heavy quark kinetic equilibration from the lattice[1]

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general background

what are we talking about?

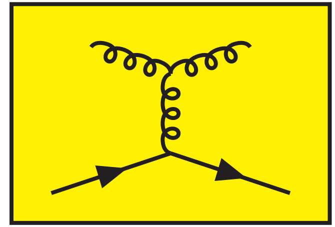
⇒ charm and bottom quarks generated in a heavy ion collision

⇒ "non-equilibrium": gluons and light quarks thermalized at $T \sim (150 - 450)$ MeV, heavy quarks probes in this background

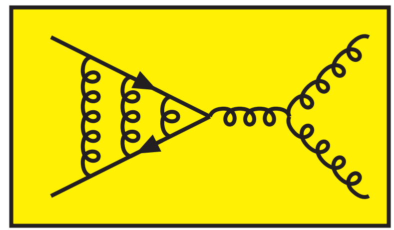
⇒ bottom quark is non-relativistic $[m_b \sim (10 - 30)\ T]$, charm quark is a borderline case $[m_c \sim (2 - 6)\ T]$

conceptual motivations

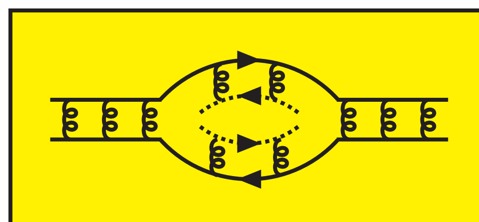
kinetic equilibration rate:
how fast does velocity adjust to hydrodynamic flow?



chemical equilibration rate:
how fast does number density adjust to boltzmann weight?



quarkonium dissociation:
do $q\bar{q}$ states propagate as scattering or bound states?



phenomenological motivations

kinetic equilibration rate:
how can D mesons show large v_2 in heavy ion collisions?

chemical equilibration rate:
would we have $N_{\rm f} = 2 + 2$ in future heavy ion experiments?

bottomonium dissociation:
precision studies of Υ spectra in the LHC era?

charmonium dissociation:
eternal questions on the fate of J/ψ?

$\Rightarrow$ can we understand ingredients for these from lattice QCD?

technical challenges

⇒ “usual” lattice systematics:

statistical signal for suppressed observables; finite-volume effects; continuum limit; light dynamical sea quarks; topological freezing; non-perturbative renormalization; ...

⇒ specific issues for thermal real-time rates:

“easy”: derivation of kubo relations, i.e. expressing non-equilibrium physics in terms of equilibrium two-point correlators

“exponentially hard”: analytic continuation from euclidean to real time, particularly if spectral function contains narrow peaks[2]

[2] G. Cuniberti, E. De Micheli and G.A. Viano, *Reconstructing the thermal Green functions at real times from those at imaginary times*, cond-mat/0109175

how to address kinetic equilibration?

basic idea: make use of effective theory concepts

by focussing on essential degrees of freedom, the measurement and control of systematic uncertainties might get facilitated

the practical implementation is quite different from vacuum field theory, however — analogues are often with plasma physics (fast↔slow) rather than with quantum mechanics (heavy↔light)

low-energy dynamics: the langevin equation[3]

just like in brownian motion, momenta are changed by thermal friction and by random kicks,

$$\dot{p}_i(t) = -\eta\, p_i(t) + \xi_i(t)\ , \quad \langle\, \xi_i(t')\, \xi_j(t)\,\rangle = \kappa\, \delta_{ij}\, \delta(t - t')$$

the parameters η, κ satisfy a fluctuation-dissipation relation,

$$\eta \approx \frac{\kappa \langle \mathbf{v}^2 \rangle}{6T^2}$$

$\Rightarrow$ task is to measure the "momentum diffusion coefficient", κ (and also to estimate the static $\langle \mathbf{v}^2 \rangle$)

