

Femtoscscopy of the origin of proton's mass

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Origin of the Visible Universe: Unraveling the Proton Mass
INT Seattle - June 13 - 17 2022

Outline

1. Motivation

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2. Trace anomaly, proton mass

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3. Quarkonium-nucleon scattering

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6. Conclusions & Perspectives

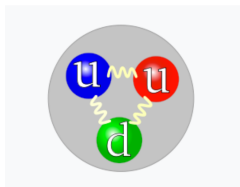
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Work with Thiago C. Peixoto, Few Body Syst. 61, 4 (2020)

1. Motivation

What is the origin of the mass of protons and neutrons (nucleons)?



Computers gave an answer to the question



Nucleon mass comes from
the quarks and gluons

Light-hadron masses

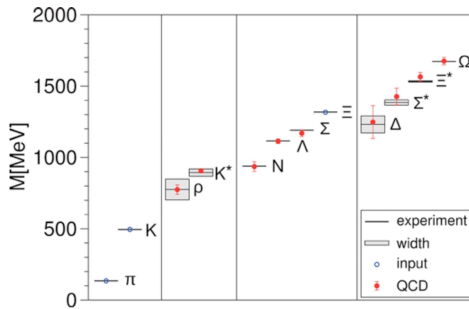
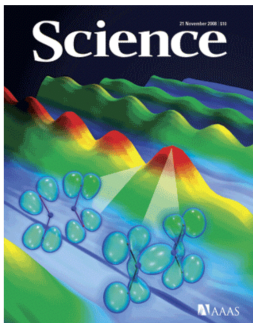
Science

2008

Ab Initio Determination of Light Hadron Masses

S. Dürer, Z. Fodor, J. Frison, C. Hoelbling, R. Hoffmann, S. D. Katz, S. Krieg, T. Kurth, L. Lellouch, T. Lippert, K. K. Szabo and G. Vulvert

Science **322** (5905), 1224-1227.
DOI: 10.1126/science.1163233



Hadron-mass differences

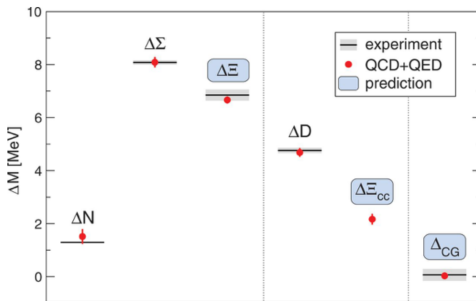
Science

2015

Ab initio calculation of the neutron-proton mass difference

Sz. Borsanyi, S. Durr, Z. Fodor, C. Hoelbling, S. D. Katz, S. Krieg, L. Lellouch, T. Lippert, A. Portelli, K. K. Szabo and B. C. Toth

Science **347** (6229), 1452-1455.
DOI: 10.1126/science.1257050



Nucleon weak axial charge

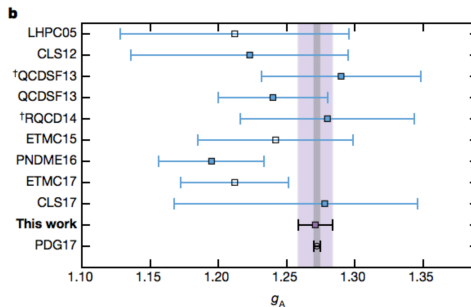


2018

A per-cent-level determination of the nucleon axial coupling from quantum chromodynamics

C. C. Chang, A. N. Nicholson, E. Rinaldi, E. Berkowitz, N. Garron, D. A. Brantley, H. Monge-Camacho, C. J. Monahan, C. Bouchard, M. A. Clark, B. Joó, T. Kurth, K. Orginos, P. Vranas & A. Walker-Loud

Nature 558, 91–94 (2018)



Yet, we are not satisfied

We want to know more:

How did it happen?*

*F. Wilczek, *The lightness of being: Mass, ether, and the unification of forces*
(Basic Books, 2008)

2. Nucleon mass – trace anomaly

$$m_N = \frac{\beta(g)}{2g} \langle N | G_{\mu\nu}^a G^{a\mu\nu} | N \rangle + \sum_{l=u,d,s} \langle N | m_l (1 + \gamma_{m_l}) \bar{q}_l q_l | N \rangle$$

\Downarrow $\simeq 760 \text{ MeV}$ \Downarrow $\simeq 80 \text{ MeV}$ (σ -term)

$$\beta(g) \simeq -b \frac{g^3}{16\pi^2}, \quad b = 11 - \frac{2n_l}{3} \quad (\text{heavy quarks integrated out})$$

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Experimental access to $\langle N | G_{\mu\nu}^a G^{a\mu\nu} | N \rangle$?

