Perturbative adventures in Chiral EFT

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INT 26-1: Nuclear Hamiltonians for Advancing Nuclear Physics and Beyond

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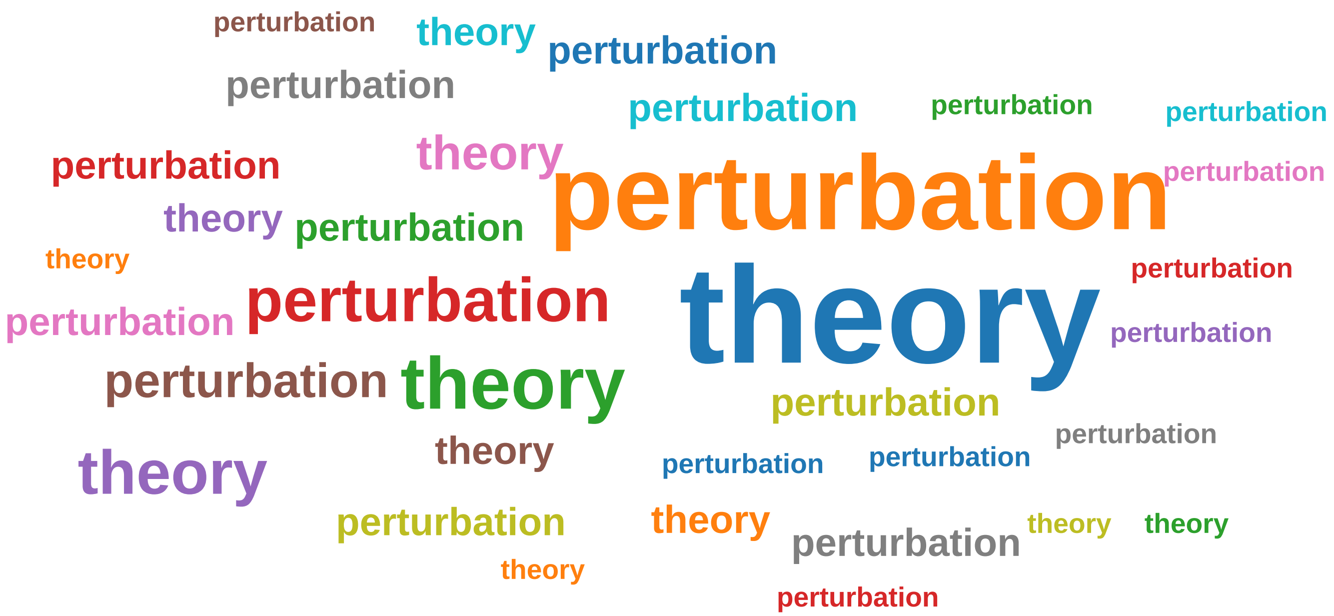
Lyu, Zuo, Peng, SK + Long, 2511.12522 [nucl-th]
Andis, Lyu, Peng, Long, SK, 2512.12823 [nucl-th]

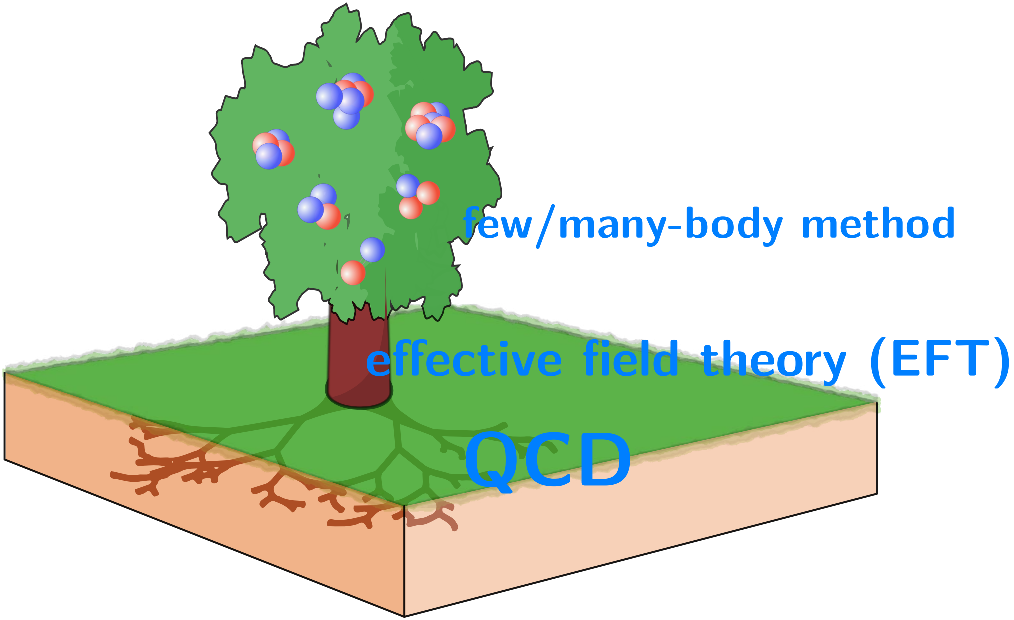
Outline



Kaufman et al., Being John Malkovich, Univ. Pic. Intl. (1999)

Outline

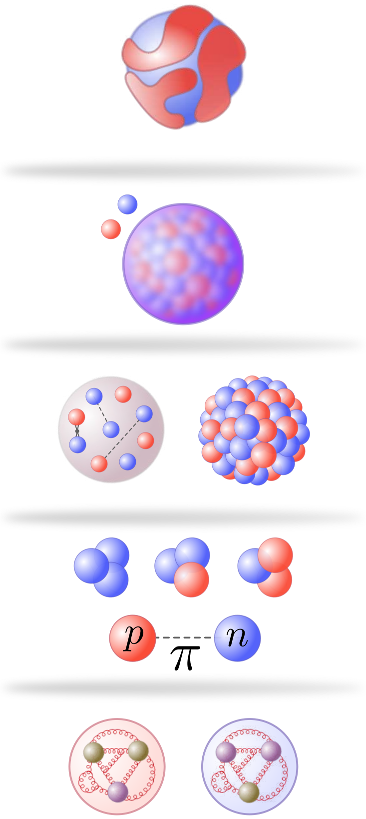
Nuclear theory tower



few/many-body method

effective field theory (EFT)

QCD

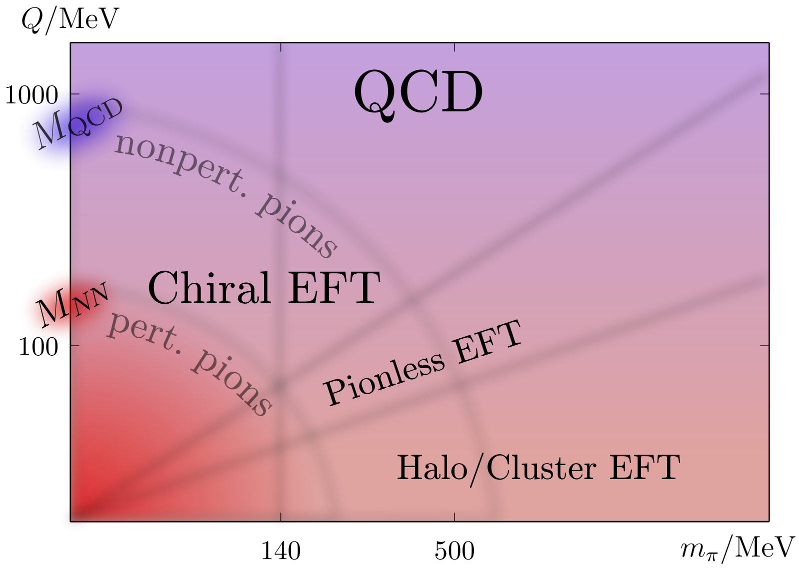


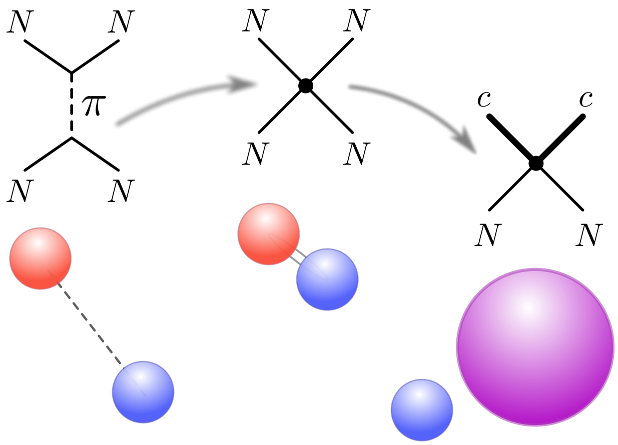
- **QCD** = underlying theory of strong interaction
- **EFT** = effective description in terms of hadrons
- **degrees of freedom depend on resolution scale**

Nuclear effective field theories

- choose **degrees of freedom** approriate to energy scale
- only restricted by **symmetry**, ordered by **power counting**

Hammer, SK, van Kolck, RMP **92** 025004 (2020)





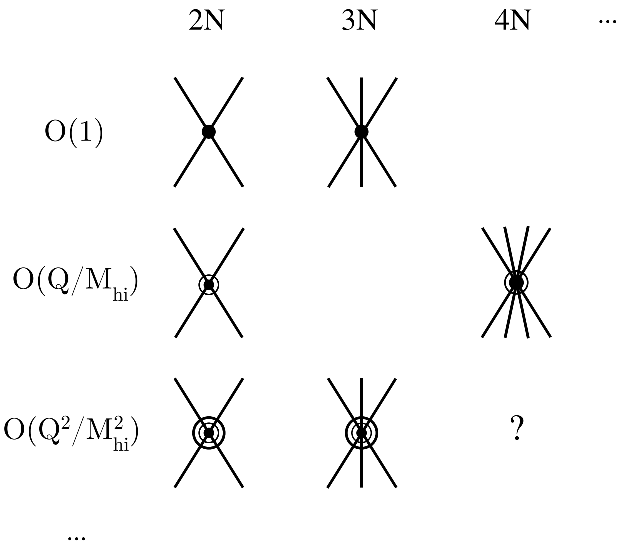
- degrees of freedom here: nucleons (and/or clusters thereof)
- even more effective d.o.f.: rotations, vibrations

Papenbrock, NPA **852** 36 (2011); ...

