Initial State Modelling of Fixed-Target Heavy Ion Collisions

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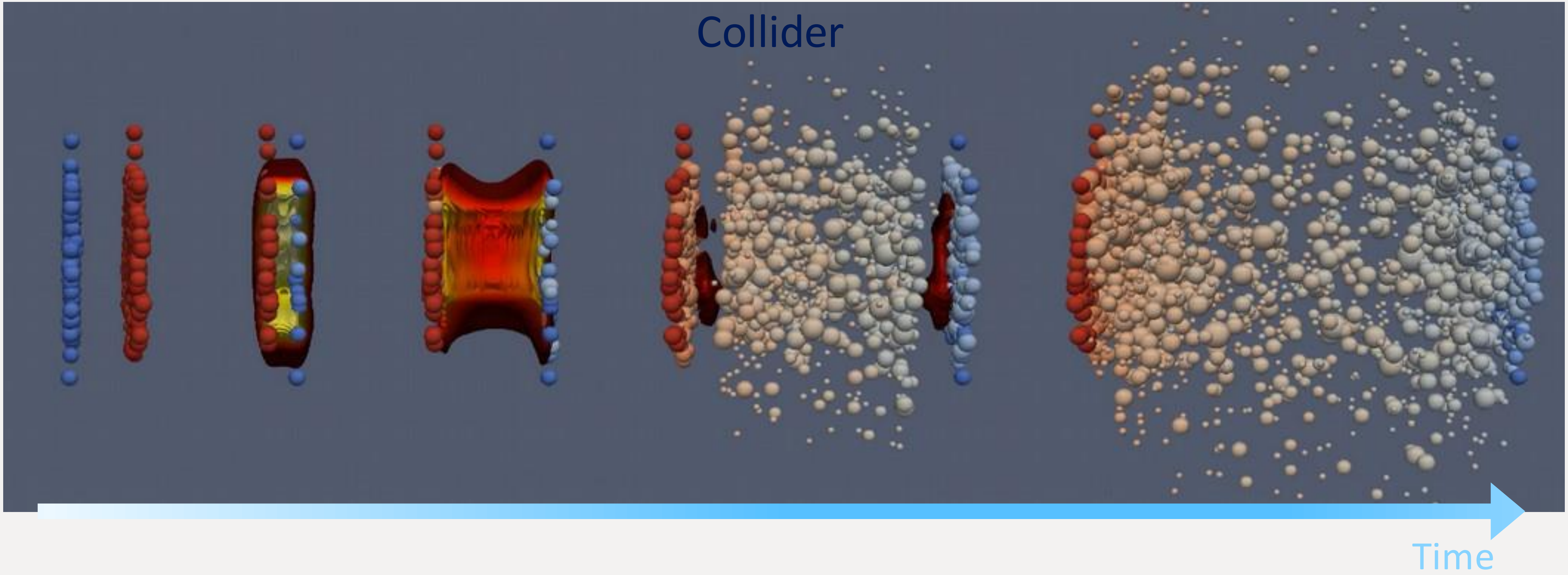
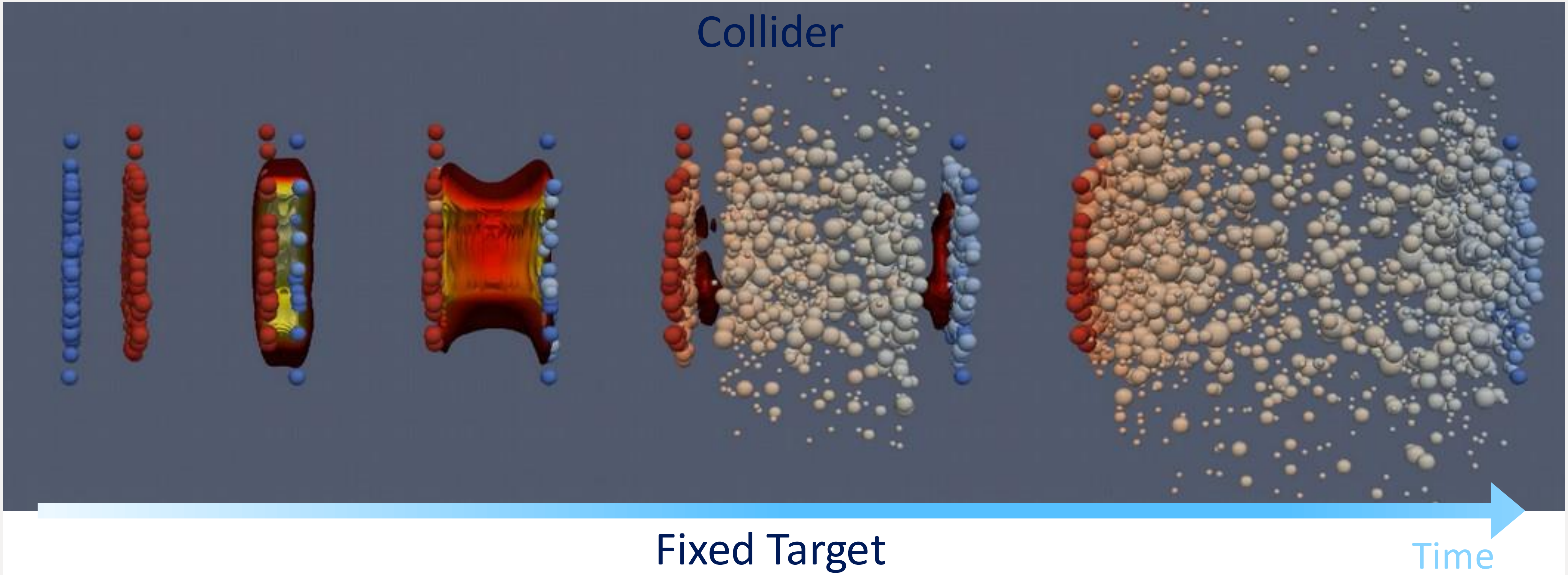
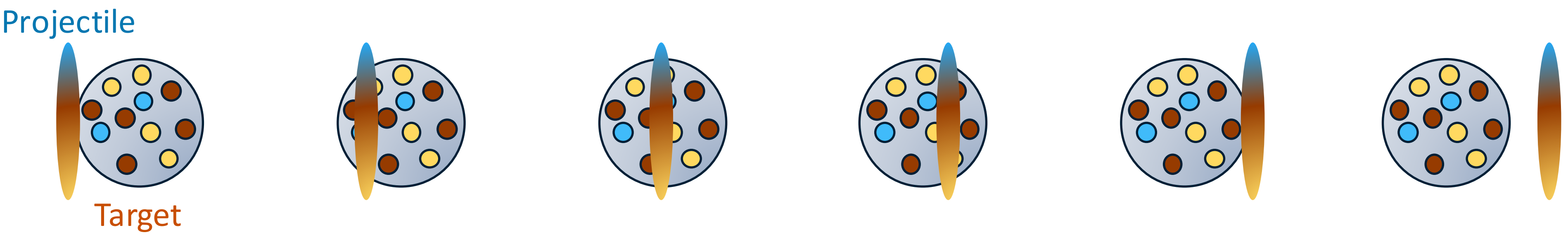
Collider vs. Fixed Target Kinematics (in lab frame)



