M. Ciacco, S. Kundu, V. A. Kuznietsov, M. Puccio and V. Vovchenko, Arxiv: 2605.30710

Canonical statistical hadronization with local charge conservation

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QCD matter

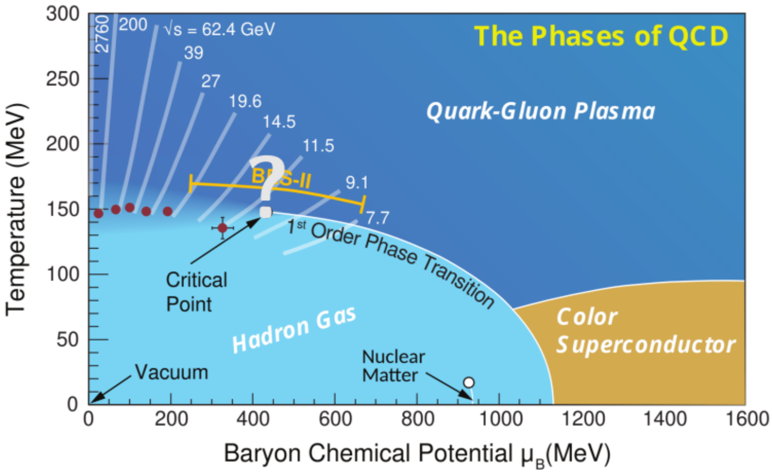
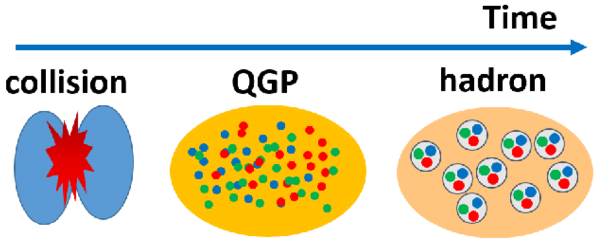
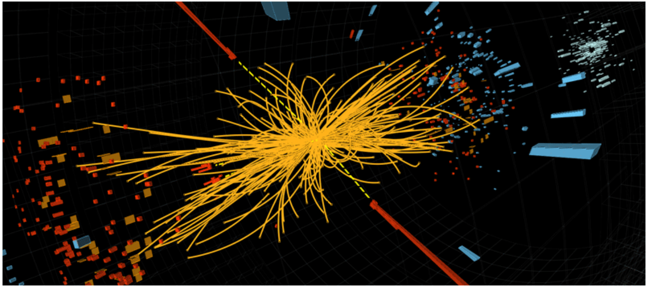
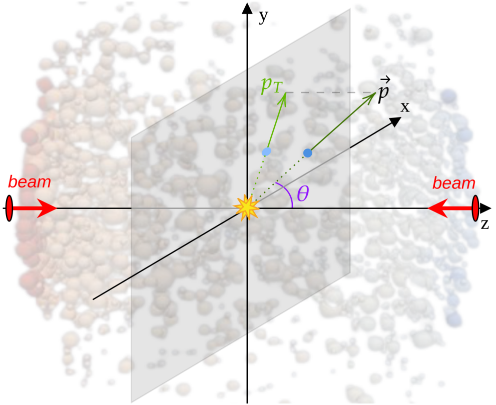


Figure from Bzdak et al., Phys. Rept. 2020 & 2015 Long Range plan

Experimental QCD studies







Fluctuations as CP signature

V. A. Kuznietsov et al., Phys. Rev. C 105, 044903 (2022)

In GCE density cumulants shows singularity behavior in the critical point.

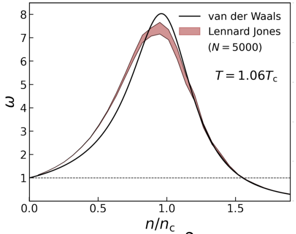
$$\ln Z^{\mathrm{gce}}(T,V,\mu) = \ln\left[\sum_N e^{\mu N} Z^{\mathrm{ce}}(T,V,N)\right],$$

$$\kappa_n \cong \frac{\partial^n(\ln Z^{\mathrm{gce}})}{\partial(\mu_N)^n}.$$

The real expression for Z^{gce} is unknown in QCD matter.

