Proton-deuteron correlation functions in pionless effective field theory

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INT 23-1a: Intersection of nuclear structure and high-energy nuclear collisions

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FRIB Theory Alliance

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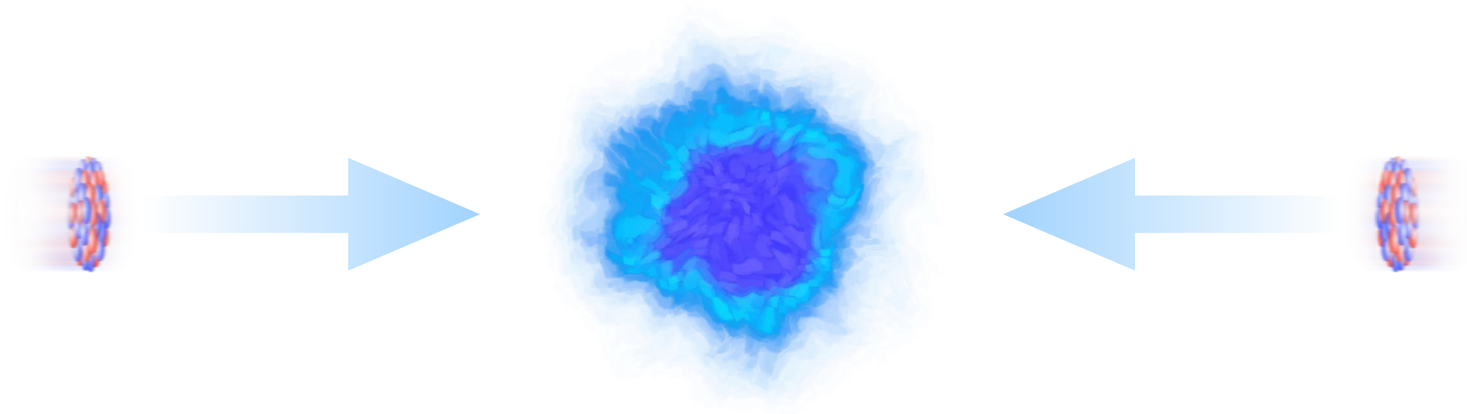
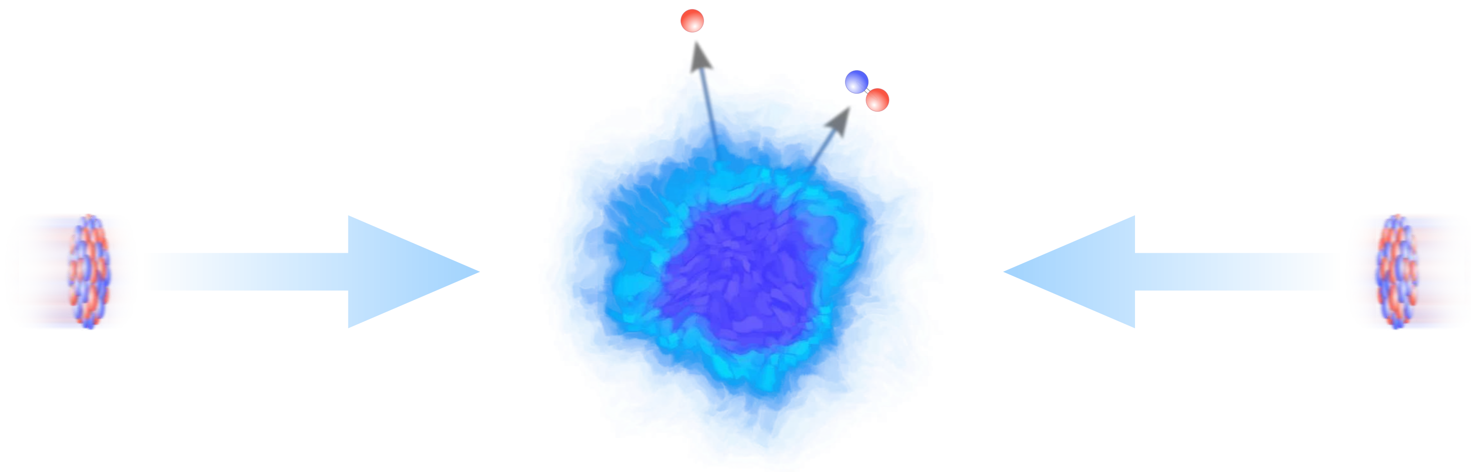
Femtoscopy

Femtoscopy

Femtoscopy

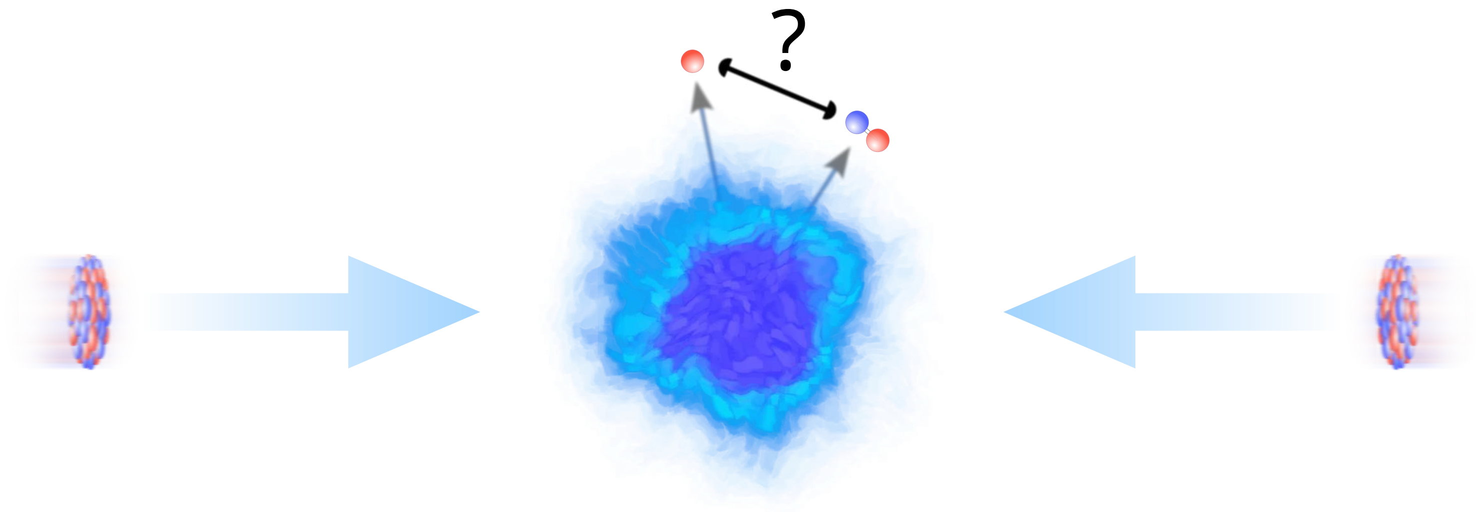


Femtoscopy



- heavy ion collisions produce hardons in the final state, including light nuclei

Femtoscopy



- heavy ion collisions produce hardons in the final state, including light nuclei
- **this can be used to study nuclear and hypernuclear interactions**

Correlation functions

Basic formalism

- consider a two-hadron system with relative center-of-mass momentum k
- let $\psi(k,r)$ be the the scattering wavefunction for the system
- then the two-particle **correlation function** can be written as

$$C(k) = 4\pi \int \mathrm{d}r\, r^2\, S(r)\, \left|\psi(k,r)\right|^2$$

where $S(r)$ is a normalized Gaussian source function

General idea

- in the absence of interactions, $C(k) \to 1$
- $C(k) \neq 1$ therefore encodes properties of the interaction between the particles
- **high-energy collisions can be analyzed to extract $C(k)$ experimentally**
  - ratio of correlated vs. uncorrelated pairs

Koonin, PLB **70** 43 (1977); Pratt, PRL **53** 1219 (1984); ...

Mihaylov et al., EPJC **78** 394 (2018); Haidenbauer, NPA **981** 1 (2019); ...

Motivation

- high-energy collisions can be used to study low-energy interactions
- this provides an interesting source for otherwise difficult to measure observables, such as scattering involving hyperons
- for $A > 2$ systems one can investigate multi-hadron forces

For this program, one should make sure that the three-nucleon system is properly understood!