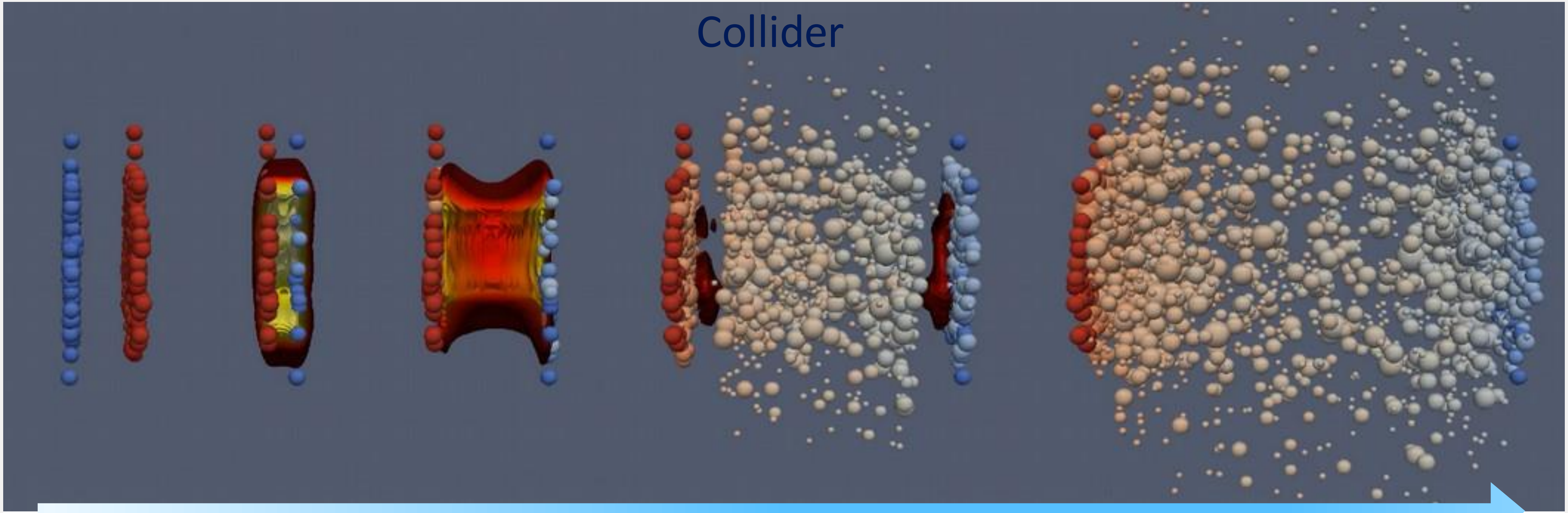
Collider vs. Fixed Target Kinematics (in lab frame)



Fixed Target

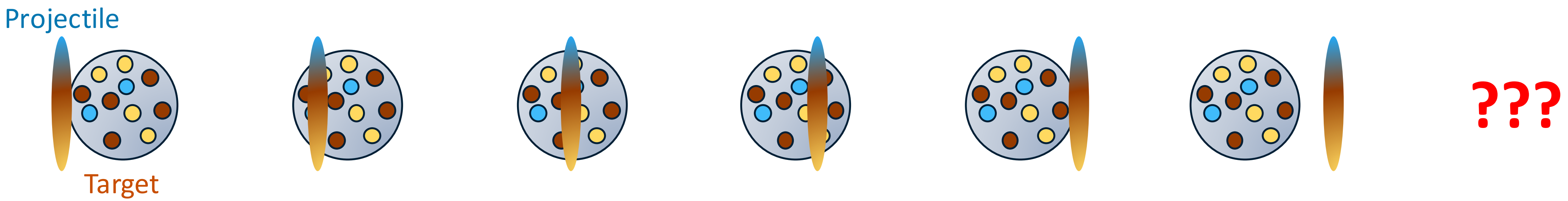


Collider vs. Fixed Target Kinematics (in lab frame)

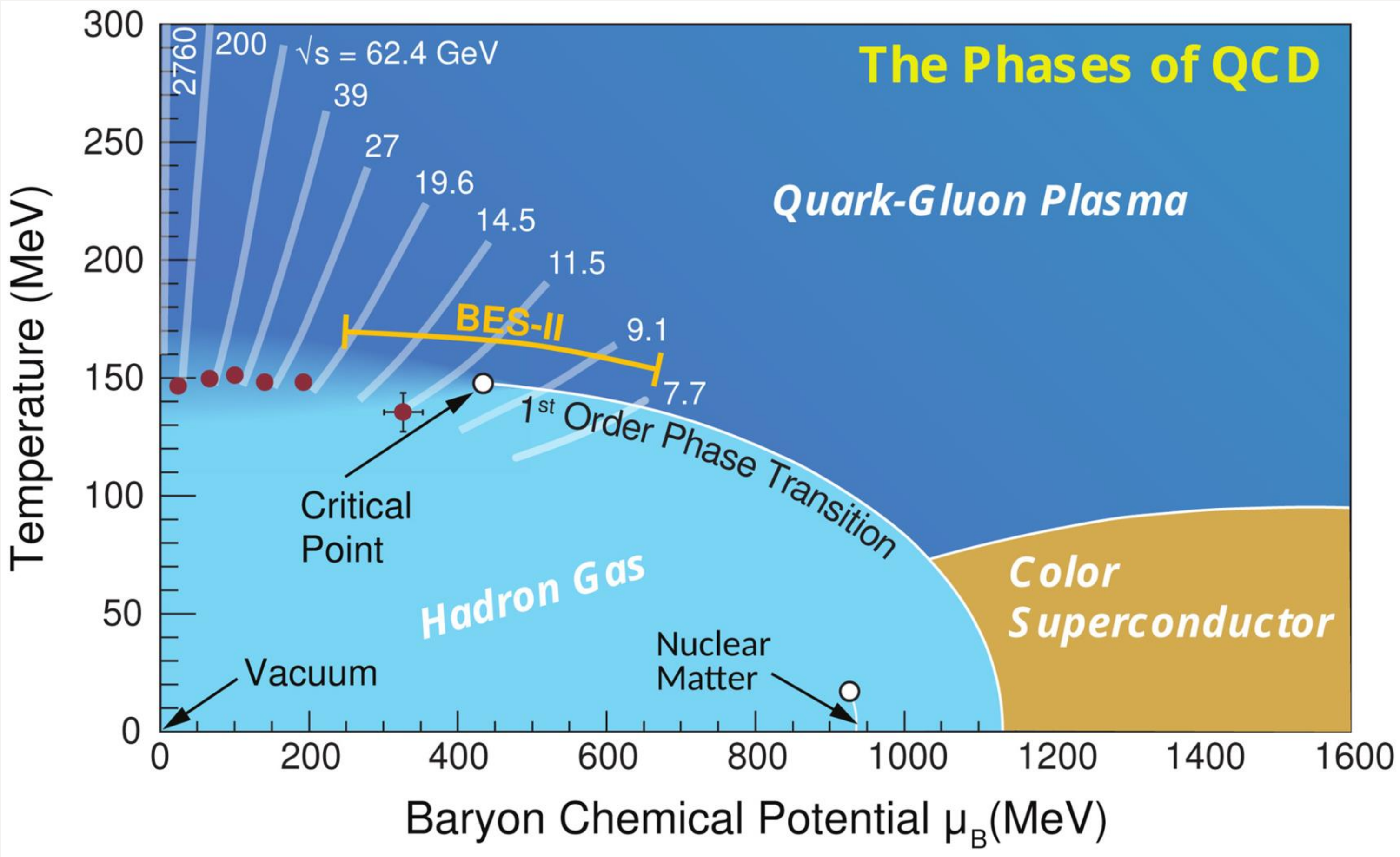


Time

Fixed Target



Why study fixed target collisions?



