

The high-density equation of state and maximum mass of neutron stars

Philippe Landry ♦ Canadian Institute for Theoretical Astrophysics

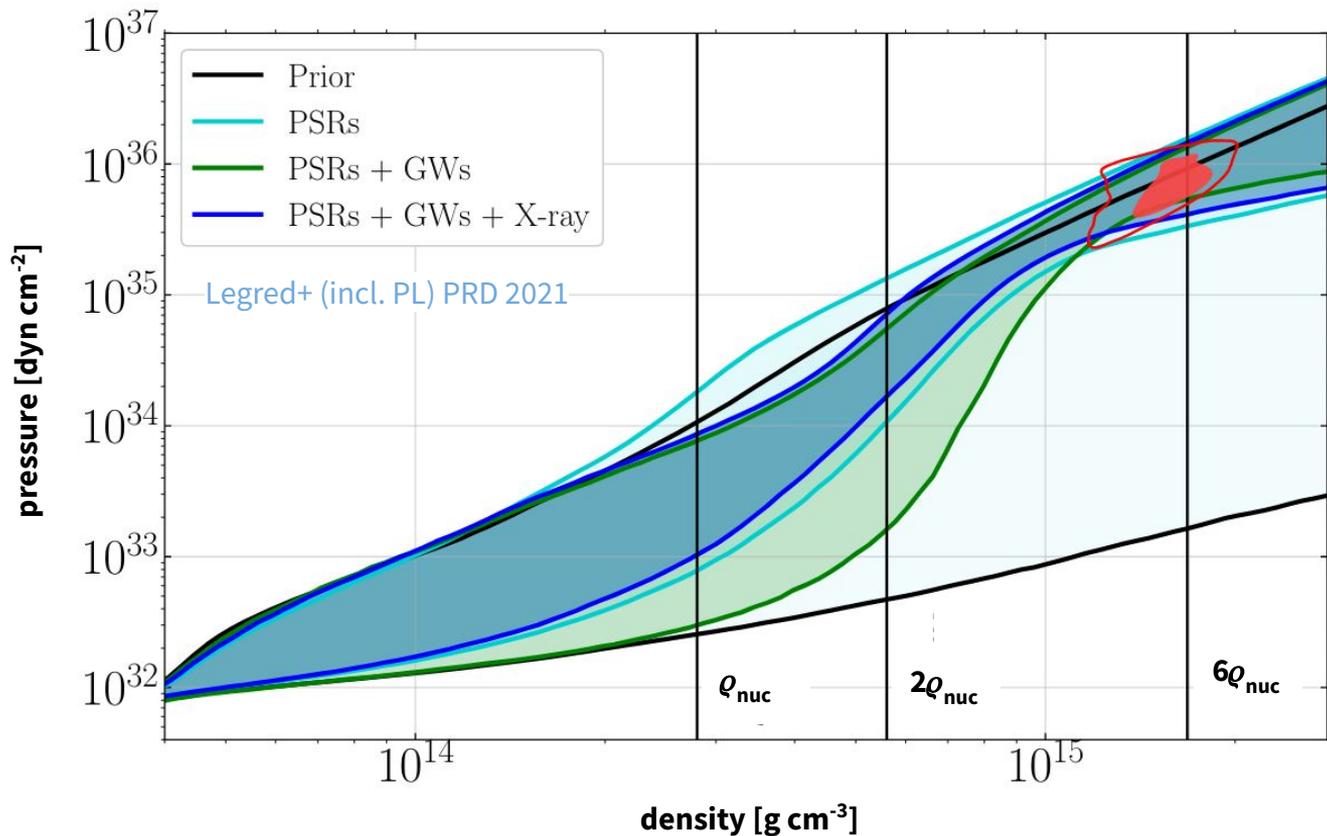
Legred, Chatziioannou, Essick, Han & Landry PRD 104 063003 (2021), arXiv:2106.05313



INT ♦ 24 May 2022

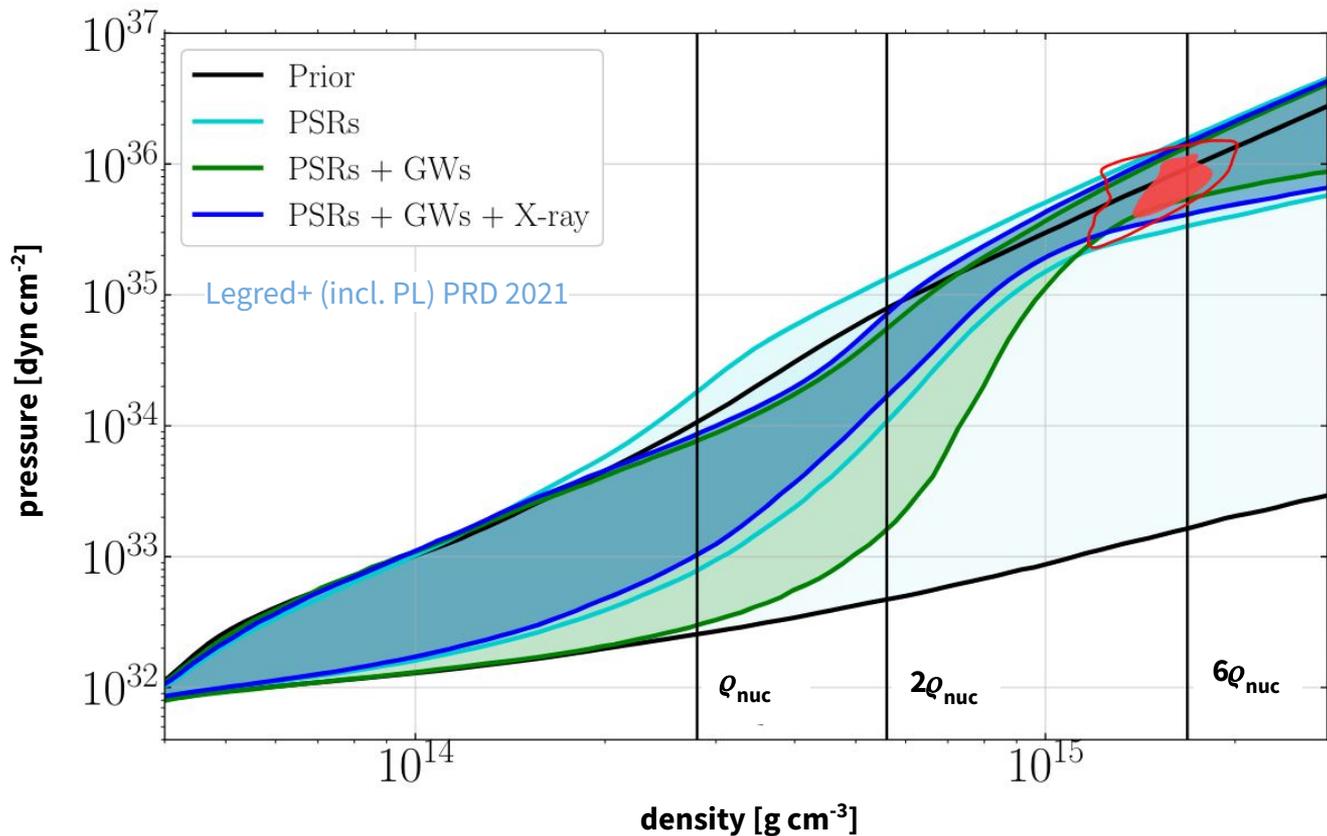


Observational constraints on the EOS



Current knowledge of the cold neutron star equation of state, with 90% credible error envelope

Observational constraints on the EOS



Current knowledge of the cold neutron star equation of state, with 90% credible error envelope

Causality + stability

(Chiral EFT + nuclear experiments)

Masses from radio PSRs

BNS tides with GWs

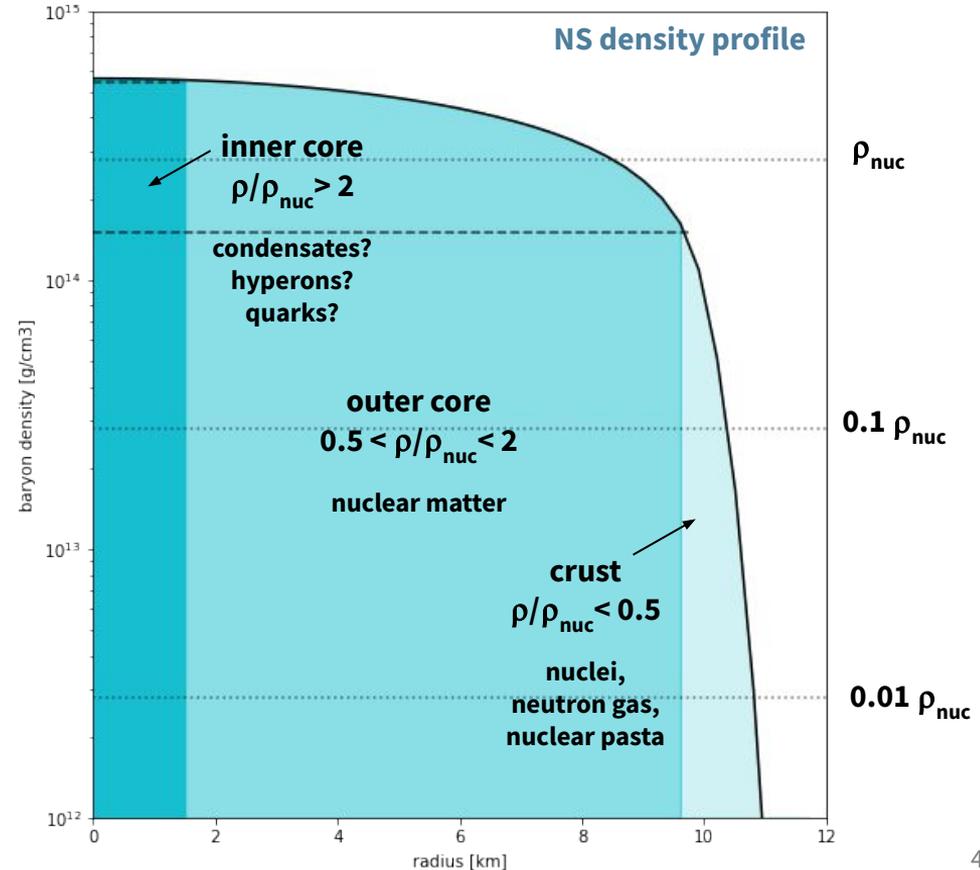
NS radii w/ X-ray hotspots

High-density neutron star matter

What pressures and densities are reached in neutron star cores?

What is the phase structure of matter inside neutron stars?