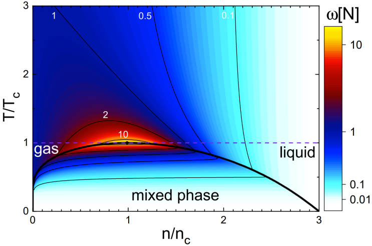
V. Vovchenko, D. V. Anchishkin, M. I. Gorenstein and R. V. Poberezhnyuk, Phys. Rev. C92 054901 (2015)

Scaled variance



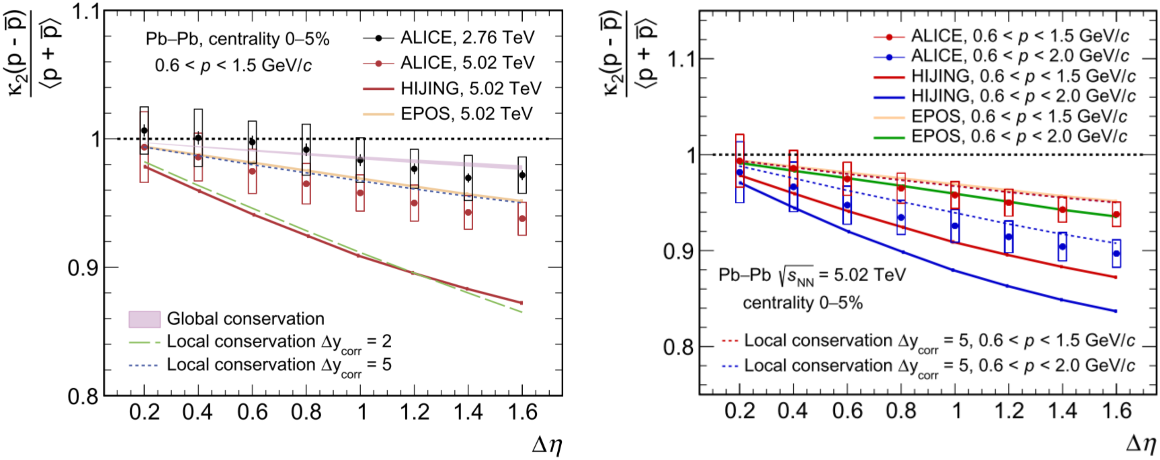
$\omega = \kappa_2/\kappa_1$

$\kappa_2 \sim \xi^2$



Motivation

ALICE, PLB 844 (2023) 137545



Formalism

As it was proposed in **V. Vovchenko., PRC 110, L061902 (2024)**, one can write n-point density correlator as

$$C_1(\mathbf{r}_1) = \rho(\mathbf{r}_1), \quad C_n(\mathbf{r}_1, \dots, \mathbf{r}_n) = \left\langle \prod_{2\le i\le n} \delta\rho(\mathbf{r}_i) \right\rangle, \quad \text{where} \quad \delta\rho(\mathbf{r}_i) = \rho(\mathbf{r}_i) - \langle \rho(\mathbf{r}_i) \rangle .$$

We would focus on a cumulants up to the 4th order, namely

$$C_2(\mathbf{r}_1, \mathbf{r}_2) = \langle \delta\rho_1 \delta\rho_2 \rangle , \quad C_3(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) = \langle \delta\rho_1 \delta\rho_2 \delta\rho_3 \rangle$$

$$C_4(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4) = \langle \delta\rho_1 \delta\rho_2 \delta\rho_3 \delta\rho_4 \rangle - \langle \delta\rho_1 \delta\rho_2 \rangle \langle \delta\rho_3 \delta\rho_4 \rangle - \langle \delta\rho_1 \delta\rho_3 \rangle \langle \delta\rho_2 \delta\rho_4 \rangle - \langle \delta\rho_1 \delta\rho_4 \rangle \langle \delta\rho_2 \delta\rho_3 \rangle .$$

