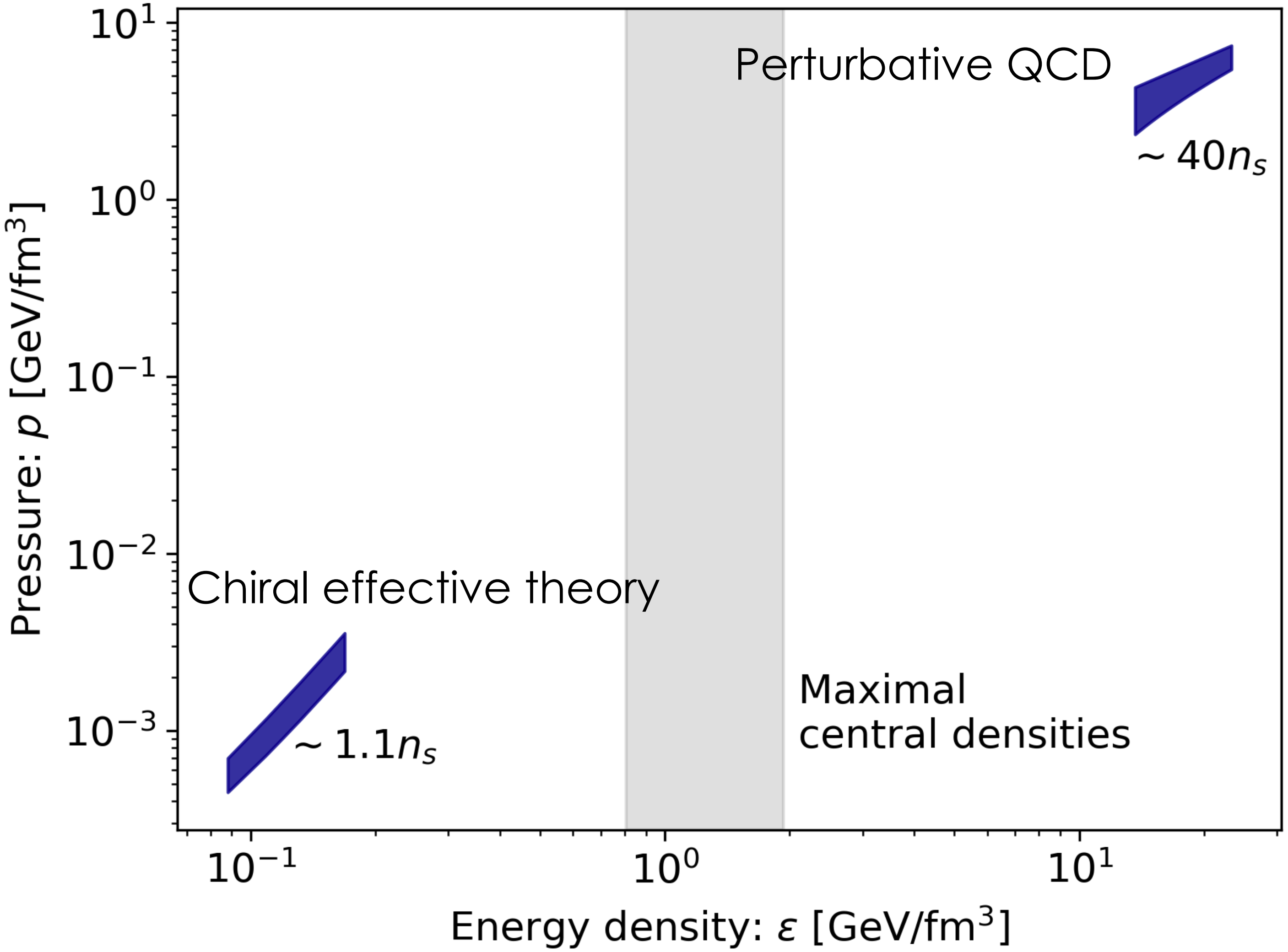
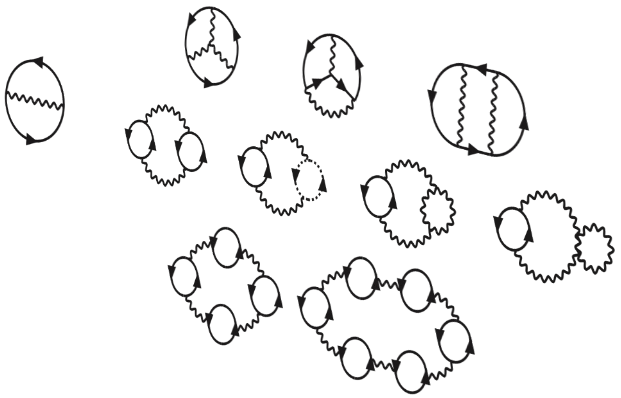
How pQCD constrains EoS at low densities:





How pQCD constrains EoS at low densities:

- **Why** does QCD at $40n_s$ constrain the EoS at NS densities:

How pQCD constrains the equation of state at neutron star densities

Komoltsev & AK, PRL128 (2022) 20, 2111.05350

- **How** QCD affects EoS infrerence

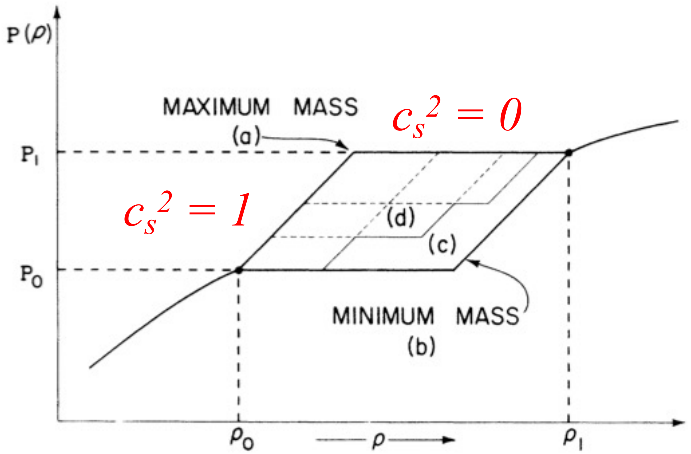
Ab-initio QCD calculations impact the inference of neutron-star equation of state

Gorda, Komoltsev & AK 2204.11877

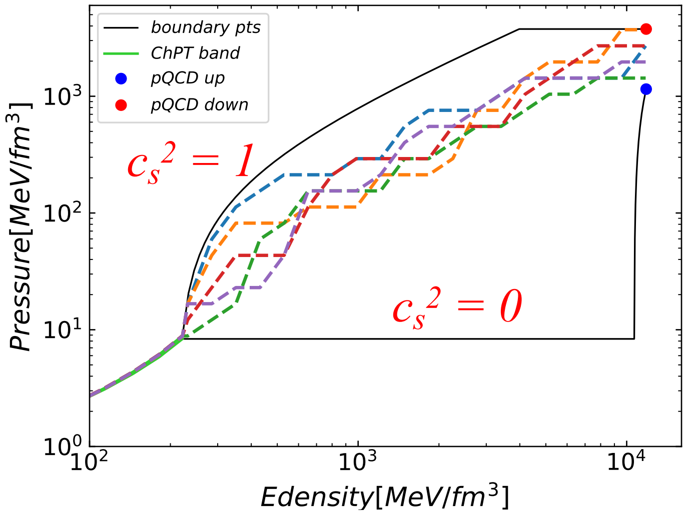
Robust EoS constraints:

General considerations:

- Mechanical stability: $c_s^2 > 0$
- Causality: $c_s^2 < 1$



Rhoades & Ruffini, *Phys.Rev.Lett.* 32 (1974)



Lope-Oter, Windisch, Llanes-Estrada, Alford, *J. Phys. G (2019)*
Lope-Oter, Llanes-Estrada, EPJA 58 (2022)

Robust EoS constraints:

General considerations:

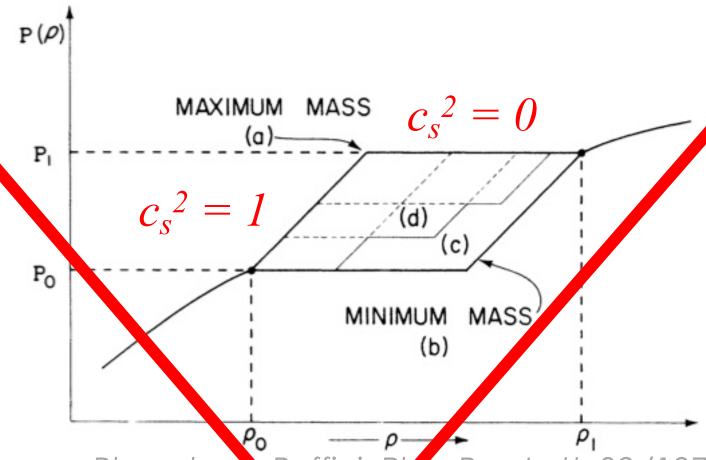
- Mechanical stability: $c_s^2 > 0$
- Causality: $c_s^2 < 1$
- Consistency:

$$P(\epsilon) \ \ vs. \ \Omega(\mu)$$

Reduced EoS

Full EoS

Information of $\{P, \epsilon, n\}$



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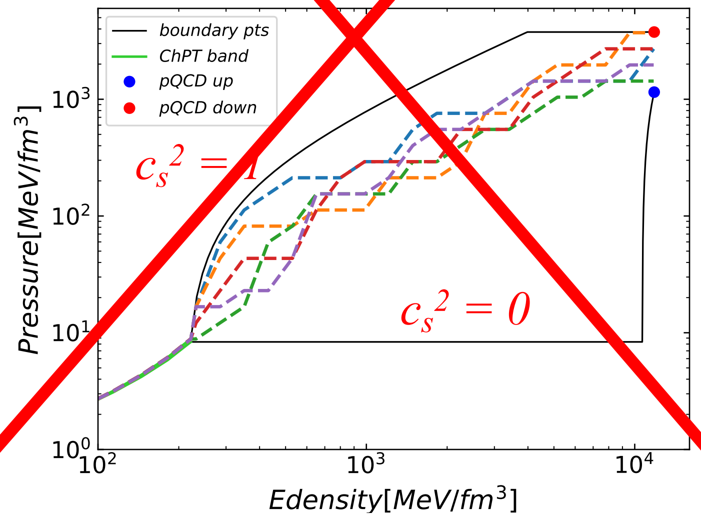

Lope-Oter, Windisch, Llanes-Estrada, Alford, J. Phys. G (2019)
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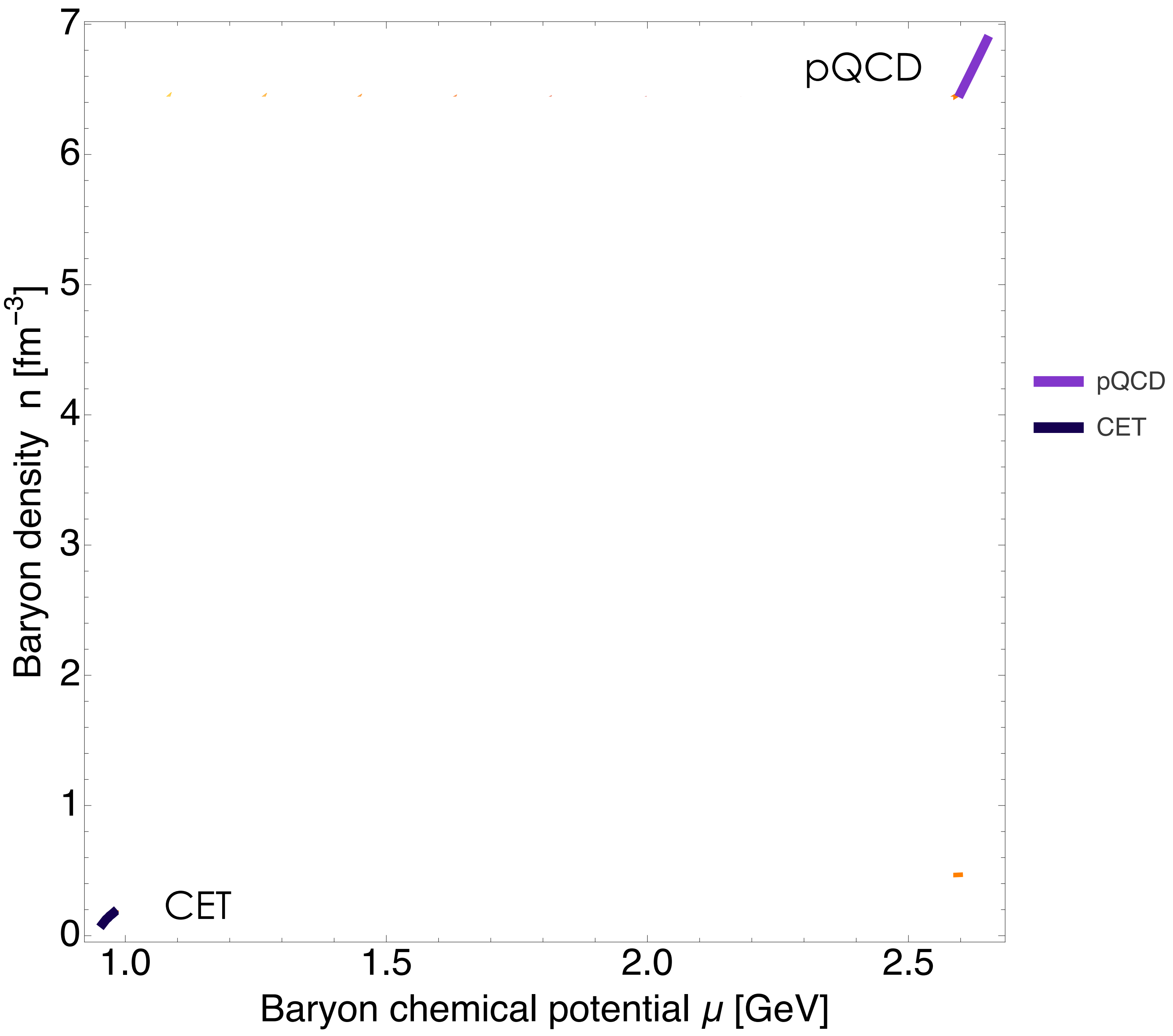
Setup:

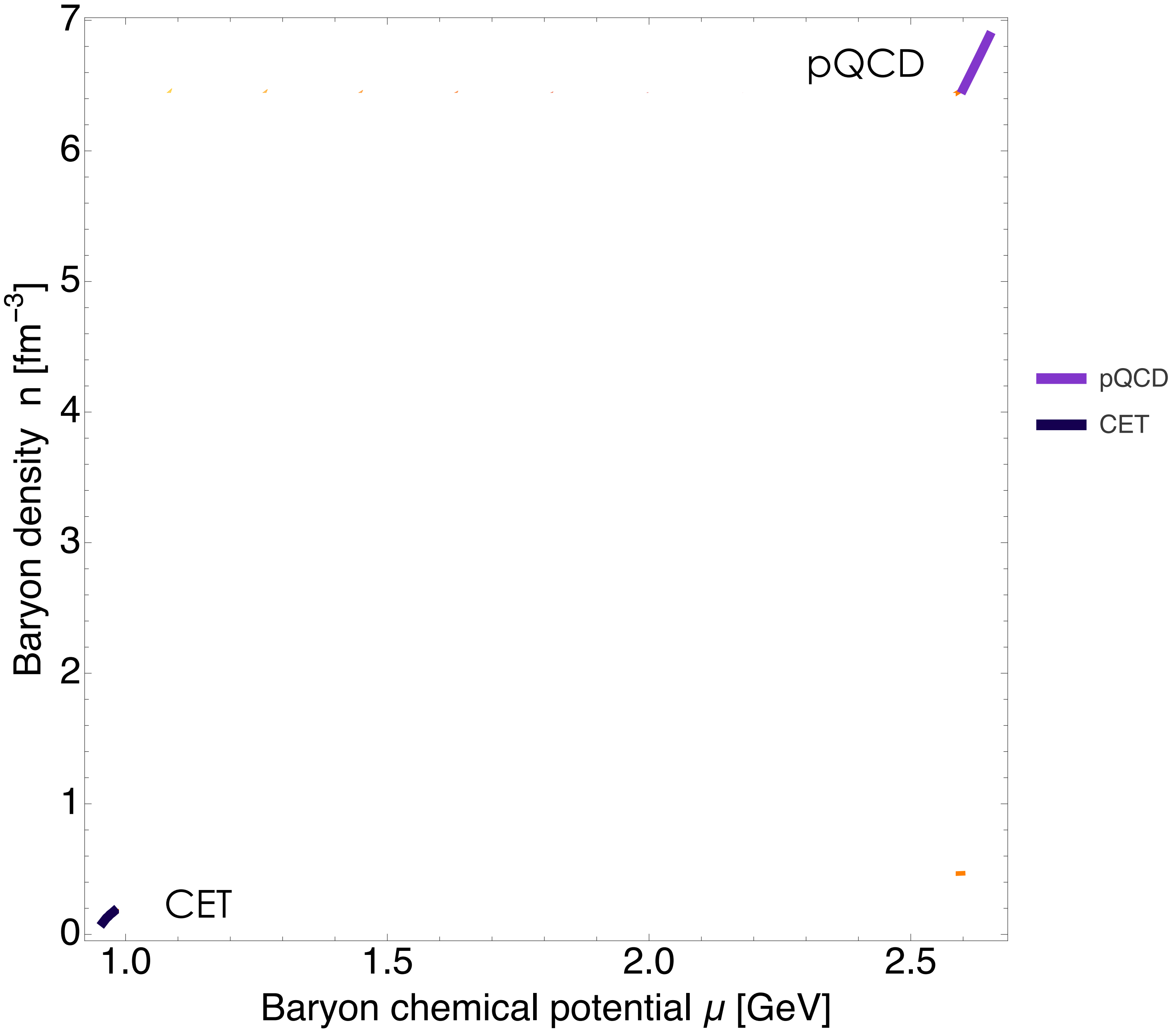


Komoltsev & AK, PRL128 (2022)

Setup:

- Stability

$$\partial_\mu^2 \Omega(\mu) \leq 0 \quad \Rightarrow \quad \partial_\mu n(\mu) \geq 0$$