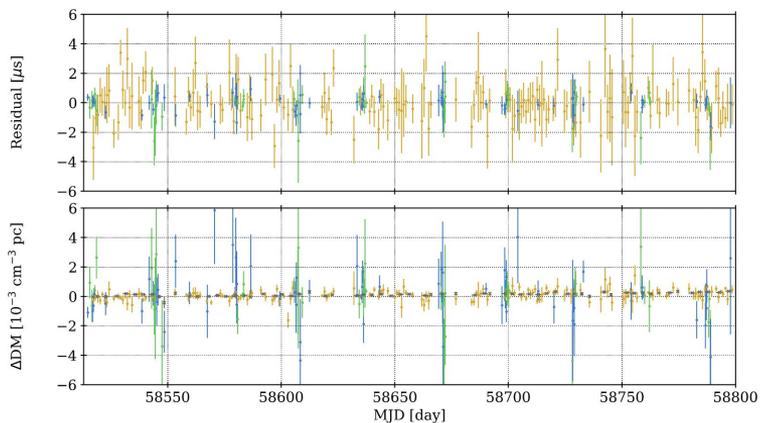
How much mass can be supported against gravitational collapse?



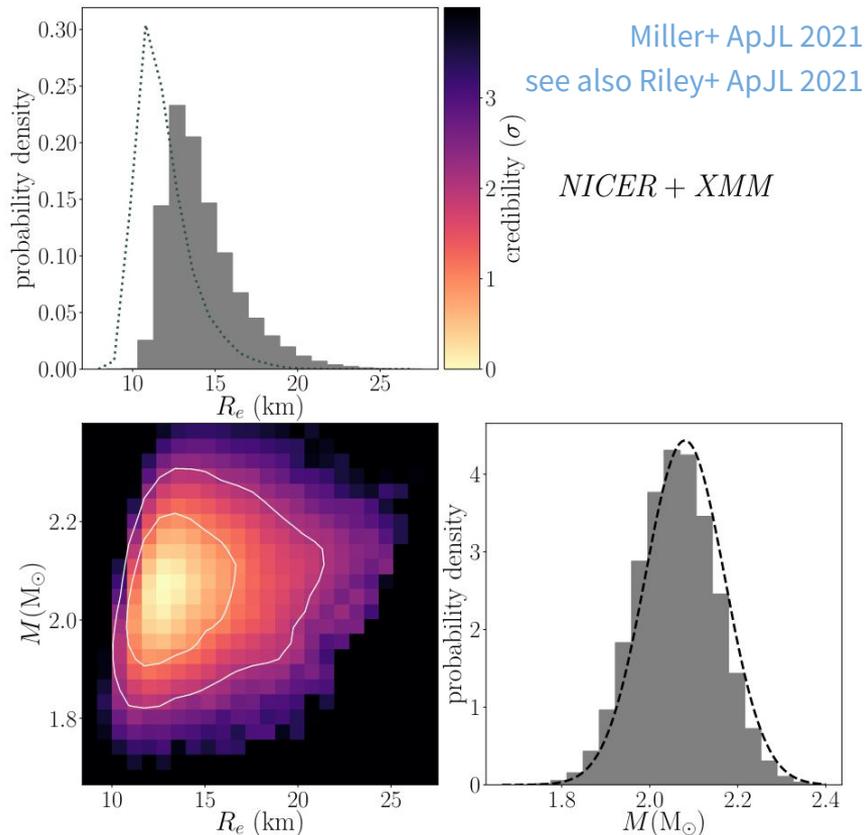
What's the maximum neutron star mass?

The heaviest known neutron star, PSR J0740+6620, has a mass of about $2.1 M_{\odot}$.

Fonseca+ ApJL 2021

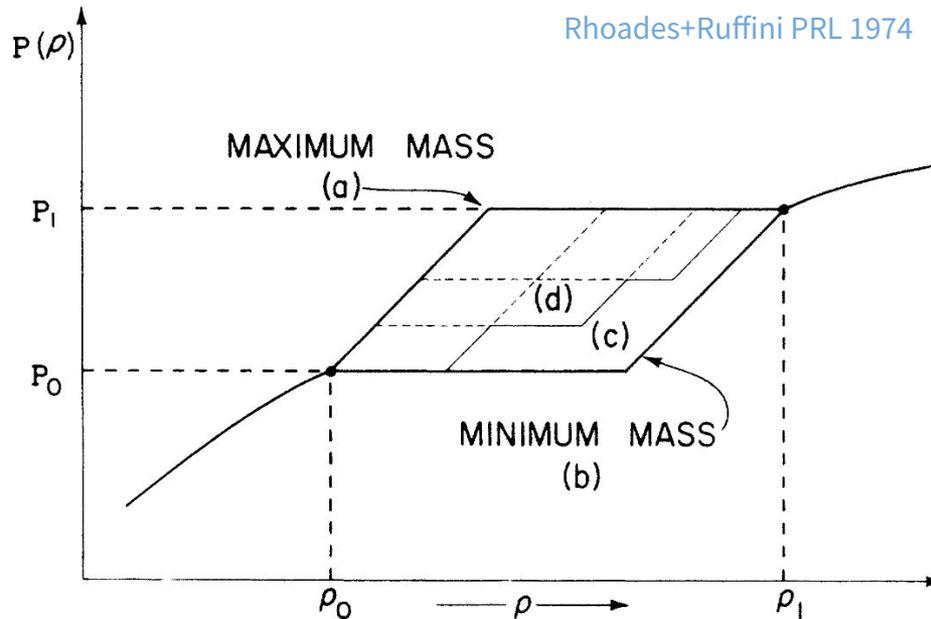


Pulsar mass measured via Shapiro delay of radio pulses:
 $m = 2.08 \pm 0.07 M_{\odot}$



What's the maximum neutron star mass?

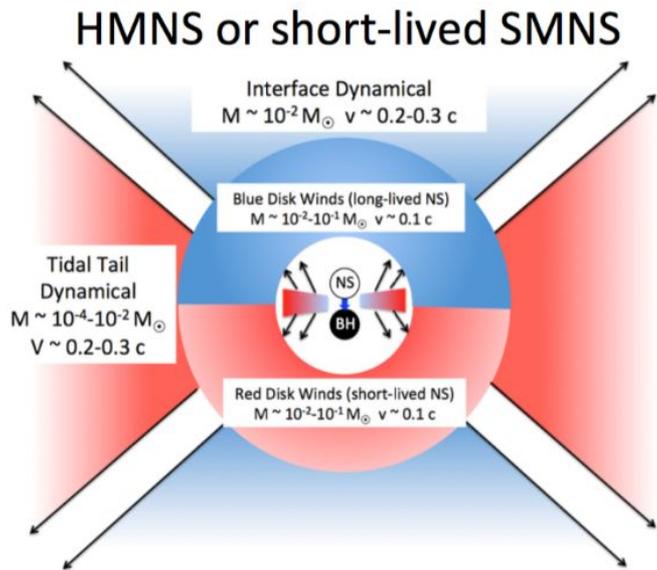
For nonrotating neutron stars, causality considerations set an upper bound of about $3 M_{\odot}$ on the maximum mass.



Neutron stars that exceed the Tolman-Oppenheimer-Volkoff (TOV) mass, M_{TOV} are unstable against radial perturbations

What's the maximum neutron star mass?

GW170817's electromagnetic counterpart suggests that the merger remnant collapsed to a black hole, bounding the threshold mass for collapse below $2.7 M_{\odot}$.



Margalit+Metzger ApJL 2017

LVC (incl. PL) PRX 2019

GW170817 remnant mass:

$$M_{\text{tot}} = 2.73^{+0.04}_{-0.01} M_{\odot}$$

Rotation can stabilize a neutron star up to 20% more massive than M_{TOV} Cook+ ApJ 1994



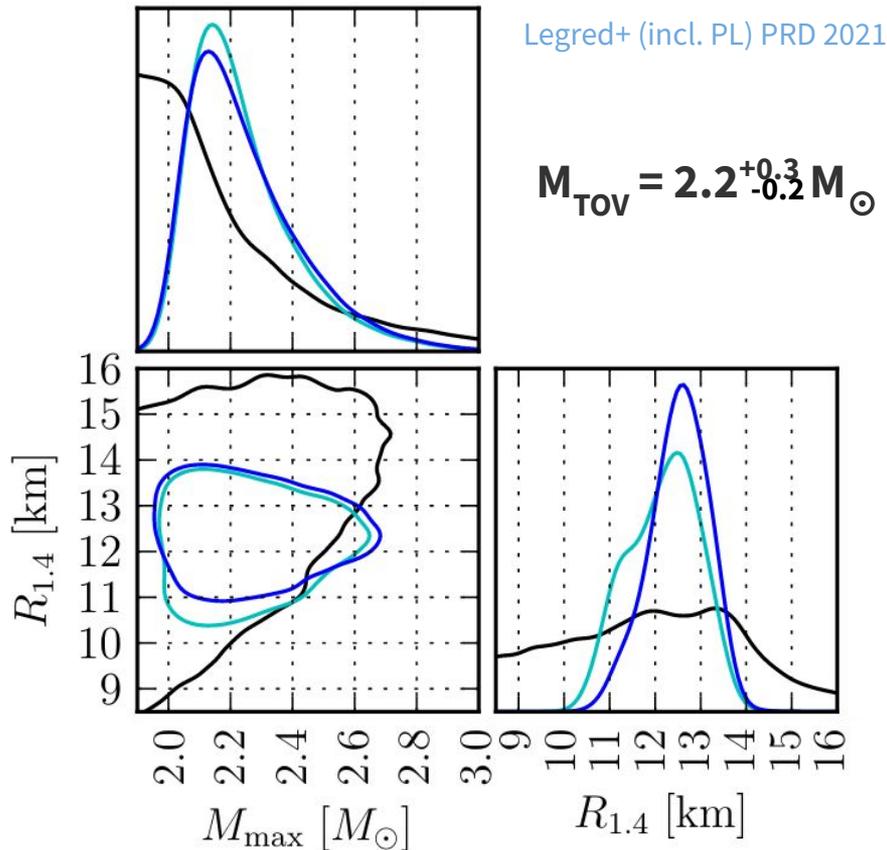
The threshold mass constraint maps to an upper bound of approximately $2.3 M_{\odot}$ on M_{TOV}

LVC (incl. PL) CQG 2020

What's the maximum neutron star mass?

Inference of the equation of state from gravitational-wave and pulsar observations of neutron stars constrains M_{TOV} to be approximately $2.2 M_{\odot}$.

