Heavy-Ion Collisions and the QCD Phase Diagram

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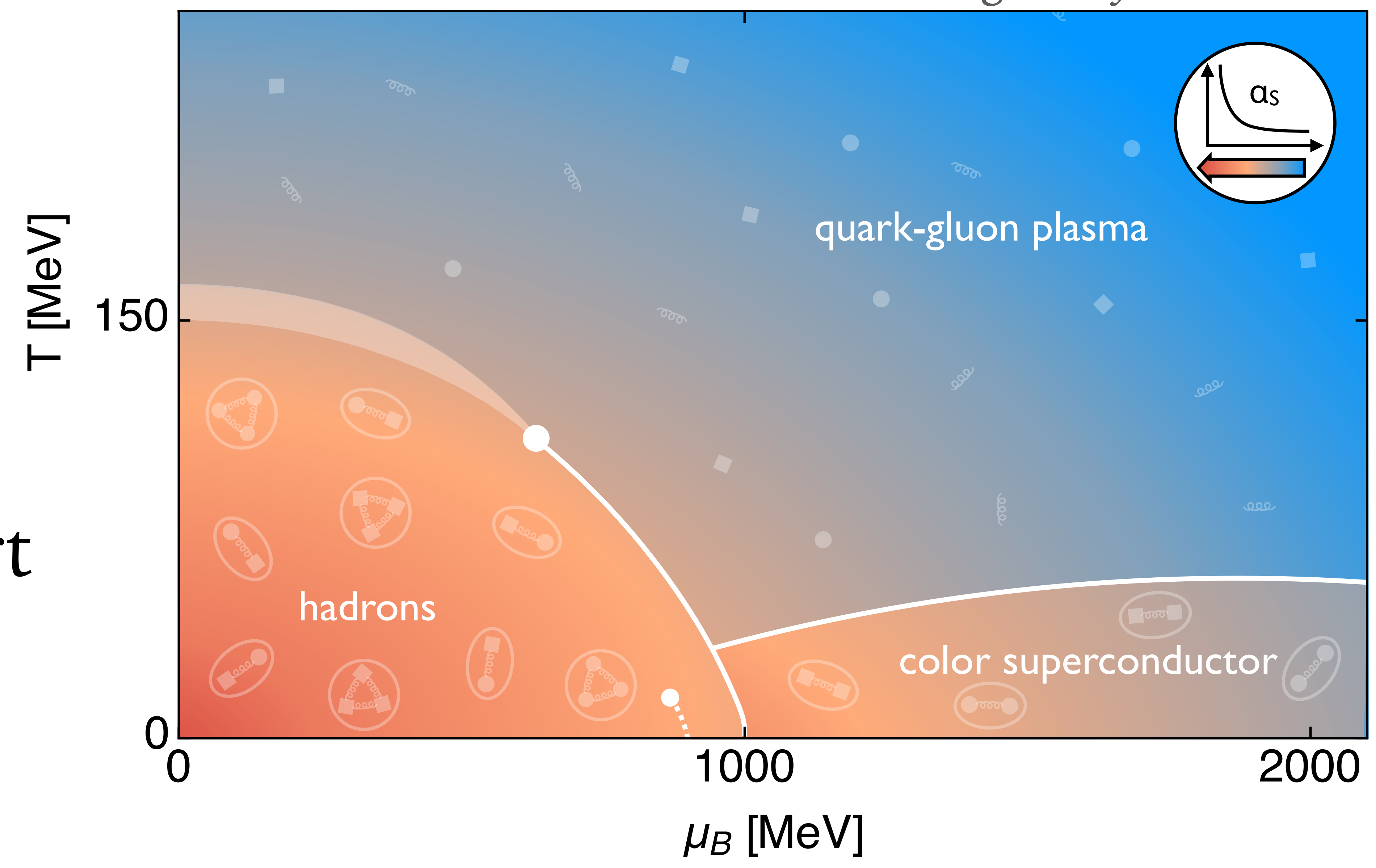
Phase Diagram (Sketch)

- Knowledge of the equation of state of strongly-interacting matter in equilibrium is crucial for:
  - Fluctuations, via derivatives of the pressure
  - The hadronic spectrum, i.e. the composition of the system in HICs, via thermal models
  - Hydrodynamic simulations
  - Hadronic transport simulations
  - Merger simulations
  - The behavior of the bulk viscosity & transport
  - The interior composition of neutron stars
  - …