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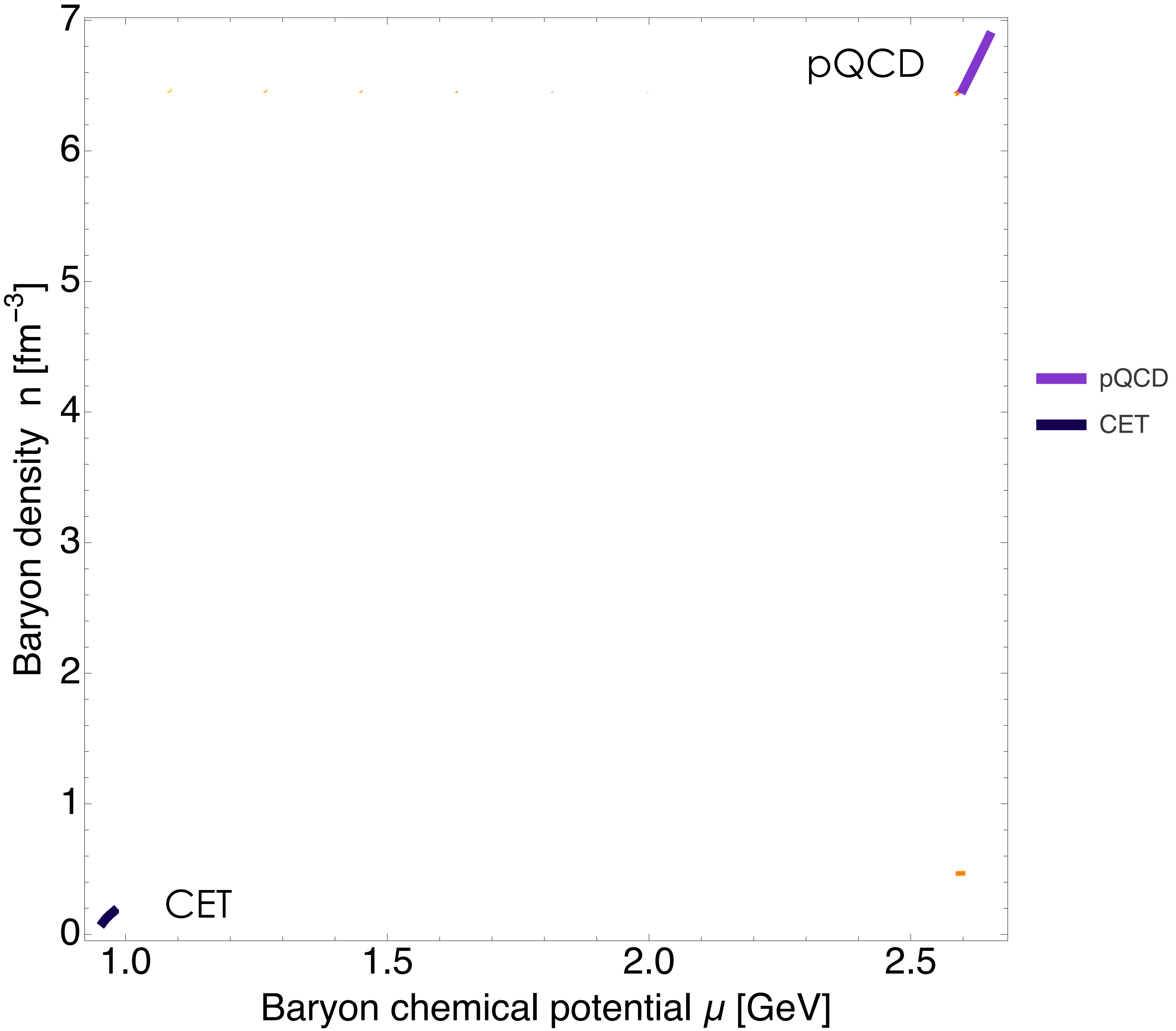

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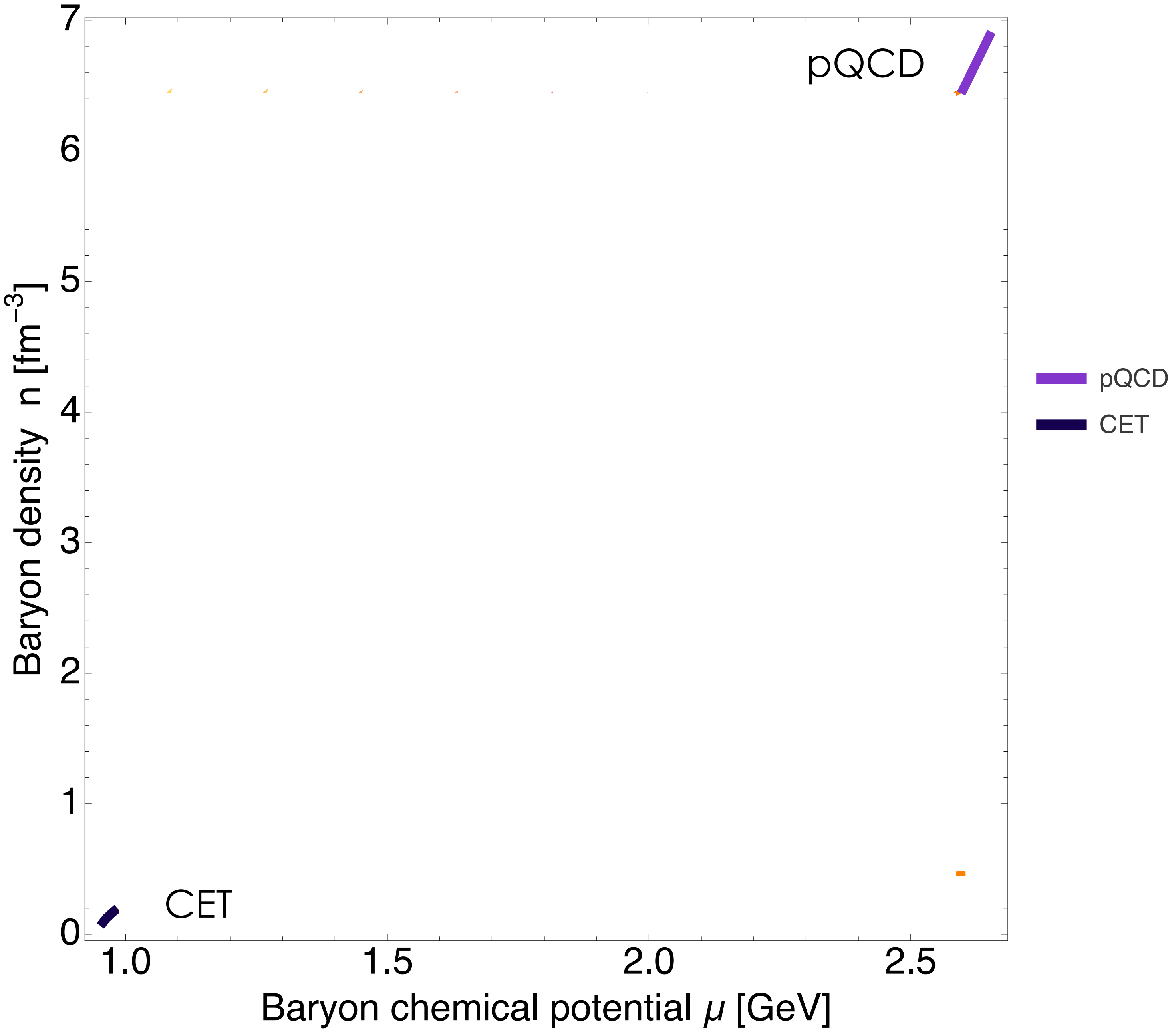

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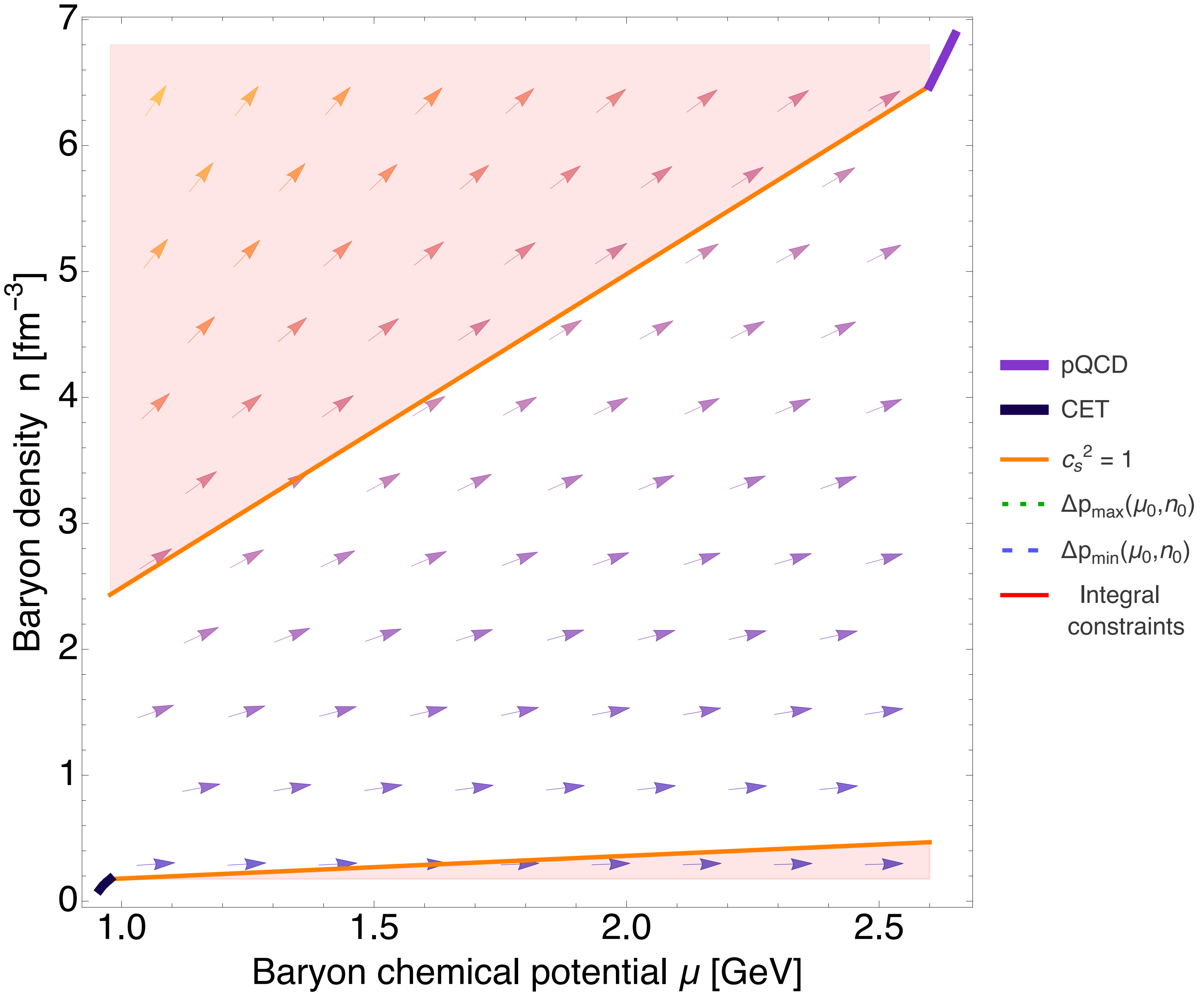

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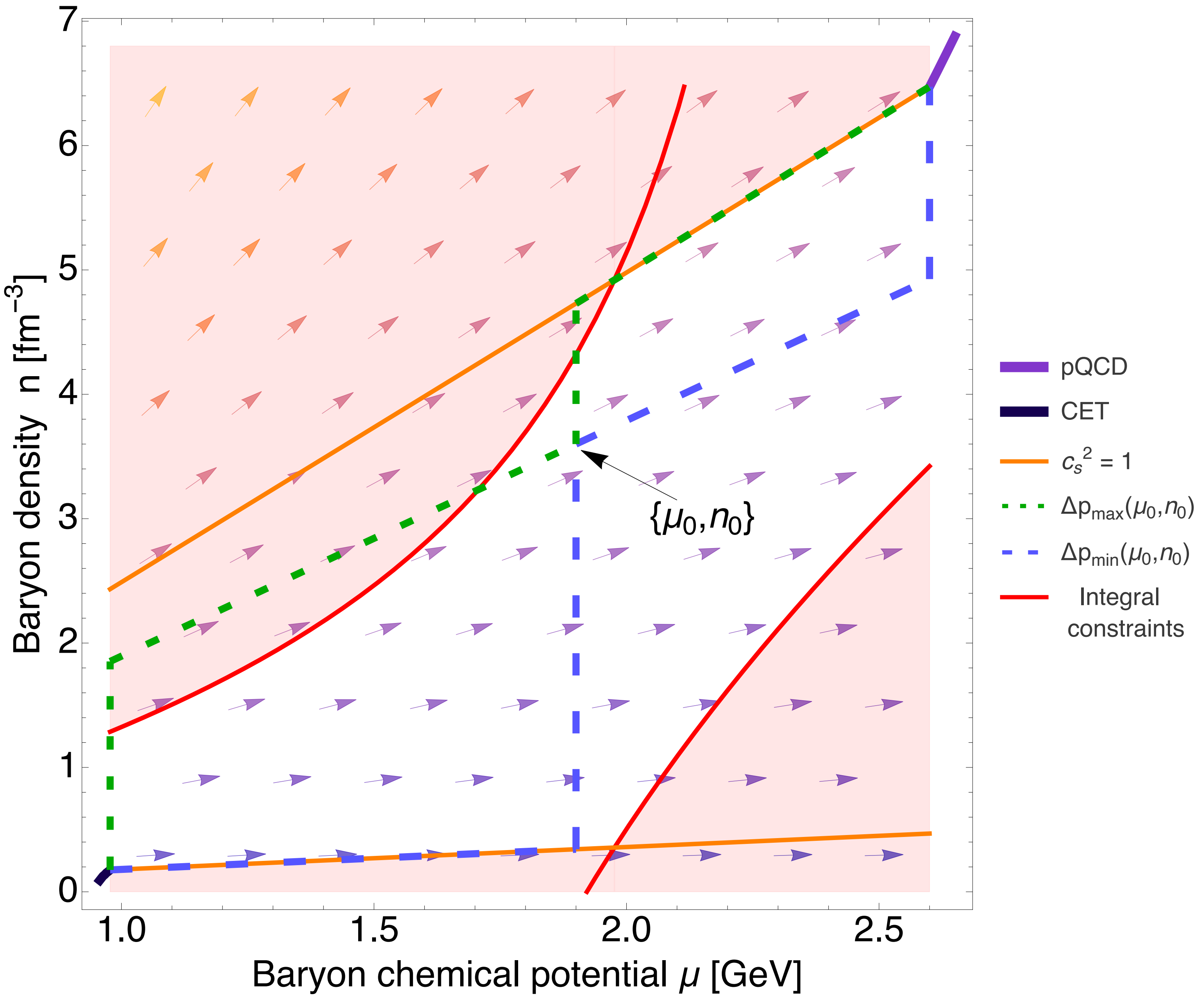

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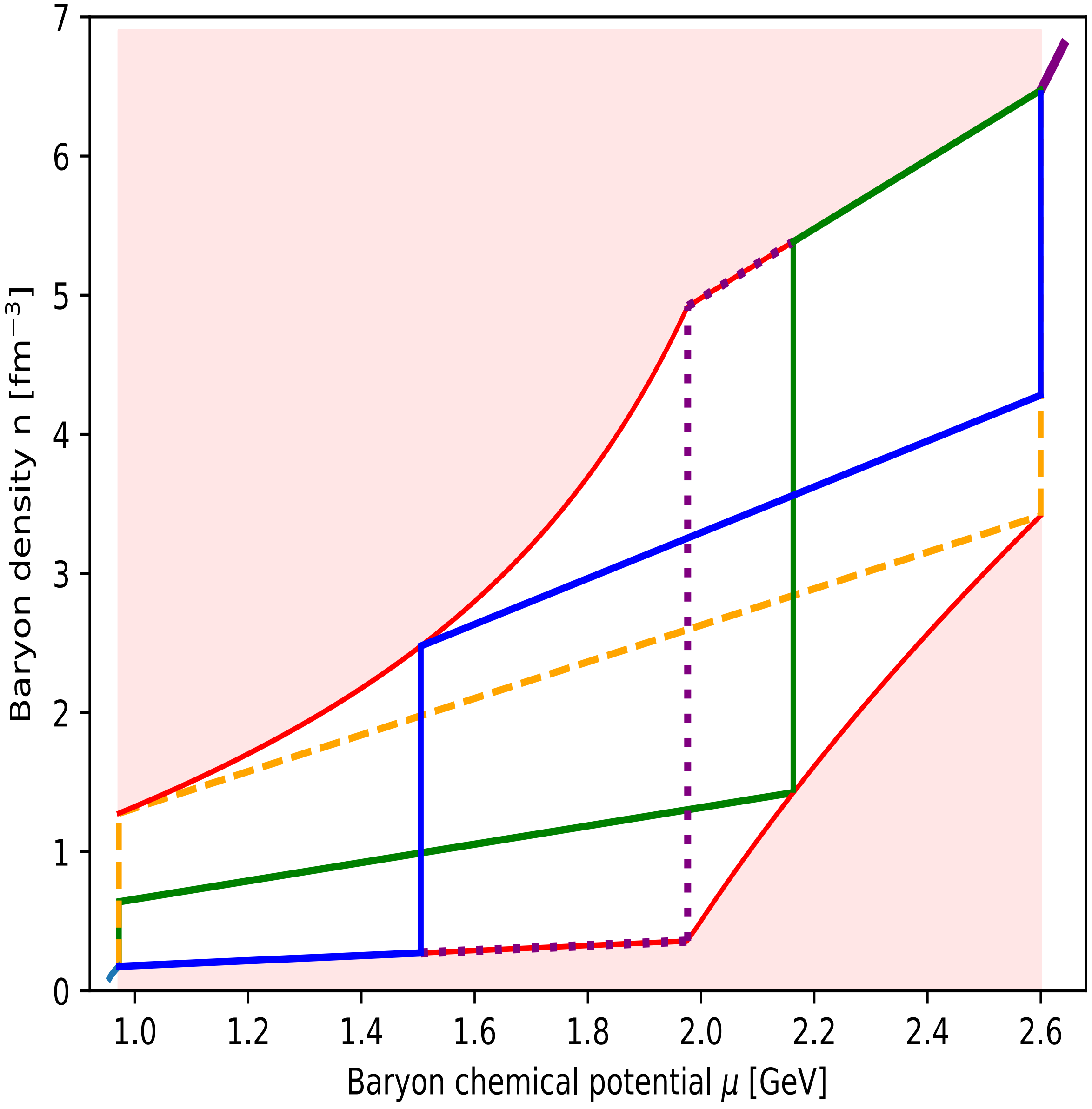
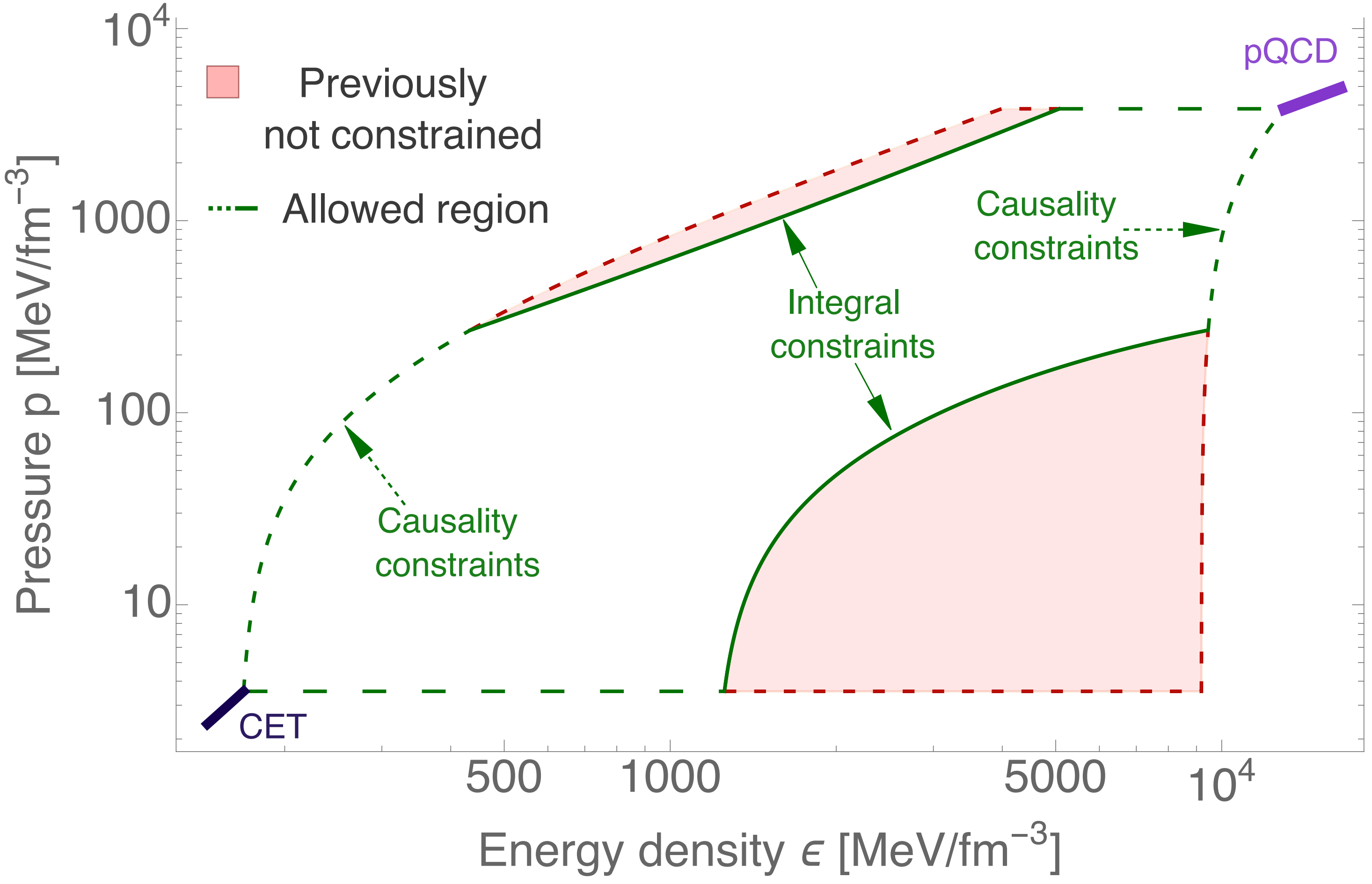