Relies on an EOS model to extrapolate up to the high densities relevant for the maximum mass



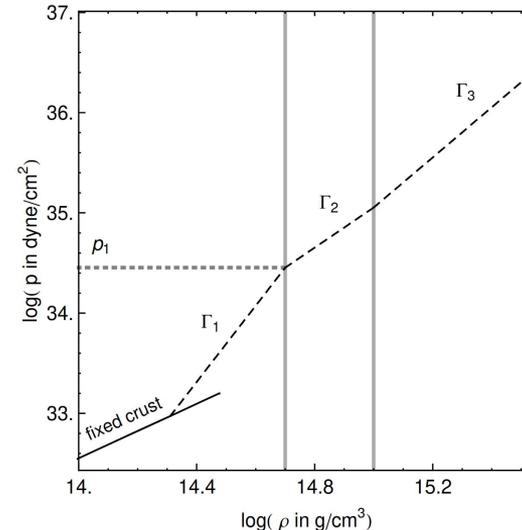
Modeling + inferring the EOS

Hierarchical Bayesian inference of the EOS

$$P(\text{eos}|d) \propto \underbrace{P(\text{eos})}_{\text{eos prior}} \prod_i \int \underbrace{P(d_i|m_{1,2}^i, \Lambda_{1,2}^i)}_{\text{gw likelihood}} \underbrace{P(m_{1,2}^i, \Lambda_{1,2}^i|\text{eos})}_{\text{prior on gw params}} dm_{1,2}^i d\Lambda_{1,2}^i$$

One way to prescribe an EOS prior is via parameterization, e.g.

- *Piecewise polytrope* [Read+ PRD 2008](#)
- *Spectral decomposition of Γ* [Lindblom PRD 2010](#)
- *Sound speed extension* [Tews+ PRC 2018](#)
- ...



piecewise polytrope

[Read+ PRD 2008](#)

Modeling + inferring the EOS

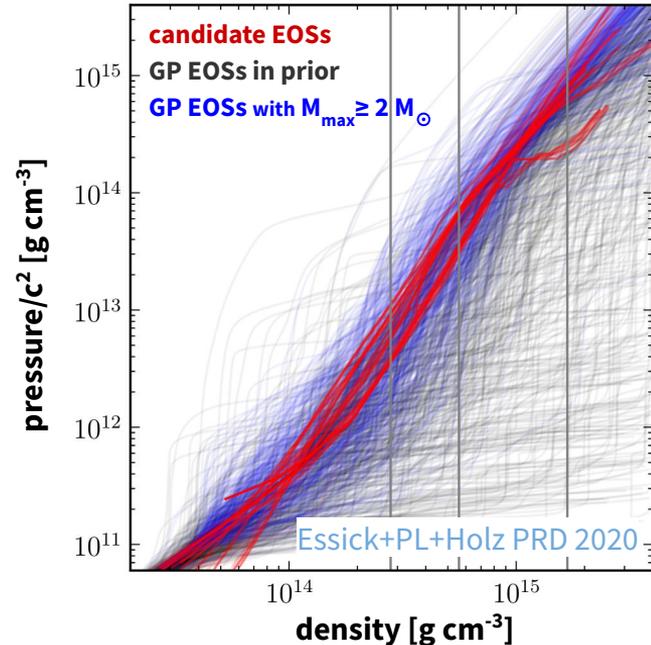
Hierarchical Bayesian inference of the EOS

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Another approach is nonparametric, representing the EOS prior as a Gaussian process (GP) [Landry+Essick PRD 2019](#)

The GP is a probability distribution over causal and thermodynamically stable functions $p(e)$ with Gaussian covariance kernel

$$K_{\text{se}}(x_i, x_j; \sigma, l) = \sigma^2 \exp\left(-\frac{(x_i - x_j)^2}{2l^2}\right)$$

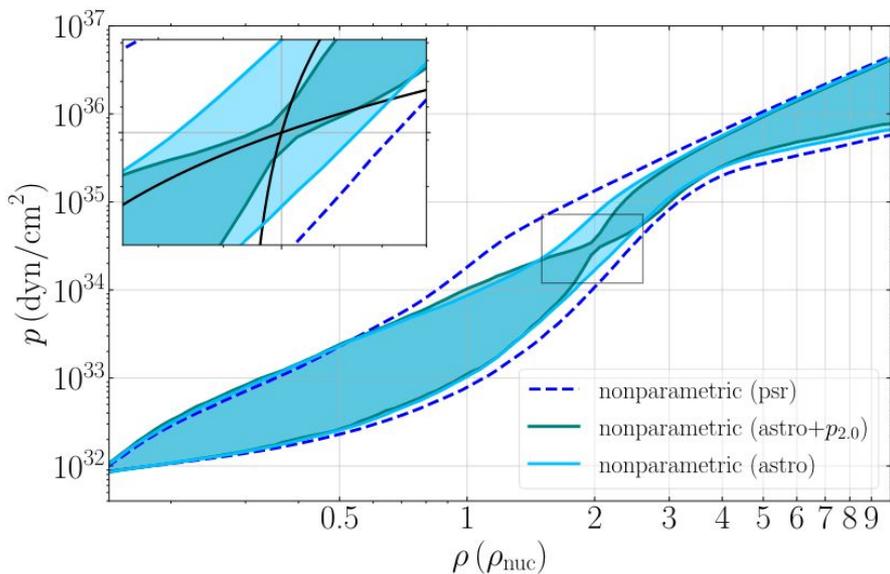


Modeling + inferring the EOS

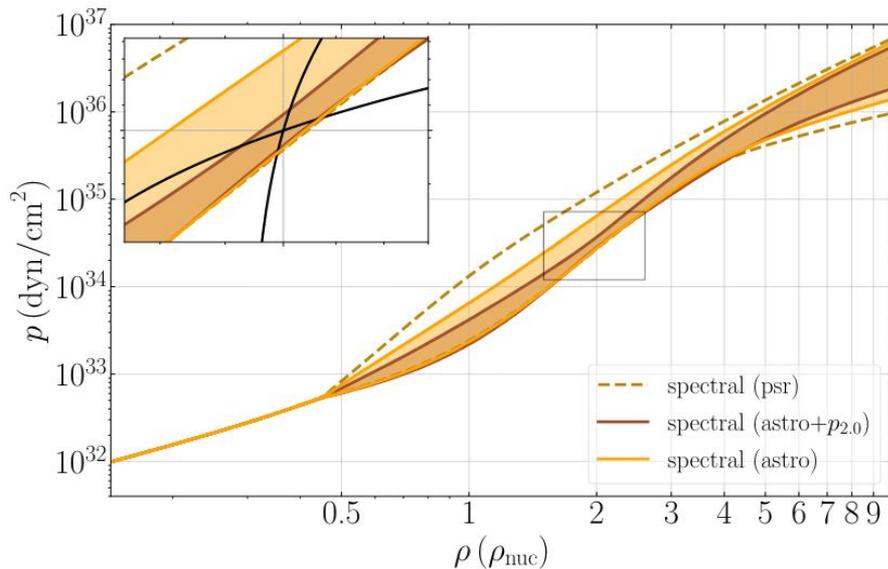
Parametric EOS representations can introduce artificial correlations between different densities; mitigate this with EOSs generated from a Gaussian process

Legred+ (incl. PL) PRD 2022

nonparametric



parametric

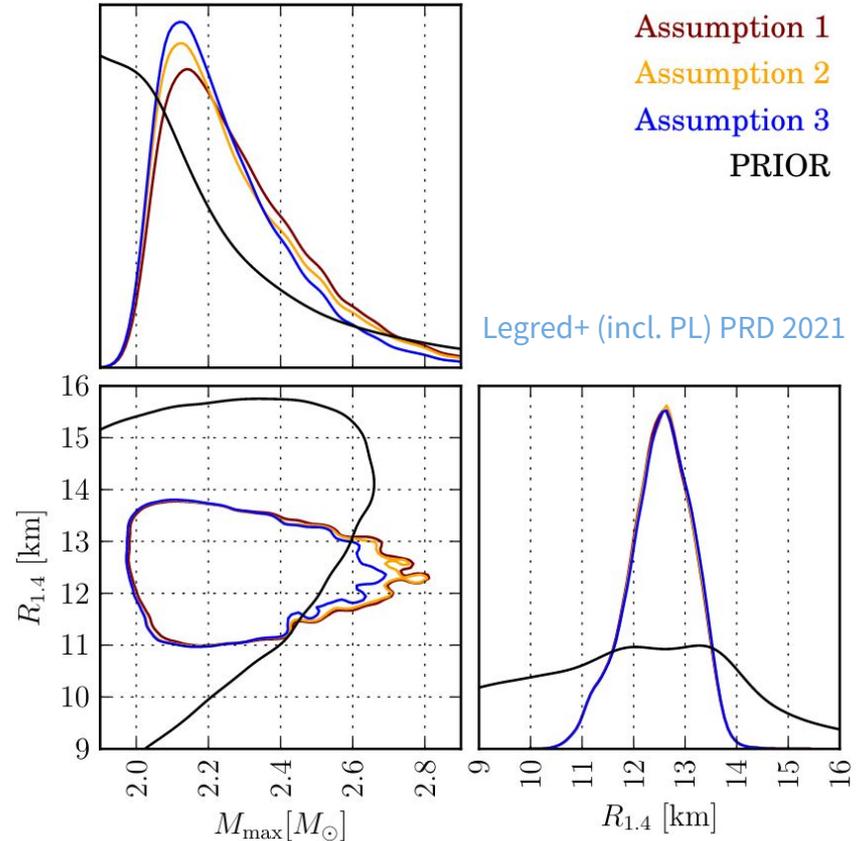


EOS inference + the neutron star population

EOS inference relies (either explicitly or implicitly) on a choice of population model to prescribe the prior on source properties for each observation.

$$P(m|\text{eos}) = \frac{\Theta(M_{\text{lower}} \leq m)\Theta(m \leq M_{\text{upper}})}{M_{\text{upper}} - M_{\text{lower}}}$$

- 1. PSR J0740 may be a black hole**
- 2. PSR J0740 is known to be a NS, but the heaviest NSs with masses near M_{TOV} aren't observable as pulsars**
- 3. PSR J0740 is known to be a NS, and pulsar masses can be as large as M_{TOV}**



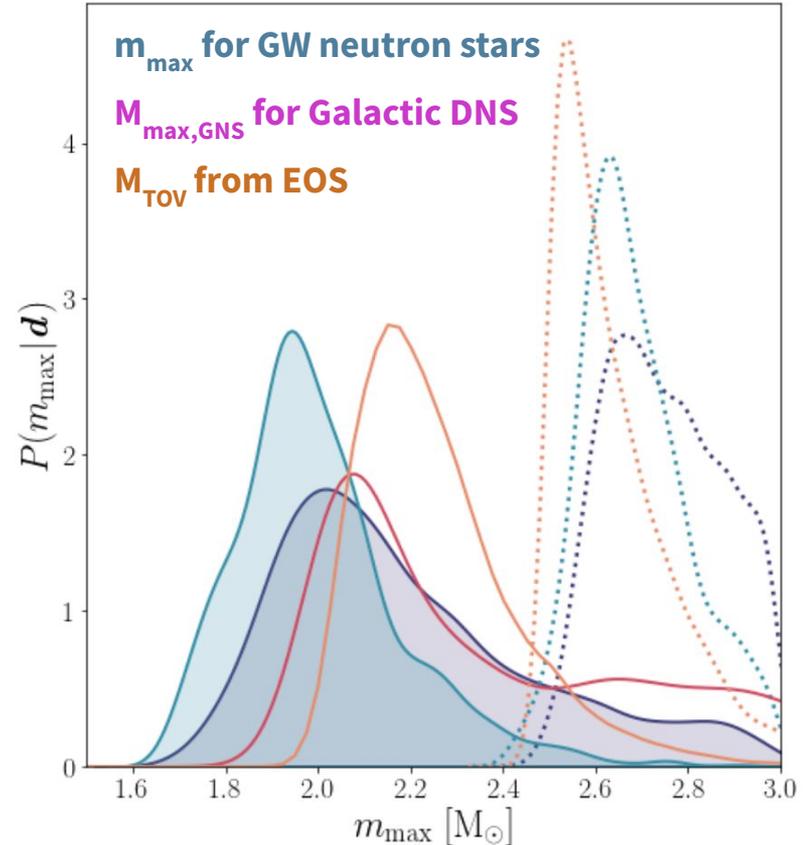
EOS inference + the neutron star population

Landry+Read ApJL 2021

EOS inference relies (either explicitly or implicitly) on a choice of population model to prescribe the prior on source properties for each observation.