Phase diagram by F. Rennecke



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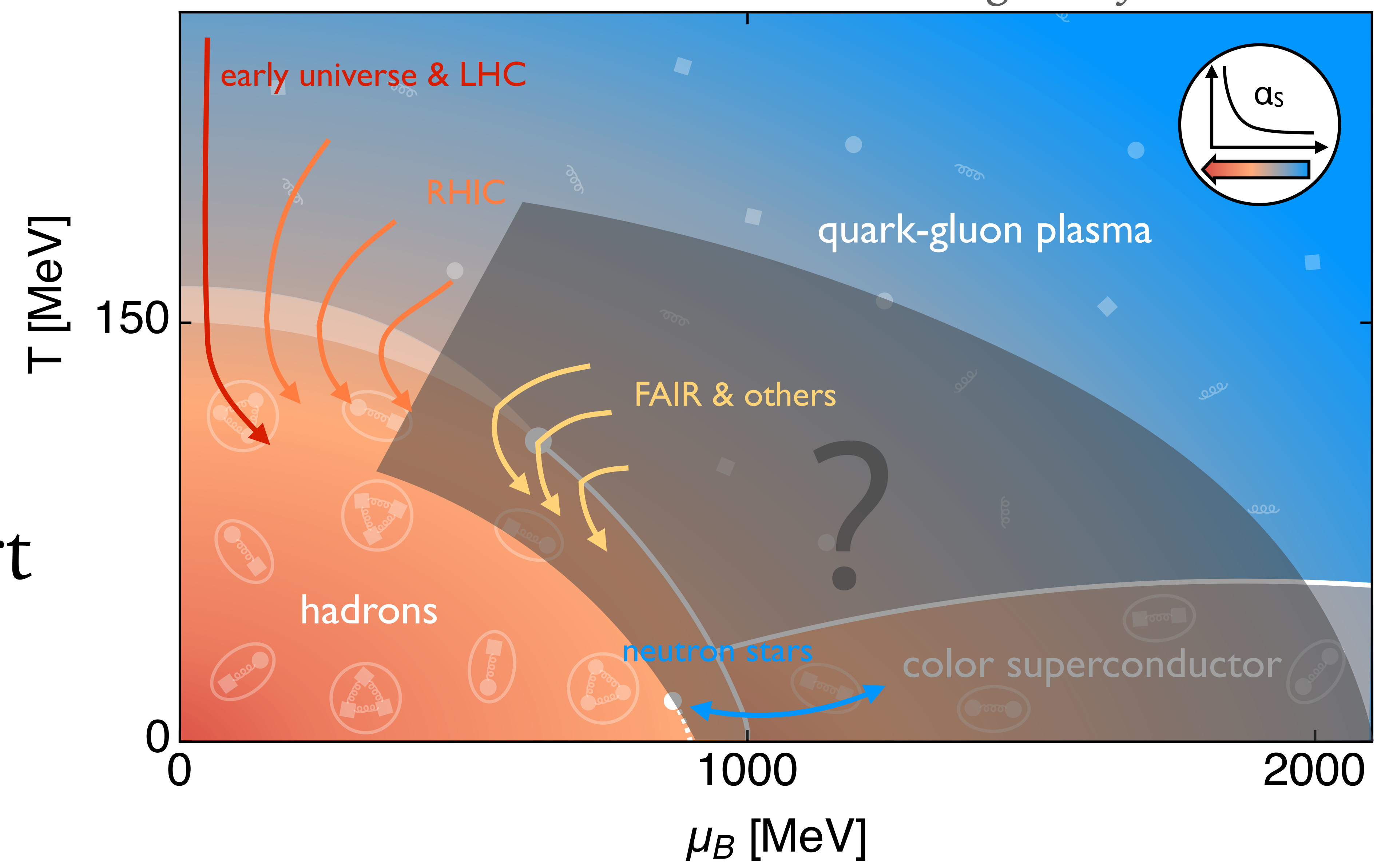
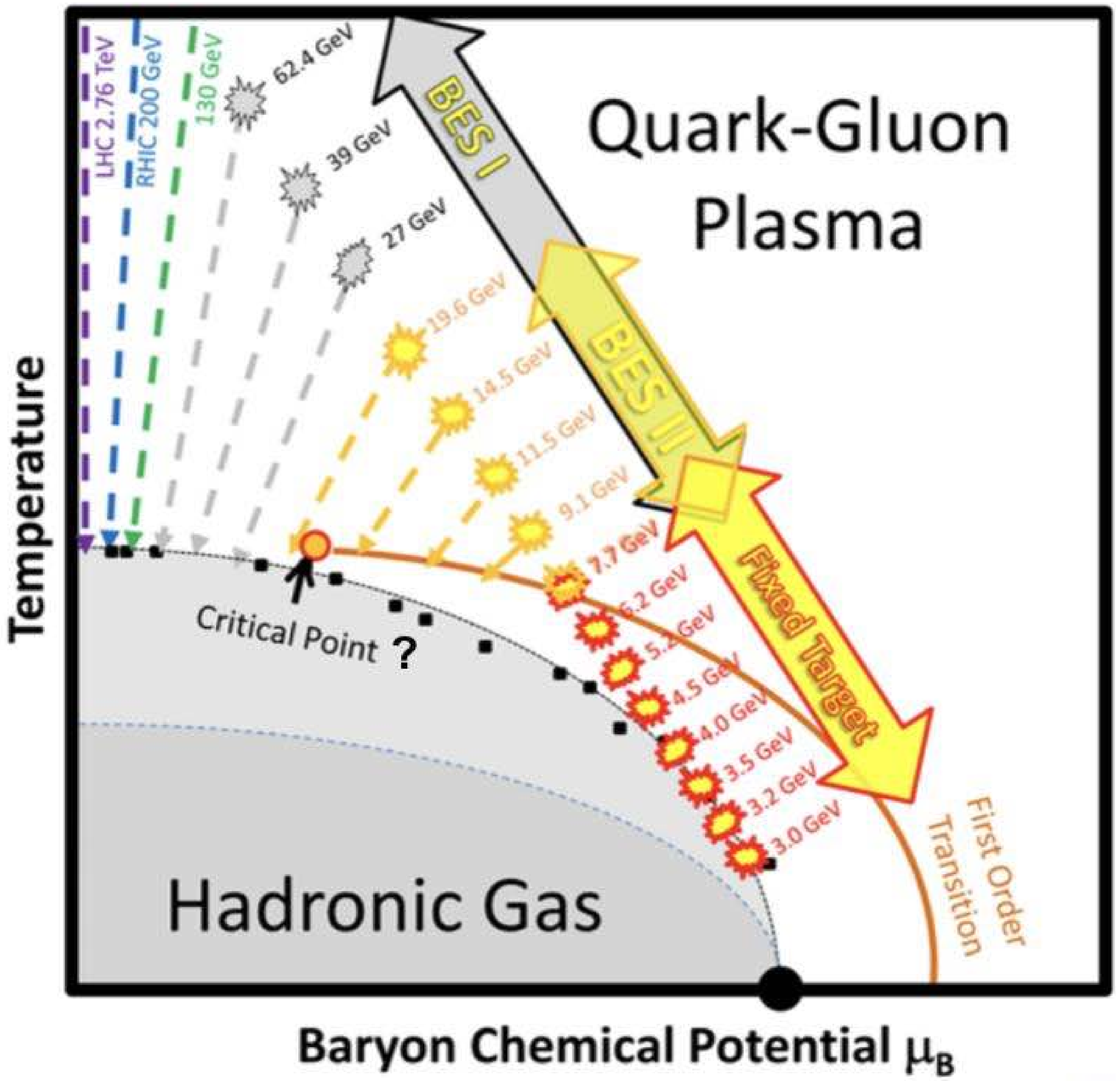
Phase diagram by F. Rennecke

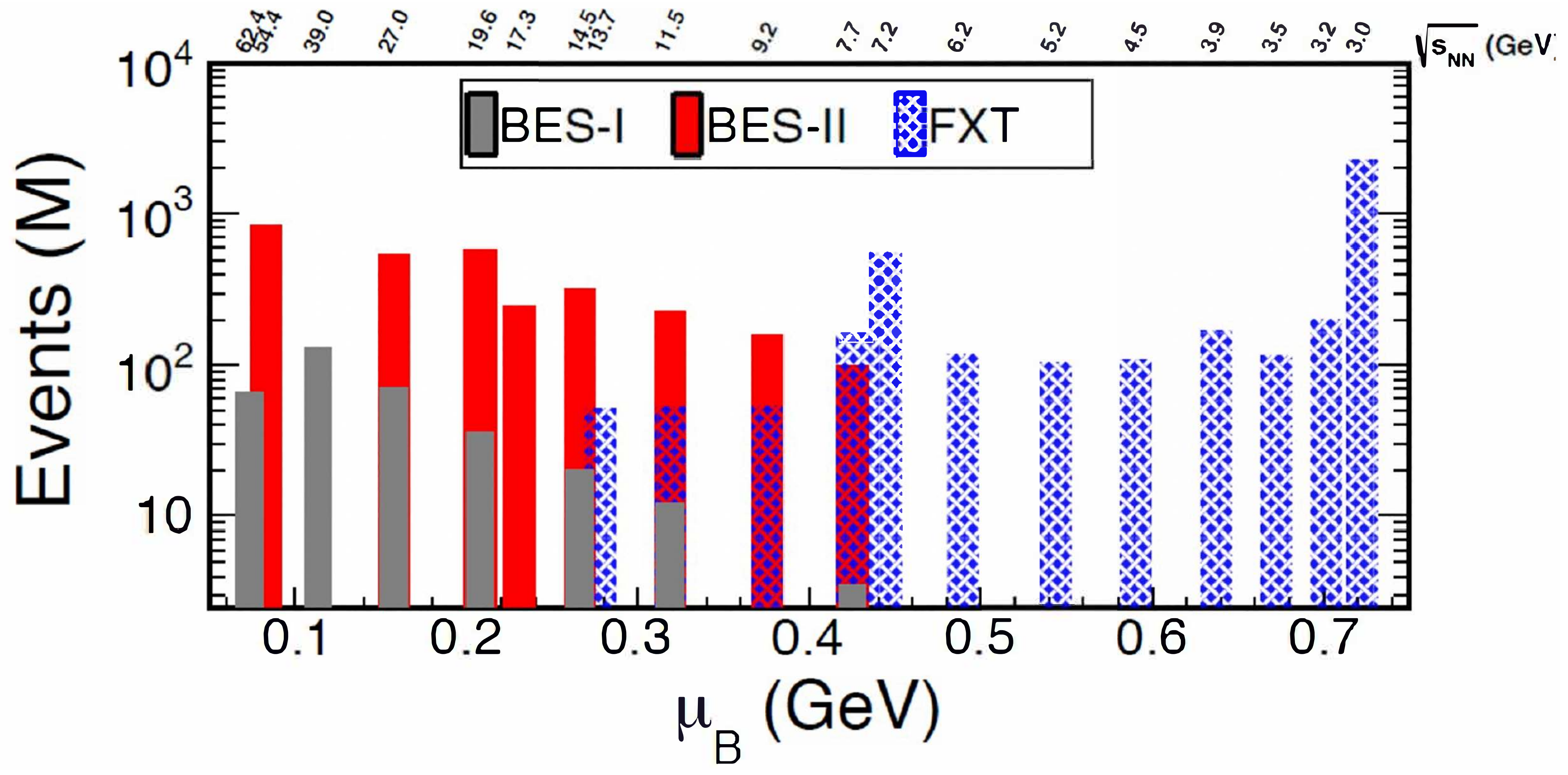


What do we know from heavy-ion collisions?