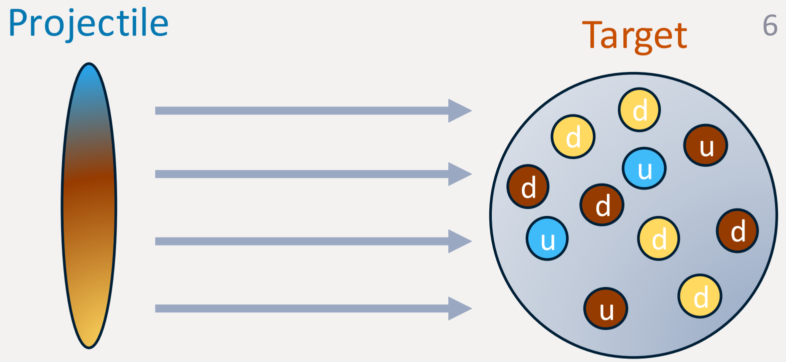
Bzdak, Esumi, *et. al.;*
Physics Reports **853**, 1 (2020).

Fixed target collisions allow for direct investigation of the far-forward "fragmentation region"

- The fragmentation region is host to the most *quark-rich* physics
- Lower energies + higher net quark density = larger $^{\mu_B}/_{T}$
  - Fruitful avenue for observing QCD critical point physics

Kolbé's Fixed-Target Model†

Classical model proposed in 2021 based in the Color Glass Condensate (CGC) framework* (see Farid's talk for more details)



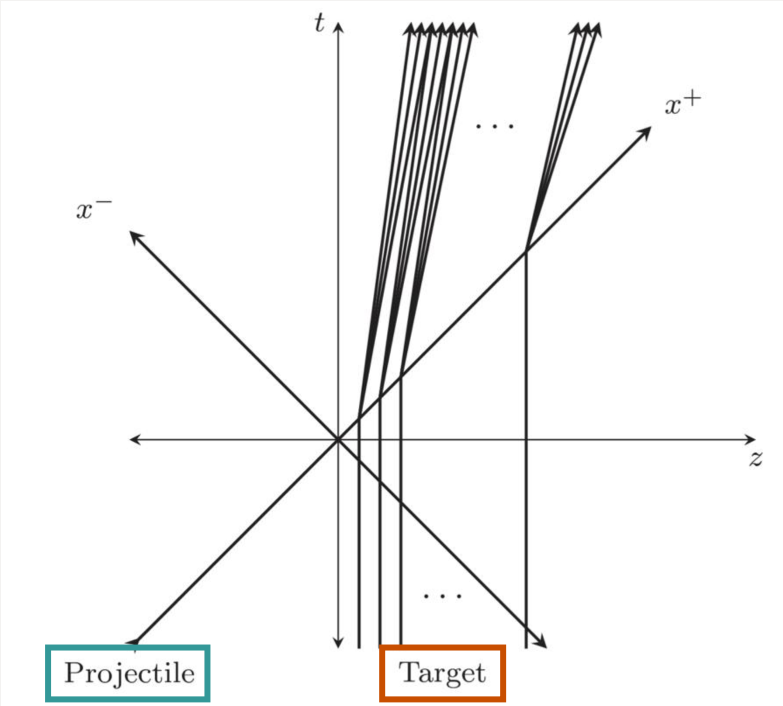
Study a static quark (target) impinged by a colored sheet (projectile):

- Source current: $J_a^\mu = \delta^{\mu +}\delta(x^-)\rho_a(x_\perp)$
- Chromodynamic fields evolve according to Yang-Mills equations:

$$D_\mu F_a^{\mu\nu} = J_a^\nu$$

- Quark momentum p^μ determined from *Wong's Equations*:

$$\frac{dp^\mu}{d\tau} = g T_a F_a^{\mu\nu} u_\nu$$



After some math...

$$\boxed{p^z = \frac{p_T^2}{2m}, \qquad \frac{dP(p^z)}{dp^z} = \frac{1}{\overline{p}^z} e^{-p^z/\overline{p}^z} \text{ with mean } \overline{p}^z = Q_s^2/2m.}$$

