Chiral Nuclear Forces with Gradient Flow Regulator

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Nuclear Hamiltonians for Advancing Nuclear Physics and Beyond
INT workshop, Seattle
May 12, 2026

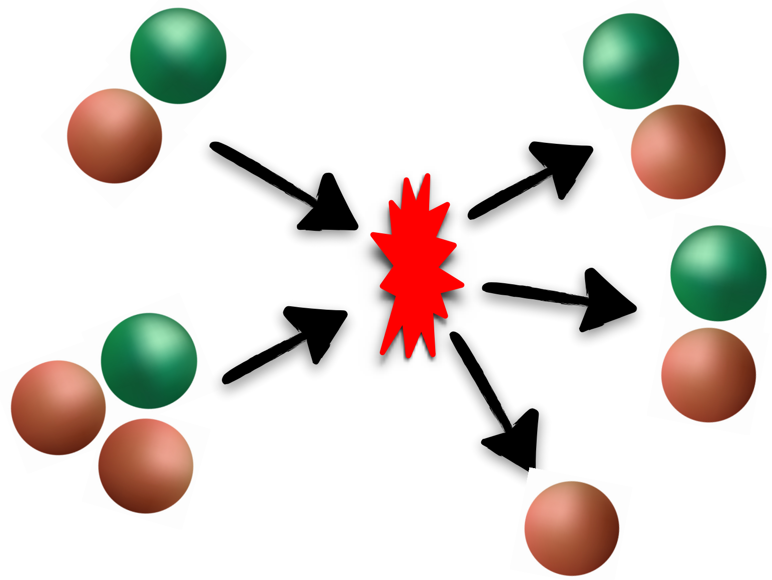
Exploratory Research for Advanced Technology

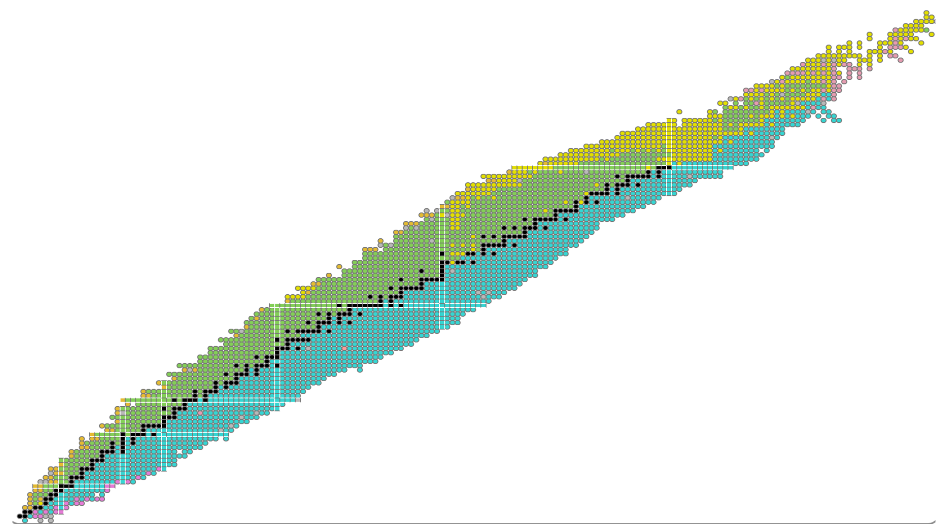
European Research Council

With Evgeny Epelbaum, Henri Huesmann, Patrick Walkowiak, Victor Springer, Andreas Nogga, Kai Hebeler

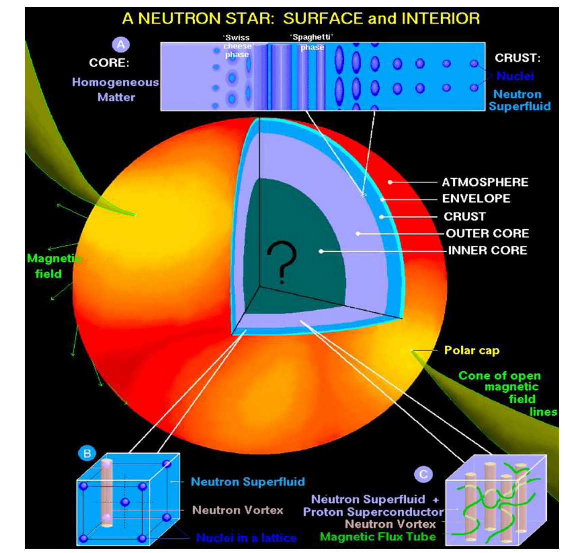
Outline

- Three-pion-exchange for NN at N^3LO within UT
- Status report on 3NF
  - Method for derivation of nuclear forces in chiral EFT
  - Pion-nucleon scattering within GF
  - PWD of 3NF at N^3LO
  - Short-range 3NF-LECs up to N^4LO





Livechart, IAEA: https://www-nds.iaea.org

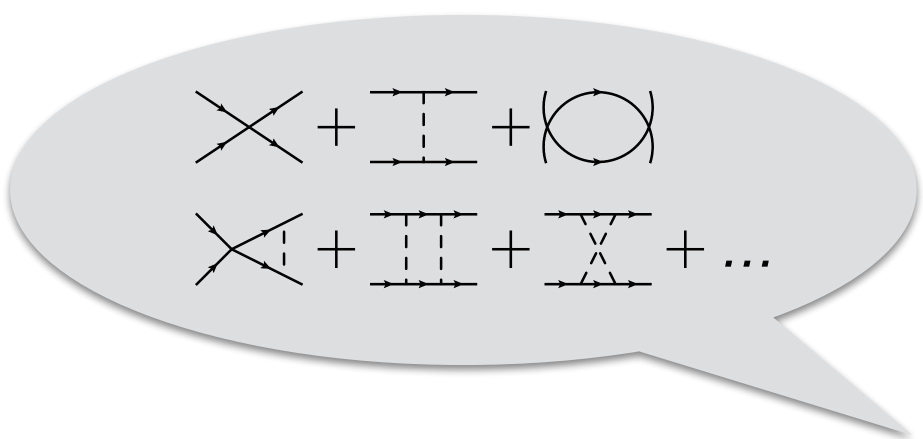


Lattimer: NAR54 (2010) 101

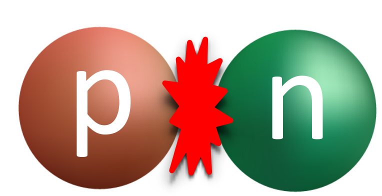
QM A-body problem

$$\left[\left(\sum_{i=1}^{A}\frac{-\vec{\nabla}_i^2}{2m_N}+\mathcal{O}(m_N^{-3})\right)+\underbrace{V_{2N}+V_{3N}+V_{4N}+\ldots}_{\textit{derived within ChPT}}\right]|\Psi\rangle = E|\Psi\rangle$$

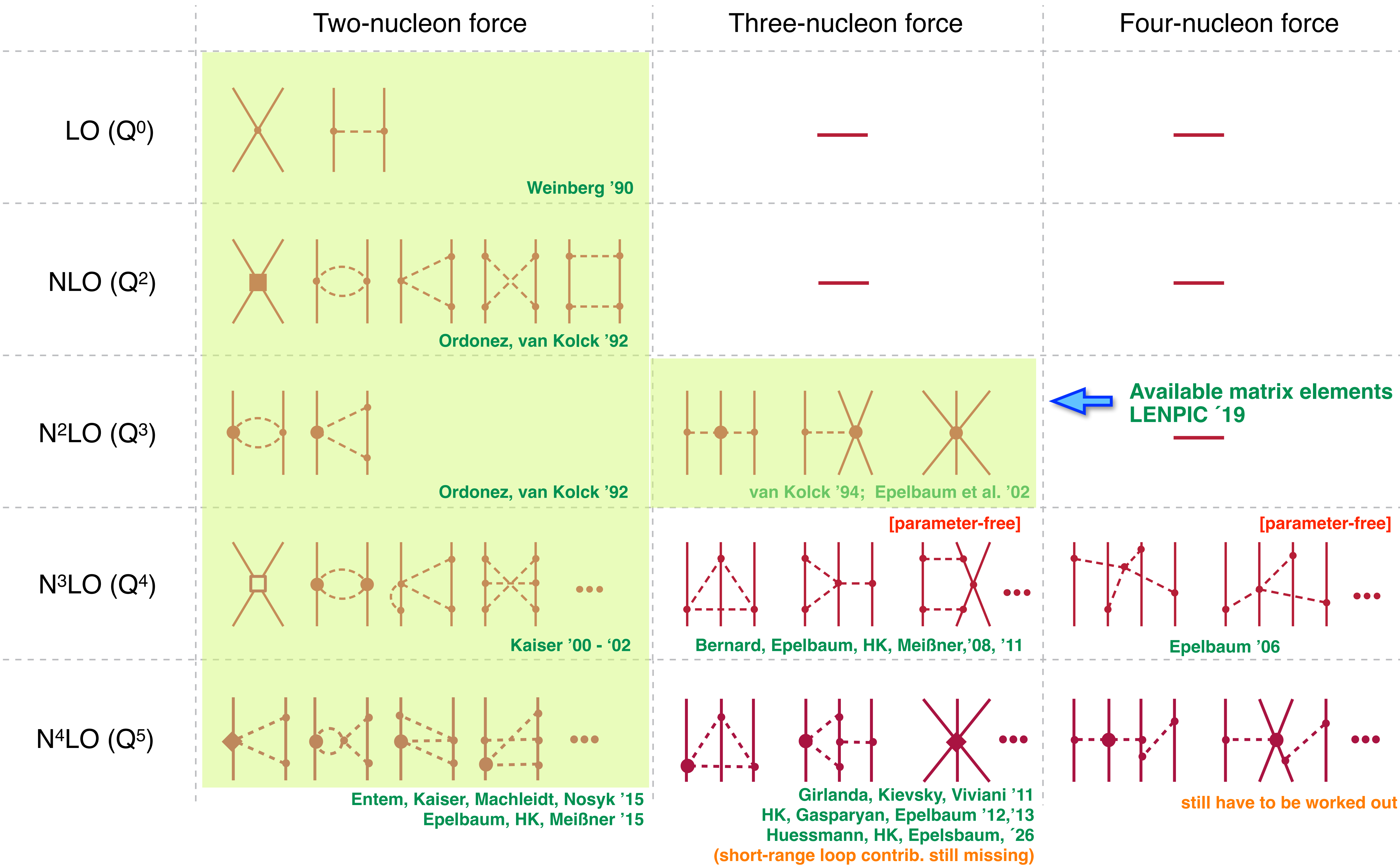
Weinberg '91



Chiral EFT is a systematic tool for derivation of nuclear forces below pion-production threshold



