Beta decay matrix elements, strengths, and spectra with quantum Monte Carlo methods

INT Program 23-1b
“New Physics Searches at the Precision Frontier”

May 1st, 2023

Garrett King
Washington University in St. Louis

Advisors: Saori Pastore and Maria Piarulli

Postdoc: Lorenzo Andreoli

Collaborators: Baroni, Brown, Carlson, Cirigliano, Gandolfi, Hayen, Mereghetti, Schiavilla, Schmitt, Wiringa, Zegers

Beta decay for fundamental symmetries

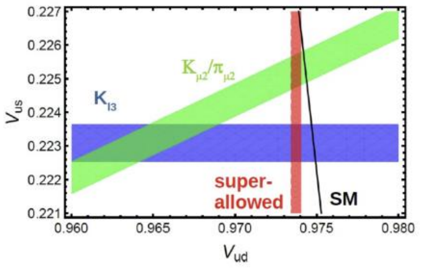
Beta decay has a rich history as a probe of the SM as the fundamental theory of the weak interaction and that continues to this day

Beta decay for fundamental symmetries

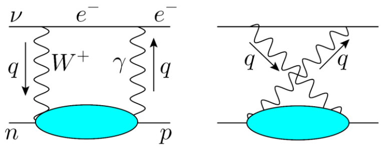
Beta decay

Beta decay for fundamental symmetries

Tests of CKM unitarity



Seng arXiv:2207.10492



Seng et al. PRD 100, 013001 (2019)

$$|V_{ud}|^2_{0^+} \propto \left[\mathcal{F}t(1+\Delta_R^V)\right]$$

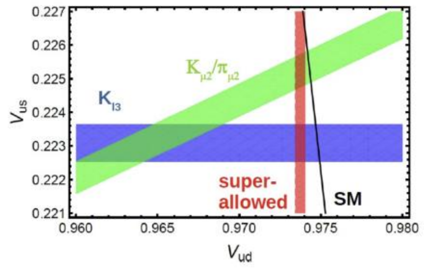
$$\mathcal{F}t = ft(1+\delta_R')(1+\delta_{\mathrm{NS}}-\delta_{\mathrm{C}})$$

Beta decay

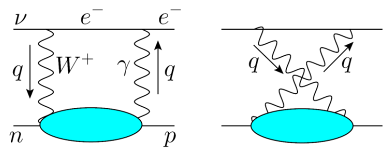
Beta decay for fundamental symmetries

Beta decay

Tests of CKM unitarity



Seng arXiv:2207.10492

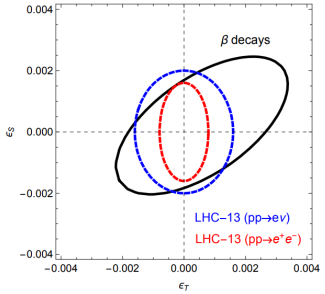


Seng et al. PRD 100, 013001 (2019)

$$|V_{ud}|^2_{0+} \propto \left[\mathcal{F}t(1+\Delta^V_R)\right]$$

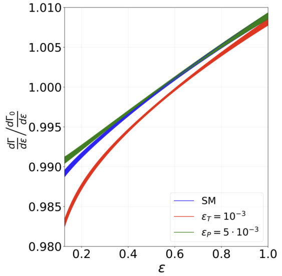
$$\mathcal{F}t = ft(1+\delta'_R)(1+\delta_{\mathrm{NS}}-\delta_{\mathrm{C}})$$

Falkowski et al. J. High Energ. Phys. 2021, 126 (2021)



He6 CRES

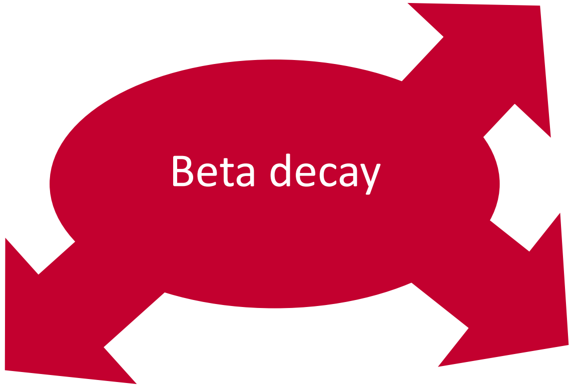
GBK, Baroni, Mereghetti et al. PRC 107, 015503 (2023)



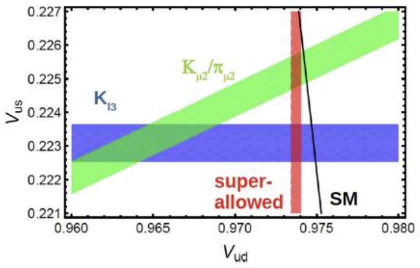
FRIB

Probe of non-standard CC weak currents

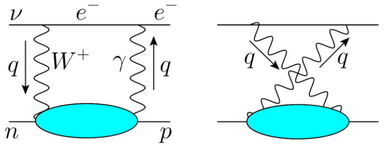
Beta decay for fundamental symmetries



Tests of CKM unitarity



Seng arXiv:2207.10492

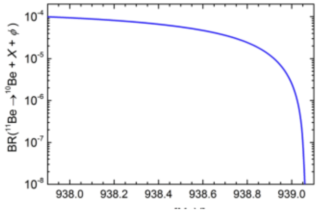


Seng et al. PRD 100, 013001 (2019)

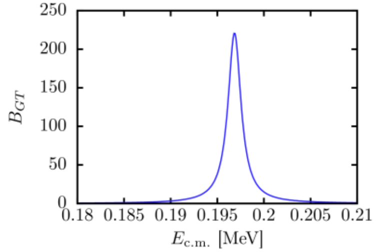
$$|V_{ud}|^2_{0^+} \propto \left[\mathcal{F}t(1+\Delta_R^V)\right]$$

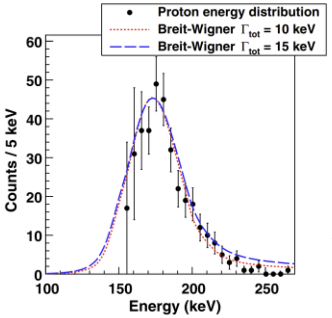
$$\mathcal{F}t = ft(1+\delta'_R)(1+\delta_{\rm NS}-\delta_{\rm C})$$

Pfützner and Riisager PRC 97, 042501(R) (2018)



Atkinson et al. PRC 105, 054316 (2022)

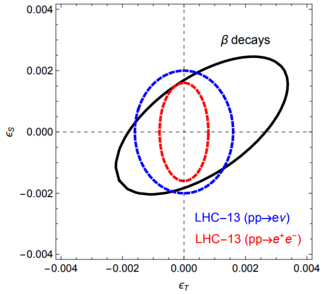




Ayyad et al. PRL 123, 082501 (2019)

Understanding possible exotic decay signals

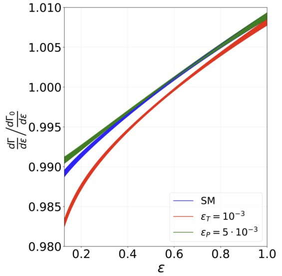
Falkowski et al. J. High Energ. Phys. 2021, 126 (2021)



He6 CRES

FRIB

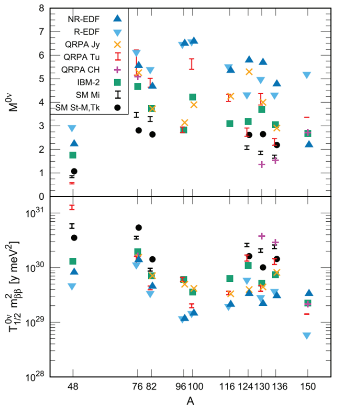
GBK, Baroni, Mereghetti et al. PRC 107, 015503 (2023)

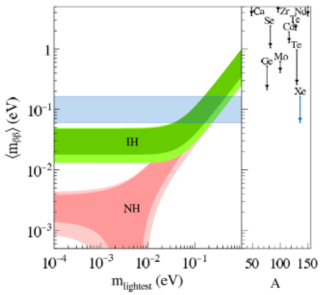


Probe of non-standard CC weak currents

Beta decay for fundamental symmetries

Improve NMEs for LNV searches

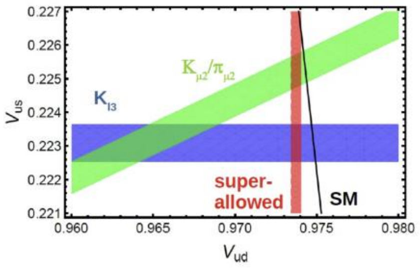




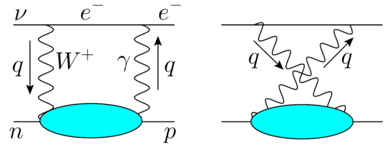
$$\Gamma_{0\nu} \propto (\text{NME})^2 \times \langle m_{\beta\beta}\rangle^2$$

Engel and Menéndez Rep. Prog. Phys. 80 046301 (2017)

Tests of CKM unitarity



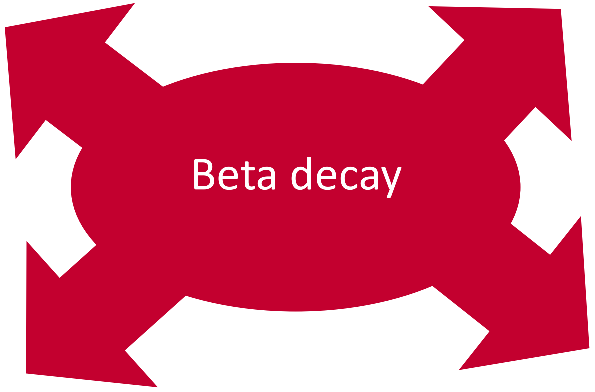
Seng arXiv:2207.10492



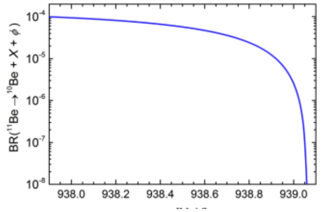
Seng et al. PRD 100, 013001 (2019)

$$|V_{ud}|^2_{0^+} \propto \left[\mathcal{F}t(1+\Delta_R^V)\right]$$

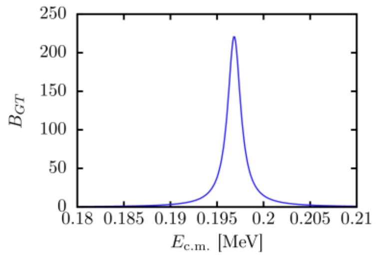
$$\mathcal{F}t = ft(1+\delta_R')(1+\delta_{\text{NS}}-\delta_{\text{C}})$$

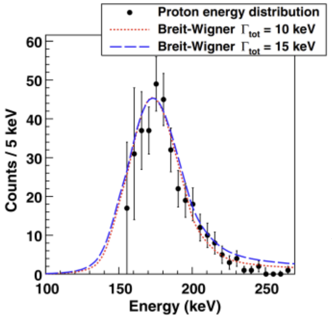


Pfützner and Riisager PRC 97, 042501(R) (2018)



Atkinson et al. PRC 105, 054316 (2022)

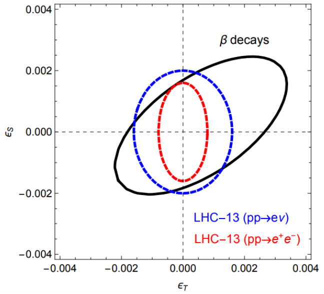




Ayyad et al. PRL 123, 082501 (2019)

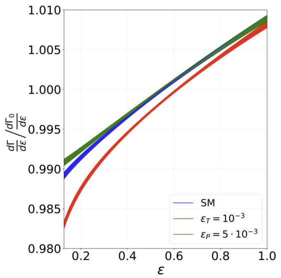
Understanding possible exotic decay signals

Falkowski et al. J. High Energ. Phys. 2021, 126 (2021)



He6 CRES

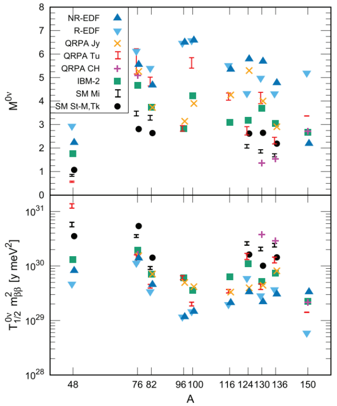
GBK, Baroni, Mereghetti et al. PRC 107, 015503 (2023)

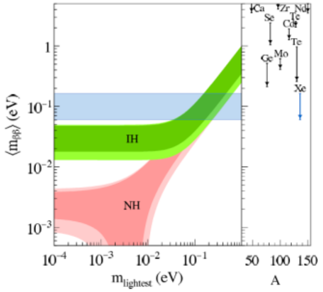


Probe of non-standard CC weak currents

Beta decay for fundamental symmetries

Improve NMEs for LNV searches

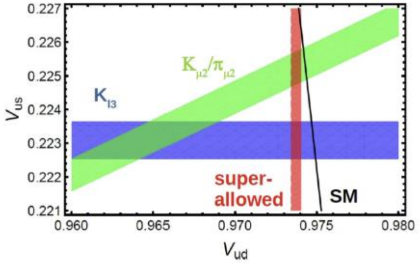




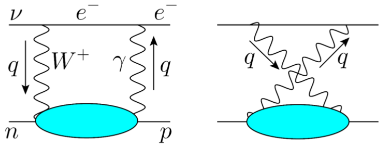
$$\Gamma_{0\nu} \propto (\mathrm{NME})^2 \times \langle m_{\beta\beta}\rangle^2$$

Engel and Menéndez Rep. Prog. Phys. 80 046301 (2017)

Tests of CKM unitarity



Seng arXiv:2207.10492



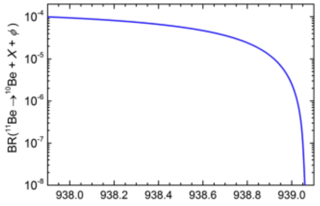
Seng et al. PRD 100, 013001 (2019)

$$|V_{ud}|^2_{0^+} \propto \left[\mathcal{F}t(1+\Delta^V_R)\right]$$

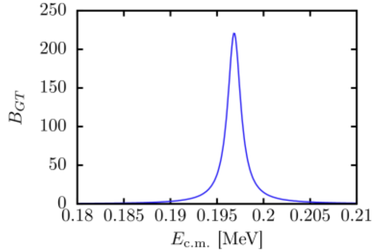
$$\mathcal{F}t = ft(1+\delta'_R)(1+\delta_{\mathrm{NS}}-\delta_{\mathrm{C}})$$

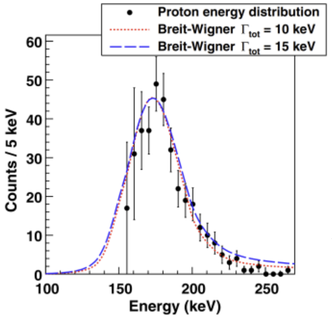
An accurate understanding of nuclear structure and dynamics is required to disentangle new physics effects from nuclear effects

Pfützner and Riisager PRC 97, 042501(R) (2018)



Atkinson et al. PRC 105, 054316 (2022)

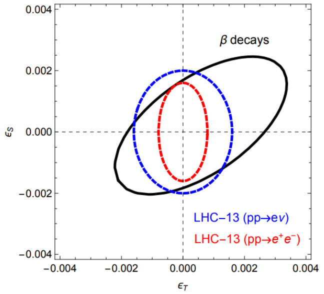




Ayyad et al. PRL 123, 082501 (2019)

Understanding possible exotic decay signals

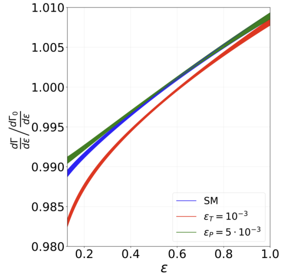
Falkowski et al. J. High Energ. Phys. 2021, 126 (2021)



He6 CRES

FRIB

GBK, Baroni, Mereghetti et al. PRC 107, 015503 (2023)



Probe of non-standard CC weak currents

Strategy

Validate nuclear physics model on readily-available experimental data:

- Energy spectra, form factors, moments
- EM and beta decay rates
- Muon capture rates
- Electron-nucleus scattering cross sections

Predict experimentally relevant quantities with validated model:

- Beta decay spectra
- Neutrinoless double beta decay
- Neutrino-nucleus scattering cross sections

Strategy

Validate nuclear physics model on readily-available experimental data:

- Energy spectra, form factors, moments
- EM and beta decay rates
- Muon capture rates
- Electron-nucleus scattering cross sections

Predict experimentally relevant quantities with validated model:

- Beta decay spectra
- Neutrinoless double beta decay
- Neutrino-nucleus scattering cross sections

Microscopic (or *ab initio*) description of nuclei

Comprehensive theory that describes quantitatively and predictably nuclear structure and reactions

Requirements:

- An accurate understanding of the interactions/correlations between nucleons in pairs, triplets, … **(two- and three-nucleon forces)**
- An accurate understanding of the electroweak interactions of external probes (electrons, neutrinos, photons) with nucleons, correlated pairs of nucleons, … **(one- and two-body electroweak currents)**
- **Computational Methods** to solve the nuclear many-body problem