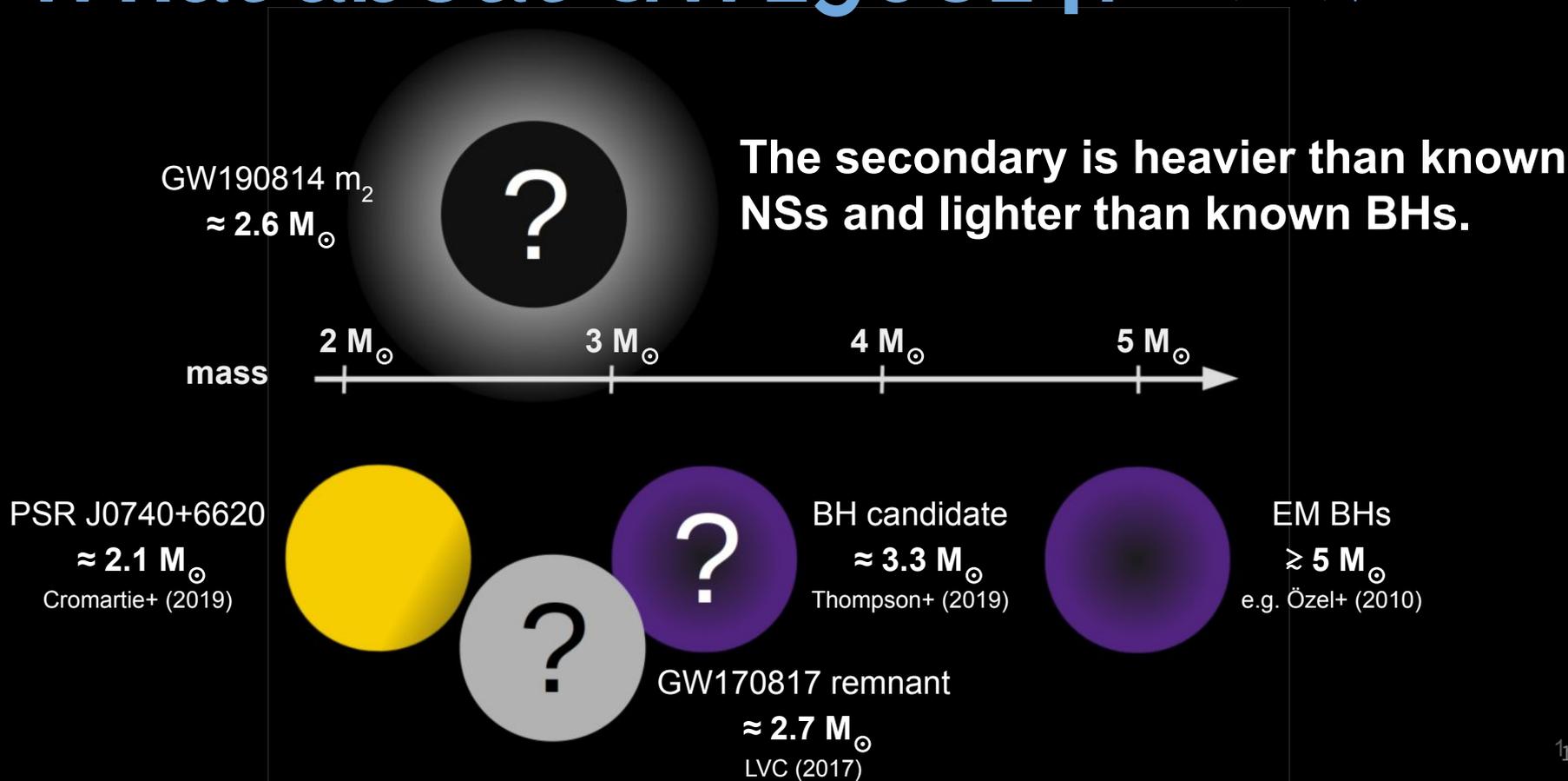
The correct approach is to simultaneously infer the neutron star population and the EOS!

- 1. PSR J0740 may be a black hole**
- 2. PSR J0740 is known to be a NS, but the heaviest NSs with masses near M_{TOV} aren't observable as pulsars**
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What about GW190814?

LVC (incl. PL) ApJL 2020



What about GW190814?

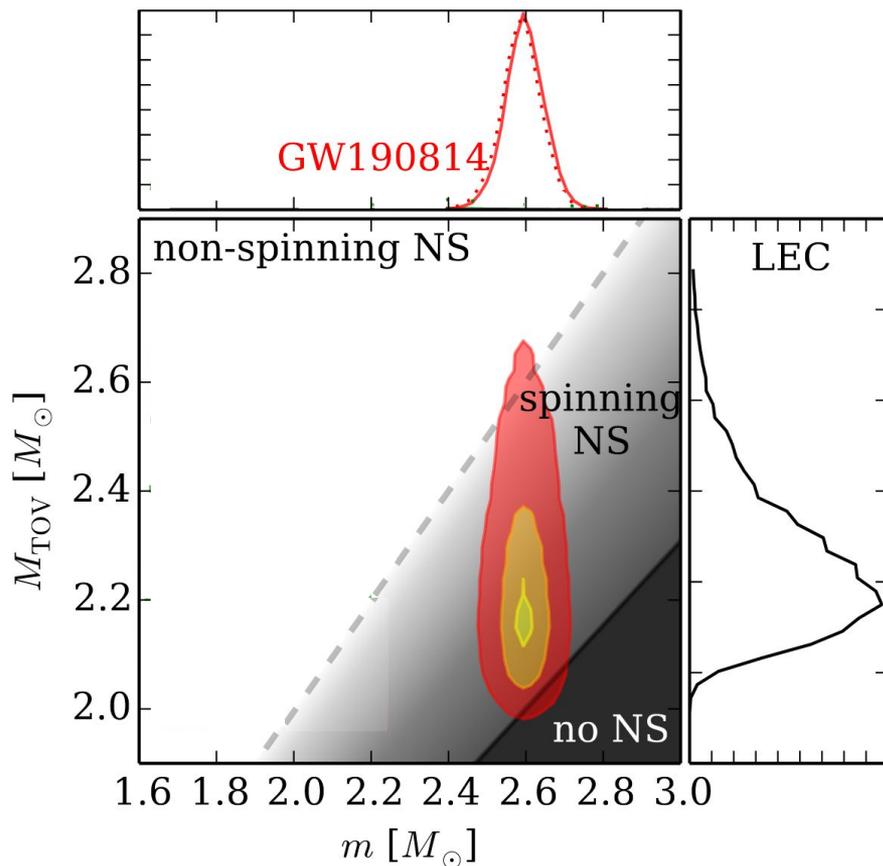
Comparison with M_{TOV} indicates that GW190814's $2.59 \pm 0.09 M_{\odot}$ secondary is probably too heavy to be a nonrotating neutron star.

$$P(m_2 \leq M_{\text{TOV}}) \approx 5\%$$

Similar conclusions reached via

- *Parametric EOS inference* [Godzieba+ ApJ 2021](#)
- *Density functional theory* [Fattoyev+ PRC 2020](#)
- *Chiral EFT* [Tews+ ApJL 2021](#)
- ...

Essick+Landry ApJ 2020



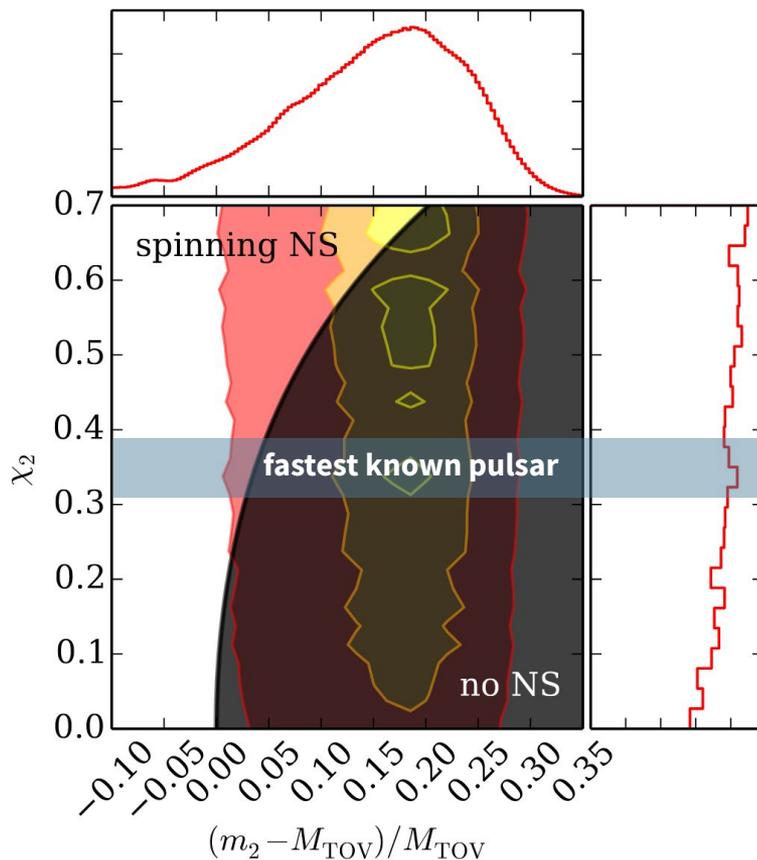
What about GW190814?