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[3] B. Svetitsky, *Diffusion of charmed quarks in the quark-gluon plasma*, PRD 37 (1988) 2484; G.D. Moore and D. Teaney, *How much do heavy quarks thermalize in a heavy ion collision?*, hep-ph/0412346

for intuition: electrodynamics

consider particle motion under the lorentz force

$$\dot{\mathbf{p}} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})(t) \;\equiv\; \mathbf{F}(t)$$

with $\langle v_i v_j \rangle = \frac{1}{3}\langle \mathbf{v}^2 \rangle\, \delta_{ij}$, its 2-point correlator can be written as

$$\begin{aligned} \langle F_i(t')F_j(t)\rangle \quad &= \quad q^2 \left\{ \langle E_i(t')E_j(t)\rangle \right. \\ &+ \quad \left. \frac{1}{3}\langle \mathbf{v}^2 \rangle\, \langle \delta_{ij} B_k(t')B_k(t) - B_j(t')B_i(t)\rangle \right\} \end{aligned}$$

the low-energy constant κ is given by the IR limit $\omega \to 0$

$$\kappa = \lim_{\omega \to 0} \int_{-\infty}^{\infty} \mathrm{d}t'\, e^{i\omega(t'-t)} \frac{1}{3} \sum_i \langle F_i(t')F_i(t)\rangle$$

infinite-mass limit $M \gg T$

basic ingredients

view the problem as an expansion in T/M (“HQET”)

for $\langle \mathbf{v}^2 \rangle \sim T/M \to 0$, κ originates from the electric field [4]

[4] J. Casalderrey-Solana and D. Teaney, *Heavy quark diffusion in strongly coupled* $\mathcal{N}=4$ *Yang-Mills*, hep-ph/0605199; S. Caron-Huot, ML and G.D. Moore, *A way to estimate the heavy quark thermalization rate from the lattice*, 0901.1195

state of the art (until recently)

measurements with multilevel and other such techniques[5]

perturbative renormalization available up to NLO[6]

continuum limit taken within the quenched theory[7]

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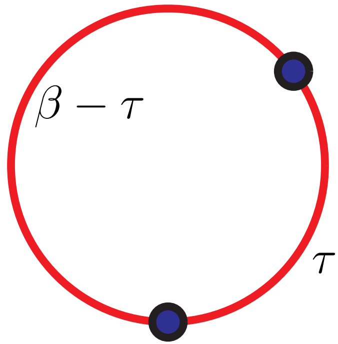
[5] H.B. Meyer, *The errant life of a heavy quark in the quark-gluon plasma*, 1012.0234; D. Banerjee, S. Datta, R. Gavai and P. Majumdar, *Heavy Quark Momentum Diffusion Coefficient from Lattice QCD*, 1109.5738

[6] C. Christensen and ML, *Perturbative renormalization of ...*, 1601.01573

[7] A. Francis *et al*, *Non-perturbative estimate of the heavy quark ...*, 1508.04543

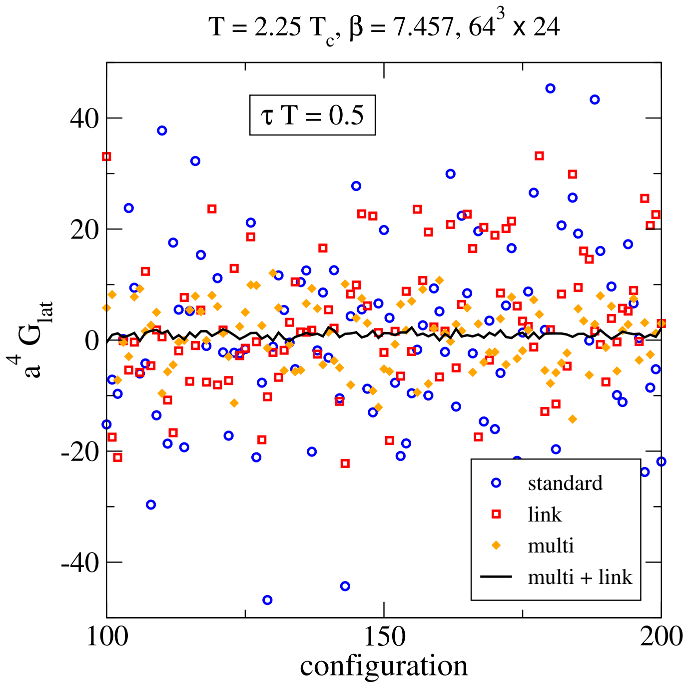
colour-electric correlator

$$G_E(\tau) \overset{\beta\equiv 1/T}{\equiv} -\frac{1}{3}\sum_{i=1}^{3} \frac{\langle \mathrm{Re\,Tr}[U_{\beta;\tau}\, gE_i(\tau)\, U_{\tau;0}\, gE_i(0)] \rangle}{\langle \mathrm{Re\,Tr}[U_{\beta;0}] \rangle}$$



