INT workshop ``Origin of the Visible Universe: Unraveling the Proton Mass”

Seattle, June 13-17, 2022

Mass radius of the proton

Dmitri Kharzeev

Based on: DK, Phys. Rev. D104 (2021) 5, 054015 [arXiv:2102.00110]

+ ongoing work

Stony Brook University | Center for Nuclear Theory

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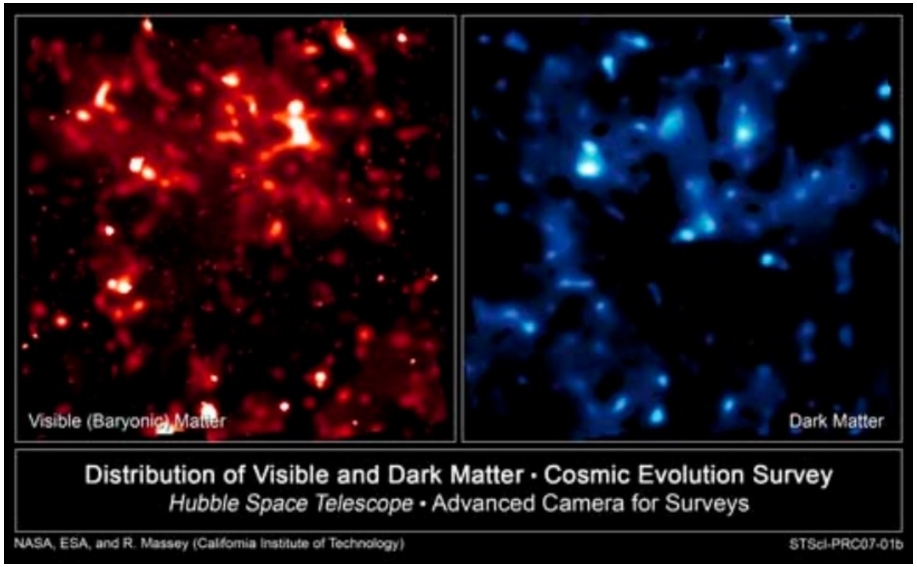
Gravitational formfactors and the mass distribution



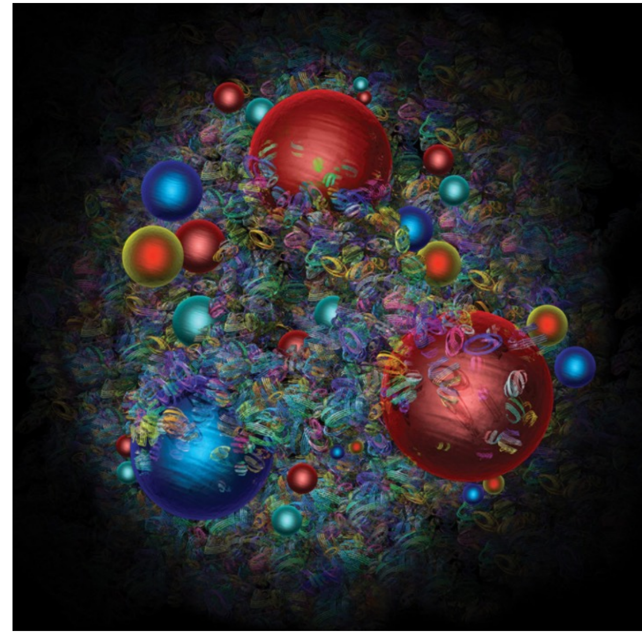


Image: CERN

What is the origin of the proton mass?

How is the mass distributed inside the proton?

Is it associated with quarks ("visible matter")
or with gluons ("dark matter")?

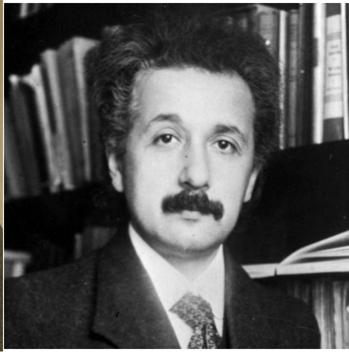
How can we measure the mass distribution?

Outline

- Gravitational formfactors and the mass distribution
- Scale invariance and scale anomaly in QCD
- Measuring the mass radius of the proton in quarkonium photoproduction near the threshold
- The mass radius puzzle?

Gravitational formfactors and the mass distribution:

scalar (Nordstrom) vs tensor (Einstein) gravity



Gunnar Nordstrom
1881-1923

Albert Einstein
1879-1955

Consider Einstein gravity:

Ricci curvature tensor $\rightarrow$
$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G\, T_{\mu\nu}$$

Take the trace with metric tensor:

$$-R = 8\pi G\, T \qquad T \equiv T^{\mu}_{\mu}$$

Cf Nordstrom 1912;
Einstein 1913; Einstein-Fokker 1914

Non-relativistic, weak gravitational field limit:

$$g_{00} = 1 + 2\varphi, \qquad T^{\nu}_{\mu} = \mu\, u_\mu u^\nu, \qquad u_0 = u^0 = 1, \quad u_i = 0.$$

Therefore, in this limit, the distributions of mass and of T coincide:

$$T^0_0 = \mu; \qquad T \equiv T^{\mu}_{\mu} = T^0_0 = \mu$$

[45] Einstein, Albert and Fokker, Adriann, D., "Die Nordströmsche Gravitationstheorie vom Standpunkt des absoluten Differentialkalküls", *Annelen der Physik* 44, 1914, pp. 321-328; p. 321.

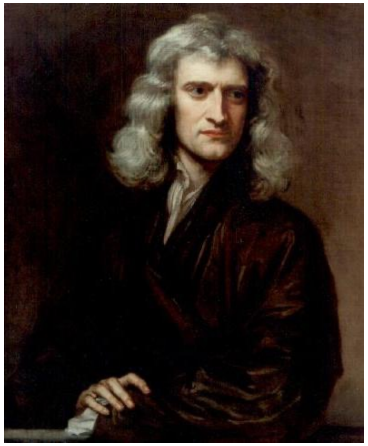
Gravitational formfactors and the mass distribution

Newtonian limit:

$$R_0^0 = \frac{\partial^2 \varphi}{\partial x^{\mu 2}} \equiv \Delta\varphi,$$

Einstein equation: $$R_{\mu\nu} = 8\pi G(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T);$$

The only non-vanishing component: $$R_0^0 = 4\pi G\mu,$$



Isaac Newton
1643-1727

Therefore, the distribution of mass determines the gravitational potential:

$$\Delta\varphi = 4\pi G\mu. \quad \Rightarrow \quad \varphi = -G \int \frac{\mu\, dV}{R} \quad \Rightarrow \quad F_g = -m\, \partial\varphi/\partial R$$

$$M = \int \mu dV$$

$$\boxed{F_g = -G\, \frac{mM}{R^2}.}$$

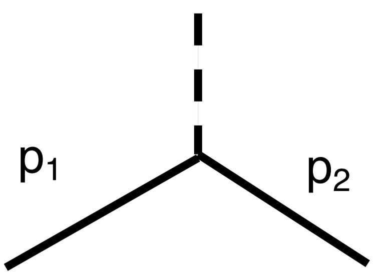
Gravitational formfactors and the mass distribution

The mass distribution is encoded in the gravitational formfactors.

