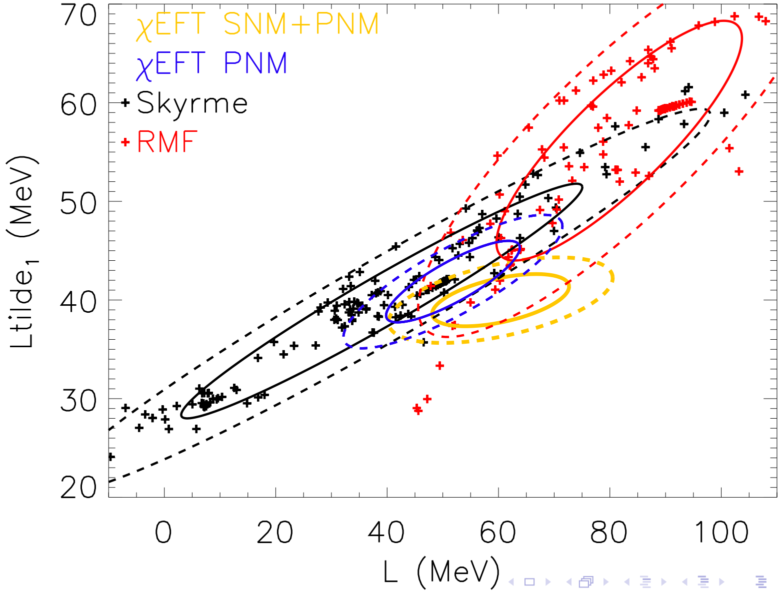
Since $S(u) \simeq S_V + (u-1)L/3$, this is similar to the mass correlation.