Nuclear Forces

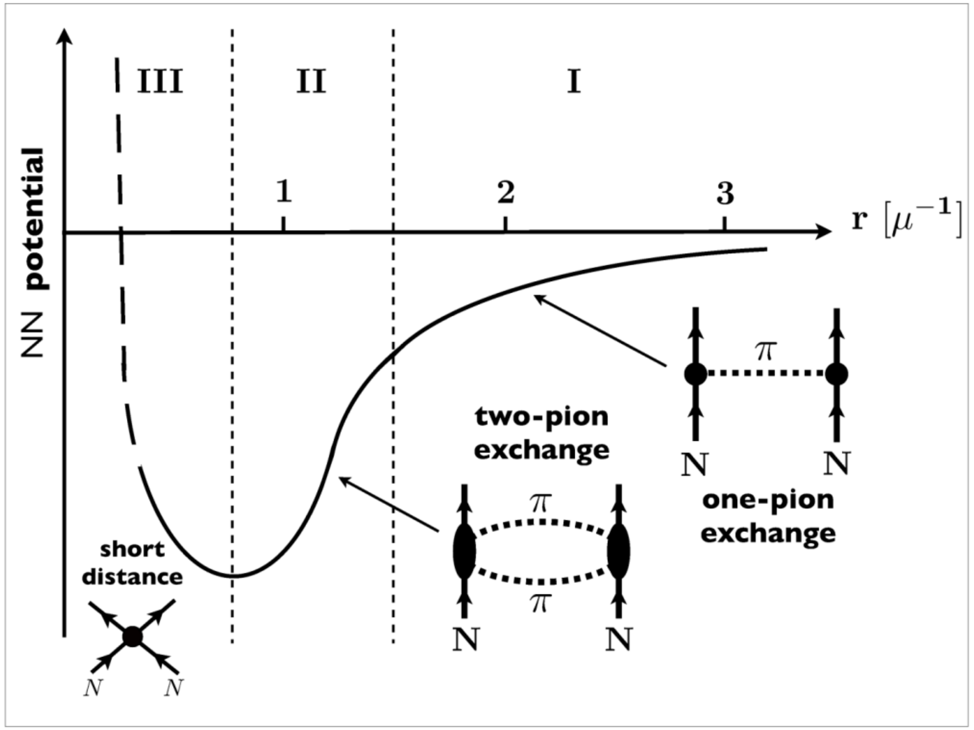
Exchange of the lightest meson, the pion, results in long-range attraction

Intermediate attraction of two-pion range

Repulsive core with short distance physics containing heavier meson exchanges

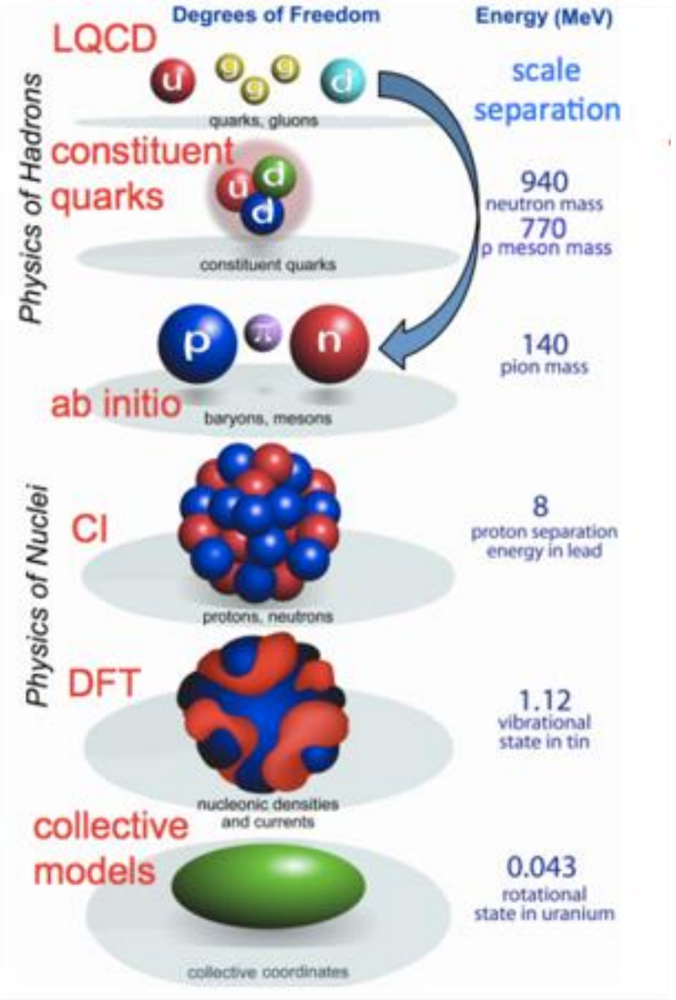
3NF needed to reproduce binding energies already in the trinucleons

Successful phenomenological potentials built upon this paradigm (AV18, CD-Bonn, …)



Holt et al. PPNP 73 (2013)

Chiral Effective Field Theory (χEFT)



Method to obtain the nuclear many-body Hamiltonian

Procedure to derive the nuclear interaction containing all low-energy symmetries of QCD

Provides a hierarchy of two- and many-nucleon forces and electroweak currents

Pions, Nucleons, possibly Δ(1232), and external fields may be retained as the relevant degrees of freedom

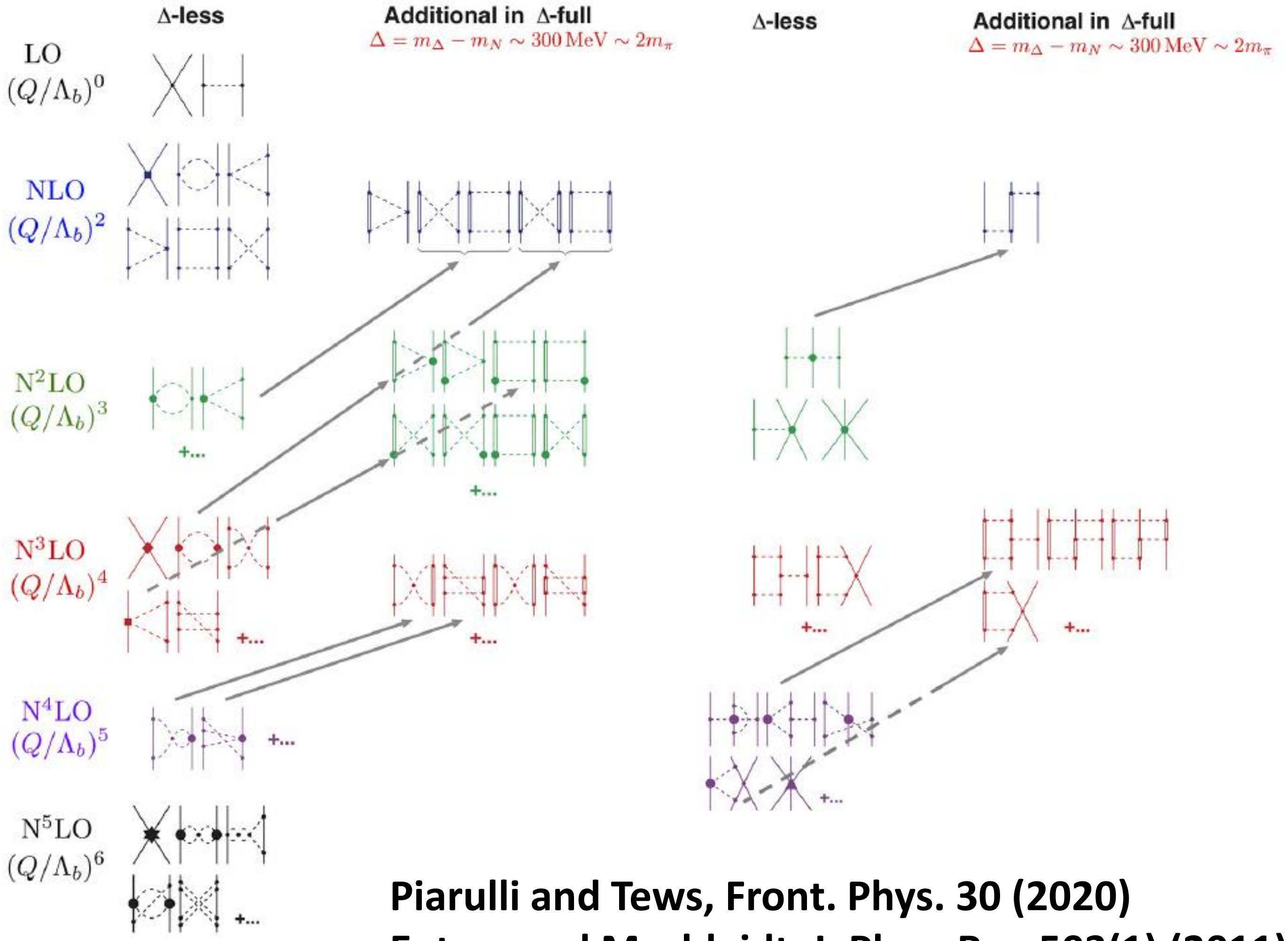
Weinberg, van Kolck, Ordóñez, Epelbaum, Hammer, Meißner, Entem, Machleidt, …

Chiral Effective Field Theory (χEFT)

Pions are the pseudo-Goldstone boson of spontaneously broken chiral symmetry

Low-energy constants (LECs) subsume the underlying QCD

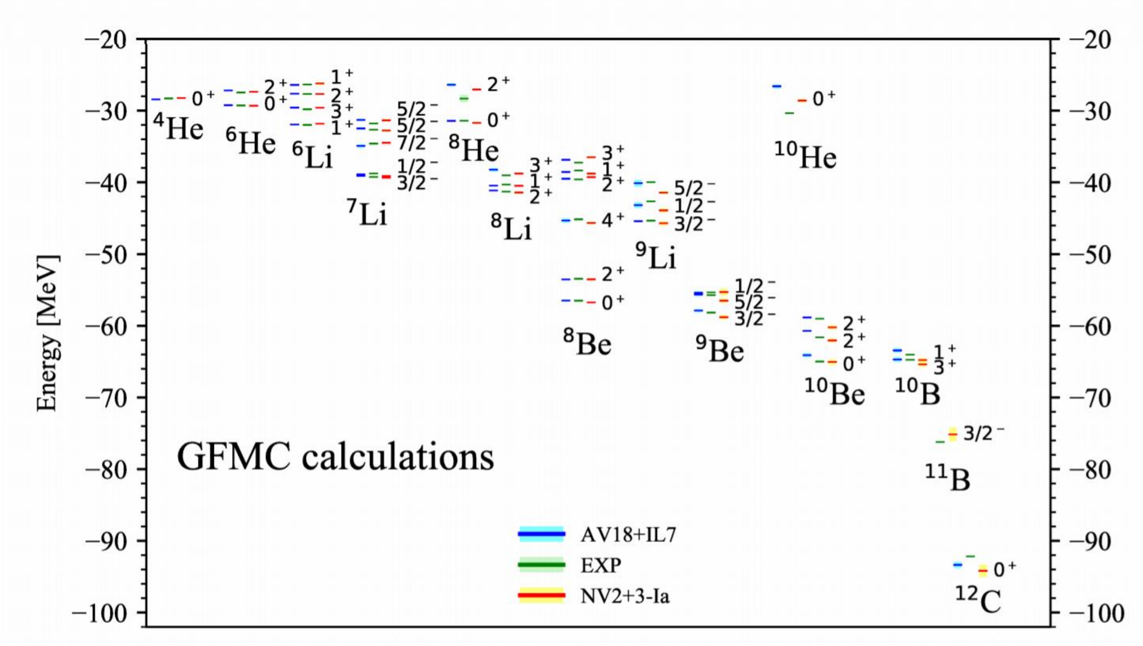
Power expansion in the typical nucleon momentum Q ~ pion mass insertions ~ N-to-Δ mass splittings over the QCD scale (Λ ~ 1 GeV)



Piarulli and Tews, Front. Phys. 30 (2020)
Entem and Machleidt, J. Phys. Rep 503(1) (2011)

The Norfolk (NV2+3) Interaction

$$H = \sum_i K_i + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk}$$



Piarulli et al. PRL 120, 052503 (2018)

Derived in χEFT with pion, nucleon, and delta degrees of freedom

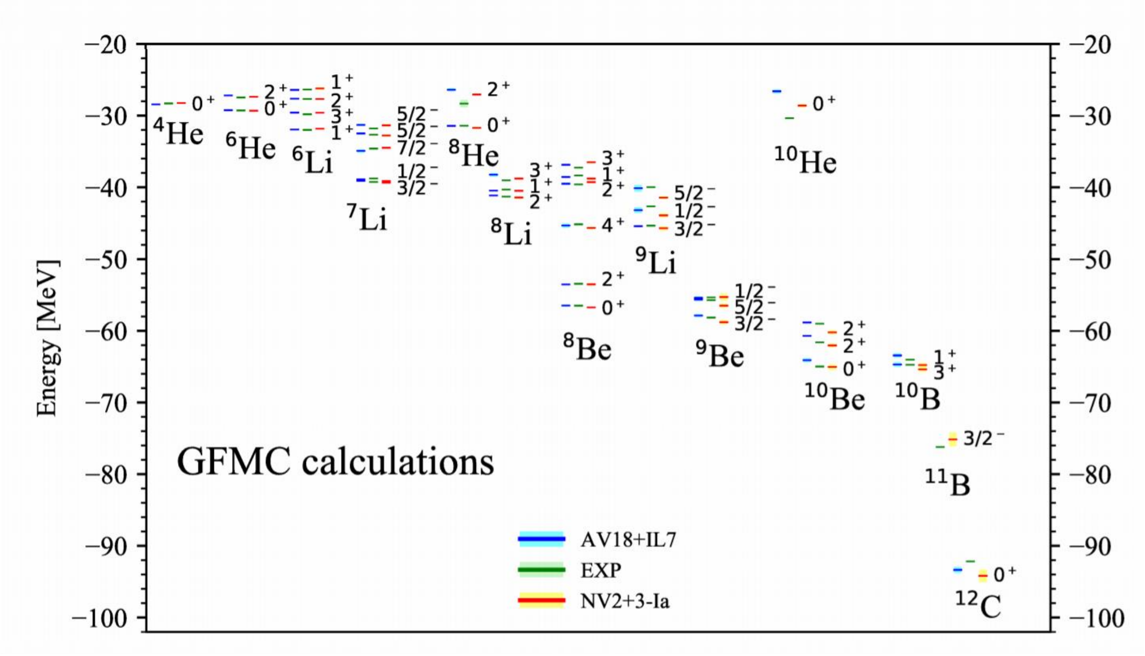
NV2 is fully local chiral interaction to N2LO (including some N3LO contributions) containing 26 unknown contact LECs

NV3 includes two long-range interactions and two contact interactions introducing two new unknown LECs, form as derived by **van Kolck PRC 49, 2932 (1994)** and **Epelbaum et al PRC 66, 064001 (2002)**

The Norfolk (NV2+3) Interaction

$$H = \sum_i K_i + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk}$$

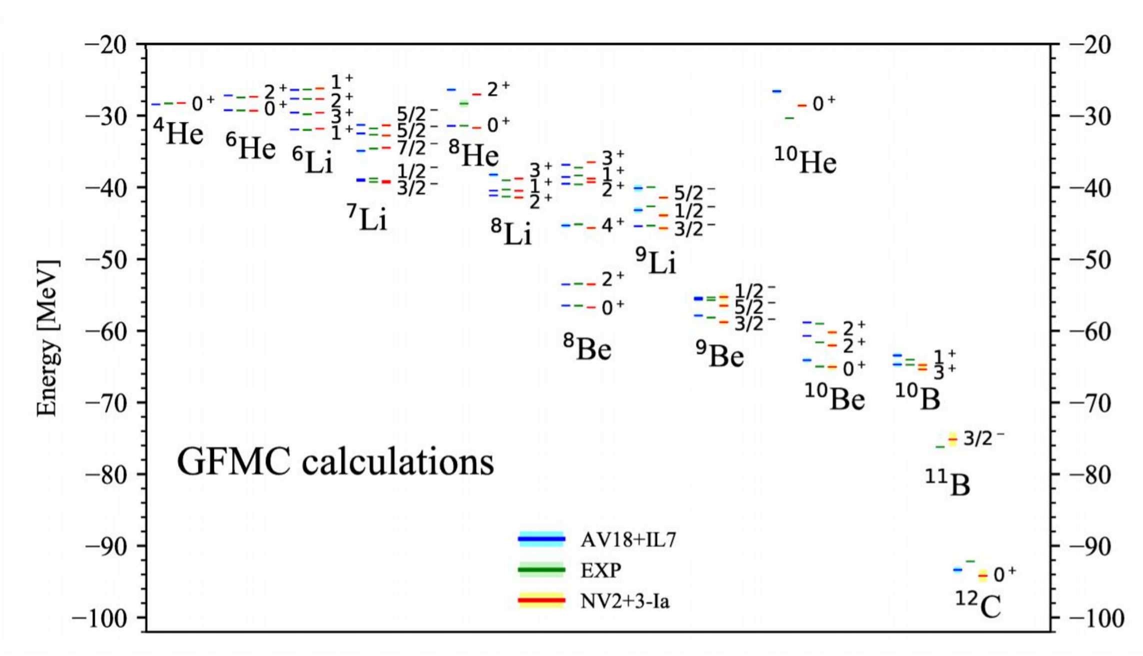
Eight different Model classes:



Piarulli et al. PRL 120, 052503 (2018)