RHIC Beam Energy Scan

- Map out the phase diagram by colliding at different CM energies
- A stated goal of the program: to locate or constrain the location of the QCD CP

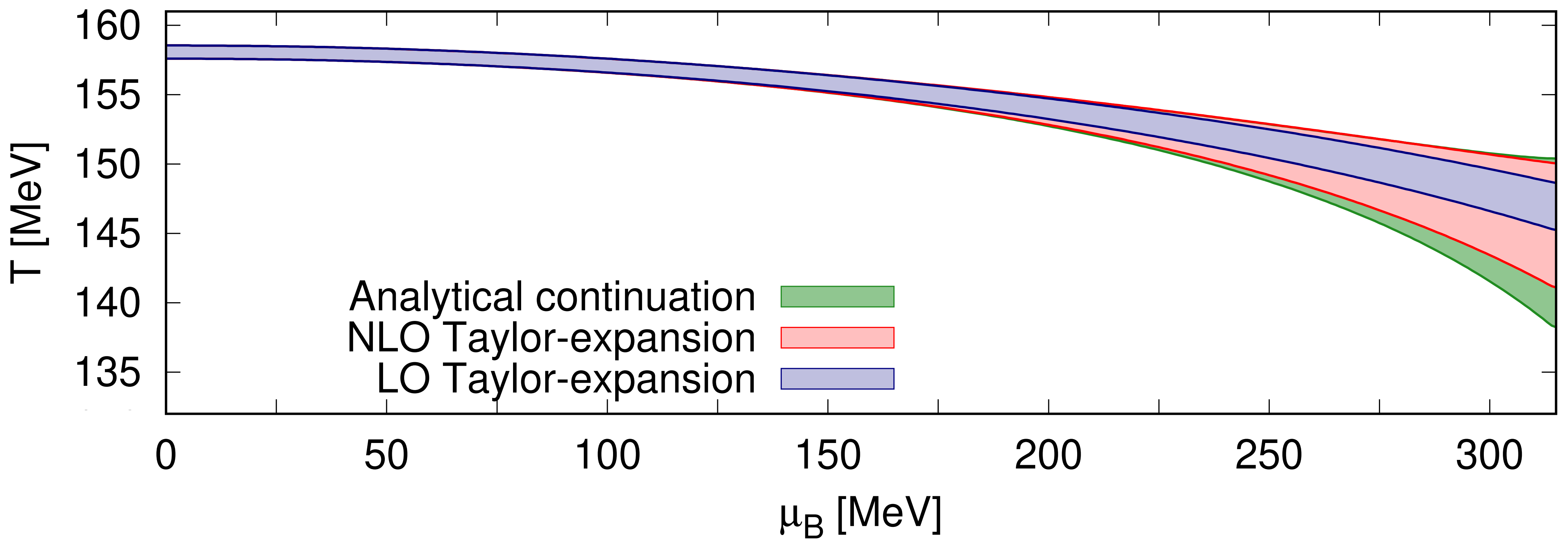


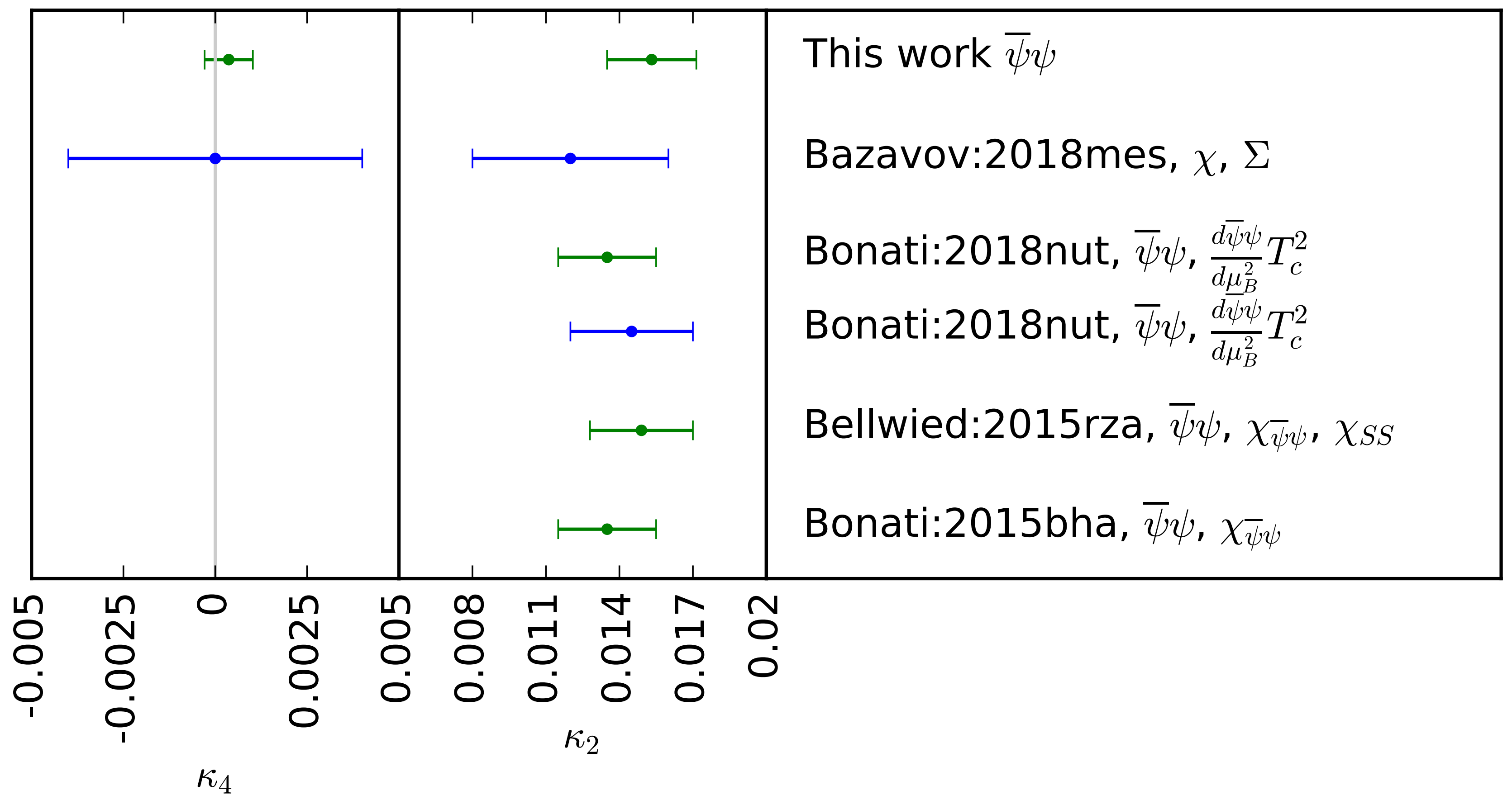


Lattice QCD

➤ Precise knowledge of the QCD pseudocritical temperature and characterization of transition line at finite chemical potential

$$\frac{T_c(\mu_B)}{T_c(\mu_B=0)} = 1 - \kappa_2 \left(\frac{\mu_B}{T_c(\mu_B)}\right)^2 - \kappa_4 \left(\frac{\mu_B}{T_c(\mu_B)}\right)^4$$