Outline

Introduction ✔

Effective field theory

Faddeev formalism

Results and discussion

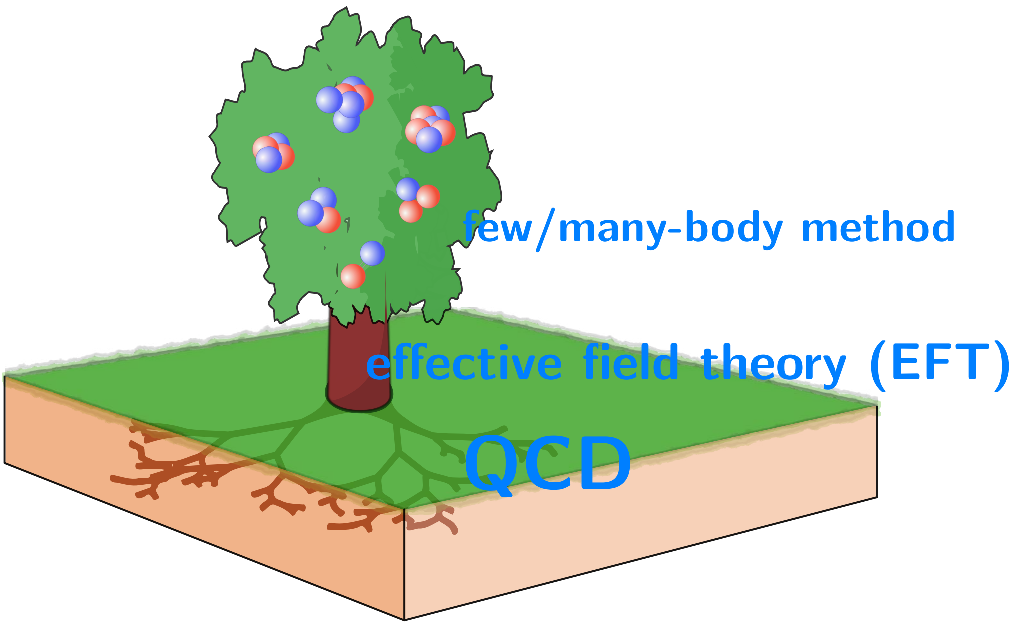
Part I

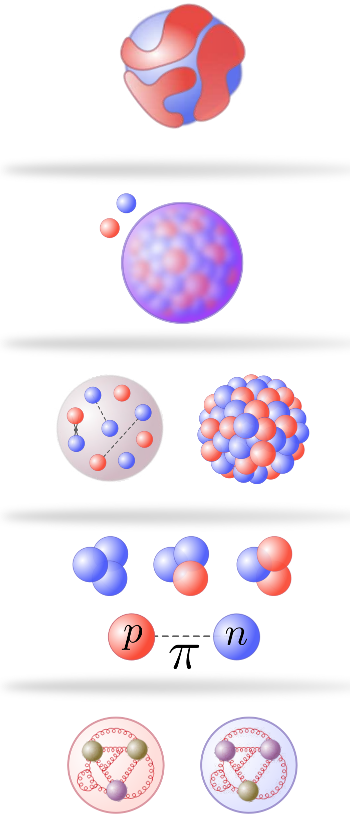
Effective Field Theory

Part I

Pionless Effective Field Theory

Nuclear theory tower



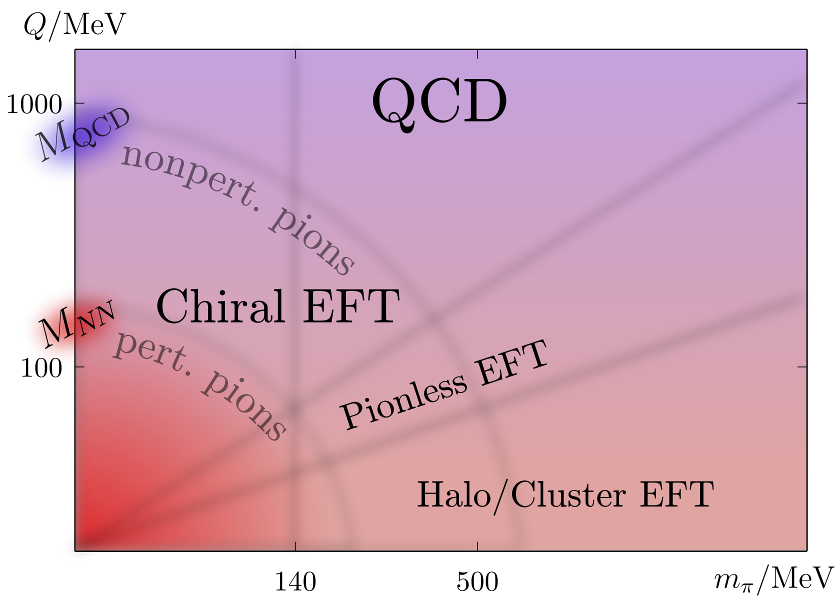


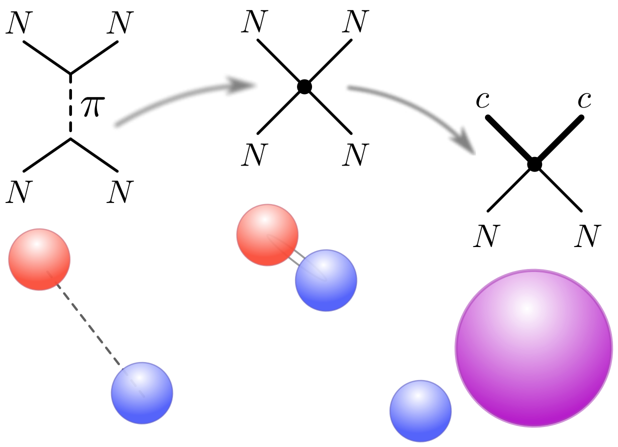
- **QCD** = underlying theory of strong interaction
- **EFT** = effective description in terms of hadrons
- **degrees of freedom depend on resolution scale**

Nuclear effective field theories

- choose **degrees of freedom** approriate to energy scale
- only restricted by **symmetry**, ordered by **power counting**

Hammer, SK, van Kolck, RMP **92** 025004 (2020)



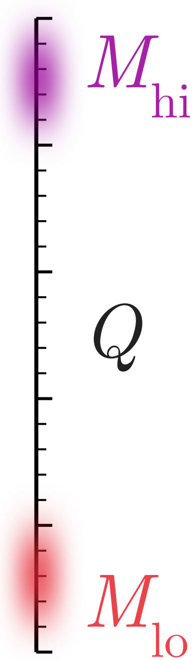


- degrees of freedom here: nucleons (and/or clusters thereof)
- even more effective d.o.f.: rotations, vibrations

Papenbrock, NPA **852** 36 (2011); ...

- most effective theory depends on energy scale and nucleus of interest

Effective Field Theory 101



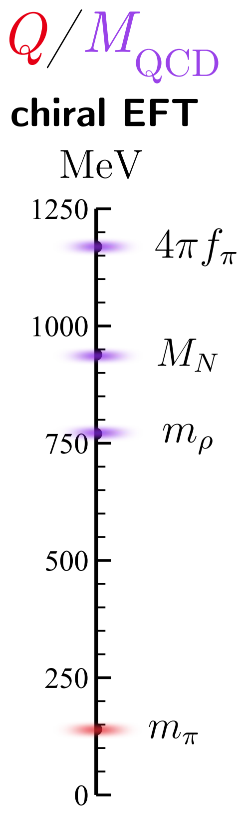
- identify **relevant symmetries** (nonrel. boosts, chiral, gauge, ...)
- identify **low and high-energy scales** (M_{lo}, M_{hi})
- identify **typical momentum scale** for given process (Q)
- pick a convenient **regulator** (e.g. cutoff) (Λ)

$$\textbf{amplitude} \;\; T(Q) \sim \sum_{\nu=0}^{\infty} \left(\frac{Q}{M_{\text{hi}}}\right)^{\nu} F^{(\nu)}\left[\ldots;\gamma^{(\nu)}\right]$$

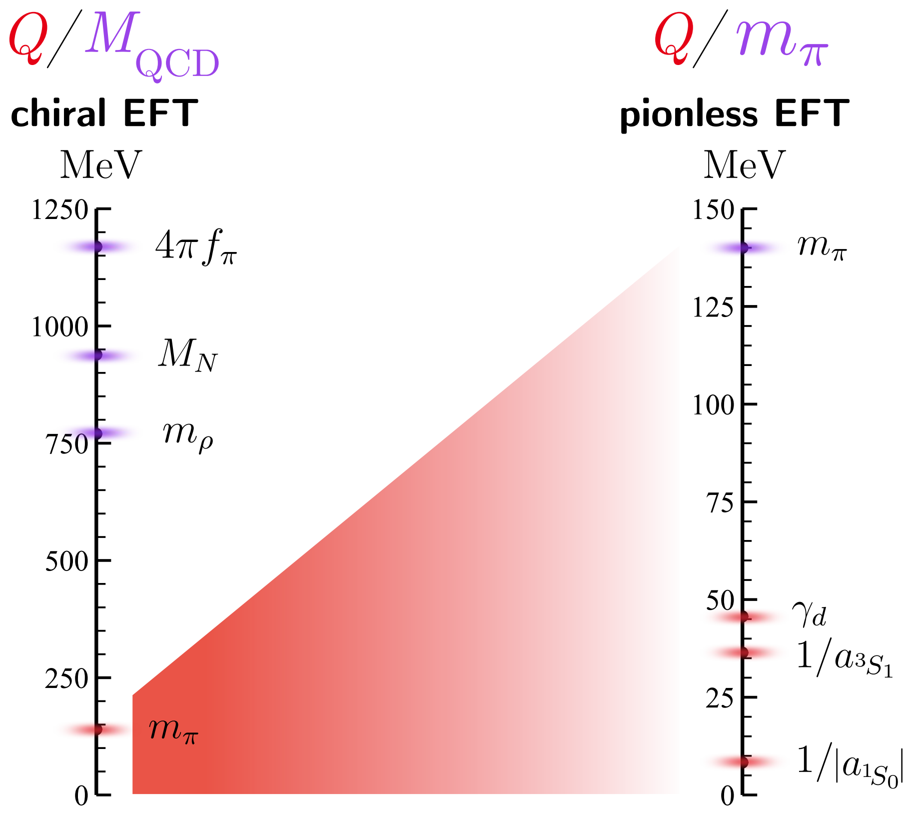
- combination of low-energy constants: $\gamma^{(\nu)}\left(\frac{M_{\text{lo}}}{M_{\text{hi}}}, \frac{\Lambda}{M_{\text{hi}}}\right)$
- encoded low-energy dynamics: $F^{(\nu)}\left(\frac{Q}{M_{\text{hi}}}, \frac{Q}{\Lambda}; \gamma^{(\nu)}\right)$

- **power counting** relates the ν to terms in effective Lagrangian
- **RG invariance** means $T = \text{const.} + \mathcal{O}(1/\Lambda)$