The Norfolk (NV2+3) Interaction

$$H = \sum_i K_i + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk}$$



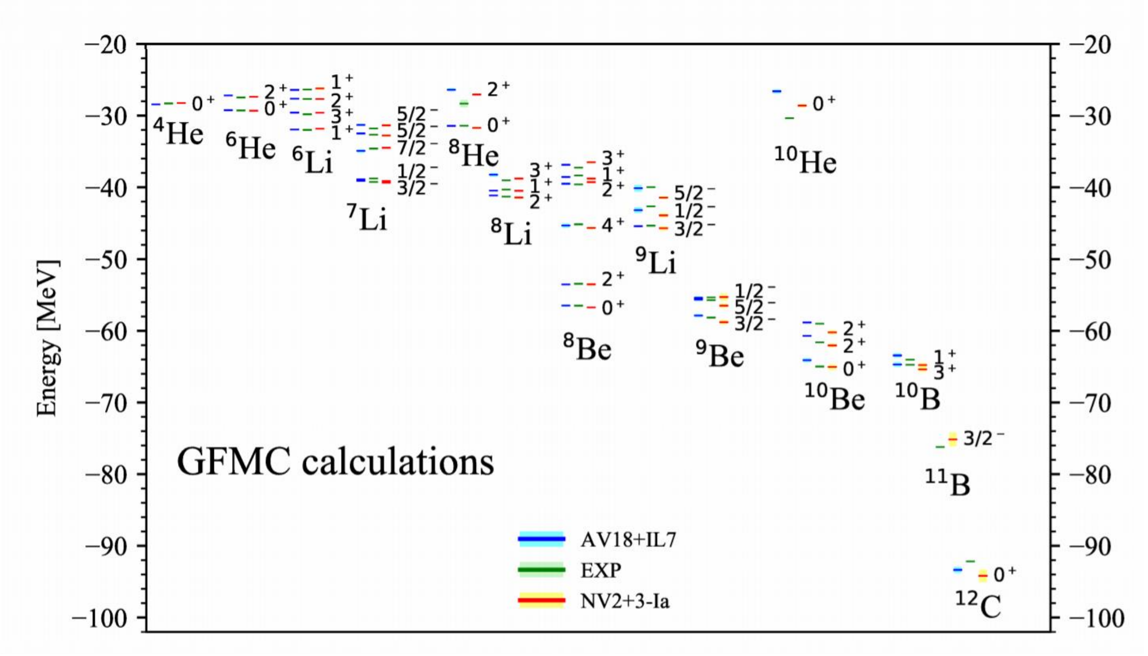
Piarulli et al. PRL 120, 052503 (2018)

Eight different Model classes:

- I [II]: NN scattering to fit two-body interaction from 0 to 125 [200] MeV

The Norfolk (NV2+3) Interaction

$$H = \sum_i K_i + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk}$$



Piarulli et al. PRL 120, 052503 (2018)

Eight different Model classes:

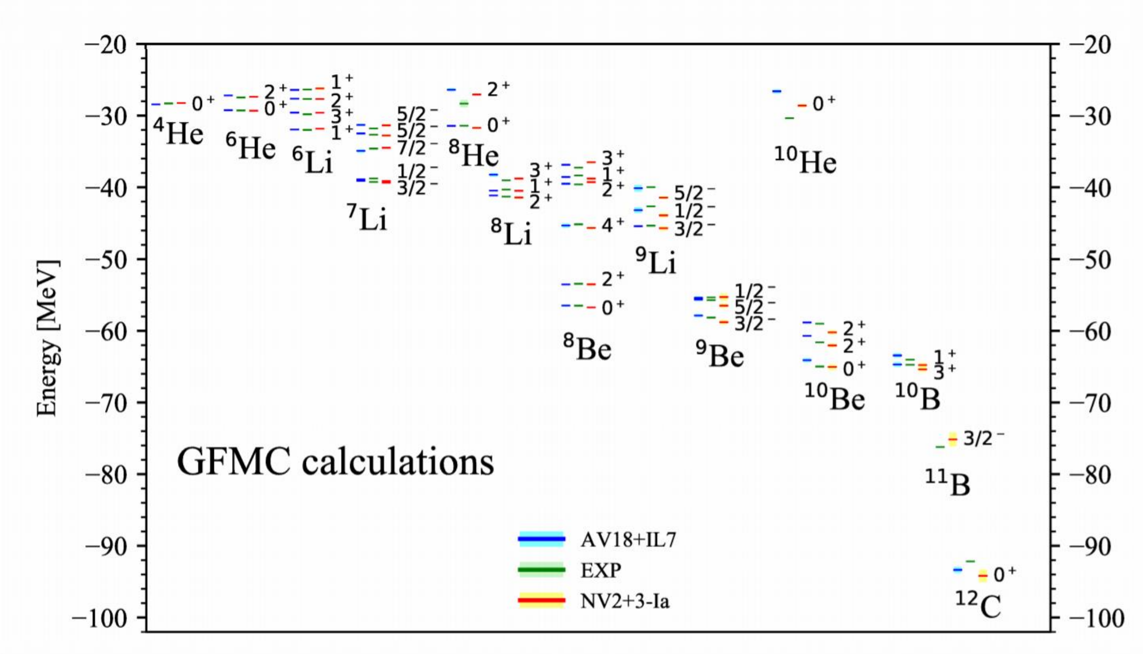
- I [II]: NN scattering to fit two-body interaction from 0 to 125 [200] MeV
- a [b]: Long- and short-range regulators (R_L,R_S) = (1.2 fm, 0.8 fm) [(1.0 fm, 0.7 fm)]

$$C_{R_L}(r) = 1 - \frac{1}{(r/R_L)^6 e^{(r-R_L)/a_L} + 1}$$

$$C_{R_S}(r) = \frac{1}{\pi^{3/2} R_S^3} e^{-(r/R_S)^2}$$

The Norfolk (NV2+3) Interaction

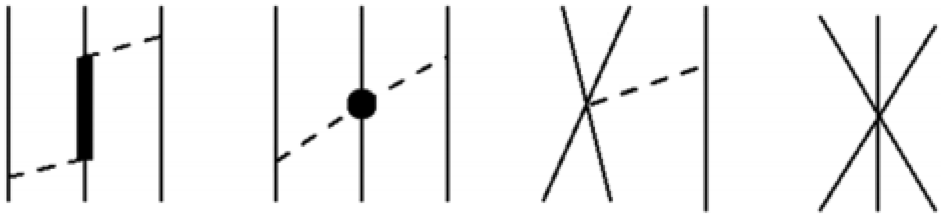
$$H = \sum_i K_i + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk}$$



Piarulli et al. PRL 120, 052503 (2018)

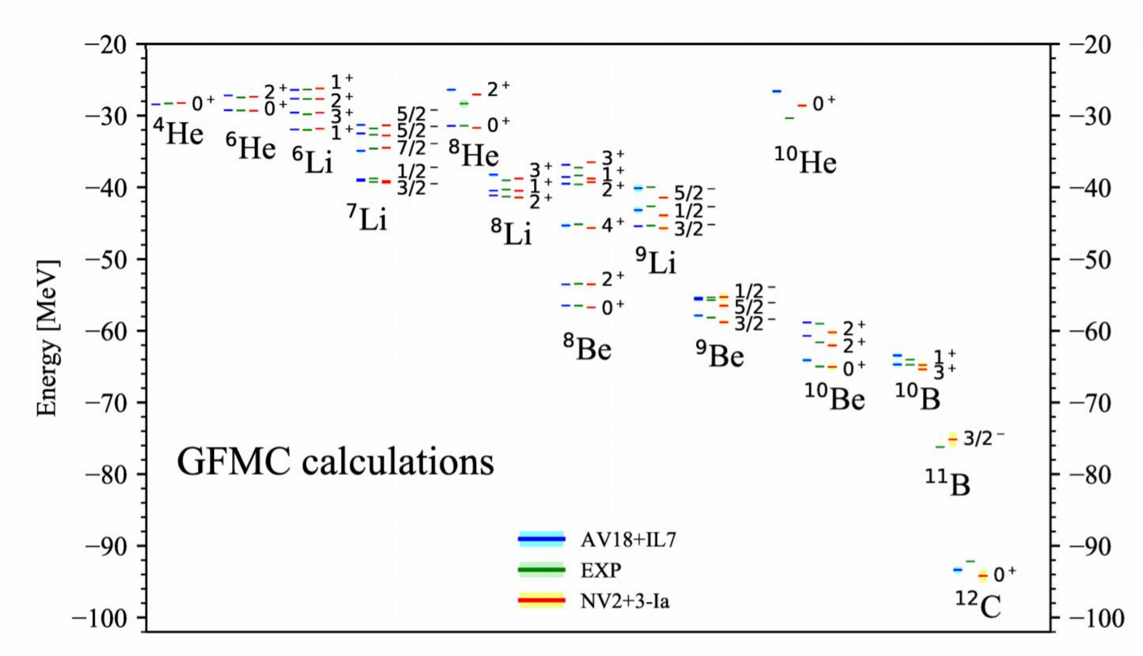
Eight different Model classes:

- I [II]: NN scattering to fit two-body interaction from 0 to 125 [200] MeV
- a [b]: Long- and short-range regulators (R_L,R_S) = (1.2 fm, 0.8 fm) [(1.0 fm, 0.7 fm)]
- Unstarred: Three-body term constrained with strong data only
- Star: Three-body term constrained with strong and weak data



The Norfolk (NV2+3) Interaction

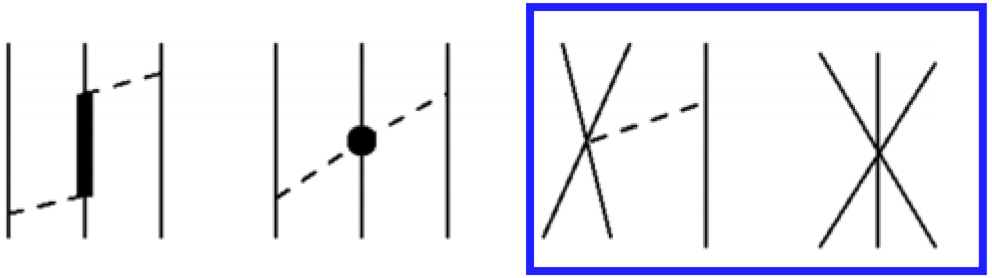
$$H = \sum_i K_i + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk}$$



Piarulli et al. PRL 120, 052503 (2018)

Eight different Model classes:

- I [II]: NN scattering to fit two-body interaction from 0 to 125 [200] MeV
- a [b]: Long- and short-range regulators (R_L,R_S) = (1.2 fm, 0.8 fm) [(1.0 fm, 0.7 fm)]
- Unstarred: Three-body term constrained with strong data only
- Star: Three-body term constrained with strong and weak data



NV2+3 Charge and Currents

Need nuclear vector and axial current operators to study weak processes in light nuclei

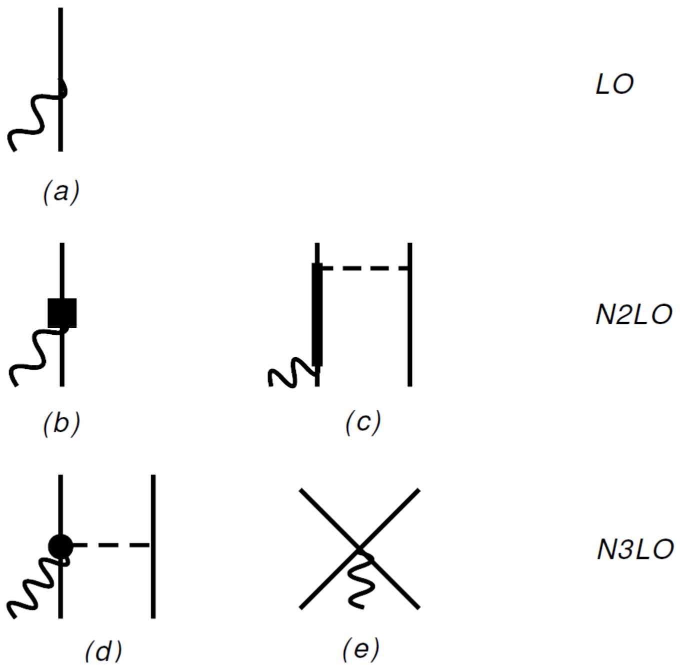
Schematically:

$$\rho = \sum_{i=1}^{A} \rho_i + \sum_{i<j} \rho_{ij} + \dots$$
$$\mathbf{j} = \sum_{i=1}^{A} \mathbf{j}_i + \sum_{i<j} \mathbf{j}_{ij} + \dots$$

External field interacts with single nucleons and correlated pairs of nucleons

Use vector and axial currents consistent with NV2+3 derived by JLAB-Pisa group: Pastore et al. PRC 80, 034004 (2009), Pastore et al. PRC 84, 024001 (2011), Piarulli et al. PRC 87, 014006 (2013), Schiavilla et al. PRC 99, 034005 (2019), Baroni et al. PRC 93, 049902 (2016), …

Example: Axial current at zero momentum transfer



Variational Monte Carlo (VMC)

Want to solve: $H\Psi(JMTT_z) = E\Psi(JMTT_z)$ with $H = \sum_i K_i + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk}$

$$|\Psi_V\rangle = \left[\mathcal{S}\prod_{i<j}(1 + U_{ij} + \sum_{k\neq i,j} U_{ijk})\right]\left[\sum_{i<j} f_c(r_{ij})\right]|\Phi_A(JMTT_z)\rangle$$

Slater determinant of nucleons in s- and p-shell coupled to the appropriate quantum numbers

Pair correlation operator encoding appropriate cluster structure

Two- and three-body correlation operator to reflect impact of nuclear interaction at short distances

Variational Monte Carlo (VMC) is used to find wavefunctions that minimize: $E_V = \frac{\langle\Psi_V|H|\Psi_V\rangle}{\langle\Psi_V|\Psi_V\rangle} \geq E_0$

Carlson et al. Rev. Mod. Phys. 87, 1607 (2015)

Green's Function Monte Carlo (GFMC)

The variational estimate can be further improved by acting with an imaginary time propagator

$$\Psi(\tau) = e^{-(H-E_0)\tau}\Psi_V = \left[e^{-(H-E_0)\Delta\tau}\right]^n \Psi_V$$

In general, variational state can be expanded in exact eigenstates of the Hamiltonian

$$|\Psi_V\rangle = \sum_{i=1}^{n} c_n|\psi_n\rangle$$

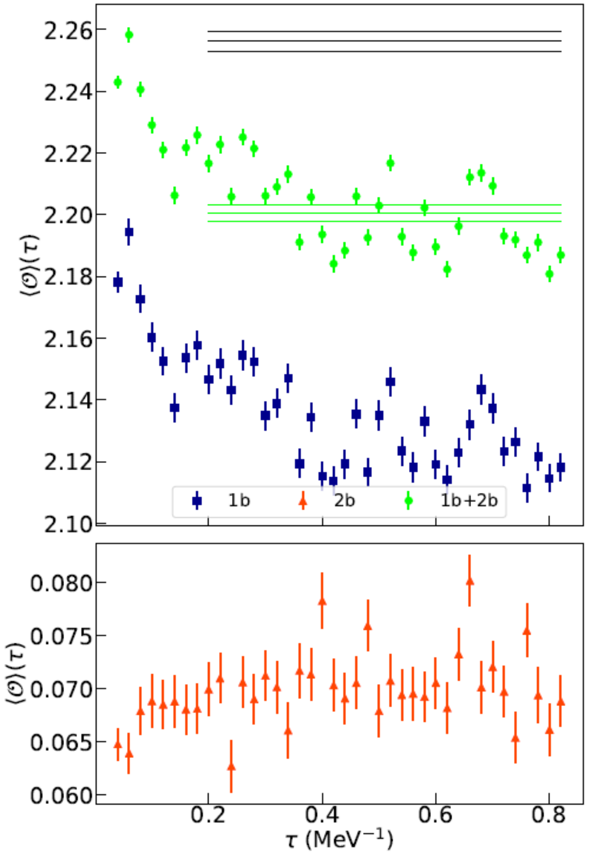
In the limit of infinite imaginary time

$$\lim_{\tau\to\infty} e^{-(H-E_0)\tau}\Psi_V \to c_0\psi_0$$

Carlson et al. Rev. Mod. Phys. 87, 1607 (2015)

Transition Matrix Element from GFMC

^{6}He → ^{6}Li GT RME extrapolation



Assume small correction to VMC: $\Psi(\tau) = \Psi_V + \delta\Psi$

Mixed estimate for off-diagonal transitions:

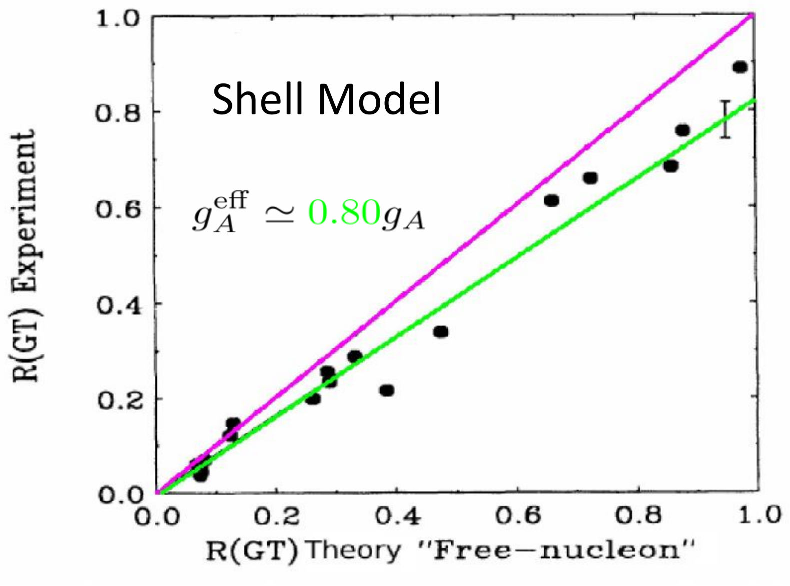
$$\langle \mathcal{O}(\tau)\rangle = \frac{\langle \Psi^f(\tau)|\mathcal{O}|\Psi^i(\tau)\rangle}{\sqrt{\langle \Psi^f(\tau)|\Psi^f(\tau)\rangle}\sqrt{\langle \Psi^i(\tau)|\Psi^i(\tau)\rangle}}$$
$$\simeq \langle \mathcal{O}(\tau)\rangle_{M_f} + \langle \mathcal{O}(\tau)\rangle_{M_i} - \langle \mathcal{O}\rangle_{\mathrm{VMC}}$$

where

$$\langle \mathcal{O}(\tau)\rangle_{M_f} = \frac{\langle \Psi^f(\tau)|\mathcal{O}|\Psi^i_V\rangle}{\langle \Psi^f(\tau)|\Psi^i_V\rangle}\,\frac{\sqrt{\langle \Psi^f_V|\Psi^f_V\rangle}}{\sqrt{\langle \Psi^i_V|\Psi^i_V\rangle}}$$

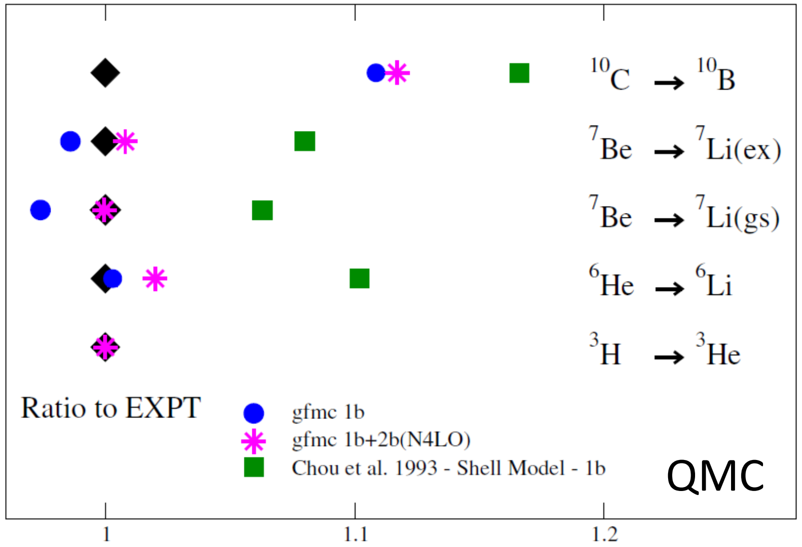
Pervin, Pieper, and Wiringa PRC 76, 064319 (2007)

Validation with GT reduced matrix elements

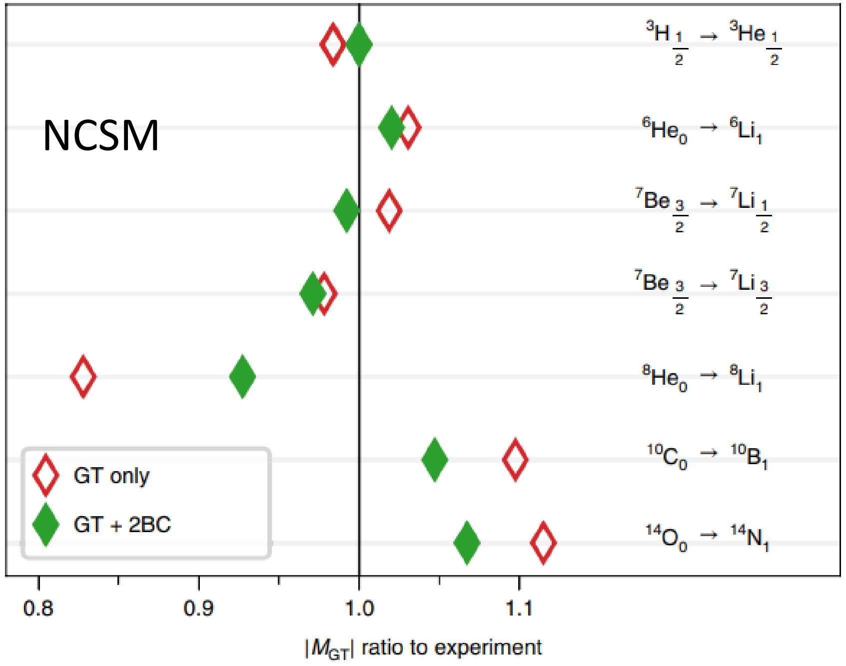


Chou et al. PRC 47, 167 (1993)

The "g_A quenching" problem for shell model calculations of beta decay rates resolved by the inclusion of two-body electroweak transition operators



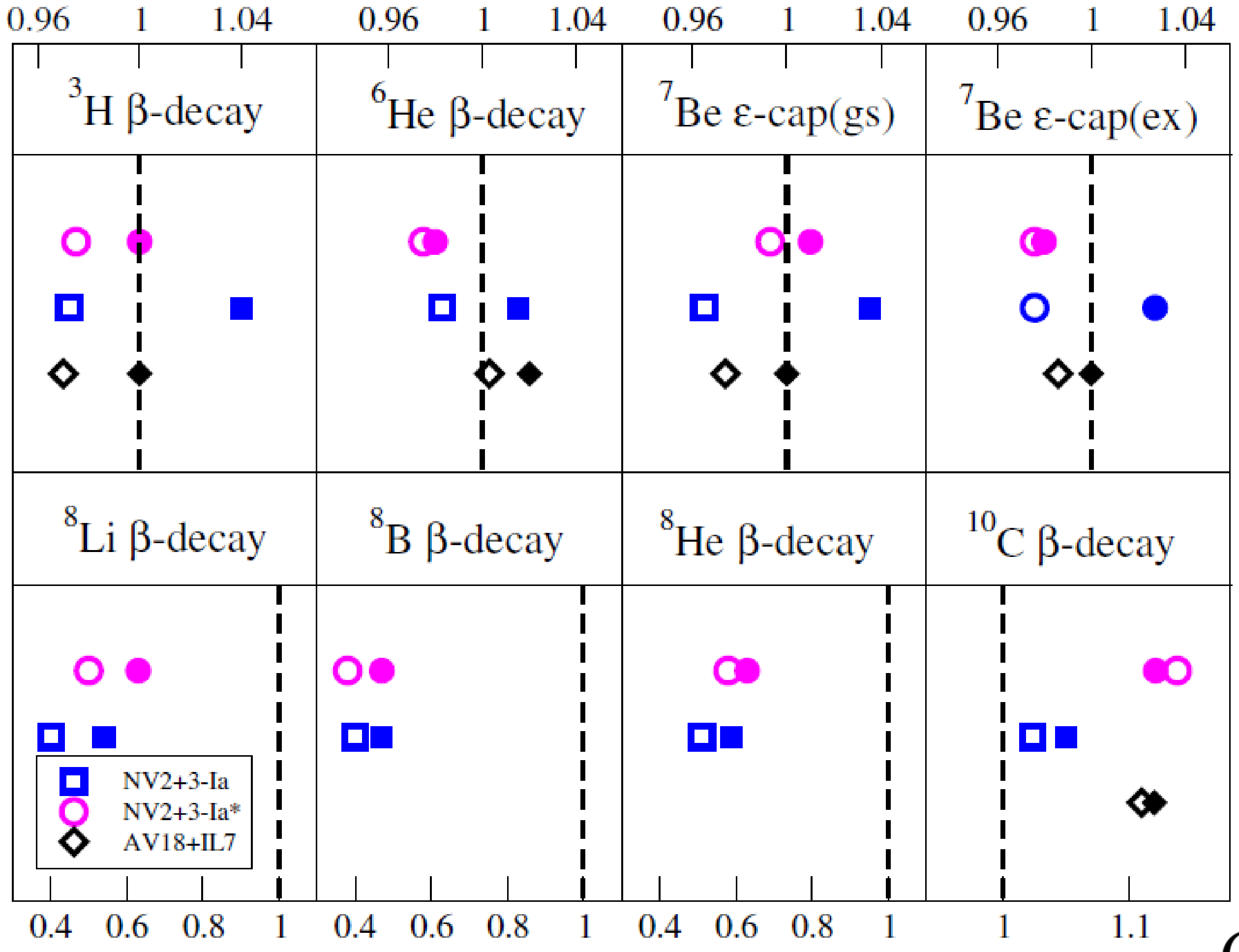
Pastore et al. PRC 97, 022501 (2018)



P. Gysbers et al. Nature Physics 15, 428 (2019)

ω ~ few MeV, q ~ 0

GFMC GT Reduced Matrix Elements



NV2+3-Ia: Three-body constrained with only strong data

NV2+3-Ia*: Three-body constrained with strong and weak data

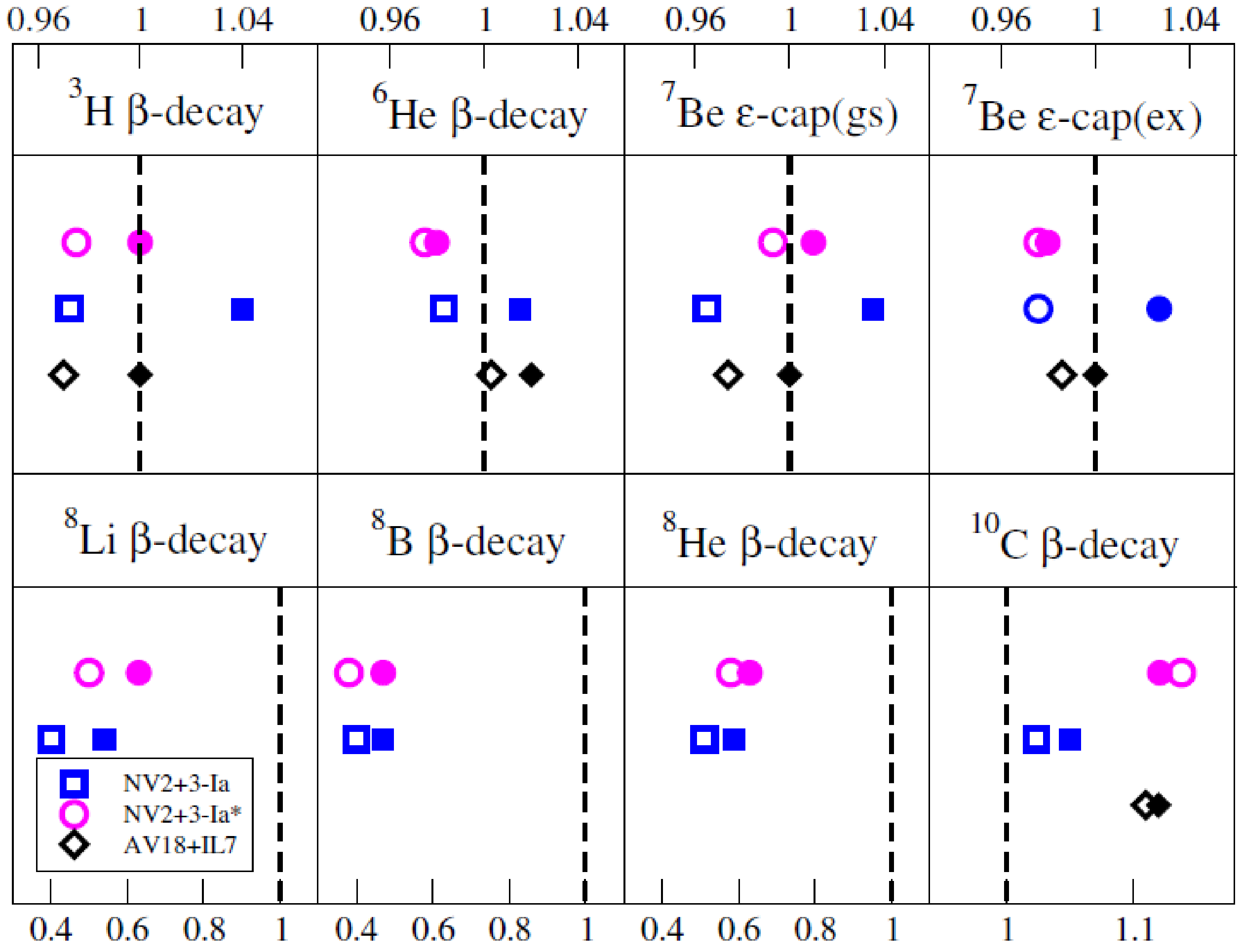
GFMC GT matrix elements compared with results using the AV18+IL7 in **Pastore et al. PRC 97, 022501 (2018)**

Empty symbols are results up to LO, solid symbols up to N3LO

$$\text{GT RME} = \frac{\sqrt{2J_f+1}}{g_A}\frac{\langle J_f\,M|j^z_{\pm,5}(\mathbf{q}\to 0)|J_i\,M\rangle}{\langle J_i\,M,10|J_f\,M\rangle}$$

King et al. PRC 121, 025501 (2020)

GFMC GT Reduced Matrix Elements



In most cases, two-body correction is small (~ few %) and additive

NV2+3-Ia has an enhanced two-body correction relative to the NV2+3-Ia*

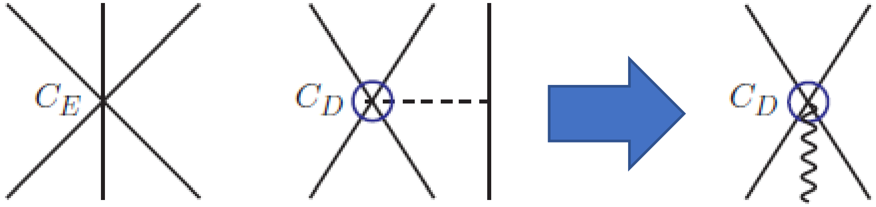
A=8 suppressed at leading order, larger two-body corrections

A=10 for NV2+3-Ia* has a negative two-body correction

$$\text{GT RME} = \frac{\sqrt{2J_f+1}}{g_A}\frac{\langle J_f M | j^z_{\pm,5}(\mathbf{q}\to 0)|J_i M\rangle}{\langle J_i M, 10 | J_f M\rangle}$$

King et al. PRC 121, 025501 (2020)

Three-body LECs and N3LO-CT



$$\mathbf{j}_{5,a}^{\mathrm{N3LO}}(\mathbf{q};\mathrm{CT}) = z_0 \mathrm{e}^{i\,\mathbf{q}\cdot\mathbf{R}_{ij}} \frac{\mathrm{e}^{-z_{ij}^2}}{\pi^{3/2}} \left(\boldsymbol{\tau}_i \times \boldsymbol{\tau}_j\right)_a \left(\boldsymbol{\sigma}_i \times \boldsymbol{\sigma}_j\right)$$

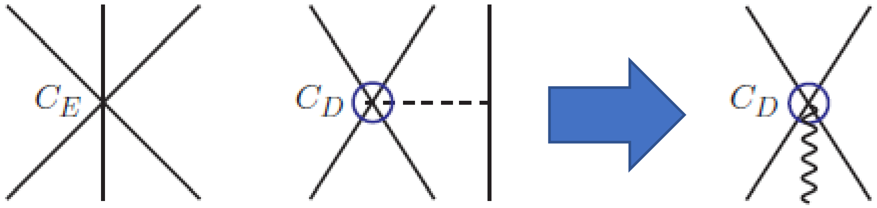
$$z_0 = \frac{g_A}{2}\,\frac{m_\pi^2}{f_\pi^2}\,\frac{1}{(m_\pi R_{\mathrm{S}})^3}\left[-\frac{m_\pi}{4\,g_A\,\Lambda_\chi}\,c_D + \frac{m_\pi}{3}\,(c_3+2c_4) + \frac{m_\pi}{6\,m}\right]$$

The NV2+3-Ia model fits the LECS using the *nd* doublet scattering length and trinucleon energies

The NV2+3-Ia* model fits trinucleon energies and the triton Gamow-Teller matrix element

Gardestig and Phillips PRL 96, 232301 (2006)

Three-body LECs and N3LO-CT



$$\mathbf{j}^{\mathrm{N3LO}}_{5,a}(\mathbf{q};\mathrm{CT}) = z_0 \mathrm{e}^{i\,\mathbf{q}\cdot\mathbf{R}_{ij}} \frac{\mathrm{e}^{-z_{ij}^2}}{\pi^{3/2}} \left(\boldsymbol{\tau}_i \times \boldsymbol{\tau}_j\right)_a \left(\boldsymbol{\sigma}_i \times \boldsymbol{\sigma}_j\right)$$