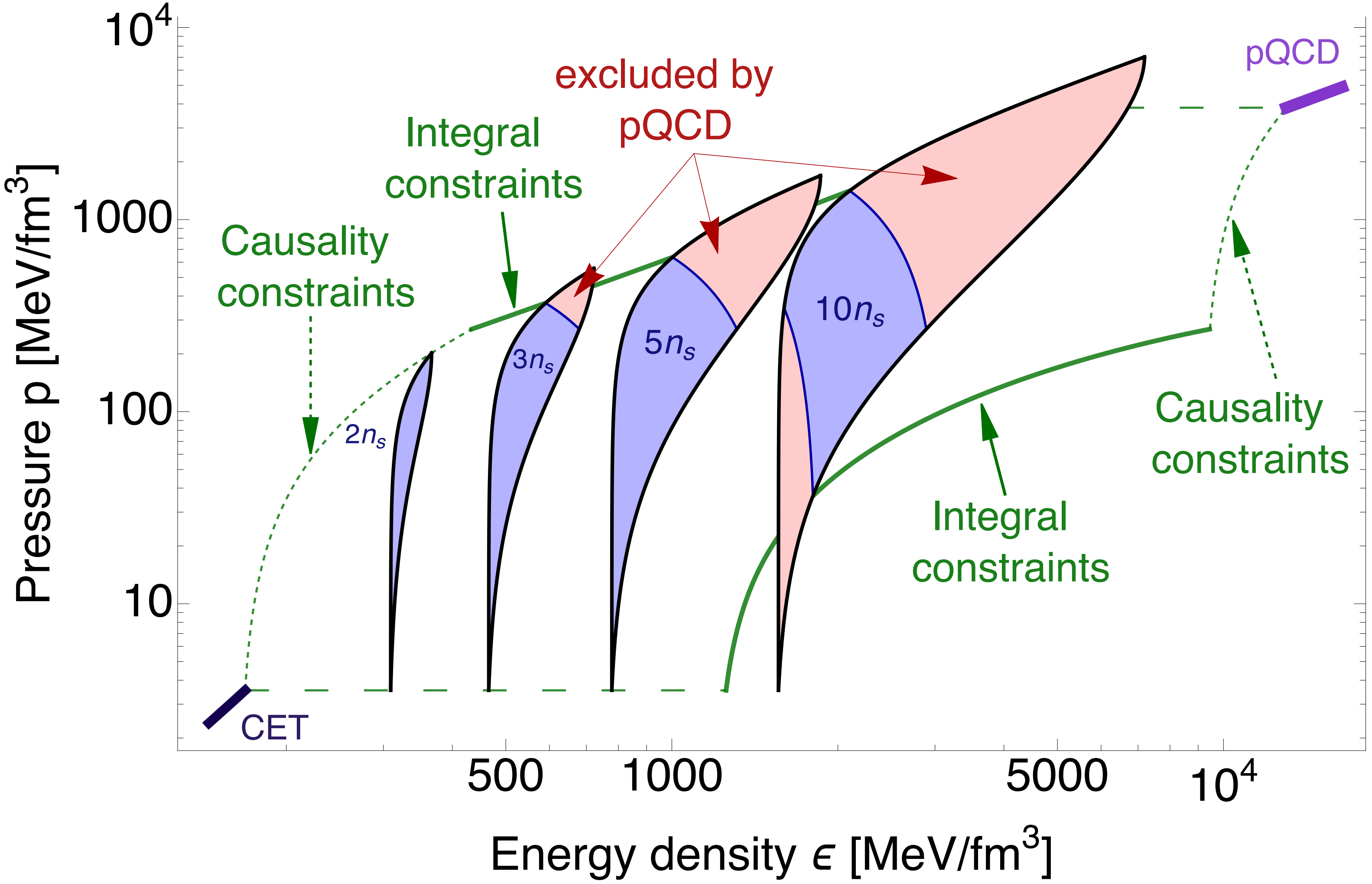
Komoltsev & AK, PRL128 (2022)

Mapping to $\epsilon - p$ -plane



Komoltsev & AK, PRL128 (2022)

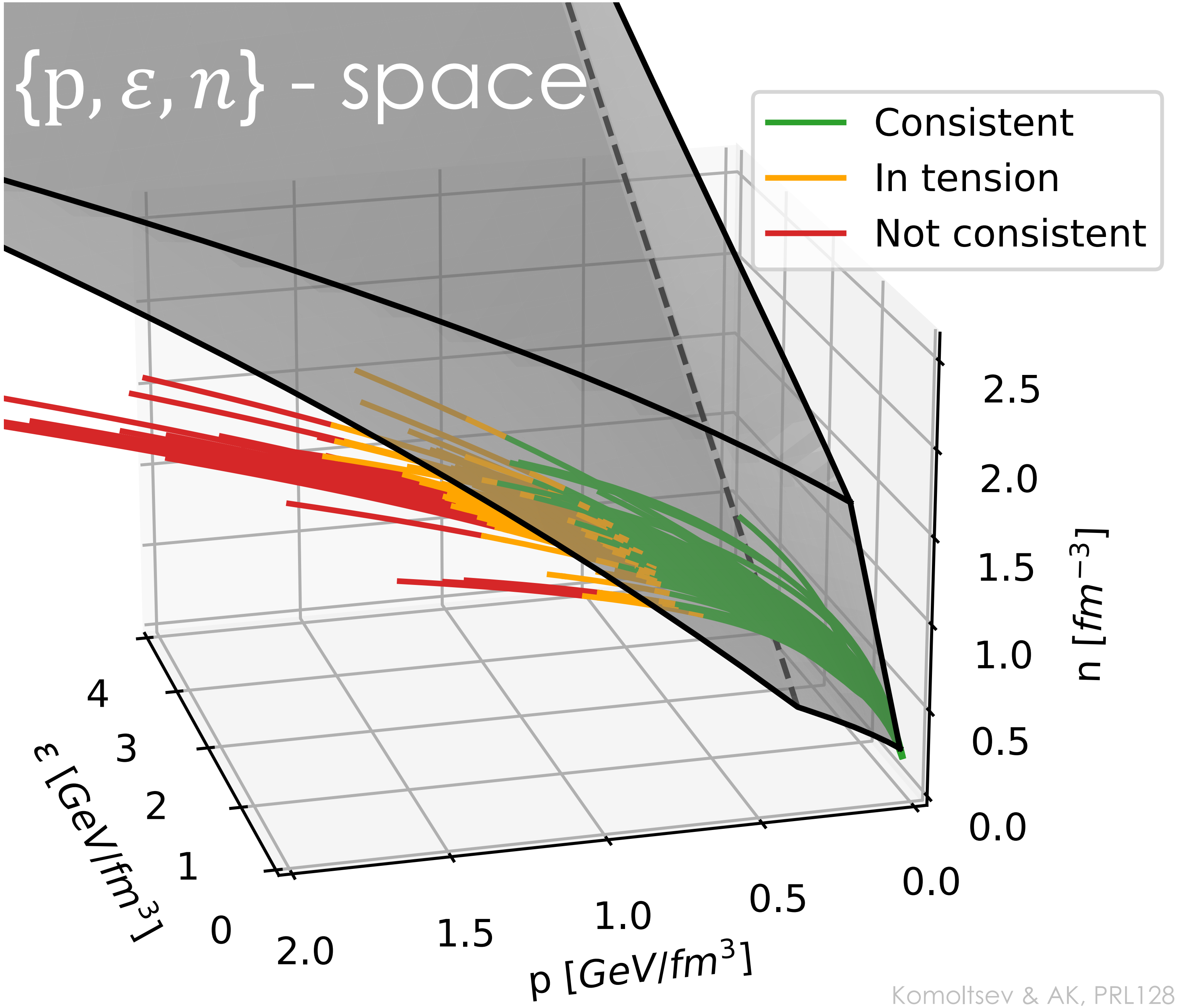
Constraints for fixed n on $\epsilon - p$ -plane



Komoltsev & AK, PRL128 (2022)

Constraints in $\{p, \varepsilon, n\}$ - space

Models from CompOSE database



Komoltsev & AK, PRL128 (2022)

Summary I:

- $\{n, p, \varepsilon\}$ carries more information than $p(\varepsilon)$
- Stability, causality and consistency
- QCD at $n = 40\ n_s$ offers a robust constraint down to $n = 2.5\ n_s$

How pQCD constrains at low densities:

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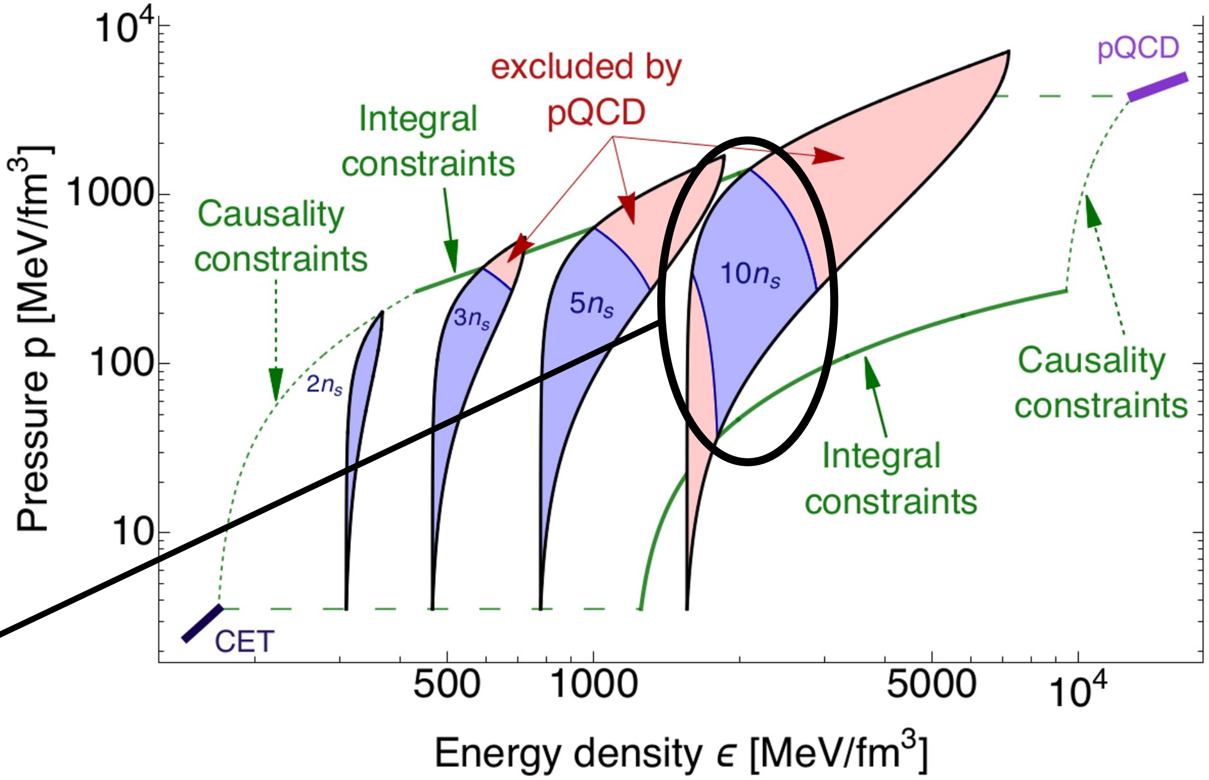
Gorda, Komoltsev & AK 2204.11877

Implementing pQCD to EoS inference:

- Standard EoS Inference setup where QCD can be turned on/off

  Gaussian process similar to Landry & Essick PRD 99 (2019), but for function of n instead of ε. Details on demand

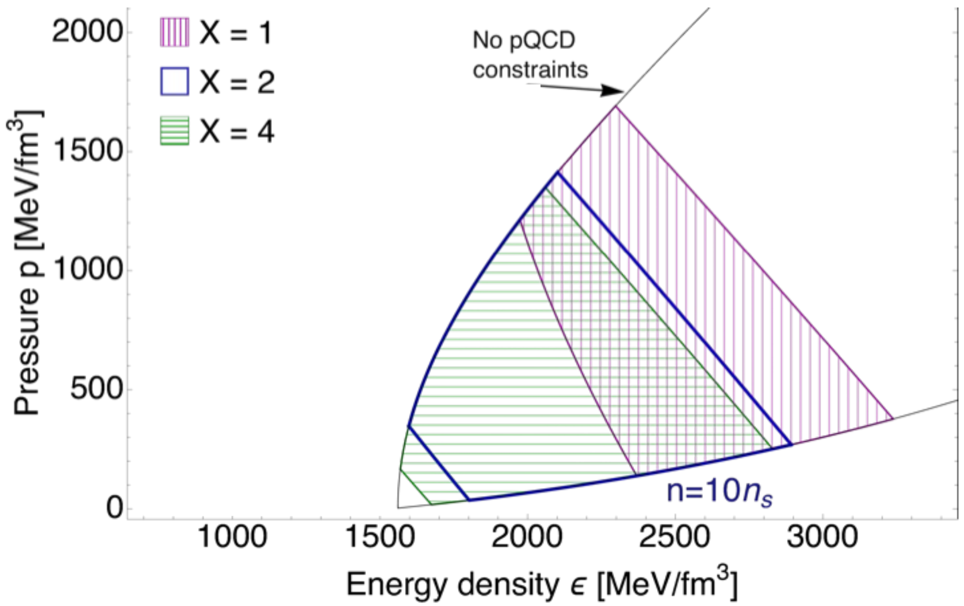
- **Conservative** QCD likelihood function



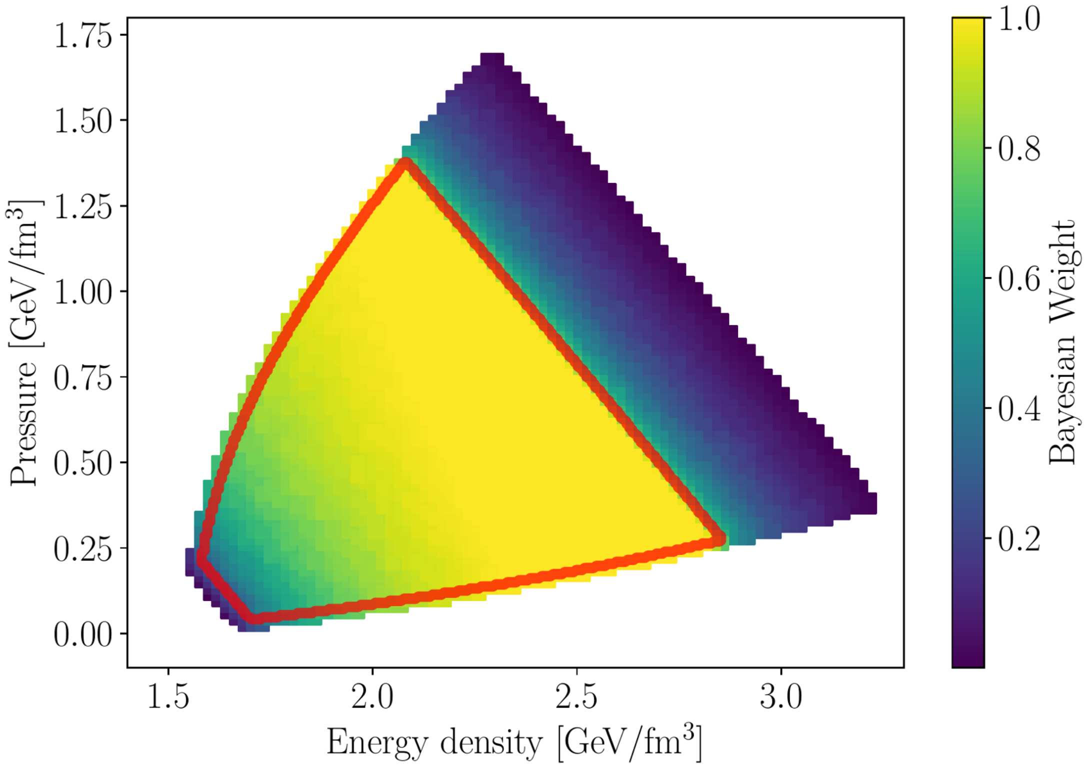
Use this area to construct a likelihood function

$$P(\mathrm{QCD}\,|\,\mathrm{EoS})$$

QCD likelihood function:



$\Longrightarrow$



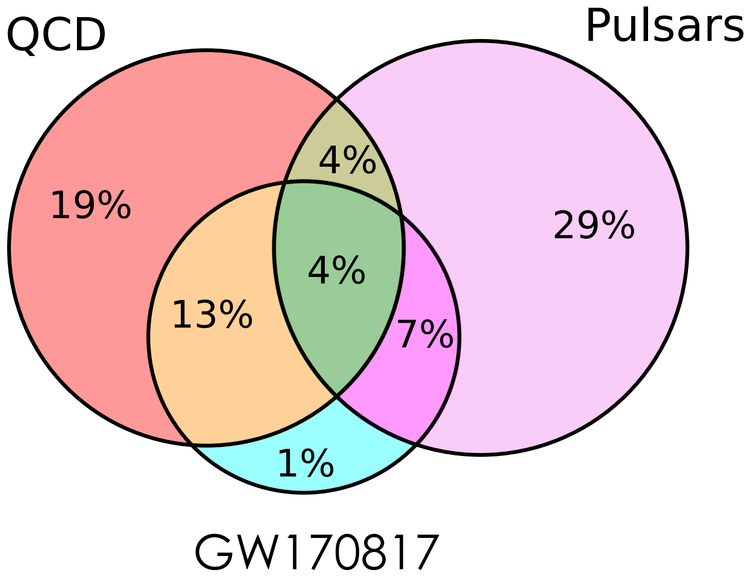
- Uncertainty in pQCD given by *renormalization scale variation:* $X = \frac{\bar{\Lambda}}{\mu_q}$