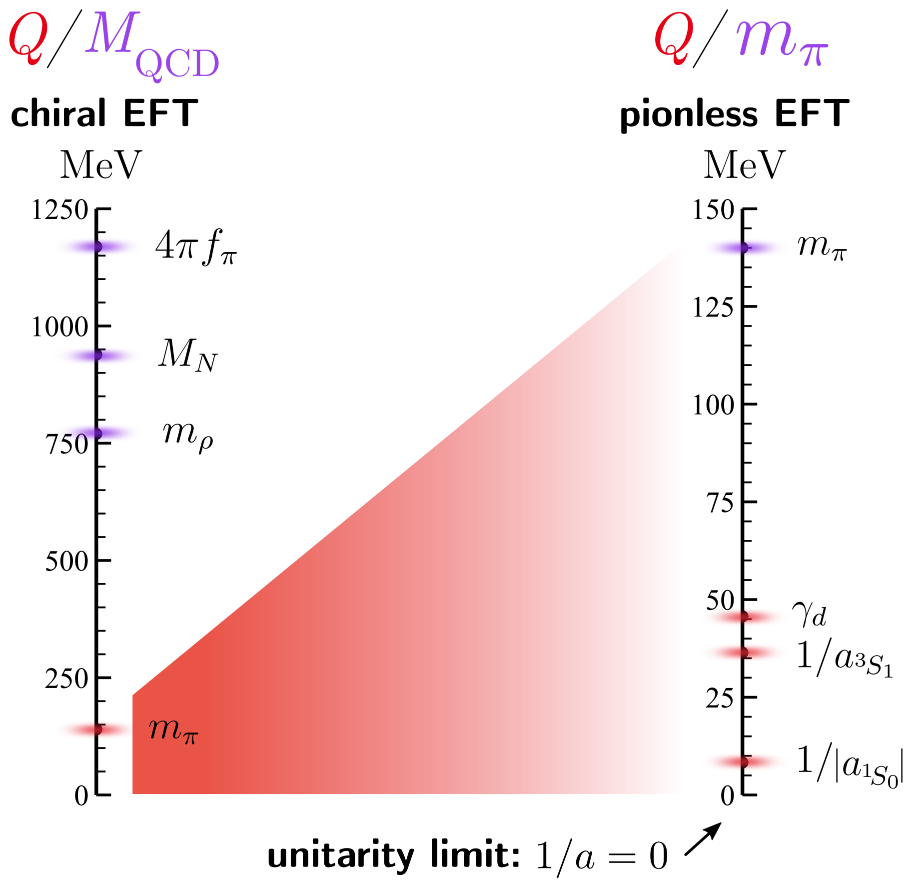
Nuclear scales



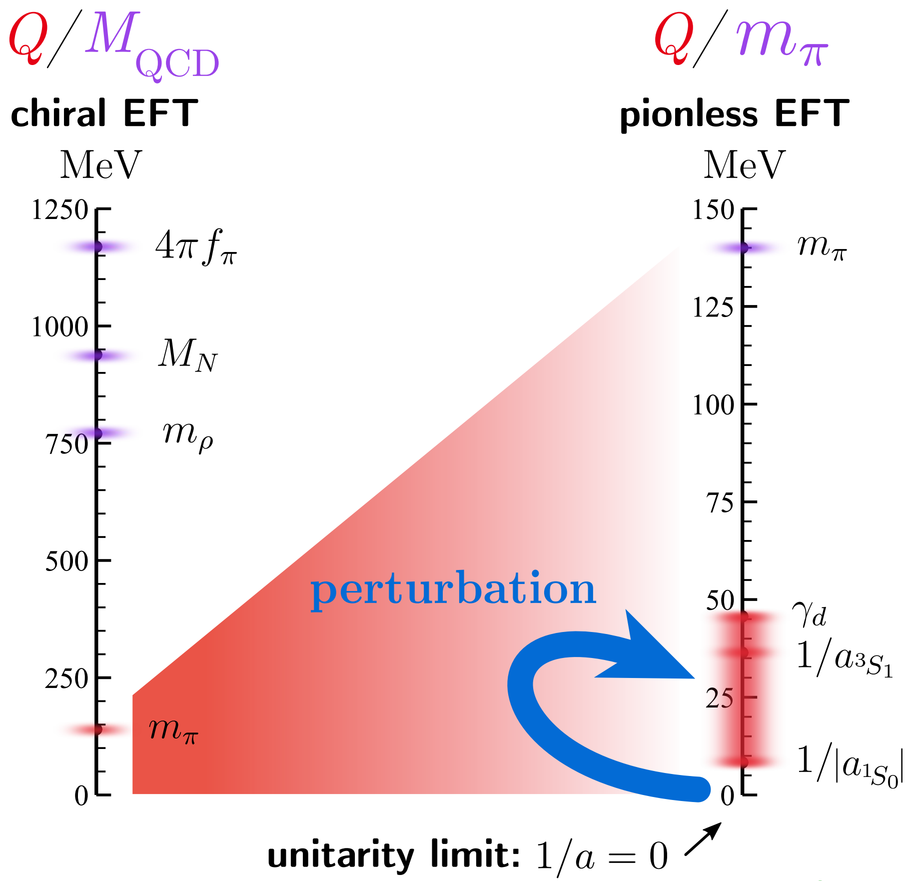
Nuclear scales



Nuclear scales



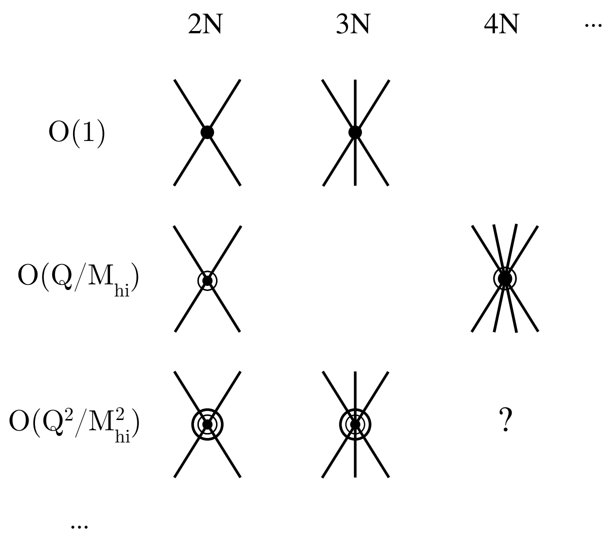
Nuclear scales



SK et al. PRL **118** 202501 (2017)

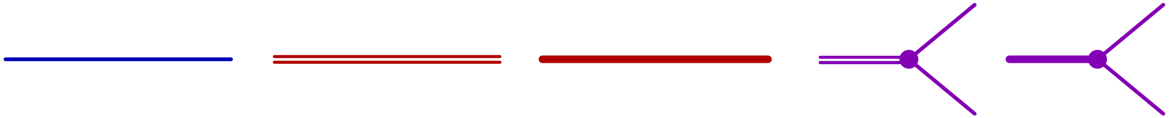
Pionless EFT

- only contact (zero-range) forces (plus electromagnetism)
- closely linked to universality for large scattering lengths
- excels at low energies, exact range of validity still an open question



Effective Lagrangian

$$\mathcal{L} = N^\dagger \left(\mathrm{i}D_0 + \frac{\boldsymbol{D}^2}{2M_N} \right) N$$
$$- d^{i\dagger} \left[\sigma_d + \left(\mathrm{i}D_0 + \frac{\boldsymbol{D}^2}{4M_N} \right) \right] d^i + t^{A\dagger} \left[\sigma_t + \left(\mathrm{i}D_0 + \frac{\boldsymbol{D}^2}{4M_N} \right) \right] t^A$$
$$+ y_d \left[d^{i\dagger} \left(N^T P_d^i N \right) + \mathrm{h.c.} \right] + y_t \left[t^{A\dagger} \left(N^T P_t^A N \right) + \mathrm{h.c.} \right] + \cdots$$



Ingredients

- nucleon field N, doublet in spin and isospin space
- dibaryon fields d (spin triplet, isospin singlet) and t (spin singlet, isospin triplet)
- coupling between nucleons and dibaryons parametrized by $\sigma_{d,t}$ and $y_{d,t}$
  - at leading order, these combine to a single coupling C_0 per spin-isospin channel
  - dibaryon fields can be intergrated out!
- **covariant derivative** $(D_0, \boldsymbol{D})$ includes coupling to photon field
- ellipses include three-nucleon forces and further electromagnetism

Parameter fixing

- like any effective field theory, Pionless EFT involves a priori unknown parameters
- these "low-energy constants" need to be fixed to input data
  - this process is provided by regularization and renormalization
  - running coupling constants are fixed by renormalization conditions
  - input can be from experiment or lattice QCD

Effective range expansion

- in contrast to theories like QED, nuclear EFTs are **nonperturbative** at low energy
- this means a certain class of diagrams needs to be resummed to all orders
- Pionless EFT in the two-nucleon sector is then constructed to reproduce the **effective range expansion (ERE)**

$$k\cot\delta_0(k) = -\frac{1}{a} + \frac{r}{2}k^2 + \mathcal{O}(k^4)$$

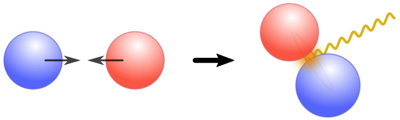
  - $a_d \rightsquigarrow \sigma_{d,t}^{(0)}, y_{d,t}^{(0)}$ at leading order (LO)
  - $r_d \rightsquigarrow \sigma_{d,t}^{(1)}, y_{d,t}^{(1)}$ at next-to-leading order (NLO)

Resummation = infinite iteration

- instead of adding a finite number of diagrams, we need to **solve an equation**
- for bound states, this is the Schrödinger equation
- for two-nucleon scattering, it is the Lippmann-Schwinger equation

Two-nucleon applications

Low-energy capture reactions



- $np \to d\gamma$ capture is very relevant for constraining big-bang nucleosynthesis
- pionless calculations provide very precise predictions ($< 1\%$ uncertainty)

Chen + Savage, PRC **60** 065205 (1999); Rupak, NPA **678** 405 (2000)