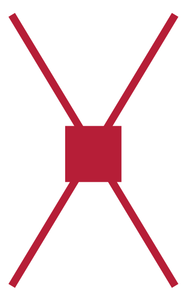
Chiral Expansion of the Nuclear Forces



Adjustable Parameters in NN

Reinert, HK, Epelbaum PRL126 (2021) 092501

Couplings of short-range interactions are fixed from NN - data



LO [Q^0]: 2 operators (S-waves)
NLO [Q^2]: + 7 operators (S-, P-waves and ε_1)
N^2LO [Q^3]: no new terms
N^3LO [Q^4]: + 12 operators (S-, P-, D-waves and ε_1, ε_2)
N^4LO [Q^5]: + 5 IB operators
N^4LO^+ [Q^6]: + 4 operators (F-waves)

# of adjustable LECs = 25 IC + 5 IB + 3 πN constants = 33 parameters

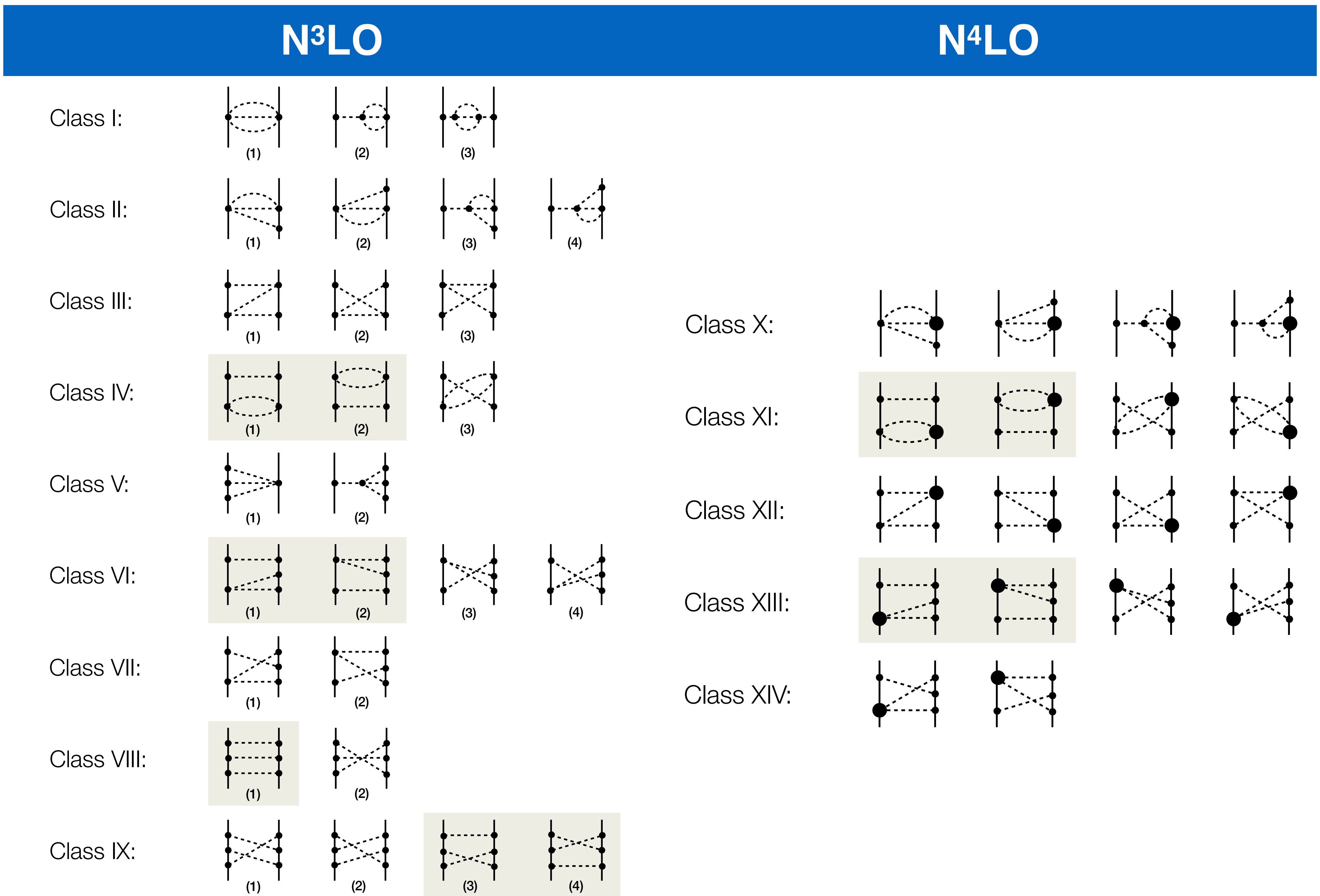
Summary on NN

- Employed a Bayesian approach to account for statistical and systematic uncertainties
- Extracted πN couplings from NN data within chiral EFT
- Achieved a statistically perfect description of NN data
  χ^2/dat = 1.005 for ~5000 data in the energy range E_{lab} = 0 - 280 MeV

Possible Improvements in NN Sector

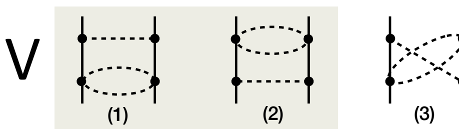
1/m correction to 2PE is scheme dependent → Scheme-dependence of 3PE

3PE calculated by Kaiser `00 - `02 can not be used in unitary transformation approach

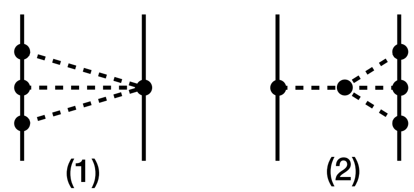


Two Schemes Results: Summarized

- For the Classes VI, VIII and IX we get for most of the potentials stronger 3PE contributions

- Despite reducible-like diagrams we do not see any deviation for the Class IV 

- We reproduced all results of Kaiser with one exception:

Class V: 

Different sign in **Kaiser PRC 62 (2000) 024001, Eq. (8)**

$$\mathrm{Im}\, W_T^{\mathrm{V}}(i\mu) = \frac{1}{\mu^2}\mathrm{Im}\, W_S^{\mathrm{V}}(i\mu) - \frac{g_A^4\left(\mu^2 - M_\pi^2\right)^{-1}}{\mu^2\left(8\pi F_\pi^2\right)^3}\iint_{z^2<1} d\omega_1 d\omega_2 \left[\left(6\mu^2 + 2M_\pi^2\right)(\omega_1+\omega_2)\right.$$

$$\left. -\mu\left(4\mu^2 + 3M_\pi^2\right)\right]\left[\left(\left(\mu^2 + M_\pi^2\right)\left(2\omega_1 - \frac{\mu}{2}\right) - 2\mu\omega_1\omega_2\right)\frac{\arccos(-z)}{l_1 l_2\sqrt{1-z^2}} + \mu + 2z\omega_1\frac{l_2}{l_1}\right]$$

- At N^4LO we don’t see any deviation for all classes of diagrams

Remains to be seen if we observe an evidence of 3PE from NN scattering data.

Work in progress

Methods for Derivation of 3NF beyond N^2LO

HK, Epelbaum, PRC110 (2024) 4, 044003

HK, Epelbaum, PRC 110 (2024) 4, 044004

Symmetry Preserving Regulator

HK, Epelbaum, PRC 110 (2024) 4, 044004

Gradient-Flow Equation (GFE)

Balitsky, Yung, PL168B (1986) 113; Irwin, Manton, PLB 385 (1996) 187

Yang-Mills gradient flow in QCD: Lüscher, JHEP 04 (2013) 123

$$\partial_\tau B_\mu = D_\nu G_{\nu\mu} \quad \text{with} \quad B_\mu|_{\tau=0} = A_\mu \quad \& \quad G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu + [B_\mu, B_\nu]$$

B_μ is a regularized gluon field

- Apply this idea to ChPT: HK, Epelbaum, PRC 110 (2024) 4, 044004

(Proposed in various talks by D. Kaplan for nuclear forces)

Introduce a smoothed pion field W with $W|_{\tau=0} = U$ satisfying GFE

$$\partial_\tau W = i\, w\, \mathrm{EOM}(\tau)\, w \quad \text{with} \quad w = \sqrt{W} \quad \text{and} \quad \mathrm{EOM}(\tau) = [D_\mu, w_\mu] + \frac{i}{2}\chi_- - \frac{i}{4}\mathrm{Tr}(\chi_-)$$

$$w_\mu = i(w^\dagger(\partial_\mu - i\, r_\mu)w - w(\partial_\mu - i\, l_\mu)w^\dagger), \quad \chi_- = w^\dagger \chi w^\dagger - w\chi^\dagger w, \quad \chi = 2B(s + ip)$$

Note: The shape of regularization is dictated by the choice of the right-hand side of GFE

- Our choice is motivated by a Gaussian regularization of one-pion-exchange in NN