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Experimental access to $\langle N | G_{\mu\nu}^a G^{a\mu\nu} | N \rangle$?

Inequality (almost saturated, chromomagnetic part suppressed)*:

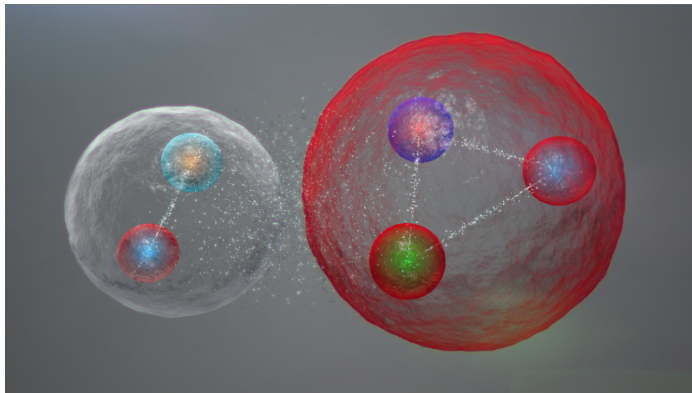
$$-\frac{1}{2} \langle N | g^2 G_{\mu\nu}^a(x) G^{a\mu\nu}(x) | N \rangle = \langle N | [(g\mathbf{E}^a)^2 - (g\mathbf{B}^a)^2] | N \rangle \leq \langle N | (g\mathbf{E}^a)^2 | N \rangle$$

Shifman, Vainshtein & Zakharov (1978)

*Sibirtsev & Voloshin (2005)

3. Quarkonium-nucleon scattering

Experimental access to $\langle N|(gE^a)^2|N\rangle$
(small QN relative momentum)



Quarkonium: $\underbrace{\phi(s\bar{s})}_{\text{light}}, \underbrace{\eta_c(c\bar{c}), J/\psi(c\bar{c}), \eta_b(b\bar{b}), \Upsilon(b\bar{b})}_{\text{heavy}}$

Heavy quarkonium - nucleon (QN)

Low QN momentum interaction

- Heavy quarkonium: small object, radius r_Q
- Interacts by exchanging gluons with nucleon's light quarks
- Low relative momentum, gluon wavelength $\lambda_g \sim r_N$ (nucleon radius)
- $r_Q \ll r_N$: quarkonium small dipole in soft gluon fields
- QCD multipole expansion (\sim OPE)

Peskin (1978), Bhanot & Peskin (1978), Voloshin (1979), Novikov & Shifman (1981), Kaidalov & Volkovitsky (1992), Luke, Manohar & Savage (1992)

QN scattering amplitude

QCD multipole expansion

$$\begin{aligned} f_{QN}(\mathbf{p}, \mathbf{p}') &= \frac{\mu_{QN}}{2\pi} \frac{1}{2} \left[\frac{2T_F}{3N_c} \langle \varphi_Q | \mathbf{r} \frac{1}{E_b + H_{\text{octet}}} \mathbf{r} | \varphi_Q \rangle \right] \langle N(\mathbf{p}) | (g\mathbf{E}^a)^2 | N(\mathbf{p}) \rangle \\ &= \frac{\mu_{QN}}{2\pi} \frac{1}{2} \alpha_Q \langle N(\mathbf{p}) | (g\mathbf{E}^a)^2 | N(\mathbf{p}) \rangle \end{aligned}$$

- μ_{QN} reduced mass, \mathbf{p}, \mathbf{p}' relative c.m. momenta
- α_Q quarkonium color polarizability
- $T_F = 1/2$, $N_c = 3$
- **Note that:** forward ($\mathbf{p}' = \mathbf{p}$) amplitude enters here

Kaidalov & Volkovitsky (1992), Kharzeev (1996), Sibirtsev & Voloshin (2005)

Experimental access to $\langle N|(g\mathbf{E}^a)^2|N\rangle$

—Will focus on $Q = J/\psi$

Lattice QCD simulations and models point toward a
weakly attractive, S -wave dominated