For the spin ½ nucleon, 3 formfactors appear:

H. Pagels '66,
A. Pais, S. Epstein '49

$$\langle \mathbf{p}_1 | T_{\mu\nu} | \mathbf{p}_2 \rangle = \left(\frac{M^2}{p_{01}\ p_{02}} \right)^{1/2} \frac{1}{4M}\ \bar{u}(p_1, s_1) \Big[G_1(q^2)(p_\mu \gamma_\nu + p_\nu \gamma_\mu) + G_2(q^2) \frac{p_\mu p_\nu}{M} + $$
$$+ G_3(q^2) \frac{(q^2 g_{\mu\nu} - q_\mu q_\nu)}{M} \Big] u(p_2, s_2),$$



Energy-momentum conservation:

$$\partial^\mu T_{\mu\nu} = 0$$

$$q^\mu \langle \mathbf{p}_1 | T_{\mu\nu} | \mathbf{p}_2 \rangle = 0;$$

Satisfied for on-shell nucleons (use Dirac equation)

$$\textstyle\sum_s \bar{u}(p, s) u(p, s) = (\hat{p} + M)/2M$$

$$p_1^2 = p_2^2 = M^2$$

Gravitational formfactors and the mass distribution

For the spin ½ nucleon, 3 formfactors appear: (no G_1 for spin 0)

$$\langle \mathbf{p}_1 | T_{\mu\nu} | \mathbf{p}_2 \rangle = \left(\frac{M^2}{p_{01}\ p_{02}} \right)^{1/2} \frac{1}{4M} \bar{u}(p_1, s_1) \Big[G_1(q^2)(p_\mu \gamma_\nu + p_\nu \gamma_\mu) + G_2(q^2) \frac{p_\mu p_\nu}{M} + + G_3(q^2) \frac{(q^2 g_{\mu\nu} - q_\mu q_\nu)}{M} \Big] u(p_2, s_2),$$

Compare to
the macroscopic energy-momentum
tensor in relativistic hydrodynamics:

The Thermodynamics of Irreversible Processes

III. Relativistic Theory of the Simple Fluid

Carl Eckart
Ryerson Physical Laboratory, University of Chicago, Chicago, Illinois

(Received September 26, 1940)

C. Eckart, 1940

u_ν - matter velocity

$$\theta_{\mu\nu} = \underbrace{(w_\mu u_\nu + w_\nu u_\mu)}_{\text{Heat flow}} + \underbrace{w\ u_\mu u_\nu}_{\text{Energy density}} + \underbrace{w_{\mu\nu}}_{\text{Stress tensor}}$$

$$u_\mu\ w^\mu = 0 \qquad w \sim \epsilon$$

Gravitational formfactors and the mass distribution

For the spin ½ nucleon, 3 formfactors appear: (no G_1 for spin 0)

$$\langle \mathbf{p}_1 | T_{\mu\nu} | \mathbf{p}_2 \rangle = \left(\frac{M^2}{p_{01}\ p_{02}} \right)^{1/2} \frac{1}{4M}\, \bar{u}(p_1, s_1) \Big[G_1(q^2)(p_\mu \gamma_\nu + p_\nu \gamma_\mu) + G_2(q^2) \frac{p_\mu p_\nu}{M} + $$
$$+ G_3(q^2) \frac{(q^2 g_{\mu\nu} - q_\mu q_\nu)}{M} \Big] u(p_2, s_2),$$

Zero momentum transfer $q \to 0$:

$$\langle \mathbf{p} | T_{\mu\nu} | \mathbf{p} \rangle = \left(\frac{M^2}{p_0^2} \right)^{1/2} \bar{u}(p, s) u(p, s)\ \frac{p_\mu p_\nu}{M^2} \left[G_1(0) + G_2(0) \right]$$

(no “stress” G_3)

the Hamiltonian

$$H = \int d^3x\ T_{00}(x)$$

In the rest frame of the nucleon:

$$\langle \mathbf{p} = 0 | T_{00} | \mathbf{p} = 0 \rangle = M$$

$$\Downarrow$$

$$G_1(0) + G_2(0) = M.$$

Formfactor of the trace of the energy-momentum tensor

Let us call it "scalar gravitational formfactor", as it would be a gravitational formfactor in a scalar model of gravity:

Nordstrom 1912
Einstein 1913

$$T \equiv T^{\mu}_{\mu}$$

$$\langle \mathbf{p}_1 | T | \mathbf{p}_2 \rangle = \left(\frac{M^2}{p_{01}\ p_{02}} \right)^{1/2} \bar{u}(p_1, s_1) u(p_2, s_2)\ G(q^2),$$

Scalar gravitational formfactor:

$$G(q^2) = G_1(q^2) + G_2(q^2) \left(1 - \frac{q^2}{4M^2} \right) + G_3(q^2) \frac{3q^2}{4M^2}$$

In the rest frame of the nucleon:

$$\langle \mathbf{p} = 0 | T | \mathbf{p} = 0 \rangle = \langle \mathbf{p} = 0 | T_{00} | \mathbf{p} = 0 \rangle = M,$$

$$\Downarrow$$

$$G(0) = M$$

How to define the mass distribution in the nucleon?

At small momentum transfer $|q^2| \ll M^2$,

the formfactor of θ_{00} and the scalar gravitational formfactor coincide if

$$\frac{G_i(0)}{4M} \ll \frac{dG_i}{dt}\Big|_{t=0} \equiv G_i(0)/m_i^2$$

The origin of the difference is frame dependence of θ_{00} :

In Breit frame, $\mathbf{p}_2 = \frac{1}{2}\mathbf{q},\ \mathbf{p}_1 = -\frac{1}{2}\mathbf{q}$ the proton is moving with

$$\gamma = E/M = \sqrt{M^2 + (q^2/4)}/M = \sqrt{1 + q^2/(4M^2)},$$

so for $q \equiv |\mathbf{q}| \simeq m_i$ it is Lorentz-contracted with

$$1/\gamma \simeq (1 + m_i^2/(4M^2))^{-1/2}$$

For massive bodies, $m_i << 2M$ – size much larger than the Compton wavelength! In this limit, the formfactors of T_{00} and T coincide.

[the proton: $8M^2_p >> M^2_s$] See R.L. Jaffe, PRD103(2021) for related discussion

How to define the mass distribution in the nucleon?

At small momentum transfer $|q^2| \ll M^2$, $\frac{G_i(0)}{4M} \ll \frac{dG_i}{dt}\Big|_{t=0}$

the formfactor of θ_{00} and the scalar gravitational formfactor are close,

thus the scalar gravitational formfactor can be used to define

the **mass radius of the proton**:

$$\langle R_{\rm C}^2 \rangle = 6\, \frac{dG_{\rm EM}}{dt}\Big|_{t=0}. \quad \Rightarrow \quad \boxed{\langle R_{\rm M}^2 \rangle = \frac{6}{M}\, \frac{dG}{dt}\Big|_{t=0},}$$

In the relativistic region (mass -> energy), it is natural to consider the scalar gravitational formfactor, as T is the Lorentz scalar

How close is the scalar radius to the mass radius?

As argued above, the difference between the scalar radius $<R_S^2>$ and a ``true'' mass radius $<R_M^2>$ should be suppressed by $1/M^2$ (M is the nucleon mass).

But how small is really this difference?

$G_3(0)$

Talk by X. Ji

$$\langle R_S^2 \rangle - \langle R_M^2 \rangle = -12\frac{C(0)}{M^2}$$

$$\langle P' |T^{\mu\nu}| P \rangle = \bar{u}(P'S') \Big[A(q^2)\gamma^{(\mu}\bar{P}^{\nu)} + B(q^2)\frac{\bar{P}^{(\mu}i\sigma^{\nu)\alpha}q_\alpha}{2M} + C(q^2)\frac{q^\mu q^\nu - g^{\mu\nu}q^2}{M}\Big] u(PS)$$

Y.Guo, X.Ji, Y.Liu,
arXiv:2103.11506 [PRD]

$C(0) = -0.84 \pm 0.82$

Extracted value is consistent with zero;
But if C(0)=-1, the difference is big: $\langle R_S^2 \rangle - \langle R_M^2 \rangle \simeq 0.47\ \mathrm{fm}^2$

Need a reliable value of C(0) from lattice!