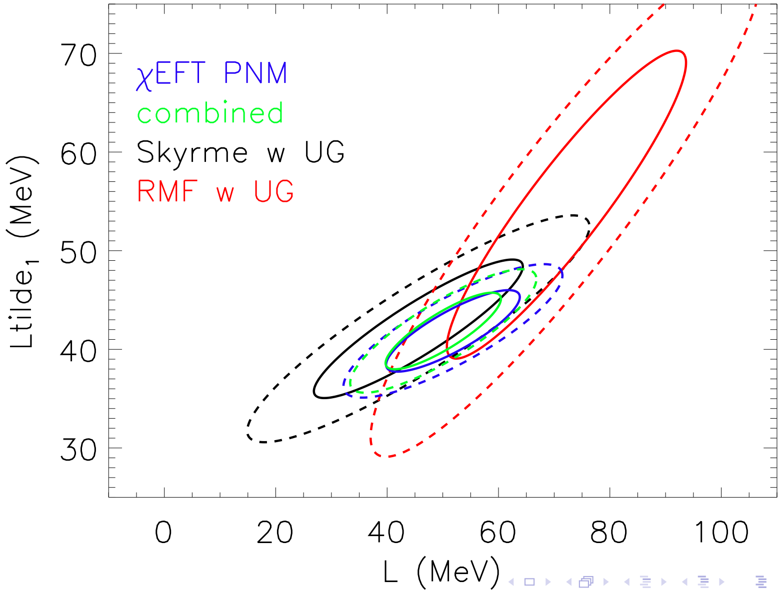
Theoretical Neutron Skin Calculations



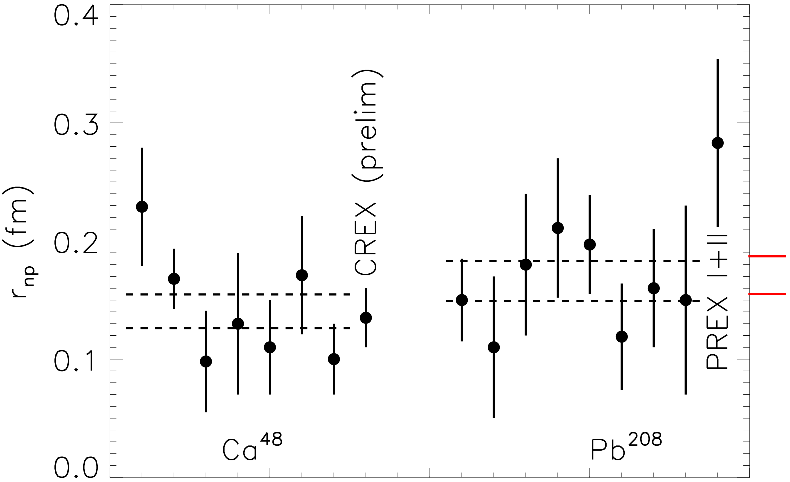
$\tilde{L}_1 - L$ Correlations



$\tilde{L}tilde_1 - L$ Correlations



Neutron Skin Data



Extracting the Physics : CREX Weak Form Factor

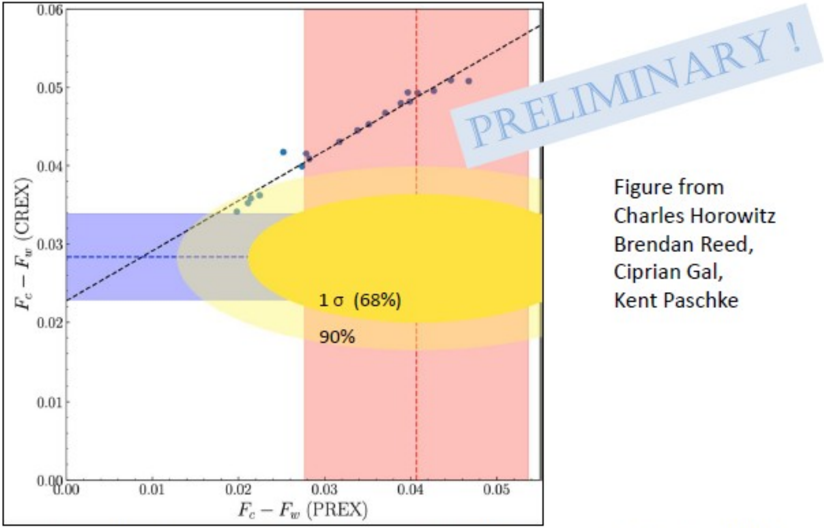
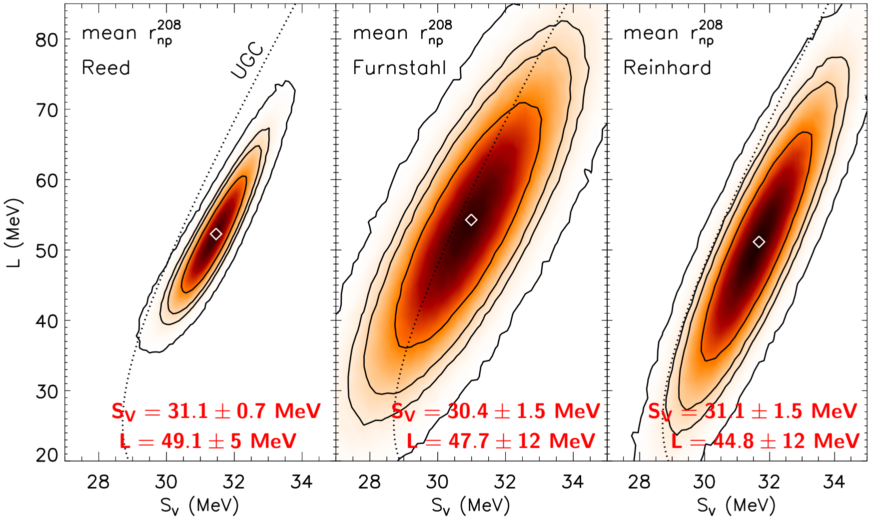