$J/\psi N$ interaction

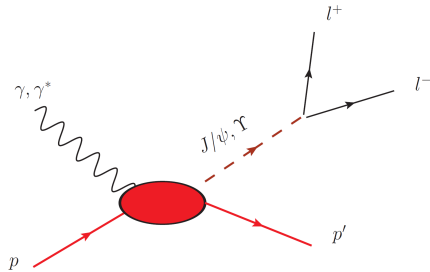
↓ small relative $J/\psi N$ momenta: $f_{\text{forw.}} \simeq -a_{J/\psi N}$

$$a_{J/\psi N} = -\frac{\mu_{J/\psi N}}{2\pi} \frac{1}{2} \alpha_{J/\psi} \langle N|(g\mathbf{E}^a)^2|N\rangle$$

Need to measure $a_{J/\psi N}$

(But to obtain $\langle N|(g\mathbf{E}^a)^2|N\rangle$ need to know $\alpha_{J/\psi}$)

Electro- and photoproduction @ JLab, EIC, EicC



Analyses of recent Glue-X experiment*

- Extracted very small values of scattering length
 $0.003 \text{ fm} \leq |a_{J/\psi N}| \leq 0.025 \text{ fm}$
100 times smaller than some of earlier theoretical estimates
- **Issues:**
No forward scattering, $t_{\text{thr.}} \simeq 1.5 \text{ GeV}^2$
Vector meson dominance problematic, not enough time for J/ψ to be formed

* I.I. Strakovsky, D. Epifanov, and L. Pentchev, PRD 101, 042201 (2020)

L. Pentchev and I.I. Strakovsky, Eur. Phys. J. A 57, 56 (2021)

4. J/ψ -nucleon @ LHC - Femtoscopy

Production yields and leptonic decay rates relatively high
Knowledge from lattice QCD and phenomenological models

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Knowledge from lattice QCD and phenomenological models

Lattice:

- $J/\psi N$ interaction: attractive, not very strong
- Used quenched confs. or large quark masses, need extrapolation to physical masses
- Extrapolation: use effective field theory (EFT) - QNEFT*
- QNEFT degrees of freedom: $J/\psi, N = (p, n), \pi$

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Models:

- Phenomenological spherical well, simple but insightful
- QCD multipole expansion + chiral soliton model (χ CSQM)

*J. T. Castellà & GK, Phys. Rev. D 98, 014029 (2018)

Femtoscscopy: basics

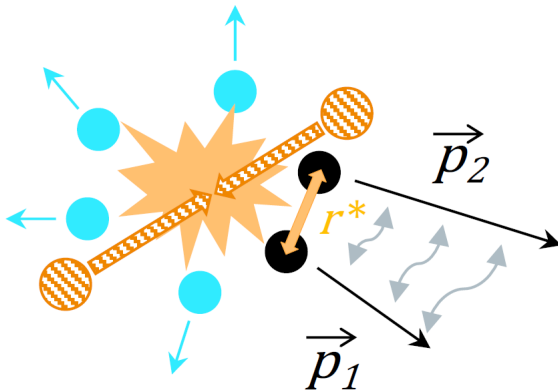


Figure from:
Unveiling the strong interaction among hadrons at the LHC
ALICE Coll., Nature 588, 232 (2020)

Momentum correlation function

Experimental extraction

— $\mathbf{p}_1, \mathbf{p}_2$: measured hadron momenta m_1, m_2 : hadron masses

$$\mathbf{P} = \mathbf{p}_1 + \mathbf{p}_2, \quad \mathbf{k} = \frac{m_2 \mathbf{p}_1 - m_1 \mathbf{p}_2}{m_1 + m_2} : \text{c.m. and relative momenta}$$

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— Pair's c.m. frame: $\mathbf{P} = 0 \rightarrow \mathbf{p}_1 = -\mathbf{p}_2 \Rightarrow \mathbf{k} = \mathbf{p}_1 = -\mathbf{p}_2$

$$C(k) = \frac{A(k)}{B(k)} \left\{ \begin{array}{l} A(k) : \text{yield from same event (coincidence yield)} \\ B(k) : \text{yield from different events (background)} \end{array} \right.$$