Many proposed alternatives to the black hole interpretation involve rapid rotation

- *Super-fast pulsar* [Zhang+Li ApJ 2020](#)
- *Rotating NS, collapsed premerger* [Most+ MNRASL 2020](#)
- *Rotating quark star* [Dexheimer+ PRC 2021](#)
- ...

The fastest known pulsar spins at 716 Hz,
or $\chi \sim 0.3-0.4$ [Hessels+ Sci 2006](#)

Supporting $2.6 M_{\odot}$ requires *very* rapid rotation that is difficult to explain from the astrophysical perspective.



Astrophysical observations favor a maximum nonrotating neutron star mass of about $2.3 M_{\odot}$

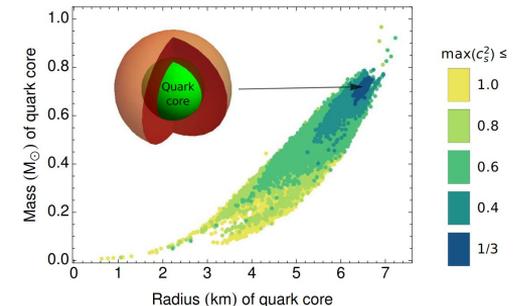
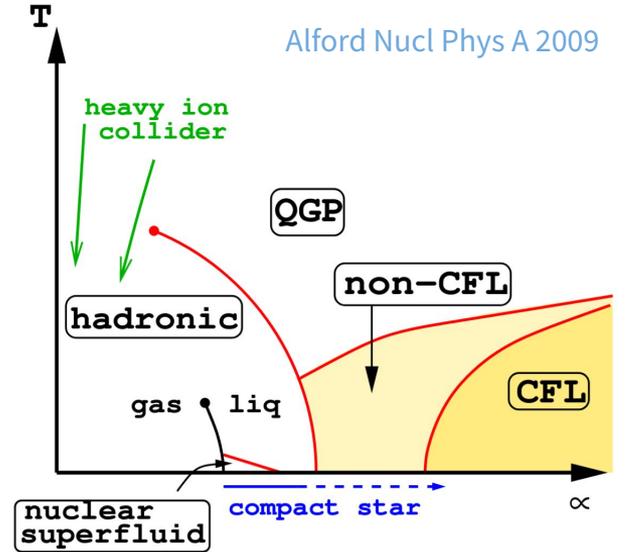
There is likely a subpopulation of light black holes with masses just above M_{TOV}

Exotic phases in neutron star cores?

At sufficiently high densities, non-hadronic degrees of freedom are expected to appear, e.g. hyperons and/or deconfined quarks

- *Distinct hadronic and exotic phases may be separated by a strong first-order phase transition* e.g. Alford+ PRD 2013
- *Hadrons and exotic particles may coexist in mixed phase with smooth crossover* e.g. Baym+ ApJ 2019

Some predictions that quark cores appear generically in the heaviest neutron stars e.g. Annala+ Nat Phys 2020



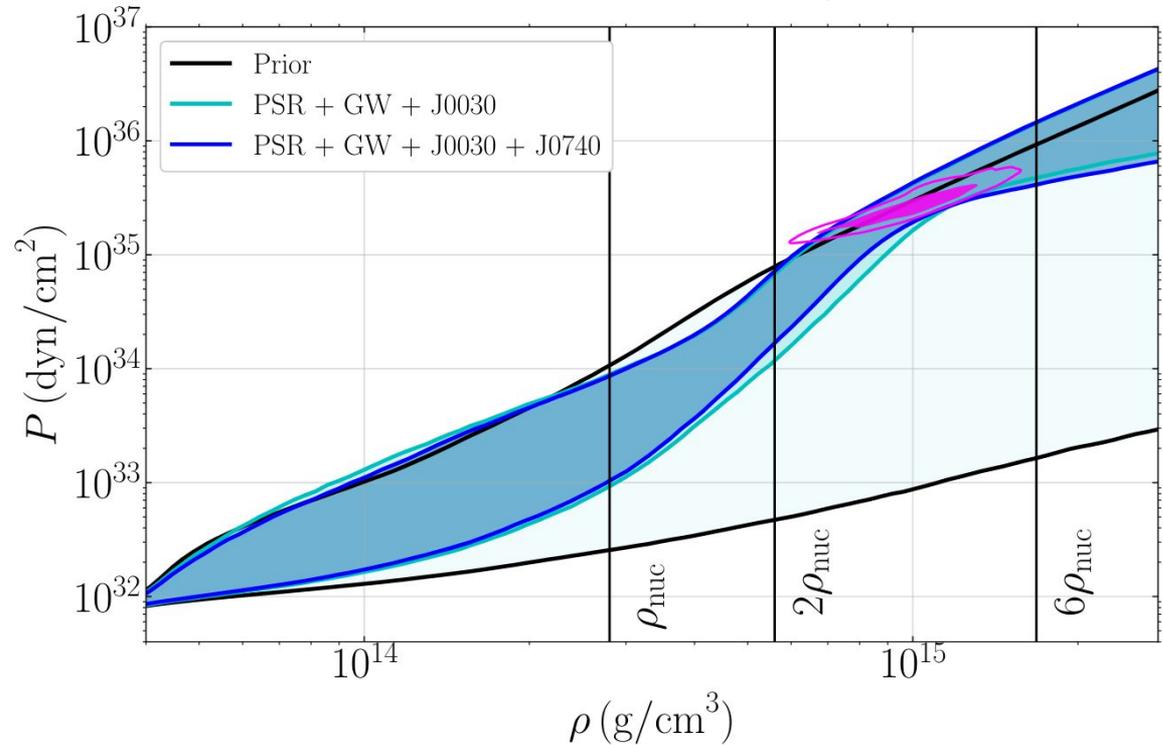
How dense can neutron stars get?

Inferred central pressure and density for PSR J0740+6620

central density

$2.3 - 4.7 \rho_{\text{nuc}}$

Legred+ (incl. PL) PRD 2021



How dense can neutron stars get?