- **most effective theory depends on energy scale (and nucleus) of interest**

Pionless EFT

- only **contact (zero-range) forces** (plus electromagnetism)
- closely linked to **universality** for **large scattering lengths**
- excels at low energies, exact range of validity still an open question



2N 3N 4N …

$O(1)$

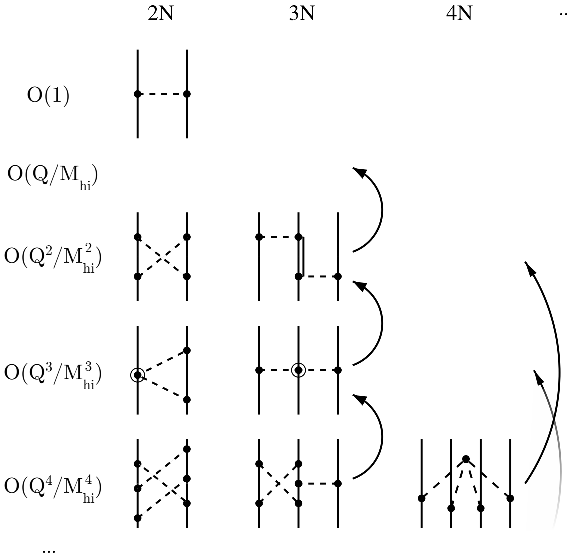
$O(Q/M_{hi})$

$O(Q^2/M_{hi}^2)$?

…

Chiral EFT

- expansion around **chiral limit** $(m_\pi = 0)$, assumes $Q \sim m_\pi$
- **pion exchange** determines nuclear interaction at large and intermediate range
- further details enter as **contact interactions** and **delta excitations**



Chiral potentials

Many remarkable results based on "Weinberg Counting"

Weinberg (90); Rho (91); Ordoñez + van Kolck (92); van Kolck (93); Epelbaum et al. (98); Entem + Machleidt (03); ...

- **expand potential** in $(Q \sim M_\pi)/M_{\mathrm{hi}}$, not the amplitude / wave function
  - $V = V^{(0)} + V^{(1)} + \cdots$
- treat one-pion exchange nonperturbatively in all partial waves
- yields a potential that can be used with established computational methods
- prevailing interaction(s) today in nuclear many-body calculations
- **undoubtedly responsible for great leap forward in nuclear structure theory**

However...

- **does not satisfy RG invariance** (amplitude $T = \mathrm{const.} + \mathcal{O}(1/\Lambda)$)
  - EFT should be independent of arbitrary cutoff/regulator scale Λ
- **challenging to reach good accuracy** for multiple observables

On the other hand:

- **approaches based on or inspired by Pionless EFT are surprisingly successful!**

SK et al., PRL **118** 202501 (2016); Kievsky et al., PRL **121** 072701 (2018); Lu et al., PLB **797** 134863 (2019); ...

How to construct an RG-invariant chiral expansion?

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It's complicated...

How to construct an RG-invariant chiral expansion?

It's complicated...

- one-pion exchange potential is **singular** (and attractive) in various partial waves
- this is a challenge for proper renormalization

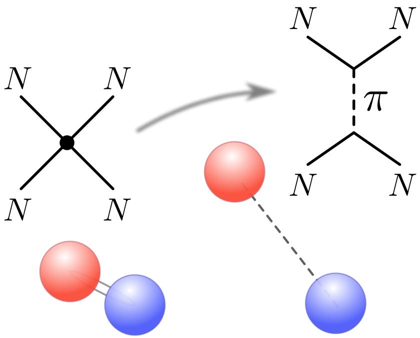
Nogga, Timmermans, van Kolck, PRC **72** 054006 (2005)

Coping mechanisms

Perturbative pions

- construct Chiral EFT on top of Pionless EFT as leading order Kaplan et al., PLB/NPB 1998
- ⇝ KSW counting, poor convergence properties Cohen+Hansen 1999; Fleming et al., NPA 2000
- recent work: **possible to construct an expansion with good convergence!**
- **more later in this talk**

Lyu, SK et al., 2511.12522 [nucl-th]



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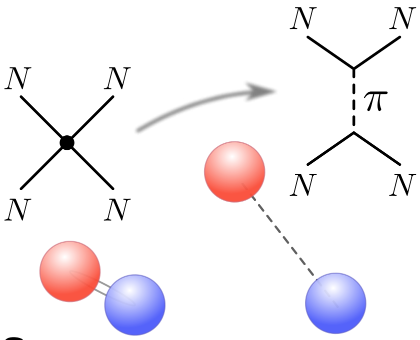


Partly perturbative pions

- **high partial waves are known to be supressed in NN scattering**
  - keep only finite number of partial waves at LO Nogga et al., PRC 2005
  - treat the rest of OPE in perturbation theory (like KSW)
- gives RG invariance for triton and nd scattering Song et al., PRC 2017
- **actively explored by different groups**

Long+Yang PRC **85** 034002 (2012), **86** 024001 (2012); Wu+Long PRC **99** 024003 (2019); Thim et al. PRC **112** 064008 (2025)

Outline

Introduction ✔

Deuteron disintegration

Structure of light nuclei

Summary and outlook

Part I: Deuteron disintegration in Chiral EFT

with partly perturbative pions

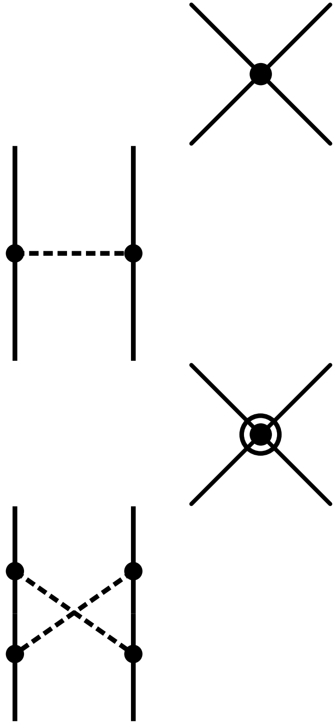
Chiral EFT for two-nucleon system

- RG invariant (MMW) chiral interactions describe NN scattering with good accuracy

  Long + Yang, PRC **85** 034002 (2012), **86** 024001 (2012); Wu + Long, PRC **99** (2019)
- static deuteron properties (form factors) also look good Shi et al., PRC **106** 015505 (2022)

Minimally non-perturbative pion scheme

a.k.a. Minimally Modified Weinberg scheme (MMW)



- **Leading order (LO): $\mathcal{O}(0)$**
  - OPE for $^3S_1$-$^3D_1$, $^1S_0$, $^3P_0$
  - contact terms for each of these
  - only for $^3P_0$ this is a promotion compared to naive counting

Wu+Long, PRC 2019

- **Next-to-leading order (NLO): $\mathcal{O}(Q/M_{hi})$**
  - OPE for other partial waves: $^1P_1$, $^3P_1$, $^3P_2$-$^3F_2$, ...
  - correction to $^1S_0$ contact term

Long+Yang, PRC 2012

- **Next-to-leading order (N2LO): $\mathcal{O}(Q^2/M_{hi}^2)$**
  - two-pion exchange (TPE)
  - **three-nucleon forces**

following Friar, Few-Body Syst. **22** 161 (1997)

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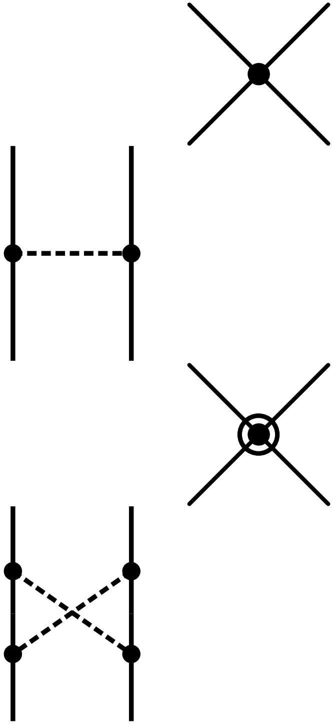

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- **all subleading corrections treated in strict perturbation theory**