Gradient-Flow Equation

Analytic solution is possible of $1/F$ - expanded gradient flow equation:

$$W = 1 + i\tau \cdot \phi(1 - \alpha\phi^2) - \frac{\phi^2}{2}\left[1 + \left(\frac{1}{4} - 2\alpha\right)\phi^2\right] + \mathcal{O}(\phi^5), \quad \phi_b = \sum_{n=0}^{\infty} \frac{1}{F^n}\phi_b^{(n)}$$

In the absence of external sources we have

$$\left[\partial_\tau - (\partial_\mu^x \partial_\mu^x - M^2)\right]\phi_b^{(1)}(x,\tau) = 0, \quad \phi_b^{(1)}(x,0) = \pi_b(x)$$

$$\left[\partial_\tau - (\partial_\mu^x \partial_\mu^x - M^2)\right]\phi_b^{(3)}(x,\tau) = (1-2\alpha)\partial_\mu \phi^{(1)} \cdot \partial_\mu \phi^{(1)} \phi_b^{(1)} - 4\alpha \partial_\mu \phi^{(1)} \cdot \phi^{(1)} \partial_\mu \phi_b^{(1)} + \frac{M^2}{2}(1-4\alpha)\phi^{(1)} \cdot \phi^{(1)} \phi_b^{(1)}, \quad \phi_b^{(3)}(x,0) = 0$$

Iterative solution in momentum space: $\tilde{\phi}^{(n)}(q,\tau) = \int d^4x\, e^{iq\cdot x} \phi_b^{(n)}(x,\tau)$

$$\tilde{\phi}_b^{(1)}(q) = e^{-\tau(q^2+M^2)}\tilde{\pi}_b(q)$$

$$\tilde{\phi}_b^{(3)}(q) = \int \frac{d^4q_1}{(2\pi)^4}\frac{d^4q_2}{(2\pi)^4}\frac{d^4q_3}{(2\pi)^4}(2\pi)^4\delta(q - q_1 - q_2 - q_3)\int_0^\tau ds\, e^{-(\tau-s)(q^2+M^2)} e^{-s\sum_{j=1}^3 (q_j^2+M^2)} \times \left[4\alpha\, q_1 \cdot q_3 - (1-2\alpha) q_1 \cdot q_2 + \frac{M^2}{2}(1-4\alpha)\right]\tilde{\pi}(q_1)\cdot\tilde{\pi}(q_2)\tilde{\pi}_b(q_3)$$

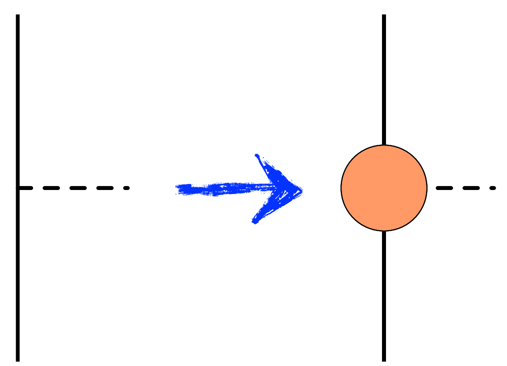
Integration over momenta of pion fields with Gaussian prefactor introduces smearing

Regularization for Nuclear Forces

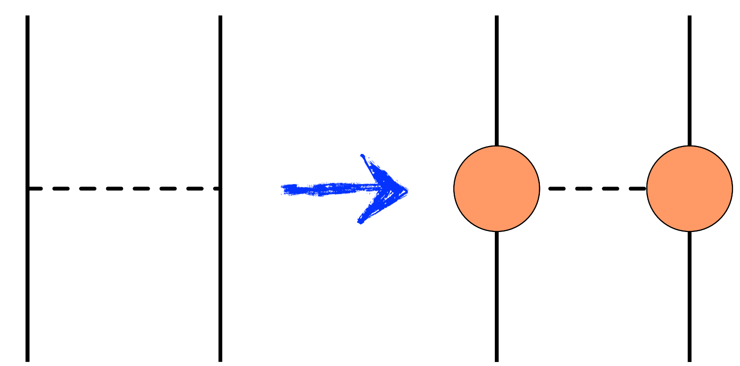
To regularize long-range part of the nuclear forces and currents

- Leave pionic Lagrangians $\mathscr{L}_\pi^{(2)}$ & $\mathscr{L}_\pi^{(4)}$ unregularized (essential)
- Replace all pion fields in pion-nucleon Lagrangians $\mathscr{L}_{\pi N}^{(1)}, \ldots, \mathscr{L}_{\pi N}^{(4)}$: $U \to W$

$$\mathscr{L}_{\pi N}^{(1)} = N^\dagger \Big(D^0 + g\, u \cdot S \Big) N \to N^\dagger \Big(D_w^0 + g\, w \cdot S \Big) N$$



$$\sim e^{-\tau(q^2+M^2)}$$



$$\sim e^{-2\tau(q^2+M^2)} \frac{1}{q^2+M^2}$$

For $\tau = \dfrac{1}{2\Lambda^2}$ this regulator reproduces SMS regularization of OPE

Path-Integral Framework for Derivation of Nuclear Forces

HK, Epelbaum, PRC110 (2024) 4, 044003

Path-integral Approach

We start with generating functional:

$$Z[\eta^\dagger, \eta] = \int [DN^\dagger][DN][D\pi] \exp\left(i \int d^4x \left(\mathcal{L}_\pi + \mathcal{L}_{\pi N} + \mathcal{L}_{NN} + \mathcal{L}_{NNN} + \eta^\dagger(x) N(x) + N^\dagger(x) \eta(x) \right) \right)$$

- Integrate over pion fields via loop-expansion of the action

  → expansion of the action around the classical pion solution

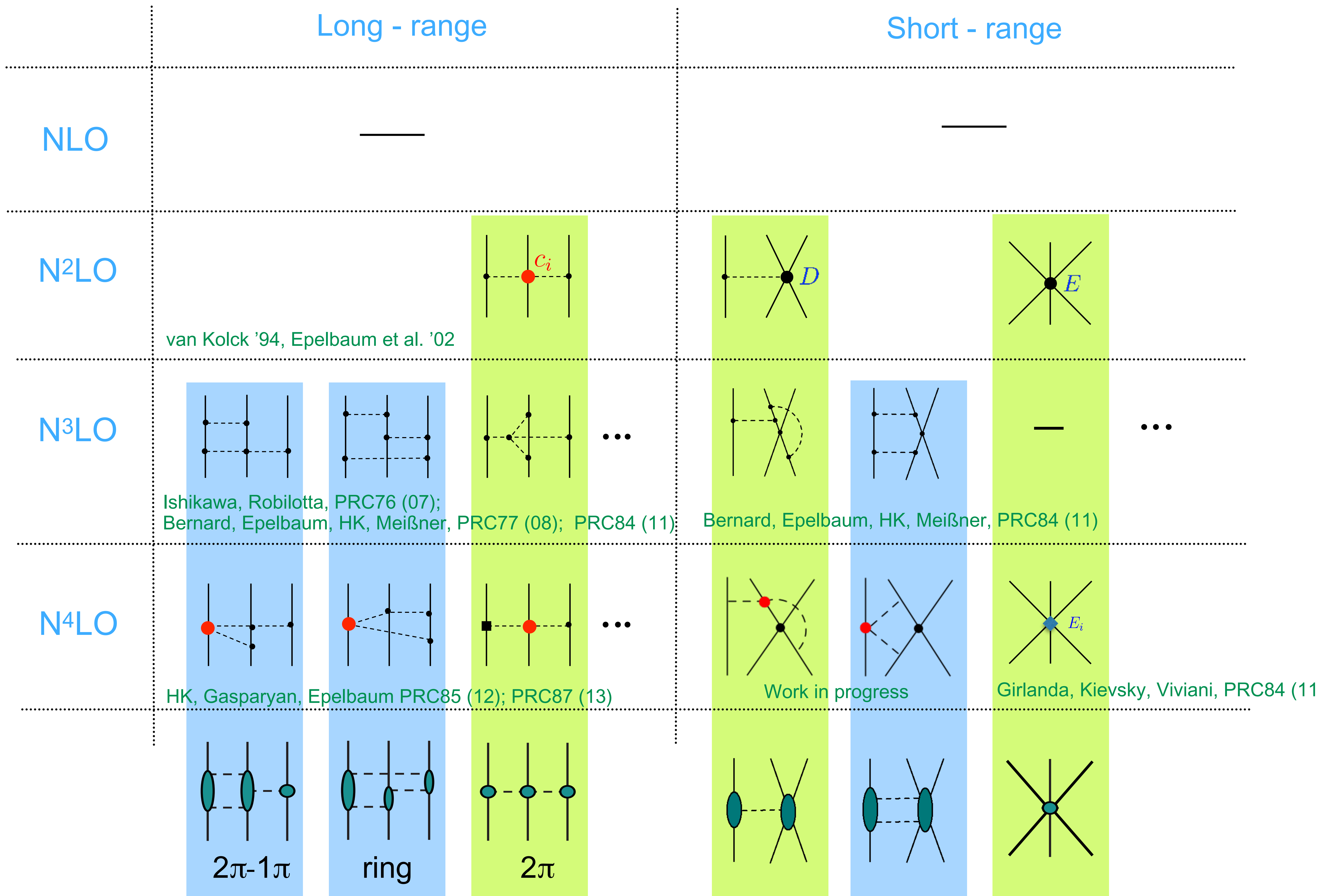
- Perform instant decomposition of the remaining interactions between nucleons
- Perform nucleon-field redefinitions to eliminate non-instant part of the interaction
- Calculate functional determinant to get one-loop corrections to few-nucleon forces

Checks in dimensional regularization

Unitary transformation (Okubo) & path-integral approaches lead to the same chiral EFT nuclear forces up to N^4LO