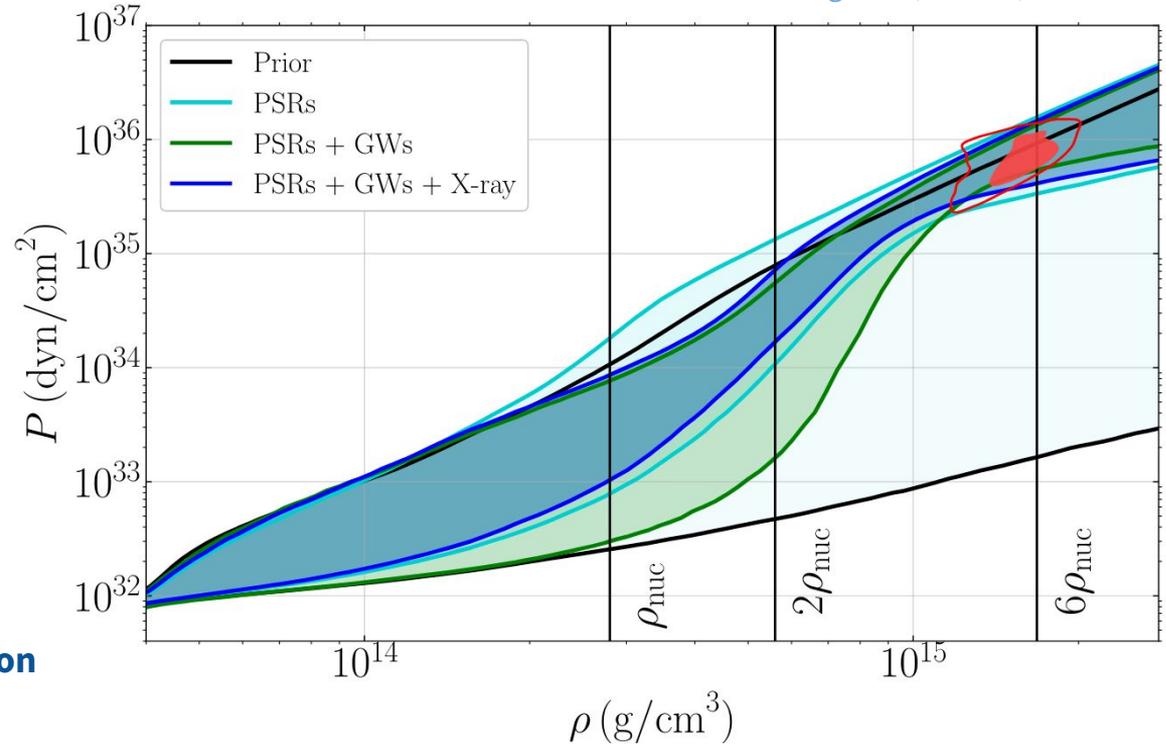
Legred+ (incl. PL) PRD 2021

Inferred central pressure and density for maximum-mass neutron stars

central density

$3.9 - 6.4 \rho_{\text{nuc}}$

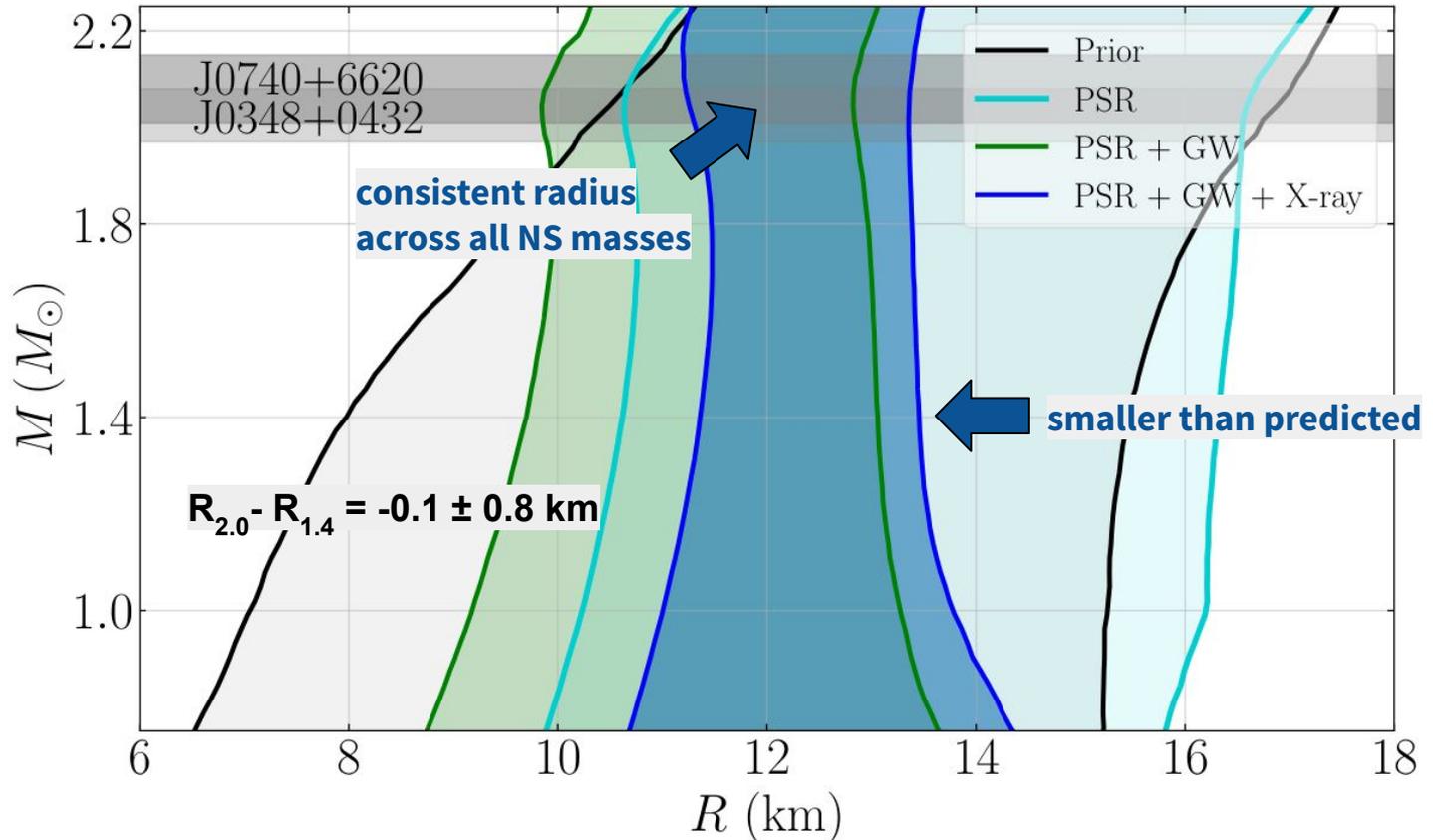
For exotic phases to occur in neutron star cores, phase transition onset must be below about $6 \rho_{\text{nuc}}$



Neutron star radii at high and low masses

Legred+ (incl. PL) PRD 2021

small radius
difference
disfavors
high-density
softening of
EOS associated
with exotic
phases



Similar radius for PSR J0740 and $1.4 M_{\odot}$
neutron stars, less extreme central
densities disfavor exotic cores

... but exotic phases by no means ruled
out yet

Sound speed in neutron star matter

At asymptotically high densities, expect matter to be described by (perturbative) QCD calculations

- *Sound speed of $1/\sqrt{3}$ for ultra-relativistic massless particles*
- *Sound speed reduced by finite particle masses, weak interactions*

Conjecture that sound speed in neutron star matter interpolates between low-density limit $c_s \ll 1$ and perturbative QCD limit $c_s = 1/\sqrt{3}$ [Bedaque+Steiner PRL 2015](#)

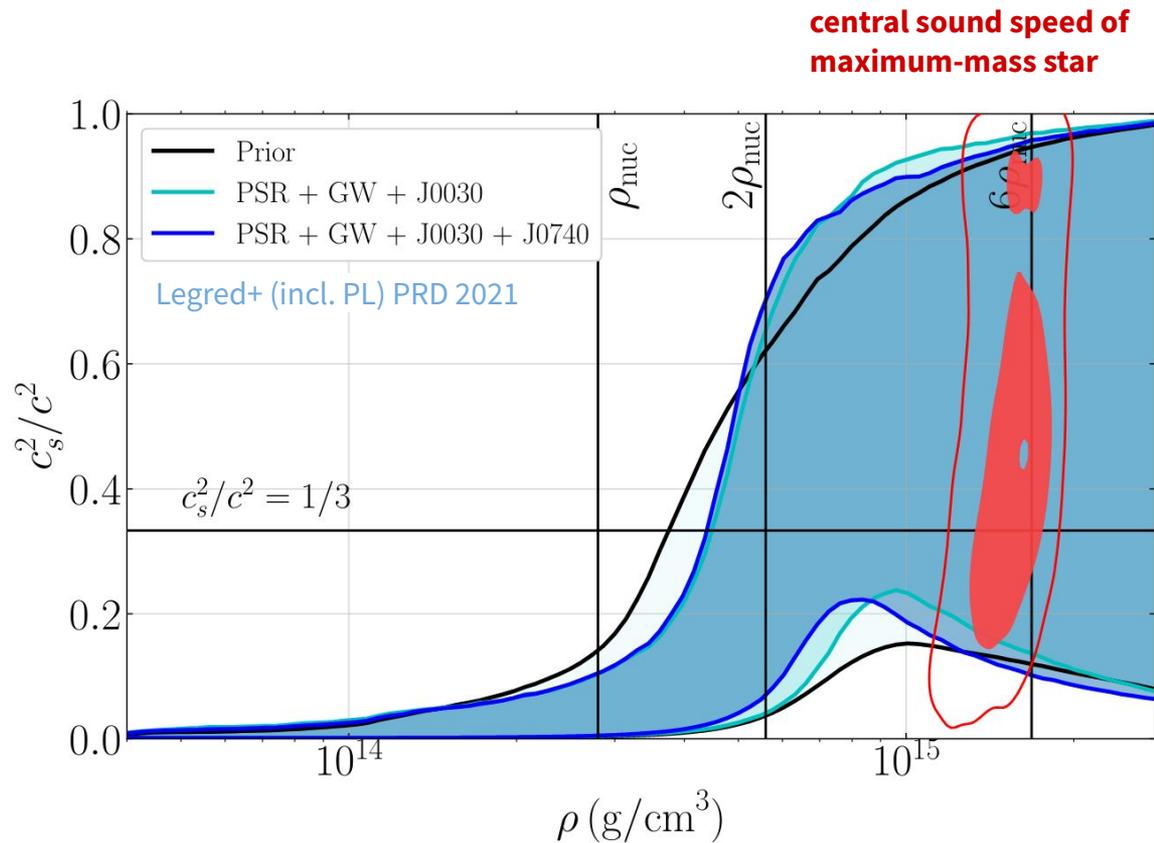
A sound speed above $1/\sqrt{3}$ indicates strongly coupled, non-conformal matter

- *E.g. quarkyonic matter* [McLerran+Reddy PRL 2019](#)

$2 M_{\odot}$ pulsars are in tension with this conformal bound [Bedaque+Steiner PRL 2015](#)

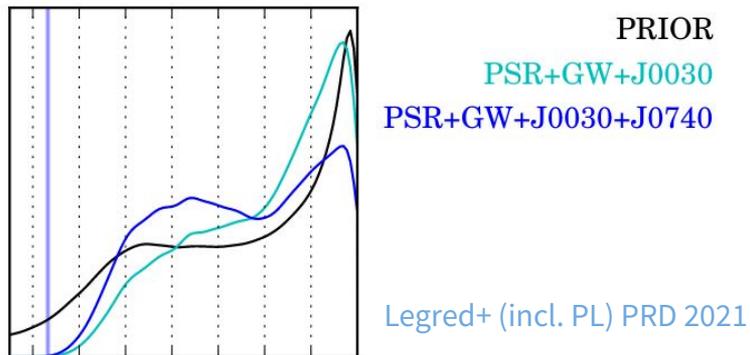
Sound speed in neutron star matter

EOSs that satisfy the conformal bound are disfavored relative to the prior



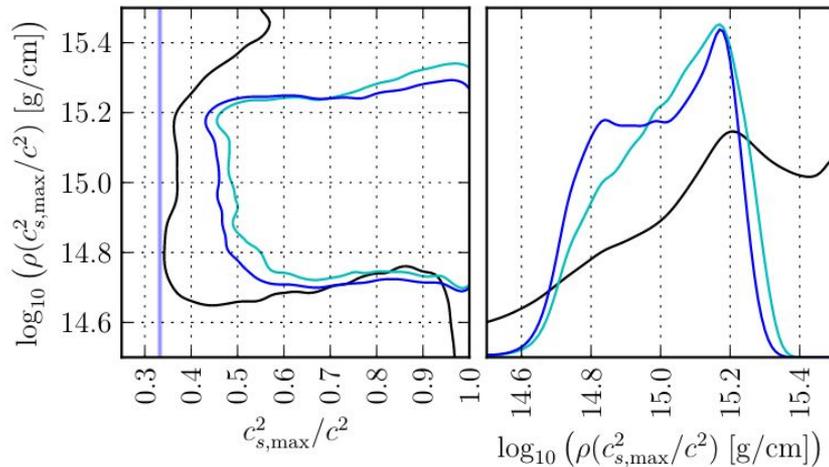
Sound speed in neutron star matter

maximum sound speed



EOSs that violate conformal limit are preferred with Bayes factor of $O(1000)$

See Landry+ PRD 2020, Drischler+ PRC 2022, Altiparmak+ arXiv:2203.14974 for similar conclusions

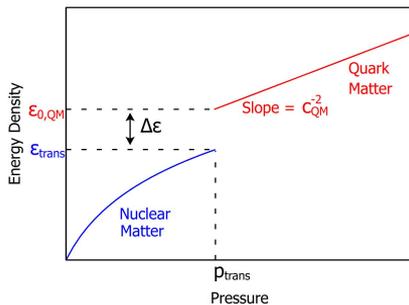


**Conjectured conformal bound on the
sound speed likely violated inside
neutron stars**

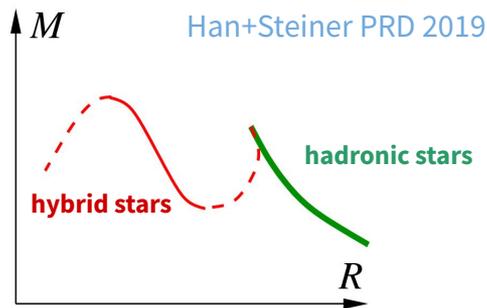
**Possible indication of a strongly
interacting phase**