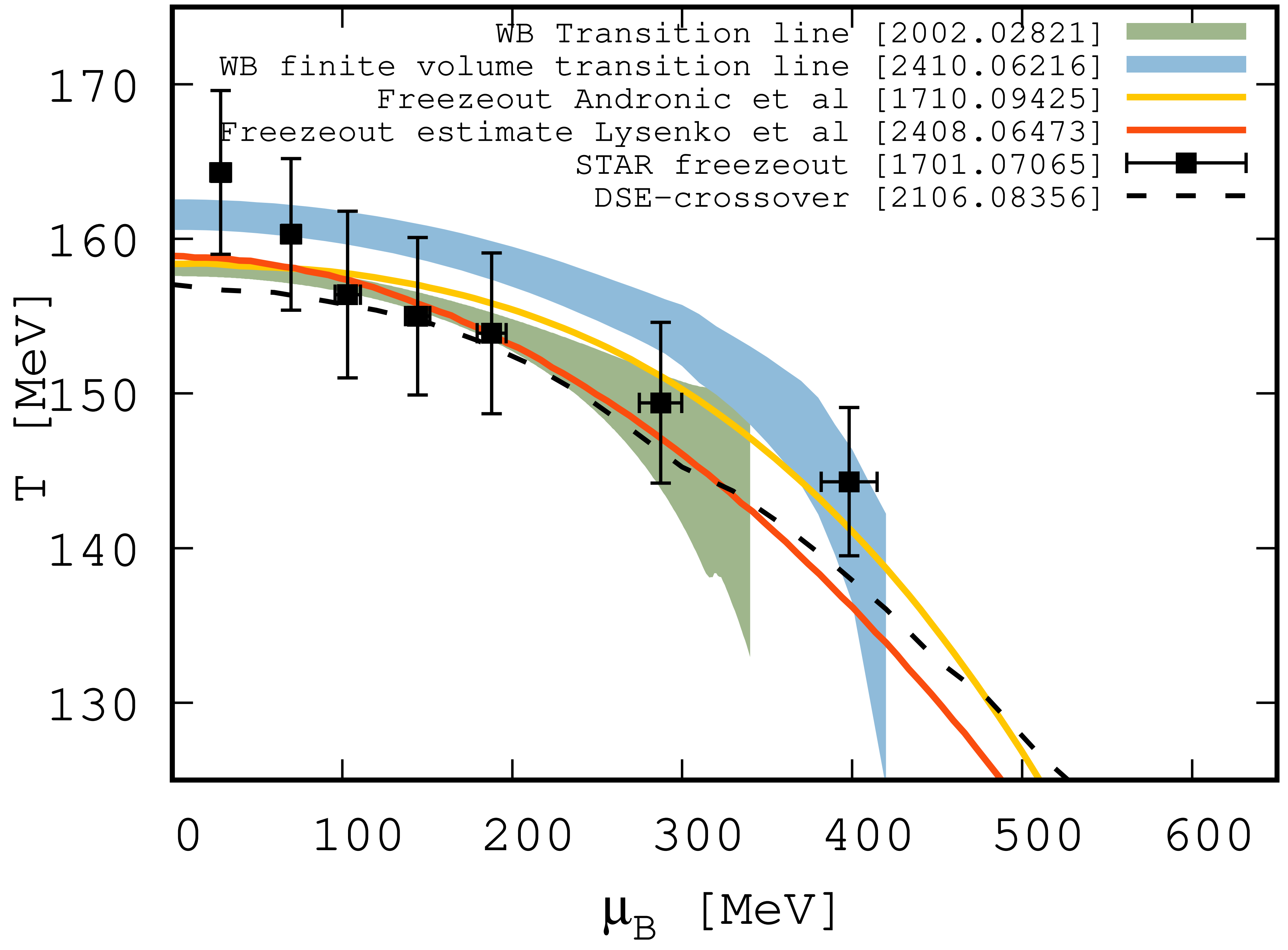




S. Borsanyi et al, PRL (2020)
See also: A. Bazavov et al, PLB (2019)

(Less Sketchy) Phase Diagram

- Much of the current knowledge of the QCD phase diagram from *ab initio* theory and experiment from QCD crossover transition & freeze-out