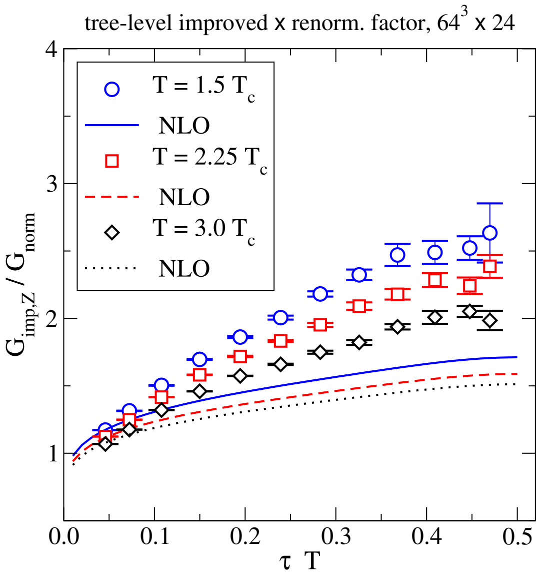
$$G_E(\tau) = \int_0^\infty \frac{\mathrm{d}\omega}{\pi} \rho_E(\omega) \frac{\cosh[(\frac{\beta}{2} - \tau)\omega]}{\sinh\left(\frac{\omega\beta}{2}\right)} \Rightarrow \kappa_E \equiv \lim_{\omega\to 0} \frac{2T\rho_E(\omega)}{\omega}$$

getting a signal is non-trivial



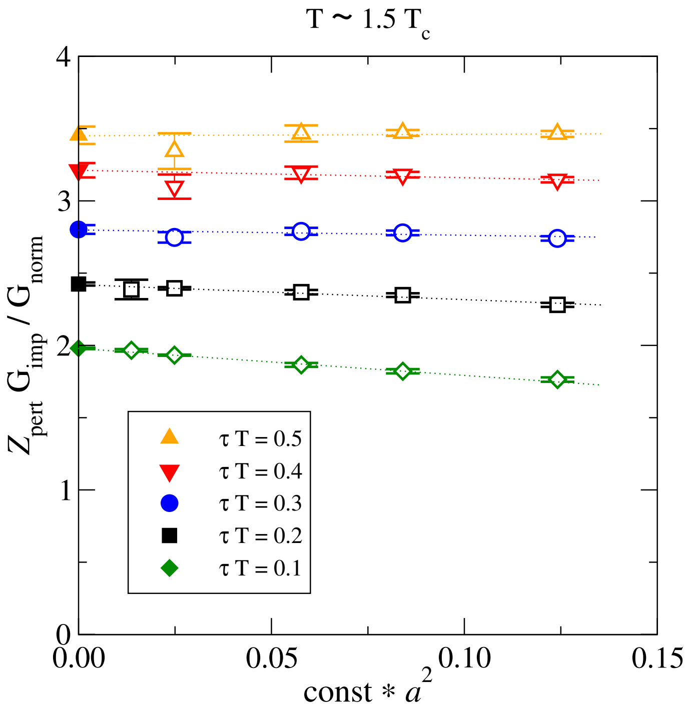
but in the end it can be obtained

with multilevel algorithm and tree-level improvement:

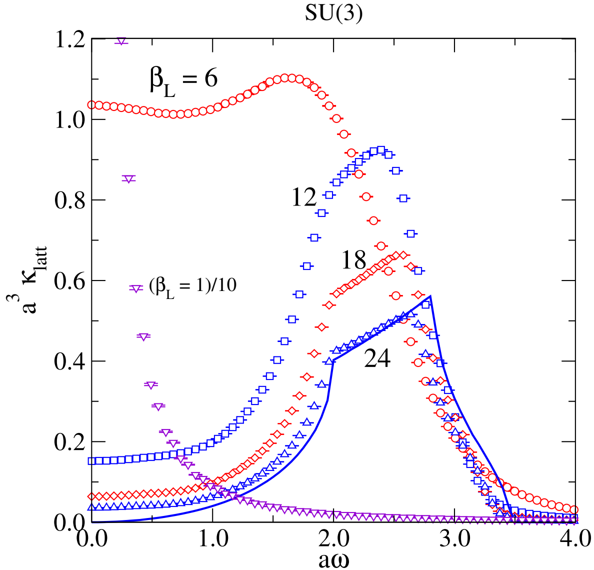


clear enhancement at large time separations

extrapolation to the continuum limit



classical simulations yield insight about the spectral shape[8]



[8] ML, G.D. Moore, O. Philipsen and M. Tassler, *Heavy Quark Thermalization in Classical Lattice Gauge Theory: Lessons for Strongly-Coupled QCD*, 0902.2856

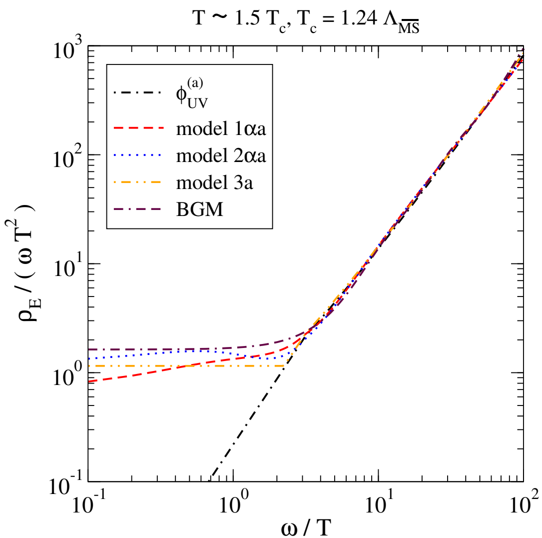
⇒ **take a similar ansatz (but now in continuum)**

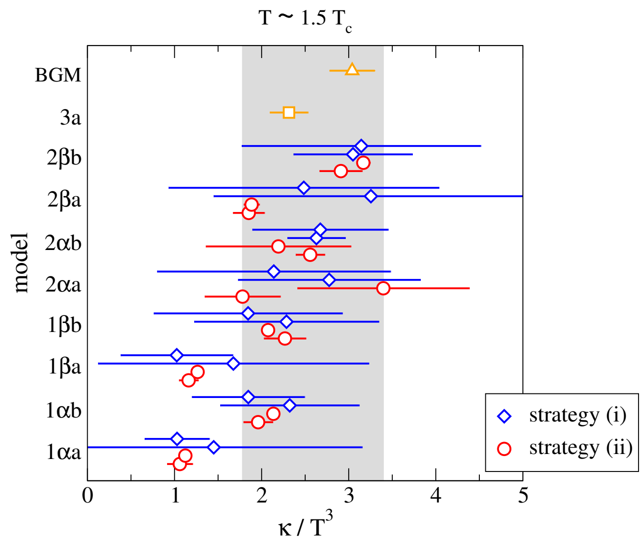
$$\phi_{\rm IR}(\omega) \;\equiv\; \frac{\kappa_E\,\omega}{2T}$$

$$\phi^{(a)}_{\rm UV}(\omega) \;\equiv\; \frac{g^2(\bar\mu_\omega) C_{\rm F}\,\omega^3}{6\pi}\;, \quad \bar\mu_\omega \equiv \mathsf{max}(\omega, \pi T)$$

$$\rho^{(2\mu i)}_{\rm E}(\omega) \;\equiv\; [1 + \sum_{n=1}^{n_{\rm max}} c_n e^{(\mu)}_n(y)]\sqrt{[\phi_{\rm IR}(\omega)]^2 + [\phi^{(i)}_{\rm UV}(\omega)]^2}$$

fitting with such interpolations yields $\kappa_E > T^3$





$$\kappa_E = (1.8 - 3.4)\, T^3$$

in the meanwhile other studies find similar results[9]