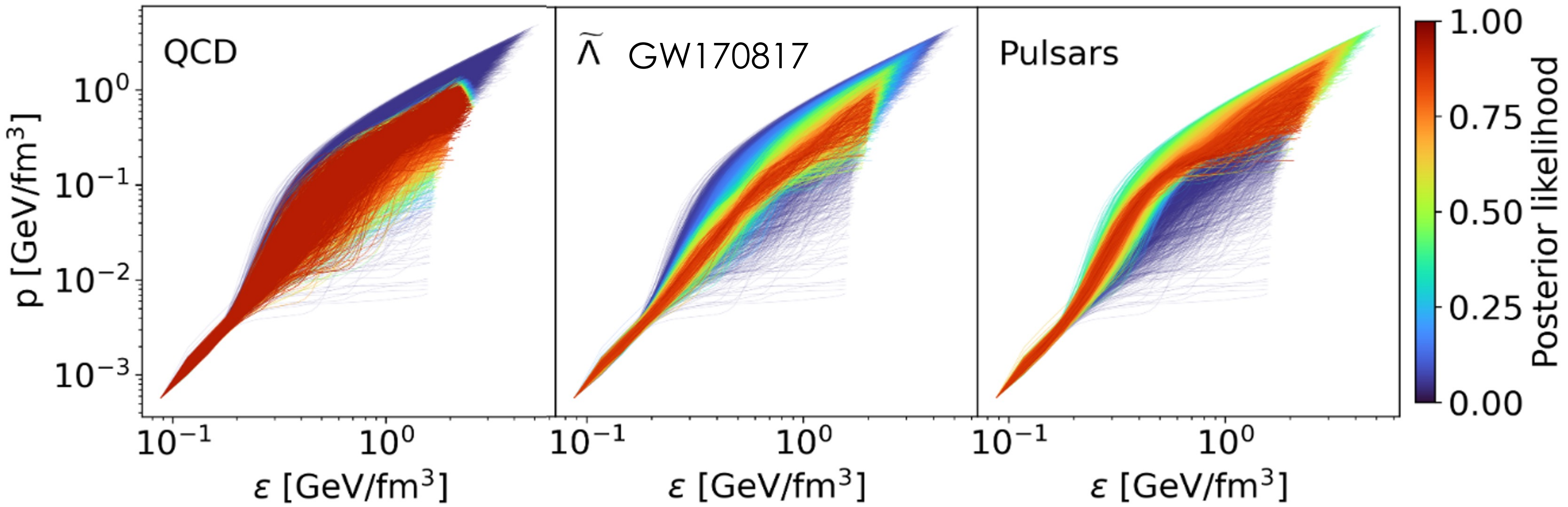
Cacciari & Houdeau, JHEP 09, (2011), Duhr et al. JHEP 122, (2021)

- Bayesian interpretation of scale variation error: scale marginalization, Log-uniform in X

Effect of QCD:

QCD input complement NS observations





Gorda, Komoltsev & AK 2204.11877

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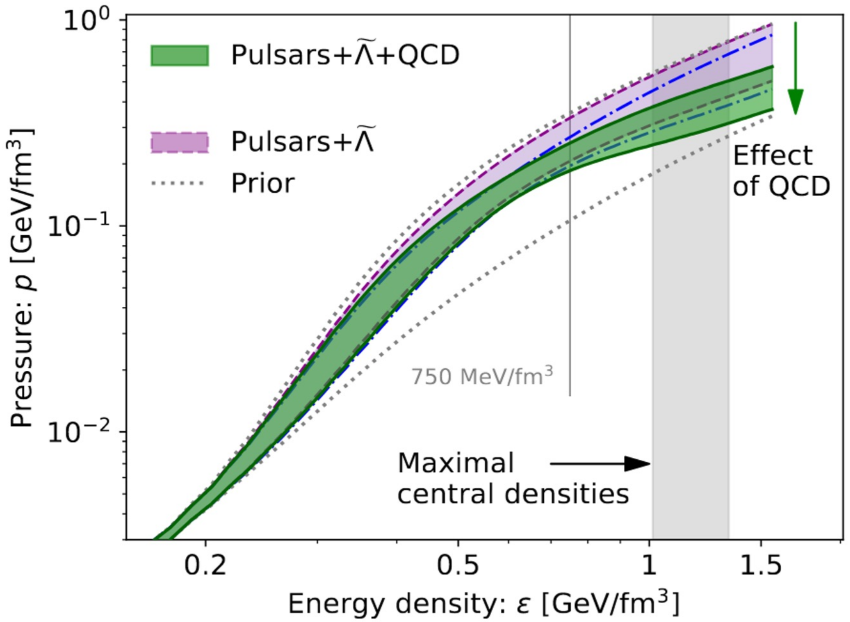

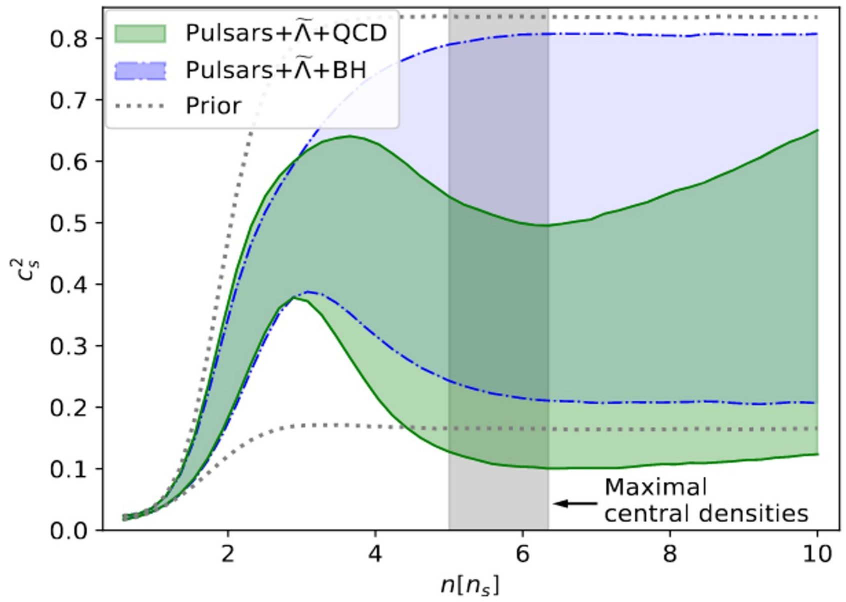

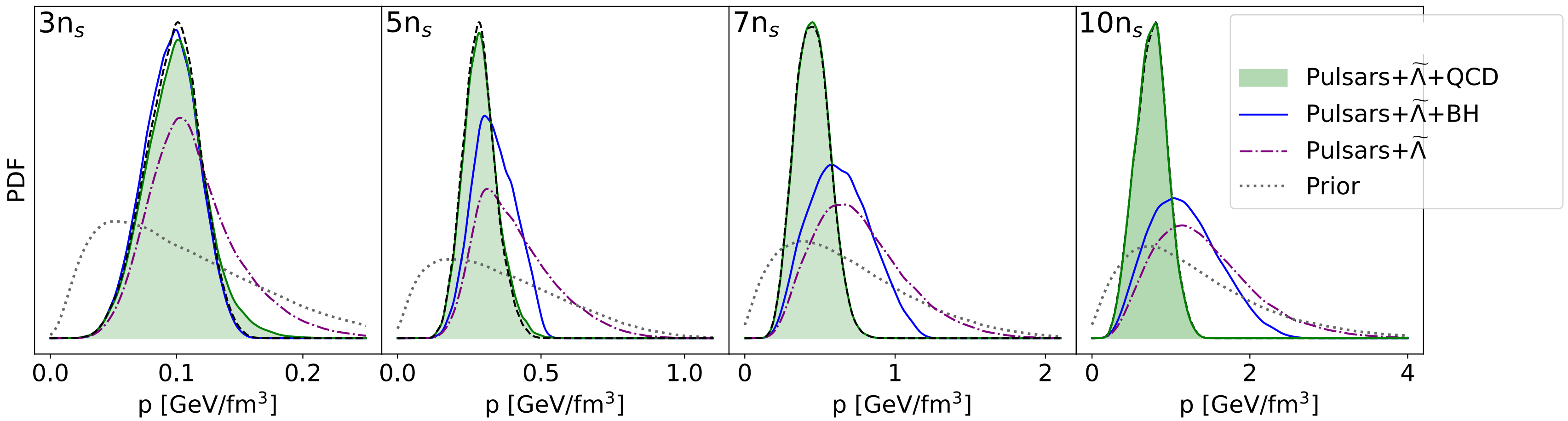

Gorda, Komoltsev & AK 2204.11877

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Summary II:

- Results support findings of earlier works with QCD

Annala et al. Phys.Rev.X 12 (2022), Altiparmak, Ecker, Rezzolla 2112.08157, …

- QCD offers ***complementary*** info at NS densities.
- QCD softens the EoS at high densities. Quark Matter?

Annala, Gorda, Kurkela, Nättilä, Vuorinen, *Nature Phys.* 16 (2020)

Discussion:

- Complementary systematics. No model uncertainites

  no transport models, no stellar models, no extrapolation in proton fraction, no GR …

- The propagation of pQCD to NS densities the **most conservative possible**, but can include assumptions

  How long of a density range can be c_s^2=1? How large phase transition is in the cards?

Discussion:

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- The propagation of pQCD to NS densities the **most conservative possible**, but can include assumptions

  How long of a density range can be c_s^2=1? How large phase transition is in the cards?

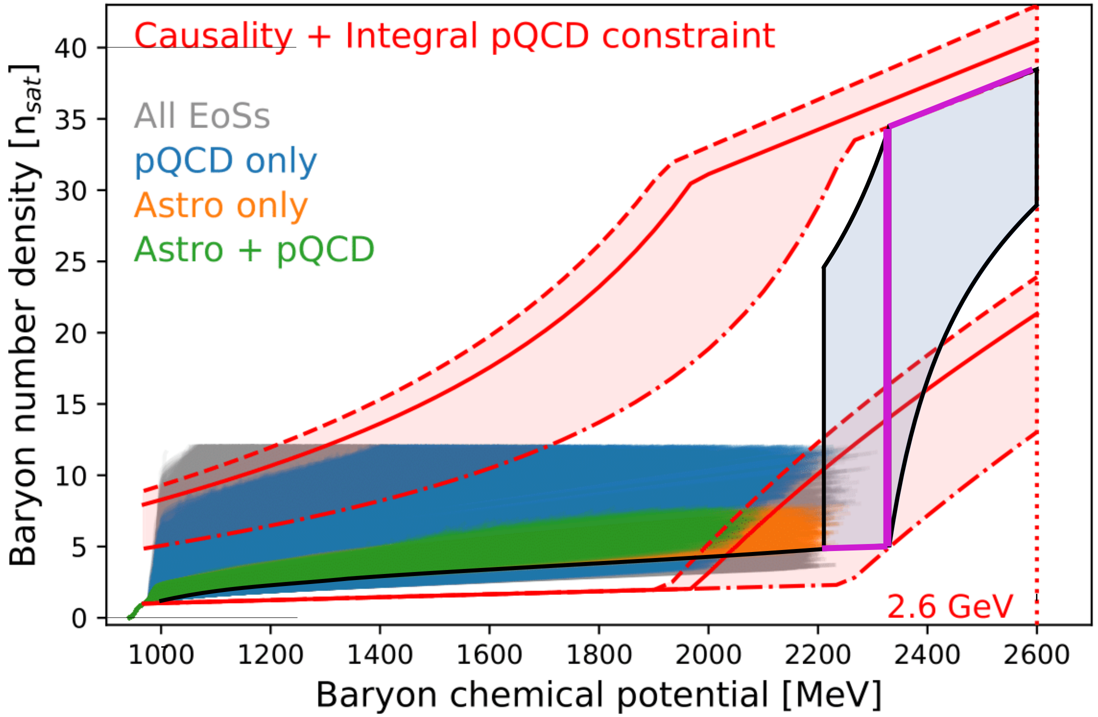
Conclusion:

QCD input should be part of any complete EoS inference setup

Jupyter notebook available on Github: OKomoltsev/QCD-likelihood-function

Extra slides

Comparison with recent work



PT at n_{TOV}+0.2 n_s of Δn = 30 n_s, followed by $c_s^2$1 until pQCD at 40 n_s

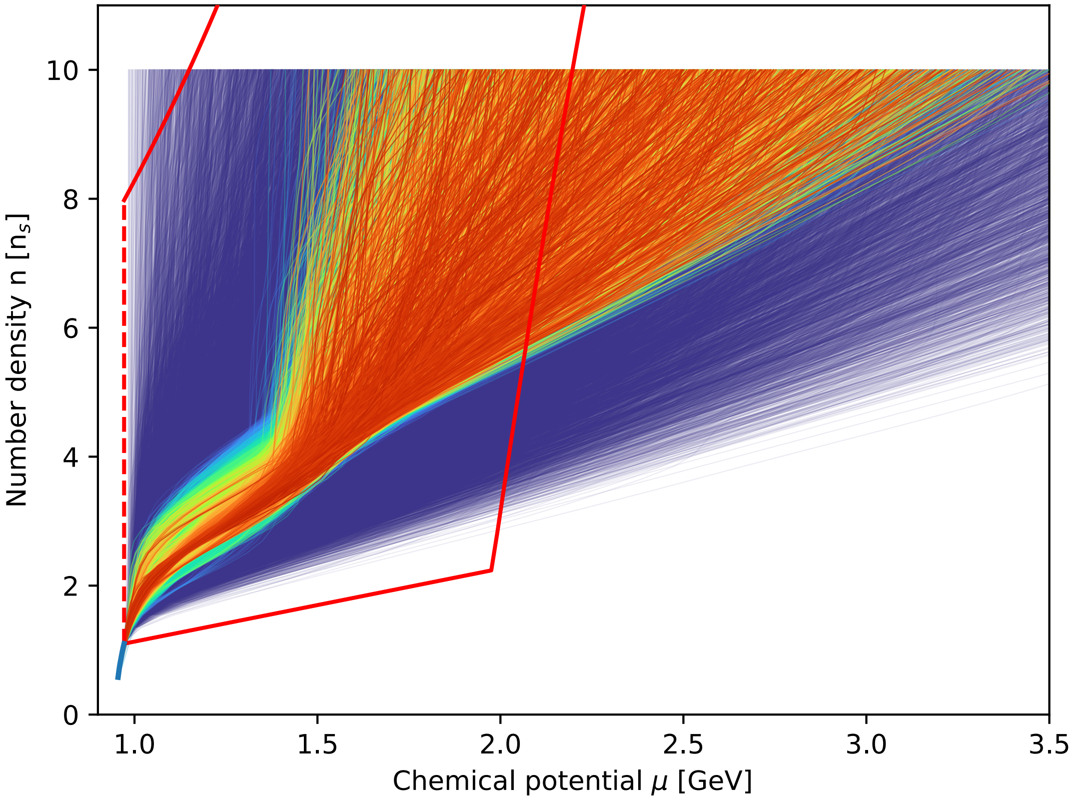
Somasundaram, Tews, Margueron (2204.14038) perform conservative analysis with QCD input:

- **Results broadly consistent with us**
- Different:
  - No Bayesian treatment of input
  - Apply QCD input at $n = n_{TOV}$ instead of $n = 10n_s$
- For QCD not to constrain:
  - Extreme value of X = 1-1.3
  - Very extreme behavior immediately after nTOV

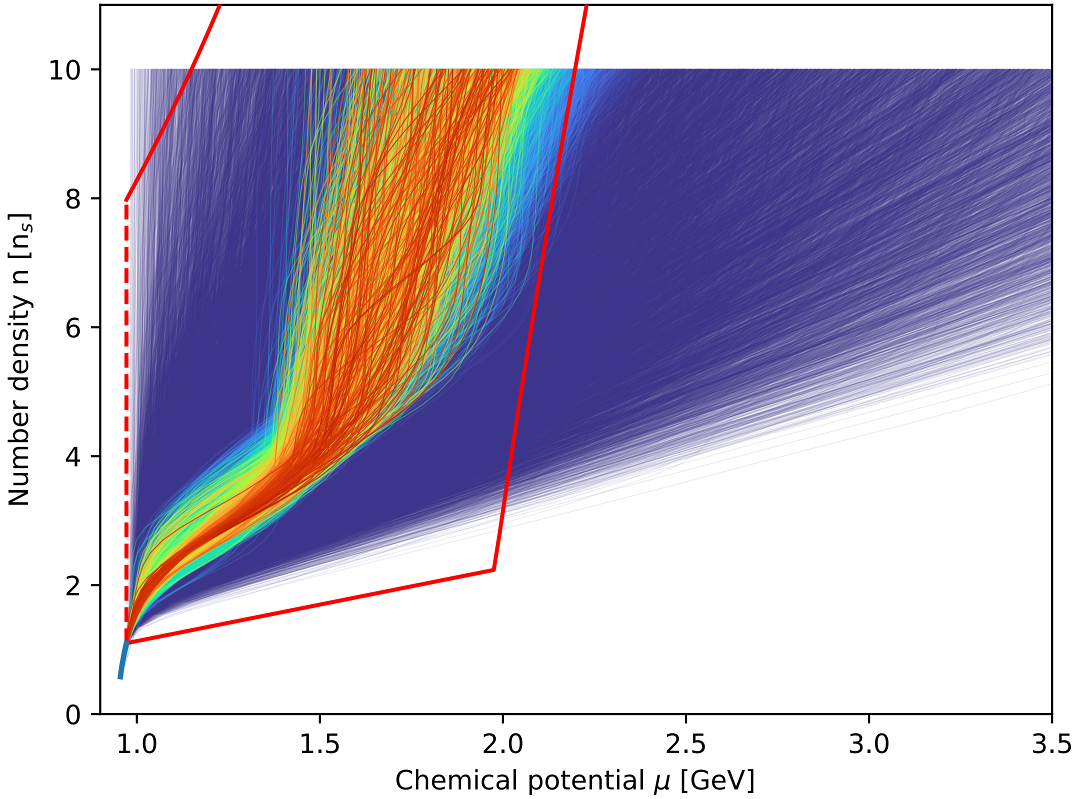
c.f. Fujimoto + 2205.03882 for signatures of such PTs

Density-chemical potential posterior

Astro only



Astro with QCD



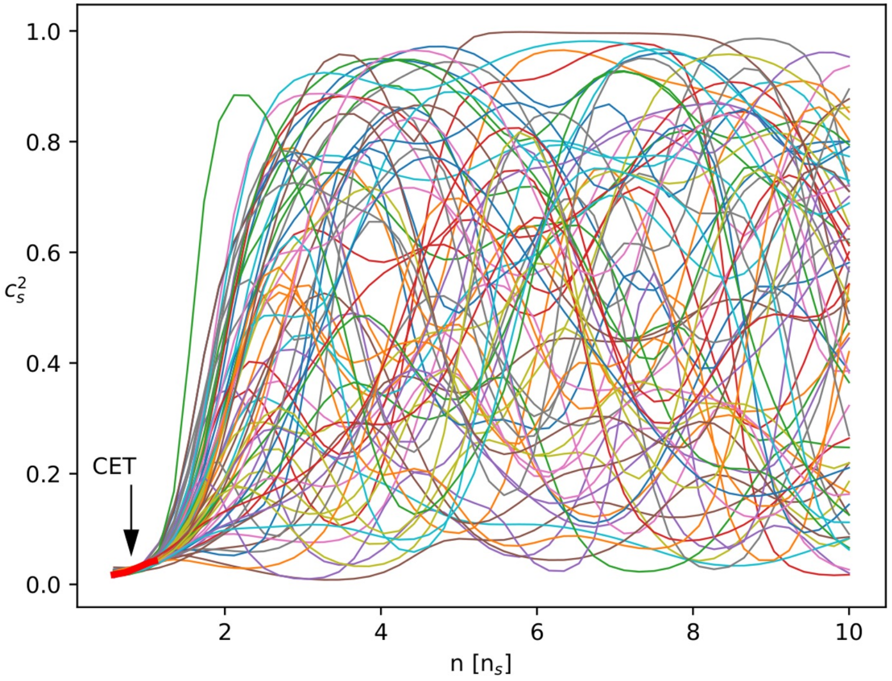
Setup:

$$P(\mathrm{EoS}\,|\,\mathrm{data}) = \frac{P(\mathrm{EoS})\,P(\mathrm{data}\,|\,\mathrm{EoS})}{P(\mathrm{data})}$$