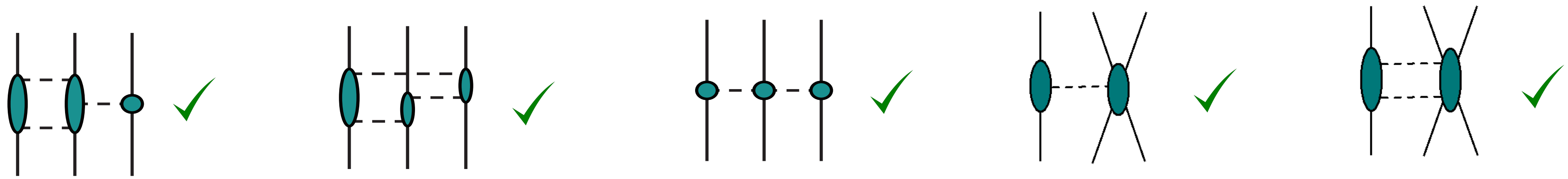
Status Report on 3NF

3NF up to N^4LO



Status Report on 3N at N³LO

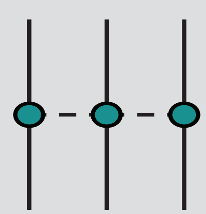
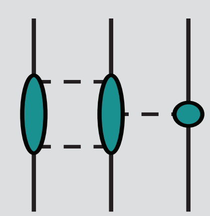
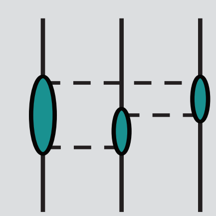
- We calculated all long- and short-range contributions to 3NF & 4NF at N^3LO



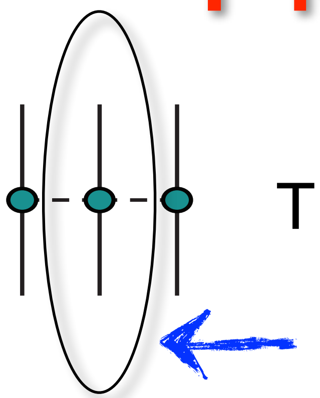
3NF's are given in terms of integrals over Schwinger parameters

$$V_{3N}^{2\pi-1\pi} = \tau_1 \cdot \tau_2 \times \tau_3 \vec{q}_1 \cdot \vec{\sigma}_1 \times \vec{\sigma}_2 \vec{q}_3 \cdot \vec{\sigma}_3 \frac{e^{-\frac{q_3^2+M_\pi^2}{\Lambda^2}}}{q_3^2+M_\pi^2} \left(-\frac{g_A^4}{F_\pi^6} \frac{q_1}{2048\pi} \int_0^\infty d\lambda \, \mathrm{erfi}\left(\frac{q_1 \lambda}{2\Lambda\sqrt{2+\lambda}} \right) \frac{\exp\left(-\frac{q_1^2+4M_\pi^2}{4\Lambda^2}(2+\lambda) \right)}{2+\lambda} + \dots \right) + \dots$$

Dimension of integrals over Schwinger parameters depends on topology

Space			
Momentum	2	1	3
Coordinate	4	1	0

Pion-Nucleon Scattering up to Q³

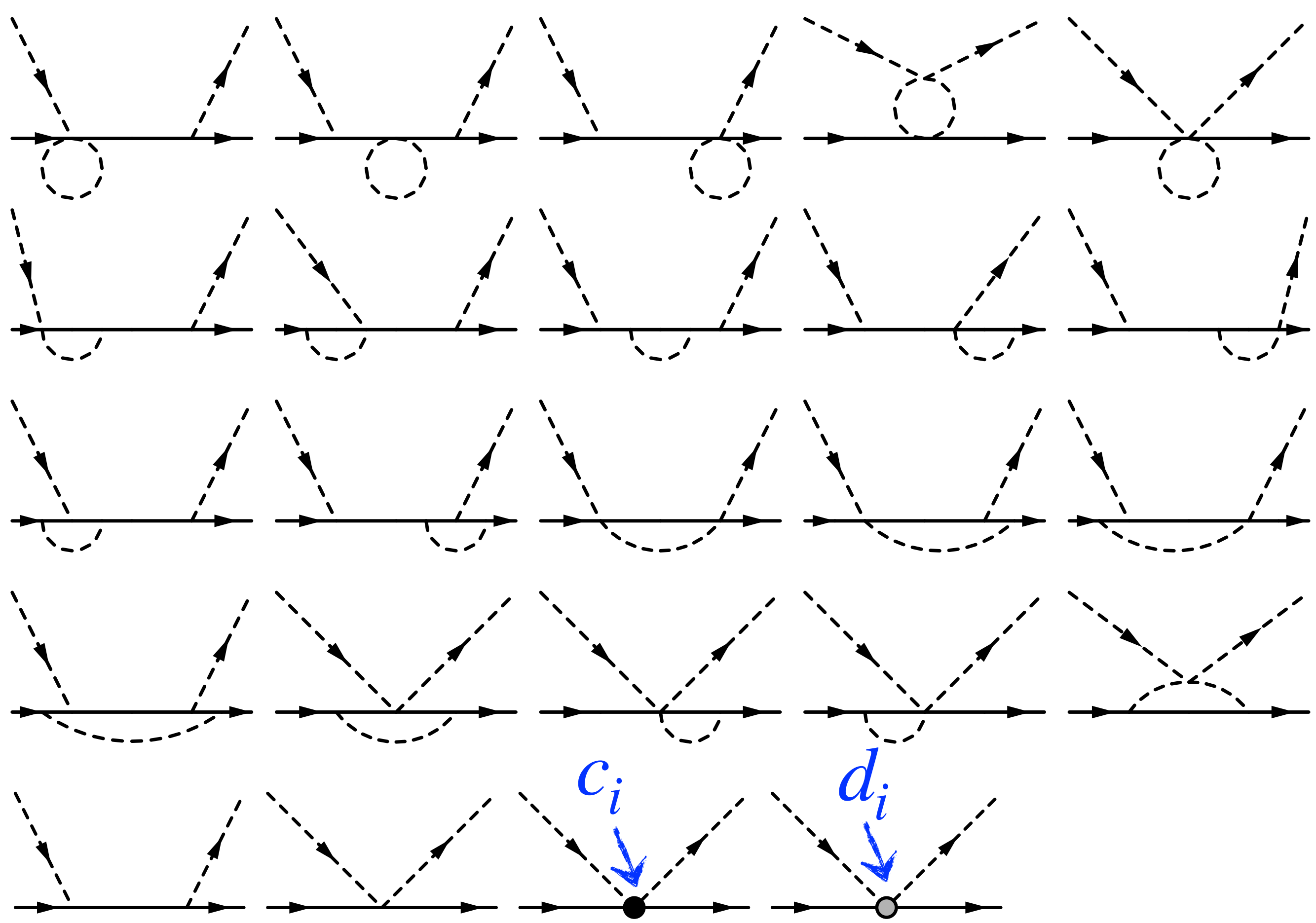


- TPE topology includes pion-nucleon amplitude as a subprocess

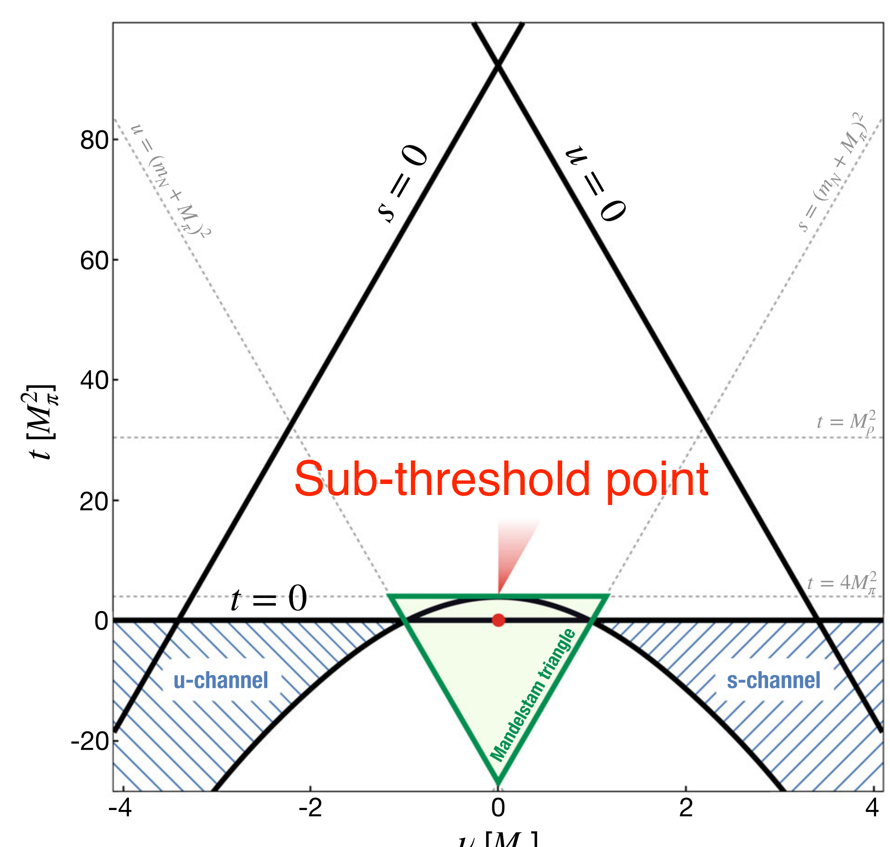
Pion-nucleon amplitude with gradient-flow regulator depends on c_i's