$\Rightarrow$ "gradient flow" has turned out to be a useful tool

$\Rightarrow$ the current drive is towards unquenching

[9] ...; N. Brambilla, V. Leino, P. Petreczky and A. Vairo, *Lattice QCD constraints on the heavy quark diffusion coefficient*, 2007.10078; L. Altenkort, A.M. Eller, O. Kaczmarek, L. Mazur, G.D. Moore and H.-T. Shu, *Heavy quark momentum diffusion from the lattice using gradient flow*, 2009.13553; *Spectral reconstruction details of a gradient-flowed color-electric correlator*, 2109.11303; D. Banerjee, R. Gavai, S. Datta and P. Majumdar, *Temperature dependence of the static quark diffusion coefficient*, 2206.15471

reminder: why is unquenching important even at high T?

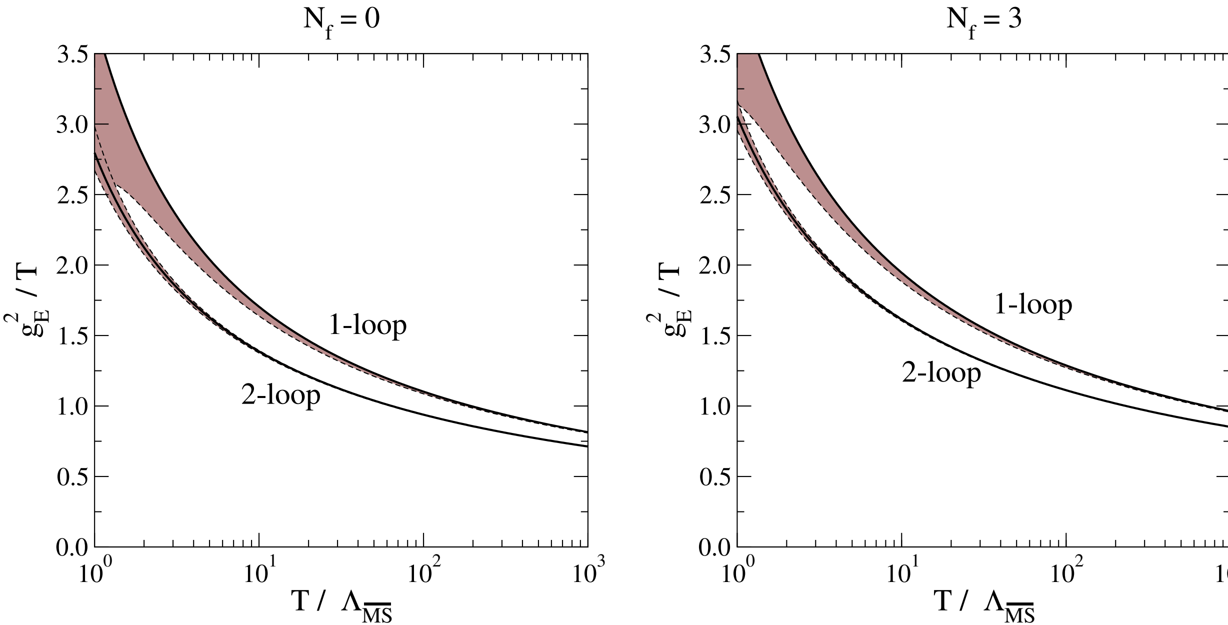
$N_{\rm f} = 0$: $T_{\rm c} \approx 1.24\Lambda_{\overline{\rm MS}}$

$m_{0_{++}} \gg 1$ GeV $\Rightarrow$ need to heat the system "a lot" in order for something to happen $\Rightarrow$ $T_{\rm c}$ is large

$N_{\rm f} = 3$: $T_{\rm c} \approx 0.45\Lambda_{\overline{\rm MS}}$

$m_{\pi} \ll 1$ GeV $\Rightarrow$ already a mild heating excites (exponentially) many hadrons $\Rightarrow$ $T_{\rm c}$ is low

this leads to quite different values of an effective coupling [10]



$$\Longrightarrow \alpha_{\rm s}\big|_{T\simeq T_{\rm c}, N_{\rm f}=0} \simeq 0.2\ , \qquad \alpha_{\rm s}\big|_{T\simeq T_{\rm c}, N_{\rm f}=3} > 0.3$$