- Gaussian process between 1-10 n_s: $\varphi(n) = -\ln\left(c_s^{-2}(n) - 1\right)$

$$\varphi(n) \sim \mathcal{N}\left(-\ln\left(\bar{c}_s^{-2} - 1\right), K(n,n')\right),\ K(n,n') = \eta e^{-(n-n')^2/2l^2}$$

Similar to Landry & Essick PRD 99 (2019), but for function of n instead of ε



- Hierarchial model:

$$\bar{c}_s^2 \sim \mathcal{N}(0.5, 0.25^2),\ l \sim \mathcal{N}\left(1.0 n_s, (0.25 n_s)^2\right),\ \eta \sim \mathcal{N}(1.25, 0.25^2).$$

- Conditioned to CET at n ~1.1 n_s

+ flat prior for masses of stars

Setup:

$$P(\mathrm{QCD} \mid \mathrm{EoS})$$

1. Scale variation introduces uncertainty:

$$\vec{\beta}_{\mathrm{QCD}}(X) = \{p_{\mathrm{QCD}}(\mu_H, X), n_{\mathrm{QCD}}(\mu_H, X), \mu_H\}, \quad X = \frac{3\bar{\Lambda}}{2\mu_H}$$

2. Scale marginalization: Duhr et al. JHEP 122, (2021)

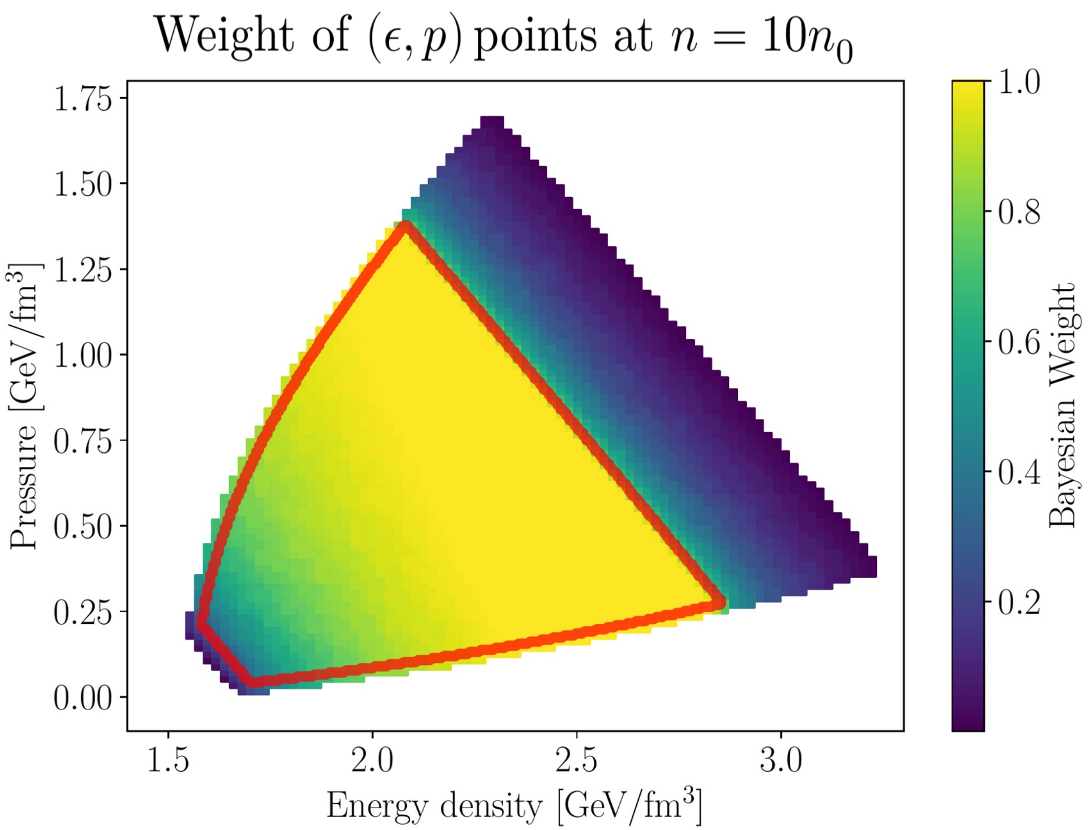
$$P(\vec{\beta}_H) = \int d(\ln X) w(\log X) \delta^{(3)}(\vec{\beta}_H - \vec{\beta}_{\mathrm{QCD}}(X))$$

Log-uniform weight: Cacciari & Houdeau, JHEP 09, (2011)

$$w(\ln X) = 1_{[\ln(1/2), \ln(2)]}(\ln X)$$

3. Compute $\Delta p_{\min}$, $\Delta p_{\max}$ between $10n_s$ and pQCD for each β_H

$$P(\mathrm{QCD} \mid \mathrm{EoS}) = \int d\vec{\beta}_H P(\vec{\beta}_H) 1_{[\Delta p_{\min}, \Delta p_{\max}]}(\Delta p)$$



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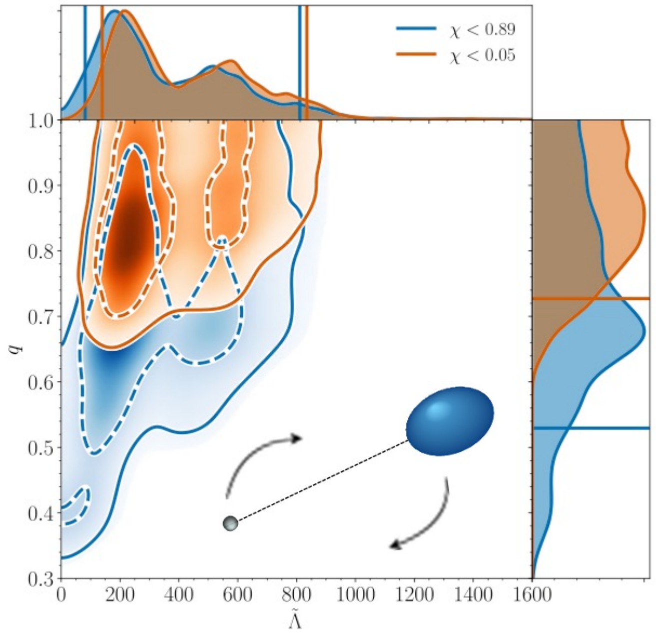
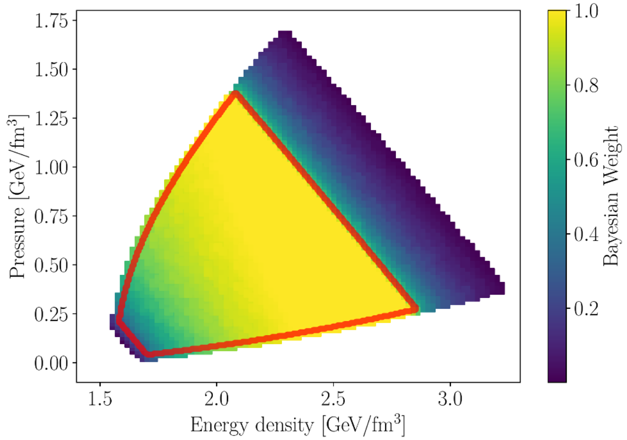
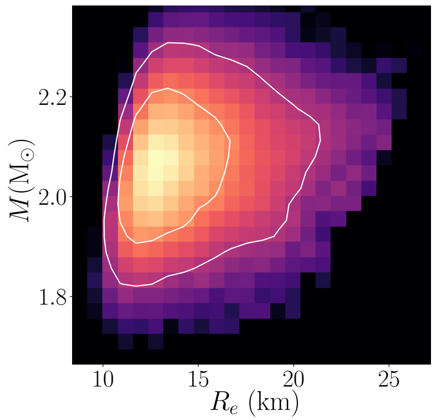


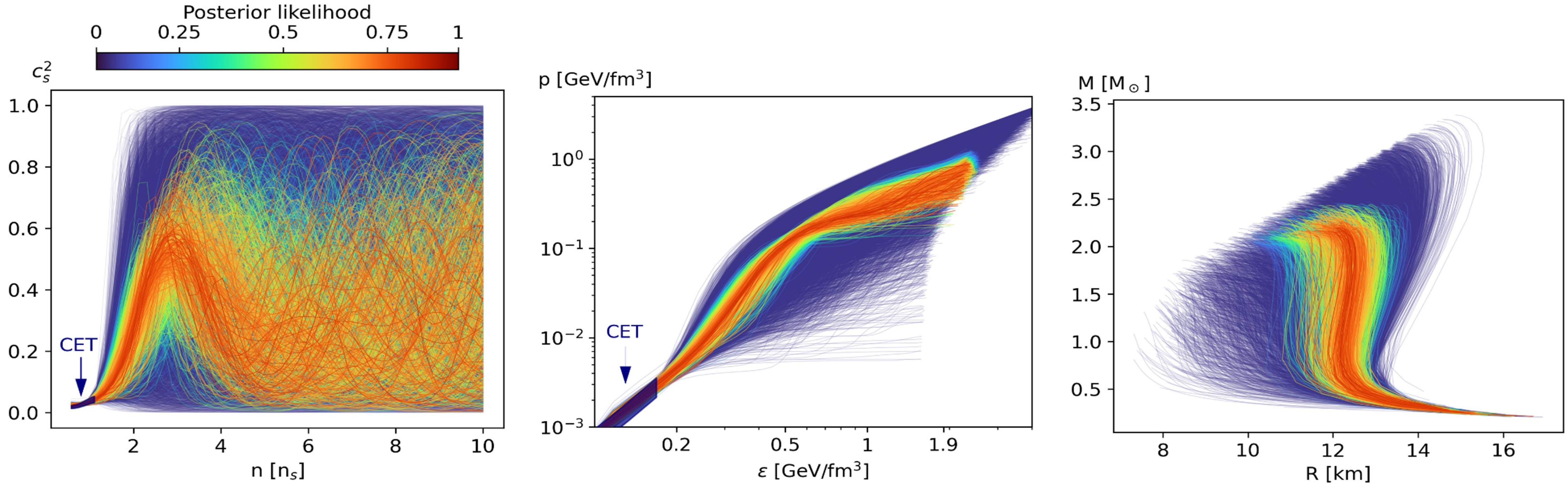
$M_{J1614\text{-}2230}$=1.908(16)
$M_{J048+0432}$ = 2.01(4)
$M_{J0740+6620}$ = 2.14(10)

$$P(\mathrm{data}\,|\,\mathrm{EoS}) = P(\mathrm{QCD}\,|\,\mathrm{EoS})P(\mathrm{Mass}\,|\,\mathrm{EoS})P(\mathrm{NICER}\,|\,\mathrm{EoS})P(\widetilde{\Lambda},\mathrm{BH}\,|\,\mathrm{EoS}).$$



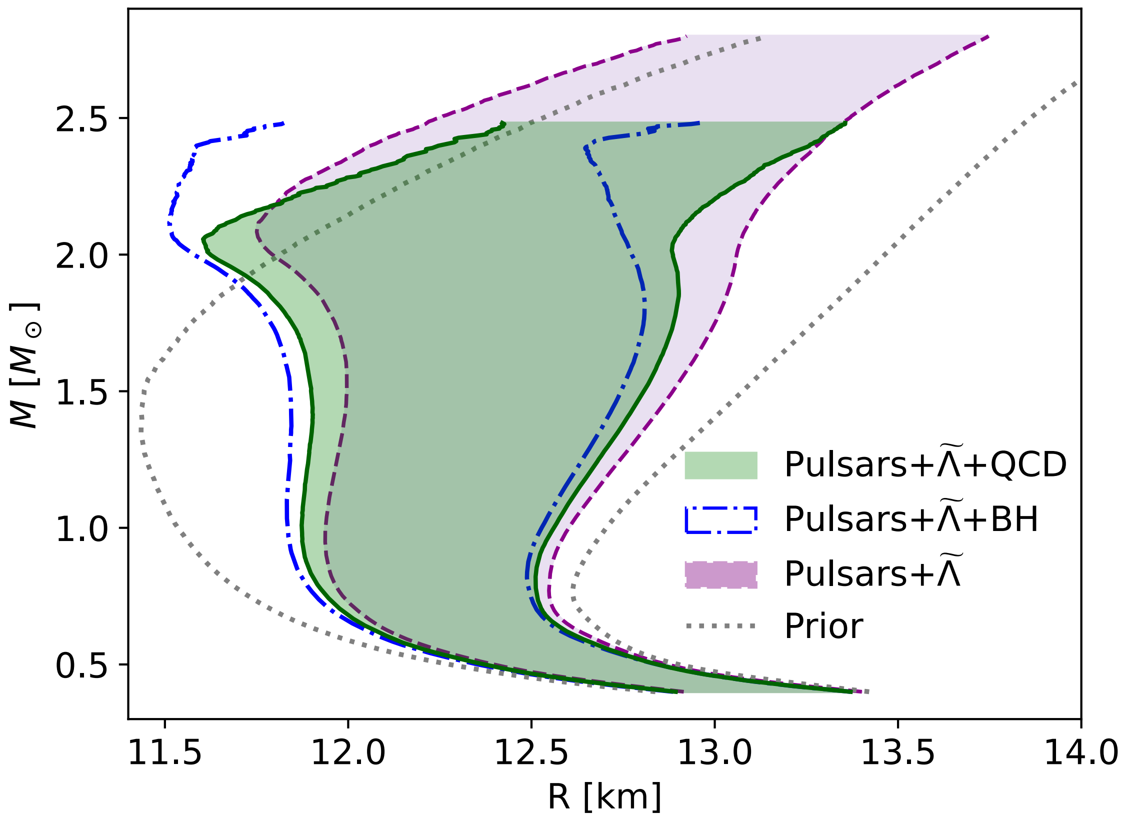


Posterior:

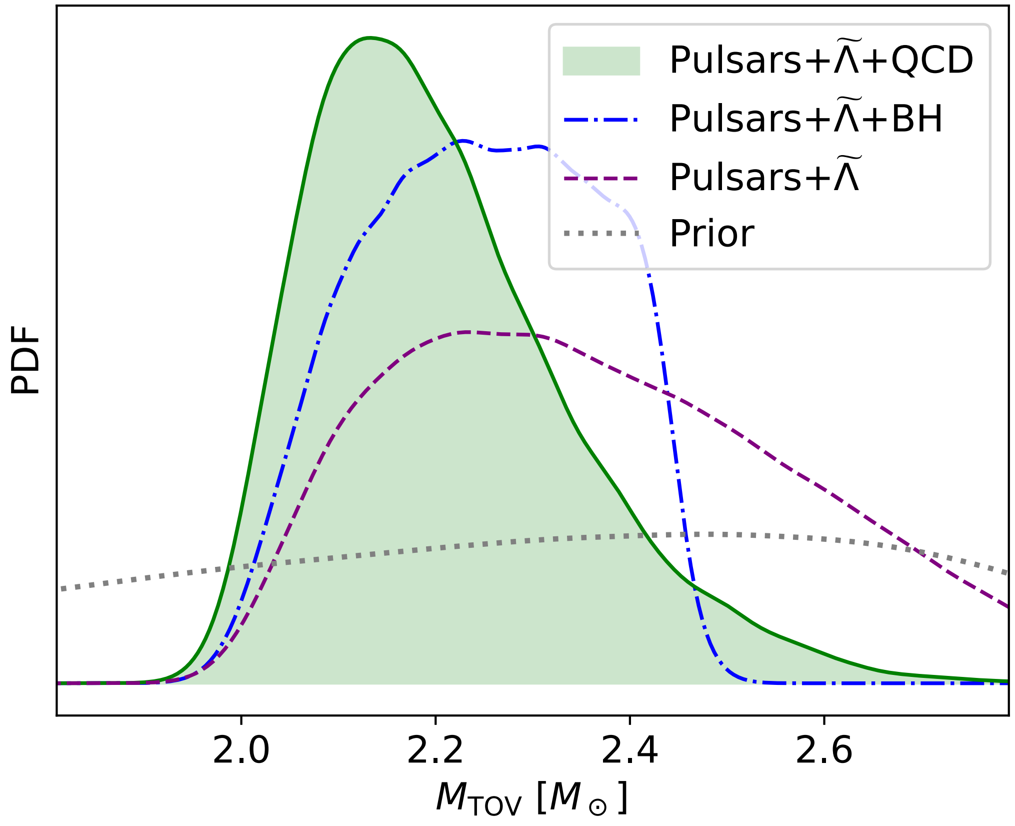


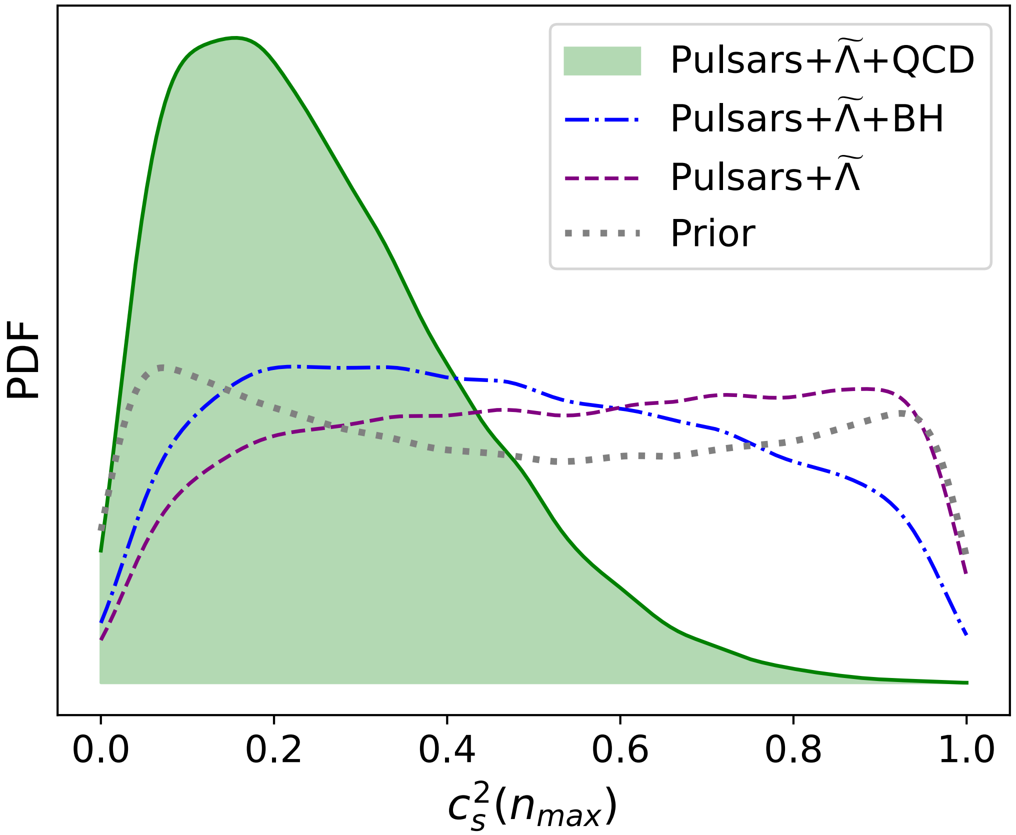
Posterior likelihood
0
0.25
0.5
0.75
1
c_s^2
1.0
0.8
0.6
0.4
0.2
0.0
CET
2
4
6
8
10
n [n_s]
p [GeV/fm^3]
10^0
10^{-1}
10^{-2}
10^{-3}
CET
0.2
0.5
1
1.9
ε [GeV/fm^3]
M [$M_\odot$]
3.5
3.0
2.5
2.0
1.5
1.0
0.5
8
10
12
14
16
R [km]