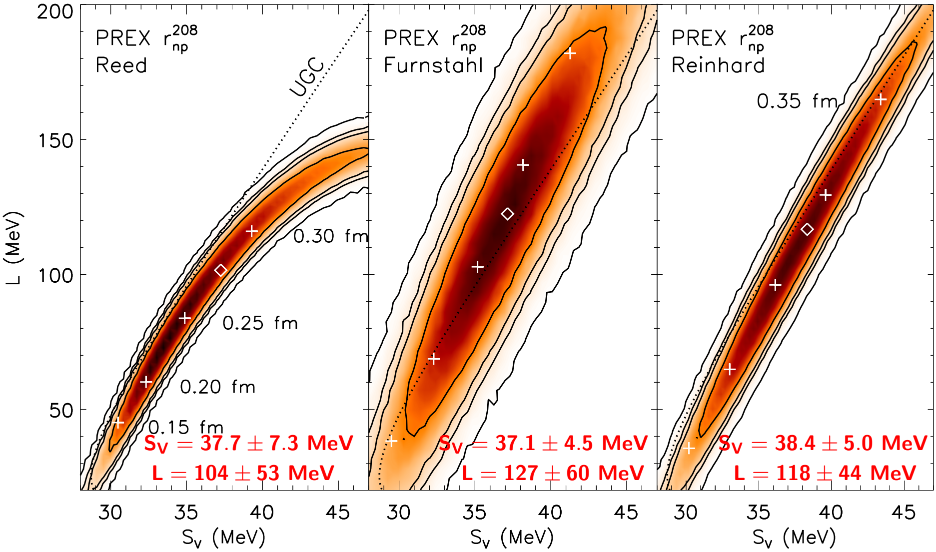
Figure from
Charles Horowitz
Brendan Reed,
Ciprian Gal,
Kent Paschke

Charge form factor F_{ch} minus weak form factor F_w for ^{48}Ca versus F_{ch} - F_w for ^{208}Pb. The error bands show the CREX and PREX-2 results. The points show a collection of nonrelativistic and relativistic density functional results.

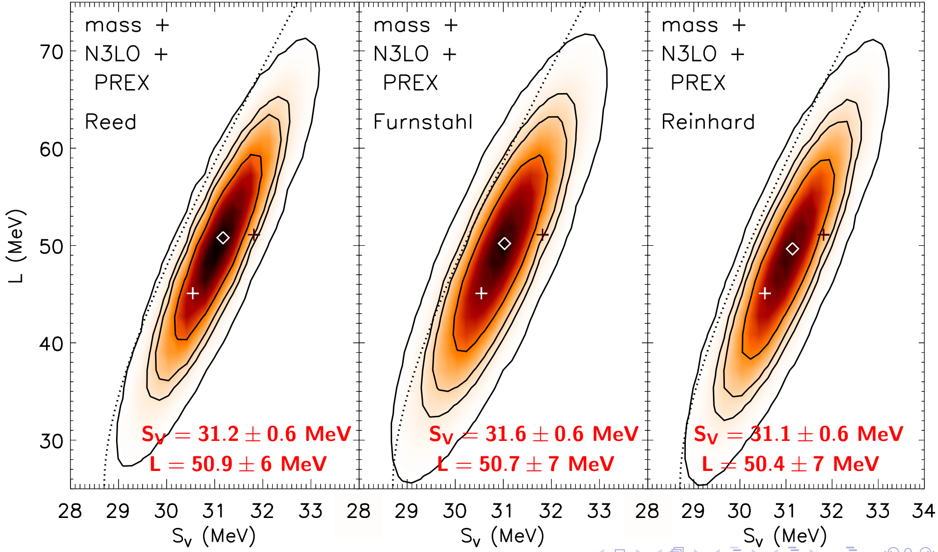
Inferred $L - S_V$ with mean r_{np}^{208}



Inferred $L - S_V$ with PREX r_{np}^{208}



Masses + N3LO + PREX



Electric Dipole Polarizability

Trippa et al. (2005) found central energy of giant dipole resonance of ^{208}Pb has highest correlation with symmetry energy at $n_1 = 0.1$ fm^{-3}, with $S(n_1) = 24.1 \pm 0.8$ MeV.