[10] ML and Y. Schröder, ... *gauge coupling at high temperatures*, hep-ph/0503061

corrections of $\mathcal{O}(T/M)$

motivation

with $m_c \sim (2-6)\,T$, the expansion parameter $\sim T/M$ is not small, so the convergence of the HQET expansion is questionable

(as is the case for charm quarks also in vacuum settings)

most important corrections at $\mathcal{O}(T/M)$

including the magnetic field part of the lorentz force yields[11]

$$\kappa_{\text{tot}} \simeq \kappa_E + \frac{2}{3}\langle \mathbf{v}^2 \rangle \, \kappa_B$$

here the velocity reads

$$\langle \mathbf{v}^2 \rangle \approx \frac{3T}{M_{\text{kin}}} \left(1 - \frac{5T}{2M_{\text{kin}}} + \ldots \right)$$

a more general definition of $\langle \mathbf{v}^2 \rangle$ can be given in terms of the ratio of the almost constant (τ-independent) part of the spatial vector current correlator, normalized to the susceptibility[12]

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[11] A. Bouttefeux and ML, *Mass-suppressed effects in heavy quark diffusion*, 2010.07316

[12] S. Caron-Huot, ML and G.D. Moore, *A way to estimate the heavy quark thermalization rate from the lattice*, 0901.1195

colour-magnetic correlator

$$G_B(\tau) \;\equiv\; \frac{1}{3}\sum_{i=1}^{3} \frac{\langle \mathrm{Re}\,\mathrm{Tr}[U_{\beta;\tau}\, gB_i(\tau)\, U_{\tau;0}\, gB_i(0)]\rangle}{\langle \mathrm{Re}\,\mathrm{Tr}[U_{\beta;0}]\rangle}$$

magnetic fields are best discretized as clovers $\widehat{F}_{jk}$,

$$gB_i \;\equiv\; \frac{\sum_{j,k}\epsilon_{ijk}\widehat{F}_{jk}}{2\imath}$$

κ_B again originates from the IR limit,

$$G_B(\tau)=\int_0^\infty \frac{\mathrm{d}\omega}{\pi}\rho_B(\omega)\frac{\cosh[(\frac{\beta}{2}-\tau)\omega]}{\sinh\left(\frac{\omega\beta}{2}\right)} \Rightarrow \kappa_B \equiv \lim_{\omega\to 0}\frac{2T\rho_B(\omega)}{\omega}$$

renormalization

unlike the electric field, gB_i displays a logarithmic divergence

the physical correlator is obtained by multiplying the $\overline{\mathrm{MS}}$ one [13] with c_B^2, where[14]

$$c_B = 1 + \frac{g^2 N_\mathrm{c}}{(4\pi)^2}\left[2\ln\left(\frac{\bar{\mu}e^{\gamma_\mathsf{E}}}{4\pi T}\right) - 2\right] + \mathcal{O}(g^4)$$

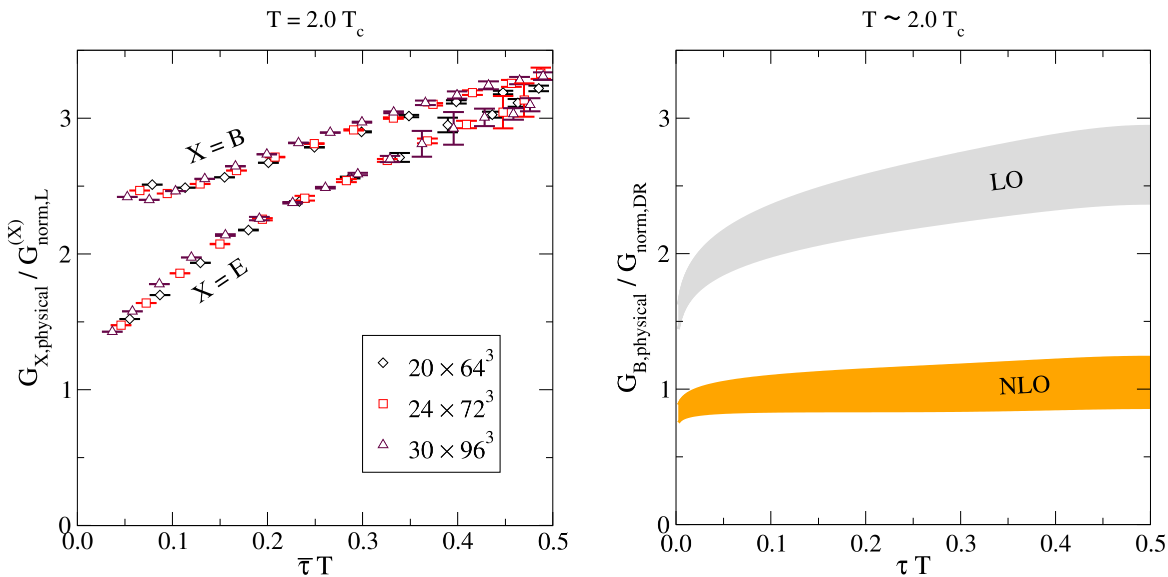
this implies that $\overline{\mathrm{MS}}$ operators need to be run to the scale