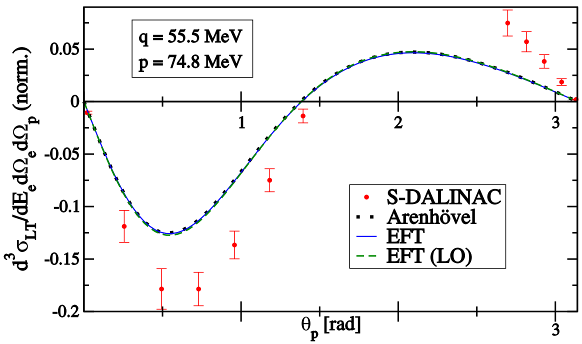
- similarly, proton-proton fusion is very relevant for solar fusion
- again, pionless EFT can provide very precise predictions

Kong + Ravndal, PRC **64** 044002 (2001); De-Leon + Gazit, 2207.10176 [nucl-th]

Deuteron electrodisintegration

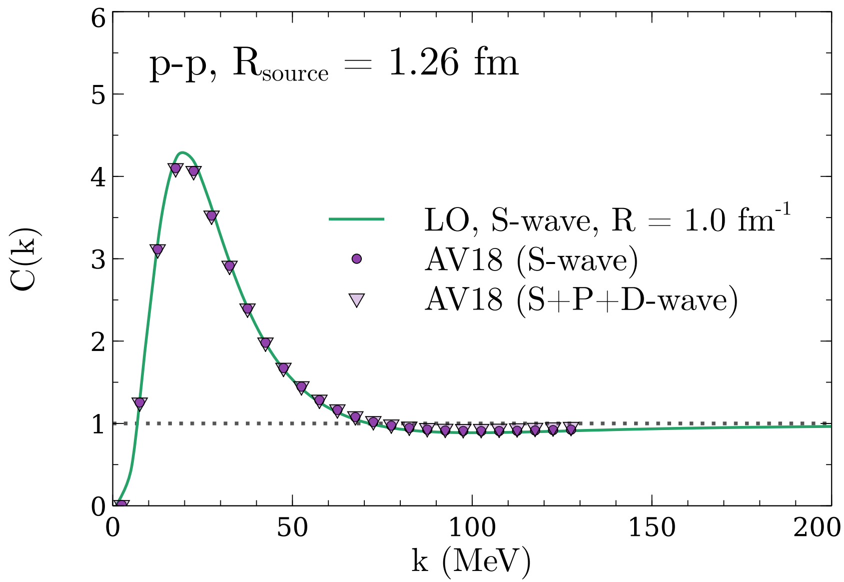
- tension between measurements at TU Darmstadt and potential-model calculations
- pionless EFT confirmed calculations...
  - ...with proper theory uncertainty estimate!
- helped identify an issue in the experimental analysis

Christlmeier + Grießhammer, PRC **77** 064001 (2008)



Proton-proton correlation function

- in coordinate space, we can implement pionless EFT with a local Gaussian potential
  - range R provides the UV regulator, choose $R > 1/m_\pi$ arbitrarily
  - add a repulsive Coulomb potential (more details about this later!)
  - fit strength $C_0(R)$ to pp scattering length $a_{pp} = -7.806$ fm
- **with just this, we can calculate the proton-proton correlation function**



- excellent agreement with AV18 CATS calculation

Mihaylov et al., EPJC **78** 394 (2018)

Relation to Lednický model

- a simple model by Lednický and Lyuboshitz assumes the wavefunction is given everywhere by its asymptotic form

Lednicky + Lyuboshitz, Sov. J. Nucl. Phys. **35** 770 (1982)

$$\psi_\ell(k,r) \propto \frac{1}{kr} C_{\eta,\ell} \left[F_\ell(\eta,kr) + \frac{G_\ell(\eta,kr) + \mathrm{i}F_\ell(\eta,kr)}{\cot\delta_\ell(k) - \mathrm{i}} \right]$$

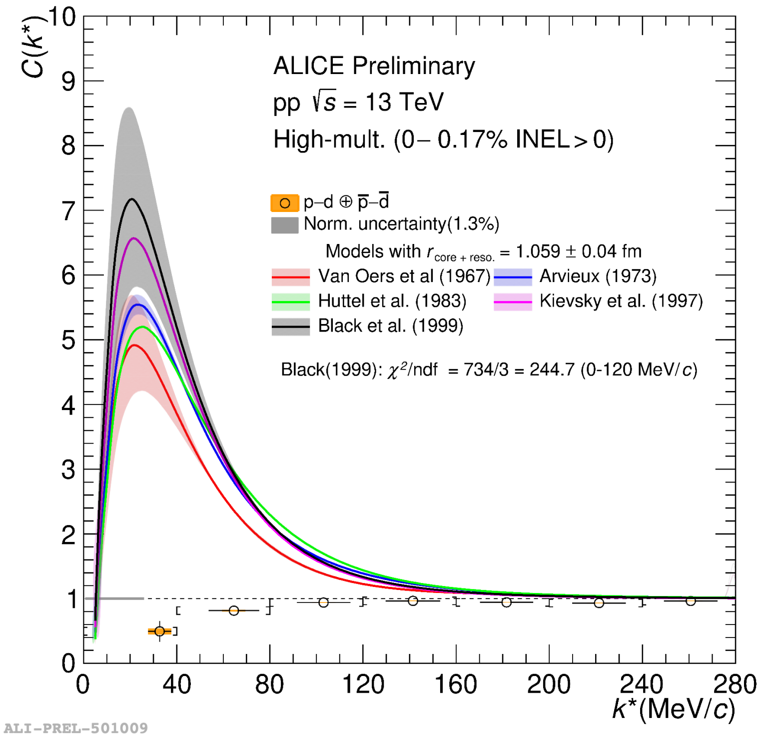
  - $F_\ell(\eta,kr)$ and $G_\ell(\eta,kr)$ are the regular and irregular Coulomb wavefunctions
  - the nuclear interaction is encoded in $\cot\delta_\ell(k)$
  - alternative (equivalent) formulations write this in term of the scattering amplitude
- into this one can insert the **Coulomb-modified effective range expansion**:

$$C_{\eta,\ell}^2 k^{2\ell+1} \cot\delta_\ell(k) = -\frac{1}{a_{pp}} + \frac{r_{pp}}{2} k^2 + \cdots$$

  - truncating this after the first term, the model provides a description in terms of the scattering length a_{pp} alone
  - pionless EFT for the p-p system recovers this model in a systematic way
  - **for p-d correlations, the simple two-body picture may not work!**
  - important to study full three-nucleon dynamics

Lednický model for p-d correlation function

- **for the proton-deuteron system, the Lednický model does not describe data**

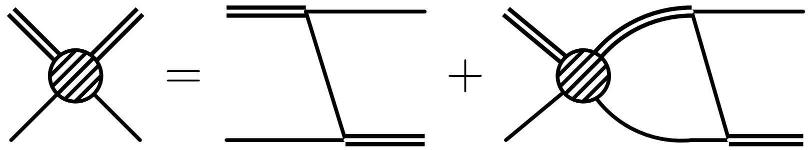


- this clearly indicates that a description as an effective two-body system is invalid

Singh, SK et al., in preparation

Neutron-deuteron scattering

- the dibaryon formalism is particularly convenient to discuss three-nucleon scattering
- specifically, we consider first the **scattering amplitude** for elastic Nd scattering
- diagrammatically, this is represented as a blob with incoming and outgoing legs:



- this diagrammatic equation translates into an **integral equation** for $\mathcal{T}(E;k,p)$:

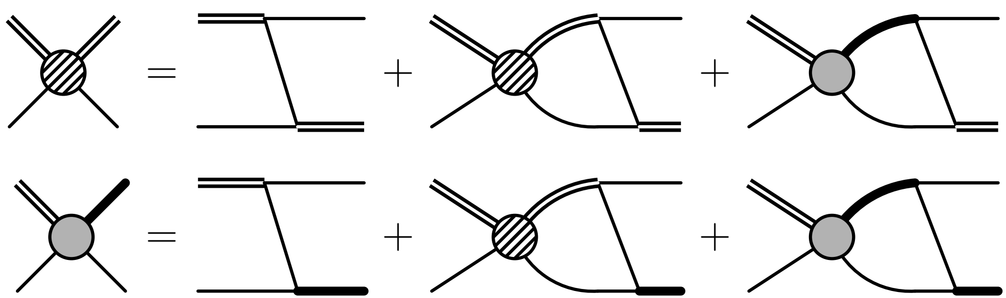
$$\mathcal{T}(E;k,p) = K(E;k,p) + \int dq\,q^2\,\mathcal{T}(E;k,q)D_d(E;q)K(E;q,p)$$

- q is the momentum associated with the loop diagram
- the kernel function $K(E;k,p)$ is given by the **one-nucleon exchange diagram**
- $D_d(E;q)$ describes the **deuteron propagation** (in terms of Δ_d)

More neutron-deuteron scattering

- the equation we wrote down applies to the spin quartet channel
  - in this case, all three nucleon spins can be aligned (maximum projection)
  - only the deuteron (with spin 1) can appear in the intermediate state
- in general, the spins can also couple to an overall spin doublet
  - in this configuration it is possible to have an ${}^1S_0$ NN intermediate state

- this leads to a **coupled-channel equation structure:**



Note

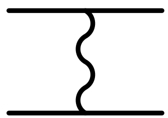
In the doublet S-wave, there is also a three-nucleon force!