Calculation of pion-nucleon scattering with gradient-flow regulator required

→ Patrick Walkowiak's master thesis



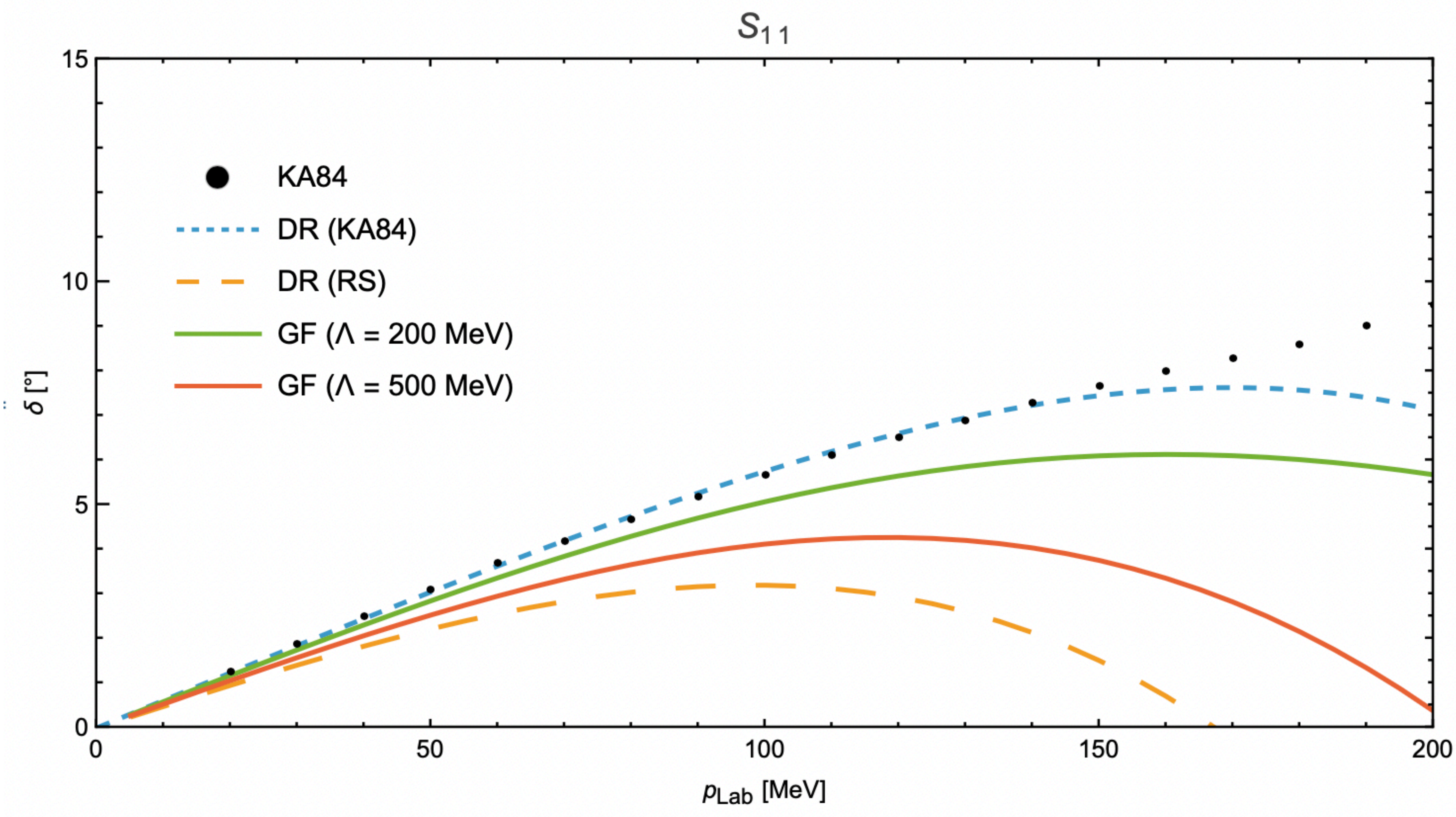
Fit LECs to pion-nucleon sub-threshold coefficients which are determined from Roy-Steiner equation



Pion-Nucleon Scattering up to Q^4

S_{11} phase-shift with different fits of LECs (RS vs DR)

- Better convergence at lower cutoff Λ

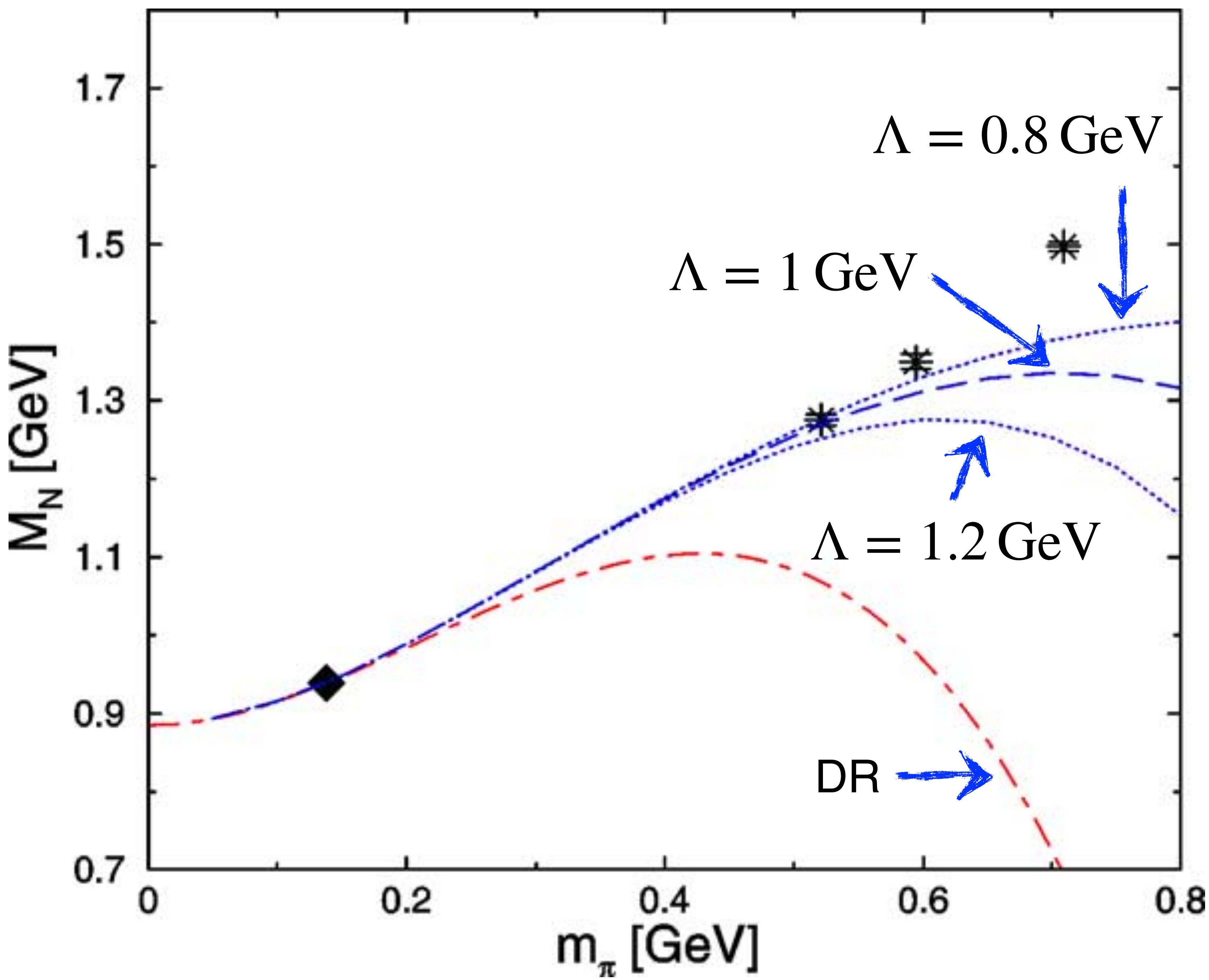


[1] R. Koch, *A calculation of low-energy πN partial waves based on fixed-t analyticity*, Nucl. Phys. A 448, 707 (1986)
[2] H. Krebs et al., *Chiral three-nucleon force at N4LO I: Longest-range contributions*, Phys.Rev.C 85 (2012), 054006

Chiral Extrapolation of Nucleon Mass

Bernard, Hemmert, Meißner, NPA 732 (2004) 149

- Chiral extrapolation of the nucleon mass to Q^3 in HB approach at various cutoffs



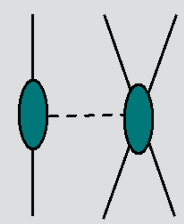
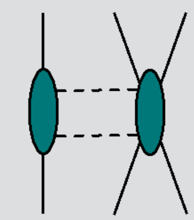
Convergence improvement of chiral expansion within cut-off scheme has also been explored in SU(3) baryon - CHPT

Donoghue, Holstein, Borasoy, PRD 59 (1999) 036002

Short Range 3NF at N^3LO

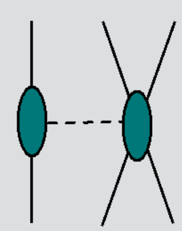
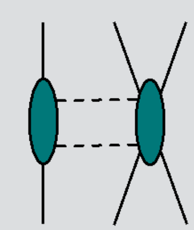
Two versions of 3NF

Version 1: Non-local short-range 3NF which can be used with SMS potential

Space		
Momentum	1	1

2.4 MB

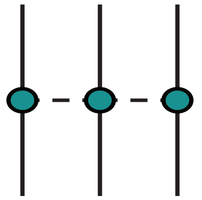
Version 2: Local short-range 3NF to be used with the new NN potential