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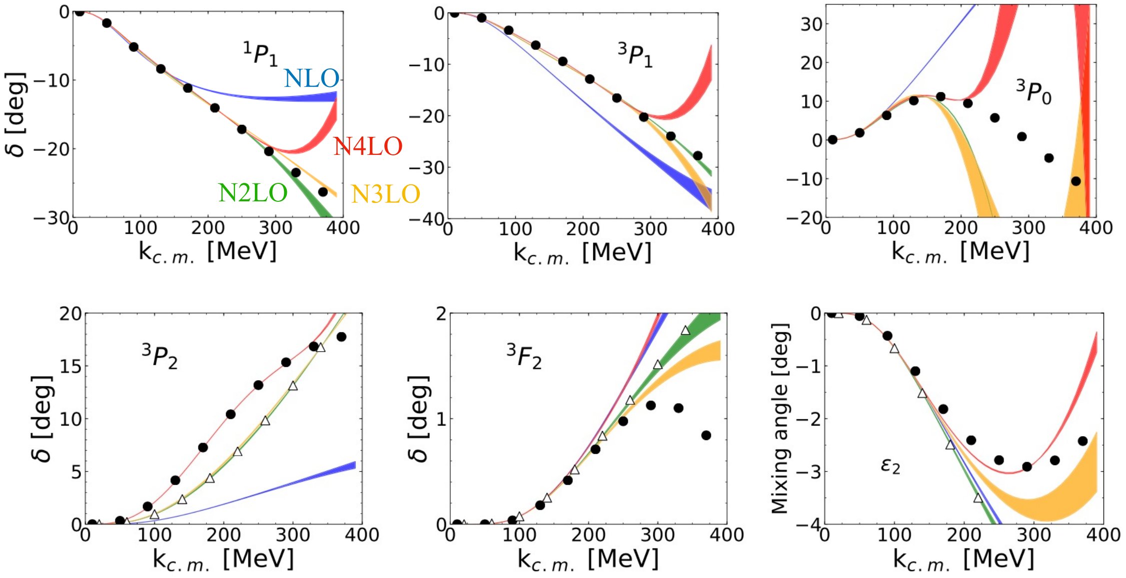

- **good convergence of two-nucleon P-waves (except $^3P_0$) up to $k \sim 300$ MeV**
- similar picture for yet higher partial waves
- triangles: OPE + once iterated, circles: SAID data

Wu + Long (2019); SAID: gwdac.phys.gwu.edu

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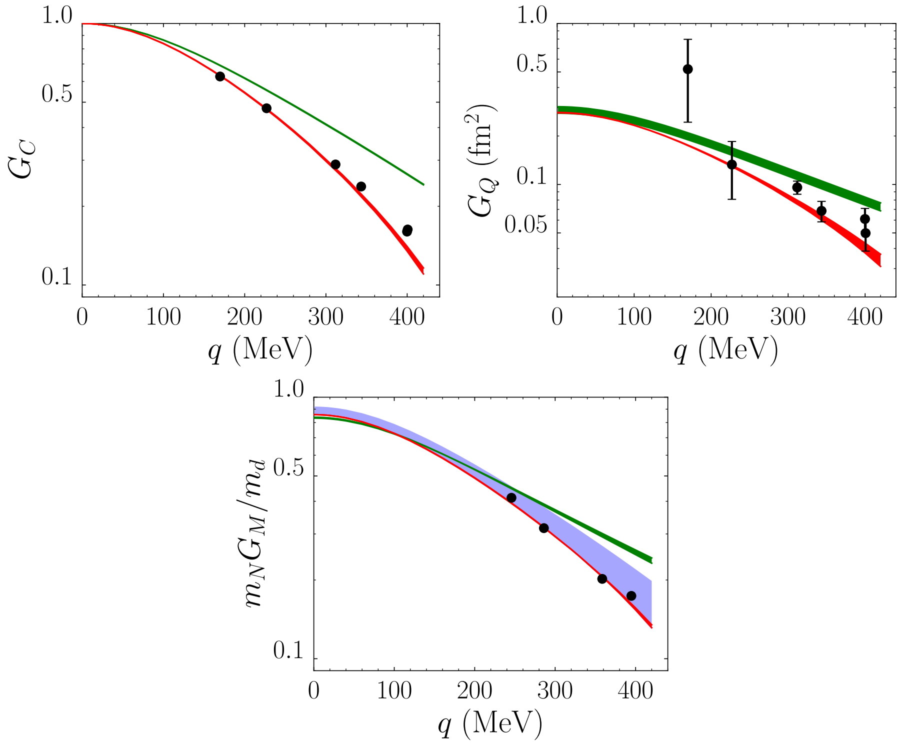

plots from Shi et al. (2022); with data from Abbot et al., EPJA **7** 421 (2000), Simon et al., NPA **364** 285 (1981)

Chiral EFT for two-nucleon system

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- static deuteron properties (form factors) also look good Shi et al., PRC **106** 015505 (2022)
- next important test: **breakup dynamics!**

Chiral EFT for two-nucleon system

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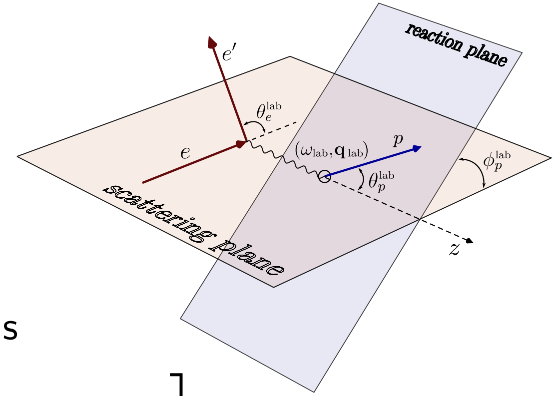
Deuteron electrodisintegration



- consider electrons scattering of a deuteron target
- virtual photon transfers energy ω and momentum $\mathbf{q}$
- cross section parameterized by **response functions**
- separated into longitudinal and transverse polarizations

$$\frac{d\sigma}{d\Omega d\omega} = \sigma_{\text{Mott}}\left[\frac{Q^4}{\mathbf{q}^4}\mathcal{R}_{\text{L}}(\omega,\mathbf{q}) + \left(\frac{Q^2}{2\mathbf{q}^2} + \tan\frac{\theta_e}{2}\right)\mathcal{R}_{\text{T}}(\omega,\mathbf{q})\right]$$

note: $\mathcal{R}_{\text{L}}$ calculated in Weinberg scheme by Yang + Phillips, EPJA **49** 122 (2013)

- these can be elegantly calculated via the **Lorentz Integral Transform (LIT)**
  - transforms the continuum problem to an effective bound-state calculation
  - can be implemented with a variety of methods

Efros et al., PLB **338** 130 (1994); JPG **34** R459 (2007)

The Lorentz Integral Transform (LIT)

Basic idea

- integrate the response function with a Lorentzian kernel
- depends on an arbitrary width parameter σ_I

$$\Phi(\sigma) = \int_{\omega_{\text{th}}}^{\infty} \mathrm{d}\omega\, \mathcal{R}(\omega) L(\omega,\sigma) \quad , \quad L(\omega,\sigma) = \frac{1}{(\omega - \sigma_R)^2 + \sigma_I^2}$$

Efros et al., PLB **338** 130 (1994); JPG **34** R459 (2007)

Practical implementation

- calculate the LIT $\Phi(\sigma)$ directly!
  - essentially: **solve an inhomogeneous Schrödinger-like equation**
  - schematically: $(H - E - \sigma^*)|\Phi\rangle = \rho|\Psi\rangle$
  - $|\Psi\rangle$ = initial bound state with energy E, $\Phi(\sigma) = \langle\Phi|\Phi\rangle - |\langle\Psi|\Phi\rangle|^2$

Final step

- **invert the LIT** to obtain the response function of interest
- this a delicate procedure that requires careful regularization
  - in principle, the inversion belongs to a class of ill-posed problems

Perturbative expansion

Premise

- consider the **expansion of the potential** (following the EFT power counting)

$$V = V^{(0)} + V^{(1)} + V^{(2)} + \cdots$$

  - only the leading order part is treated nonperturbatively
  - everything else is treated in strict perturbation theory
- all other quantities have an **induced perturbative expansion**
  - binding energy $B = B^{(0)} + B^{(1)} + B^{(2)} + \cdots$
  - bound state $|\Psi\rangle = |\Psi^{(0)}\rangle + |\Psi^{(1)}\rangle + |\Psi^{(2)}\rangle + \cdots$
- the **current operator is expanded consistently with the potential**

$$\rho = \rho^{(0)} + \rho^{(1)} + \rho^{(2)} + \cdots$$

Computational approach

- solve **inhomogeneous equations** to obtain state corrections
- then everything else can be obtained as matrix elements

Example

Consider the bound-state expansion

- at leading order, we just have the **standard Schrödinger equation:**

$$(-B_0 - H_0)|\Psi^{(0)}\rangle = V^{(0)}|\Psi^{(0)}\rangle$$

  - **note:** H_0 here is just the free Hamiltonian
- higher-order corrections are solutions of **inhomogeneous Schrödinger equations:**

$$(-B^{(0)} - H_0 - V^{(0)})|\Psi^{(1)}\rangle = (V^{(1)} + B^{(1)})|\Psi^{(0)}\rangle$$

$$(-B^{(0)} - H_0 - V^{(0)})|\Psi^{(2)}\rangle = (V^{(1)} + B^{(1)})|\Psi^{(1)}\rangle + (V^{(2)} + B^{(2)})|\Psi^{(0)}\rangle$$

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- **this is just standard textbook perturbation theory!**
  - but we are not solving explicitly for $|\Psi^{(1)}\rangle$, $|\Psi^{(2)}\rangle$, etc.
  - because solving linear equations is numerically more stable and efficient

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- **this is just standard textbook perturbation theory!**
  - but we are not solving explicitly for $|\Psi^{(1)}\rangle$, $|\Psi^{(2)}\rangle$, etc.
  - because solving linear equations is numerically more stable and efficient
- **useful variation:** reformulate in terms of Green's function $G_0 = (-B^{(0)} - H_0)^{-1}$

$$(G_0 V^{(0)} - 1)|\Psi^{(1)}\rangle = G_0\,(E^{(1)} - V^{(1)})|\Psi^{(0)}\rangle \quad \text{etc.}$$

How to do this for the LIT?