Zhang et al. (2015) found dipole polarizability has highest correlation at $n_2 = 0.05$ fm^{-3}, and $S(n_2) = 16.54 \pm 1.0$ MeV.

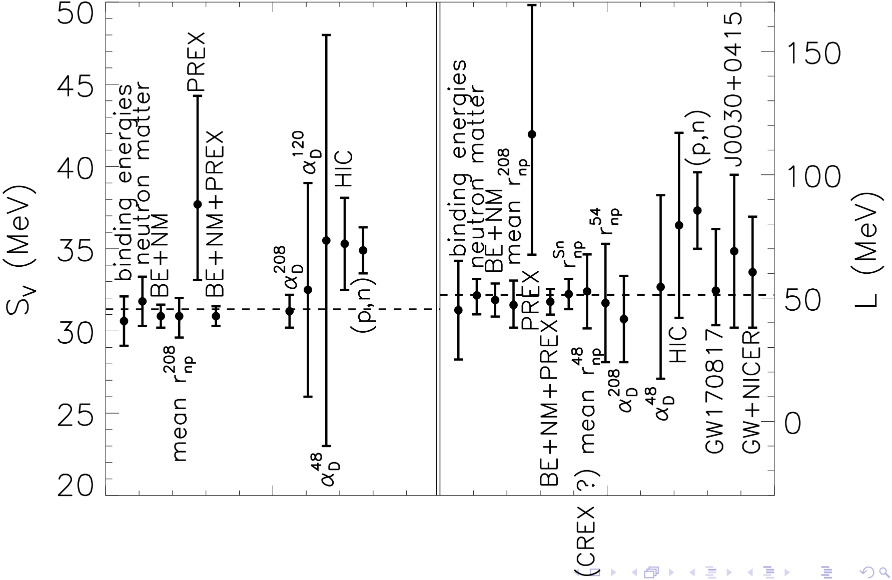
$\Rightarrow L = 10.9 S_V - 287.3 \pm 8.9 \mathrm{~MeV},\ L = 8.05 S_V - 206.9 \pm 9.6 \mathrm{~MeV}$

Hashimoto (2015) Sn: $\alpha_D^{120} = 8.59 \pm 0.37 \mathrm{~fm}^3$
Tamii (2012) Pb: $\alpha_D^{208} = 19.6 \pm 0.6 \mathrm{~fm}^3$
Birkhan (2017) Ca: $\alpha_D^{48} = 2.07 \pm 0.22 \mathrm{~fm}^3$

Roca-Maza et al. (2015) showed (using liquid droplet model as justification) $\alpha_D S_V \propto r_{np}$:

$$\begin{aligned}
\alpha_D^{48} S_V &= (355 \pm 44)\,(r_{np}^{48}/\mathrm{fm}) + 12 \pm 19 \mathrm{~MeV~fm}^3, \\
\alpha_D^{120} S_V &= (1234 \pm 93)\,(r_{np}^{120}/\mathrm{fm}) + 115 \pm 36 \mathrm{~MeV~fm}^3, \\
\alpha_D^{208} S_V &= (1922 \pm 73)\,(r_{np}^{208}/\mathrm{fm}) + 301 \pm 32 \mathrm{~MeV~fm}^3,
\end{aligned}$$

Summary of Constraints on S_V and L



Measuring Neutron Star Masses and Radii

- Pulsar timing in binary systems
  - Accurately measure masses, most being $1.2M_\odot - 1.5M_\odot$. The lowest well-measured mass is $1.174 \pm 0.004M_\odot$, the highest are $2.08 \pm 0.07M_\odot$ and $2.01 \pm 0.04M_\odot$.
  - Moment of inertia measurements via spin-orbit coupling
- X-ray observations yield radii, but are uncertain to a few km.
  - Quiescent binary sources in globular clusters
  - Thermonuclear explosions on accreting neutron stars in binaries leading to photospheric radius expansion bursts
  - Pulse profile modeling of hot spots on rapidly rotating neutron stars (NICER).
- Gravitational waves from merging binaries measure masses, tidal deformabilites, and give insights about $M_{\rm max}$.
- Gravitational collapse supernova neutrinos measure proto-neutron star masses, radii and possibly $M_{\rm max}$.