Space		
Momentum	1	1
Coordinate	0	0

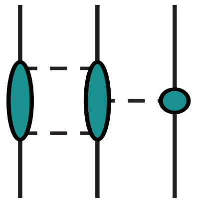
0.4 MB

Partial Wave Decomposition of 3NF

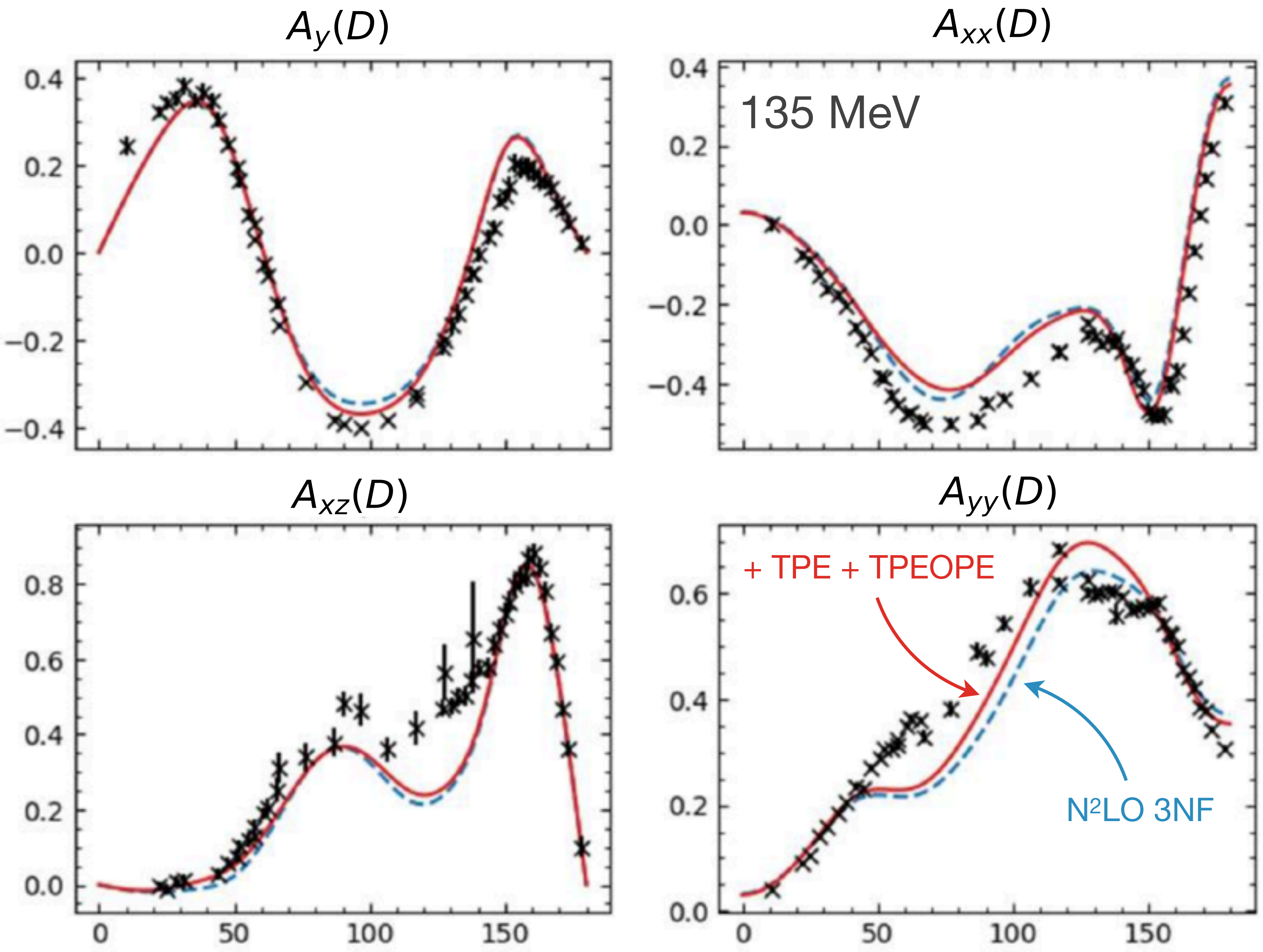
- PWD all N3LO contributions to 3NF finished **Kai Hebeler, Andreas Nogga**
- first experience with TPE & TPE-OPE: sizable, but cancelations



$\delta B_{3H} = -315\,\mathrm{keV}$ (repulsive)

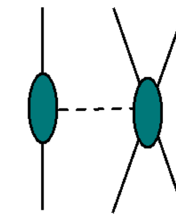


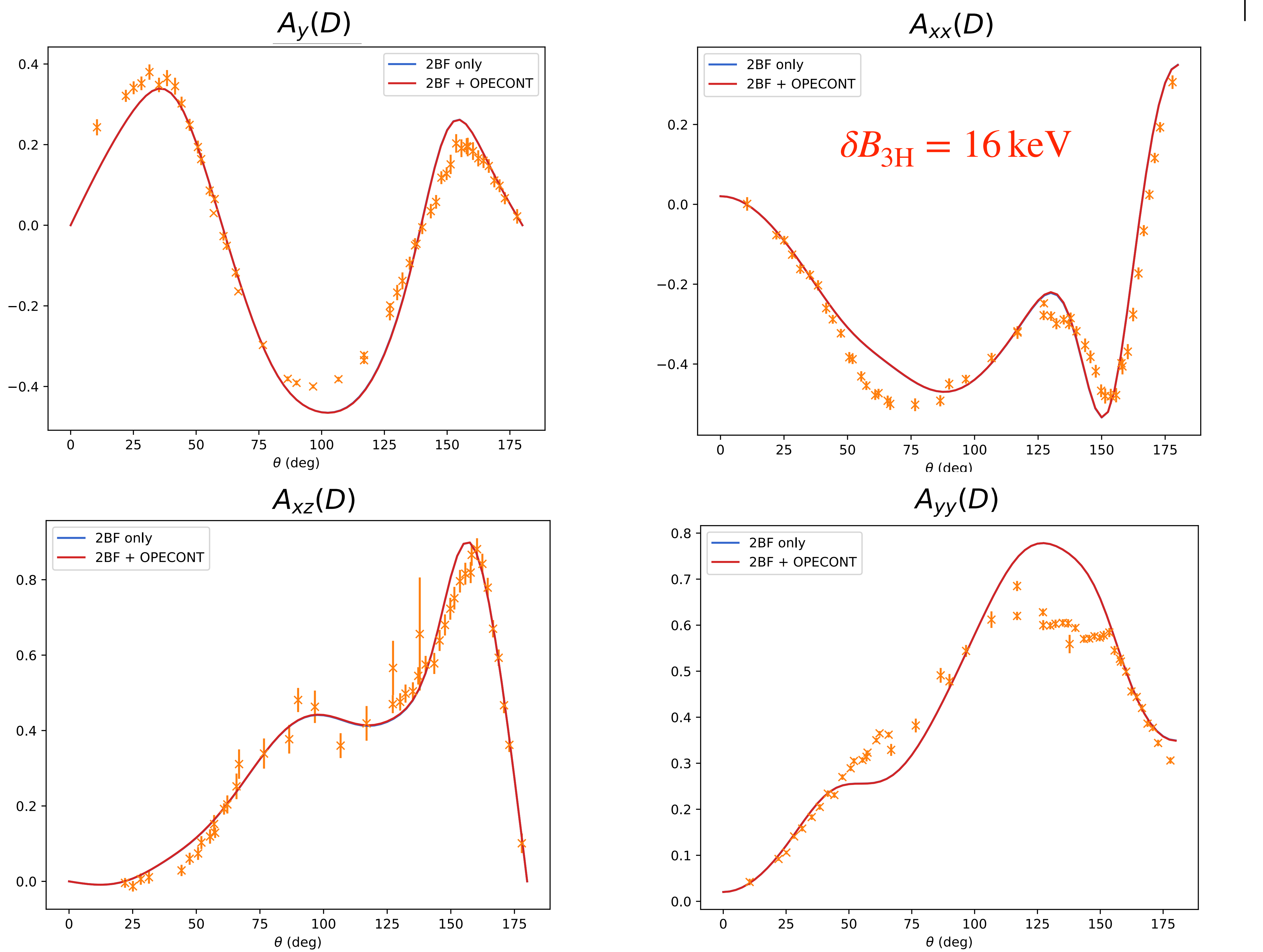
$\delta B_{3H} = 308\,\mathrm{keV}$ (attractive)