$$\bar{\mu} \approx 19.179T$$

[13] D. Guazzini, H.B. Meyer and R. Sommer, *Non-perturbative renormalization of the chromo-magnetic operator in Heavy Quark Effective Theory and the B^* – B mass splitting*, 0705.1809

[14] ML, *1-loop matching of a thermal Lorentz force*, 2103.14270

example of a measurement [after renormalization][15]



⇒ comparing with pQCD, there is a huge non-perturbative effect

[15] D. Banerjee, S. Datta, ML, *Lattice study of a magnetic contribution to heavy quark momentum diffusion*, 2204.14075

our data set

β	N_τ	N_S	subs	sub-ups	confs	streams	τ_{int}	r_0/a	T/T_c
6.860	20	48	5	500	500	5	13	17.7	1.2
	20	56	5	500	905	5	13	17.7	1.2
	20	64	5	500	1020	6	11	17.7	1.2
7.010	24	64	6	500	1465	10	7	21.3	1.2
	24	72	6	500	1051	15	4	21.3	1.2
7.050	20	48	5	500	875	4	5	22.4	1.5
	20	56	5	500	706	5	5	22.4	1.5
	20	64	5	500	1000	8	8	22.4	1.5
7.135	28	84	7	1000	1225	9	11	24.8	1.2
7.192	24	60	4	500	1530	9	5	26.6	1.5
	24	72	4	500	1448	7	5	26.6	1.5
	30	96	5	1000	1256	12	12	26.6	1.2
7.300	20	48	4	500	1000	4	5	30.2	2.0
	20	64	4	500	1120	8	5	30.2	2.0
7.330	28	84	7	1000	1256	10	11	31.3	1.5
7.457	24	60	4	500	1645	9	4	36.4	2.0
	24	72	4	500	1038	7	4	36.4	2.0
7.634	30	96	5	1000	1130	9	7	44.9	2.0

spectral modelling

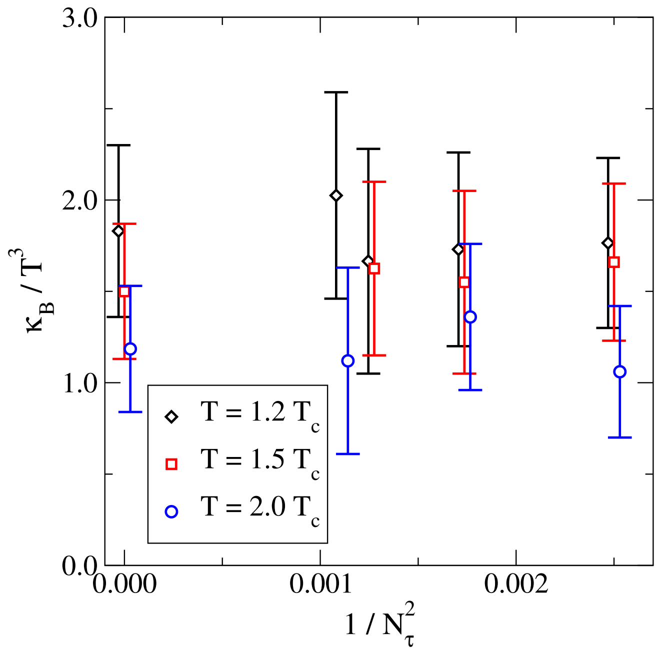
employ ansätze with two parameters like for ρ_E, e.g.