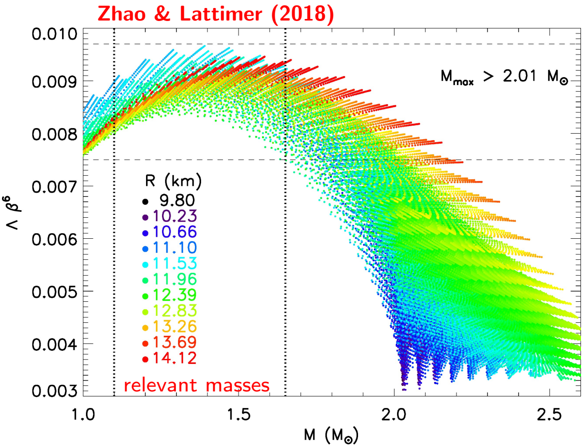
Λ is Highly Correlated With M and R

$$R = 13.9 \left(\frac{M}{1.4 M_\odot}\right)\left(\frac{\Lambda}{800}\right)^{1/6} \text{km}$$

- $\Lambda = a\beta^{-6}$
  $\beta = GM/Rc^2$

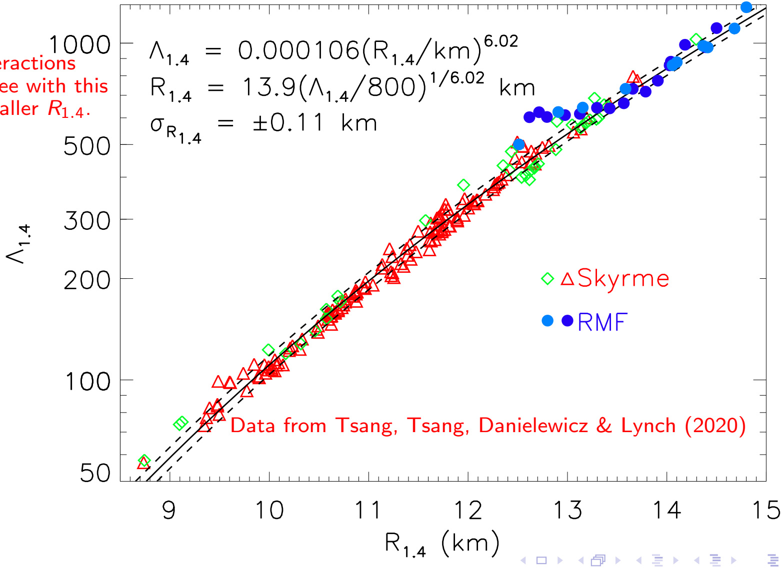
  $a = 0.0086 \pm 0.0011$
  for
  $M = (1.35 \pm 0.25)\ M_\odot$

- If $R_1 \simeq R_2 \simeq R_{1.4}$,
  it follows that
  $\Lambda_2 \simeq q^{-6}\Lambda_1$.



186 Skyrme Interactions

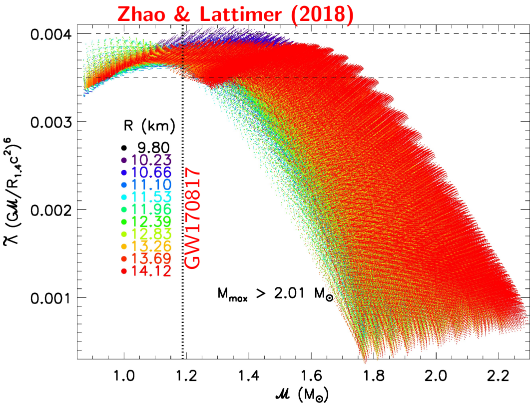
RMF interactions don't agree with this fit for smaller $R_{1.4}$.