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- Corrections: nonfemtoscopic correlations, momentum resolution, etc $\leftarrow \xi(k)$

$$C(k) = \xi(k) \frac{A(k)}{B(k)}$$

Correlation function

Theoretical interpretation

- Kooning-Pratt formula

$$C(k) = \xi(k) \frac{A(k)}{B(k)} = \int d^3r S_{12}(\mathbf{r}) |\psi(\mathbf{k}, \mathbf{r})|^2$$

$S_{12}(\mathbf{r})$: source, pair's relative distance distribution function (in pair's frame)

$\psi(\mathbf{k}, \mathbf{r})$: pair's relative wave function

- One needs here $\psi(\mathbf{k}, \mathbf{r})$ for $0 \leq r \leq \infty$, not asymptotic as in scattering
- $\psi(\mathbf{k}, \mathbf{r})$: properties of the interaction

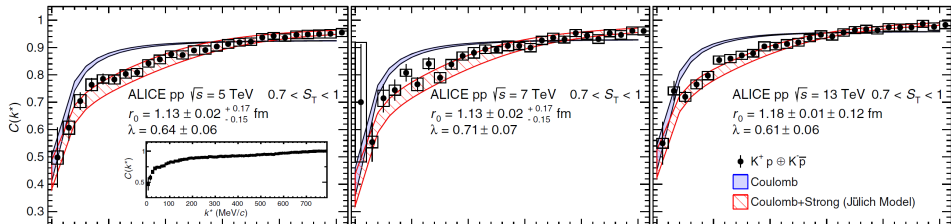
Femtoscscopy of the KN interaction

PHYSICAL REVIEW LETTERS **124**, 092301 (2020)

Scattering Studies with Low-Energy Kaon-Proton Femtoscopy in Proton-Proton Collisions at the LHC

S. Acharya *et al.**

(A Large Ion Collider Experiment Collaboration)



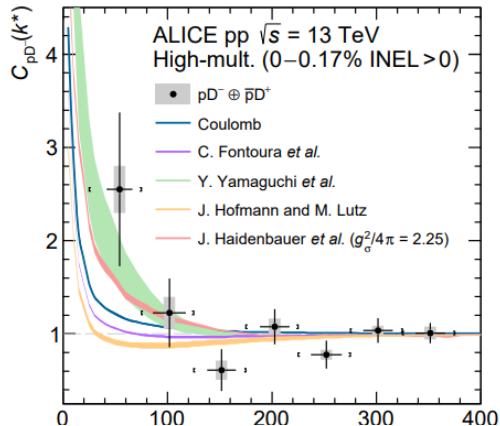
Red band (theory prediction):

J. Haidenbauer, G. Krein, U.-G. Meißner and L. Tólos
Eur. Phys. J. A 47, 18 (2011)

Femtoscscopy of the DN interaction

First study of the two-body scattering involving charm hadrons

ALICE Collaboration*



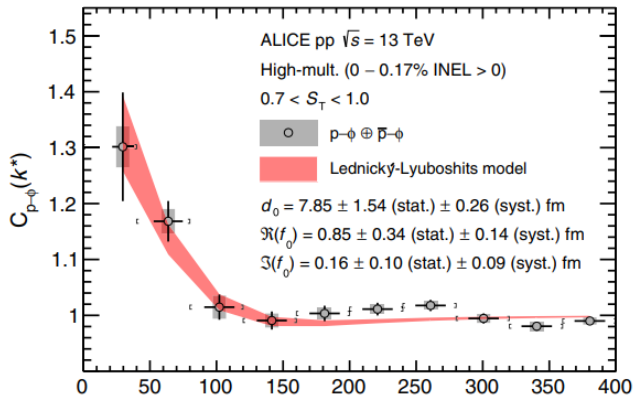
Femtoscscopy of the ϕN interaction

PHYSICAL REVIEW LETTERS 127, 172301 (2021)