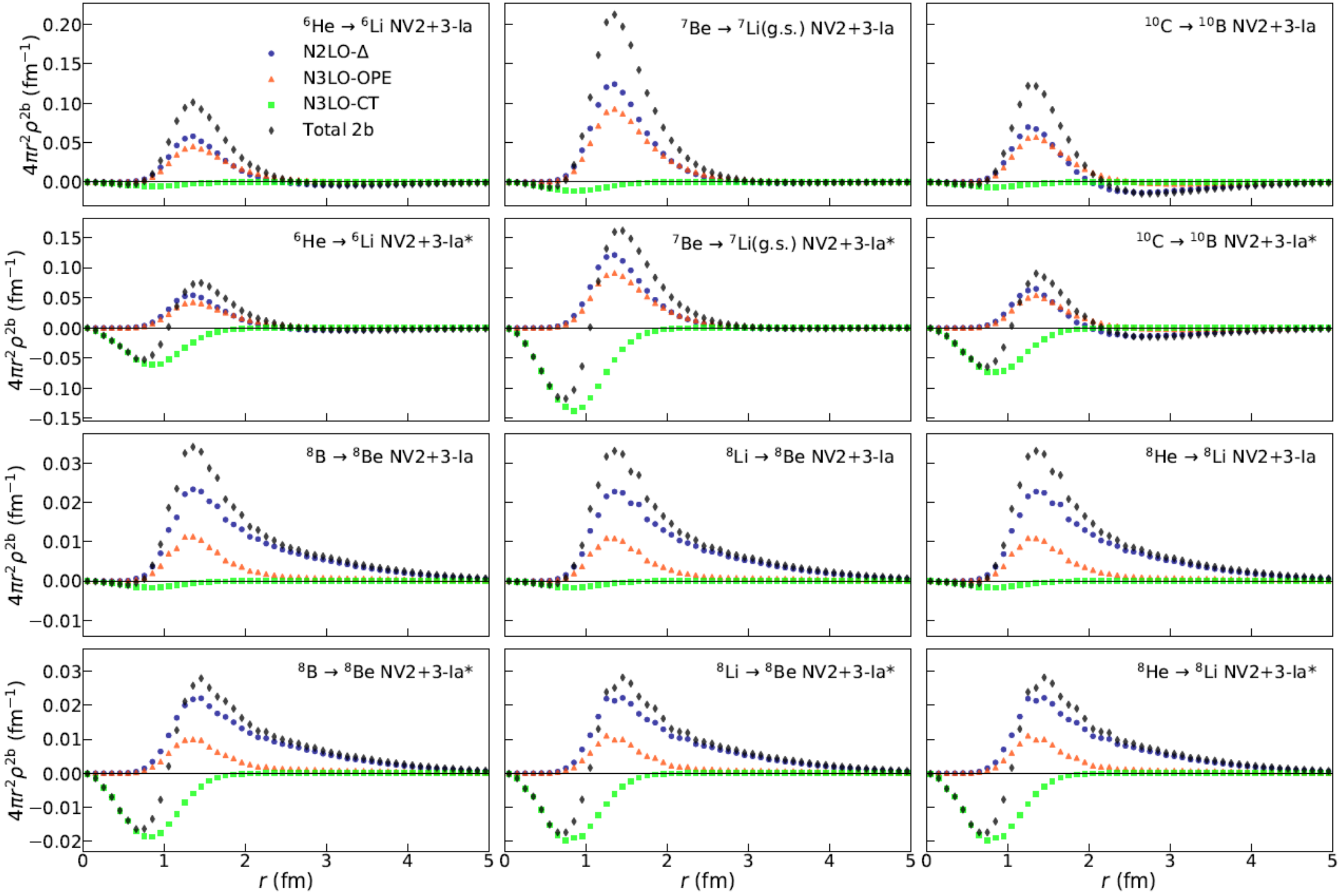
$$z_0 = \frac{g_A}{2}\frac{m_\pi^2}{f_\pi^2}\frac{1}{\left(m_\pi R_{\mathrm{S}}\right)^3}\left[\boxed{-\frac{m_\pi}{4\,g_A\,\Lambda_\chi}c_D} + \frac{m_\pi}{3}\left(c_3 + 2c_4\right) + \frac{m_\pi}{6\,m}\right]$$

The NV2+3-Ia model fits the LECS using the *nd* doublet scattering length and trinucleon energies

The NV2+3-Ia* model fits trinucleon energies and the triton Gamow-Teller matrix element

Gardestig and Phillips PRL 96, 232301 (2006)

Two-body VMC transition densities



Ia

Ia*

Ia

Ia*

The N3LO-CT term is a negative contribution is enhanced in the NV2+3-Ia*

N2LO-Δ and N3LO-OPE terms are consistent independent of the data used to constrain the three-body force

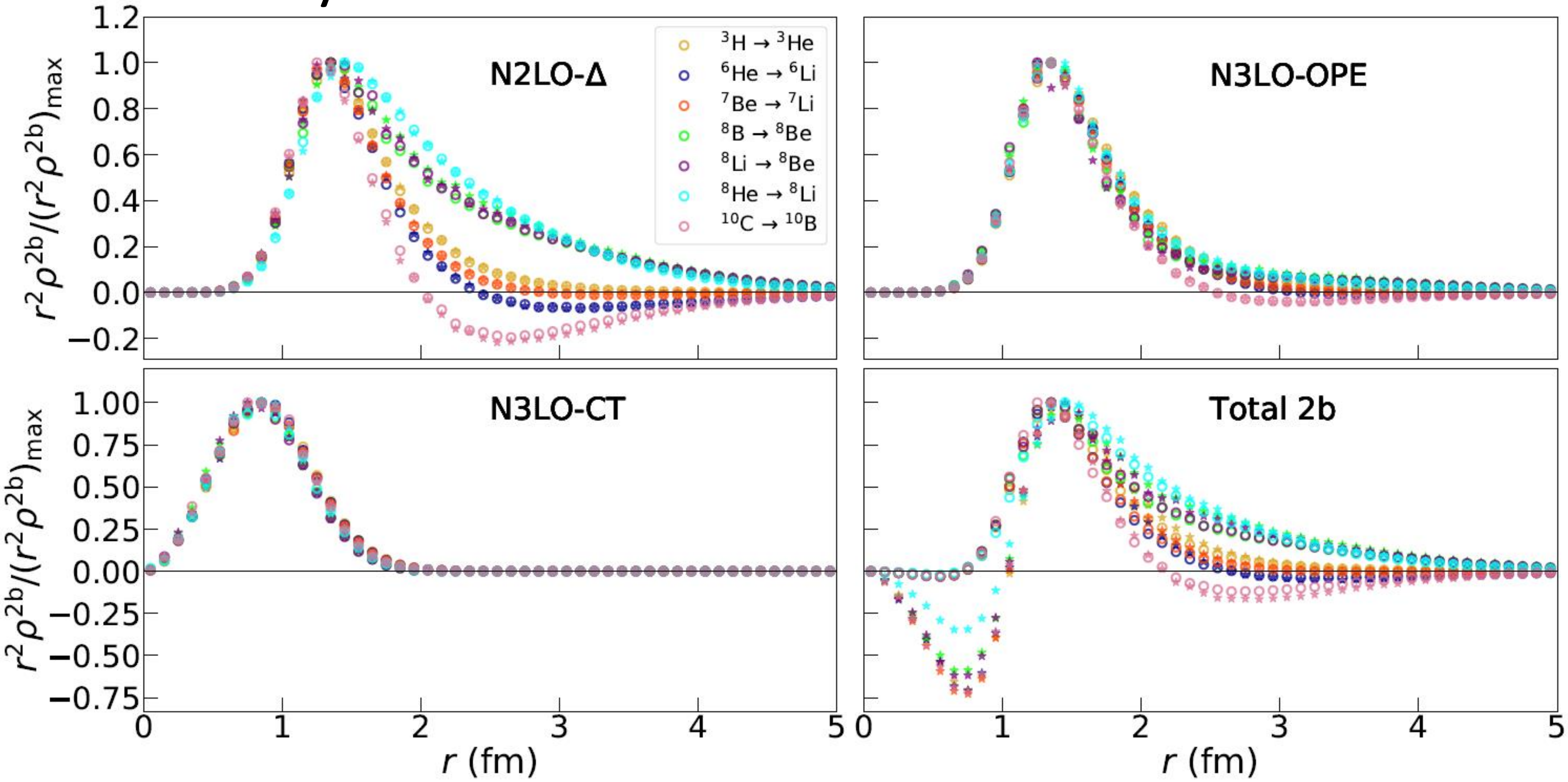
$$\mathrm{RME(2b)} = \int dr_{ij} 4\pi r_{ij}^2 \rho^{2b}(r_{ij})$$

King et al. PRC 121, 025501 (2020)

Scaled Two-body Transition Densities

Long-range N2LO-Δ and N3LO-OPE are transition dependent

Universal shape of the short-range transition density



King et al. PRC 121, 025501 (2020)

$$\mathrm{RME(2b)} = \int dr_{ij} 4\pi r_{ij}^2 \rho^{2b}(r_{ij})$$

B(GT) for A=11 nuclei

$$\mathrm{GT} = \frac{\sqrt{2J_f+1}}{g_A}\frac{\langle J_F M | j^z_{\pm,5}(\mathbf{q}\to 0)| J_i M\rangle}{\langle J_i M, 10 | J_f M\rangle}$$

$$B(GT) = \frac{|\mathrm{GT}|^2}{2J_i+1}$$

Reduced matrix elements from QMC can be used to obtain transition strengths to exclusive final states

B(GT) may be obtained from charge exchange reactions at zero momentum transfer

Do not depend on any model assumptions for the structure of the system

Tests quality of *ab initio* wave functions and many-body methods

B(GT) for A=11 nuclei

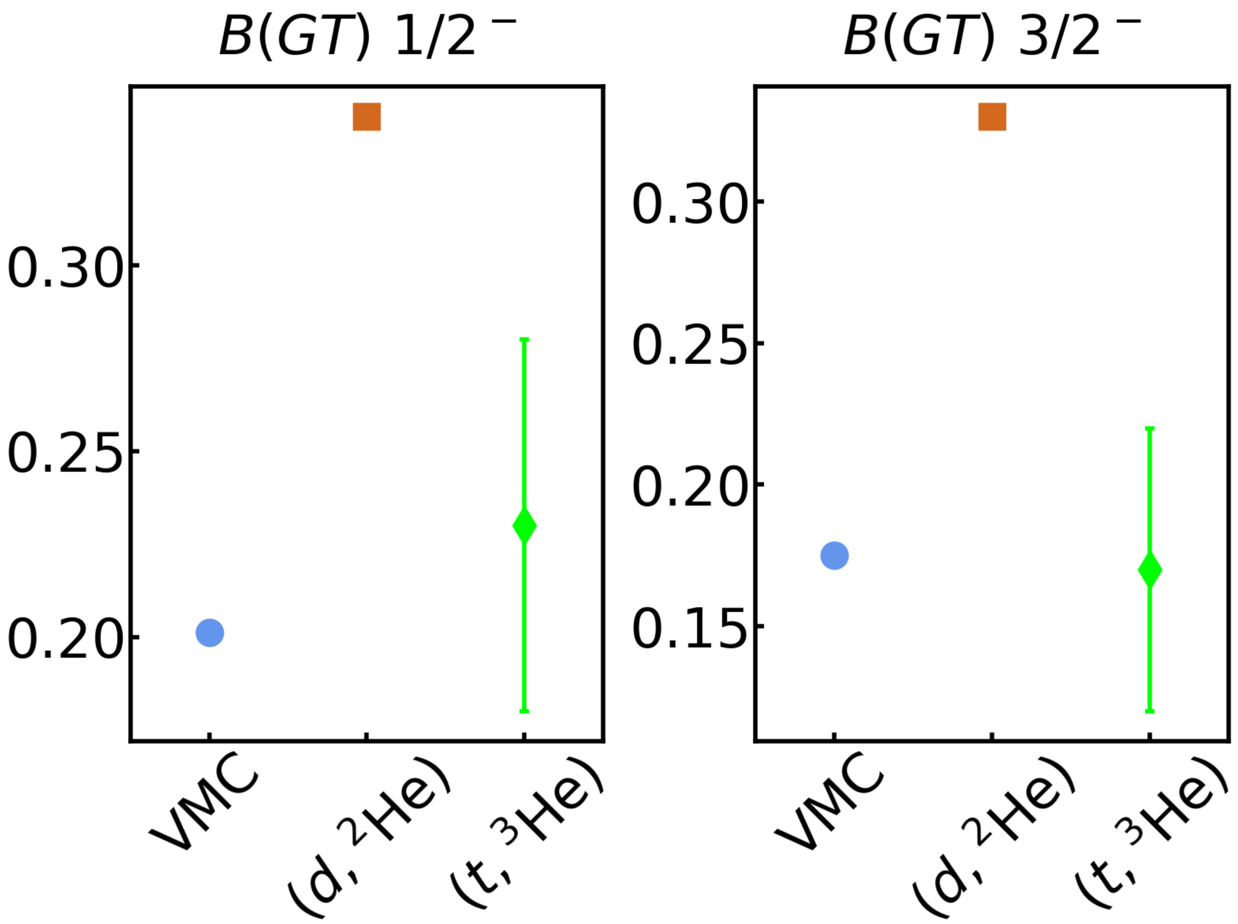
^{11}B(g.s.) → ^{11}Be*

NV2+3-Ia* VMC agrees well with the value extracted from *(t,^{3}He)*

(d,^{2}He) data consistent with unquenched shell model calculation

Two-body effects ~2%-3% and subtractive

$$B(GT) = \frac{|\mathrm{GT}|^2}{2J_i+1}$$



(d,^{2}He) **– Ohnishi et al., Nucl. Phys. A 687 (2001)**
(t,^{3}He) **– Daito et al., Phys. Lett. B (1998)**

Schmitt, GBK, et al. PRC 106, 054323 (2022)

$B(GT)$ for A=11 nuclei

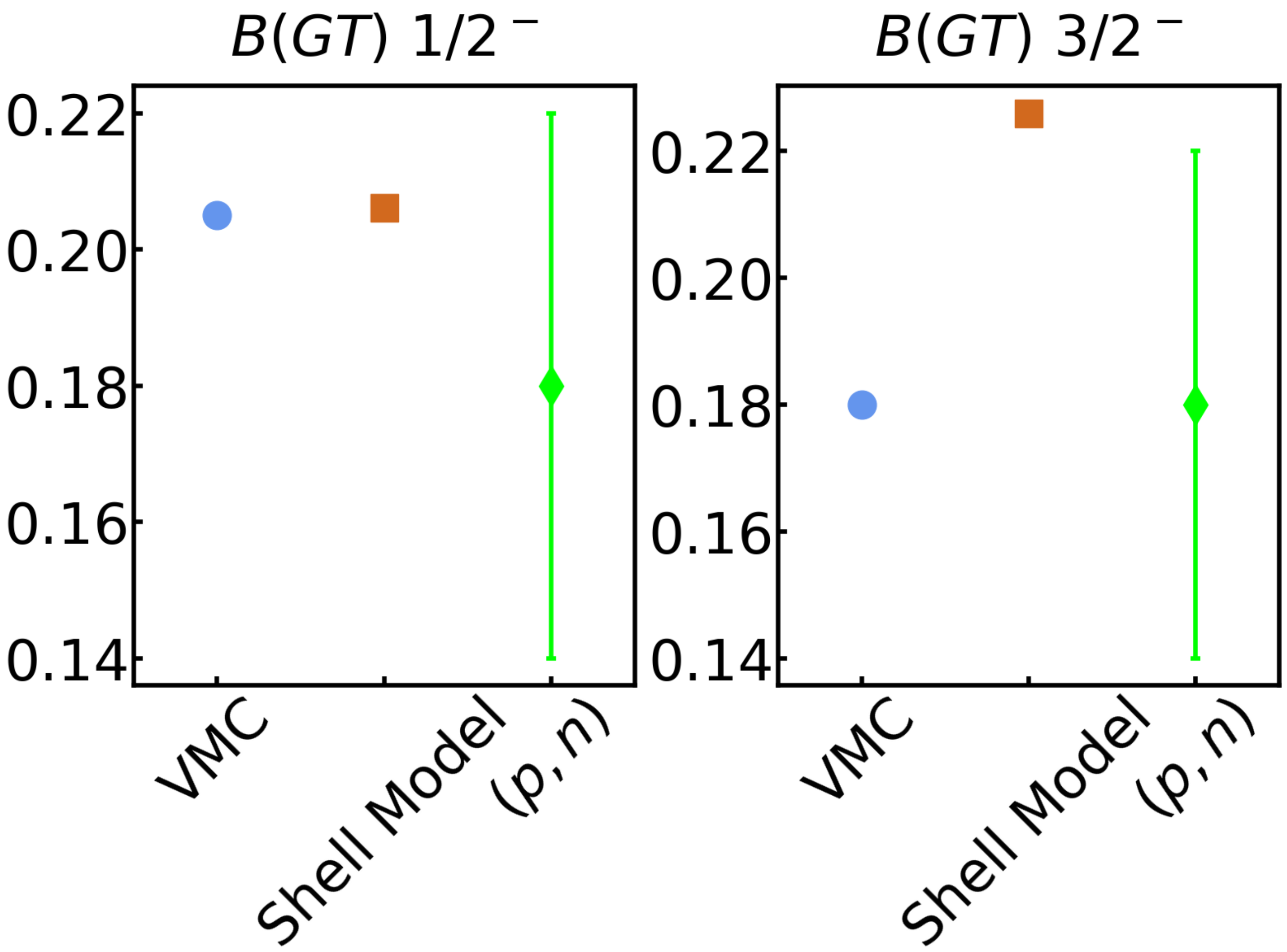
^{11}C(g.s.) → ^{11}N*

NV2+3-Ia* VMC result consistent under isospin symmetry when studying mirror transition

Good agreement between central value of VMC and experimental error bars

Two-body effects ~2%-4% and subtractive

$$B(GT) = \frac{|\mathrm{GT}|^2}{2J_i+1}$$



Shell Model – B. A. Brown (MSU)
***(p,n)* – J. Schmitt (MSU)**

Schmitt, GBK, et al. PRC 106, 054323 (2022)