Perturbative LIT calculation

Starting point

- $\Phi(\sigma) = \langle\Phi|\Phi\rangle - |\langle\Psi|\Phi\rangle|^2$ with $(H - E - \sigma^*)|\Phi\rangle = \rho|\Psi\rangle$
- employ **multiple expansion**: $\Phi(\sigma) = \sum_L \Phi_L(\sigma)$
  - solve equation separately for each multipole $L \rightsquigarrow |\Phi_L\rangle$

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First step

- solve perturbatively expanded LIT equations

$$\big(G_0 V^{(0)} - 1\big)|\Phi^{(0)}\rangle = G_0 \rho^{(0)} |\Psi^{(0)}\rangle$$

$$\big(G_0 V^{(0)} - 1\big)|\Phi^{(1)}\rangle = G_0 \big(E^{(1)} - V^{(1)}\big)|\Phi^{(0)}\rangle + G_0 \Big[\rho^{(1)}|\Psi^{(0)}\rangle + \rho^{(0)}|\Psi^{(1)}\rangle\Big]$$

$$\big(G_0 V^{(0)} - 1\big)|\Phi^{(2)}\rangle = \cdots$$

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\big(G_0 V^{(0)} - 1\big)|\Phi^{(2)}\rangle &= \cdots
\end{aligned}$$

Second step

- consistently expand the overlap matrix elements!
  - define $\mathcal{A}^{(0)} = \langle\Phi^{(0)}|\Phi^{(0)}\rangle$, $\mathcal{A}^{(1)} = \langle\Phi^{(0)}|\Phi^{(1)}\rangle + \langle\Phi^{(1)}|\Phi^{(0)}\rangle$, etc.
  - similarly: $\mathcal{B}^{(n)}$ for the $\langle\Phi|\Psi\rangle$ overlaps
- then $\Phi^{(0)} = \mathcal{A}^{(0)} - |\mathcal{B}^{(0)}|^2$, $\Phi^{(1)} = \mathcal{A}^{(1)} - \mathcal{B}^{(0)}\mathcal{B}^{(1)*} - \mathcal{B}^{(1)}\mathcal{B}^{(0)*}$, etc.

The current operator

- **consistent expansion of external currents is a key feature of EFT**

$$\rho = \rho^{(0)} + \rho^{(1)} + \rho^{(2)} + \cdots$$

  - the chiral e.m. current has been studied in great detail

    see e.g. Kölling et al. PRC **80** 054008 (2011), Pastore et al. PRC **84** 024001 (2011); ...; Krebs EPJA **56** 234 (2020)

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Leading order (LO)

- **just the simple one-nucleon current:**

$$\langle \mathbf{p} | \rho^{(0)}(\mathbf{q}) | \mathbf{p}' \rangle \sim \frac{1+\tau_3}{2} \delta^{(3)} \left(\mathbf{p} - \mathbf{p}' - \mathbf{q}/2 \right)$$

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Next-to-leading order (NLO)

- no correction to charge density (but to other operators)

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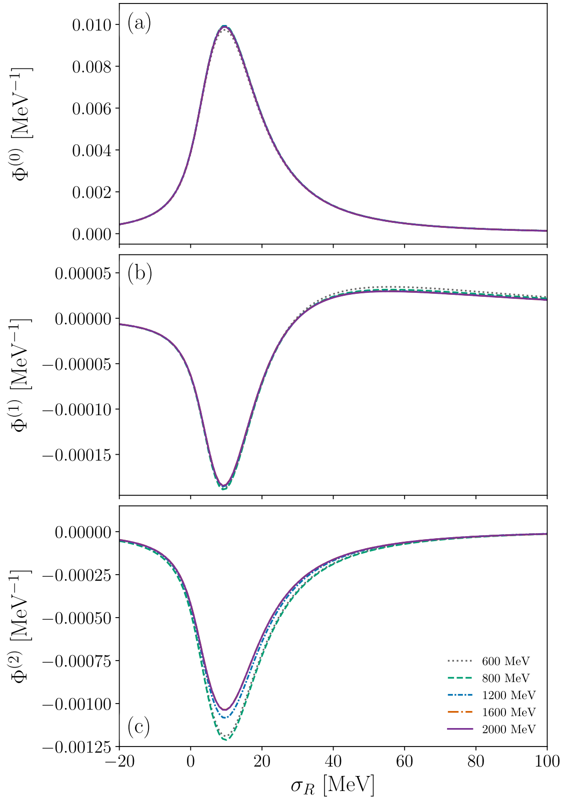
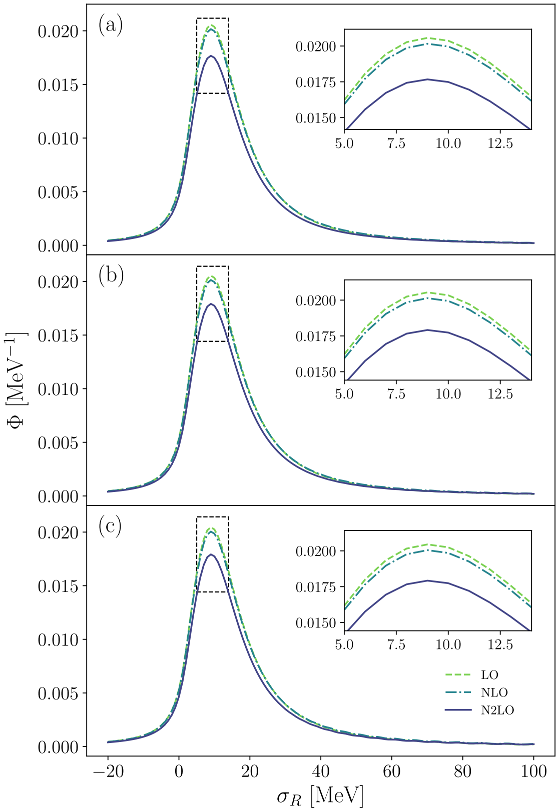
Next-to-next-to-leading order (N2LO)

- **here we get various corrections:**
  - $\rho^{(2)}_{\text{str}}(\mathbf{q};\mathbf{p},\mathbf{p}') \sim -q^2 \langle r_s^2 \rangle / 6 \times \langle \mathbf{p} | \rho^{(0)}(\mathbf{q}) | \mathbf{p}' \rangle$
  - $\rho^{(2)}_{\text{rel}}(\mathbf{q};\mathbf{p},\mathbf{p}') \sim -\frac{1}{8M_N^2}\left(\frac{1}{2} + \kappa_s + (\frac{1}{2} + \kappa_v)\tau_3\right)\left(q^2 + 2i\mathbf{q}\cdot\boldsymbol{\sigma}\times\mathbf{K}\right)\delta^{(3)}(\cdots)$
  - **boost correction** (deuteron calculated in rest frame!)

EFT convergence

- **overall, the calculation converges well**
  - with respect to increasing the cutoff Λ (RG invariance ✔)
  - with respect to increasing EFT order (power counting ✔)





Perturbative LIT results

- LIT is a powerful tool that can be used with different few- and many-body methods

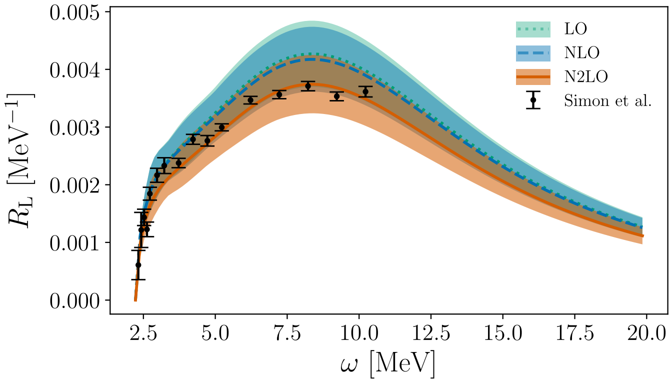
Bacca et al., PRL **89** 052502 (2002); Miorelli et al., PRC **94** 034317 (2016); ...