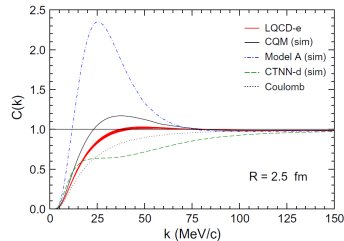
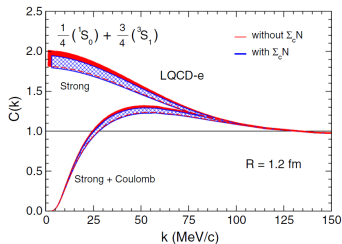
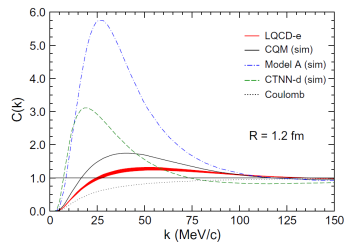
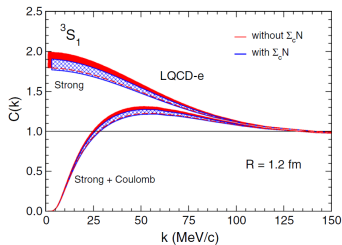
Editors' Suggestion

Experimental Evidence for an Attractive $p\text{-}\phi$ Interaction

S. Acharya *et al.*^{*}
(ALICE Collaboration)



Femtoscscopy of the $\Lambda_c N$ - prediction



Femtoscscopy of J/ψ -nucleon

- Interaction: weakly attractive, S -wave dominated

$$\psi(\mathbf{k}, \mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}} + \psi_0(k, r) - j_0(kr)$$

$\psi_0(k, r)$ contains the effects of the interaction

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- Simplification (not unrealistic):

$$S_{12}(r) = \frac{1}{(4\pi R^2)^{3/2}} e^{-r^2/4R^2}$$

Normally used: $R = 1 \text{ fm} - 1.3 \text{ fm}$ ($p\bar{p}$), $R = 1.5 \text{ fm} - 4.0 \text{ fm}$ (pA, AA)

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- Correlation function:

$$C(k) = 1 + \frac{4\pi}{(4\pi R^2)^{3/2}} \int_0^\infty dr r^2 e^{-r^2/4R^2} [|\psi_0(k, r)|^2 - |j_0(kr)|^2]$$

Source size \times interaction range

If emission happens outside “interaction range”: $\psi_0(k, r) \rightarrow \psi_0^{\text{asy}}(k, r)$

$$\psi_0^{\text{asy}}(k, r) = \frac{\sin(kr + \delta_0)}{kr} = e^{-i\delta_0} \left[j_0(kr) + f_0(k) \frac{e^{ikr}}{r} \right]$$

$$f_0(k) = \frac{e^{i\delta_0} \sin \delta_0}{k} \underset{k \rightarrow 0}{\approx} \frac{1}{-1/a_0 + r_0 k^2/2 - ik}$$

Lednicky-Lyuboshits (LL) model

$$C(k) = 1 + \frac{|f_0(k)|^2}{2R^2} \left(1 - \frac{r_0}{2\sqrt{\pi}R} \right) + \frac{2\text{Re}f_0(k)}{\sqrt{\pi}R} F_1(2kR) - \frac{\text{Im}f_0(k)}{R} F_2(2kR)$$

$$F_1(x) = \frac{1}{x} \int_0^x dt e^{t-x}, \quad F_2(x) = \frac{1}{x} \left(1 - e^{-x^2} \right)$$

Validity: $r_0 \ll R$

Universal formula, independent of interaction details

Correlation and $\langle (g\mathbf{E})^2 \rangle_N$

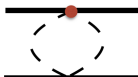
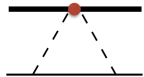
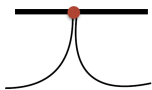
LL for small k :

$$C(k) = 1 - \frac{1}{2\pi^{3/2}} \left(1 - \frac{8}{3} k^2 R^2 \right) \frac{\mu_{J/\psi N} \alpha_{J/\psi} \langle (g\mathbf{E})^2 \rangle_N}{R}$$

$C(k)$ gives direct access to $\langle (g\mathbf{E})^2 \rangle_N^*$

*Under validity of LL model

Quarkonium-nucleon EFT (QNEFT)



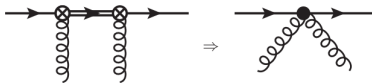
Degrees of freedom – Scales – Power counting

- **DOF:** nucleons (N), quarkonia (ϕ), pions (π)
- **Scales:** $E_N, E_\phi, E_\pi \ll \Lambda_\chi \simeq 1 \text{ GeV}$
- **Power counting:** powers of $\frac{m_\pi}{\Lambda_\chi}$
- **Loops:** dimensional regularization

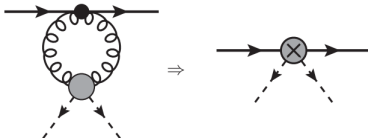
J. T. Castellà & GK, Phys. Rev. D 98, 014029 (2018)