$B(GT)$ for A=11 nuclei

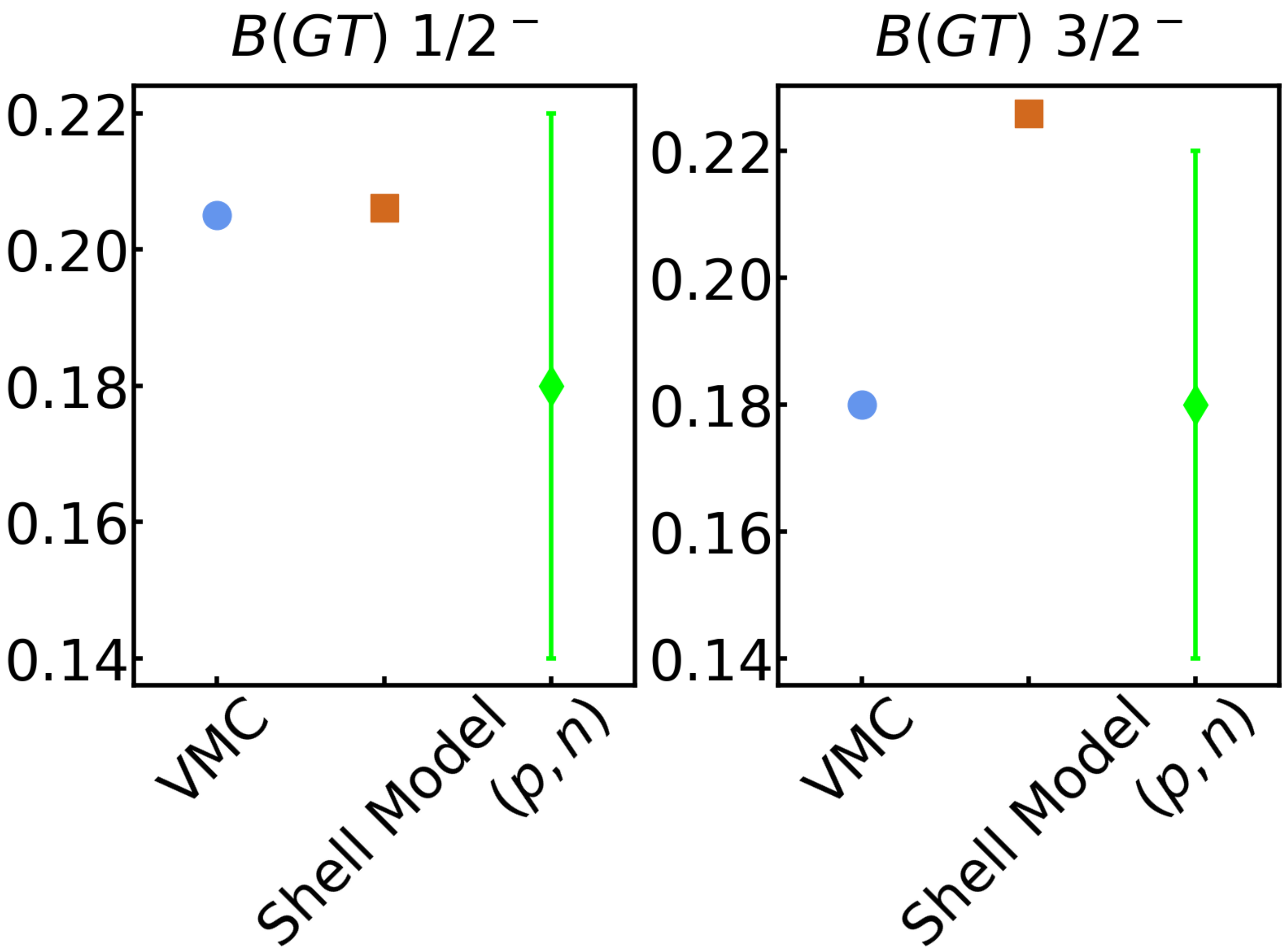
^{11}C(g.s.) → $^{11}N^*$

NV2+3-Ia* VMC result consistent under isospin symmetry when studying mirror transition

Good agreement between central value of VMC and experimental error bars

Two-body effects ~2%-4% and subtractive

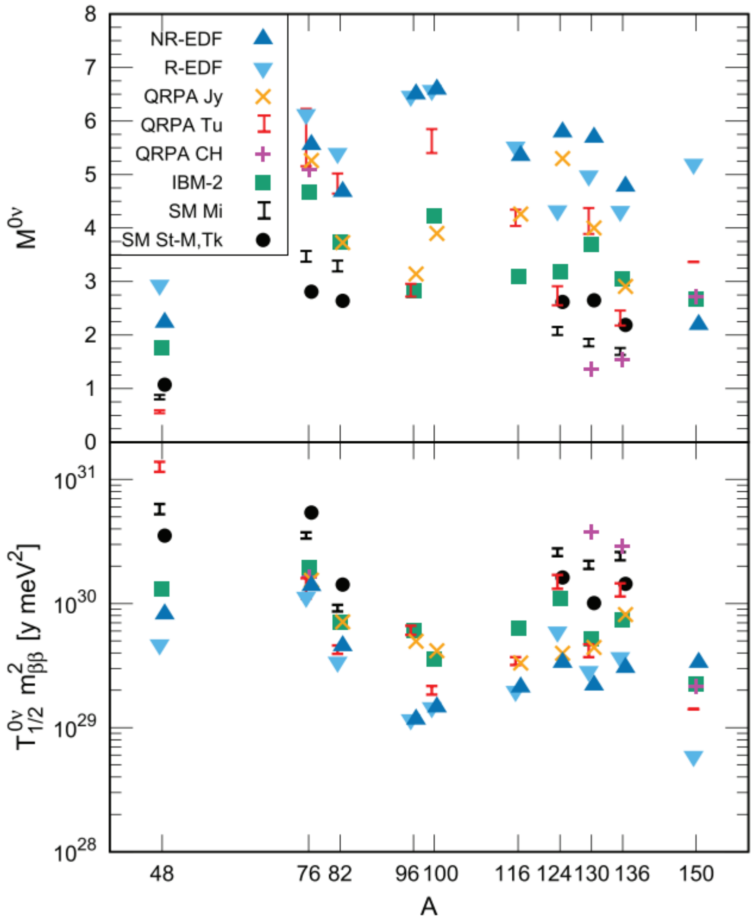
Outlook: Systematic study of GT transitions for nuclei with A ≥ 11 at GFMC level



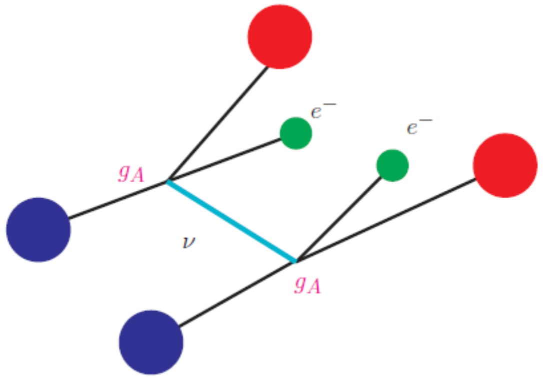
Shell Model – B. A. Brown (MSU)
***(p,n)* – J. Schmitt (MSU)**

Schmitt, GBK, et al. PRC 106, 054323 (2022)

Validation of vector current



Engel and Menéndez Rep. Prog. Phys. 80 046301 (2017)



ω ~ few MeV, q ~ 100 MeV: Muon capture, Neutrinoless ββ-decay

Goal: Validate weak transition operators with less restrictive selection rules at a non-zero momentum transfer with muon capture

Partial Muon Capture Rates with QMC

$$H_W = \frac{G_V}{\sqrt{2}} \int d\mathbf{x} e^{-i\mathbf{k}_\nu \cdot \mathbf{x}} \tilde{l}_\sigma(\mathbf{x}) j^\sigma(\mathbf{x})$$

Momentum transfer q ~ 100 MeV

Validation of vector and axial charges and currents

For light nuclei, you can approximate the muon as at rest in a Hydrogen-like 1s orbital

Partial Muon Capture Rates with QMC

Assuming a muon at rest in a Hydrogen-like 1s orbital:

$$\Gamma = \frac{G_V^2}{2\pi}\frac{|\psi_{1s}^{\rm av}|^2}{(2J_i+1)}\frac{E_\nu^{*2}}{\rm recoil}\sum_{M_f,M_i} |\langle J_f,M_f|\rho(E_\nu^*\hat{\mathbf{z}})|J_i,M_i\rangle|^2 + |\langle J_f,M_f|\mathbf{j}_z(E_\nu^*\hat{\mathbf{z}})|J_i,M_i\rangle|^2$$

$$+ 2\,{\rm Re}\left[\langle J_f,M_f|\rho(E_\nu^*\hat{\mathbf{z}})|J_i,M_i\rangle\,\langle J_f,M_f|\mathbf{j}_z(E_\nu^*\hat{\mathbf{z}})|J_i,M_i\rangle^*\right] + |\langle J_f,M_f|\mathbf{j}_x(E_\nu^*\hat{\mathbf{z}})|J_i,M_i\rangle|^2$$

$$+ |\langle J_f,M_f|\mathbf{j}_y(E_\nu^*\hat{\mathbf{z}})|J_i,M_i\rangle|^2 - 2\,{\rm Im}\left[\langle J_f,M_f|\mathbf{j}_x(E_\nu^*\hat{\mathbf{z}})|J_i,M_i\rangle\,\langle J_f,M_f|\mathbf{j}_y(E_\nu^*\hat{\mathbf{z}})|J_i,M_i\rangle^*\right]$$

King et al. PRC 105, L042501 (2022)

Partial Muon Capture Rates with QMC

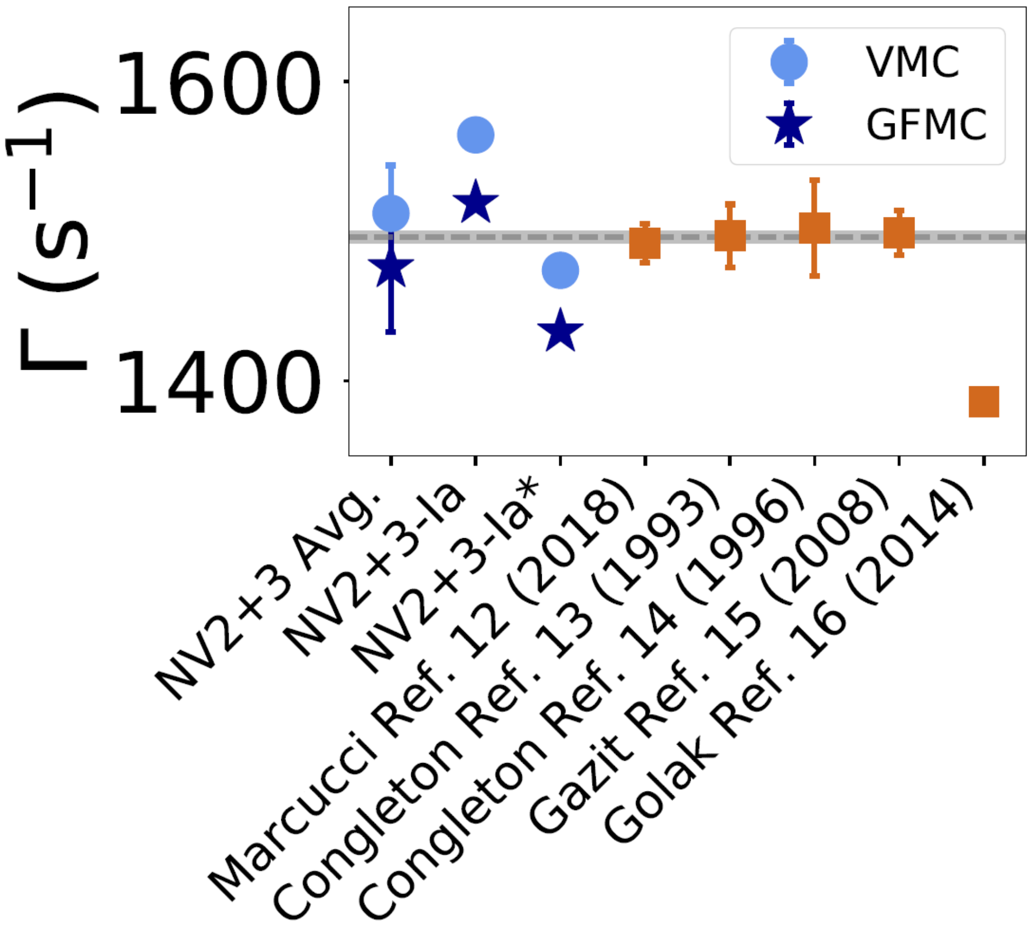
QMC rate for ^{3}He(1/2^{+};1/2) → ^{3}H(1/2^{+};1/2)

Γ_{VMC} = 1512 s^{-1} ± 32 s^{-1}

Γ_{GFMC} = 1476 s^{-1} ± 43 s^{-1}

Γ_{expt} = 1496.0 s^{-1} ± 4.0 s^{-1}

[Ackerbauer et al. Phys. Lett. B 417 (1998)]



VMC uncertainty estimates:

- Cutoff: 8 s^{-1} (0.7%)
- Energy range of fit: 11 s^{-1} (0.5%)
- Three-body fit: 27 s^{-1} (1.8%)
- Systematic: 9 s^{-1} (0.6%)

King et al. PRC 105, L042501 (2022)

Partial Muon Capture Rates with QMC

QMC rates for ^{6}Li(g.s.) → ^{6}He(g.s.):

Γ_{VMC}(avg.) = 1243 s^{-1} ± 59 s^{-1}

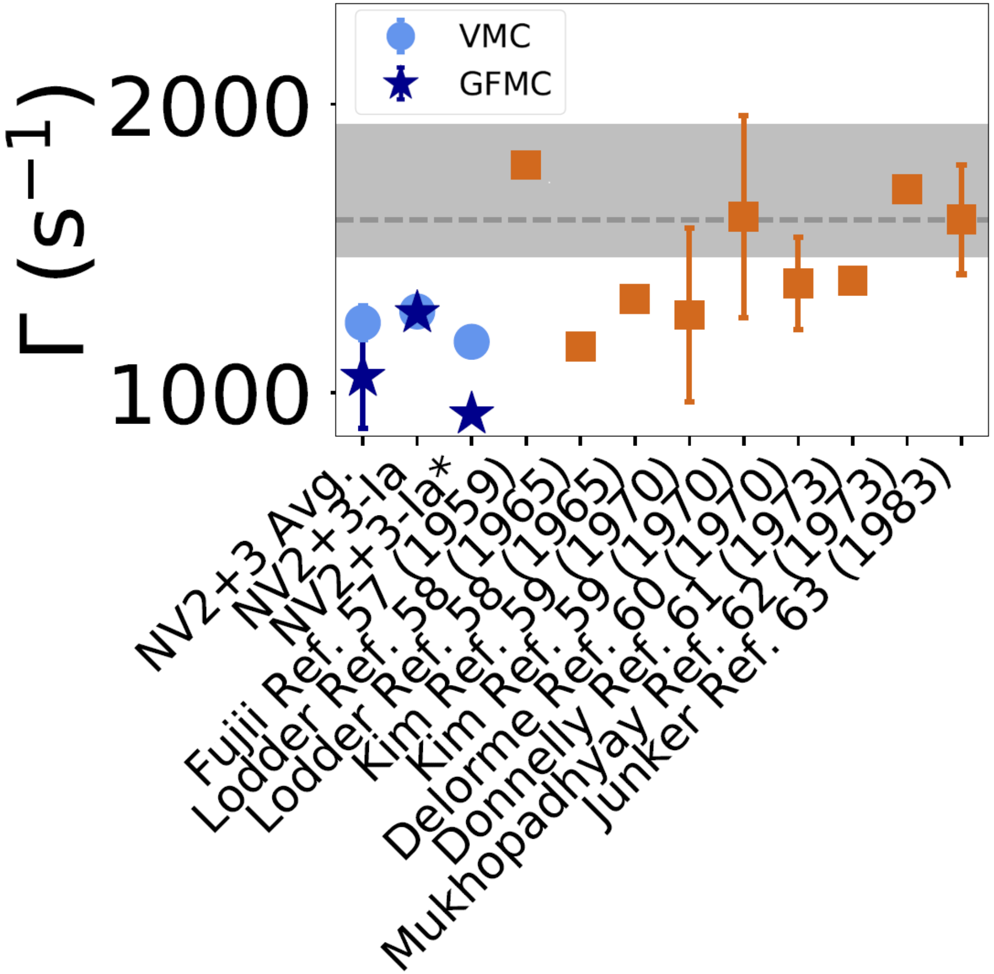
Γ_{GFMC}(avg.) = 1102 s^{-1} ± 176 s^{-1}

Γ_{expt} = 1600 s^{-1} +330/-129 s^{-1}

[Deutsch et al. Phys. Lett. B26, 315 (1968)]

VMC uncertainty estimates:

- Cutoff: 36 s^{-1} (2.9%)
- Energy range of fit: 36 s^{-1} (2.9%)
- Three-body fit: 30 s^{-1} (2.4%)
- Systematic: 7 s^{-1} (0.6%)