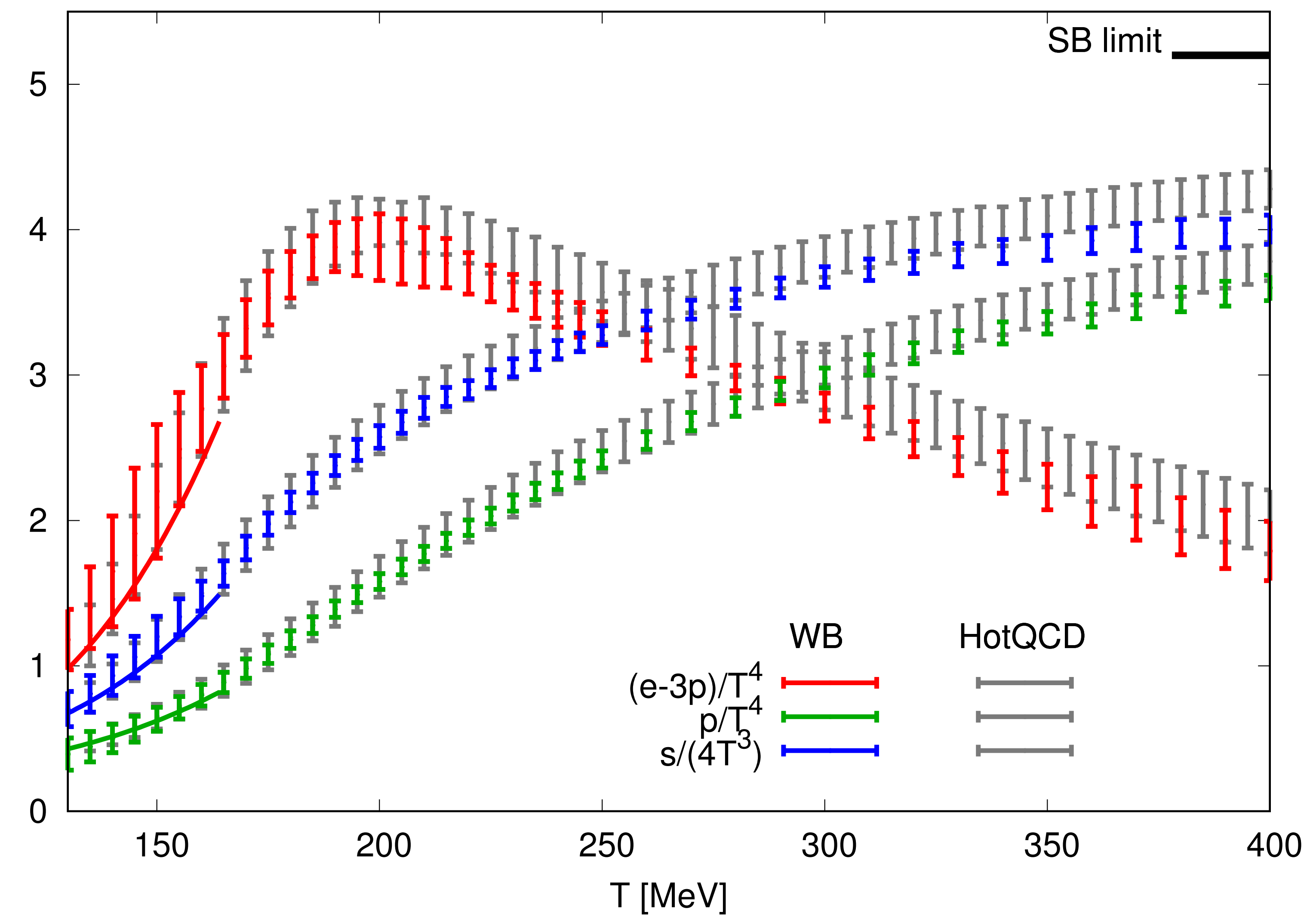


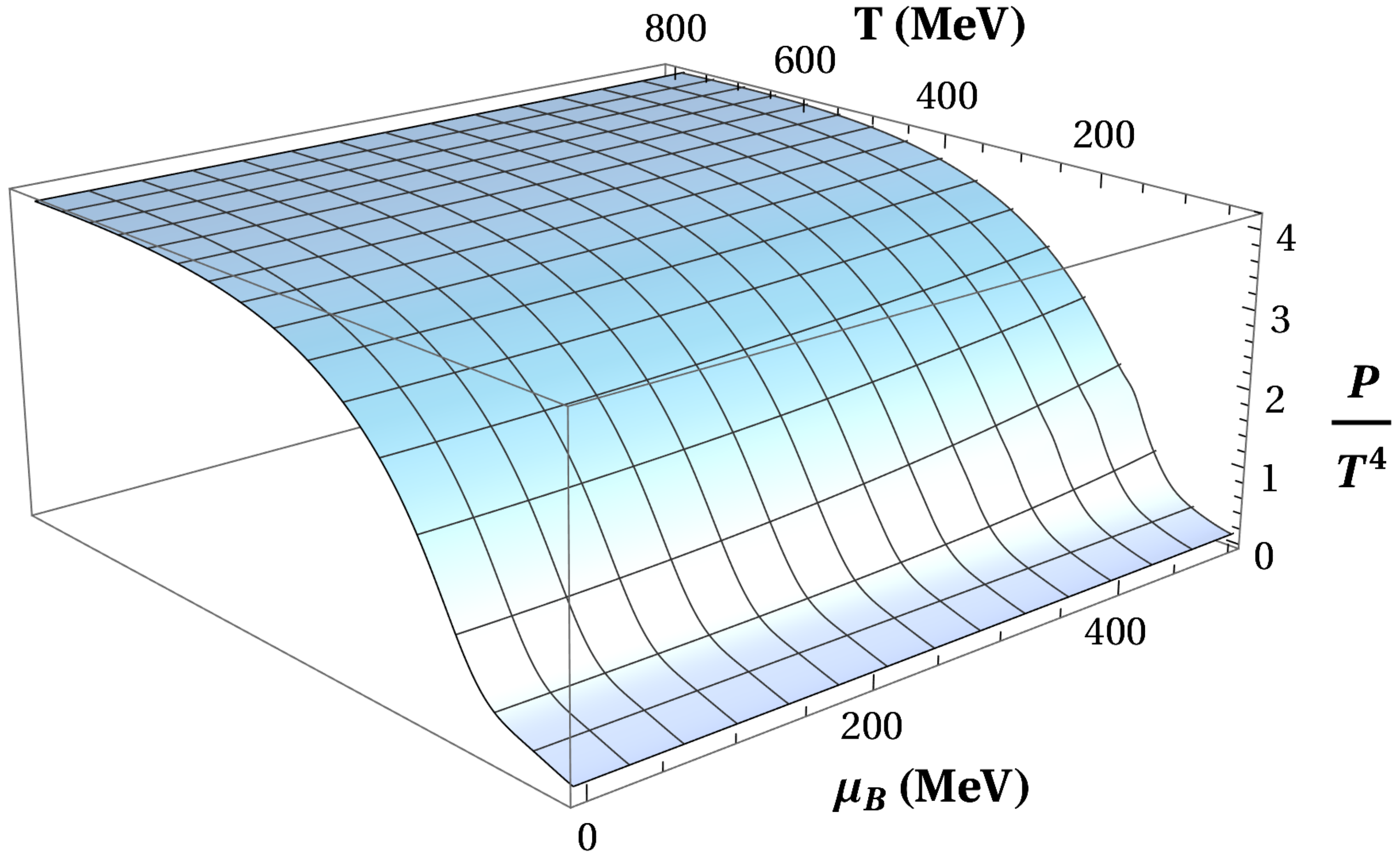
S. Borsanyi et al, arXiv:2502.10267

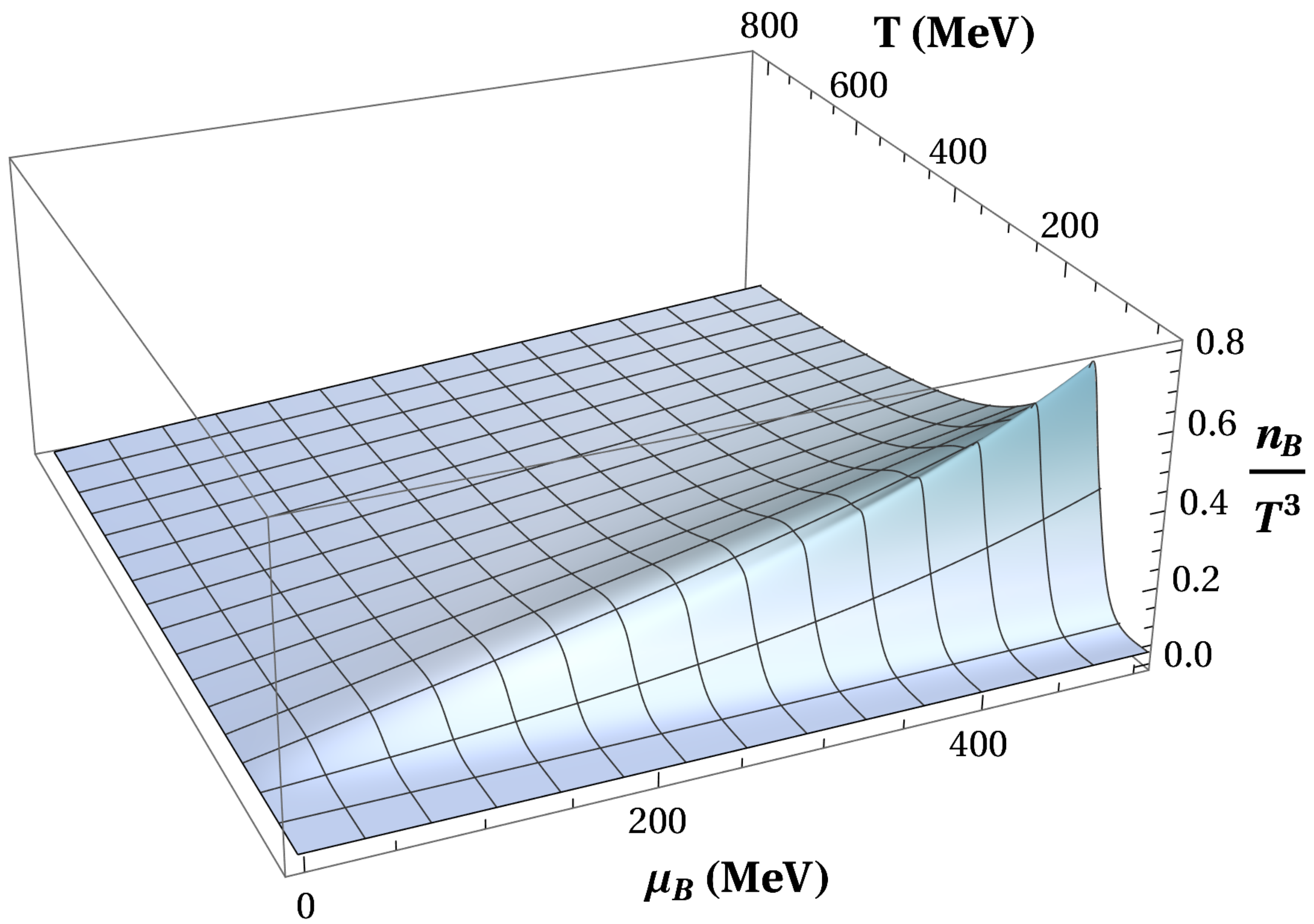
Lattice EoS at Finite T & μ_B

➤ Equilibrium thermodynamics calculated from first principles lattice QCD computations are well-established with good agreement amongst techniques