QNEFT input: $\phi - \pi$ vertex

pNRQCD \rightarrow gWEFT (J/ψ polarizability)



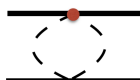
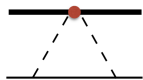
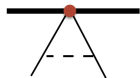
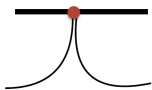
gWEFT \rightarrow χ EFT (trace anomaly)



¹ A. Vairo, in QCHS IV, ed. W. Lucha and K. M. Maung (World Scientific, 2002)

² N. Brambilla, GK, J. T. Castellà, A. Vairo, Phys. Rev. D 93 054002 (2016)

QNEFT predictions



QNEFT: J/ψ polarizability + χ EFT

- Weakly attractive
- Tail: van der Waals type of force

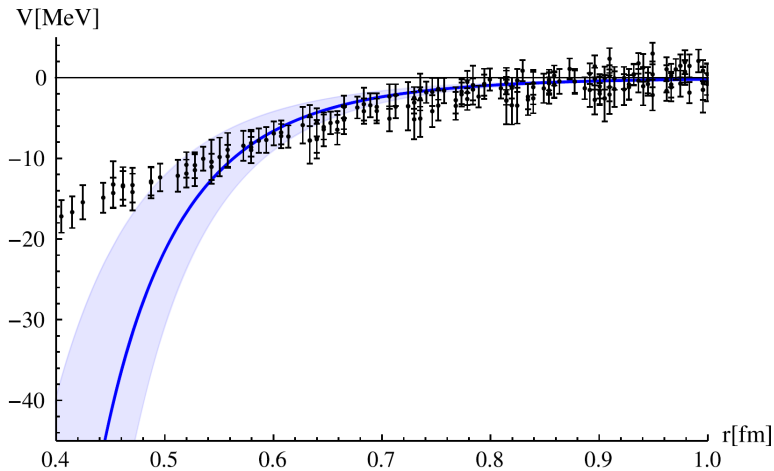
$$V_{\text{vdW}}(r) \xrightarrow{r \gg 1/2m_\pi} \frac{3g_A^2 m_\pi^4 (c_{di} + c_m)}{128\pi^2 F^2} \frac{e^{-2m_\pi r}}{r^2}$$

- S -wave dominated:

Effective range expansion (ERE):

$$f_0(k) = \frac{1}{k \cot \delta - ik} = \frac{1}{-\frac{1}{\mathbf{a}_0} + \frac{1}{2} \mathbf{r}_0 k^2 - ik} \begin{cases} -0.71 \text{ fm} \leq \mathbf{a}_0 \leq -0.35 \text{ fm} \\ 1.29 \text{ fm} \leq \mathbf{r}_0 \leq 1.35 \text{ fm} \end{cases}$$

$J/\psi N$ long range tail (Latt-QNEFT)



Phenomenological models

Finite well¹:

$$V(r) = \begin{cases} -\frac{2\pi}{3} \left(\frac{\alpha_{J/\psi}}{R_N^3} \right) m_N & \text{for } r < R_N \\ 0 & \text{for } r > R_N \end{cases}$$

Multipole expansion + χ SQM²

$$V(r) = -\alpha_{J/\psi} \frac{4\pi^2}{b} \left(\frac{g^2}{g_s^2} \right) [\nu \rho_E(r) - 3p(r)] \begin{cases} \rho_E(r), p(r) : \text{energy density, pressure} \\ b = 27/3, \quad g^2/g_s^2 = 1, \quad \nu = 1.5 \end{cases}$$

¹ J. Ferretti, E. Santopinto, M. N Anwar and M. Bedolla, Phys. Lett. B 789, 562 (2019)

² M.I. Eides, V.Y. Petrov and M.V. Polyakov, Eur. Phys. J. C 78, 36 (2018)

ERE parameters (phenom. models)