Correlators are connected to the cumulants of conserved charge through the integral over acceptance

$$\kappa_n = \int d\mathbf{r}_1 \cdots \int d\mathbf{r}_n \; C_n(\mathbf{r}_1, \dots, \mathbf{r}_n)$$

Formalism

It can be shown, that from the charge conservation correlators can be written as a sum of symmetric and balancing parts,

$$C_i(\mathbf{r}_1, \ldots, \mathbf{r}_i) = C_{sym,i}(\mathbf{r}_1, \ldots, \mathbf{r}_i) + C_{bal,i}(\mathbf{r}_1, \ldots, \mathbf{r}_i),$$

both of which must be symmetric with respect to the variables of integration. The phenomenological expressions for the second cumulnat, satisfying these properties, reads

$$C_2(\mathbf{r}_1, \mathbf{r}_2) = \chi_2 \delta_{1,2} - \chi_2 V^{-1},$$

where χ_2 - GCE susceptibilities. Similar expressions could be obtained for the higher order correlators.

Local conservation

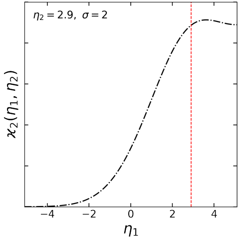
To introduce effects of local conservation, one introduce a phenomenological functions $\varkappa_{1,2}$

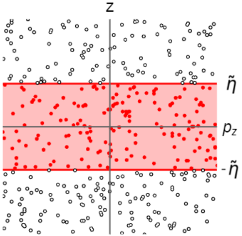
$$V^{-k} \to V^{-k}\varkappa_{k+1}, \quad \text{with} \quad \int_V d\mathbf{r}_i \ \varkappa_{k+1}(\mathbf{r}_1, \ldots, \mathbf{r}_{k+1}) = V\varkappa_k$$

For the case of 1D **longitudinal acceptance** of fireball with $V = 2\eta_{\max}$ one can write for second order kernel

$$\varkappa_2 = \frac{\sqrt{2\pi}\sigma}{4\eta_{\max}}\left[\theta_3\left(\frac{\pi(\eta_1-\eta_2)}{4\eta_{\max}}, e^{-\pi^2\sigma^2/(8\eta_{\max}^2)}\right) + \theta_3\left(\frac{\pi(\eta_1+\eta_2-2\eta_{\max})}{4\eta_{\max}}, e^{-\pi^2\sigma^2/(8\eta_{\max}^2)}\right)\right]$$

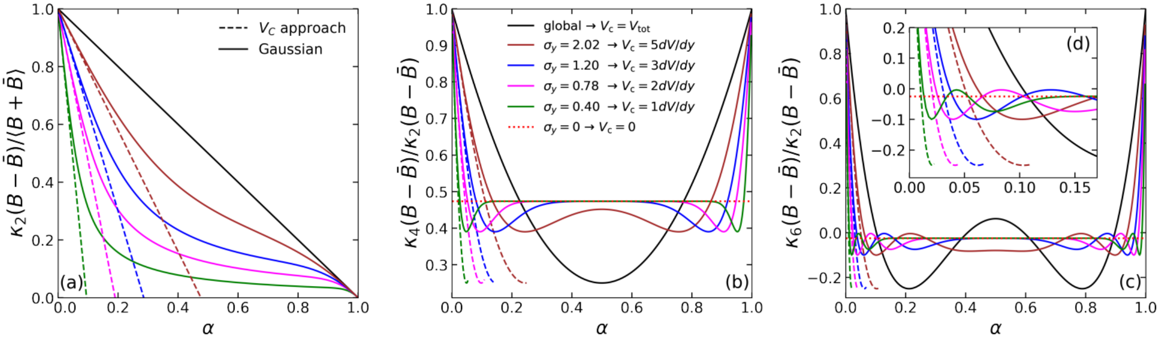
with the anti-periodic boundary conditions. Higher orders could be found numerically as k-dim Poisson sum of gaussians with the same type of BC.