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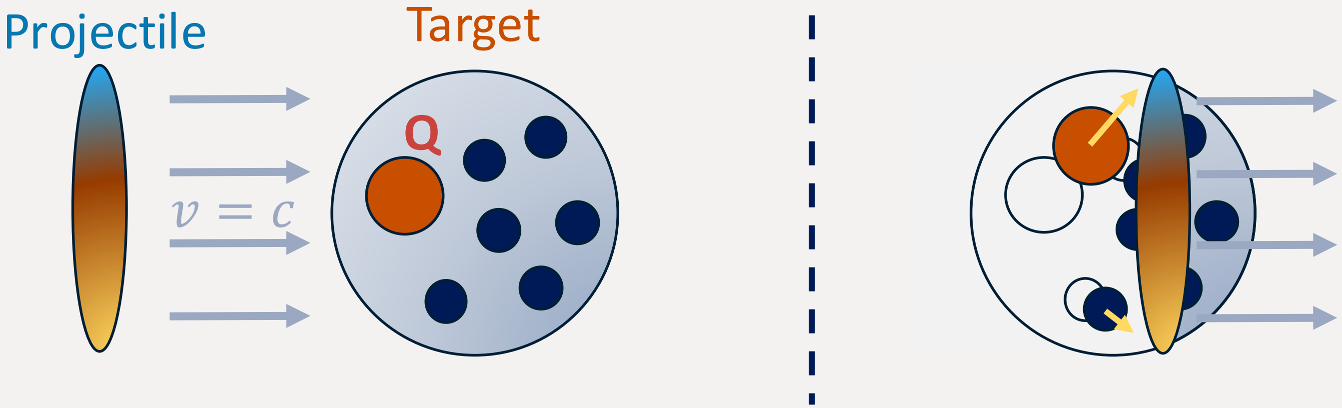
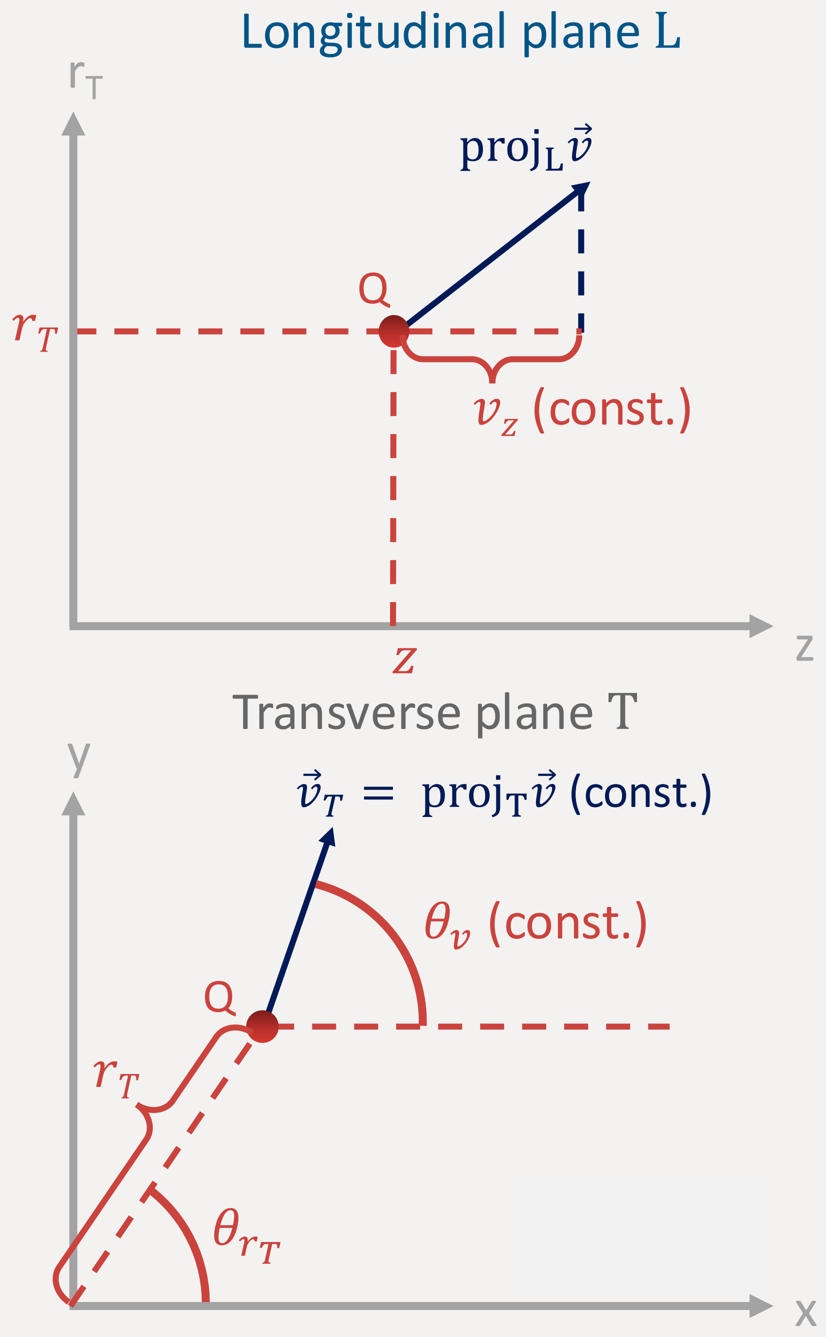
†Kolbé, Lushozi, McLerran, & Yu; Phys. Rev. C **103**, 044908 (2021).
*Kajantie, McLerran, & Paatelainen; Phys. Rev. D **100**, 054011 (2019).

From single- to multi-particle dynamics





Describe the state of a post-collision particle Q with **5 random variables:**

$$z,\ r_T,\ v_z,\ \theta_v, \theta_{r_T}.$$

Evolve the probability density $P(z, r_T, v_z, \theta_v, \theta_{r_T})$ via **free-streaming.**

For a uniform initial distribution, integration over θ_v, θ_{r_T} can be performed analytically:

$$\boxed{P'(z', r'_T, v'_z) = \frac{3}{2\pi R^3} r'_T \left|1 - v'_z \frac{\partial \Delta t}{\partial z'}\right| \Theta(R + z' - v'_z \Delta t)\Theta(R - z' + v'_z \Delta t)\,\mathrm{ModArccos}\left(\frac{r'^2_T + (v'_T \Delta t)^2 + (z' - v'_z \Delta t)^2 - R^2}{2 r'_T v'_T \Delta t}\right) \rho(v'_z)}$$

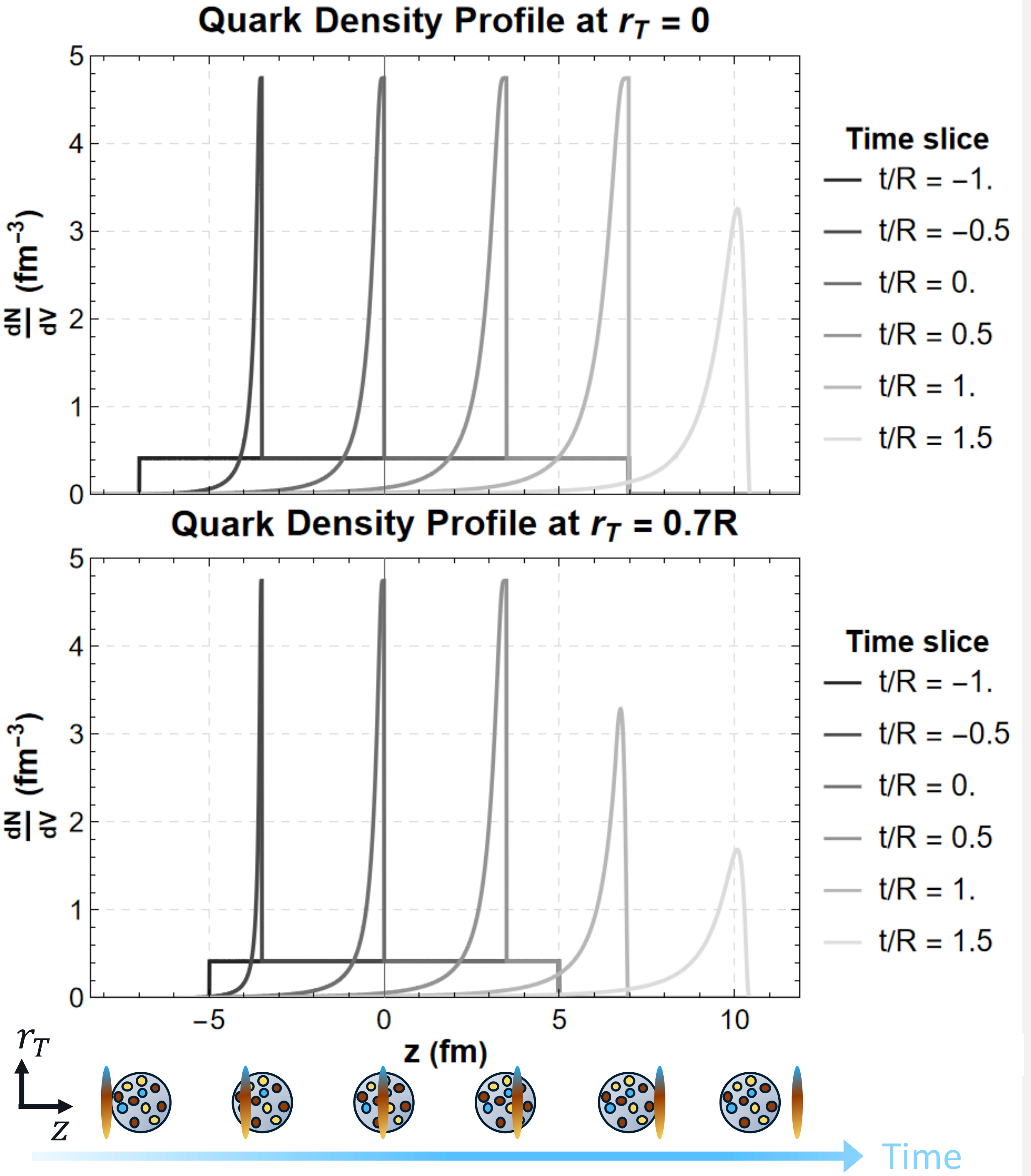
Quark Density Profiles

Results for Au-Au collisions at high-end RHIC energies:

- Significant compression in forward region
  - Peak density enhancement: $\zeta = 1 + \frac{Q_s^2}{2m_q^2}$

    $$\begin{aligned} &\text{@RHIC: } \zeta \sim 12 \longrightarrow \zeta_{\mathrm{LRF}} \sim 4 \\ &\text{@LHC: } \zeta \sim 27 \longrightarrow \zeta_{\mathrm{LRF}} \sim 8 \end{aligned}$$

- Estimate local baryon densities from free EoS, find $\mu_{\mathrm{B}} \sim 600 - 800\ \mathrm{MeV}$ at the wavefront.
  - Potential to observe critical point physics

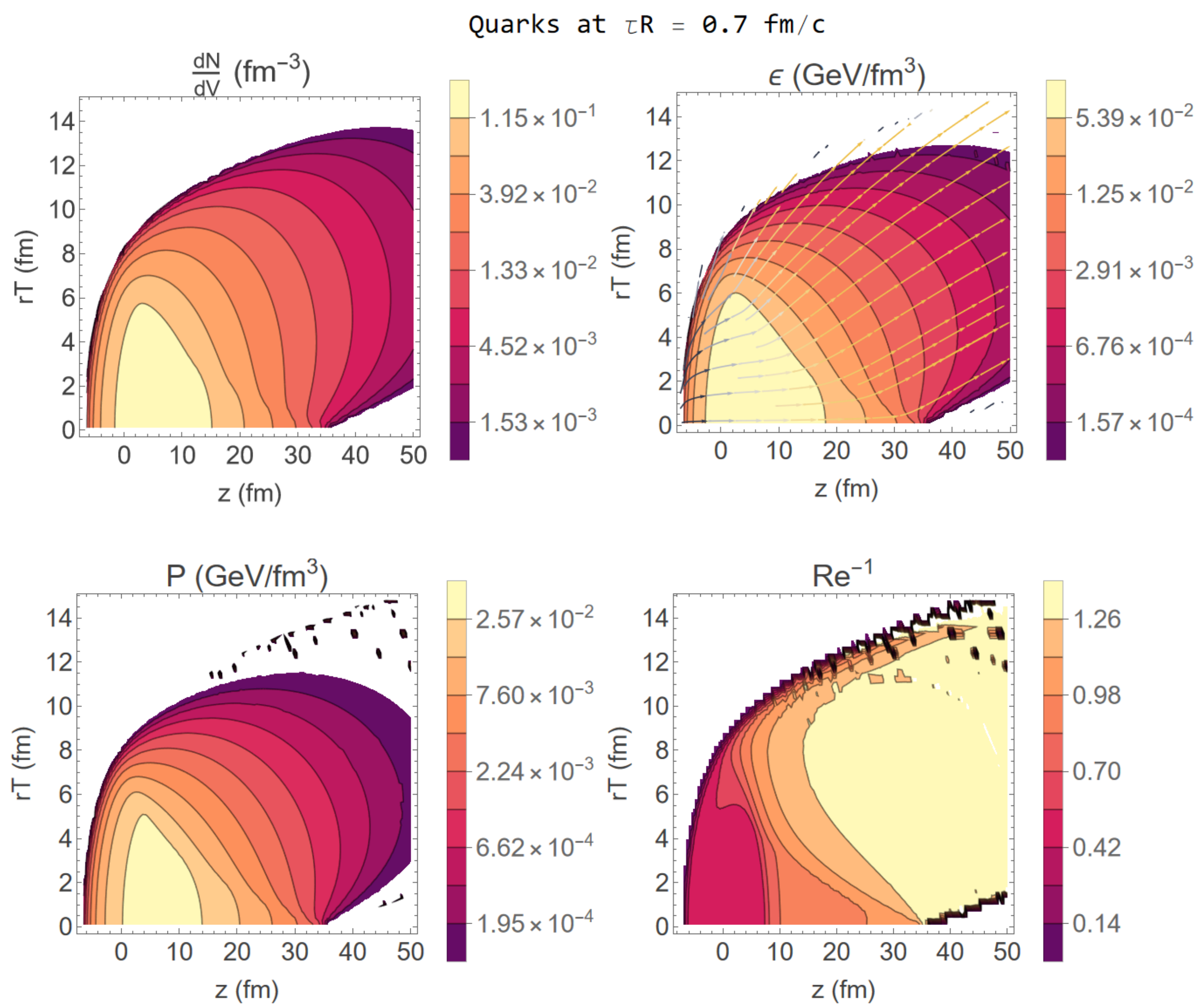


Current + Future Work

Study hydrodynamization, evolution to local equilibrium:

- Use Landau matching to calculate local energy density ϵ and flow velocity u^{μ}.
- Use QCD equation of state to calculate thermodynamic quantities, e.g. T, μ_B.
  - Hydrodynamic attractor: Inverse Reynolds number Re^{-1}

Eventually connect to a 3D Hydro simulation pipeline, e.g. 3D MUSIC



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Thank you!