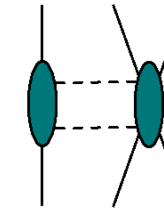
Partial Wave Decomposition of 3NF

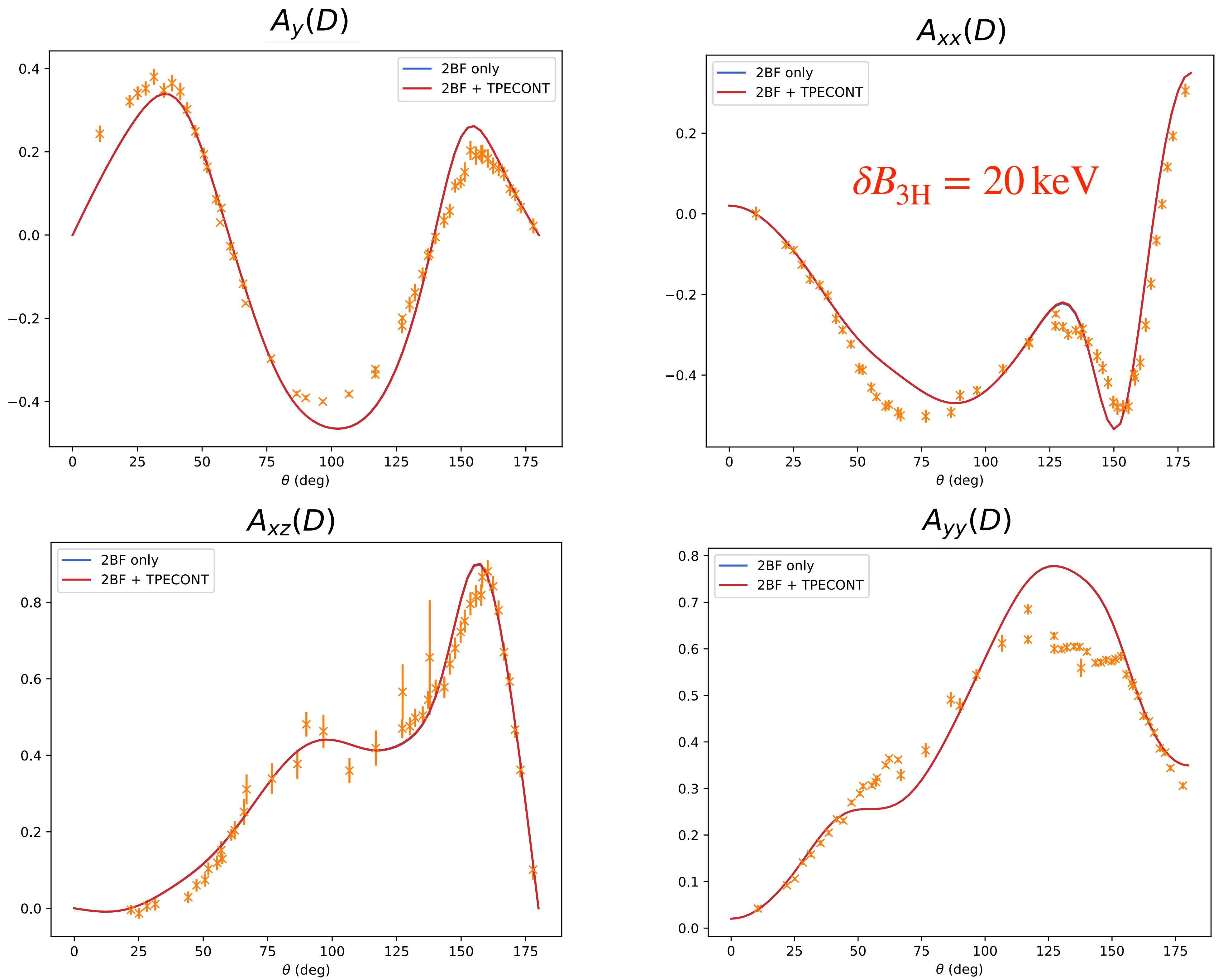
- Small contribution to ND-scattering at $E = 135\,\mathrm{MeV}$ from short-range part





Partial Wave Decomposition of 3NF

- Small contribution to ND-scattering at $E = 135\,\mathrm{MeV}$ from short-range part  $\sim C_T$



Three-Nucleon Forces

Counter Terms at N^4LO

Induced 3NF from Short-Range NN UT

Girlanda, Kievsky, Viviani, PRC84 (2011) 014001; PRC102 (2020) 019903(E)

$$U = \exp\Big(-\sum_j \beta_j T_j\Big) \rightarrow \delta H = U^\dagger H U - H_0 \simeq \sum_j \Big[(H_0 + V^{\rm LO}_{\rm short-range} + H^{\rm LO}_{1\pi}), \beta_j T_j\Big]$$

$$\langle \vec p_1^{\,\prime} \vec p_2^{\,\prime} | T_1 | \vec p_1 \vec p_2 \rangle = \vec k \cdot \vec q, \quad \langle \vec p_1^{\,\prime} \vec p_2^{\,\prime} | T_2 | \vec p_1 \vec p_2 \rangle = \vec k \cdot \vec q \, \vec\sigma_1 \cdot \vec\sigma_2, \quad \langle \vec p_1^{\,\prime} \vec p_2^{\,\prime} | T_3 | \vec p_1 \vec p_2 \rangle = \vec\sigma_1 \cdot \vec k \vec\sigma_2 \cdot \vec q + \vec\sigma_1 \cdot \vec q \vec\sigma_2 \cdot \vec k$$

$$\langle \vec p_1^{\,\prime} \vec p_2^{\,\prime} | T_4 | \vec p_1 \vec p_2 \rangle = i\,(\vec\sigma_1 - \vec\sigma_2) \cdot (\vec P \times \vec k), \quad \langle \vec p_1^{\,\prime} \vec p_2^{\,\prime} | T_5 | \vec p_1 \vec p_2 \rangle = \frac{1}{2}\big(\vec\sigma_1 \cdot \vec P \, \vec\sigma_2 \cdot \vec q - \vec\sigma_1 \cdot \vec q \, \vec\sigma_2 \cdot \vec P\big)$$

lead to $O(m^2)$ enhanced boost operators:
incompatible with assumed non-relativistic limit for interacting nucleons
→ $\beta_{4,5} = O(m^{-1} F_\pi^{-2} \Lambda_b^{-2})$ → Induced 3NF is N⁵LO

δH has the form of N⁴LO 3NF, however $\beta_i = O(m\, F_\pi^{-2} \Lambda_b^{-4})$

$$m/\Lambda_b = (m/Q)(Q/\Lambda_b) \sim (Q/\Lambda_b)^{-2}(Q/\Lambda_b) = (Q/\Lambda_b)^{-1}$$

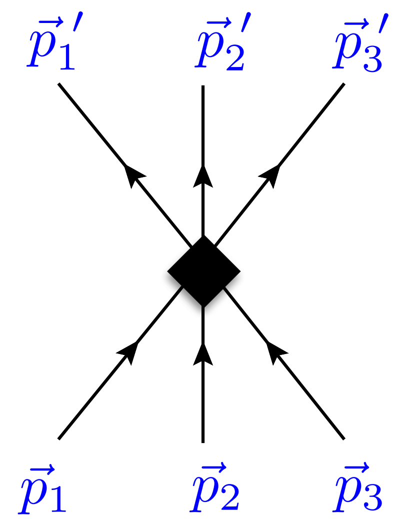
→ Induced 3NFs contribute already at N³LO

E-Like 3NF LECs at N4LO

Girlanda, Kievsky, Viviani, PRC84 (2011) 014001; PRC102 (2020) 019903(E)

Complete set of independent operators

$$\mathcal{O}_1 = -\,\vec{q}_1^{\,2},$$
$$\mathcal{O}_2 = -\,\vec{q}_1^{\,2}\,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2,$$
$$\mathcal{O}_3 = -\,\vec{q}_1^{\,2}\,\vec{\sigma}_1\cdot\vec{\sigma}_2,$$
$$\mathcal{O}_4 = -\,\vec{q}_1^{\,2}\,\vec{\sigma}_1\cdot\vec{\sigma}_2\,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2,$$
$$\mathcal{O}_5 = -\,(3\,\vec{q}_1\cdot\vec{\sigma}_1\,\vec{q}_1\cdot\vec{\sigma}_2 - \vec{q}_1^{\,2}\,\vec{\sigma}_1\cdot\vec{\sigma}_2),$$
$$\mathcal{O}_6 = -\,(3\,\vec{q}_1\cdot\vec{\sigma}_1\,\vec{q}_1\cdot\vec{\sigma}_2 - \vec{q}_1^{\,2}\,\vec{\sigma}_1\cdot\vec{\sigma}_2)\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2,$$
$$\mathcal{O}_7 = -\,i\left((\vec{k}_1-\vec{k}_2)\times\vec{q}_1\right)\cdot(\vec{\sigma}_1+\vec{\sigma}_2)/2,$$
$$\mathcal{O}_8 = -\,i\left((\vec{k}_1-\vec{k}_2)\times\vec{q}_1\right)\cdot(\vec{\sigma}_1+\vec{\sigma}_2)\,\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3/2,$$
$$\mathcal{O}_9 = -\,\vec{q}_1\cdot\vec{\sigma}_1\,\vec{q}_2\cdot\vec{\sigma}_2,$$
$$\mathcal{O}_{10} = -\,\vec{q}_1\cdot\vec{\sigma}_1\,\vec{q}_2\cdot\vec{\sigma}_2\,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2,$$
$$\mathcal{O}_{11} = -\,\vec{q}_1\cdot\vec{\sigma}_2\,\vec{q}_2\cdot\vec{\sigma}_1,$$
$$\mathcal{O}_{12} = -\,\vec{q}_1\cdot\vec{\sigma}_2\,\vec{q}_2\cdot\vec{\sigma}_1\,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2,$$
$$\mathcal{O}_{13} = -\,\vec{q}_1\cdot\vec{\sigma}_2\,\vec{q}_2\cdot\vec{\sigma}_1\,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_3\,.$$



$$\vec{q}_j = \vec{p}_j' - \vec{p}_j$$
$$\vec{k}_j = (\vec{p}_j' + \vec{p}_j)/2$$

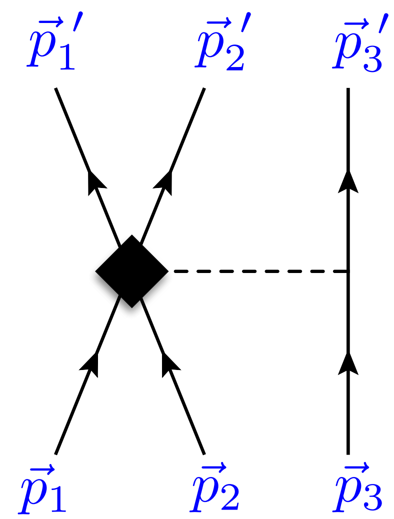
$$V = \sum_{j=1}^{13} E_j \mathcal{O}_j + 5 \text{ perm}$$