Binary Deformability and the Radius

$$\tilde{\Lambda}=\frac{16}{13}\frac{(1+12q)\Lambda_1+q^4(12+q)\Lambda_2}{(1+q)^5}\simeq\frac{16a}{13}\left(\frac{R_{1.4}c^2}{G\mathcal{M}}\right)^6\frac{q^{8/5}(12-11q+12q^2)}{(1+q)^{26/5}}$$

- $\tilde{\Lambda}=a'(R_{1.4}c^2/G\mathcal{M})^6$
  $a'=0.0035\pm0.0006$
  for
  $\mathcal{M}=(1.2\pm0.2)\ M_\odot$
- $R_{1.4}=$
  $(11.5\pm0.3)\frac{\mathcal{M}}{M_\odot}\left(\frac{\tilde{\Lambda}}{800}\right)^{1/6}$ km
- For GW170817:
  $a'=0.00375\pm0.00025$
  $R_{1.4}=(13.4\pm0.1)\left(\frac{\tilde{\Lambda}}{800}\right)^{1/6}$ km



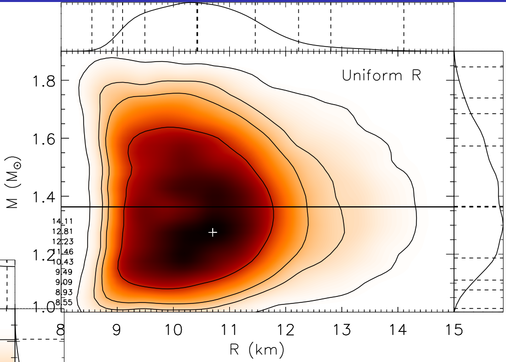
68.3%, 90%, 95.4% and 99.7% Confidence Bounds

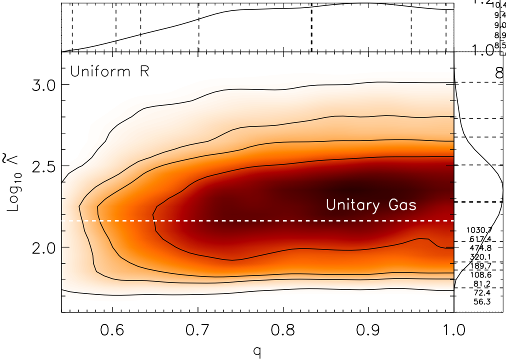
Waveform analysis by De et al. (2018)

Zhao and Lattimer (2021)

Uniform ln Λ and causality priors

$$R = 10.4^{+1.1}_{-0.9}\ \text{km}$$





$$\tilde{\Lambda} = 190^{+130}_{-81}$$

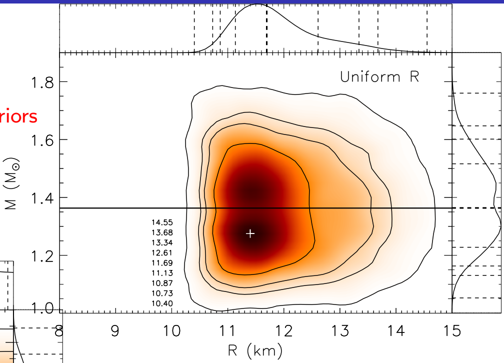
68.3%, 90%, 95.4% and 99.7% Confidence Bounds

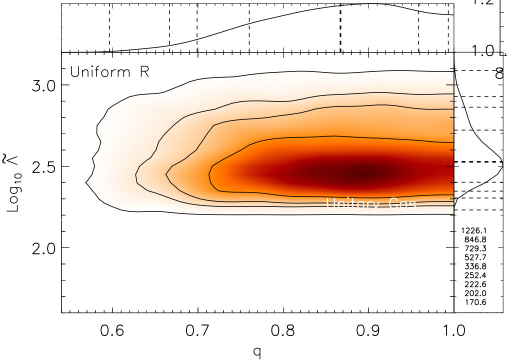
Waveform analysis by De et al. (2018)