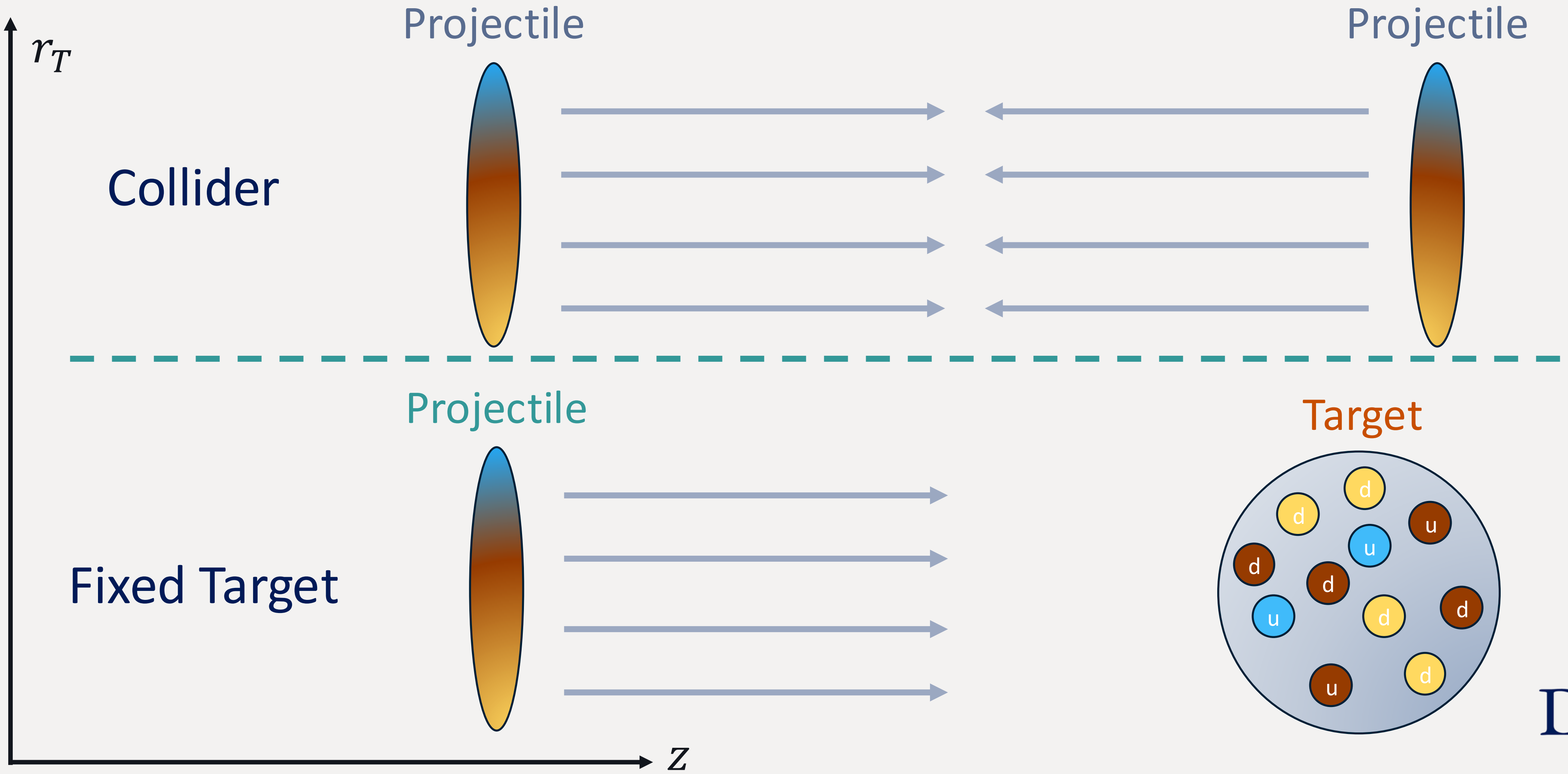
Questions?*

*time permitting

Backup Slides

Collider vs. Fixed Target Pictures

Consider a high-energy **(Collision Energy ~ 10s-100s GeV)** *heavy ion* collision in the lab frame:



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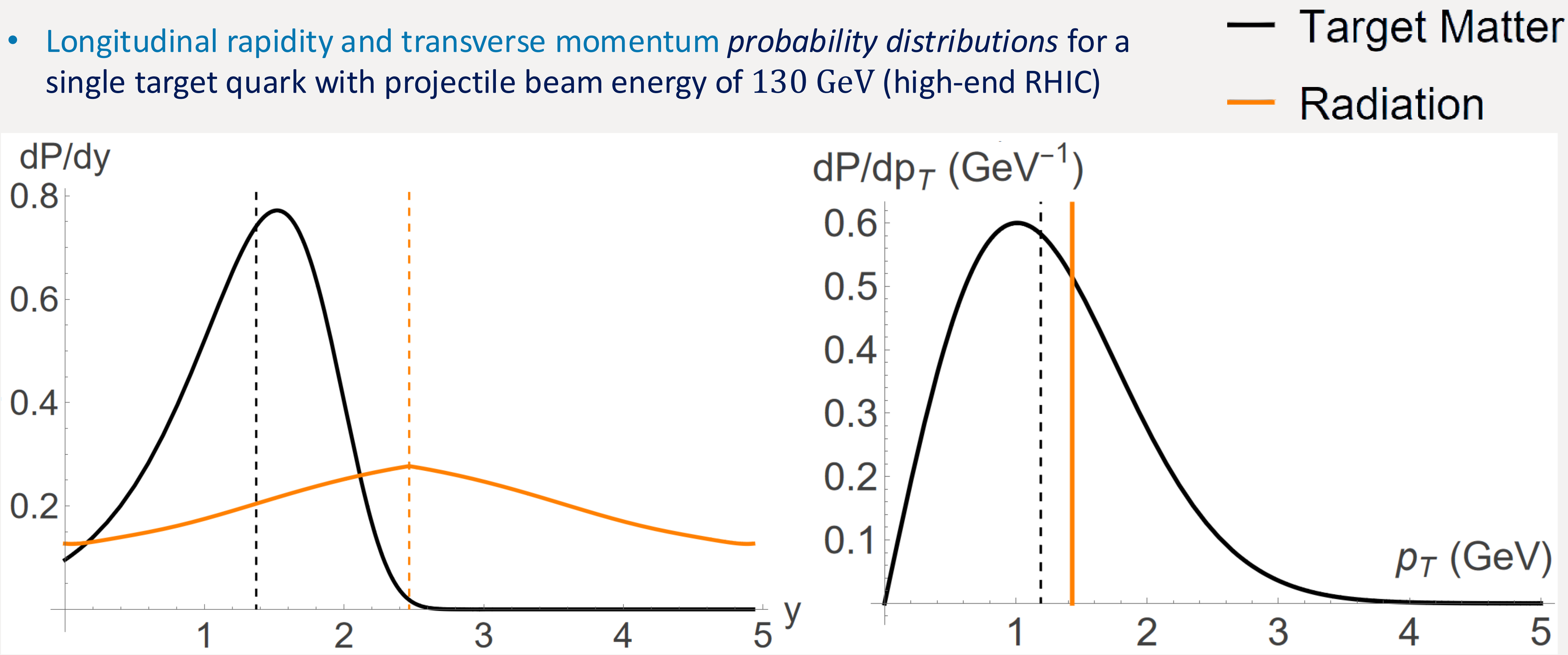
Single-particle distributions for Au-Au

- Longitudinal rapidity and transverse momentum *probability distributions* for a single target quark with projectile beam energy of 130 GeV (high-end RHIC)