$$\frac{p(T,\mu_B)}{T^4} = \frac{p(T,0)}{T^4} + \sum_{n=1}^{\infty} \frac{1}{(2n)!} \frac{\mathrm{d}^{2n}(p/T^4)}{d(\frac{\mu_B}{T})^{2n}}\bigg|_{\mu_B=0} \left(\frac{\mu_B}{T}\right)^{2n} = \sum_{n=1}^{\infty} \frac{1}{(2n)!} \chi^B_{2n} \left(\frac{\mu_B}{T}\right)^{2n}$$







See also T'-expansion scheme - S. Borsanyi et al, PRL (2021) and T-μ_B-μ_Q-μ_S EoS - A. Abuali et al, arXiv:2504.01881

A. Bazavov PRD (2014), S. Borsanyi PLB (2014)

Evolution of a Heavy-ion Collision

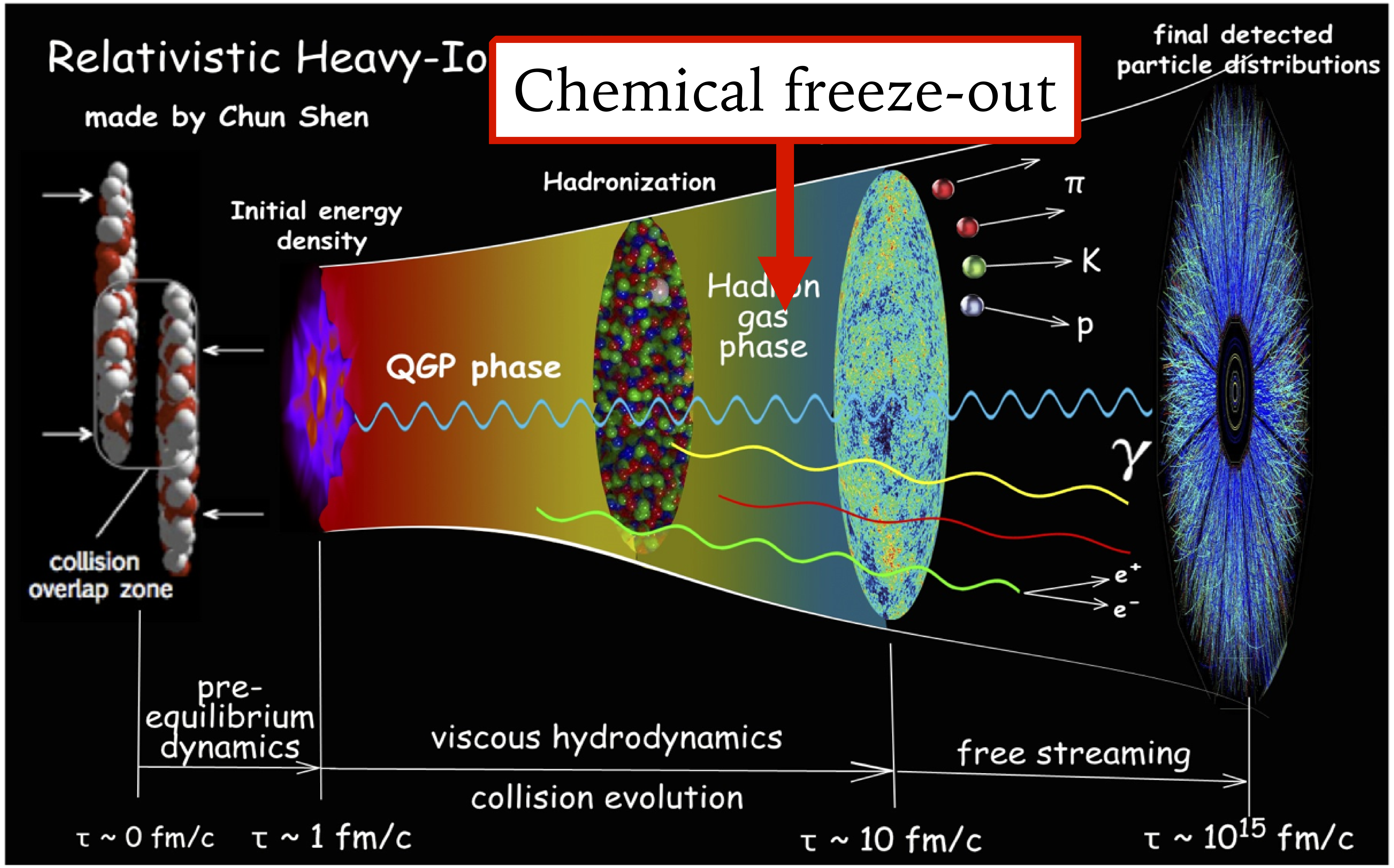
- Strongly-interacting matter proceeds through several different phases during a collision event → HIC modeling/phenomenology

Nuclear initial conditions

$\langle n_Q \rangle = 0.4 \langle n_B \rangle$

$\langle n_S \rangle = 0$

(strangeness neutrality)