Hybrid stars and strong phase transitions

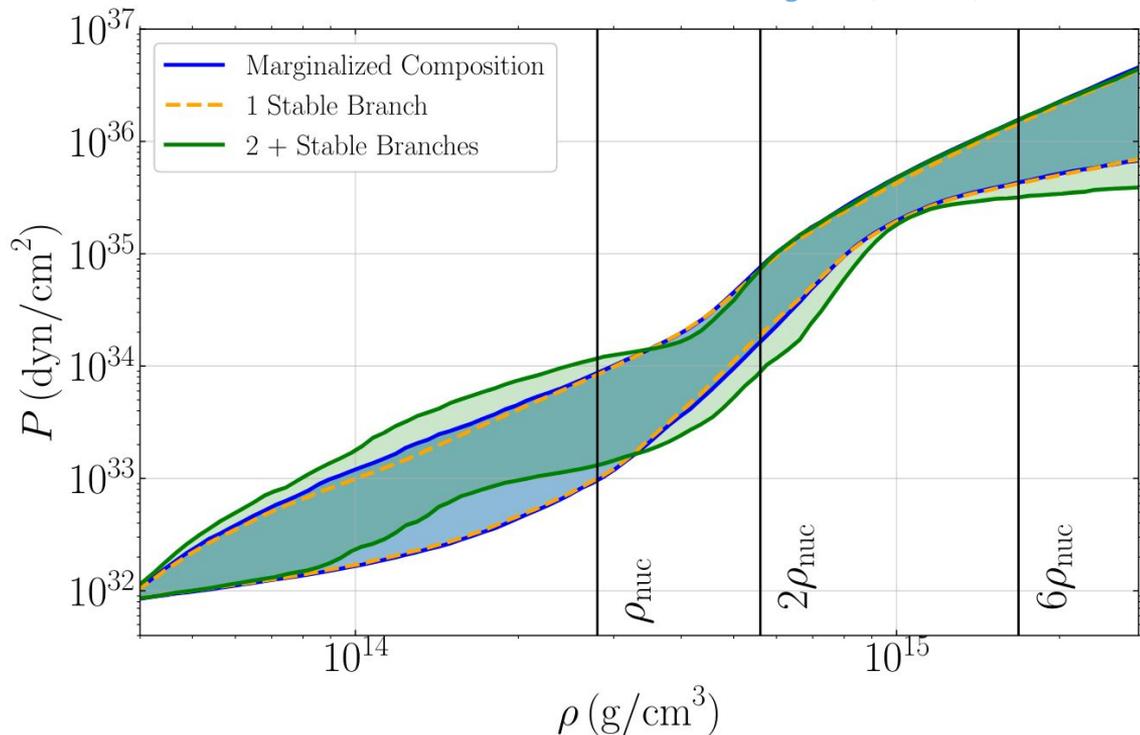
- Energy jump in the EOS from strong first-order phase transitions



- Large enough jumps produce a disconnected hybrid star branch

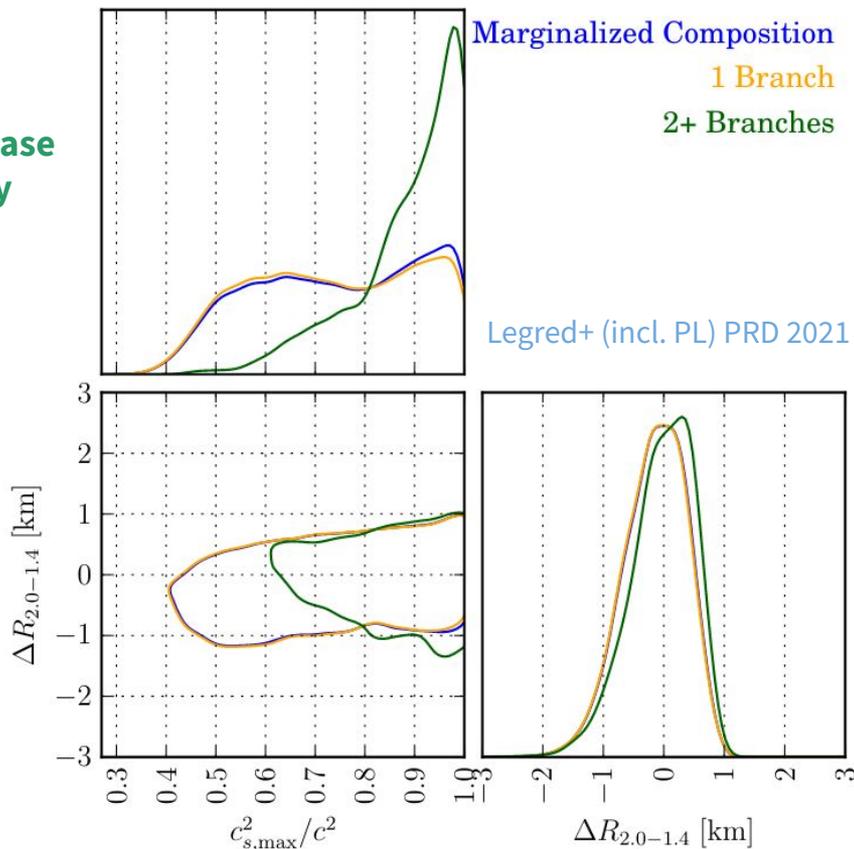


Legred+ (incl. PL) PRD 2021



Hybrid stars and strong phase transitions

EOSs with a strong phase transition reach a very large sound speed



radius difference tightly constrained

EOSs that have a disconnected hybrid star branch are disfavored with a Bayes factor of 7

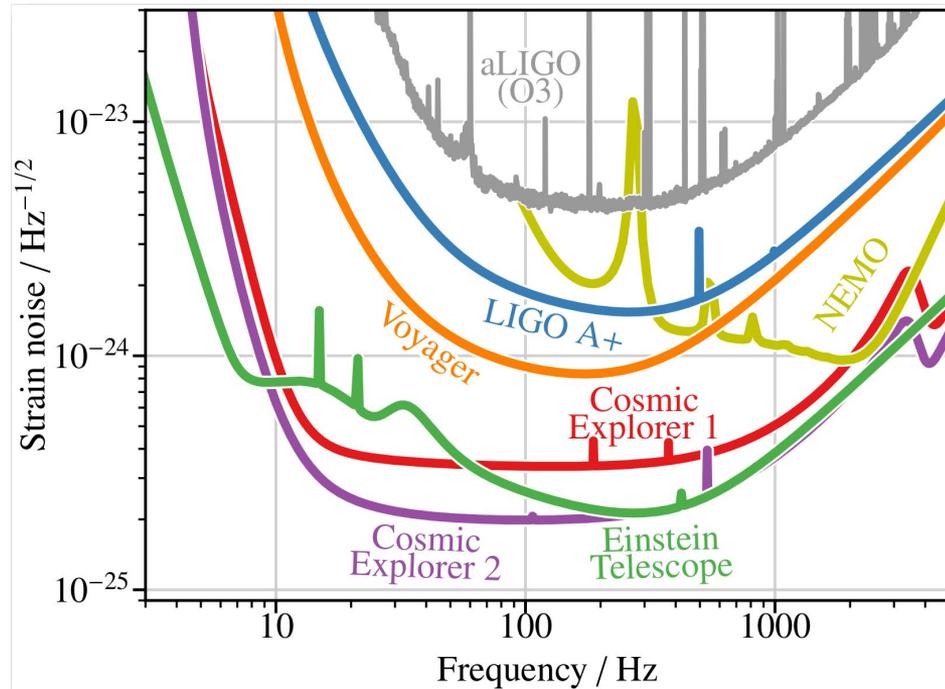
**PSR J0740's radius measurement
reduces the probability of a strong
phase transition supporting a
disconnected hybrid star branch
relative to past results**

... but only mildly disfavored

LIGO A+ & 3G GW observatories

Kuns+ 2020

- During O4, at design sensitivity, LIGO & Virgo expected to detect ~ 4 BNSs with $\text{SNR} > 20$
- During O5, at A+ sensitivity, LIGO & Virgo expected to detect $\mathcal{O}(10)$ BNSs with $\text{SNR} > 20$
- Cosmic Explorer expected to detect $\mathcal{O}(100)$ BNSs with $\text{SNR} > 100$ per year
- CE will also capture complete BNS population out to $z \sim 2$, have a horizon of $z \sim 10$



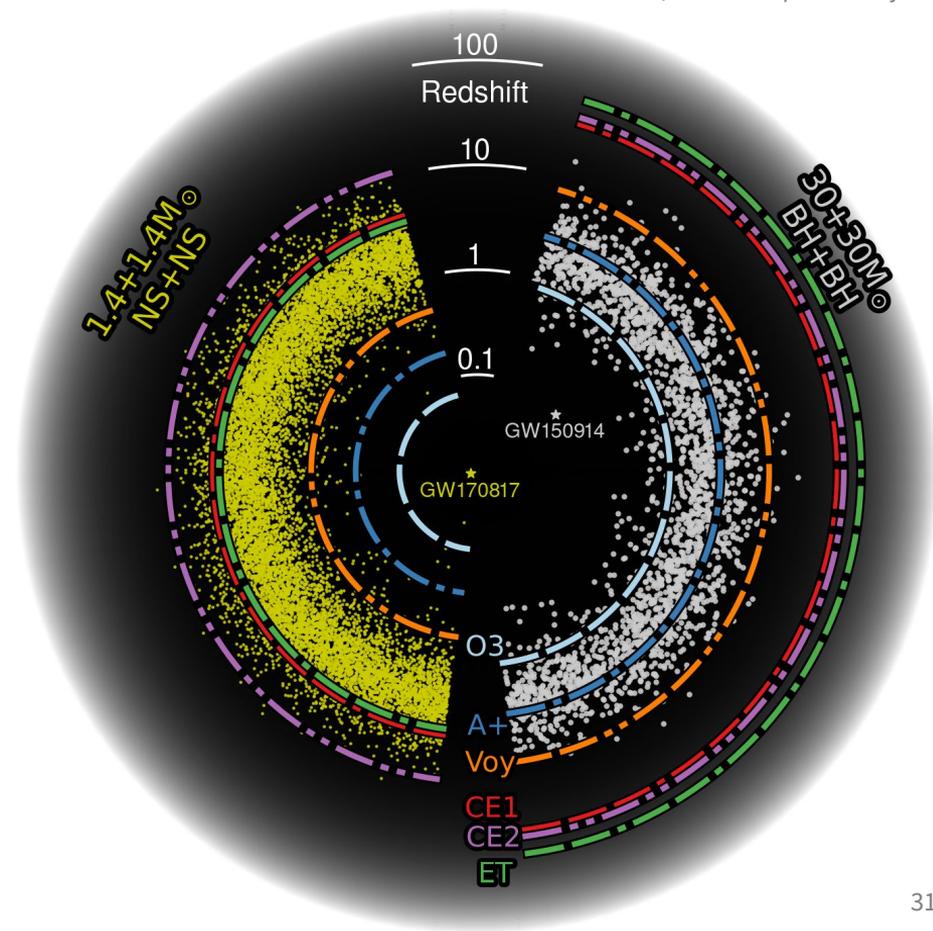
CE Horizon Study: [Evans+ \(+PL\) arXiv:2109.09882](#)