Zhao and Lattimer (2021)

Uniform ln Λ and Unitary Gas Conjecture priors

$R = 11.7^{+0.9}_{-0.5}$ km





$\tilde{\Lambda} = 337^{+191}_{-85}$

186 Skyrme Interactions

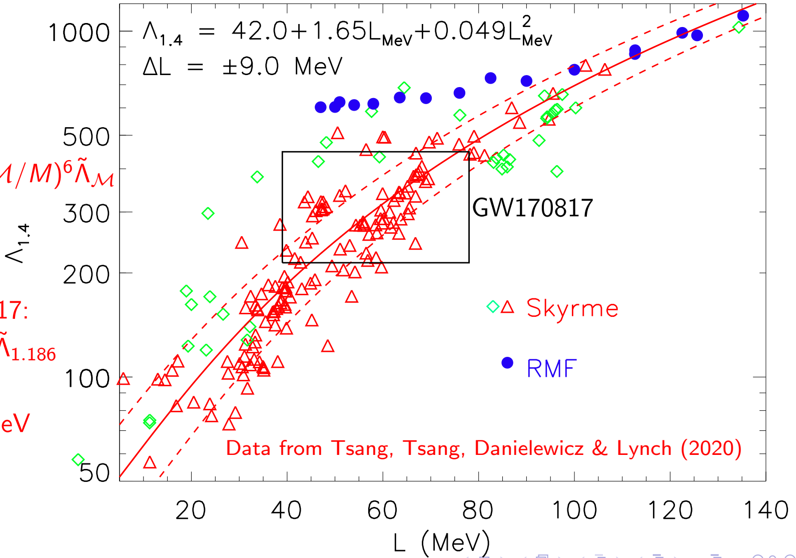
$\Lambda_M \simeq 2^{6/5}(\mathcal{M}/M)^6\tilde{\Lambda}_{\mathcal{M}}$

For GW170817:

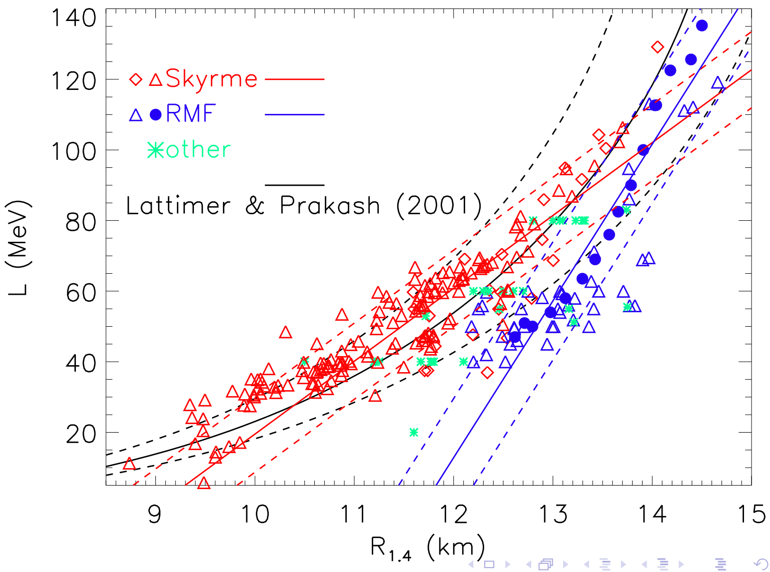
$\Lambda_{1.4} \simeq 0.849\tilde{\Lambda}_{1.186}$

$= 286^{+162}_{-72}$

$L = 53^{+25}_{-14}$ MeV



Relation Between $R_{1.4}$ and L



Model-Dependence of $R_{1.4} - L$ Relation

Many studies show that $R_{1.4}$ is most sensitive to the pressure at about $2n_s$.
The usual energy expansion for dense matter ($u = n/n_s, x = n_p/n$) is

$$E(u, x \simeq 0) = -B + \frac{K_0}{18}(u-1)^2 + (1-2x)^2\left[S_V + \frac{L}{3}(u-1) + \frac{K_{\rm sym}}{18}(u-1)^2 + \cdots\right],$$