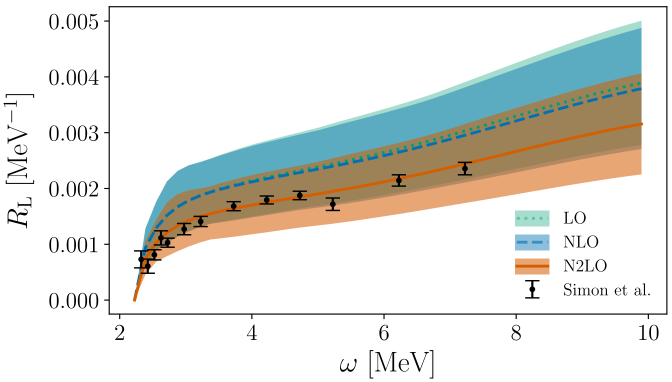
- essentially solves the continuum problem with bound-state methods
- now adapted to systematically treat the **EFT expansion in perturbation theory**

Andis, Lyu, Long, SK, 2512.12823 [nucl-th]

$$\Phi_L(\sigma) = \Phi_L(\sigma)^{(0)} + \Phi_L(\sigma)^{(1)} + \Phi_L(\sigma)^{(2)} + \cdots$$

- based on integral formulation: $(G_0 V - 1)|\Phi\rangle = G_0 \rho |\Psi\rangle$ Martinelli et al., PRC **52** 1778 (1995)
- all parts expanded consistently, including charge density ρ and bound state $|\Psi\rangle$
  - includes first relativistic corrections at N2LO





- good agreement with experimental data at $q^2 = 0.6$ (left) and 1.0 (right) fm^{-2}

Simon et al., NPA **324** 277 (1979)

Part II: Light nuclei with perturbative pions

Part II: Light nuclei with perturbative pions

Yes, fully perturbative pions now.

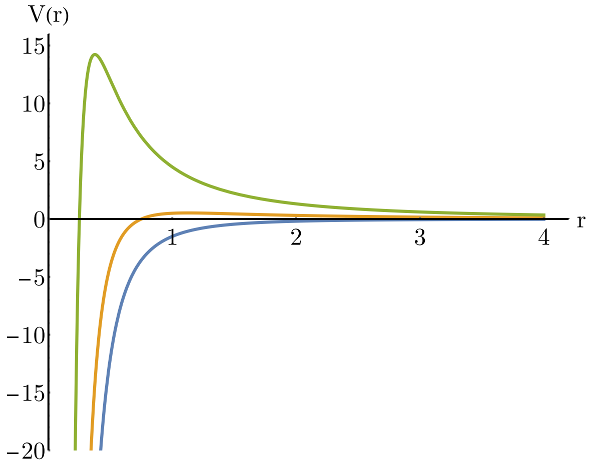
Perturbative pions?

- consider the effective total potential:

$$V(r) = V_{\mathrm{OPE}}(r) + \frac{L(L+1)}{r^2}$$

- for $L > 0$, this "shields" the nucleons from the singular attraction of $V_{\mathrm{OPE}}(r)$
- critical momentum characterizes perturbativeness

Birse, PRC 2006



KSW counting

- argument above does not apply in NN S waves, of course
- however, it can be argued by dimensional analysis that OPE is overall perturbative
    - S waves nonperturbative only in the short-range interaction Kaplan et al., PLB/NPB 1998
- compelling idea, alas: poor convergence properties Cohen+Hansen 1999; Fleming et al., NPA 2000
- more recent work: problematic only in low partial waves! Wu + Long 2019; Kaplan, PRC 2020

Perturbative pions!

Observation 1

- KSW scheme works for two-nucleon partial waves with $l > 0$, except ${}^3P_0$
  - for center-of-pass momenta up to ∼300 MeV

Wu + Long (2019); Kaplan, PRC (2020)

- however, Born approximation with OPE + contact works for ${}^3P_0$ scattering
  - **${}^3P_0$ converges with a P-wave contact promoted to NLO!**

Peng, Lyu, Long (2020)

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- ${}^3S_1$-${}^3D_1$ mixing angle vanishes in the combined unitarity+chiral limit
  - $1/a_{{}^3S_1} \to 0$, $m_\pi \to 0$ implies $\epsilon \to 0$
  - this happens despite the tensor force still being strong in the chiral limit
  - however, this is an on-shell only effect

Lyu, Zuo, Peng, SK + Long, 2511.12522 [nucl-th]

- at N2LO, there is still a large correction to the mixing angle (and ${}^3D_1$ phase shift)
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Bottom line

- **together, these two modifications fix perturbative pions for 2N scattering**

The PPI scheme

Leading order (LO) -- standard Pionless EFT

- two NN S-wave contact interactions ($^3S_1$, $^1S_0$)
- **3N contact interaction** (fit to reproduce ^{3}H energy)

Next-to-leading order (NLO)

- **one-pion exchange** in partial waves up to D waves (plus coupled channels)
  - acommpanied by momentum-dependent contact interactions as needed
- **promoted SD mixing contact interaction (N2LO → NLO)**
- **terms from standard Pionless EFT**
  - S-wave contact interactions with derivatives (→ effective range)
  - **four-body contact interaction**

Bazak, Kirscher, SK et al., PRL **122** 143001 (2019)

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Bazak, Kirscher, SK et al., PRL **122** 143001 (2019)

Note

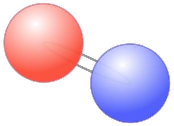
- **related recent work takes a different perspective:**
  - unitarity limit / SU(4) symmetry more important than chiral symmetry
  - tensor component of OPE demoted to higher orders

Teng + Grießhammer, EPJA **61** 9 (2025); Grießhammer, PoS CD2024

Numerical approach

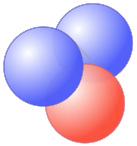
Unified (2-, 3-, 4-body) numerical framework

Two-body system



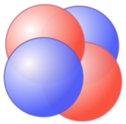
- separable regulator for contact interactions: $V = C_0|g\rangle\langle g|$
- can be solved analytically to get scattering amplitudes
- numerical calculation for other observables (like breakup, just discussed)

Three-body system



- Faddeev equations: $|\psi\rangle = G_0 t P|\psi\rangle + (G_0 + G_0 t G_0)V_3|\Psi\rangle$
- full wave function: $|\Psi\rangle = (1+P)|\psi\rangle$

Four-body system



- Faddeev-Yakubowsky equations: two components $|\psi_{A,B}\rangle$
- full wave function involves both components:
  - $|\Psi\rangle = (1-P_{34}-PP_{34})(1+P)|\psi_A\rangle + (1+P)(1+\tilde{P})|\psi_B\rangle$

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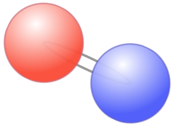

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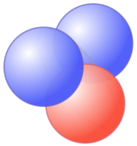

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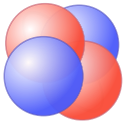

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- **rigorous perturbation theory:** $|\Psi\rangle = |\Psi^{(0)}\rangle + |\Psi^{(1)}\rangle + \cdots$

SK, EPJA **56** 113 (2020)

Perturbative Faddeev scheme

Basic setup

SK, EPJA **56** 113 (2020)

- full wave function: $|\Psi\rangle = (1 + P)|\psi\rangle$
- perturbative expansion: $|\Psi\rangle = |\Psi^{(0)}\rangle + |\Psi^{(1)}\rangle + \cdots$
- Faddeev equation: $|\psi^{(0)}\rangle = K^{(0)}|\psi^{(0)}\rangle$ with $K^{(0)} = G_0 t^{(0)} P$

$\rightsquigarrow$ NLO energy shift $B^{(1)} = \langle\Psi^{(0)}|V^{(1)}|\Psi^{(0)}\rangle$

Wave-function correction

- set $K^{(1)} = B^{(1)}(G_0 + G_0 t^{(0)} G_0) + G_0 t^{(1)} P$
- then $(\mathbf{1} - K^{(0)})|\psi^{(1)}\rangle = K^{(1)}|\psi^{(0)}\rangle$
  - singular part from LO solution projected out
- **need only solve linear system with same kernel as LO equation!**

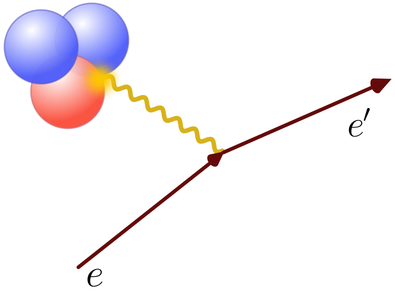
$\rightsquigarrow$ N2LO energy corrections $B^{(2)} = \langle\Psi^{(1)}|V^{(1)}|\Psi^{(0)}\rangle + \langle\Psi^{(0)}|V^{(2)}|\Psi^{(0)}\rangle$

$\rightsquigarrow$ **NLO corrections for operators (form factor → radius)**

- **Works essentially the same way for Faddeev-Yakubowsky equations!**

Charge radii

- **calculate charge form factors**
  - $F_C(q) = \langle\Psi|\rho(q)|\Psi\rangle \rightsquigarrow \langle r^2\rangle = -\frac{1}{6}\frac{d^2}{dq^2}F_C(q)$, $q \to 0$
  - $|\Psi\rangle =$ full few-body wavefunction
  - $\rho(q) =$ charge density operator
- point charge radii: subtract effects from r_p and r_n