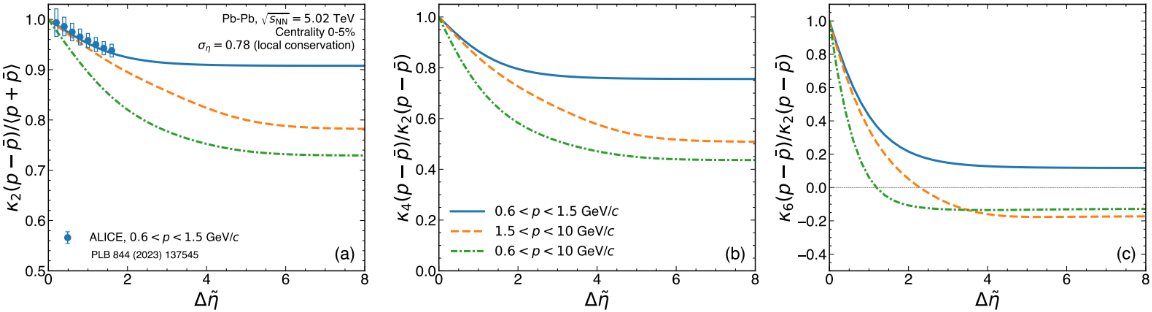
Cumulants with local conservation (coordinate space)

$\alpha = V_{\rm obs}/V_{\rm tot}$ - observed system fraction



Cumulants with local conservation (momentum space)

$\Delta\tilde{\eta}$ - pseudorapidity acceptance, $\sigma = 0.78$



Summary

- Local conservation formalism with gaussian-like kernels was extended to higher orders of cumulants
- Full antiperiodic picture developed for taking into account sum of the distribution tails
- With help of blast wave model kinematic weights with realistic acceptance parameters kinematic cuts were obtained and used to find $\kappa_{2,3,4}$ as a function of momentum space acceptances $\tilde{\eta}$, including experimentally accessible region

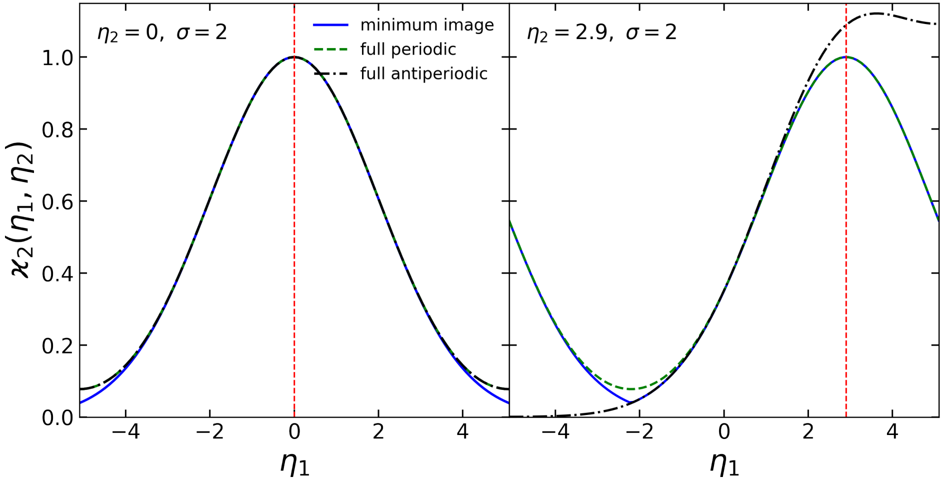
Future directions:

- Local conservation can be incorporated to the Balance function formalism and connected to the new experimental measurements
- Develop a mapping to interacting QCD matter as was done for C_2 with maximum entropy method

THANK YOU FOR ATTENTION!