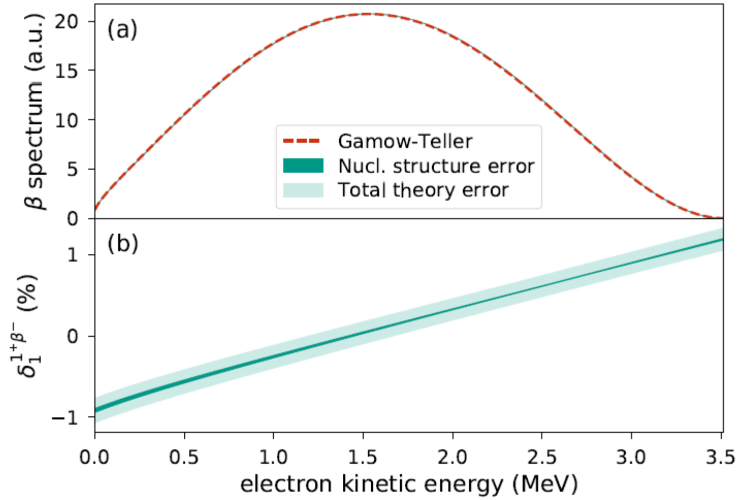
King et al. PRC 105, L042501 (2022)

^{6}He Beta Decay Spectrum

Beta decay in light nuclei is important for experiments searching for beyond standard model physics

Goal: Predict beta decay spectrum for ^{6}He retaining one- and two-body electroweak currents

^{6}He beta decay spectrum from NCSM



Glick-Magid et al. Phys. Lett. B 832 (2022)

NSCL

He6 CRES

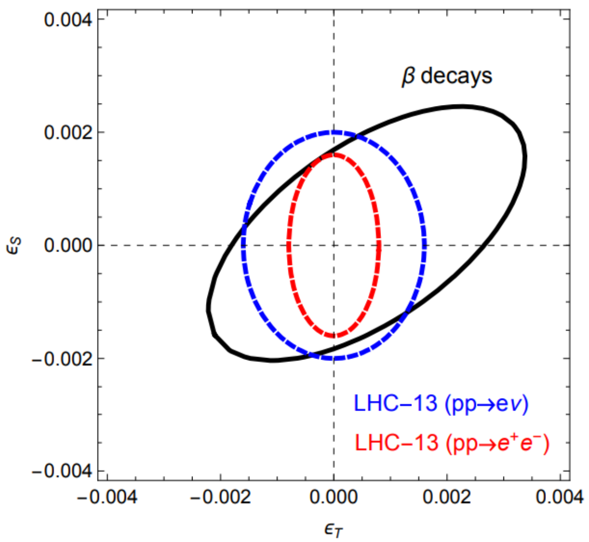
lpc caen

^{6}He beta decay to be used for BSM searches with NSCL, He6-CRES, LPC-Caen

Cirigliano et al. arXiv:1097.02164 (2019)

Nuclear electroweak interactions are crucial for future fundamental physics searches on the precision and high energy frontiers

ω ~ few MeV, q ~ 0: β-decay, ββ-decay



Falkowsi, González-Alonso, and Naviliat-Cuncic J. High Energ. Phys. 2021, 126 (2021)

^{6}He Beta Decay Spectrum: Overview

Differential beta decay rate:

$$d\Gamma = \frac{2\pi}{2J_i + 1} \sum_{s_e, s_\nu} \sum_{M_i, M_f} |\langle f | H_W | i \rangle|^2 \delta(\Delta E) \frac{d^3 k_e}{(2\pi)^3} \frac{d^3 k_\nu}{(2\pi)^3}$$

Traces of lepton tensor appearing in the rate depend on the electron and neutrino kinematics

Integrating over neutrino energy and relative angle between neutrino and electron gives the rate

$$\frac{d\Gamma}{d\varepsilon} = \frac{d\Gamma_0}{d\varepsilon} \times (1 + \mathrm{corrections}) \qquad \varepsilon = \frac{E_e}{\omega}$$

where Γ_0 is the decay rate in the limit $q \to 0$ with radiative corrections included and the additional corrections are determined by retaining q dependence in NME

We treat the spectrum with one- and two-body currents retaining momentum dependence and (some) isospin breaking effects

^{6}He Beta Decay Spectrum: Multipoles

$$
\begin{aligned}
C_1(q;A) &= \frac{i}{\sqrt{4\pi}}\langle {}^6\mathrm{Li},10|\rho_+^\dagger(q\hat{\mathbf{z}};A)|{}^6\mathrm{He},00\rangle \\
L_1(q;A) &= \frac{i}{\sqrt{4\pi}}\langle {}^6\mathrm{Li},10|\hat{\mathbf{z}}\cdot\mathbf{j}_+^\dagger(q\hat{\mathbf{z}};A)|{}^6\mathrm{He},00\rangle \\
E_1(q;A) &= -\frac{i}{\sqrt{2\pi}}\langle {}^6\mathrm{Li},10|\hat{\mathbf{z}}\cdot\mathbf{j}_+^\dagger(q\hat{\mathbf{x}};A)|{}^6\mathrm{He},00\rangle \\
M_1(q;V) &= -\frac{1}{\sqrt{2\pi}}\langle {}^6\mathrm{Li},10|\hat{\mathbf{y}}\cdot\mathbf{j}_+^\dagger(q\hat{\mathbf{x}};V)|{}^6\mathrm{He},00\rangle
\end{aligned}
$$

The multipoles in the differential rate can be written in terms of matrix elements that can be evaluated with QMC

Schiavilla and Wiringa PRC 65, 054302 (2002)

^{6}He Beta Decay Spectrum: Multipoles

$$
\begin{aligned}
C_1(q;A) &= -i\frac{qr_\pi}{3}\left(C_1^{(1)}(A) - \frac{(qr_\pi)^2}{10}C_1^{(3)}(A) + \mathcal{O}\left((qr_\pi)^4\right)\right) \\
L_1(q;A) &= -\frac{i}{3}\left(L_1^{(0)}(A) - \frac{(qr_\pi)^2}{10}L_1^{(2)}(A) + \mathcal{O}\left((qr_\pi)^4\right)\right) \\
M_1(q;V) &= -i\frac{qr_\pi}{3}\left(M_1^{(1)}(V) - \frac{(qr_\pi)^2}{10}M_1^{(3)}(V) + \mathcal{O}\left((qr_\pi)^4\right)\right) \\
E_1(q;A) &= -\frac{i}{3}\left(E_1^{(0)}(A) - \frac{(qr_\pi)^2}{10}E_1^{(2)}(A) + \mathcal{O}\left((qr_\pi)^4\right)\right)
\end{aligned}
$$

The multipoles in the differential rate can be written in terms of matrix elements that can be evaluated with QMC

Because q is limited by the reaction Q-value, it is limited to small values (<< m_π) and thus one can consider the multipoles expanded for small q

Multipoles have standard definitions in terms of Bessel functions and so can be shown to be purely even or odd in q

$$r_\pi = 1/m_{\pi^+} = 1.41382 \text{ fm}$$

$$qr_\pi \lesssim 0.03$$

GBK, Baroni, Mereghetti et al. PRC 107, 015503 (2023)

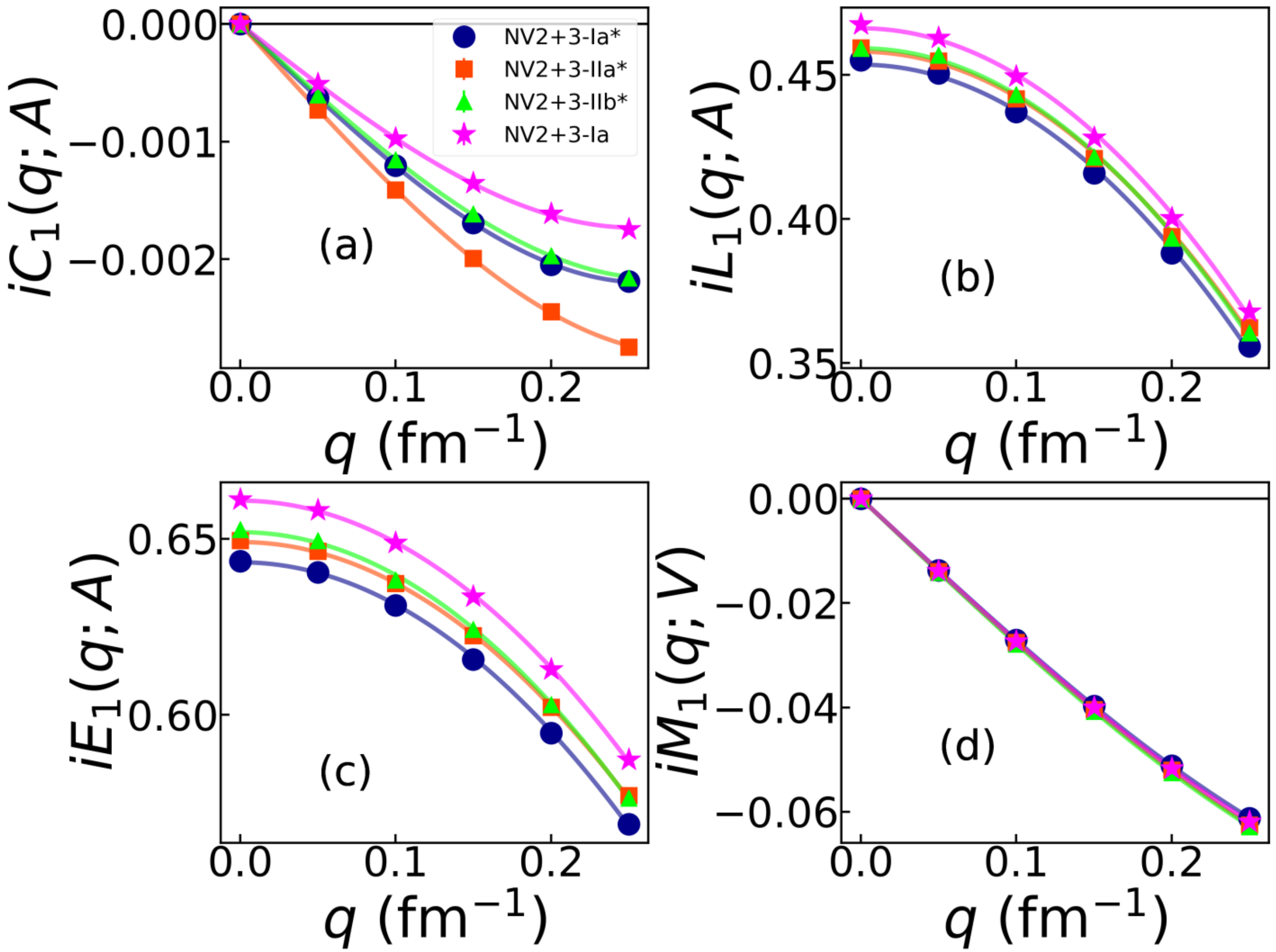
^{6}He Beta Decay Spectrum: SM Results

The strategy: Calculate necessary matrix elements for several small q values and fit the small q expansions

Dominant terms $L_1^{(0)}$ and $E_1^{(0)}$ have model dependence of ~1% to ~2%

Linear term model dependencies ~few percent

Quadratic expansion coefficients have significant model dependence, suppressed by q^2 in the differential rate



VMC multipoles

GBK, Baroni, Mereghetti et al. PRC 107, 015503 (2023)

^{6}He Beta Decay Spectrum: SM Results

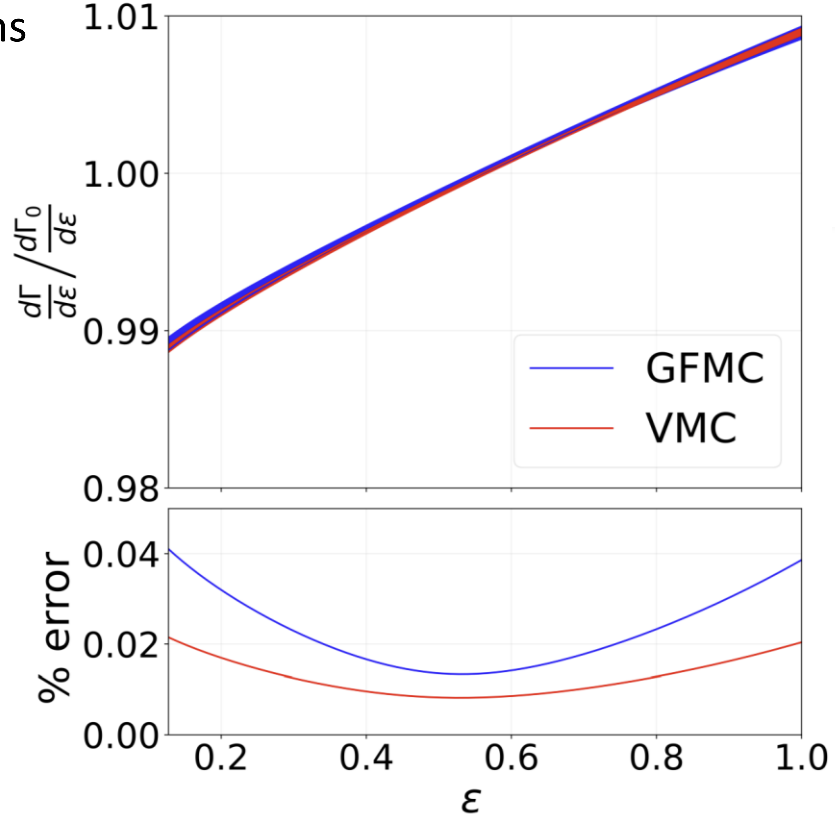
Coefficients fit to VMC and GFMC calculations can be plugged back into decay rate formula

~1% correction arising from retaining *q* dependence in the Standard Model

VMC and GFMC average rates:

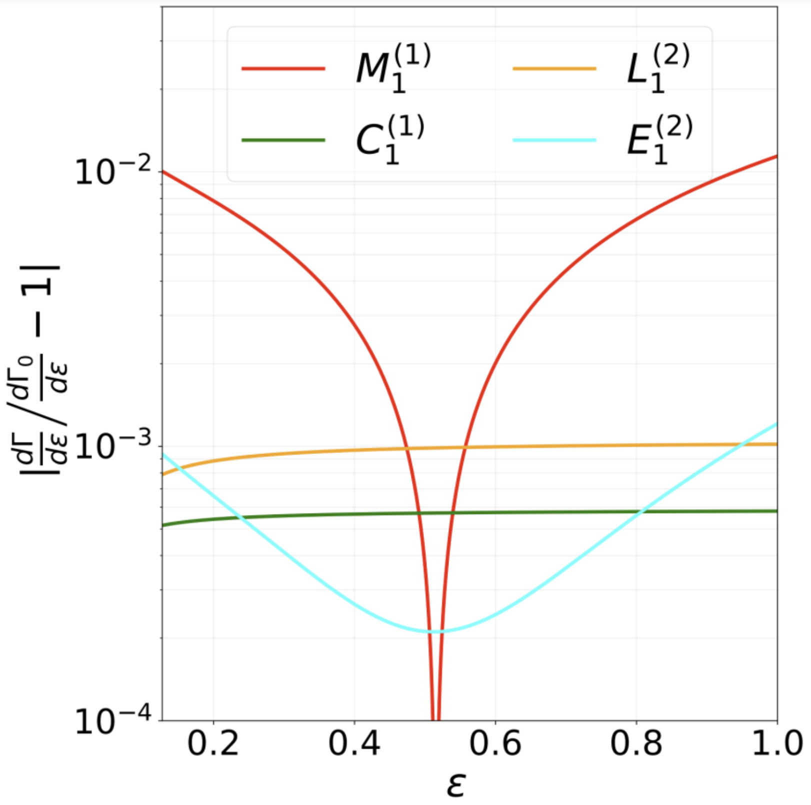
τ_{VMC} = 762 +/- 11 ms
τ_{GFMC} = 808 +/- 24 ms
$\tau_{Expt.}$ = 807.25 +/- 0.16 +/- 0.11 ms
[Kanafani et al. PRC 106, 045502 (2022)]

Cutoff, energy range, and 3N force uncertainties estimated on multipoles from the available model calculations