Perturbative corrections

- **need corrections to wavefunctions:** $|\Psi\rangle = |\Psi_0\rangle + |\Psi_1\rangle + \cdots$
- can be obtained from inhomogeneous Faddeev/Faddeev-Yakubowsky equations
  - $[1 - G_0t_0P]\,|\psi_1\rangle = B_1(G_0 + G_0t_0G_0)|\psi_0\rangle + G_0t_1P|\psi_0\rangle$
  - inclusion of three-body forces somewhat tedious but straightforward
  - note: LO kernel is singular at $E = -B_0$
  - calculate $|\Psi_1\rangle = (1+P)|\psi_1\rangle$, enforce orthogonality to $|\Psi_0\rangle$

- $F_{C,0}(q) = \langle\Psi_0|\rho(q)|\Psi_0\rangle$, $F_{C,1}(q) = 2\langle\Psi_1|\rho(q)|\Psi_0\rangle$, $\ldots$
- analogous expansion for $\langle r^2\rangle$

SK, EPJA **56** 113 (2020)

A note on computational efficiency

- in the F/FY scheme, the interaction determines which "channels" contribute
  - "channel" $=$ momentum space 3/4-body partial-wave state
  - e.g., $|u_1, u_2; s\rangle$ with $|s\rangle = |\,\big(l_2\big((l_1 s_1)j_1\tfrac{1}{2}\big)s_2\big)J;\,\big(t_1\tfrac{1}{2}\big)T\,\rangle$ for 3N calculation
- leading oder setup has few channels with perturbative pions (partly of PPI)
- but at NLO the number of channels increases fast
- **state corrections span the full NLO space!**

NLO equation setup

- for $|\psi\rangle = |\psi^{(0)}\rangle + |\psi^{(1)}\rangle + \cdots$, the relevant equations are (schematically):

$$\begin{aligned}(1-K^{(0)})|\psi^{(0)}\rangle &= 0\\ (1-K^{(0)})|\psi^{(1)}\rangle &= K^{(1)}|\psi^{(0)}\rangle\end{aligned}$$

- $|\psi^{(1)}\rangle$ is non-vanishing in more channels than $|\psi^{(0)}\rangle$
- naively this seems to imply that the "kernel" $K^{(0)}$ grows with order...
- **...but there is a way to avoid this!**

A note on computational efficiency

- **partition the total set of channels at NLO:**
  - $\mathcal{S}_0$ = channels already in LO kernel, $\mathcal{S}_1$ = new channels at NLO
- then the kernel can be written in block form

$$\left[\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} K_{00}^{(0)} & K_{01}^{(0)} \\ 0 & 0 \end{pmatrix}\right] \begin{pmatrix} |\psi_0^{(1)}\rangle \\ |\psi_1^{(1)}\rangle \end{pmatrix} = \begin{pmatrix} K_{00}^{(1)} & K_{01}^{(1)} \\ K_{10}^{(1)} & K_{11}^{(1)} \end{pmatrix} \begin{pmatrix} |\psi_0^{(0)}\rangle \\ 0 \end{pmatrix}$$

- the second row of this can be solved trivially: $|\psi_1^{(1)}\rangle = K_{10}^{(1)}|\psi_0^{(0)}\rangle$
- inserting this into the first row gives for the other part:

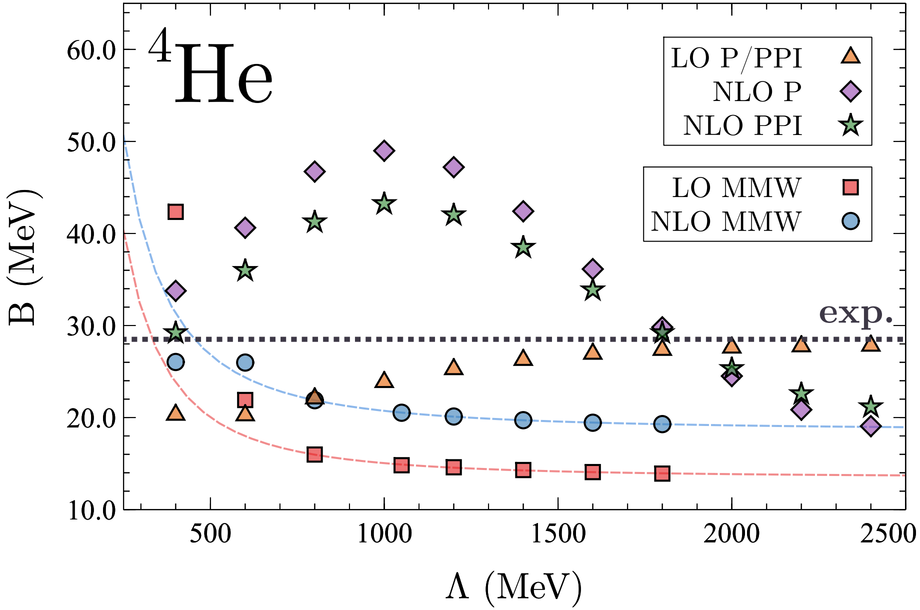
$$(1 - K_{00}^{(0)})|\psi_1^{(1)}\rangle = (K_{00}^{(1)} + K_{01}^{(0)}K_{10}^{(1)})|\psi_0^{(0)}\rangle$$

Bottom line

- nonperturbative solving is only ever required in the $\mathcal{S}_0$ space
- everything else can be done via straightforward operator application
- **this greatly reduces both runtime and memory requirements!**

^{4}He energy at NLO

- **consider a calculation without four-nucleon force at NLO**
  - leading-order Pionless EFT (= leading order PPI) gives remarkably good result
  - shown for comparison: MMW results at LO and NLO

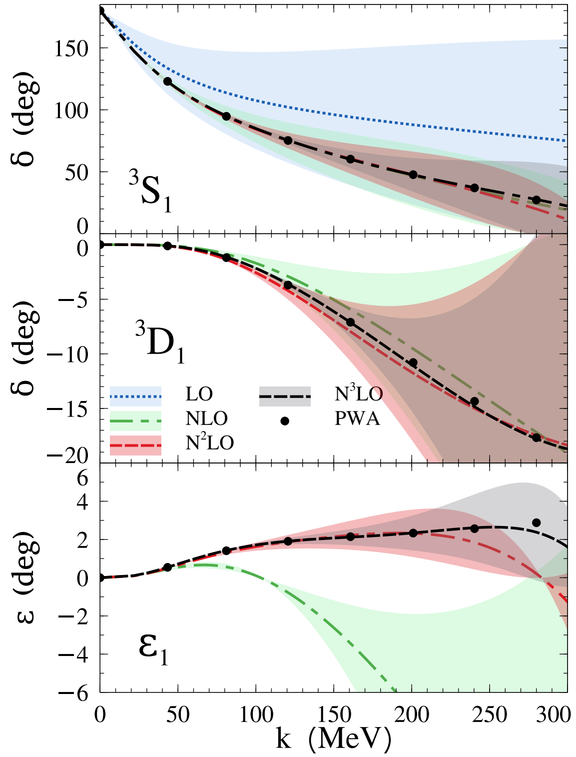


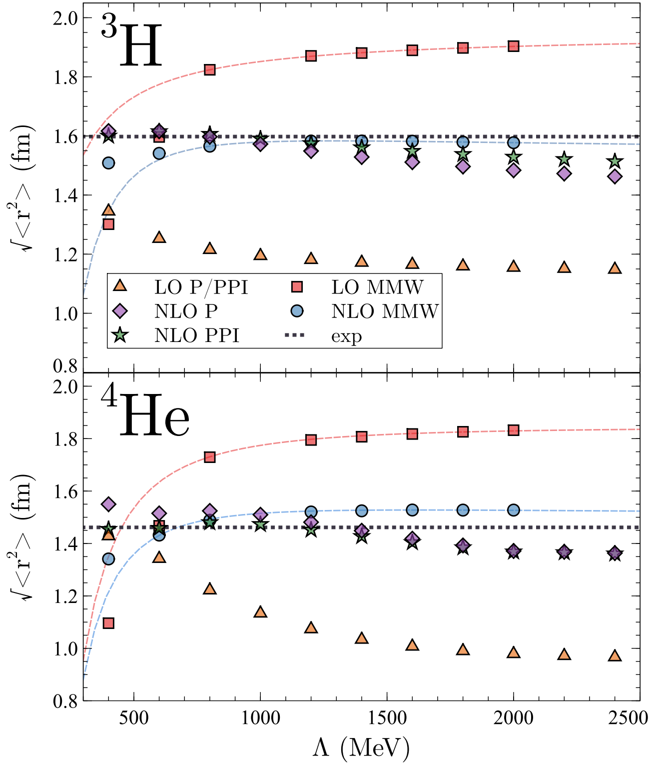
- **do perturbative pions eliminate the need for a four-nucleon force?**
  - it does not look like they do: energy does not converge at NLO

Overall results for light nuclei

- **it is possible to construct an pert. pion expansion with good convergence!**

Lyu, SK et al., 2511.12522 [nucl-th]





- uncertainty bands: $\pm(k\alpha_\pi)$ with $\alpha_\pi^{-1} \sim 270$ MeV
- almost no impact from pion exchange on 3N/4N charge radii! (from F/FY calcs.)