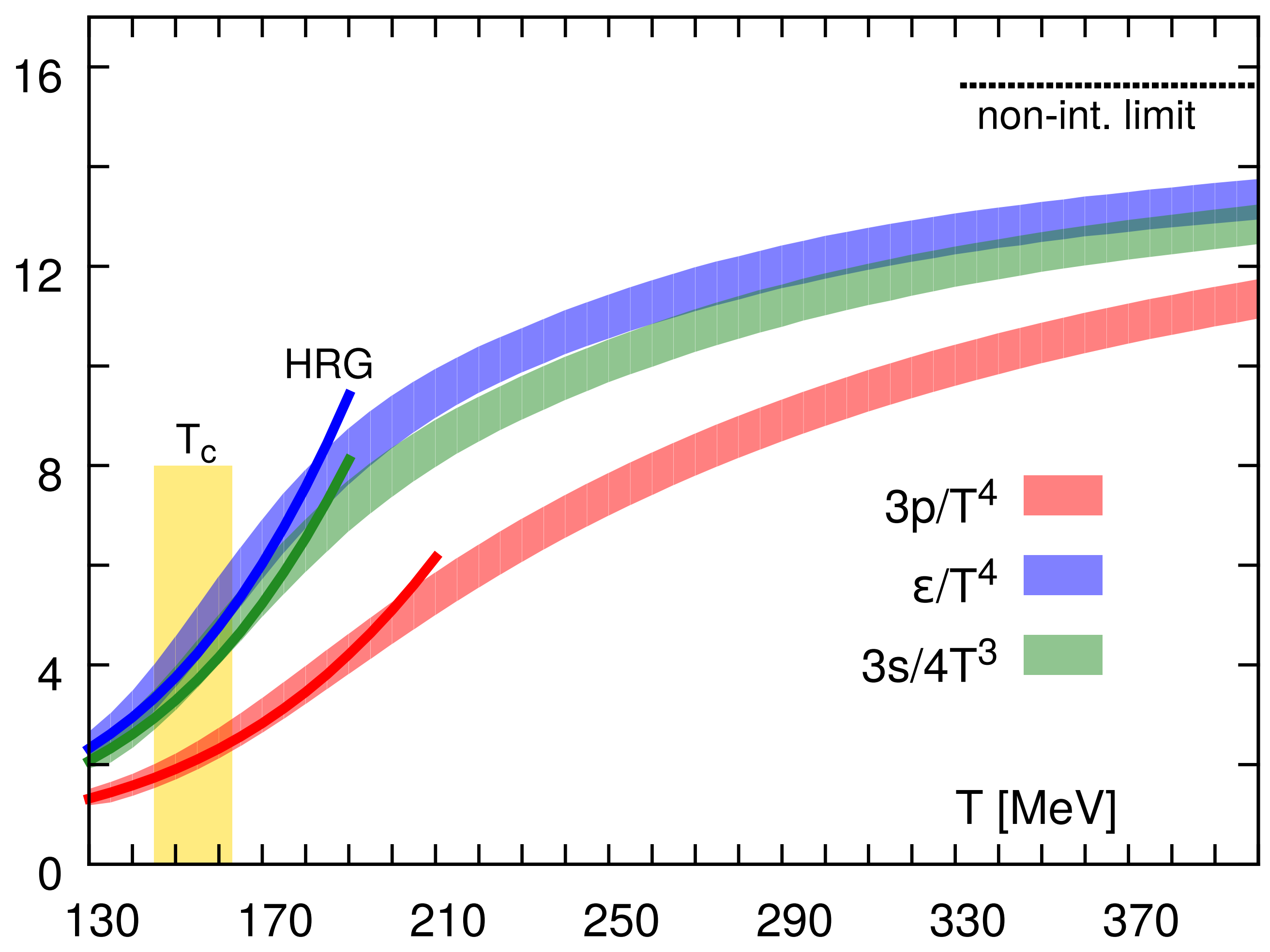


Chemical freeze-out: inelastic collisions cease; the chemical composition is fixed (particle yields and fluctuations)

Kinetic freeze-out: elastic collisions cease; spectra and correlations are fixed

Hadron Resonance Gas & Hagedorn T_H

- The low temperature thermodynamics is well-described by the Hadron Resonance Gas model but hadronic spectrum still unknown

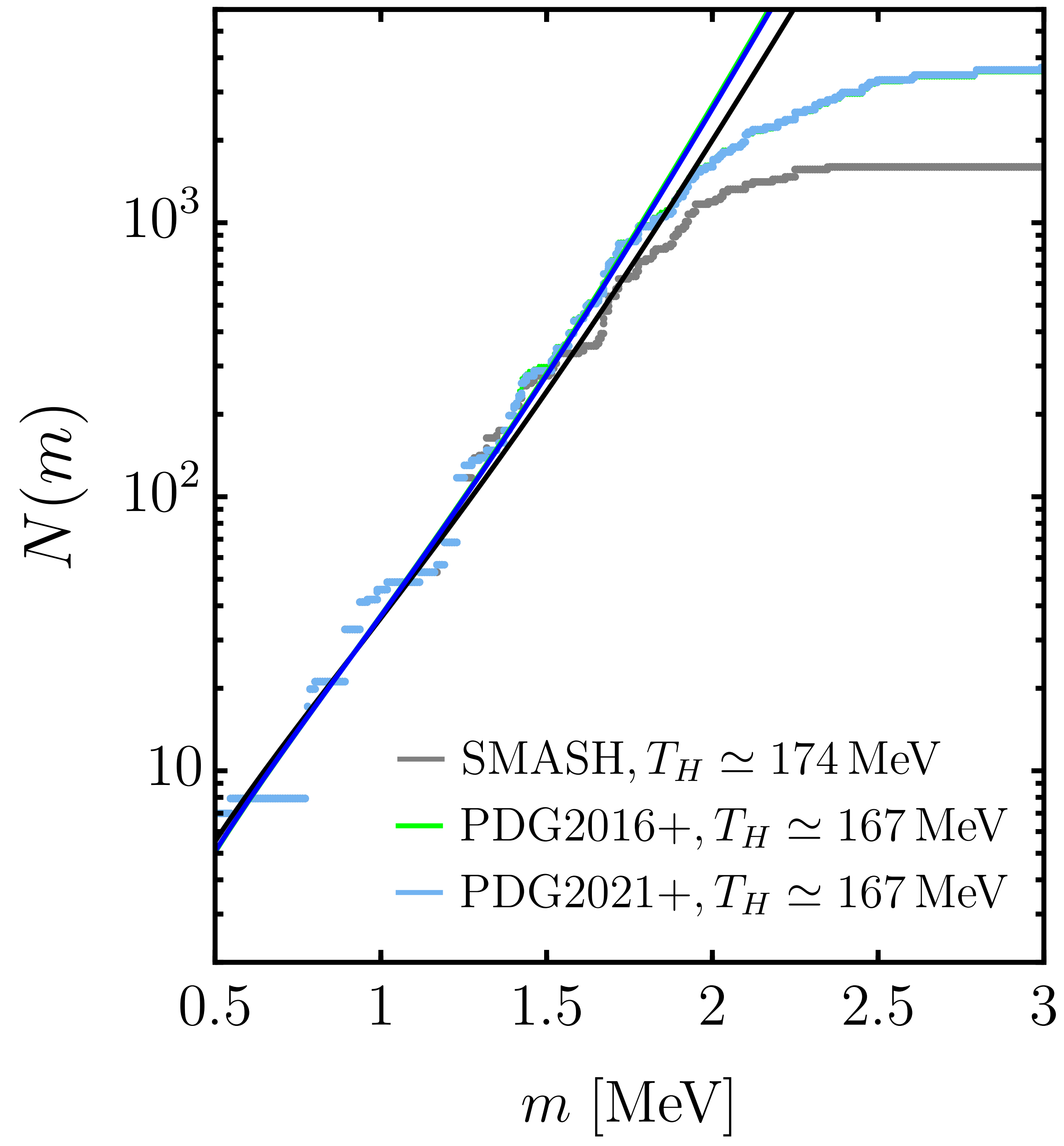


Pressure: $$\frac{P}{T^4} = \frac{1}{VT^3}\sum_i \ln Z_i(T,V,\vec{\mu})$$

$$= \sum_i (-1)^{B_i+1}\frac{g_i}{2\pi^2}\int_0^\infty p^2 \ln\left[1+(-1)^{B_i+1}e^{(-\frac{\sqrt{p^2+m_i^2}}{T}+\tilde{\mu}_i)}\right]dp$$

Density: $$\frac{n_i}{T^3} = \frac{1}{T^3}\left(\frac{\partial p}{\partial \mu_i}\right)\Bigg|_{T,\mu_j}$$

$$= \frac{g_i}{2\pi^2}\int_0^\infty p^2\left[\exp\left(\frac{\sqrt{p^2+m_i^2}}{T}-\tilde{\mu}_i\right)+(-1)^{B_i-1}\right]^{-1} dp$$