- naively one would expect this to enter only at higher order
- a phenomenon related to the Efimov effect implies a promotion to leading order

  Bedaque, Hammer, van Kolck, NPA **676** 357 (1999)
- we have not shown this explicitly in the diagrammatic equation

Inclusion of Coulomb effects

- the effective Lagrangian includes the coupling of photons to baryons
  - this is included in the covariant derivative not written out explicitly before
  - in addition there is of course a photon kinetic term
- **the leading contribution comes from so-called Coulomb photons**
  - effectively one recovers a static potential between charged particles
  - transverse photons enter systematically as higher-order corrections

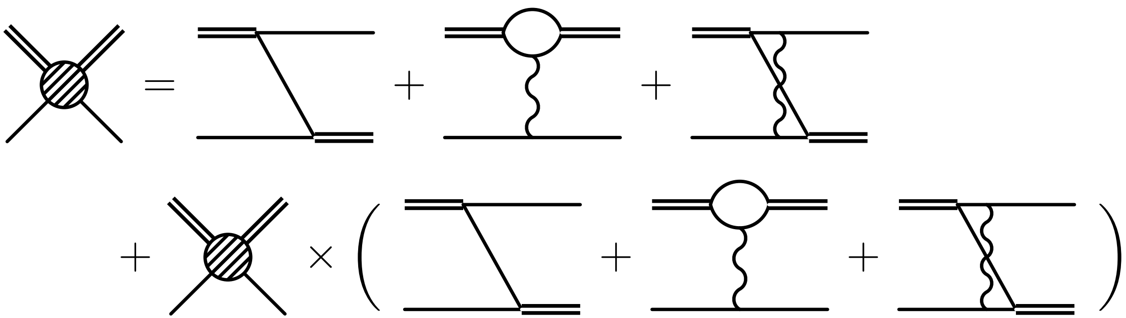


$$\sim \ (\mathrm{ie})\,\frac{\mathrm{i}}{\mathbf{q}^2}\,(\mathrm{ie}) \ \longrightarrow \ (\mathrm{ie})\,\frac{\mathrm{i}}{\mathbf{q}^2+\lambda^2}\,(\mathrm{ie})$$

- a small photon mass is introduced here to regulate the singularity at $\mathbf{q} = 0$
- coordinate-space two-body calculations can be performed with direct inclusion of the Coulomb potential
  Kong + Ravndal, NPA **665** 137 (2000)
- this is based on analytically known expressions for Coulomb wavefunctions
- pionless EFT recovers the Coulomb-modified effective range expansion
  Bethe, Phys. Rev. **76** 38 (1949)

Proton-deuteron scattering

- including Coulomb-exchange diagrams yields the equation for p-d scattering



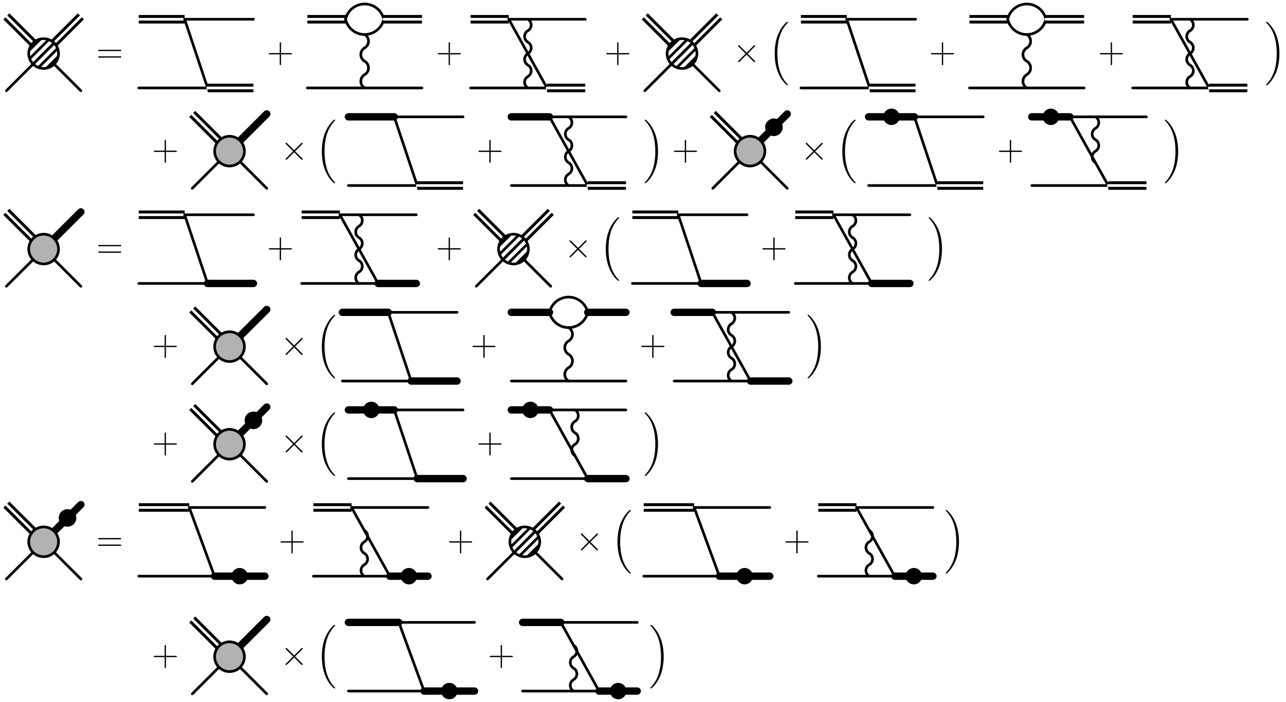
- dominant contribution is given by the "bubble diagram"
  - static Coulomb potential between deuteron and proton
  - with account for deuteron substructure (not just a point particle!)

Subtracted phase shifts

- one solves the same equation with the pure strong-interaction diagrams omitted
- the difference of scattering phase shifts is then well defined
- this is closely related to the Coulomb-modified effective range expansion

Full doublet-channel equation structure

- in the spin-doublet channel, the possibilty of pp intermediate states () leads to a **three-channel equation structure:**



- this accounts for Coulomb photon exchange in all possible places

Overview of results (selection)

- **systematic inclusion of three-nucleon Coulomb contributions**
  - predictions for proton-deuteron scattering and ^{3}He binding energy

    Rupak + Kong, NPA **717** 73 (2003); Ando + Birse, JPG **37** 105108 (2010)
    SK + Hammer, PRC **83** 064001 (2011); ...; SK et al. JPG **43** 055106 (2016)
  - lead to good understanding of perturbative vs. nonperturbative Coulomb effects
- **isospin-breaking three-nucleon force at next-to-leading order**
  - nonperturbative inclusion of Coulomb effects at LO promotes counterterm

    Vanasse, SK, et al., PRC **89** 064003 (2014)
- **fully perturbative n-d scattering up to N2LO and higher** Vanasse, PRC **88** 044001 (2014)
  - introduced efficient technique for rigourous perturbation theory
  - found good convergence properties across various partial waves
  - studied A_y puzzle with N3LO calculation Margaryan et al., PRC **93** 054001 (2016)
- **perturbative expansion of light nuclei around unitarity limit**

  SK et al., PRL **118** 202501 (2016)
  - details of two-nucleon interaction not important for gross features
  - shown to work well for $A \leq 4$ binding energies and charge radii
- **electroweak properties: magnetic moments, beta decay**

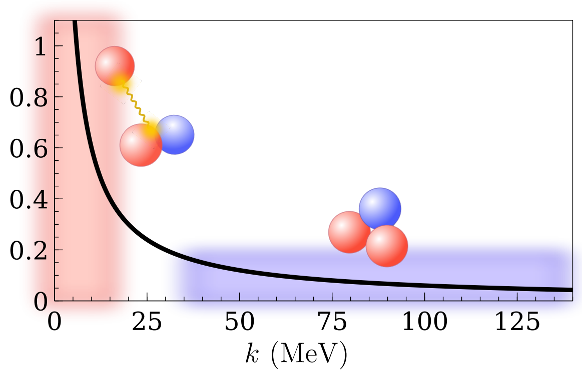
  De-Leon et al., arXiv:1902.07677 [nucl-th]; De-Leon + Gazit, 2004.11670 [nucl-th]
- **...**

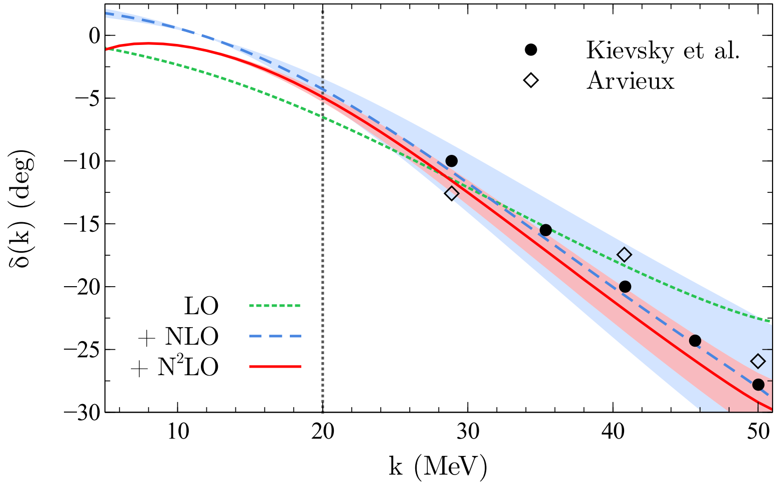
Perturbative Coulomb effects

- strength of Coulomb interaction depends on momentum scale
- perturbative effect in bound states and scattering beyond very small energies
- **to leading order, p-d and n-d scattering for $k \gtrsim 20$ MeV are degenerate!**

SK et al., JPG **43** 055106 (2016)

Coulomb strength $\eta \sim 1/k$





- Coulomb-subtracted phase shifts calculated in perturbation theory
- uncertainty bands estimated based on EFT expansion parameter
  - (with 30% leading-order band not shown explicitly)

Reflection

So we have a way to calculate proton-deuteron scattering...