Essentially one unknown parameter: $\alpha_{J/\psi}$

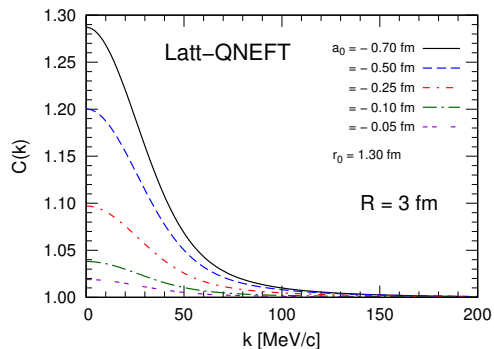
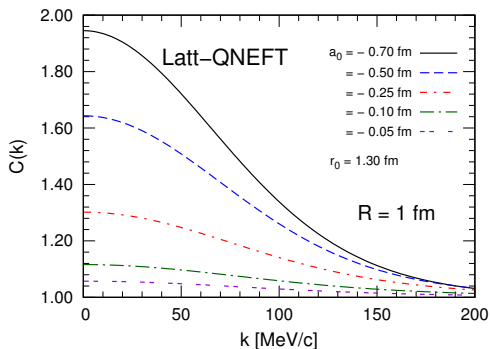
ERE parameters (in fm) for different $\alpha_{J/\psi}$ (in GeV^{-3})

$\alpha_{J/\psi}$	Finite well*		χSQM	
	a_0	r_0	a_0	r_0
2.00	-0.68	1.59	-0.42	1.86
1.60	-0.47	1.86	-0.30	2.25
0.54	-0.12	4.50	-0.08	6.00
0.24	-0.05	9.46	-0.03	13.05

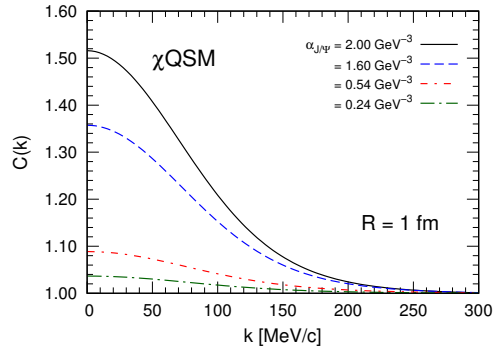
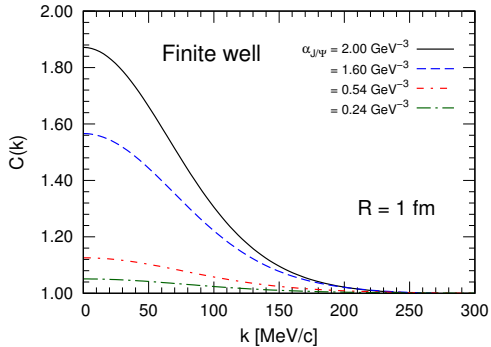
* $R_N = 1$ fm

5. Predictions J/ψ -nucleon femtoscopy

Use of ERE



Full wave function



6. Conclusions & Perspectives

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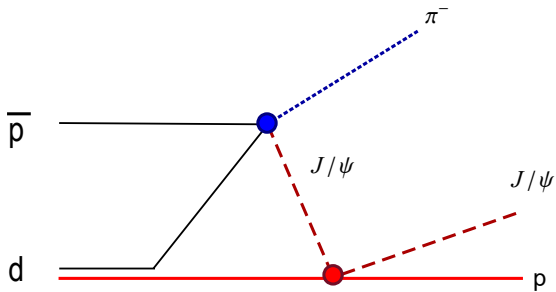
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6. Prospects: **cautiously optimistic !**

$\bar{p}d \rightarrow J/\psi p \pi^-$: $\bar{P}ANDA @ FAIR, AMBER @ CERN$



Similar to $\bar{p}d \rightarrow D\bar{D}N$: $\bar{P}ANDA @ FAIR$

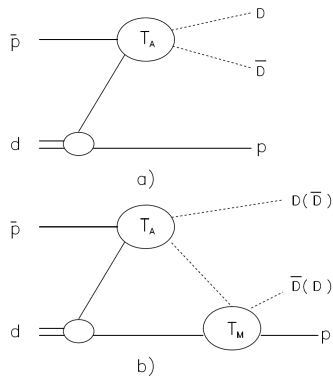


Fig. 1. Contributions to the reaction $\bar{p}d \rightarrow D\bar{D}N$: a) the Born (nucleon exchange) diagram. T_A denotes the annihilation amplitude. b) Meson rescattering diagram. T_M denotes the meson-nucleon scattering amplitude. Note that both DN and $\bar{D}N$ scatterings contribute to the reaction amplitude.

Thank you

Funding