Trace of $T^{\mu\nu}$ plays a fundamental role: link to scale invariance

Scale transformations (dilatations) are defined by

$$x \to e^{\lambda}\ x$$

the corresponding dilatational current is

$$s^{\mu} = x_{\nu} T^{\mu\nu}$$

Hermann Weyl
(1885-1955)

It is conserved (a theory is scale-invariant) if the energy-momentum is traceless:

$$\partial_{\mu} s^{\mu} = T^{\mu}_{\mu} \equiv T$$

Scale invariance

A scale-invariant theory cannot contain massive particles, all particles must be massless

For example, in Maxwell electrodynamics with action

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

the energy-momentum is traceless: (massless photons) $\quad T^{\mu}_{\mu} = 0$

Note: because of this, in scalar gravity (Nordstrom, 1912; Einstein, 1913) there would be no light bending by massive bodies!

Scale invariance in QCD

The trace of the energy-momentum tensor in QCD (computed in classical field theory) is

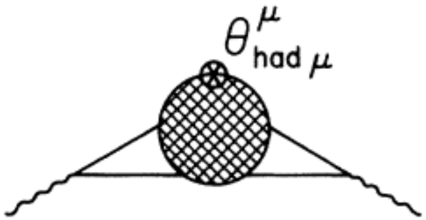
$$T^{\mu}_{\mu} = \sum_{l=u,d,s} m_l \ \bar{q}_l q_l + \sum_{h=c,b,t} m_h \ \bar{Q}_h Q_h$$

Two problems:

1. Potentially large contribution from heavy quarks to the masses of light hadrons

2. If we forget about heavy quarks, all hadron masses must be equal to zero in the chiral limit

Scale anomaly in QCD



The quantum effects (loop diagrams) modify the expression for the trace of the energy-momentum tensor:

$$T^{\mu}_{\mu} = \frac{\beta(g)}{2g}\, G^{\mu\nu a} G^{a}_{\mu\nu} + \sum_{l=u,d,s} m_l (1+\gamma_{m_l}) \bar{q}_l q_l + \sum_{h=c,b,t} m_h (1+\gamma_{m_h}) \bar{Q}_h Q_h$$

Running coupling -> dimensional transmutation -> mass scale

Gross, Wilczek; Politzer

$$\beta(g) = -b\frac{g^3}{16\pi^2} + \ldots,\ \ b = 9 - \frac{2}{3} n_h,$$

Ellis, Chanowitz; Crewther; Collins, Duncan, Joglecar; …

At small momentum transfer, heavy quarks decouple:

$$\sum_h m_h \bar{Q}_h Q_h \to -\frac{2}{3}\, n_h\, \frac{g^2}{32\pi^2} G^{\alpha\beta a} G^{a}_{\alpha\beta} + \ldots$$

so only light quarks enter the final expression

Shifman, Vainshtein Zakharov '78

$$T^{\mu}_{\mu} = \frac{\tilde{\beta}(g)}{2g}\, G^{\mu\nu a} G^{a}_{\mu\nu} + \sum_{l=u,d,s} m_l (1+\gamma_{m_l}) \bar{q}_l q_l,$$

The proton mass

At zero momentum transfer, the matrix element of the trace of the energy-momentum tensor defines the mass of the proton:

$$\langle \mathbf{p} = 0 | T | \mathbf{p} = 0 \rangle = \langle \mathbf{p} = 0 | T_{00} | \mathbf{p} = 0 \rangle = M,$$

$$T^{\mu}_{\mu} = \frac{\tilde{\beta}(g)}{2g} G^{\mu\nu a} G^{a}_{\mu\nu} + \sum_{l=u,d,s} m_l (1 + \gamma_{m_l}) \bar{q}_l q_l,$$

In the chiral limit, the only contribution is from gluons!

PHYSICAL REVIEW D **104**, 074507 (2021)

Demonstration of the hadron mass origin from the QCD trace anomaly

Fangcheng He,[1,*] Peng Sun,[2,†] and Yi-Bo Yang[1,3,4,5,‡]

(χQCD Collaboration)

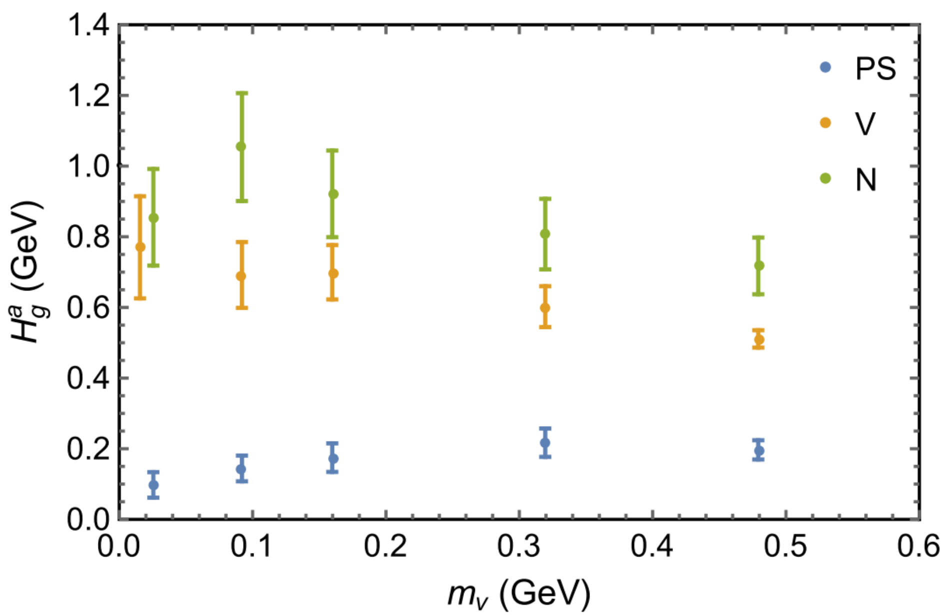


FIG. 3. The gluon trace anomaly contribution to the hadron mass. For five different quark masses, the corresponding pion masses are 0.340, 0.647, 0.864, 1.277, and 1.640 GeV. We can see that it is always small for the PS meson, while it approaches ~800 MeV for the nucleon and vector mesons in the chiral limit $m_v \to 0$.

Confinement due to scale anomaly?

$$T^{\mu}_{\mu} = \frac{\tilde{\beta}(g)}{2g}\, G^{\mu\nu a} G^{a}_{\mu\nu} + \sum_{l=u,d,s} m_l (1+\gamma_{m_l}) \bar{q}_l q_l,$$

In quantum theory, gluons gravitate; scale anomaly induces conformally flat deformation of space-time. Can this be used to describe confinement?

QCD in curved space-time: A conformal bag model

Also: JHEP06(2009)055

Dmitri Kharzeev, Eugene Levin, and Kirill Tuchin
Phys. Rev. D **70**, 054005 – Published 3 September 2004

$$\mathrm{g}_{\mu\nu}(x) \;=\; e^{h(x)}\, \delta_{\mu\nu}$$

$$S \;=\; \int d^4x \left(\frac{4\,|\epsilon_{\mathrm{v}}|}{m^2}\, e^h\, (\partial_\mu h)^2 \;-\; \frac{1}{4}\, (F^a_{\mu\nu})^2 \;+\; |\epsilon_{\mathrm{v}}|\, e^{2h} \;-\; \frac{1}{4}\, e^{2h} \left[-\frac{b\, g^2}{32\, \pi^2}\, (F^a_{\mu\nu})^2 \right] \right)$$

This model belongs to the class of confining models proposed in
'‘t Hooft hep-th/0207179:

It describes gluons in the dilaton background:

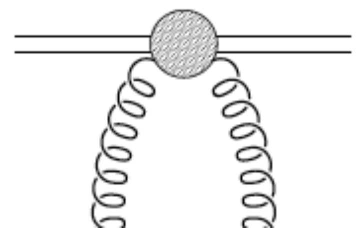
$$\mathcal{L} = -V(\chi) - Z(\chi)\frac{1}{4}\,(F^a_{\mu\nu})^2 \qquad Z(\chi) = -e^{\chi}\,(1-\chi)\,c + 1\,, \quad V(\chi) = -|\epsilon_{\mathrm{v}}|\, e^{\chi}\,(1-\chi)$$

How to measure the mass distribution inside the proton?