LIGO A+ & 3G GW observatories

E. Hall, Cosmic Explorer Project

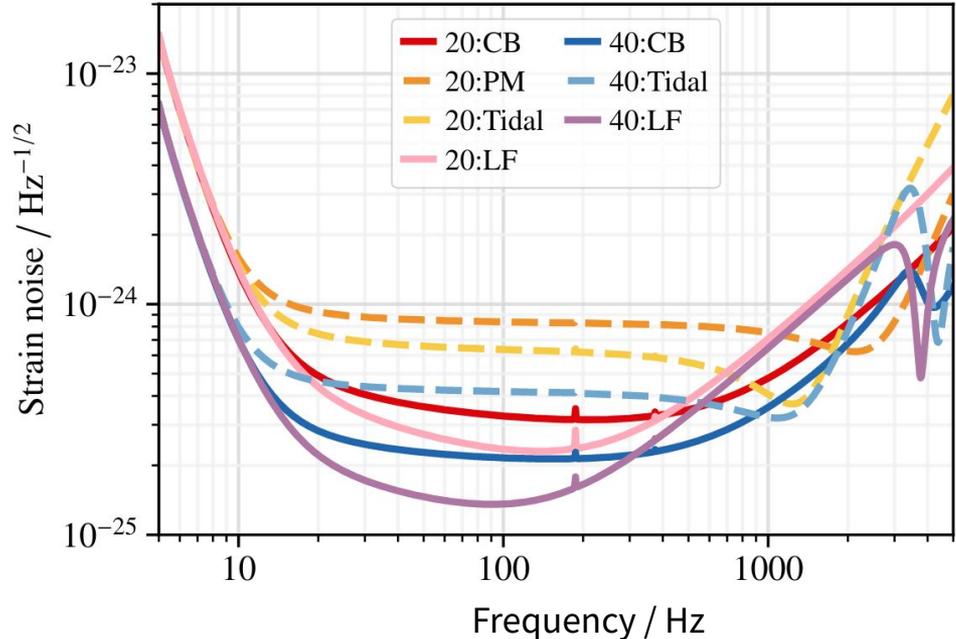
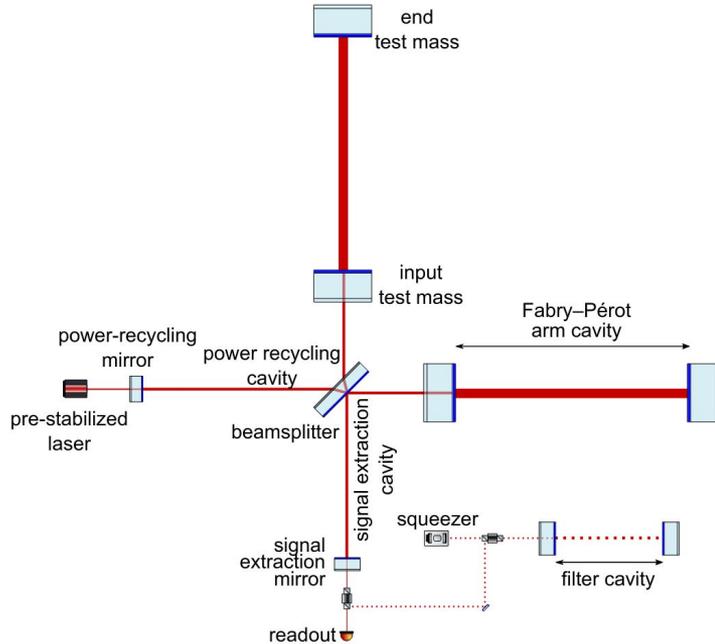
- During O4, at design sensitivity, LIGO & Virgo expected to detect ~ 4 BNSs with $\text{SNR} > 20$
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CE Horizon Study: Evans+ (+PL) arXiv:2109.09882



LIGO A+ & 3G GW observatories

It may be possible to tune the reflectivity of Cosmic Explorer's signal extraction mirror to improve sensitivity to binary neutron star inspirals (or postmerger gravitational waves). [Srivastava+ \(incl. PL\) arXiv:2201.10668](#)



The nonrotating maximum neutron star mass is approximately $2.3 M_{\odot}$

The matter density inside cold, nonrotating neutron stars likely does not exceed six times nuclear saturation density

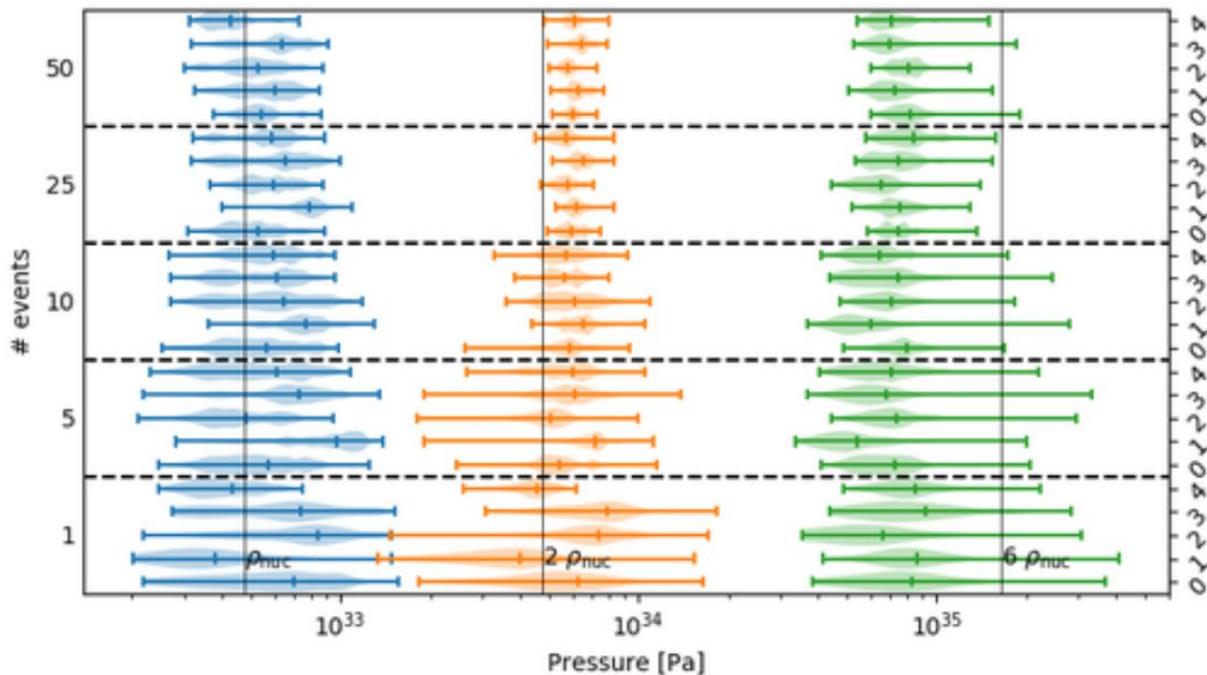
There is strong evidence that the sound speed inside neutron stars exceeds the conjectured conformal bound

Strong phase transitions aren't necessary to explain current observations, but they aren't strongly disfavored either

Mismodeling the neutron star population

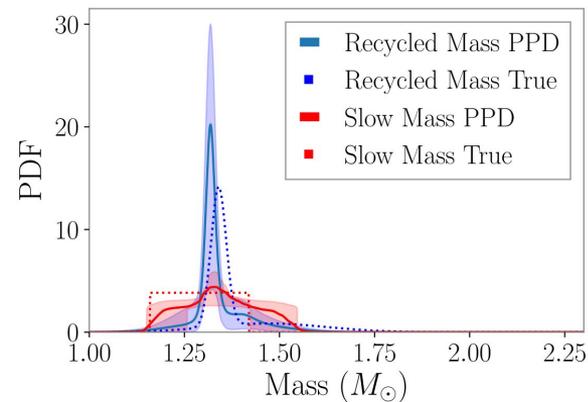
Choosing wrong population-level mass prior **biases recovered EOS** after $O(10)$ events

Wysocki+ arXiv:2001.01747

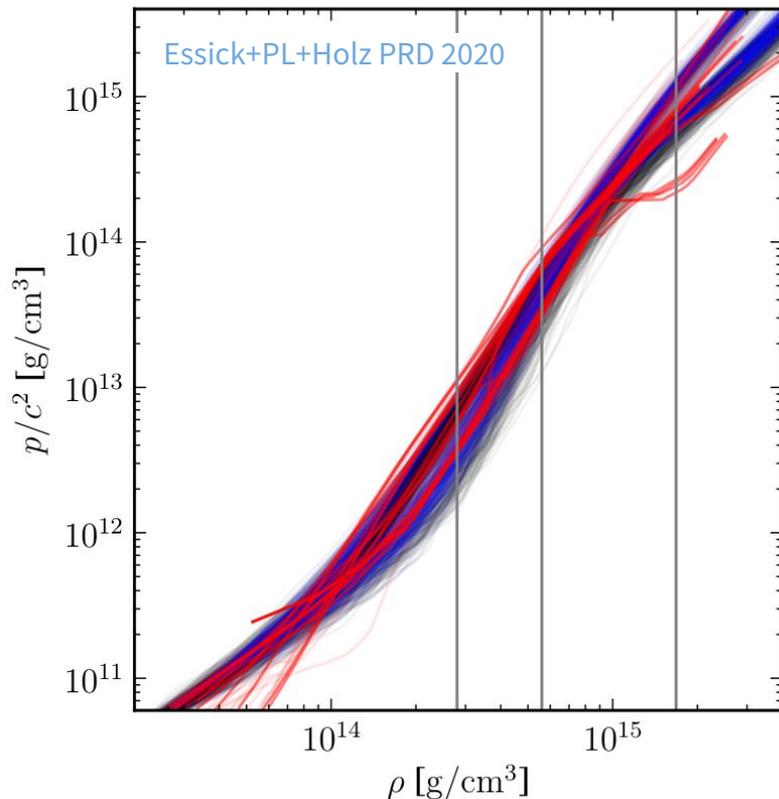


Converse also true:
mismodeling EOS biases
recovered mass distribution

Golomb+Talbot ApJ 2022



Inferring neutron star composition



A Gaussian process model for the EOS allows for model comparison between different compositions for neutron star matter within a particular theoretical framework, e.g.

- *Condition a GP on hadronic RMF models*
- *Do the same with RMF models hyperons*
- *Do the same with RMF models with quarks*

| Prior (\mathcal{H}_i) | $P(\text{Hadronic} \text{data})$ | $P(\text{Hyperonic} \text{data})$ | $P(\text{Quark} \text{data})$ |
|---------------------------|----------------------------------|-----------------------------------|-------------------------------|
| <i>informed</i> | 28% | 16% | 56% |