Questions?

Boundary conditions



Boundary conditions

Using a Poisson summation approach one can extend a gaussian to be periodic function

$$\varkappa_n^{\mathrm{P}}(\eta_1, \ldots, \eta_n) = A_n \sum_{k_2,\ldots,k_n=-\infty}^{\infty} \exp\left[-\frac{1}{n\sigma^2} \sum_{1\le i<j\le n} (\eta_i - \eta_j + 2\eta_{\max} k_i)^2\right].$$

Similar and slightly more complicated expression can be obtained for antiperiodic case.
For the second order kernels, one can find a closed analytical form for both periodic and antiperiodic cases.

Periodic

$$\varkappa_2^{\mathrm{P}} = \frac{\sqrt{2\pi}\sigma}{2\eta_{\max}} \theta_3\left(\frac{\pi(\eta_1-\eta_2)}{2\eta_{\max}}, e^{-\pi^2\sigma^2/(2\eta_{\max}^2)}\right)$$

Antiperiodic

$$\varkappa_2^{\mathrm{AP}} = \frac{\sqrt{2\pi}\sigma}{4\eta_{\max}}\left[\theta_3\left(\frac{\pi(\eta_1-\eta_2)}{4\eta_{\max}}, e^{-\pi^2\sigma^2/(8\eta_{\max}^2)}\right) + \theta_3\left(\frac{\pi(\eta_1+\eta_2-2\eta_{\max})}{4\eta_{\max}}, e^{-\pi^2\sigma^2/(8\eta_{\max}^2)}\right)\right]$$

Antiperiodic boundary condition

As simpliest example, one can extend the periodicity region twice and perform a Poisson summation of the sum of 2 functions

$$\varkappa^{\mathrm{AP}}(\eta_1, \eta_2) = \varkappa(\eta_1, \eta_2) + \varkappa(\eta_1 + \eta_{\max}, -\eta_2 + \eta_{\max})$$

For $n \geq 2$ general order of conservation function it'll take form

$$\varkappa^{\mathrm{AP}}_{1,\ldots,n}(\eta_1, \ldots, \eta_n) = A_n \sum_{k_2,\ldots,k_n} \sum_{s_2,\ldots,s_n \in \{0,1\}} \exp\left[-\frac{1}{n\sigma^2} \sum_{1 \leq i < j \leq n} (\tilde{\eta}_i - \tilde{\eta}_j)^2\right].$$

The coordinate mapping for particles $j \geq 2$ is:

$$\tilde{\eta}_j(k_j, s_j) = \begin{cases} \eta_j + 2k_j\, 2\eta_{\max} & \text{if } s_j = 0 \quad \text{(Direct)} \\ 2\eta_{\max} - \eta_j + 2k_j\, 2\eta_{\max} & \text{if } s_j = 1 \quad \text{(Reflected)} \end{cases}.$$

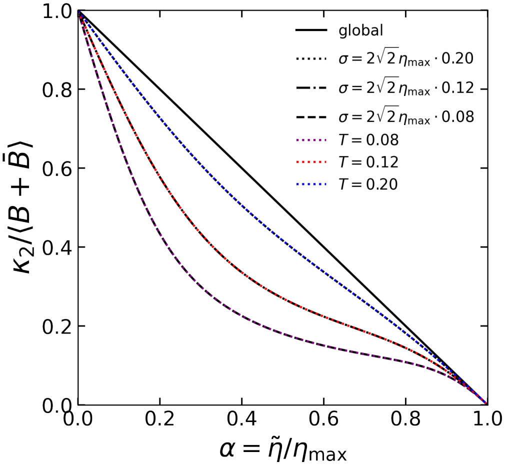
The constant A_n is the same as for periodic boundary conditions case.