- Significant number of quarks in the forward regime owing to baryon stopping

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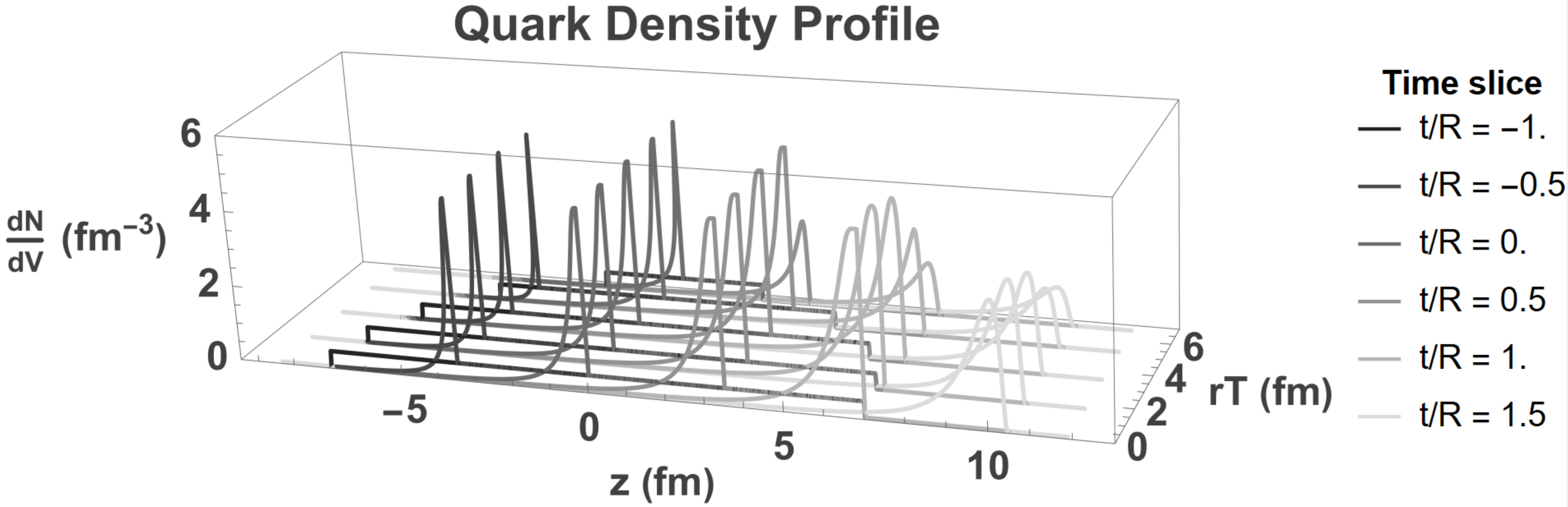
*Radiation distribution from Kharzeev & Levin; Phys. Lett. B **523**, 79 (2001).

3D Quark Density Profile



- Peak density is independent of transverse position and coordinate time
  - High μ_B achieved across entire target nucleus

Gluon Radiation Distribution

Kolbé employs an (over)simplified model for radiation[†]:

radiation first. We will consider a situation in which the production distribution of the radiation is flat as a function of the rapidity $F(y) = F$, and the multiplicity is "fitted" to experimental data. The flatness assumption is justified by the experimental observation that the meson distribution varies slowly with pseudo-rapidity [10]. We assume a picture

This poses some issues:

- Boost-invariance is iffy in fragmentation region
- No description of p_T-dependence

Instead, we take the following rapidity distribution developed by Kharzeev and Levin*:

$$\frac{dN}{dy} = c\ N_{part}\ \left(\frac{s}{s_0}\right)^{\frac{\lambda}{2}}\ e^{-\lambda|y|}\ \left[\ln\left(\frac{Q_s^2}{\Lambda_{QCD}^2}\right) - \lambda|y|\right]\ \left[1 + \lambda|y|\left(1 - \frac{Q_s}{\sqrt{s}}\ e^{(1+\lambda/2)|y|}\right)^4\right]$$

- We fix $p_T = Q_s$.
- Gluon mini-jets can be assigned a nonzero invariant mass $m \approx Q_s \cdot 1$ GeV, but we'll take $m = 0$.

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[†]Kolbé, Lushozi, McLerran, & Yu; Phys. Rev. C **103**, 044908 (2021).
*Kharzeev & Levin; Phys. Lett. B **523**, 79 (2001).

Remaining Model Parameters

We now have a complete model:

(single particle velocity/momenta distributions) + (3D kinematic spacetime evolution)

for the initial-stage time-evolution of gluons and quark degrees of freedom.

The only free parameters remaining are (with values used in this study):

- $\sqrt{s_{\mathrm{Col}}}$: invariant mass of collider-mode collision, determined by beam energy
  - We set $\sqrt{s_{\mathrm{Col}}} = 130\ \mathrm{GeV}$ (high-end RHIC beam energies)
- R: radius of target nucleus
  - We set $R = 7\ \mathrm{fm}$ (Au target nucleus)
- n_{quarks}: number of valence quarks in targets nucleus
  - We set $n_{\mathrm{quarks}} = 600$ (Au target nucleus)
- n_{gluons}: number of gluons produced in collision, fit from data
  - We set $n_{\mathrm{gluons}} = 40{,}000$ (estimated from total particle production)

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Transitioning to Hydrodynamics: Inverse Reynolds

Goal: link our phase-space distribution $\frac{dN}{dVdp}$ to a hydro simulation.

- How can we quantify when to transition? (Or if we should at all?)

Many options available:

- Knudsen number, energy attractor, Bjorken flow attractor, etc.

One common choice is the **inverse Reynold number:**

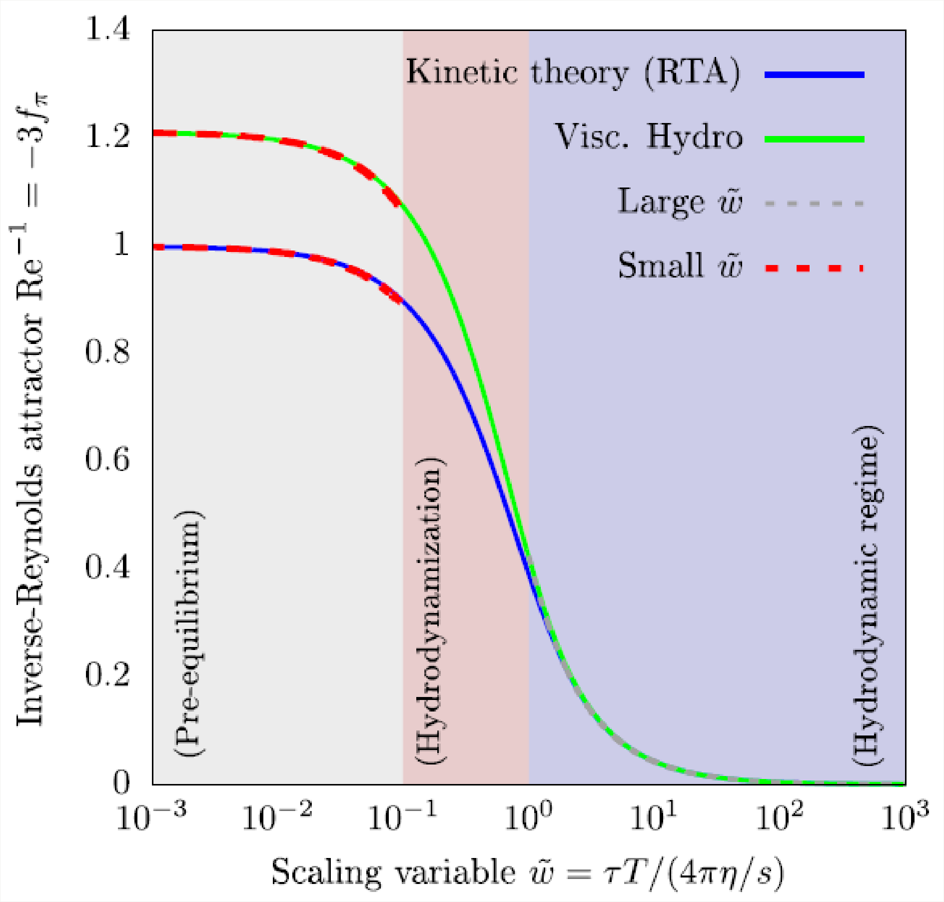
$$\mathrm{Re}^{-1}(x) = \left(\frac{6\pi^{\mu\nu}\pi_{\mu\nu}}{\varepsilon^2}\right)^{1/2}$$

where

- $\pi^{\mu\nu}$ = nonequilibrium part of energy-momentum tensor
- ε = energy density in local rest frame

Measures the relative size of dissipative forces; describes how "out-of-equilibrium" a system is (at different points in spacetime).