Mass-radius with QCD

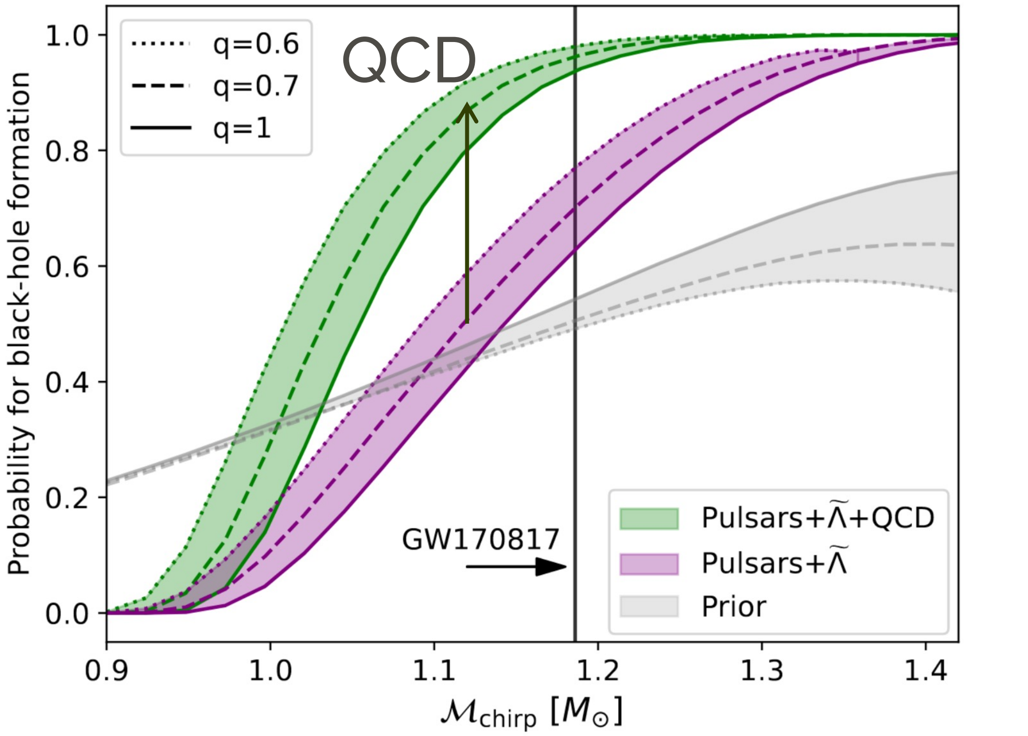


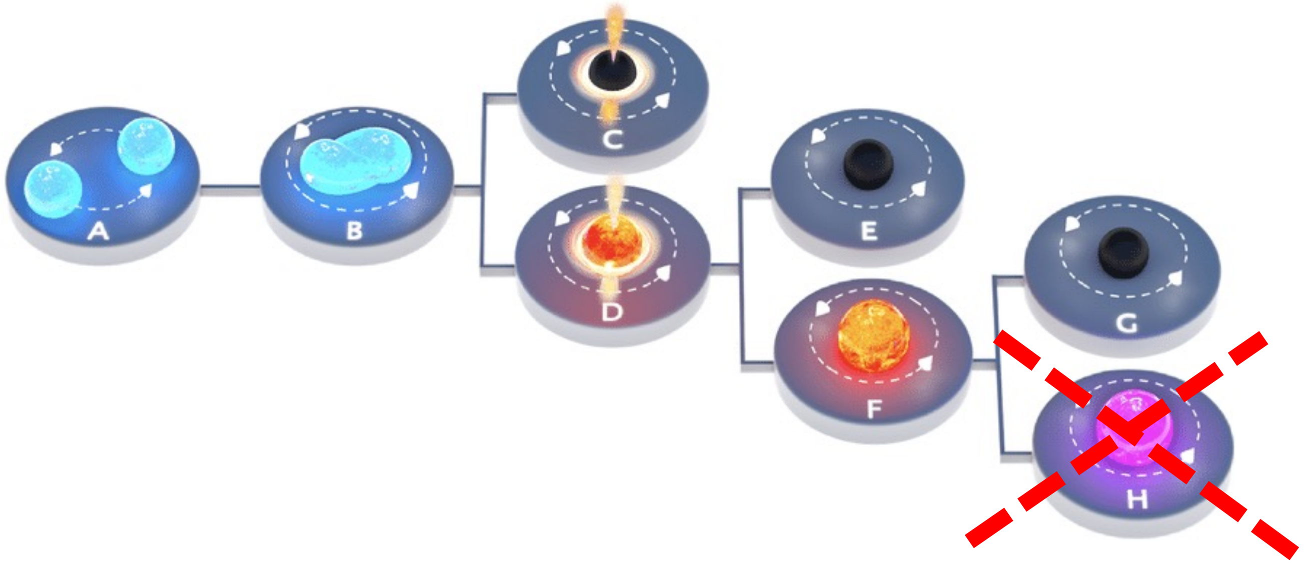
Maximal mass stars





Prediction of QCD:

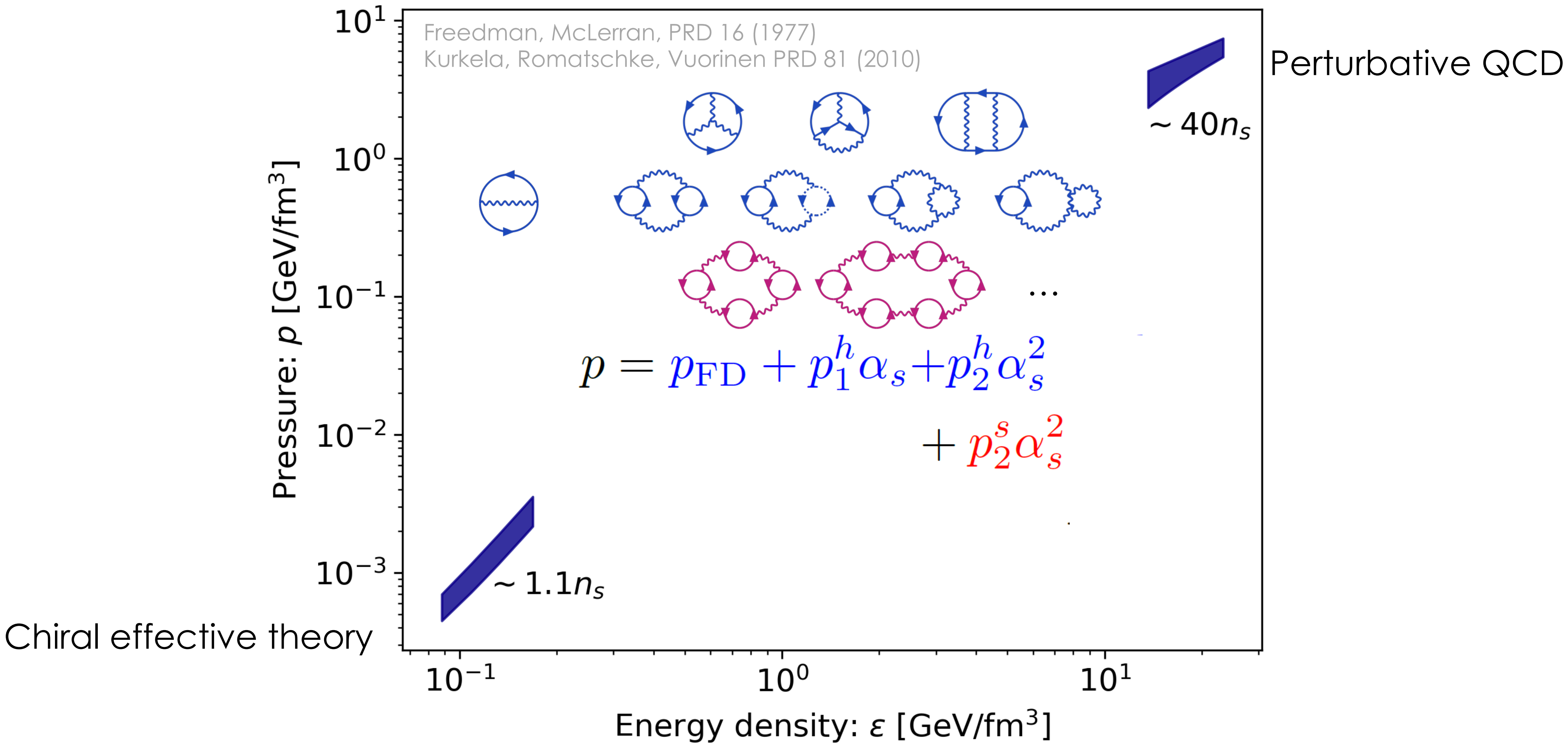




QCD predicts black hole as binary merger product

Consistent with current modelling of the electromagnetic counterpart of GW170817

Neutron star EoS:



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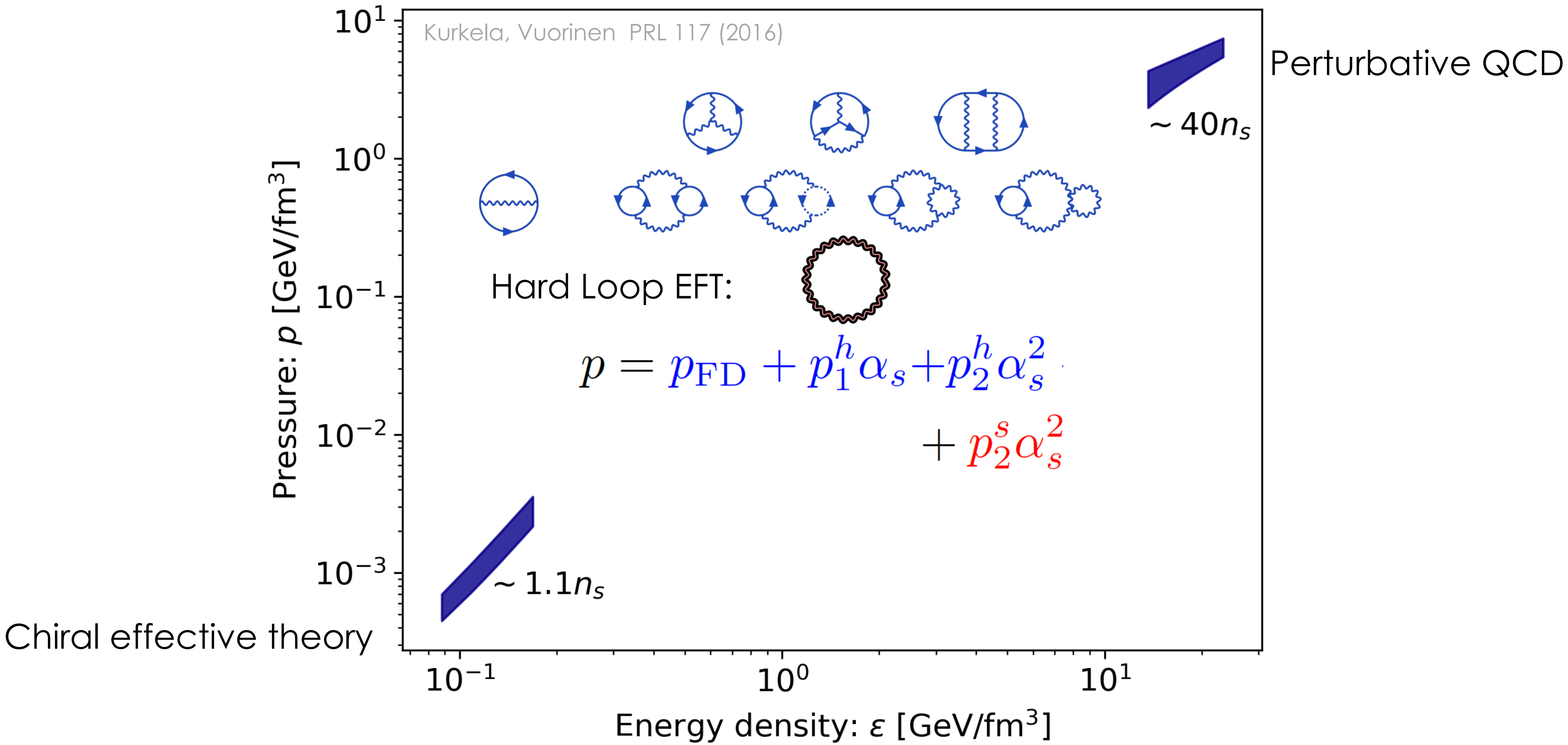
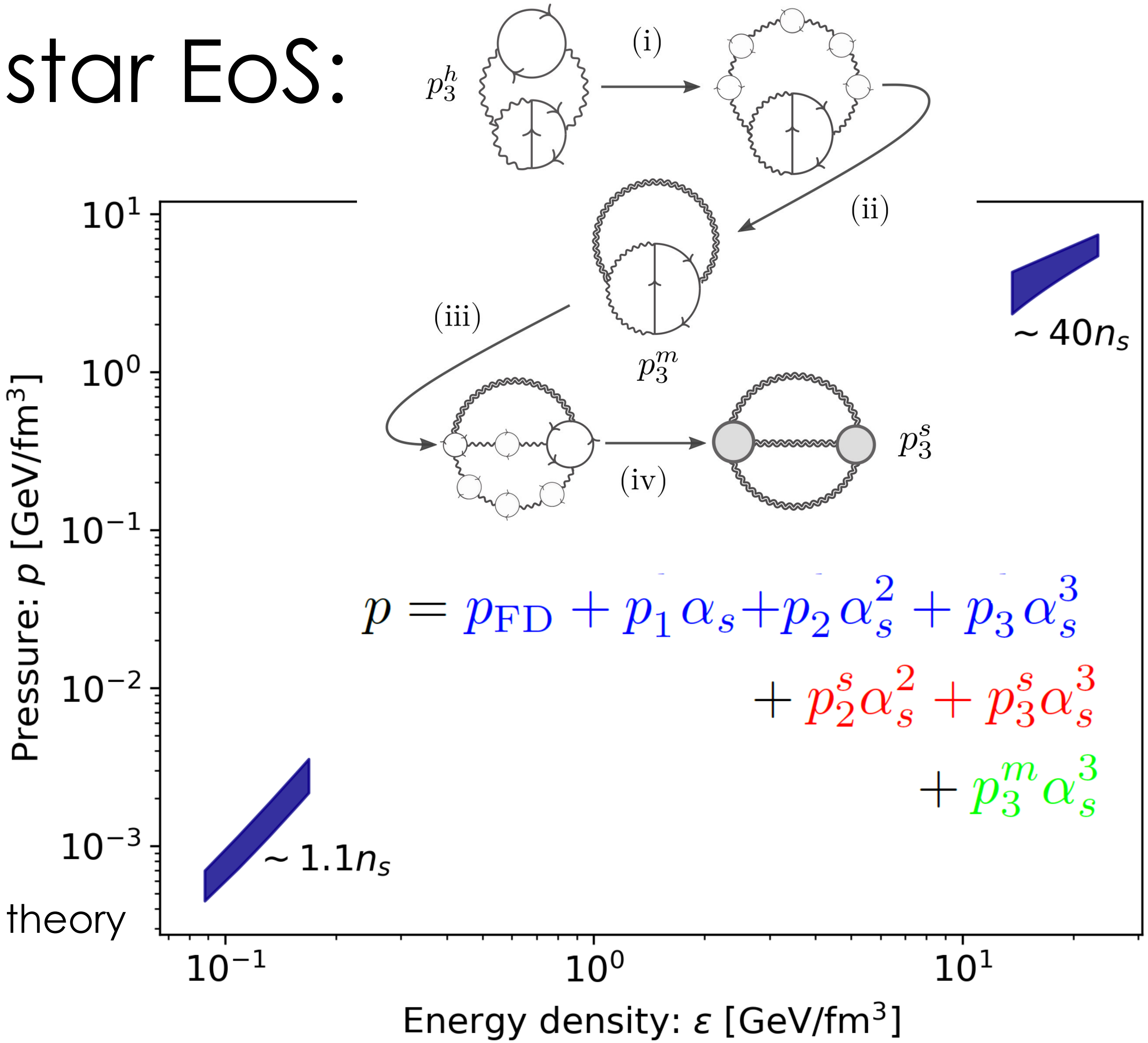


Neutron star EoS:

Gorda et al. PRD 104 (2021)

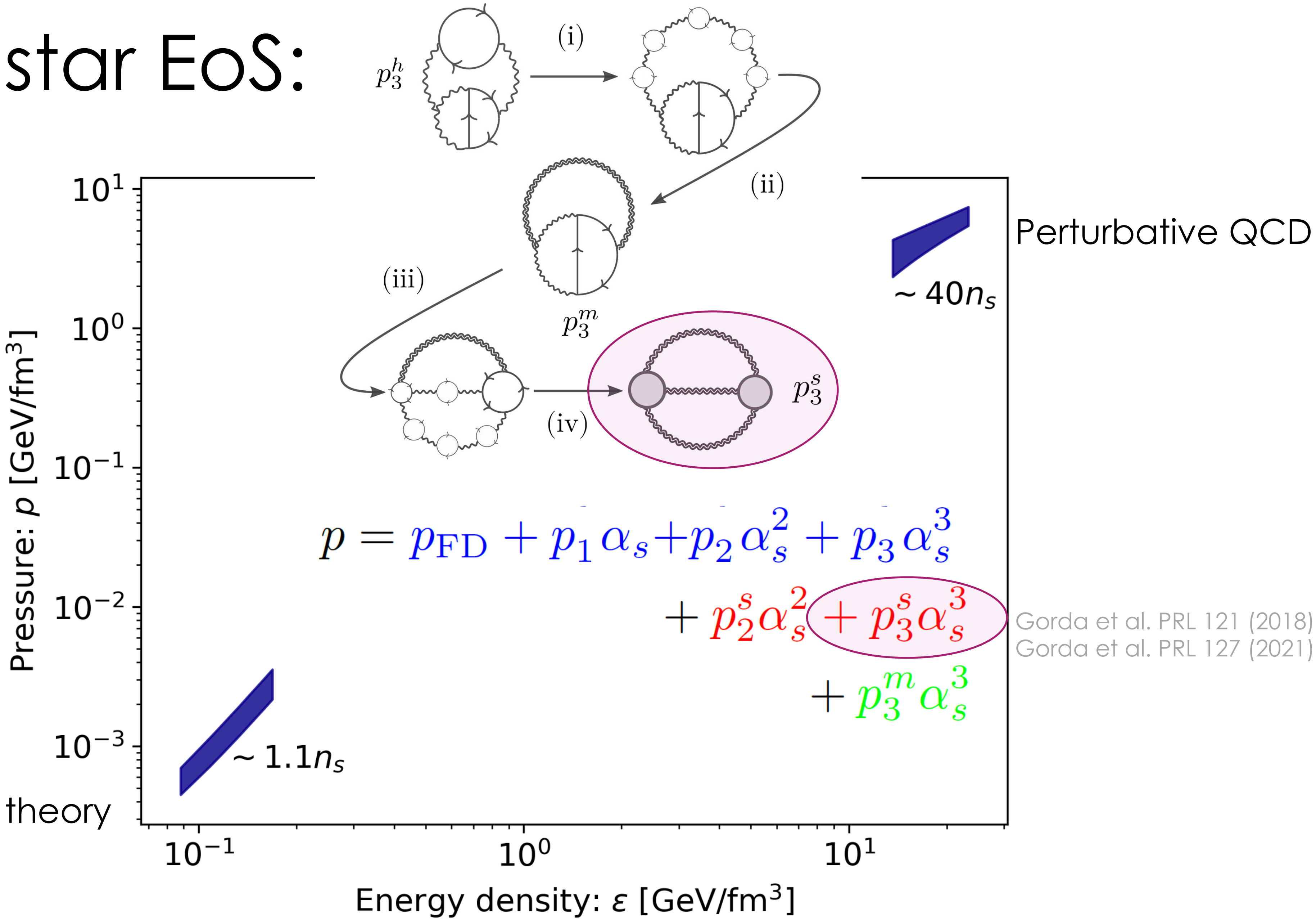


Perturbative QCD

Chiral effective theory

$$p = p_{\rm FD} + p_1 \alpha_s + p_2 \alpha_s^2 + p_3 \alpha_s^3 + p_2^s \alpha_s^2 + p_3^s \alpha_s^3 + p_3^m \alpha_s^3$$