D-Like 3NF LECs at N⁴LO

Huesmann, HK, Epelbaum, arXiv: 2602.12879v1

Complete set of independent operators

$$\mathcal{O}_1 = i\,(\vec{q}\times\vec{k})\cdot\vec{q}_3\,(\boldsymbol{\tau}_1+\boldsymbol{\tau}_2),$$
$$\mathcal{O}_2 = i\,(\vec{\sigma}_1-\vec{\sigma}_2)\cdot\vec{k}\,\vec{q}\cdot\vec{q}_3\,(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2),$$
$$\mathcal{O}_3 = i\,\vec{k}\cdot\vec{q}(\vec{\sigma}_1-\vec{\sigma}_2)\cdot\vec{q}_3\,(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2),$$
$$\mathcal{O}_4 = i\,(\vec{\sigma}_1-\vec{\sigma}_2)\cdot\vec{q}\,\vec{k}\cdot\vec{q}_3\,(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2),$$
$$\mathcal{O}_5 = q^2(\vec{\sigma}_1\times\vec{\sigma}_2)\cdot\vec{q}_3\,(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2),$$
$$\mathcal{O}_6 = (\vec{\sigma}_1\times\vec{\sigma}_2)\cdot\vec{q}\,\vec{q}\cdot\vec{q}_3\,(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2),$$
$$\mathcal{O}_7 = q^2(\vec{\sigma}_1-\vec{\sigma}_2)\cdot\vec{q}_3\,(\boldsymbol{\tau}_1-\boldsymbol{\tau}_2),$$
$$\mathcal{O}_8 = q^2(\vec{\sigma}_1+\vec{\sigma}_2)\cdot\vec{q}_3\,(\boldsymbol{\tau}_1+\boldsymbol{\tau}_2),$$
$$\mathcal{O}_9 = (\vec{\sigma}_1-\vec{\sigma}_2)\cdot\vec{q}\,\vec{q}\cdot\vec{q}_3\,(\boldsymbol{\tau}_1-\boldsymbol{\tau}_2),$$
$$\mathcal{O}_{10} = (\vec{\sigma}_1+\vec{\sigma}_2)\cdot\vec{q}\,\vec{q}\cdot\vec{q}_3\,(\boldsymbol{\tau}_1+\boldsymbol{\tau}_2),$$
$$\mathcal{O}_{11} = i\,\vec{\sigma}_2\cdot\vec{q}\,(\vec{\sigma}_1\times\vec{k})\cdot\vec{q}_3\,(\boldsymbol{\tau}_1+\boldsymbol{\tau}_2),$$
$$\mathcal{O}_{12} = i\,\vec{\sigma}_1\cdot\vec{q}_3\,(\vec{q}\times\vec{\sigma}_2)\cdot\vec{q}_3\,(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2),$$
$$\mathcal{O}_{13} = (\vec{\sigma}_1+\vec{\sigma}_2)\cdot\vec{q}_3\,\vec{q}\cdot\vec{q}_3\,(\boldsymbol{\tau}_1-\boldsymbol{\tau}_2),$$
$$\mathcal{O}_{14} = (\vec{\sigma}_1-\vec{\sigma}_2)\cdot\vec{q}_3\,\vec{q}\cdot\vec{q}_3\,(\boldsymbol{\tau}_1+\boldsymbol{\tau}_2),$$
$$\mathcal{O}_{15} = (\vec{\sigma}_1+\vec{\sigma}_2)\cdot\vec{q}\,q_3^2\,(\boldsymbol{\tau}_1-\boldsymbol{\tau}_2),$$
$$\mathcal{O}_{16} = (\vec{\sigma}_1-\vec{\sigma}_2)\cdot\vec{q}\,q_3^2\,(\boldsymbol{\tau}_1+\boldsymbol{\tau}_2)\,.$$



$$\vec{q}_3 = \vec{p}_3{}' - \vec{p}_3$$
$$\vec{q} = \vec{p}\,' - \vec{p}$$
$$\vec{k} = (\vec{p}\,' + \vec{p})/2$$
$$\vec{p}\,' = (\vec{p}_1{}' - \vec{p}_2{}')/2$$
$$\vec{p} = (\vec{p}_1 - \vec{p}_2)/2$$

$$V = \frac{g_A^2}{4F_\pi^2}\frac{\vec{\sigma}_3\cdot\vec{q}_3}{q_3^2+M_\pi^2}\boldsymbol{\tau}_3\cdot\sum_{j=1}^{16}F_j\mathcal{O}_j + 5\text{ perm}$$

Constraints:

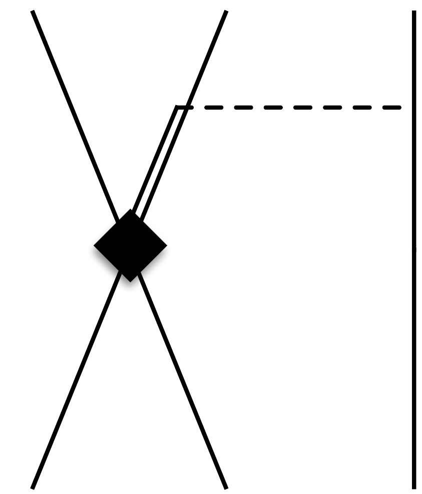
- Isospin symmetry
- Chiral symmetry
- Parity and time - reversal invariance
- Rotation - invariance in 3-dim
- Galilei - invariance

Resonance Saturation of LECs

Huesmann, HK, Epelbaum, arXiv: 2602.12879v1

Complete set of independent $NN \to N\Delta$ transition operators

$$\mathcal{O}_1^\Delta = \left[i\,(\vec{q}_3\cdot\vec{S}_2\vec{S}_2^\dagger\cdot((\vec{q}-\vec{q}_3/2)\times(\vec{k}-\vec{q}_3/4))\,\mathcal{T}_2\mathcal{T}_2^\dagger\cdot\boldsymbol{\tau}_1 + 1\leftrightarrow 2\right] - \text{h.c.},$$
$$\mathcal{O}_2^\Delta = \left[\vec{q}_3\cdot\vec{S}_2\vec{S}_2^\dagger\cdot\vec{\sigma}_1\,(\vec{q}-\vec{q}_3/2)^2\,\mathcal{T}_2\mathcal{T}_2^\dagger\cdot\boldsymbol{\tau}_1 + 1\leftrightarrow 2\right] - \text{h.c.},$$
$$\mathcal{O}_3^\Delta = \left[\vec{q}_3\cdot\vec{S}_2\vec{S}_2^\dagger\cdot(\vec{q}-\vec{q}_3/2)\,\vec{\sigma}_1\cdot(\vec{q}-\vec{q}_3/2)\,\mathcal{T}_2\mathcal{T}_2^\dagger\cdot\boldsymbol{\tau}_1 + 1\leftrightarrow 2\right] - \text{h.c.},$$
$$\mathcal{O}_4^\Delta = \left[\vec{q}_3\cdot\vec{S}_2\vec{S}_2^\dagger\cdot(k-\vec{q}_3/4)\,\vec{\sigma}_1\cdot(\vec{k}-\vec{q}_3/4)\mathcal{T}_2\mathcal{T}_2^\dagger\cdot\boldsymbol{\tau}_1 + 1\leftrightarrow 2\right] - \text{h.c.}\,.$$



$\vec{S}$ & $\mathcal{T}$ spin & isospin $1/2 \to 3/2$ transition matrices

$$V = \frac{g_A^2}{4F_\pi^2}\frac{\vec{\sigma}_3\cdot\vec{q}_3}{q_3^2+M_\pi^2}\boldsymbol{\tau}_3\cdot\sum_{j=1}^{4}\alpha_j\mathcal{O}_j^\Delta + 5\text{ perm} = \frac{g_A^2}{4F_\pi^2}\frac{\vec{\sigma}_3\cdot\vec{q}_3}{q_3^2+M_\pi^2}\boldsymbol{\tau}_3\cdot\sum_{j=1}^{16}F_j^\Delta\mathcal{O}_j + 5\text{ perm}$$

$$F_1^\Delta = -4F_2^\Delta = 4F_4^\Delta = \frac{8\alpha_1}{9\Delta},$$
$$F_3^\Delta = F_{11}^\Delta = 0,$$
$$F_5^\Delta = -\frac{4\alpha_2+2\alpha_3}{9\Delta},$$
$$F_6^\Delta = \frac{2\alpha_3+\alpha_4}{9\Delta},$$
$$F_7^\Delta = \frac{8\alpha_2-\alpha_4}{18\Delta},$$
$$F_8^\Delta = \frac{4\alpha_2}{9\Delta},$$
$$F_9^\Delta = \frac{8\alpha_3+\alpha_4}{18\Delta},$$
$$F_{10}^\Delta = 4F_{12}^\Delta = \frac{4\alpha_3-\alpha_4}{9\Delta},$$
$$F_{13}^\Delta = \frac{2\alpha_1-8\alpha_2-4\alpha_3+\alpha_4}{18\Delta},$$
$$F_{14}^\Delta = \frac{-2\alpha_1-16\alpha_2-8\alpha_3+\alpha_4}{36\Delta},$$
$$F_{15}^\Delta = \frac{-2\alpha_1-4\alpha_3+\alpha_4}{18\Delta},$$
$$F_{16}^\Delta = \frac{2\alpha_1-8\alpha_3-\alpha_4}{36\Delta},$$

Summary I

- Leftovers in NN up to N^4LO
  - 3PE expressions are worked out within UT approach
- Method for derivation of the regularized nuclear forces in chiral EFT
  - Gradient flow regularization preserves chiral symmetry
  - Path-integral approach for derivation of nuclear forces
- Calculation of 3NF at N^3LO is finished
  - Pion-nucleon scattering within GF is calculated up to Q^4
    - Chiral expansion seems to converge better at lower cutoff
  - PWD of 3NF at N^3LO is finished
    - From 3H expectation values we see sizable TPE and TPE-OPE contributions that almost cancel each other
    - Short-range contributions seem to be small (due to $\sim C_T$)

Summary II

- At N^4LO: 16 D-like counter terms worked out → 16 D-like + 13 E-like = 29 LECs
  - One can reduce the number of D-like terms by using Δ - resonance saturation
    → 4 D-like LECs ($\alpha_{1,2,3,4}$)
- At N^3LO: induced terms from short-range NN UT $\sim \beta_{1,2,3}$ need to be fitted in 3N sector