No dilatons available…
next best thing: a heavy quarkonium

M.B. Voloshin
1953-2020

QCD multipole expansion:



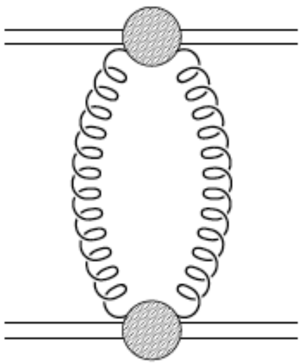
Voloshin ‘78; Appelquist, Fischler ‘78; Gottfried ‘78; Peskin ‘79; Novikov, Shifman ‘81; Leutwyler ‘81, Luke, Manohar, Savage ‘92, …

$$g^2\mathbf{E}^{a2} = \frac{g^2}{2}(\mathbf{E}^{a2} - \mathbf{B}^{a2}) + \frac{g^2}{2}(\mathbf{E}^{a2} + \mathbf{B}^{a2})$$

$$= -\frac{1}{4}g^2 G^a_{\alpha\beta}G^{a\alpha\beta} + g^2(-G^a_{0\alpha}G^{a\alpha}_0 + \frac{1}{4}g_{00}G^a_{\alpha\beta}G^{a\alpha\beta}) = \frac{8\pi^2}{b}\theta^\mu_\mu + g^2\theta^{(G)}_{00}$$

$$\theta^\mu_\mu \equiv \frac{\beta(g)}{2g}G^{a\alpha\beta}G^a_{\alpha\beta} = -\frac{bg^2}{32\pi^2}G^{a\alpha\beta}G^a_{\alpha\beta}\ , \quad \theta^{(G)}_{\mu\nu} \equiv -G^a_{\mu\alpha}G^{a\alpha}_\nu + \frac{1}{4}g_{\mu\nu}G^a_{\alpha\beta}G^{a\alpha\beta}$$

Quarkonium interactions at low energy



Perturbation theory:

at large distances,
the Casimir-Polder
interaction (retardation)

Bhanot, Peskin '78

$$V^{\rm pt}(R) = -g^4 \left(\bar{d}_2 \frac{a_0^2}{\epsilon_0}\right)^2 \frac{23}{8\pi^3} \frac{1}{R^7}$$

Fujii, DK '99

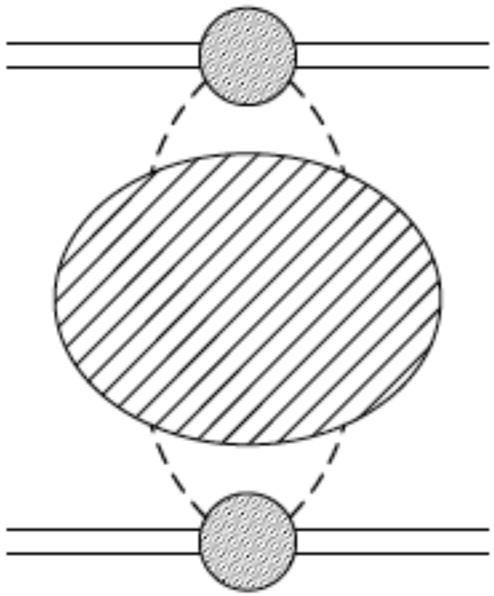
23 = 15 + 8

scalar 0^{++} tensor 2^{++}

Beyond perturbation theory, scalar is strongly enhanced due to scale anomaly

Quarkonium interactions at low energy and the scale anomaly

But, at very large distances, the interaction must be dominated by the lightest physical states - pions



conversion of gluons to pions is a (hopeless?) non-perturbative problem

…but, can use scale anomaly matching!

Voloshin, Zakharov ’80
Novikov, Shifman ‘82

Quarkonium interactions at low energy and the scale anomaly

Use RG invariance to match the EMT computed in QCD and in the chiral theory:

$$\theta^\mu_\mu = -2\,\frac{f_\pi^2}{4}\,\mathrm{tr}\,\partial_\mu U \partial^\mu U^\dagger \;-\; m_\pi^2 f_\pi^2\,\mathrm{tr}\left(U + U^\dagger\right)$$

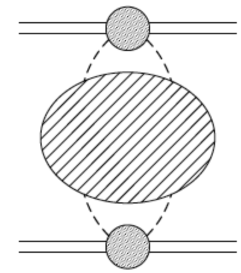
to lowest order in the pion field

$$\theta^\mu_\mu = -\partial_\mu \pi^a \partial^\mu \pi^a + 2m_\pi^2 \pi^a \pi^a + \cdots$$

In the chiral limit scale anomaly yields:

$$\langle \pi^+ \pi^- | \theta^\mu_\mu | 0 \rangle = q^2$$

Quarkonium interactions at low energy and the scale anomaly

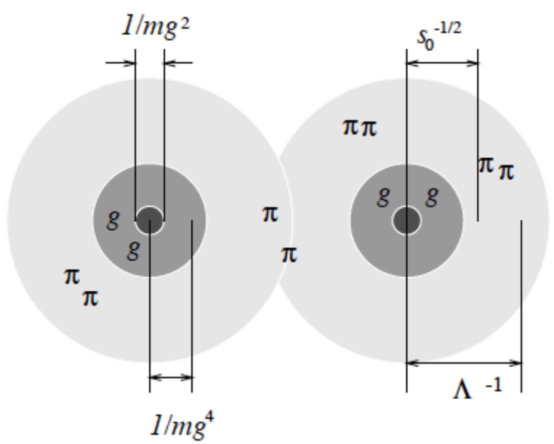


The result (long distances):

$$V^{\pi\pi}(R) \to -\left(\bar{d}_2 \frac{a_0^2}{\epsilon_0}\right)^2 \left(\frac{4\pi^2}{b}\right)^2 \frac{3}{2}(2m_\pi)^4 \, \frac{m_\pi^{1/2}}{(4\pi R)^{5/2}} \, e^{-2m_\pi R}.$$

Fujii, DK, PRD (1999)

See also A.Belitsky and X.Ji, PLB (2002)



1. Not a Yukawa potential (retardation)
2. The QCD coupling has disappeared at large distance (but not b from the beta-function)
3. Entirely due to scalar 0^{++} exchange

This two-pion tail in quarkonium interactions has just been clearly observed on the lattice:

arXiv:2205.10544

Attractive N-ϕ Interaction and Two-Pion Tail from Lattice QCD near Physical Point

Yan Lyu,[1,2,*] Takumi Doi,[2,†] Tetsuo Hatsuda,[2,‡] Yoichi Ikeda,[3,§]
Jie Meng,[1,4,¶] Kenji Sasaki,[3,**] and Takuya Sugiura[2,††]

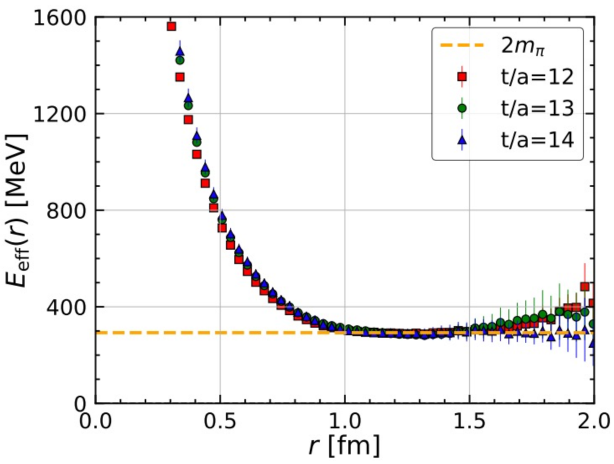


FIG. 2. (Color online). The spatial effective energy $E_{\rm eff}(r)$ as a function of separation r at Euclidean time $t/a = 12$ (red squares), 13 (green circles) and 14 (blue triangles). The orange dashed line corresponds to $2m_\pi$ with lattice pion mass $m_\pi = 146.4$ MeV.