- theory is well understood and relatively simple
- pionless EFT captures precisely what is relevant at low energies
- to actually evaluate correlation functions we need to work a bit harder

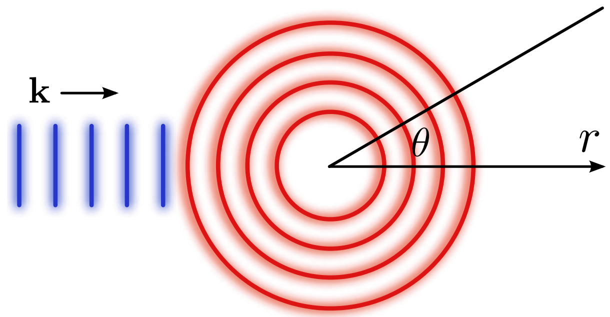
Part II

Faddeev formalism

Two-body scattering setup

- consider two particles with masses m_1 and m_2 at positions $\mathbf{r}_1$ and $\mathbf{r}_2$
- assume that the interaction does not depend on absolute particle positions
- then we can **neglect the overall center-of-mass motion** and work only with the relative coordinate $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$ and reduced mass $\mu = m_1 m_2/(m_1 + m_2)$
- for the two particles scattering off one another, we physically expect that the wavefunction describing their relative motion is given as a sum of an incoming plane wave and an outgoing spherical scattered wave:

$$\psi_{\mathbf{k}}^{(+)}(\mathbf{r}) = \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{r}} + f_k(\theta)\frac{\mathrm{e}^{\mathrm{i}kr}}{r} \tag{1}$$



- all physics information is contained in the scattering amplitude $f_k(\theta)$

The Lippmann-Schwinger equation

- consider the stationary Schrödinger equation: $H|\psi\rangle = E|\psi\rangle$ with $H = H_0 + V$
  - H_0 here is the free Hamiltonian with $H_0|\mathbf{k}\rangle = E_{\mathbf{k}}|\mathbf{k}\rangle = \frac{\mathbf{k}^2}{2\mu}|\mathbf{k}\rangle$
- this alone does not specify a **boundary condtion** for solutions $\psi(\mathbf{r}) = \langle\mathbf{r}|\psi\rangle$
  - a scattering state should be such that for $V \to 0$, $|\psi\rangle \to |\mathbf{k}\rangle$
  - moreover, it should be one that evolved from a free state in the infinite past
- both conditions can be enforced with the ansatz

$$|\psi_{\mathbf{k}}^{(+)}\rangle = |\mathbf{k}\rangle + (E_{\mathbf{k}} - H_0 + \mathrm{i}\varepsilon)^{-1} V |\psi_{\mathbf{k}}^{(+)}\rangle \tag{2}$$

- this is the **Lippmann-Schwinger equation** for the scattering state $|\psi_{\mathbf{k}}^{(+)}\rangle$

Notes

- the free Green's function $G_0(z) = (z - H_0)^{-1}$ appears in Eq. (2) with $z = E_{\mathbf{k}} + \mathrm{i}\varepsilon$
- $\varepsilon \to 0$ is implied in all equations
  - this implements the second boundary condition via adiabatic switching
- $\psi_{\mathbf{k}}^{(+)}(\mathbf{r})$ is exactly what appears in the definition of the correlation function!

The Lippmann-Schwinger equation

- consider further Eq. (2) and apply V from the left:

$$V|\psi_{\mathbf{k}}^{(+)}\rangle = V|\mathbf{k}\rangle + VG_0(E_{\mathbf{k}} + \mathrm{i}\varepsilon)V|\psi_{\mathbf{k}}^{(+)}\rangle \tag{3}$$

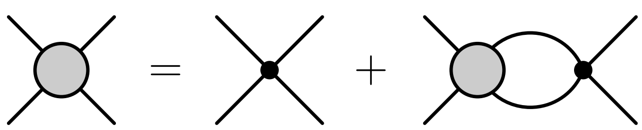
- define an operator T via $V|\psi_{\mathbf{k}}^{(+)}\rangle = T|\mathbf{k}\rangle$ to write

$$T|\mathbf{k}\rangle = V|\mathbf{k}\rangle + VG_0(E_{\mathbf{k}} + \mathrm{i}\varepsilon)T|\mathbf{k}\rangle \tag{4}$$

- this is the **Lippmann-Schwinger equation** for the operator T

Notes

- since $\mathbf{k}$ is arbitrary in Eq. (4), we postulate at the operator level: $T = V + VG_0T$
- $T = T(E_{\mathbf{k}} + \mathrm{i}\varepsilon)$ carries an implicit energy dependence via G_0
- alternative form: $T = V + TG_0V$ (seen to be equivalent by iteration)
- we can also write this in a diagrammatic representation:



Scattering wavefunctions

- from the T-matrix we can obtain **scattering wavefunctions in momentum space**
- recall the initial form of the Lippmann-Schwinger equation, Eq. (2):

$$|\psi_{\mathbf{k}}^{(+)}\rangle = |\mathbf{k}\rangle + (E_{\mathbf{k}} - H_0 + \mathrm{i}\varepsilon)^{-1} V |\psi_{\mathbf{k}}^{(+)}\rangle$$

- with $V|\psi_{\mathbf{k}}^{(+)}\rangle = T|\mathbf{k}\rangle$, we obtain directly:

$$\langle \mathbf{q}|\psi_{\mathbf{k}}^{(+)}\rangle = \langle \mathbf{q}|\mathbf{k}\rangle + \langle \mathbf{q}|(E_{\mathbf{k}} - H_0 + \mathrm{i}\varepsilon)^{-1} T|\mathbf{k}\rangle \qquad (5)$$

- from the first term $\langle \mathbf{q}|\mathbf{k}\rangle = (2\pi)^3 \delta^{(3)}(\mathbf{q} - \mathbf{k})$ it is clear that this is a distribution, not an ordinary function
- the second term, with $\varepsilon \to 0$ implied, contains a smooth part as well as a pole contribution (from the on-shell point $\mathbf{q} = \mathbf{k}$):

$$\langle \mathbf{q}|(E_{\mathbf{k}} - H_0 + \mathrm{i}\varepsilon)^{-1} T|\mathbf{k}\rangle = \frac{2\mu T(E_{\mathbf{k}}; \mathbf{q}, \mathbf{k})}{\mathbf{k}^2 - \mathbf{q}^2 + \mathrm{i}\varepsilon} \qquad (6)$$

- note that $T(E_{\mathbf{k}}; \mathbf{q}, \mathbf{k})$ is the half off-shell T-matrix

Shortcut

- in principle we could Fourier-transform $\psi_{\mathbf{k}}^{(+)}(\mathbf{q})$ to evaluate

$$C(k) = \int \mathrm{d}^3 r \, S(r) \left| \psi_{\mathbf{k}}^{(+)}(\mathbf{r}) \right|^2$$

- but we can also **stay directly in momentum space**

Source function in momentum space

- we can express the correlation function in terms of a **source operator**:

$$C(k) = \langle \psi_{\mathbf{k}}^{(+)} | \hat{S} | \psi_{\mathbf{k}}^{(+)} \rangle$$

- in configuration space, $\hat{S}$ is local:

$$\langle \mathbf{r} | \hat{S} | \mathbf{r}' \rangle = \frac{\exp\left(-r^2/R^2\right)}{(4\pi R)^{3/2}} \delta^{(3)}(\mathbf{r} - \mathbf{r}')$$

- this translates into a **non-local representation** in momentum space:

$$\langle q, \ell | \hat{S} | q', \ell' \rangle = \exp\left(-R^2(q^2 + q'^2)\right) i_\ell \left(2Rqq'\right) \delta_{\ell\ell'}$$

Tabakin + Davies, Phys. Rev. **150** 793 (1966)

- we have written this directly in partial waves
- this matches how the Lippmann-Schwinger equation is commonly solved
- ℓ denotes the angular momentum
- $i_\ell(z)$ is a modified spherical Bessel function
- $C(k)$ is obtained by summing over ℓ (and other discrete quantum numbers)

We now want to follow this path for three particles!

T-matrix $\rightarrow$ scattering wavefunction $\rightarrow$ correlation function

Faddeev equation for bound states

- consider first the Schrödinger equation for a three-nucleon bound state:

$$\Big(H_0 + \sum_{\substack{j=1,2,3\\ i<j}} V_{ij}\Big)|\Psi\rangle = E|\Psi\rangle \tag{7}$$

- we neglect the three-nucleon force and consider only pairwise interactions
- at leading order, Pionless EFT in this formulation features S-wave potentials of the form $V(p,p') = C_0^{(0)}(\Lambda)g(p)g(p')$, with $C_0(0)$ related to the $\sigma_{d,t}^{(0)}$ discussed earlier

Faddeev decomposition

- the full wavefunction can be decomposed into Faddeev components:

$$|\Psi\rangle = \sum_{k=1,2,3} |\psi_k\rangle \equiv (1+P)|\psi\rangle \quad , \quad P = P_{12}P_{23} + P_{13}P_{23}$$

- antisymmetry reduces the problem to a single component and permutations
- Eq. (7) can then be rewritten as $|\psi\rangle = G_0 t P|\psi\rangle$ with a two-body T-matrix t

Faddeev scattering

- for nucleon-deuteron scattering, we need to include an **inhomogeneous term**
- this is given as a product of a deuteron wave function φ_d and a third nulceon with relative momentum k: $|\phi\rangle = |\varphi_d k; s\rangle$
  - for neutrons, the relative motion is a plane wave (Bessel function)
  - for protons, this involves the regular pure Coulomb wave function
  - s collects all relevant quantum numbers (angular momentum, spin, isospin)
- a single Faddeev component of the scattering wavefunction is given by $|\psi\rangle = |\phi\rangle + \tilde{T}|\phi\rangle$ with $\tilde{T}$ satisfying

$$\tilde{T}|\phi\rangle = G_0 t P|\phi\rangle + G_0 t P \tilde{T}|\phi\rangle \tag{8}$$

- the full scattering wavefunction is ultimately obtained via **antisymmetrization**

$$|\Psi\rangle = (1 + P)|\psi\rangle \tag{9}$$

Notes

- $|\phi\rangle$ is kept in Eq. 8 to indicate the boundary condition
- in the literature, one typically sees $T = G_0^{-1}\tilde{T}$

see for example Hüber et al., PRC **51** 1100 (1995)

Why all this?