$$\phi_{\rm IR}(\omega) \;\equiv\; \frac{\kappa_B\,\omega}{2T}$$

$$\phi_{\rm UV}(\omega) \;\equiv\; \frac{g^2(\bar\mu_B)\,C_{\rm F}\,\omega^3}{6\pi}\;, \quad \bar\mu_B \equiv \max[\omega^{1-\frac{\gamma_0}{b_0}}(\pi T)^{\frac{\gamma_0}{b_0}}, \pi T]$$

$$\rho_B(\omega) \;\equiv\; \sqrt{\phi^2_{\rm IR}(\omega) \;+\; a_B\,\phi^2_{\rm UV}(\omega)}$$

we have tested finite-V and finite-a dependences



$\Rightarrow$ both moderate compared with spectral reconstruction

our final results

T/T_c	κ_E/T^3	κ_B/T^3	$\langle \mathbf{v}^2 \rangle_c$	$\langle \mathbf{v}^2 \rangle_b$	κ_c/T^3	κ_b/T^3	η_c/T	η_b/T
1.2	1.5-3.4	1.0-2.6	0.52	0.20	1.8-4.3	1.6-3.8	0.16-0.38	0.05-0.13
1.5	1.3-2.8	1.0-2.1	0.59	0.24	1.7-3.7	1.4-3.2	0.16-0.36	0.05-0.13
2.0	1.0-2.5	0.6-1.8	0.67	0.30	1.2-3.4	1.1-2.9	0.14-0.37	0.05-0.15

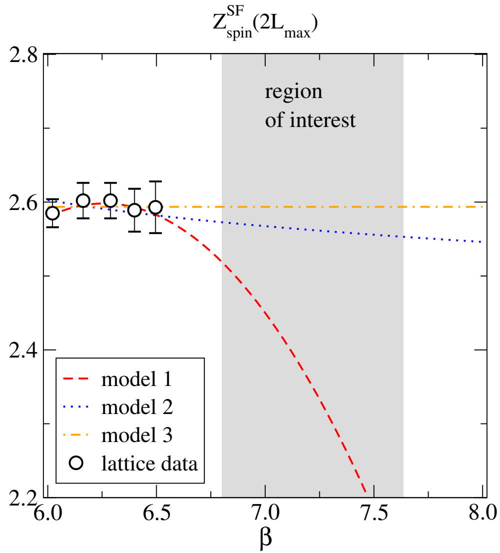
these can be contrasted with the non-EFT formulation,[16] where the spectral function contains a very narrow transport peak

$$\eta_c/T \overset{[16]}{\simeq} 0.9 - 5.3 \text{ at } T/T_c = 1.5 - 2.25$$

$$\eta_b/T \overset{[16]}{\simeq} 0.3 - 4.0 \text{ at } T/T_c = 1.3 - 2.25$$

[16] H.-T. Ding, O. Kaczmarek, A.-L. Lorenz, H. Ohno, H. Sandmeyer and H.-T. Shu, *Charm and beauty in the deconfined plasma from quenched lattice QCD*, 2108.13693

remark: renormalization involves uncertainties as well



⇒ would be nice to measure the same with gradient flow[17]

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17 J. Mayer-Steudte, N. Brambilla, V. Leino, P. Petreczky, *Chromoelectric and chromomagnetic correlators … from gradient flow*, 2111.10340; L. Altenkort, A.M. Eller, O. Kaczmarek, L. Mazur, G.D. Moore, H.-T. Shu, *Continuum extrapolation of … color-magnetic correlator at 1.5*T_c, 2111.12462; N. Brambilla, V. Leino, J. Mayer-Steudte, P. Petreczky, *Heavy quark diffusion coefficient with gradient flow*, 2206.02861

summary

$\Rightarrow$ nice playground for theoretical and numerical progress

$\Rightarrow$ $\mathcal{O}(T/M)$ is $\sim 30\%$ for charm, $\sim 10\%$ for bottom

$\Rightarrow$ systematics needs to be scrutinized further

$\Rightarrow$ unquenching is important indeed