This is a consequence of **non-perturbative** mixing between the scalar gluon and quark operators induced by spontaneous breaking of chiral symmetry. It is controlled by scale anomaly:

CERN-TH/99-278
RIKEN-BNL preprint
UT-Komaba preprint

Scalar Glueball–Quarkonium Mixing and the Structure of the QCD Vacuum

John Ellis[a], **Hirotsugu Fujii**[b] and **Dmitri Kharzeev**[c]

$$\lim_{q\to 0} i \int dx\ e^{iqx} \langle 0|T\left\{\frac{\beta(\alpha_s)}{4\alpha_s} G^2(x),\ \mathcal{O}(0)\right\}|0\rangle = (-d)\langle\mathcal{O}\rangle + O(m_q)$$

General LET:

Novikov, Shifman, Vainshtein, Zakharov '81

For a scalar quark operator and a single resonance:

$$\frac{1}{m_\sigma^2}\langle 0|\frac{\beta(\alpha_s)}{4\alpha_s}G^2|k\rangle\langle k|\sum_i m_i\bar{q}_i q_i|0\rangle = -3\langle\sum_i m_i\bar{q}_i q_i\rangle.$$

Probing the proton mass

The quarkonium-proton scattering amplitude

$$F_{\Phi h} = r_0^3 \epsilon_0^2 \sum_{n=2}^{\infty} d_n \langle h | \frac{1}{2} G^a_{0i} (D^0)^{n-2} G^a_{0i} | h \rangle$$

Wilson coefficients

$$d_n^{(1S)} = \left(\frac{32}{N}\right)^2 \sqrt{\pi}\, \frac{\Gamma(n+\frac{5}{2})}{\Gamma(n+5)}$$

M.Peskin '78

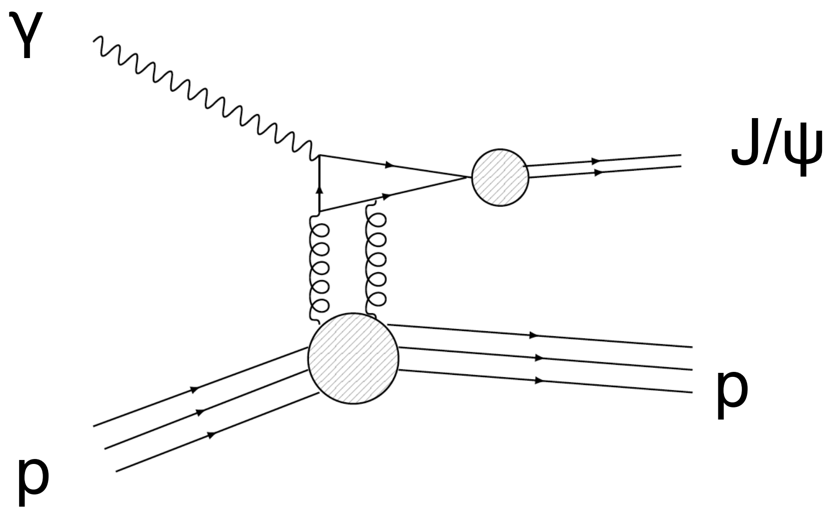
$$d_n^{(2S)} = \left(\frac{32}{N}\right)^2 4^n \sqrt{\pi}\, \frac{\Gamma(n+\frac{5}{2})}{\Gamma(n+7)} (16n^2 + 56n + 75)$$

$$d_n^{(2P)} = \left(\frac{15}{N}\right)^2 4^n 2\sqrt{\pi}\, \frac{\Gamma(n+\frac{7}{2})}{\Gamma(n+6)}$$

DK, '96

nucl-th/9601029

Threshold photoproduction of quarkonium as a probe of mass distribution inside the proton



Near threshold, dominance of

$$g^2 \mathbf{E}^{a2} = \frac{8\pi^2}{b}\theta^\mu_\mu + g^2\theta^{(G)}_{00}$$

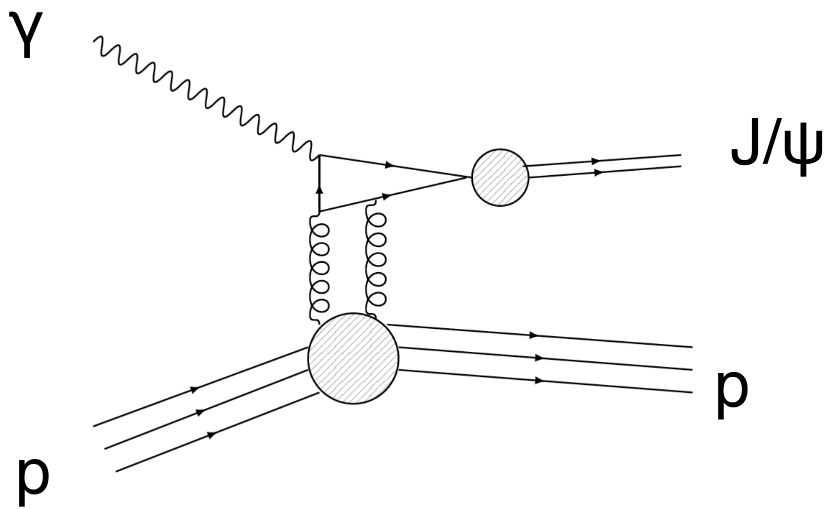
Assuming the validity of vector meson dominance, can relate photoproduction to quarkonium scattering amplitude and probe the mass of the proton

DK ‘96; DK, Satz, Syamtomov, Zinovjev ‘99

Other approaches to threshold photoproduction:

Hatta, Yang ‘18; Hatta, Rajan, Yang ‘19; Mamo, Zahed ’19-’22;
Ji, 2102.07830; Gao, Ji, Liu, 2103.11506; Sun, Tong, Yuan, 2103.12047…

Threshold photoproduction of quarkonium as a probe of mass distribution inside the proton