Neutron star EoS:



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Gorda et al. PRL 121 (2018)
Gorda et al. PRL 127 (2021)

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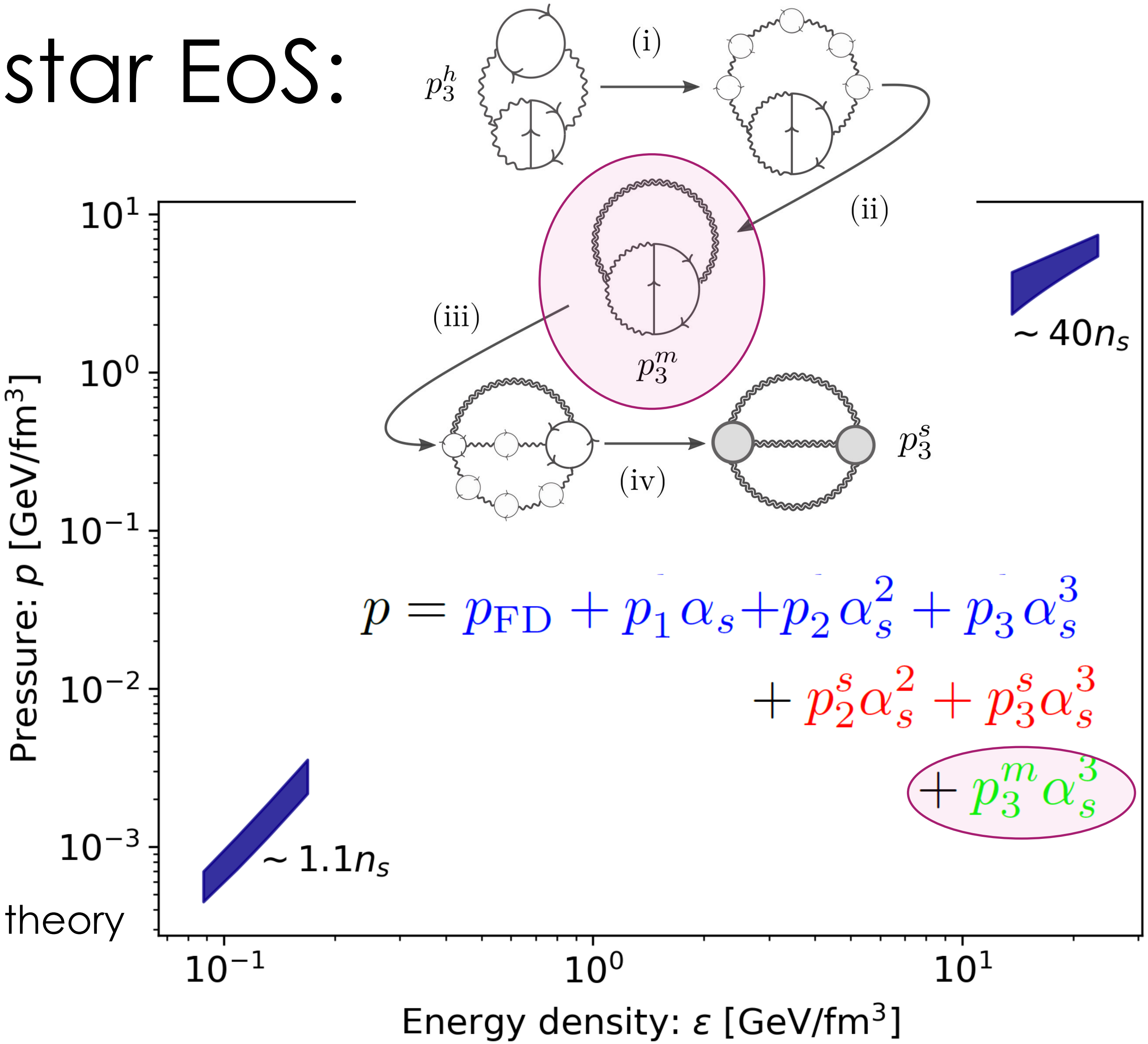
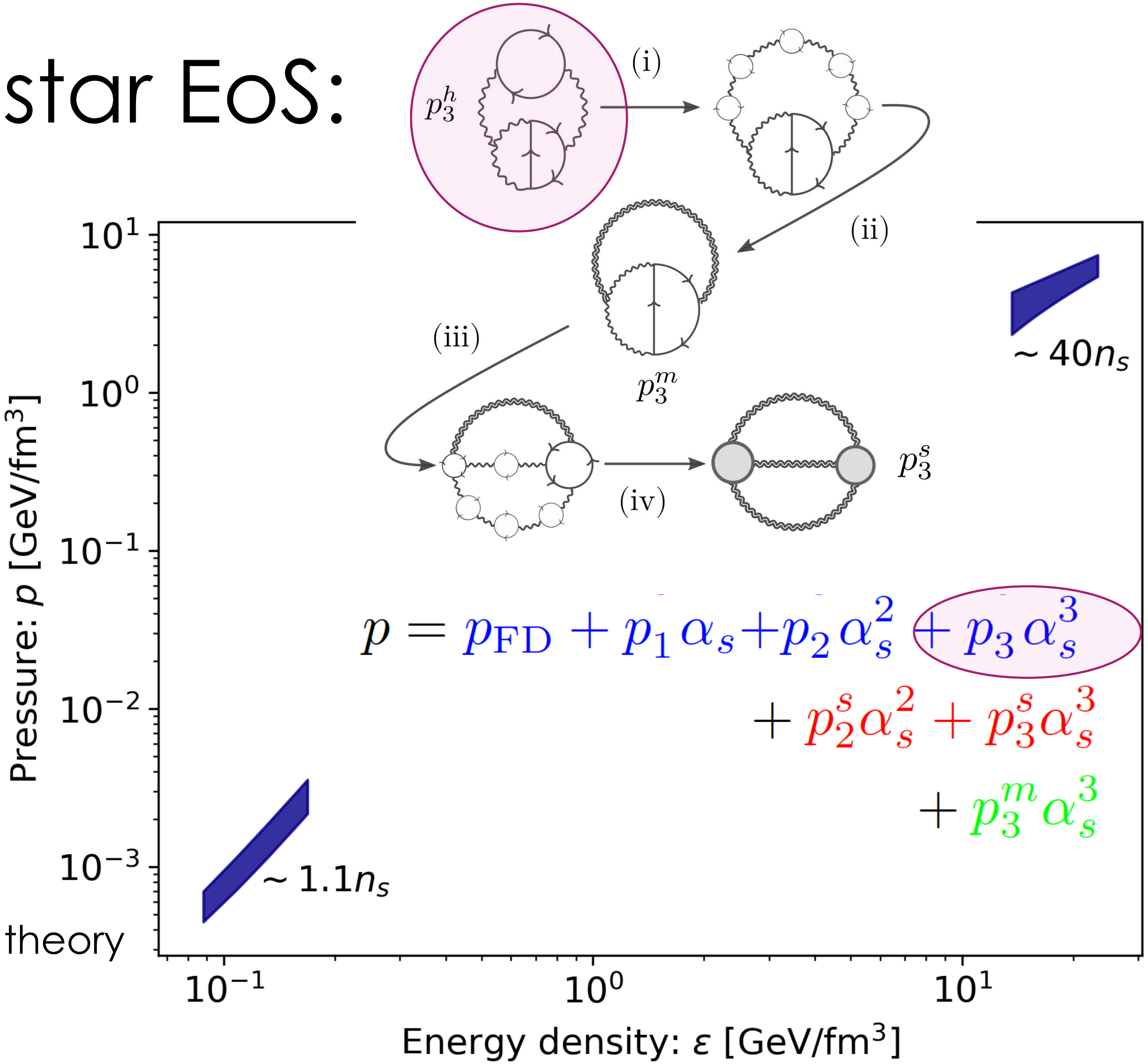


Perturbative QCD

For QED:
Gorda et al. 2204.11893

QCD to come soon...

Neutron star EoS:

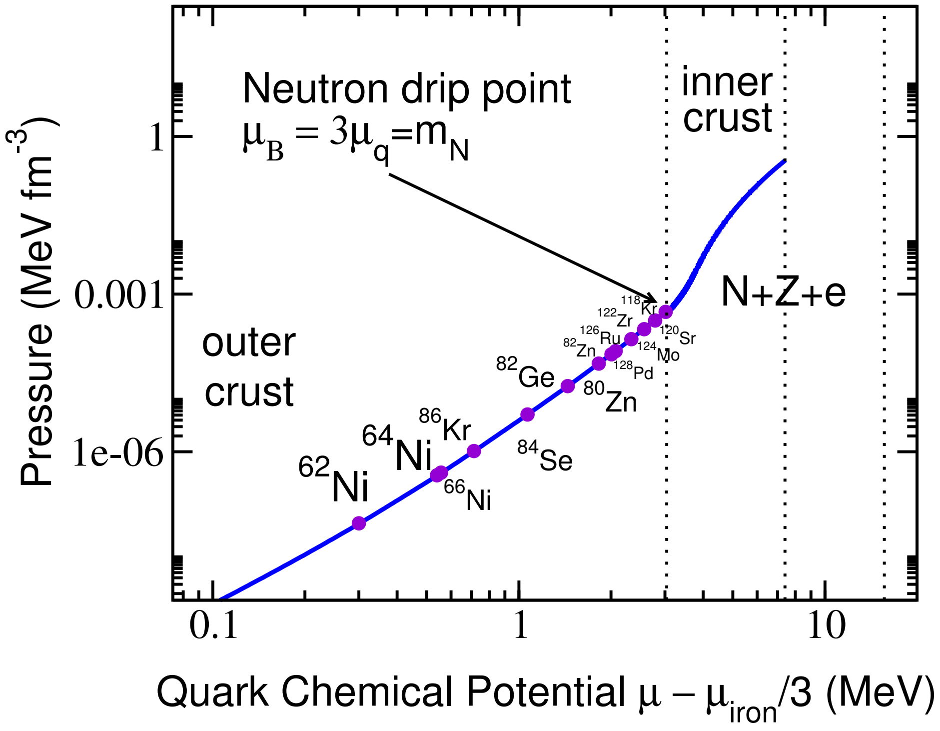


Perturbative QCD

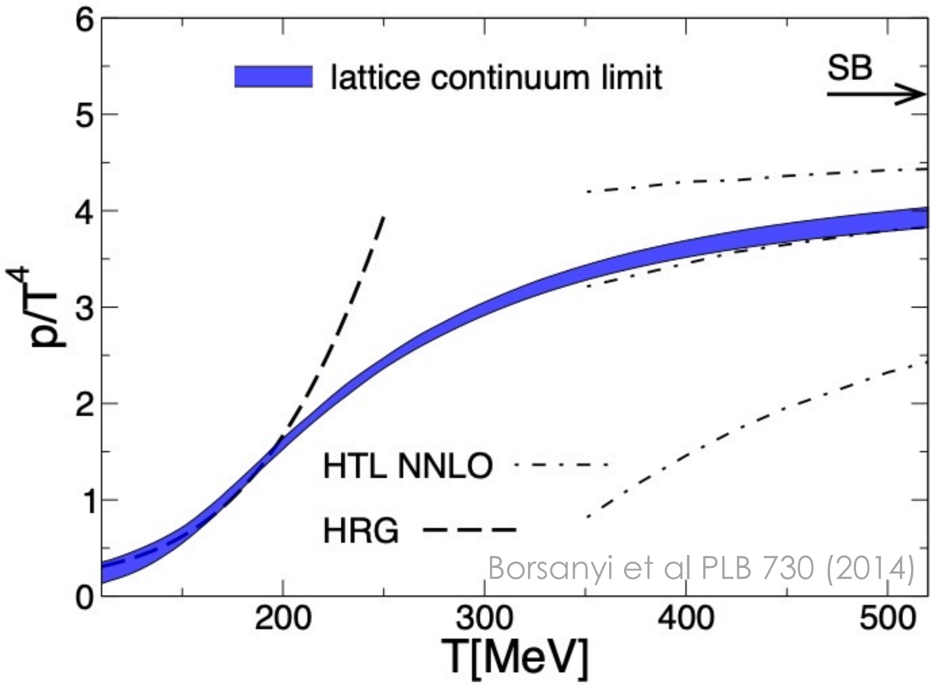
Extensive calculations underway

Chiral effective theory

EoS tells us about the phases of matter

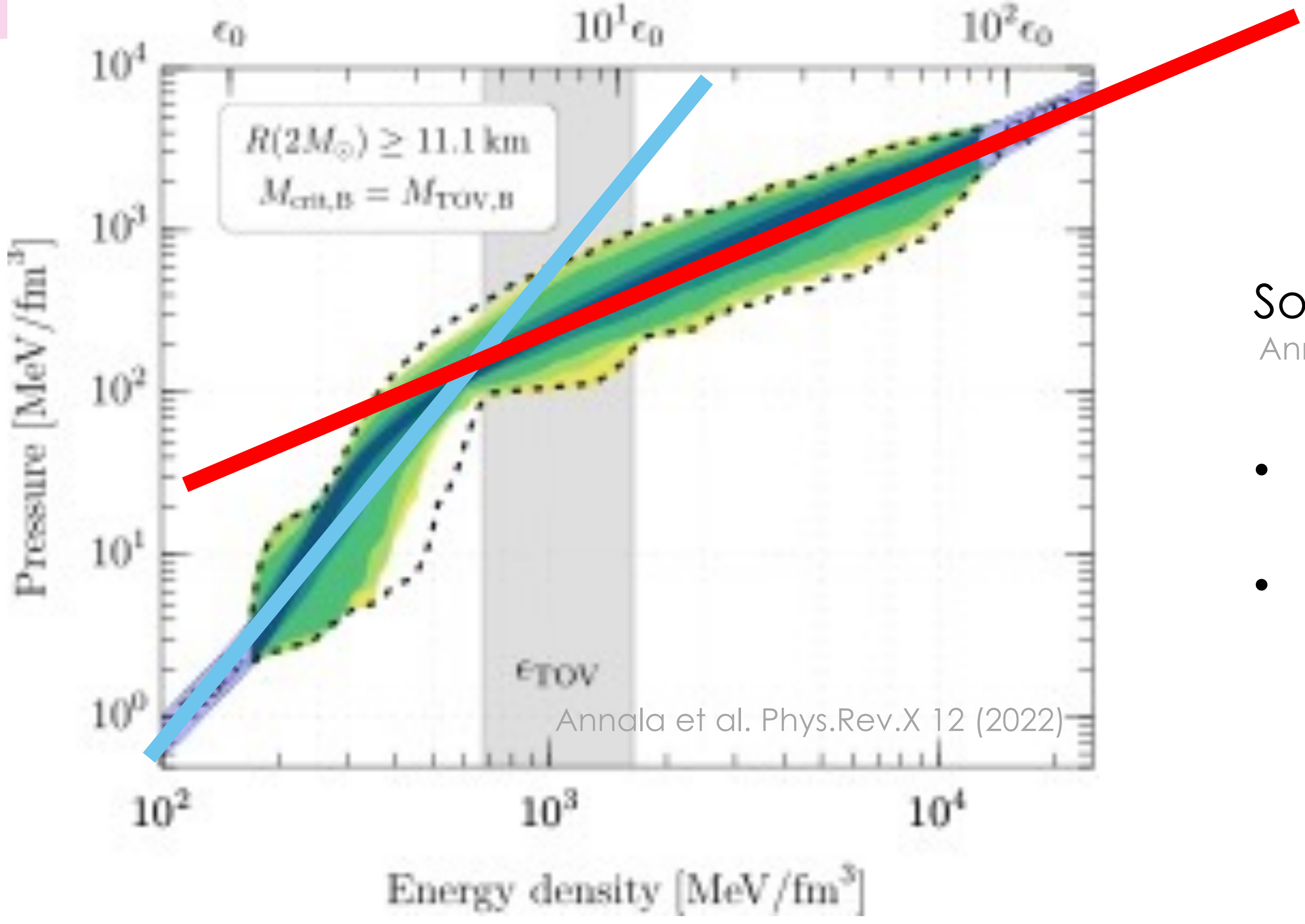


From nuclei to nuclear matter



From hadronic matter to quark gluon plasma

EoS tells us about the phases of matter



From hadronic matter to quark matter

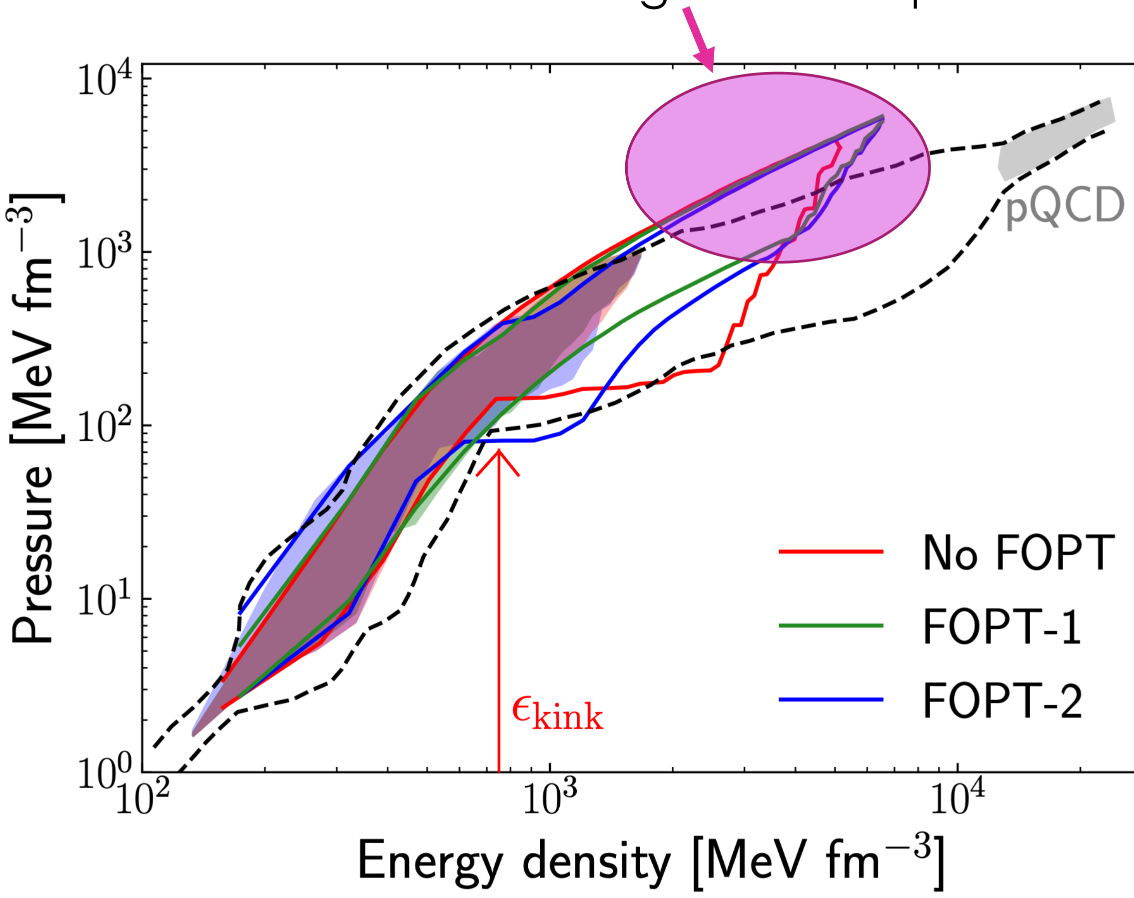
Softening as onset of Quark Matter phase

Annala, Gorda, Kurkela, Nättilä, Vuorinen, *Nature Phys.* 16 (2020)

- Is the softening feature there?
- If yes, are cores of neutron stars in QM phase?