Why all this?

- $\tilde{T}$ can be directly related to the $\mathcal{T}$ from the diagrammatic approach
- construction of the full wave function is easier to discuss in the Faddeev formalism

Strategy

1. Solve integral equation to obtain $\mathcal{T}(E(k); q, k)$

- this is where the bulk of the dynamics in the N-d system is accounted for
- uses well established and tested code for elastic scattering

2. Convert $\mathcal{T} \to \tilde{T}$ for further processing

- this step is straightforward because it just multiplies known functions
- the equation for $\mathcal{T}$ follows from the generic one for separable interactions

3. Calculate wavefunction $|\Psi\rangle$ from $\tilde{T}$

- this includes the required antisymmetrization
- ensure proper normalization with a factor $1/3$
- everything is done in momentum space, treating the plane-wave part analytically

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4. Evaluate $\langle\Psi|\hat{S}|\Psi\rangle$ to calculate correlation functinon

Three-body correlation function

- we follow a microscopic formalism starting from individual nucleon source functions

Mrówczyński, EPJST **229** 3559 (2020), Viviani, SK et al., work in progress

$$C(k) = \frac{1}{A_d} \int \mathrm{d}^3 r_1 \mathrm{d}^3 r_2 \mathrm{d}^3 r_3 S_1(\mathbf{r}_1) S_1(\mathbf{r}_2) S_1(\mathbf{r}_2) \left|\Psi(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3)\right|^2 \tag{10}$$

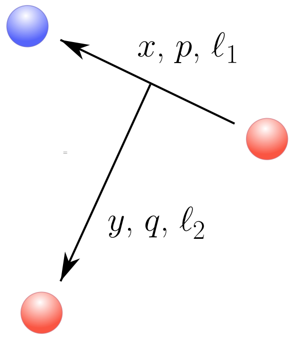
$$S_1(\mathbf{r}) = \frac{\exp\left(-r^2/(2R_M^2)\right)}{(2\pi R_M)^{3/2}} \tag{11}$$

  - note the different source radius prefactor compared to the previous definition
- the dependence on the overall center of mass $\mathbf{r} = \frac{1}{3}(\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_2)$ can be removed
  - doing the same for two particles yields exactly the previous definition of $C(k)$
  - the source is then a function of $\mathbf{x} = \mathbf{r}_2 - \mathbf{r}_1$ with same radius factor as before
- this happens also in the calculation of the deuteron formation rate

$$A_d = \int \mathrm{d}^3 r_1 \mathrm{d}^3 r_2 S_1(\mathbf{r}_1) S_1(\mathbf{r}_2) \left|\varphi_d(\mathbf{r}_1, \mathbf{r}_2)\right|^2 \tag{12}$$

- **we evaluate all these expression in momentum-space partial waves**

Momentum-space evaluation



- given the full wave function $|\Psi\rangle$, we evaluate

$$\langle\Psi|\hat{S}(R)|\Psi\rangle = A_d \times C(k)$$

- $\hat{S}(R)$ here is a product of sources in two Jacobi coordinates
  - the center-of-mass motion is factorized out from the beginning
  - the two source radii enter with different prefactors
  - this follows naturally by starting from single-particle coordinates
- to evaluate the matrix element above insert complete sets of three-body states:

$$\mathbf{1} = \sum_s |pq;s\rangle\langle pq;s| \quad , \quad s = \{\ell_1, \ell_2, \cdots\}$$

$$\langle pq;s|\hat{S}|p'q';s\rangle = \langle p;\ell_1|\hat{S}\left(\sqrt{\tfrac{4}{3}}R\right)|p';\ell_1\rangle \times \langle q;\ell_2|\hat{S}(R)|q';\ell_2\rangle \times \delta_{ss'}$$

- the normalization factors is calculated analogously:

$$A_d = \langle\varphi_d|\hat{S}\left(\sqrt{\tfrac{4}{3}}R\right)|\varphi_d\rangle$$

Higher-order corrections

- consider now the perturbative EFT expansion $T = \mathcal{T}^{(0)} + \mathcal{T}^{(1)} + \cdots$
- from this we obtain $|\Psi_k\rangle = |\Psi_k^{(0)}\rangle + |\Psi_k^{(1)}\rangle + \cdots$
- likewise, we have the deuteron expansion $|\varphi_d\rangle = |\varphi_d^{(0)}\rangle + |\varphi_d^{(1)}\rangle + \cdots$
- the leading-order correlation function is given by

$$A_d^{(0)} \times C^{(0)}(k) = \langle \Psi_k^{(0)} | \hat{S} | \Psi_k^{(0)} \rangle \quad \text{with} \quad A_d^{(0)} = \langle \varphi_d^{(0)} | \hat{S} | \varphi_d^{(0)} \rangle$$

Next-to-leading order correlation function

- **at next-to-leading order, we have two contributions:**

$$2\mathrm{Re}\big(\langle \Psi_k^{(0)} | \hat{S} | \Psi_k^{(1)} \rangle\big) = A_d^{(0)} \times C^{(1)}(k) + A_d^{(1)} \times C^{(0)}(k)$$

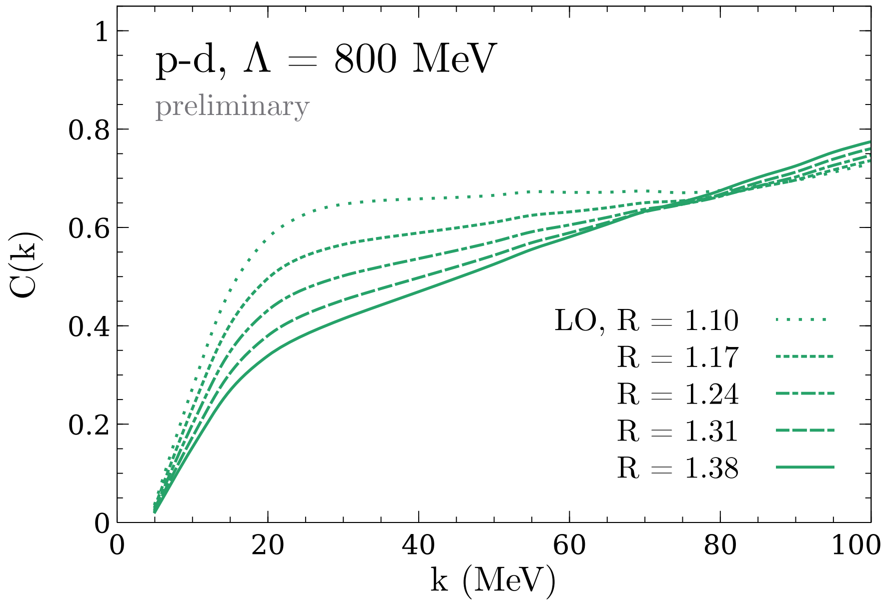
- we can extract the genuine NLO correlation function $C^{(1)}(k)$:
  - (1) from the LO calculation we already know $C^{(0)}(k)$ and $A_d^{(0)}$
  - (2) we can compute $A_d^{(1)} = 2\mathrm{Re}\big(\langle \varphi_d^{(0)} | \hat{S} | \varphi_d^{(1)} \rangle\big)$
  - (3) therefore we can just subtract the second term and divide by $A_d^{(0)}$!

Part III

Results and discussion

Leading-order correlation function

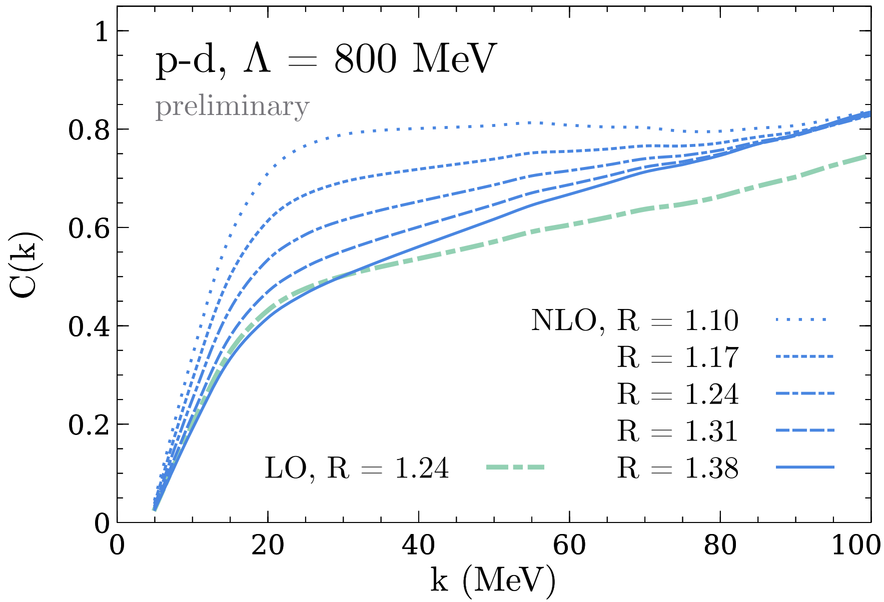
- for the overall correlation function, we explicitly include S- and P-wave pd channels
  - in pionless EFT through NLO, there is no splitting with total spin J
  - therfore, the spin quartet and doublet channels are calculated independently
  - they are summed with weight factors $2/3$ and $1/3$, respectively
- beyond P waves, we sum the "free" correlation function up to $\ell = 15$



- source radius is varied here between 1.10 fm and 1.38 fm

Next-to-leading order correction

- NLO corrections are included in strict perturbation theory, as discussed
  - this still performs the full calculation in S- and P-wave channels only
  - D and higher partial waves become gradually relevant at larger momenta
  - however, pionless EFT is not designed to describe that regime
- the magnitude of the NLO shift is natural w.r.t. expectations from power counting