Higher order correlators

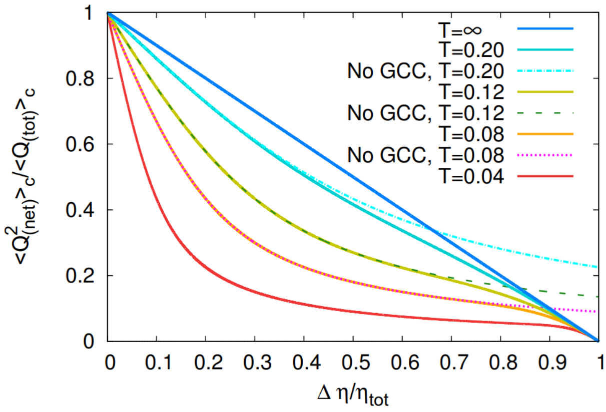
$$C_3(\eta_1,\eta_2,\eta_3)/\chi_3 = \delta_{1,2,3} - \frac{1}{V}[\varkappa_2(\eta_1,\eta_2)\delta_{1,3} + \varkappa_2(\eta_1,\eta_3)\delta_{1,2} + \varkappa_2(\eta_1,\eta_2)\delta_{2,3}]$$
$$+ \frac{2}{V^2}\varkappa_3(\eta_1,\eta_2,\eta_3),$$
$$C_4(\eta_1,\eta_2,\eta_3,\eta_4)/\chi_4 = \delta_{1,2,3,4} - \frac{1}{V}[\varkappa_2(\eta_1,\eta_2)\delta_{1,3,4} + \varkappa_2(\eta_1,\eta_3)\delta_{1,2,4} + \varkappa_2(\eta_1,\eta_4)\delta_{1,2,3}$$
$$+\varkappa_2(\eta_1,\eta_2)\delta_{2,3,4}] + \frac{1}{V^2}[\varkappa_3(\eta_1,\eta_2,\eta_3)\delta_{1,2} + \varkappa_3(\eta_1,\eta_2,\eta_4)\delta_{1,3} + \varkappa_3(\eta_1,\eta_2,\eta_3)\delta_{1,4}$$
$$+\varkappa_3(\eta_1,\eta_2,\eta_4)\delta_{2,3} + \varkappa_3(\eta_1,\eta_2,\eta_3)\delta_{2,3} + \varkappa_3(\eta_1,\eta_2,\eta_3)\delta_{3,4}] - \frac{3}{V^3}\varkappa_4(\eta_1,\eta_2,\eta_3,\eta_4),$$

Note, that for C_4 we put $\chi_3 = 0$ (LHC).

Cumulants with local conservation (coordinate space)



M. Sakaida et al., PRC 90, 064911 (2014)



Blast wave model

$$\frac{d^3N_i}{dp_T d\tilde{\eta}}(\eta) = \frac{E(\tilde{\eta},\eta)p_T^2\cosh\tilde{\eta}}{\sqrt{(p_T\cosh\tilde{\eta})^2+m^2}}\int_0^1 x\ dx\ \exp\left[-\frac{E(\tilde{\eta},\eta)\cosh\rho}{T_{\rm kin}}\right] I_0\left(\frac{p_T\sinh\rho}{T_{\rm kin}}\right)$$

where $\rho = \tanh^{-1}(\beta_s x_{\rm bw}^n)$ and

$$E(\tilde{\eta},\eta) = m_T\cosh\left(\operatorname{arcsinh}\left[\frac{p_T\sinh\tilde{\eta}}{m_T}\right]-\eta\right)$$

The acceptance probability calculated as

$$p(\eta) = \frac{\int_{p_T^{\min}}^{p_T^{\max}} dp_T \int_{-\eta_{\max}}^{\eta_{\max}} d\tilde{\eta}\ \frac{d^3N}{dp_T d\tilde{\eta}}(\eta)}{\int_0^{\infty} dp_T \int_{-\infty}^{\infty} d\tilde{\eta}\ \frac{d^3N}{dp_T d\tilde{\eta}}(\eta)}$$