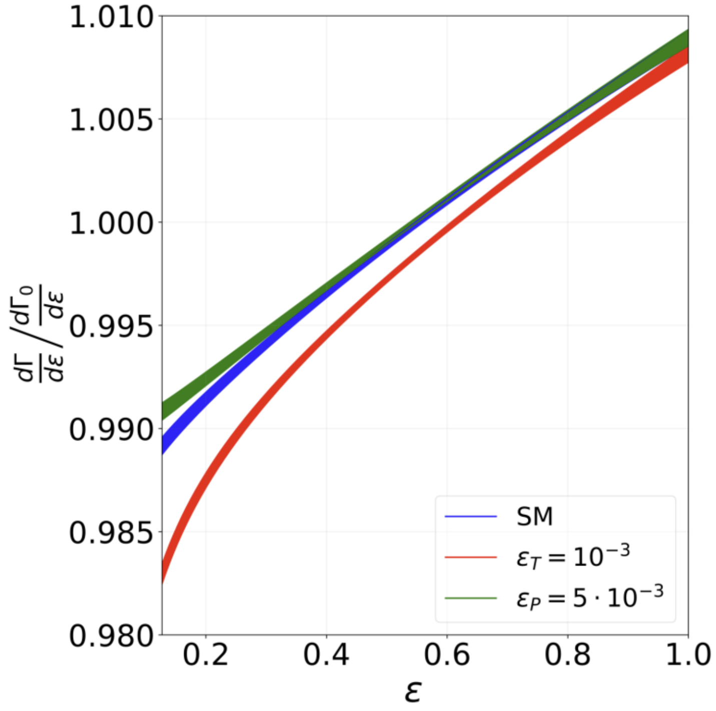
$$\varepsilon = \frac{E_e}{\omega}$$

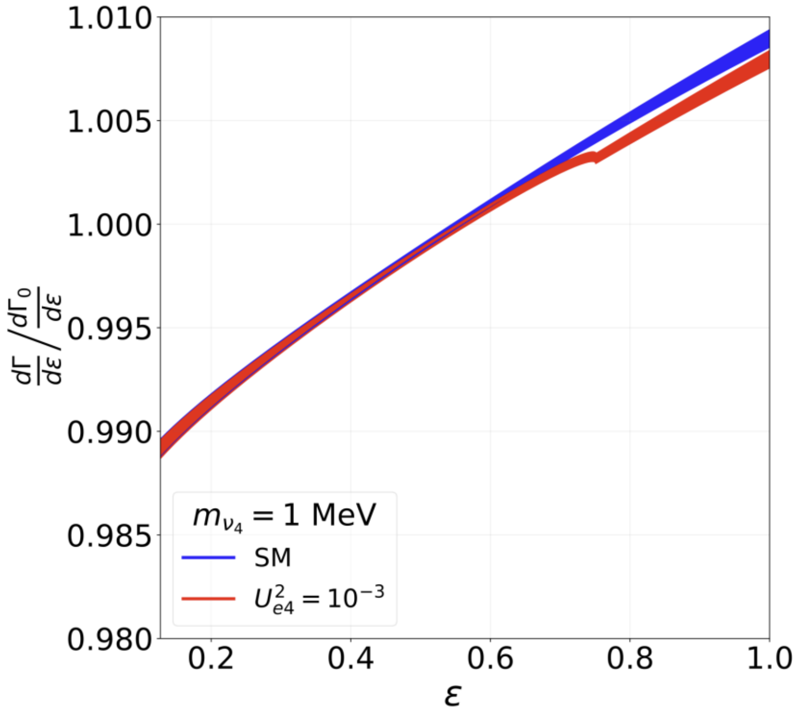
w/ radiative corrections from **Hayen**



GBK, Baroni, Mereghetti et al. PRC 107, 015503 (2023)

^{6}He Beta Decay Spectrum: BSM Connections





Investigate the signatures of BSM physics in ^{6}He β-decay spectrum

SMEFT to look at non-standard tensor and pseudoscalar currents, 1 MeV sterile neutrino **(Mereghetti+)**

Sensitive to tensor, less so to pseudoscalar, in next-gen experiments

Can put constraints on 1 MeV sterile neutrino

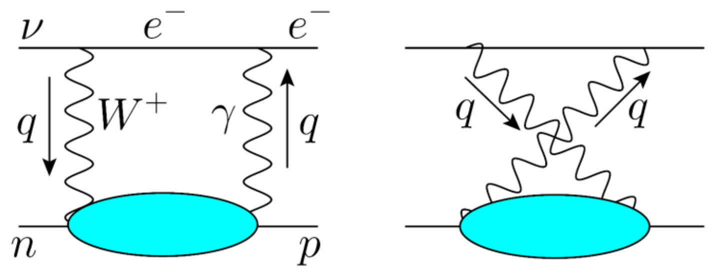
GBK, Baroni, Mereghetti et al. PRC 107, 015503 (2023)

Lepton scattering and ties to beta decay

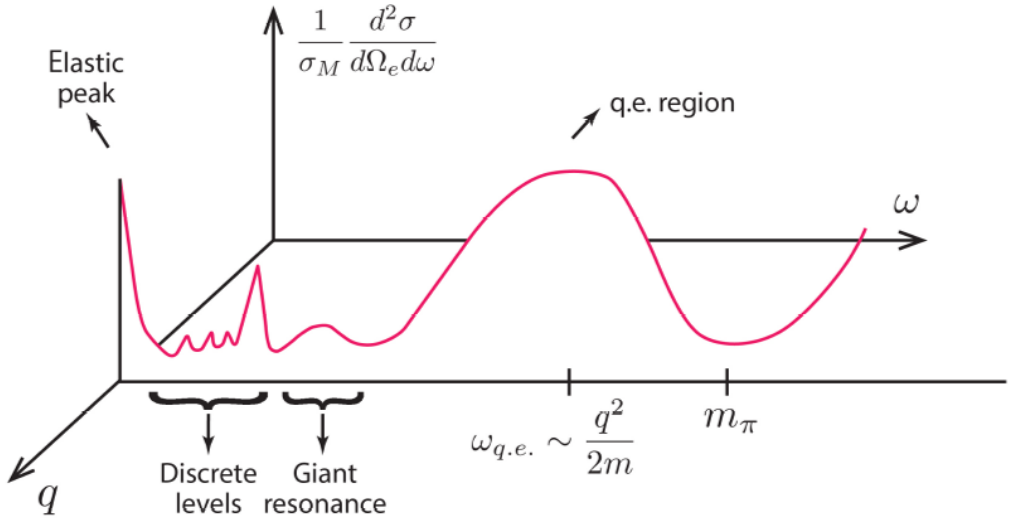
Superallowed beta decays are used to test CKM unitarity

Connection between EM and weak nuclear response functions and radiative corrections to beta decay

Radiative corrections receive contributions from the quasi-elastic regime



Seng et al. PRD 100, 013001 (2019)



The Short-time Approximation

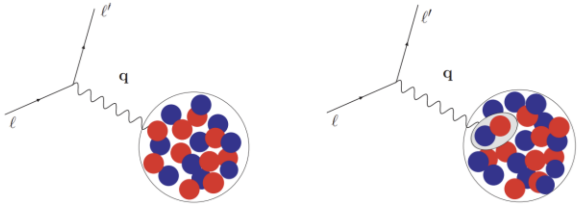
Begin from the standard response definition:

$$R(\mathbf{q},\omega) = \sum_f \langle 0|\mathcal{O}^\dagger(\mathbf{q})|f\rangle\langle f|\mathcal{O}(\mathbf{q})|0\rangle\delta(E_f - E_0 - \omega)$$

One may recast this as a real-time response:

$$R(\mathbf{q},\omega) = \int \frac{dt}{2\pi} e^{i(E_0+\omega)t}\langle 0|\mathcal{O}^\dagger(\mathbf{q})e^{-iHt}\mathcal{O}(\mathbf{q})|0\rangle$$

Short-time corresponds to at most two active particles with approximate current-current correlator **[Pastore et al. PRC 101, 044612 (2020)]:**



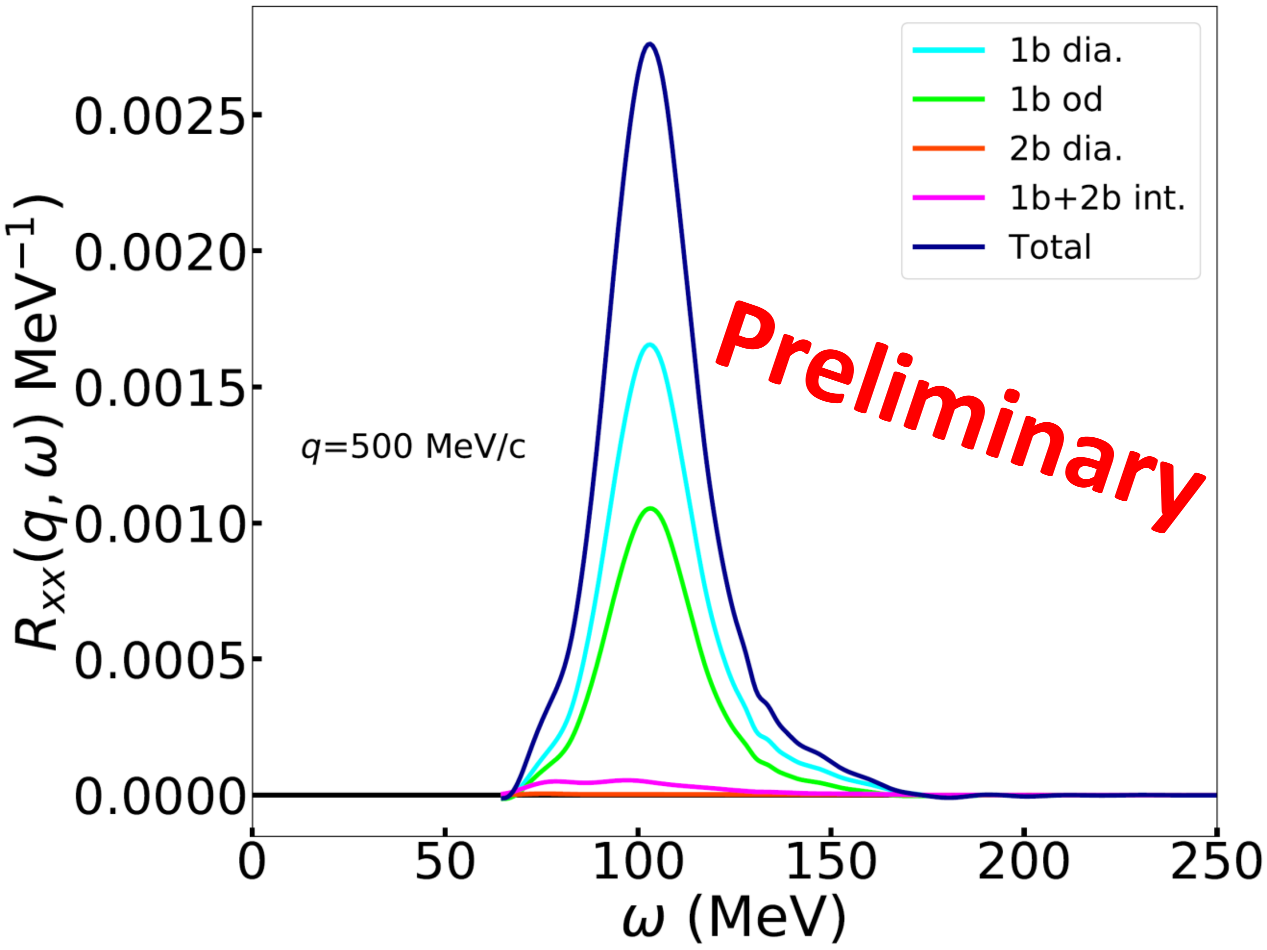
$$\mathcal{O}^\dagger(\mathbf{q})e^{-iHt}\mathcal{O}(\mathbf{q}) = \left(\sum_i \mathcal{O}_i^\dagger(\mathbf{q}) + \sum_{i<j}\mathcal{O}_{ij}^\dagger(\mathbf{q})\right) e^{-iHt}\left(\sum_{i'}\mathcal{O}_{i'}(\mathbf{q}) + \sum_{i'<j'}\mathcal{O}_{i'j'}(\mathbf{q})\right)$$

$$= \sum_i \mathcal{O}_i^\dagger(\mathbf{q})e^{-iHt}\mathcal{O}_i(\mathbf{q}) + \sum_{i\neq j}\mathcal{O}_i^\dagger(\mathbf{q})e^{-iHt}\mathcal{O}_j(\mathbf{q}) + \sum_{i\neq j}\left(\mathcal{O}_i^\dagger(\mathbf{q})e^{-iHt}\mathcal{O}_{ij}(\mathbf{q}) + \mathcal{O}_{ij}^\dagger(\mathbf{q})e^{-iHt}\mathcal{O}_i(\mathbf{q}) + \mathcal{O}_{ij}^\dagger(\mathbf{q})e^{-iHt}\mathcal{O}_{ij}(\mathbf{q})\right)$$

Retains PWIA physics, some static correlations of the ground state, and important 1b*2b interference

Does not capture physics of low-lying excitations or collective behavior

Current status of NC in STA



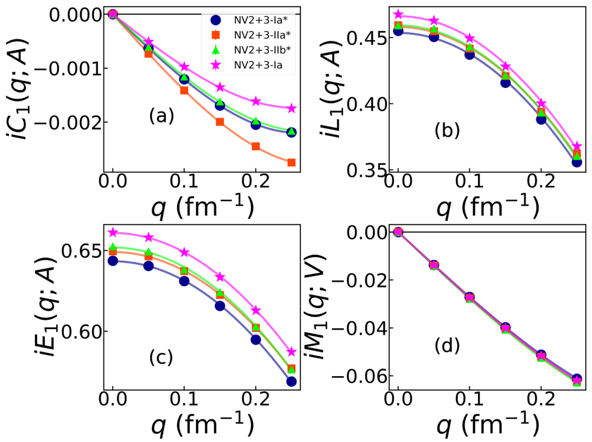
STA neutral weak current quasi-elastic responses now evaluated for ^{2}H with VMC wave functions using AV18 and consistent currents

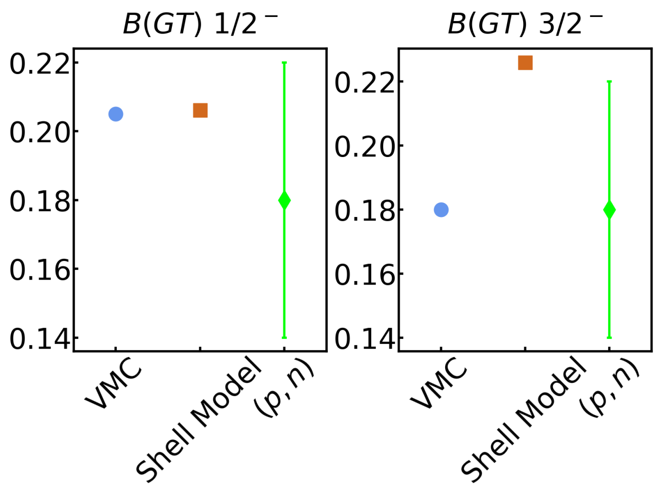
Pursuing responses in ^{4}He and ^{12}C to benchmark with GFMC

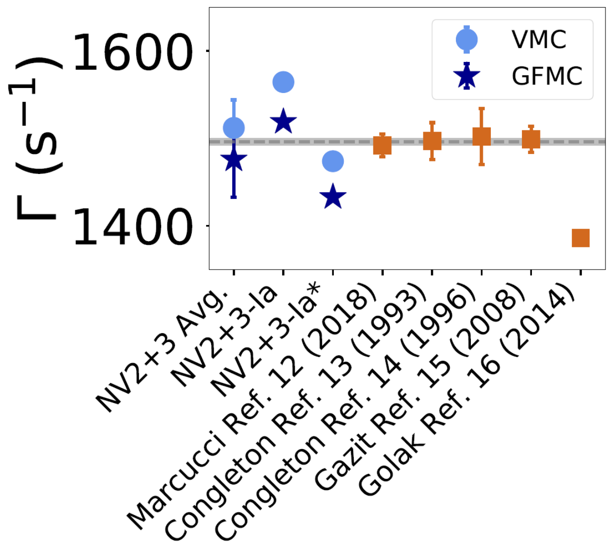
Previous work has successfully studied EM quasi-elastic response with STA **[Pastore et al. 2020, Andreoli et al. PRC 105, 014002 (2022)]**

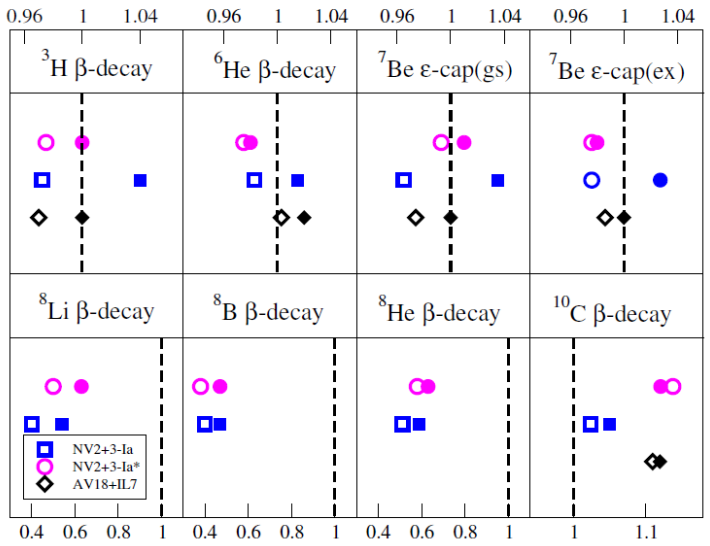
Appropriately combining weak and neutral currents in STA can provide information on radiative corrections **[Seng et al. PRD 100, 013001 (2019)]**

Conclusions and Outlook









QMC methods combined with the NV2+3 interactions provide a powerful tool to understand electroweak structure and reactions in light nuclei

Effects of several choices in chiral interactions manifest in the results seen for beta decay and muon capture

Validated model used for predictions of A=11 beta decay strengths and ^{6}He beta decay spectrum

Future work: radiative corrections to superallowed beta decay, neutrinoless double beta decay, developing robust tools for UQ in QMC calculations with NV2+3 interaction

Collaborators/Acknowledgements

Wash U QMC: Bub (GS), Chambers-Wall (GS)
Andreoli (PD), McCoy (PD), Pastore (PI), Piarulli (PI)
ANL: Wiringa
JLab+ODU: Schiavilla
LANL: Baroni, Carlson, Gandolfi, Mereghetti
MSU+FRIB: Brown, Schmitt, Zegers
NC State: Hayen
UW: Cirigliano

Funding from DOE/NNSA Stewardship Science Graduate Fellowship

STEWARDSHIP SCIENCE GRADUATE FELLOWSHIP
National Nuclear Security Administration
U.S. DEPARTMENT OF ENERGY | Office of Science
U.S. DEPARTMENT OF ENERGY INCITE LEADERSHIP COMPUTING
Argonne NATIONAL LABORATORY
Theory Alliance FACILITY FOR RARE ISOTOPE BEAMS
Washington University in St. Louis