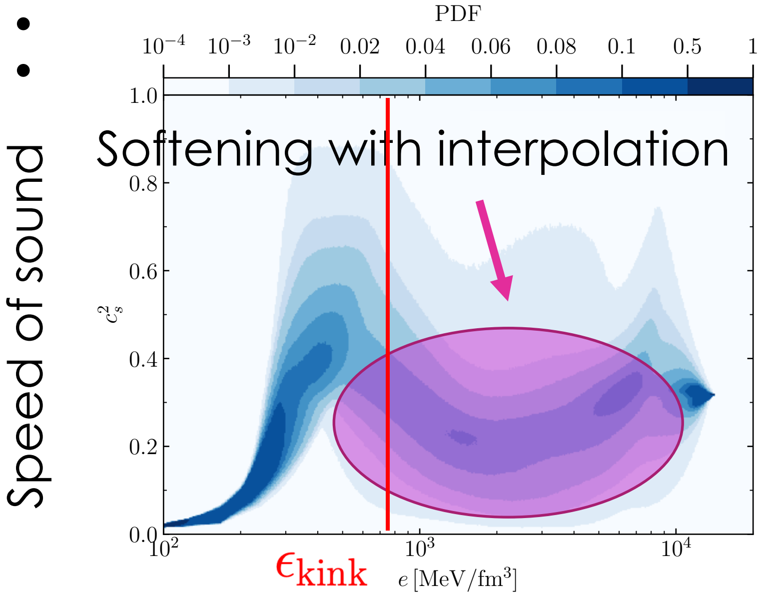
Interpolation vs. extrapolation:

No softening with extrapolation

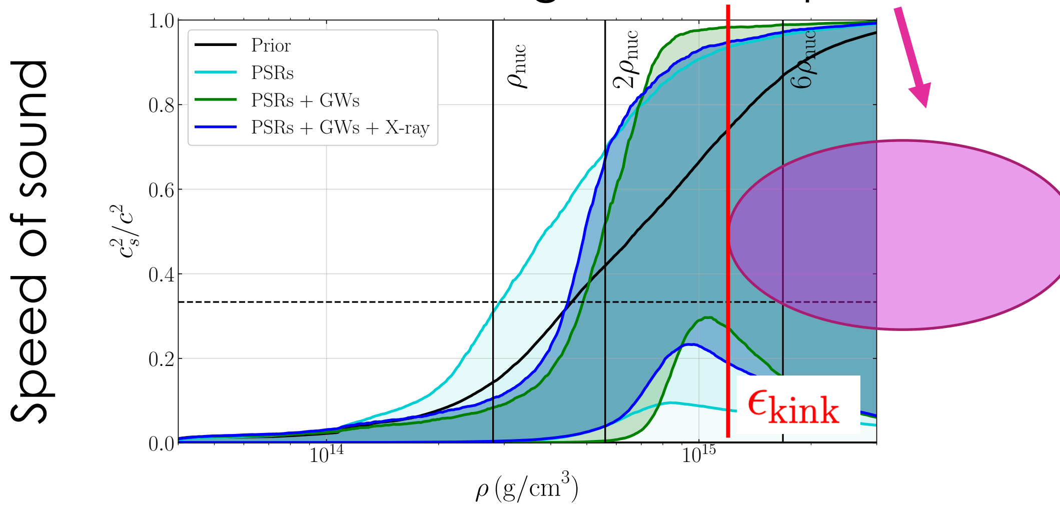


Somasundaram, Tews, Margueron 2112.08157

Altiparmak, Ecker, Rezzolla 2112.08157

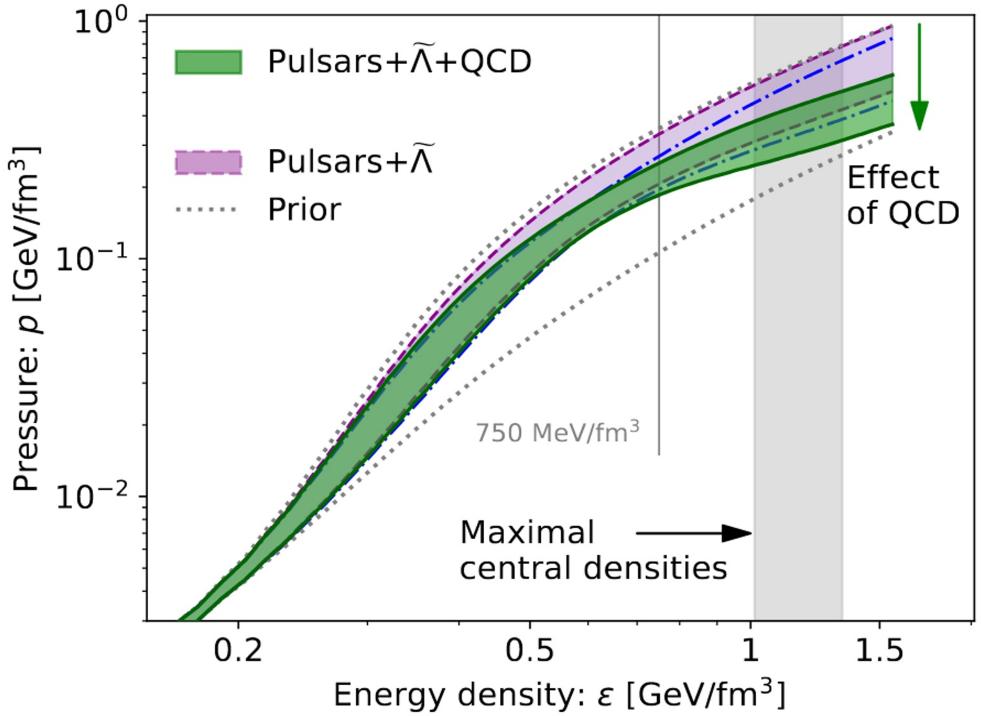


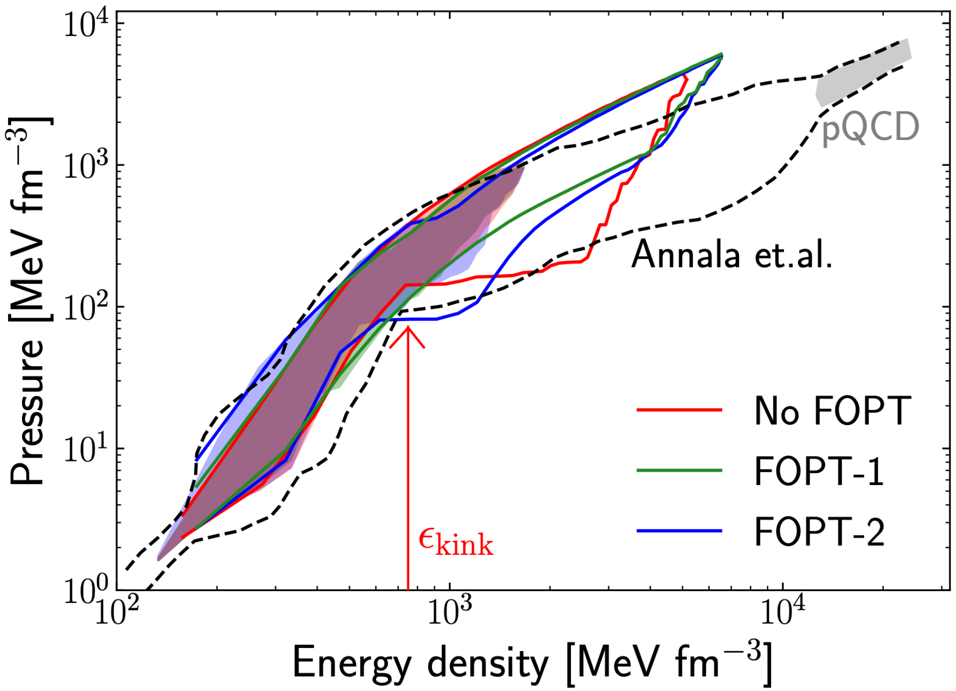
No softening with extrapolation



Landry, Essick, Chatziioannou PRD 101 (2020)

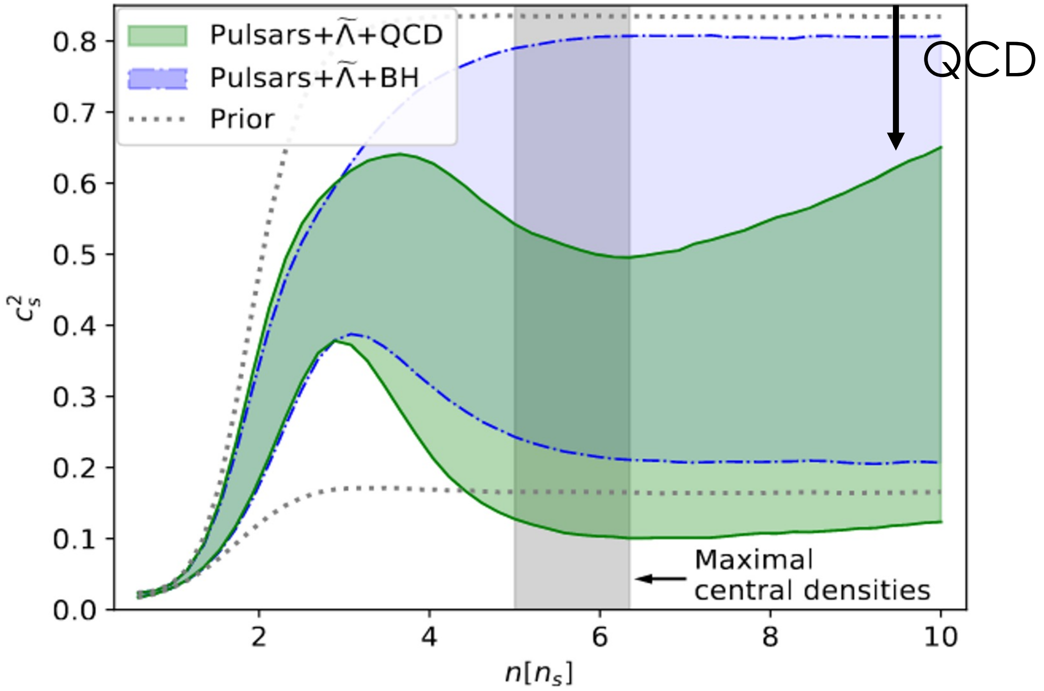
Effect of QCD:

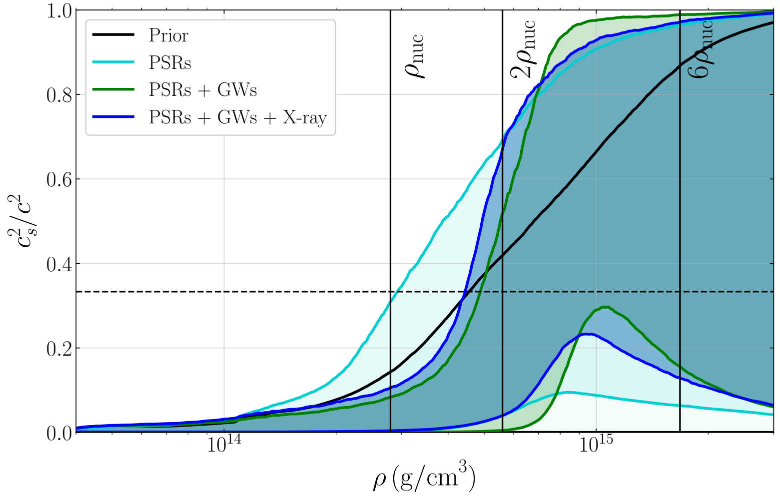


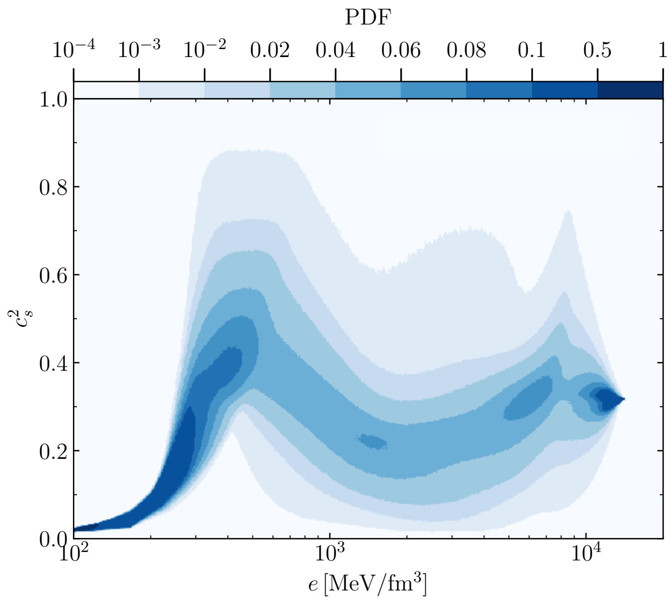


Softening caused by QCD, not by interpolation

Effect of QCD:

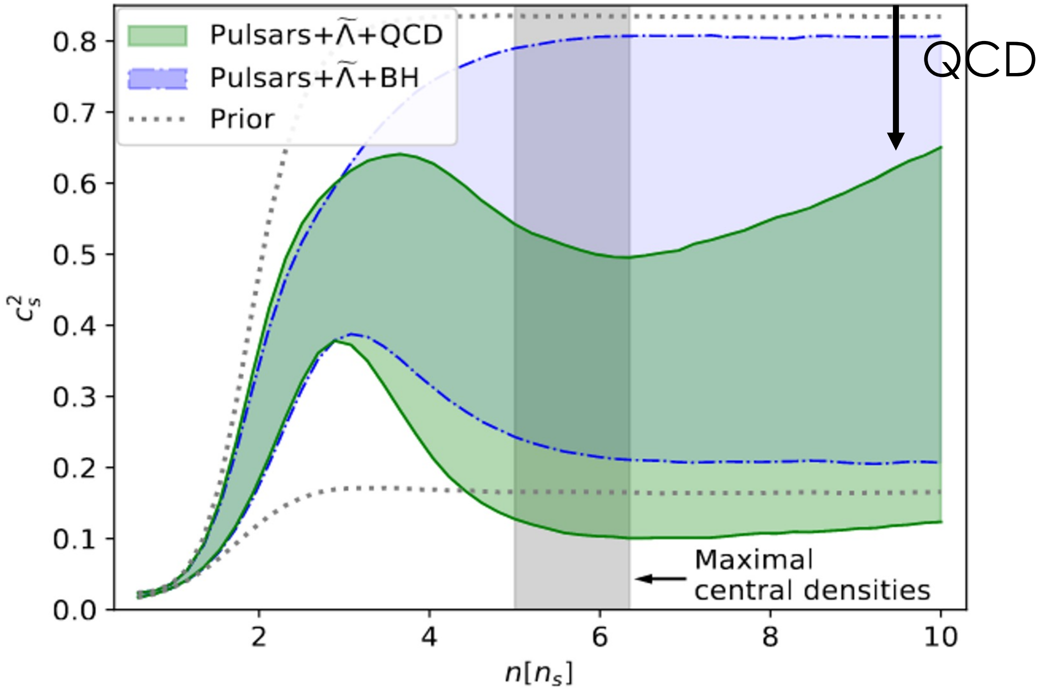




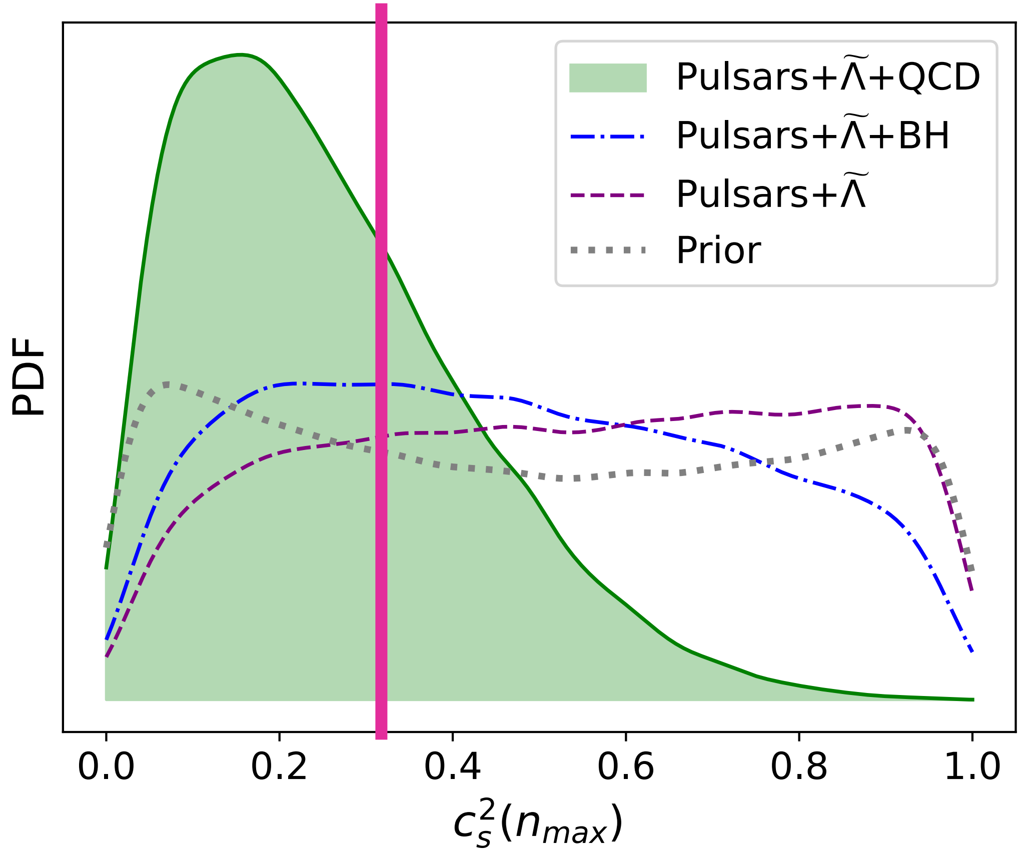


Softening caused by QCD, not by interpolation

Effect of QCD:



Quark matter



Softening caused by QCD, not by interpolation