One more thing

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What about deuteron breakup with perturbative pions?

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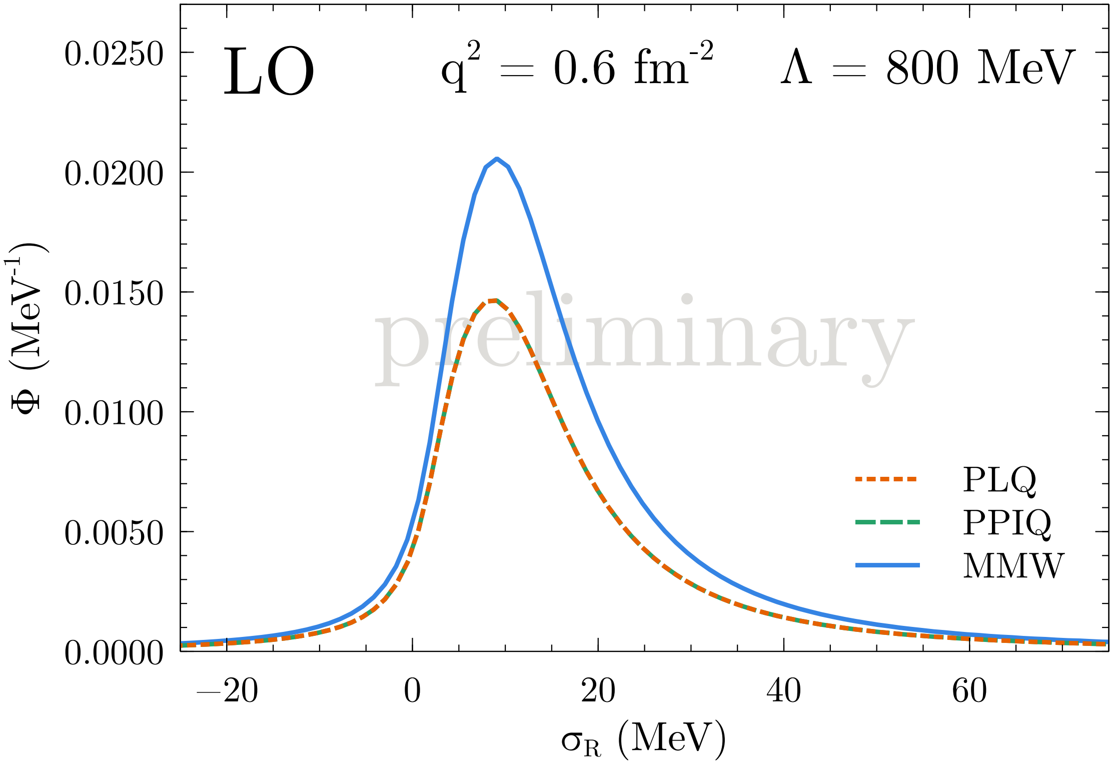
(preliminary results)

Deuteron LIT with PPI



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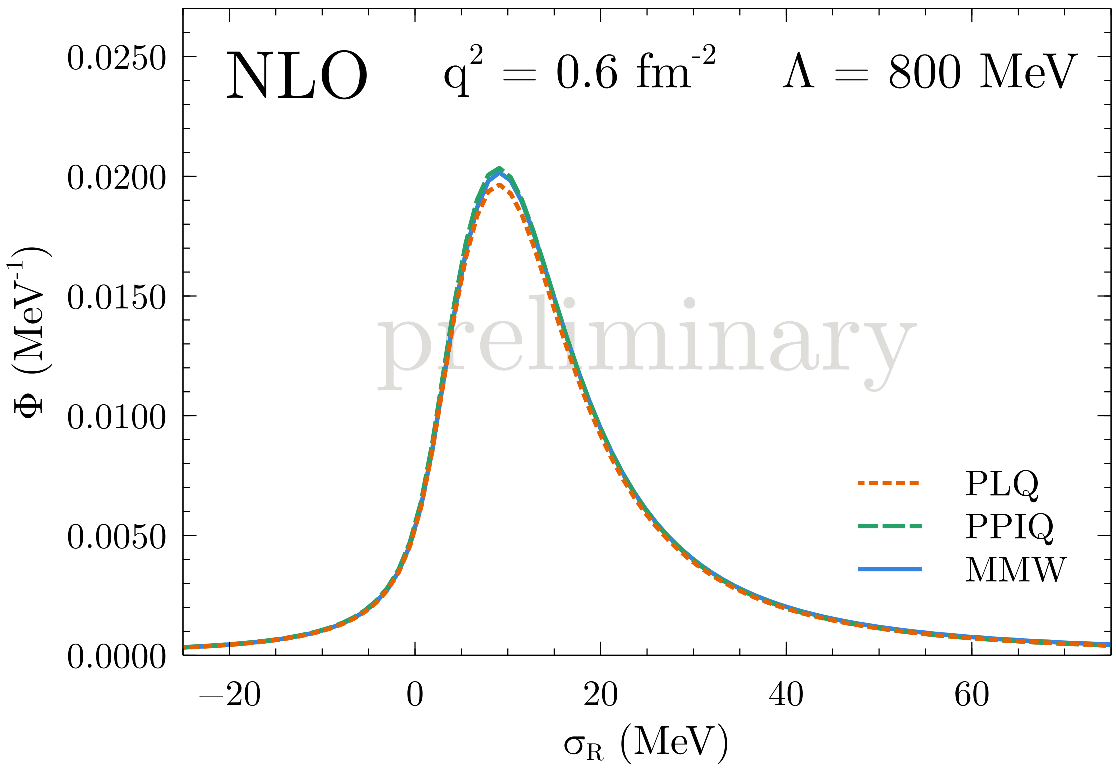

- perturbative pions almost indistinguishable from Pionless EFT at NLO

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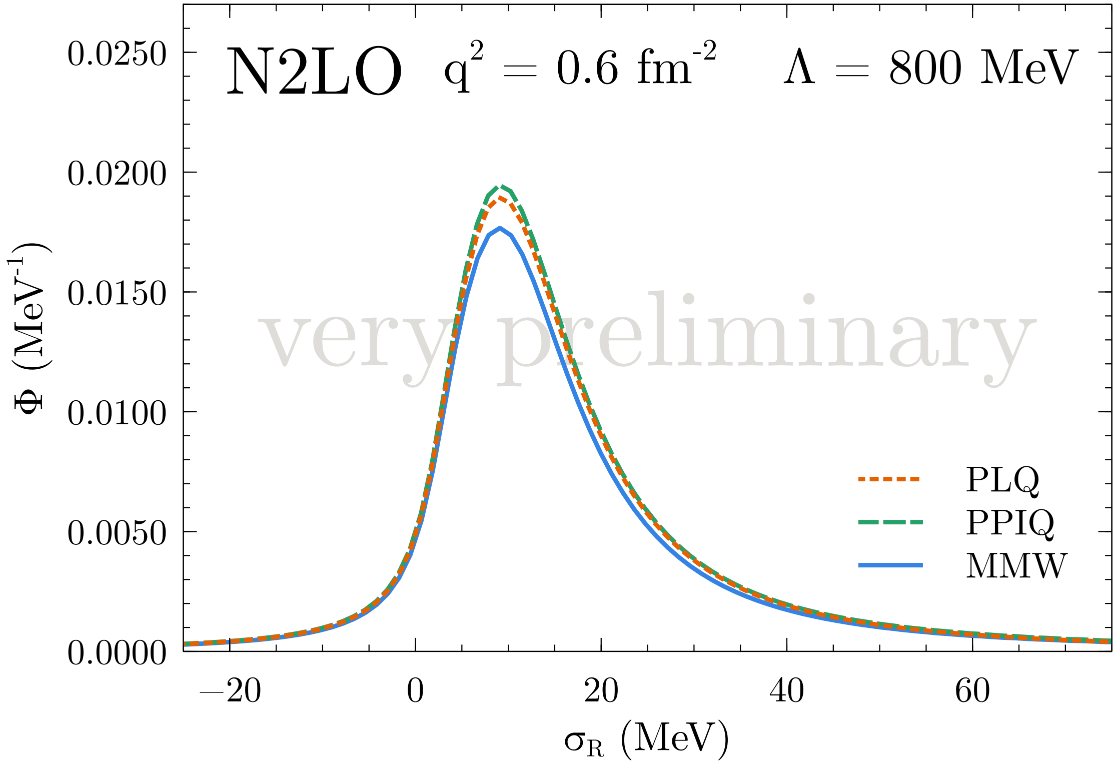