In β equilibrium, $x << 0.5$, so the pressure of neutron star matter is

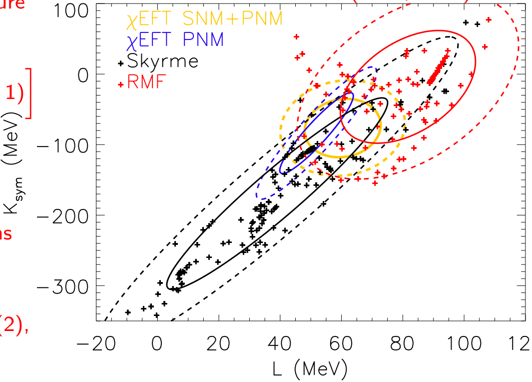
$$P_{\rm NSM}(u) = n_s u^2\left[\frac{L}{3} + \frac{K_0 + K_{\rm sym}}{9}(u-1)\right]$$

$$P_{\rm NSM}(2) = \frac{4n_s}{3}\left[L + \frac{K_0 + K_{\rm sym}}{3}\right]$$

Around $L = 50$ MeV, Skyrme interactions average $K_{\rm sym} \sim -130$ MeV, but RMF interactions average $K_{\rm sym} \sim -30$ MeV.

This produces $\simeq 40\%$ increase in $P_{\rm NSM}(2)$, and 10% larger $R_{1.4}$ for the same L.

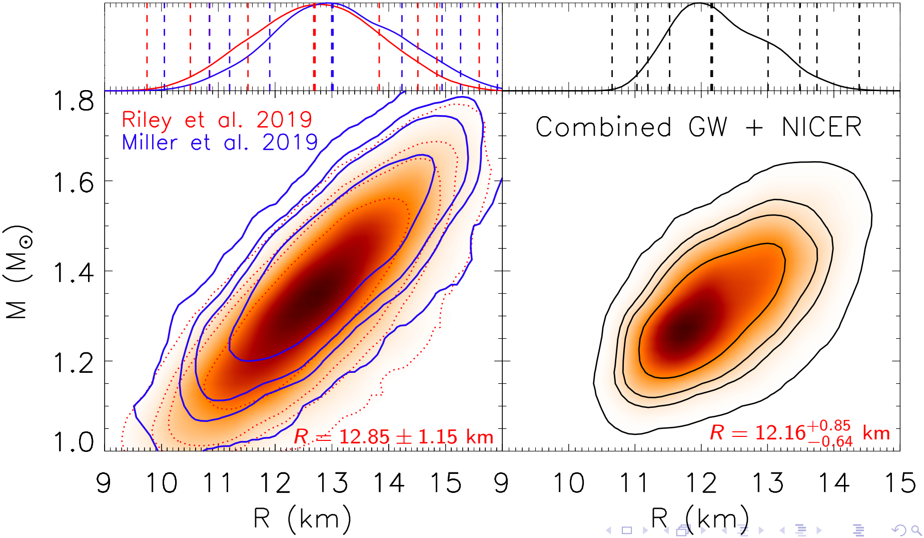
Data from Dutra et al. (2012, 2014)



For $L \gtrsim 70$ MeV, values of $K_{\rm sym}$ for Skyrme and RMF are similar, but these L values are disfavored by nuclear systematics (mass fits, neutron matter theory, neutron skin and dipole polarizability measurements).

NICER Results For PSR J0030+0451

PSR J0030+0415 and GW170817 have neutron stars with similar masses $\simeq 1.4M_\odot$, but PSR J0740+6620 has a larger mass $\simeq 2.0M_\odot$, so don't include it in this analysis.



Recent Moment of Inertia Measurement