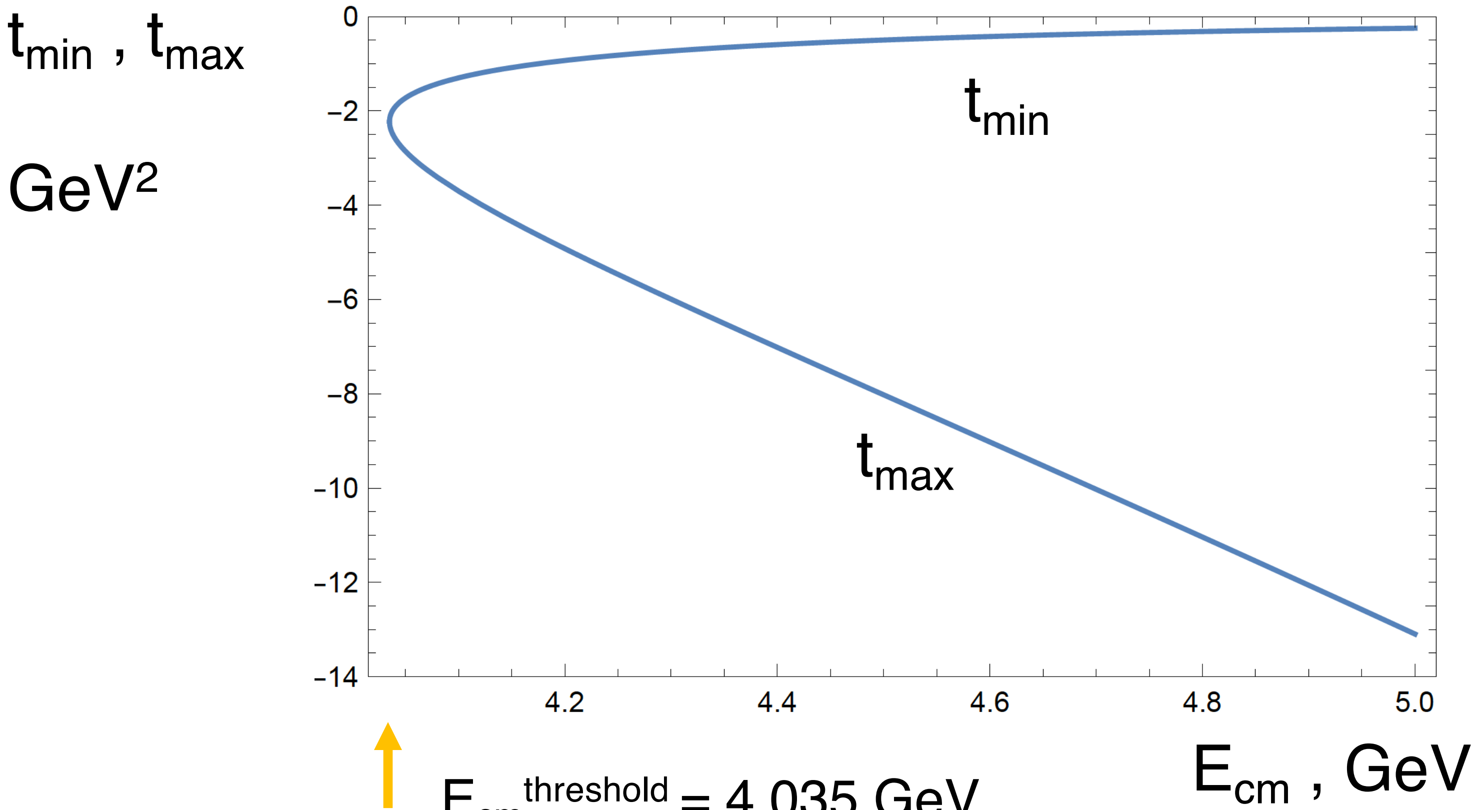


Large minimum momentum transfer at threshold:

$$t_{min} = -\frac{M_\psi^2 M}{M_\psi + M} \simeq -2.23 \text{ GeV}^2 \simeq -(1.5 \text{ GeV})^2$$

–> VDM questionable.
but, scanning the energy range near the threshold, we measure the scalar gravitational formfactor – can extract the proton mass distribution!

Threshold photoproduction of quarkonium as a probe of mass distribution inside the proton



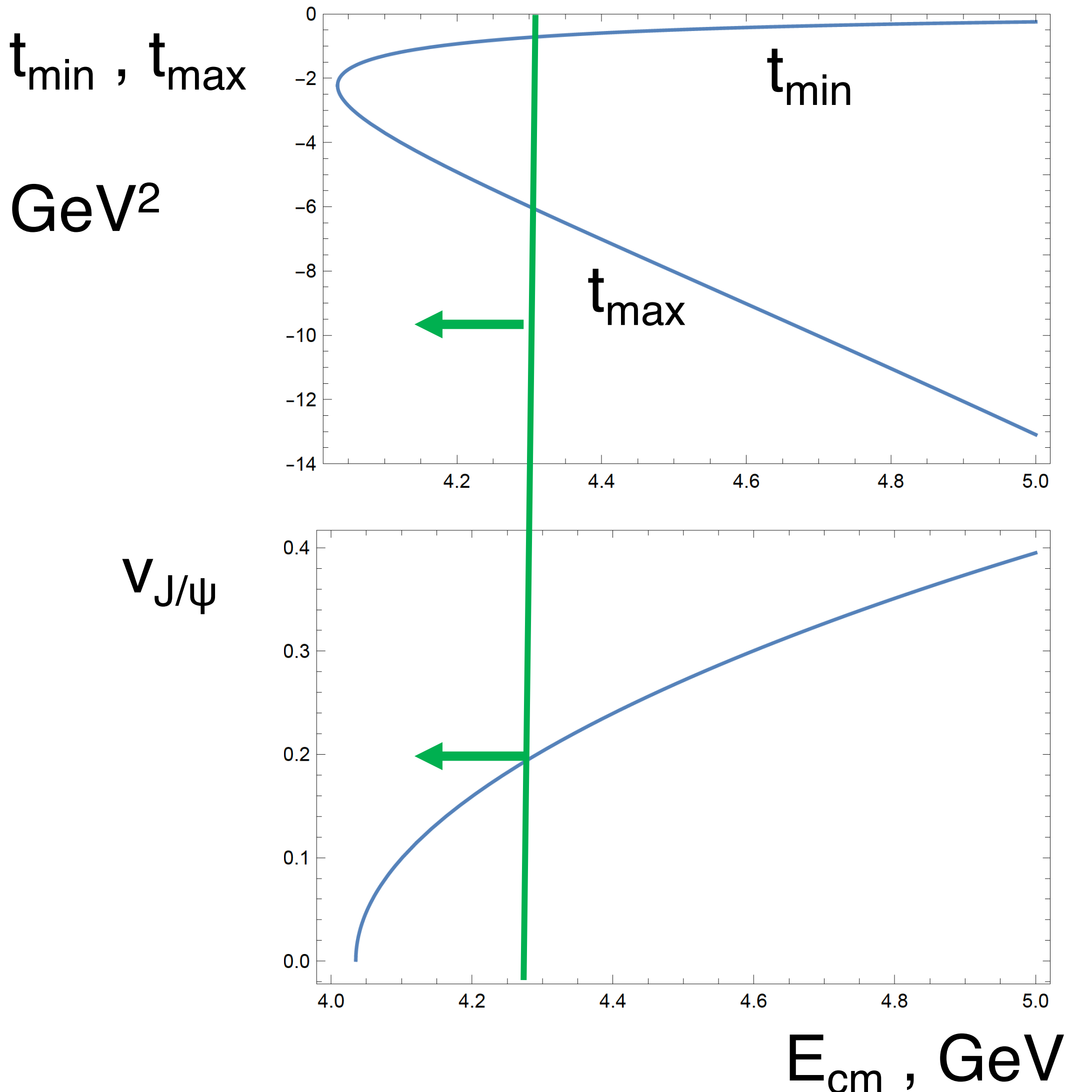
$E_{cm}^{threshold} = 4.035$ GeV

Can measure scalar gravitational formfactor in a broad kinematical domain!

At threshold:

$$t_{min} = -\frac{M_\psi^2 M}{M_\psi + M} \simeq -2.23\ \mathrm{GeV}^2 \simeq -(1.5\ \mathrm{GeV})^2$$

Threshold photoproduction of quarkonium as a probe of mass distribution inside the proton



The scalar operator dominates for small velocity of heavy quarkonium;

Limiting $v_{J/\psi} < 0.2$, (corrections $\sim v_{J/\psi}^2$) the optimal kinematical region is:

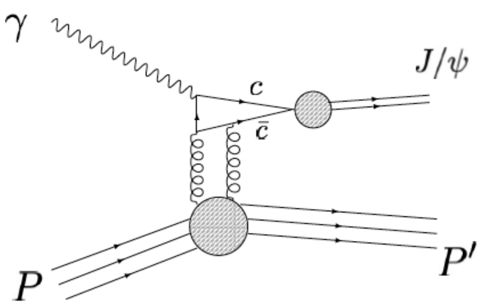
$E_{cm} < 4.25$ GeV
$E_\gamma < 9.2$ GeV

$-t < 6$ GeV^2

Threshold photoproduction of quarkonium as a probe of mass distribution inside the proton

The amplitude:

$$\mathcal{M}_{\gamma P\to\psi P}(t) = -Qe\ c_2\ 2M\ \langle P'|g^2\mathbf{E}^{a2}|P\rangle,$$



$$Qe = 2e/3$$

$$\mathcal{M}_{\gamma P\to\psi P}(t) = -Qe\ c_2\ \frac{16\pi^2 M}{b}\ \langle P'|T|P\rangle$$