- perturbative pions almost indistinguishable from Pionless EFT at NLO and N2LO

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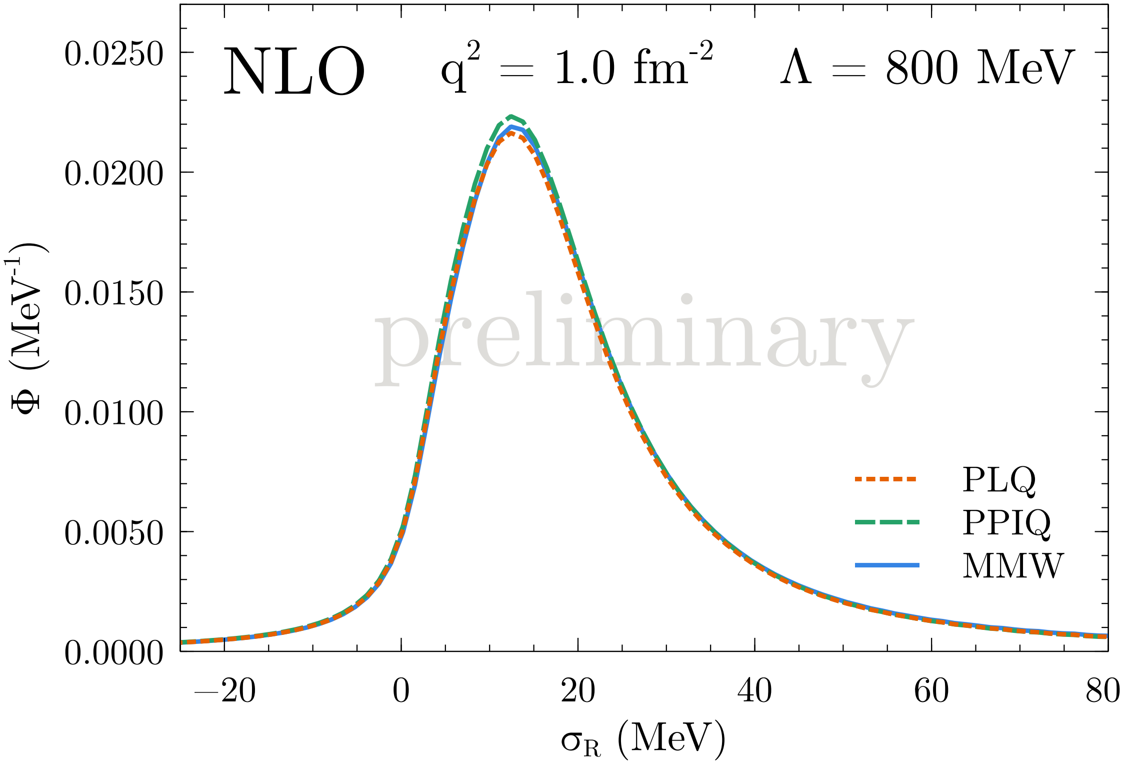

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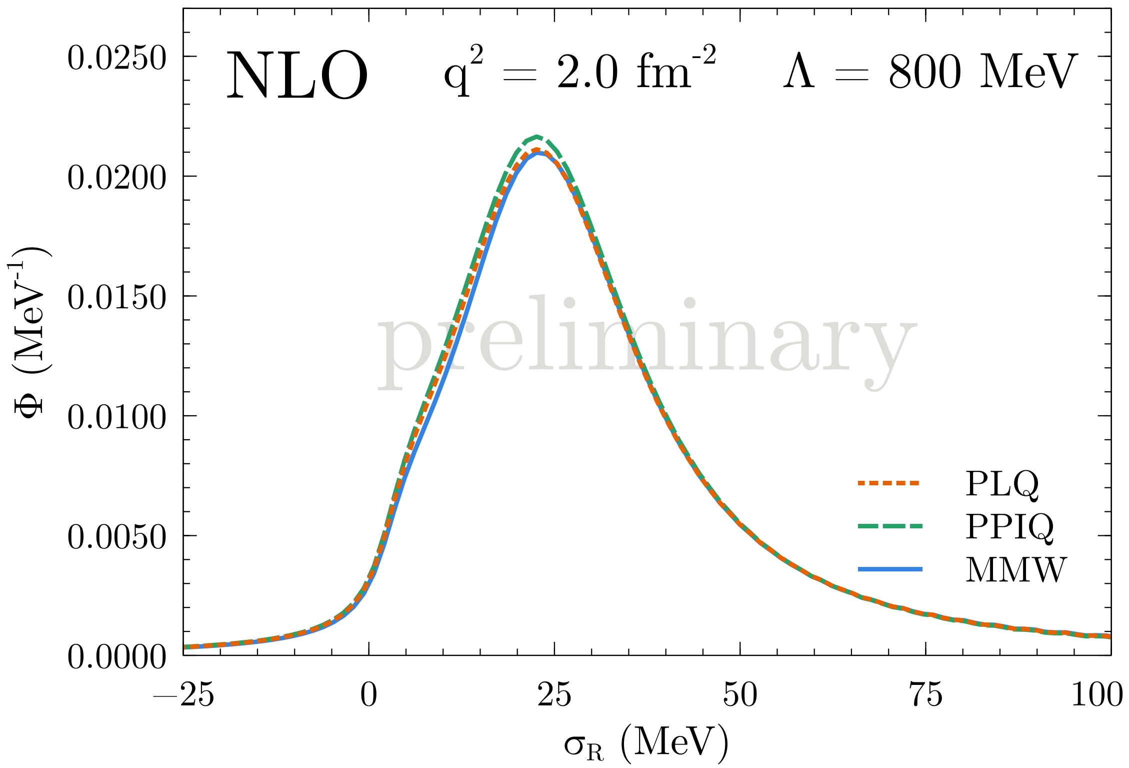

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- does not really change with increasing momentum transfer

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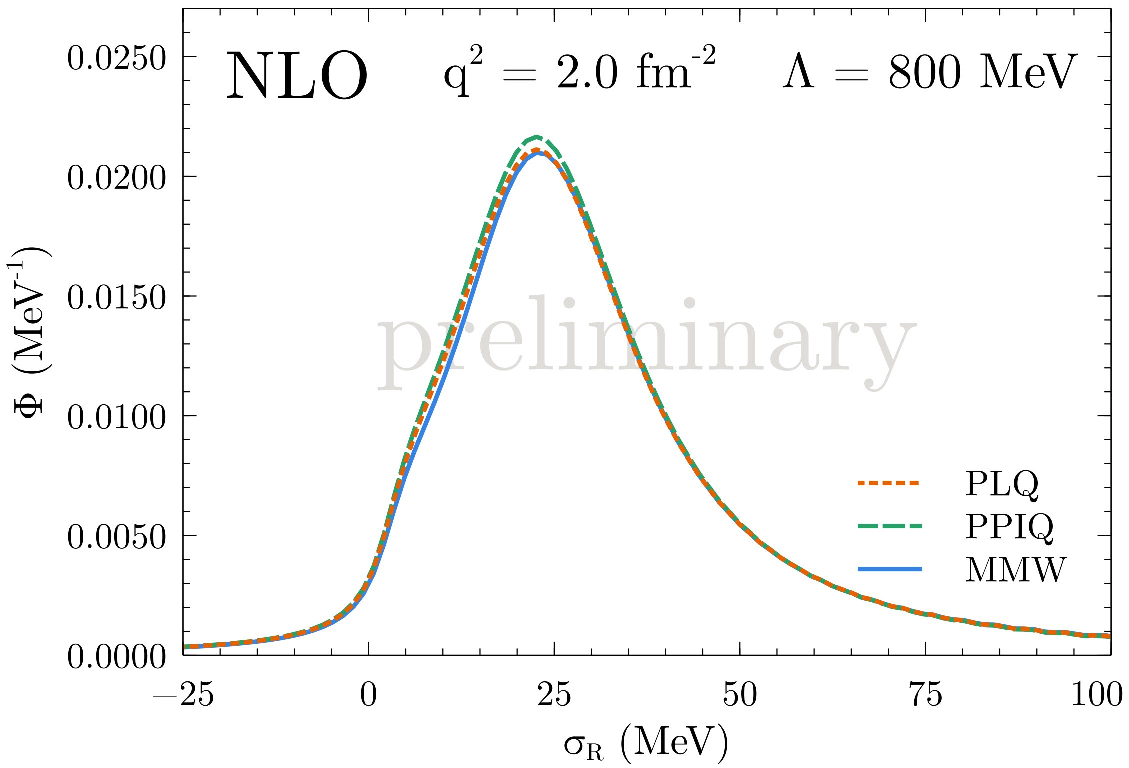

- perturbative pions almost indistinguishable from Pionless EFT at NLO
- does not really change with increasing momentum transfer
- **note:** cf. previous calculation of deuteron breakup in Pionless EFT

Christlmeier+Grießhammer PRC **77** 064001 (2008)

Summary and outlook

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- RG-invariant chiral EFT calculation deuteron breakup shows good convergence
- small modifications to KSW enable perturbative-pion expansion
- guided by simultaneous closeness to chiral limit and unitarity limit
  - good convergence for two-nucleon phase shifts
  - four-nucleon force at NLO still needed with perturbative pions
- perturbative pions have only minor effects on radii
  - ...and, apparently, on the deuteron longitudinal response function
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Outlook

- calculation of response functions for $A > 2$ systems
- comprehensive study of ^{4}He excited state
- perturbative inclusion of Coulomb repulsion
- comparison with minimally modified Weinberg counting (MMW) up to N2LO
- analysis of more observables beyond phase shifts and static properties
- heaver nuclei with PPI (and MMW)

Thanks...

...to my collaborators...

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...for support, funding, and computing time...

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...and to you, for your attention!