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Note

- this prediction of the correlation function is based on **very few input parameters**
- at LO, only S-wave 2N scattering lengths and a single 3N datum
- at NLO, 2N effective ranges and a pd or ^{3}He datum

Source radius

- the source radius R is a free parameter in the theory calculation
- it needs to be determined from experimental data
  - extraction based on transverse-mass distributions ALICE Collab., PLB **811** 135849 (2020)
- theory results can also be used to **fit R constrained by the experimental data**
- Gaussian shape is an assumption

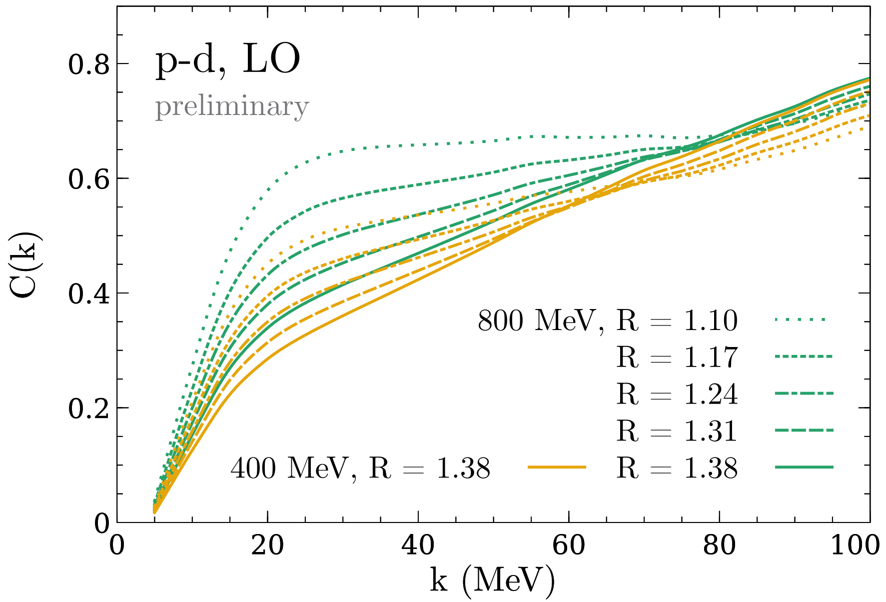
Observability

- it is an important question what extent the correlation function should be interpreted as a model-dependent quantity
- **nuclear potentials are not observable!**
  - potentials that differ at short distance can describe the same low-energy physics
  - given one potential one can obtain equivalent ones by unitary transfomations
  - varying the EFT cutoff changes unconstrained short-distance features
- **note:** integration measure suppresses short-distance contributions

$$C(k) = 4\pi \int \mathrm{d}r\, r^2\, S(r)\, \left|\psi(k,r)\right|^2$$

Cutoff dependence

- varying the EFT cutoff changes the wavefunctions
  - the long-range part should converge for $\Lambda \to \infty$
  - that is because the asymptotic behavior is governed by the scattering phase shifts
- short-distance physics is not constrained by the low-energy effective theory
- **we need to check carefully for sensitivity in the correlation function!**



- reduced cutoff dependence for large source radii

Pionless EFT vs. AV18

- with a generic Faddeev code (not limited to separable interactions), we can **calculate the correlation function also with the AV18 interaction**
- this code currently has two significant limitations
  - no inclusion of Coulomb effects
  - support only for energies below the deuteron breakup threshold ($k \leq 50$ MeV)
- nevertheless we can compare the n-d correlation function in this regime
  - EFT values are for cutoff $\Lambda = 800$ MeV, all calculations used $R = 1.24$ fm

k / MeV	Pionless LO	Pionless NLO	AV18
30	0.524	0.710	0.655
40	0.545	0.719	0.725
50	0.579	0.742	0.785

- **pionless NLO results within 10% of AV18 numbers ✓**
- a full AV18 (plus UIX 3N force) pd calculation is being worked on by the Pisa group

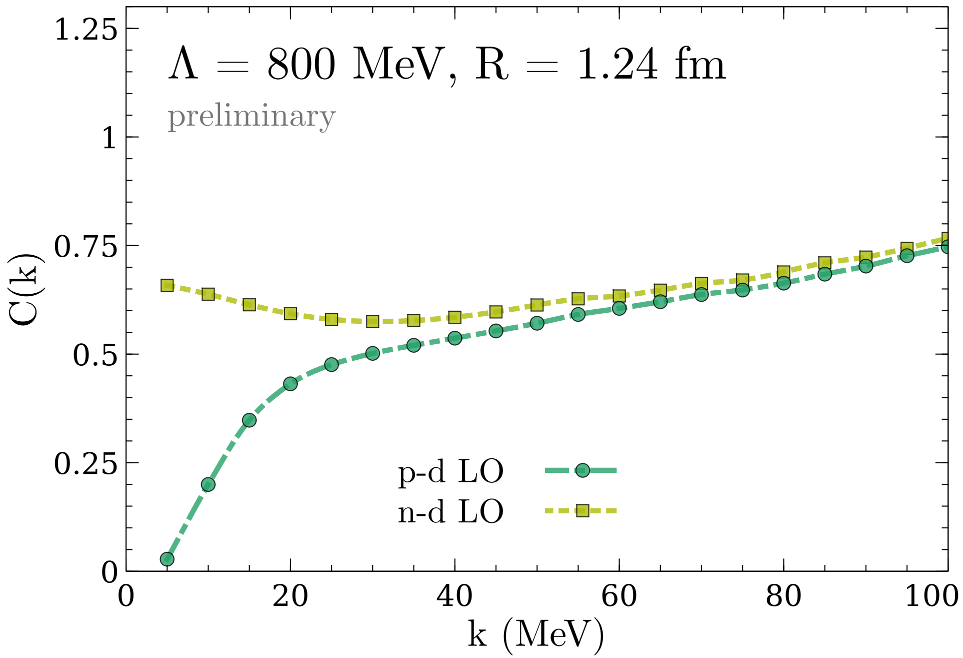
Viviani, Kievsky, Marcucci, SK et al., work in progress

- that approach uses the Hyperspherical Harmonics (HH) formalism

Kievsky et al., JPG **35** 063101 (2008)

Proton-deuteron vs. neutron-deuteron

- **Coulomb effects should be relevant primarily at very low energies**
- as discussed previously, the effective Coulomb strength is $\eta = \alpha\mu/k$
- in addition, there is some strong isospin breaking (pp vs np $^1S_0$ scattering length)



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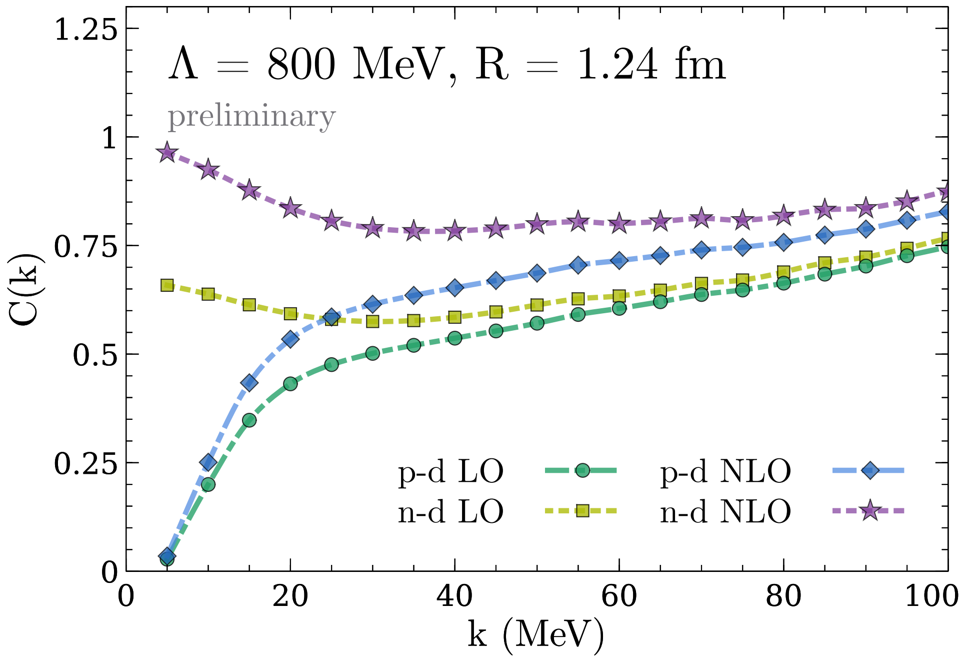

- **note:** ^{3}He binding energy fixed at NLO for p-d calculation

Vanasse, SK, et al., PRC **89** 064003 (2014)

The end

Summary

- high-energy collisions provide information about few-body systems at low energy
- for identical particles (like nucleons), a full treatment of the few-body dynamics is important to describe the correlation function
- pionless EFT can be used to make predictions based on very few input parameters

The end

Summary

- high-energy collisions provide information about few-body systems at low energy
- for identical particles (like nucleons), a full treatment of the few-body dynamics is important to describe the correlation function
- pionless EFT can be used to make predictions based on very few input parameters

Outlook

- **comparison to experiment**
  - ALICE has taken and analyzed data, paper submitted for internal review
- **more detailed comparison to AV18 calculation** (performed by Pisa group)
  - work on joint theory paper is in progress
- **analysis of further systems**
  - study in particular ppp and dd correlation functions
  - although neutrons are not measured by the relevant experiments, theory can compare and benchmark nnn, nd, ...

Thanks...

...to my collaborators...

- **B. Singh**, L. Fabbietti (TU Munich)
- M. Viviani, A. Kievsky, L. Marcucci (Pisa)
- O. Vázquez Doce (CERN)
- J. Haidenbauer (FZ Jülich)
- ...

...for support, funding, and computing time...

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...and to you, for your attention!