Differential cross section:

$$\frac{d\sigma_{\gamma P\to\psi P}}{dt} = \frac{1}{64\pi s}\ \frac{1}{|\mathbf{p}_{\gamma cm}|^2}\ |\mathcal{M}_{\gamma P\to\psi P}(t)|^2$$

DK, PRD'21

$$\sigma_{\gamma P\to\psi P}(s) = \int_{t_{min}}^{t_{max}} dt\ \frac{d\sigma_{\gamma P\to\psi P}}{dt},$$

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Editors' Suggestion

First Measurement of Near-Threshold J/ψ Exclusive Photoproduction off the Proton

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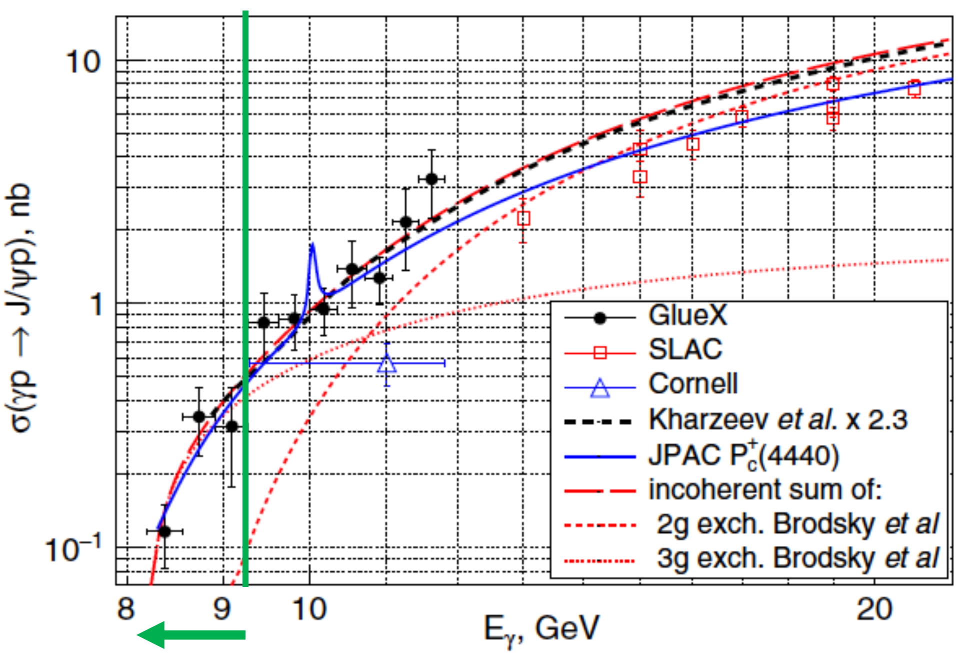
(GlueX Collaboration)



Need to focus on the threshold region!

$E_{cm} < 4.25$ GeV

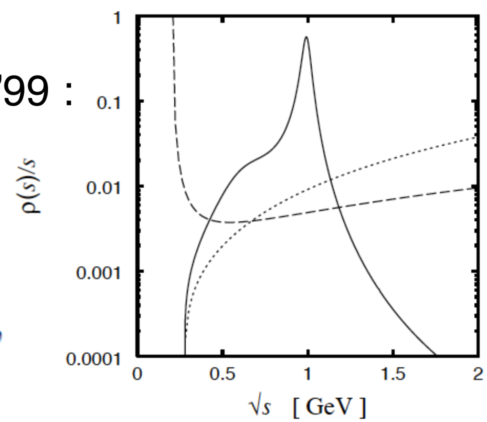
$E_\gamma < 9.2$ GeV

Threshold photoproduction of quarkonium: the effect of the scalar gravitational formfactor

The scalar gravitational formfactor can be constrained theoretically by using:

i) dispersion relations;
ii) low-energy theorems of broken scale invariance;
iii) experimental data on ππ phase shifts and scalar mesons

See e.g. Fujii, DK'99 :



However, as a first step, can try a simple dipole formfactor of the type used for electromagnetic formfactor:

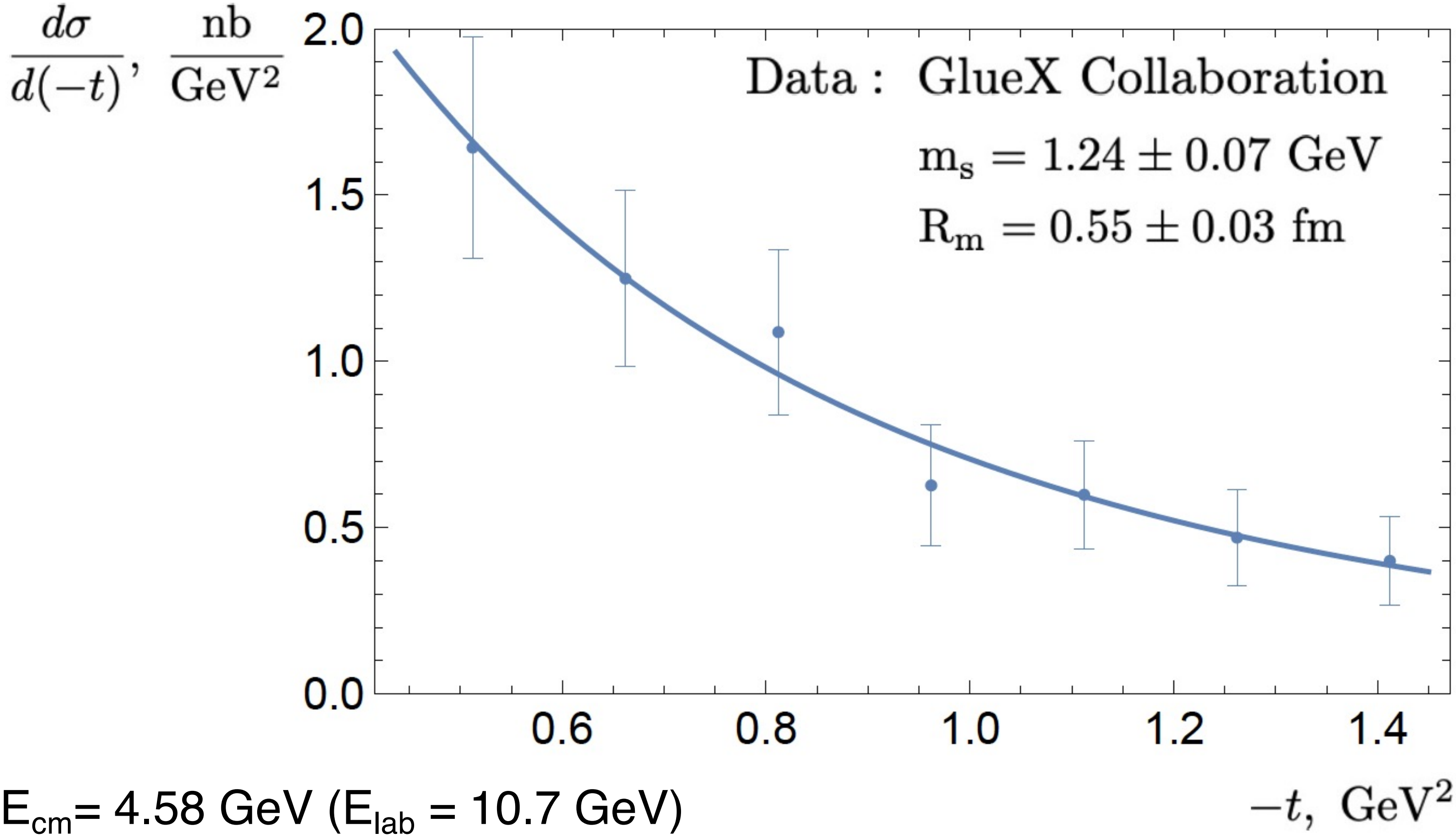
$$G(t) = \frac{M}{(1 - t/M_s^2)^2}$$ radius $$\langle R_{\rm M}^2 \rangle = \frac{6}{M} \left. \frac{dG}{dt} \right|_{t=0},$$