- Hydrodynamization is typically seen around $\mathrm{Re}^{-1} \approx 0.4 - 0.8$.*



*Ambruş, Schlichting, & C. Werthmann, Phys. Rev. D **107**, 094013 (2023).

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Calculating Re^{-1}: Landau Matching Crash Course

The nonequilibrium EM tensor field $\pi^{\mu\nu}$ is the difference between the true EM tensor field $T^{\mu\nu}$ and the equilibrium EM tensor field $T_{\mathrm{eq}}^{\mu\nu}$:

$$\pi^{\mu\nu} = T^{\mu\nu} - T_{\mathrm{eq}}^{\mu\nu}$$

- $T^{\mu\nu}(x) = \int p^\mu p^\nu \frac{dN}{d^3x d^3p} \frac{d^3p}{p^0}$ is calculable directly from the particle probability distribution function $P'\left(z', r_T', v_z', \theta_v', \theta_{r_T}'\right)$.
    - This calculation can be reduced to a **single numerical integral** at each point in spacetime (for each of the **7** independent nonzero components) in a similar fashion to our prior $P'(z', r_T')$ calculation.

- $T_{\mathrm{eq}}^{\mu\nu} = (\varepsilon + P) u^\mu u^\nu - P g^{\mu\nu}$ (in mostly minus metric) is found via **Landau matching**.
    - Find a $T_{\mathrm{eq}}^{\mu\nu}$ which best mimics the non-dissipative energy flow described by $T^{\mu\nu}$ at each point in spacetime.

- ε (local energy density) and u^μ (local energy flow 4-velocity) solve the eigenvalue equation
$$T_\nu^\mu u^\nu = \varepsilon u^\mu.$$
    - Additionally, u^μ is the *unique* timelike eigenvector of T_ν^μ: $u^\mu u_\mu = +1,$ and we expect $\varepsilon \geq 0$.

- P (local isotropic pressure) is found by equating the spatial trace of $T_{\mathrm{eq}}^{\mu\nu}$ and $T^{\mu\nu}$:
$$P = \tfrac{1}{3}\Delta_{\mu\nu} T^{\mu\nu}, \qquad \Delta_{\mu\nu} = -g_{\mu\nu} + u_\mu u_\nu.$$

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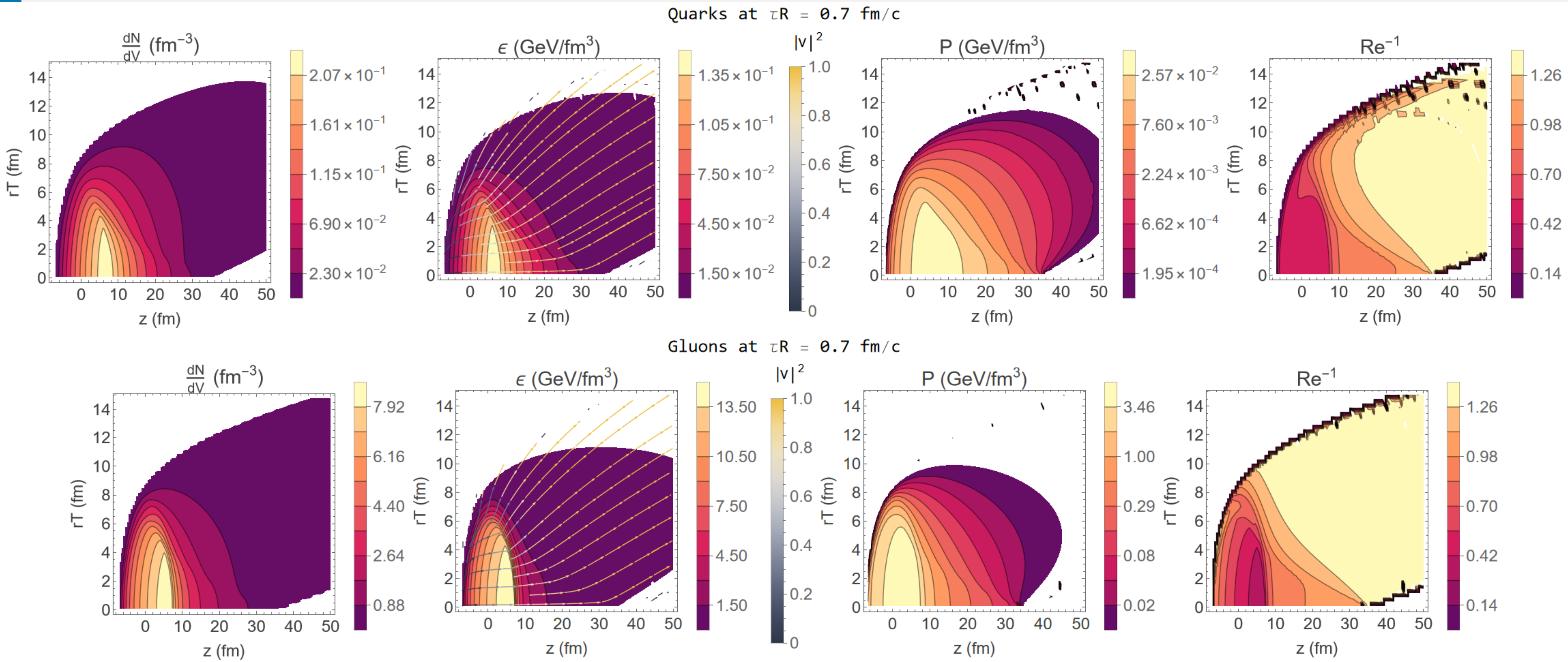

Quarks at τR = 0.7 fm/c
dN/dV (fm⁻³)
ϵ (GeV/fm³)
|v|²
P (GeV/fm³)
Re⁻¹
rT (fm)
z (fm)
2.07 × 10⁻¹
1.61 × 10⁻¹
1.15 × 10⁻¹
6.90 × 10⁻²
2.30 × 10⁻²
1.35 × 10⁻¹
1.05 × 10⁻¹
7.50 × 10⁻²
4.50 × 10⁻²
1.50 × 10⁻²
2.57 × 10⁻²
7.60 × 10⁻³
2.24 × 10⁻³
6.62 × 10⁻⁴
1.95 × 10⁻⁴
1.26
0.98
0.70
0.42
0.14
Gluons at τR = 0.7 fm/c
7.92
6.16
4.40
2.64
0.88
13.50
10.50
7.50
4.50
1.50
3.46
1.00
0.29
0.08
0.02

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