Dipole formfactor was also used for 2-gluon coupling in perturbative models

See e.g. Frankfurt, Strikman '02

Differential cross section

DK, arXiv:2102.00110



E_{cm}= 4.58 GeV (E_{lab} = 10.7 GeV)

$$|c_2|^2 = 0.043 \pm 0.006 \text{ fm}^4$$

Lattice QCD, P. Shanahan, W. Detmold PRD'19:

$$m_s = 1.13 \pm 0.06 \text{ GeV}$$ (Traceless gluon operator)

$$c_2 \sim \pi r_{c\bar{c}}^2 \qquad r_{c\bar{c}} \simeq 0.1 \text{ fm}$$

The proton mass radius

The r.m.s. "proton mass radius" from GlueX data:

DK, arXiv:2102.00110

$$R_{\mathrm{m}} \equiv \sqrt{\langle R^2_{\mathrm{m}} \rangle} = 0.55 \pm 0.03 \text{ fm}$$

Compare to the proton charge radius:

$$\bar{\mathrm{R}}_{\mathrm{c}} \equiv \sqrt{R_c^2} = 0.8409 \pm 0.0004 \quad \text{fm}$$

See J.Bernauer, EPJ 234 (2020) for review

A more compact mass distribution? Need more data!

VALUE (fm)		*DOCUMENT ID*		*TECN*	*COMMENT*
0.8409 ± 0.0004		**OUR AVERAGE**			
0.833 ±0.010	1	BEZGINOV	2019	LASR	2S-2P transition in H
0.831 ±0.007 ±0.012	2	XIONG	2019	SPEC	$e\,p \to ep$ form factor
0.84087 ±0.00026 ±0.00029		ANTOGNINI	2013	LASR	μp -atom Lamb shift
• • • We do not use the following data for averages, fits, limits, etc. • • •					
0.877 ±0.013	3	FLEURBAEY	2018	LASR	1S-3S transition in H
0.8335 ±0.0095	4	BEYER	2017	LASR	2S-4P transition in H
0.8751 ±0.0061		MOHR	2016	RVUE	2014 CODATA value
0.895 ±0.014 ±0.014	5	LEE	2015	SPEC	Just 2010 Mainz data
0.916 ±0.024		LEE	2015	SPEC	World data, no Mainz
0.8775 ±0.0051		MOHR	2012	RVUE	2010 CODATA, ep data
0.875 ±0.008 ±0.006		ZHAN	2011	SPEC	Recoil polarimetry
0.879 ±0.005 ±0.006		BERNAUER	2010	SPEC	$e\,p \to ep$ form factor

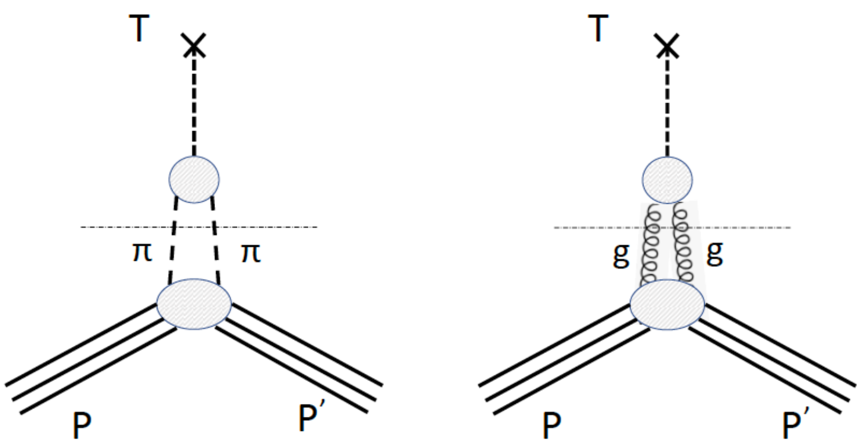
2020 Review of Particle Physics.
P.A. Zyla *et al.* (Particle Data Group), Prog. Theor. Exp. Phys. **2020**, 083C01 (2020)

Some day:
p MASS RADIUS in PDG?

Theoretical uncertainties

- Higher dimensional operators (suppressed by $1/m_c$)
- Chiral limit (we omitted the scalar quark operator)
- Gluon operators with derivatives (~ 5% close to threshold)
- t-dependence of short-distance coefficient c_2 ($\sim t/4m_c^2$)
- Dipole parameterization of formfactor

Why is proton mass radius smaller than the charge radius?



Spectral representation –

EM formfactor: $M_\rho = 0.77$ GeV

Scalar gravitational formfactor: scalar glueball M = 1.5 GeV

But: scalar gluon current mixes with the scalar quark current – σ(500) is lighter than the ρ!

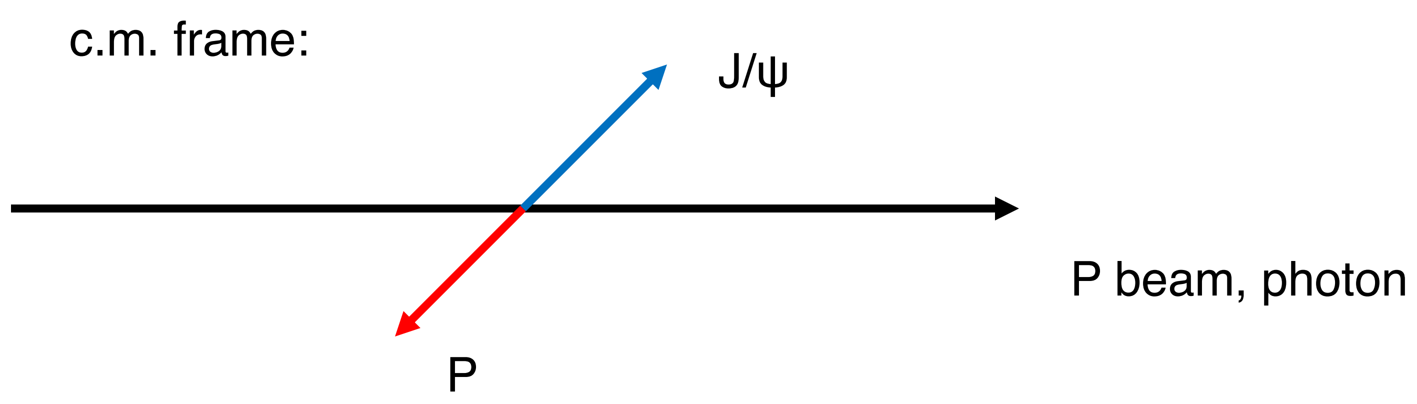
The real reason (?) – decoupling of Goldstone bosons:

$$\langle 0|T|\pi^+\pi^-\rangle = q^2$$

DK, arXiv:2102.00110

Future measurements

- GlueX has 10 times more data
- Future: SoLID@Jlab (~ 2028), EIC (including Υ !)
- Polarization? (scalar vs tensor)
- Also: ultra-peripheral collisions at RHIC?



For a fixed invariant mass (cms energy), measure the angular distribution – differential cross section of photoproduction

Summary

- The proton mass to large extent originates from quantum anomalies

- The threshold photoproduction of J/ψ probes the mass distribution inside the proton; current data and a simple dipole model favor

$$R_{\mathrm{m}} \equiv \sqrt{\langle R_{\mathrm{m}}^2 \rangle} = 0.55 \pm 0.03 \ \mathrm{fm}$$

- We need a quantitative theory of the scalar gravitational formfactor and precise data at $E_{cm} < 4.3$ GeV to understand the mass distribution inside the proton, and the origin of the proton mass!