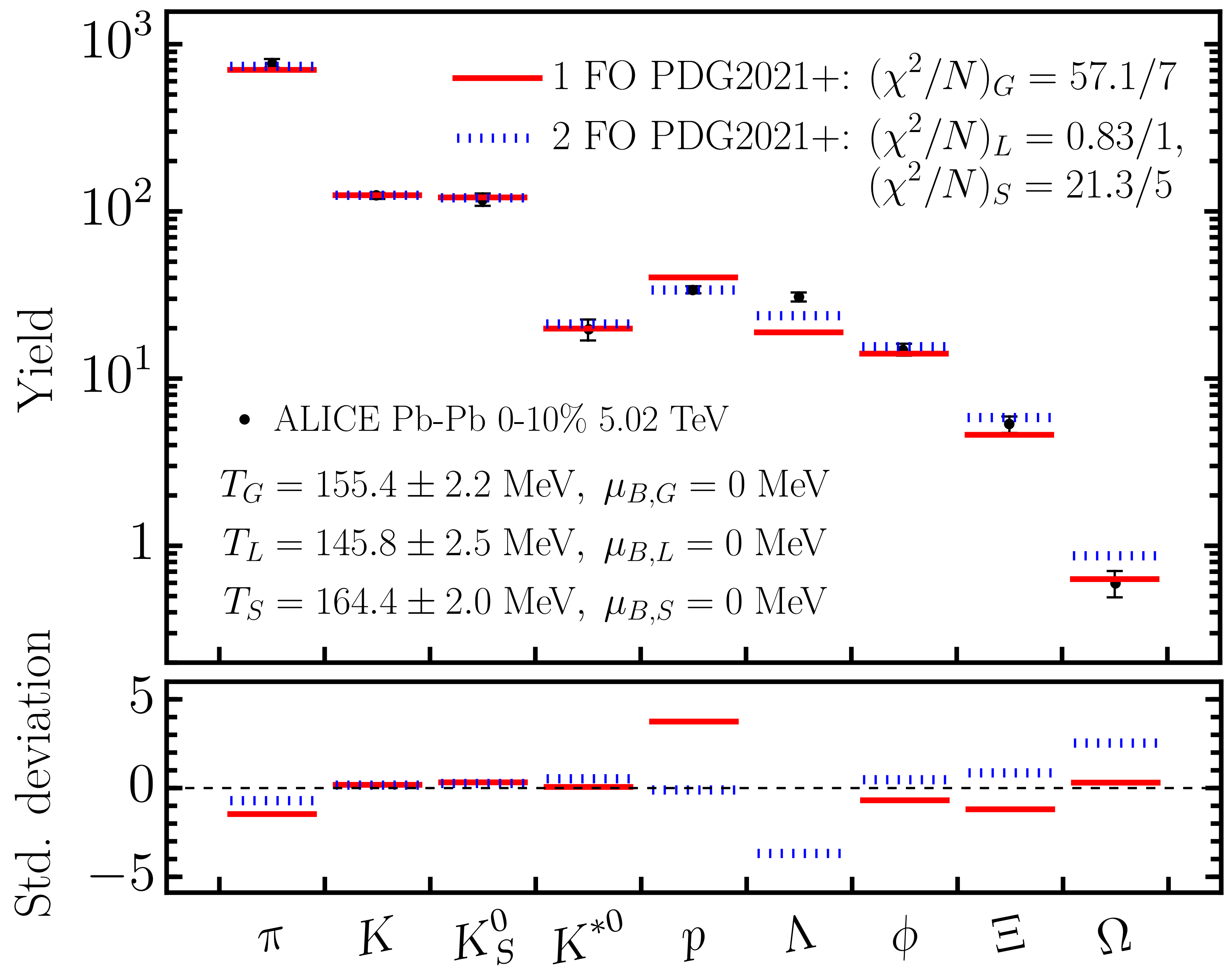
A. Bazavov et al, PRD (2014)
R. Hagedorn, Nuovo Cim. Suppl (1965, 1968)
*J. Salinas San Martin, R. Hirayama, J. Hammelmann, **J.M. Karthein** et al, arXiv:2309.01737*

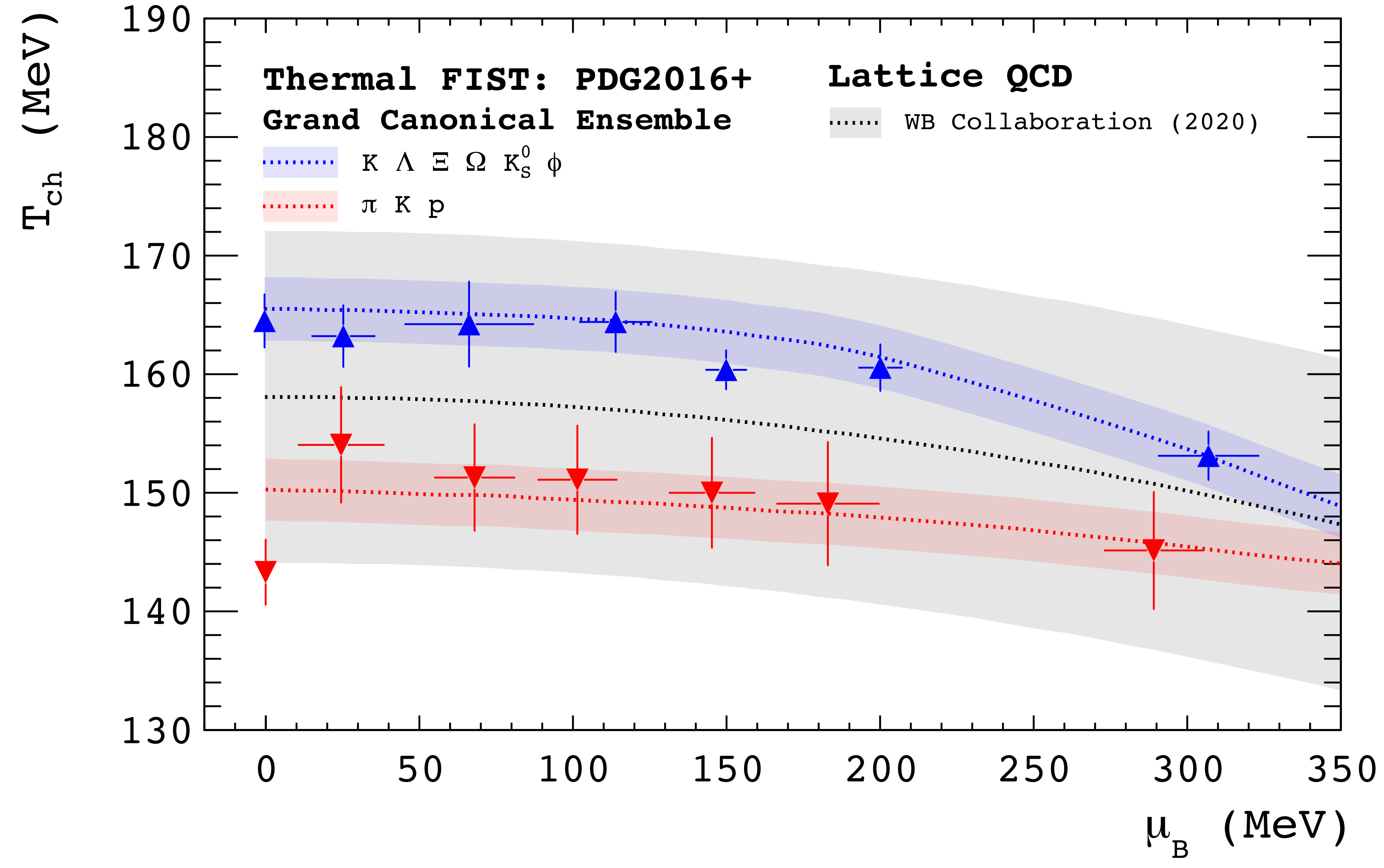
Thermal Fits

- Characterize the medium produced in heavy-ion collisions via thermal fits to locate the freeze-out points in the phase diagram

$$\langle N_i \rangle = V\, n_i + V \sum_R \langle n_i \rangle_R\, n_R$$

$$\frac{\chi^2}{N_{\rm dof}} = \frac{1}{N_{\rm dof}} \sum_{i=1}^{N} \frac{\left(N_i^{\rm exp} - N_i^{\rm HRG}\right)^2}{\sigma_i^2}$$

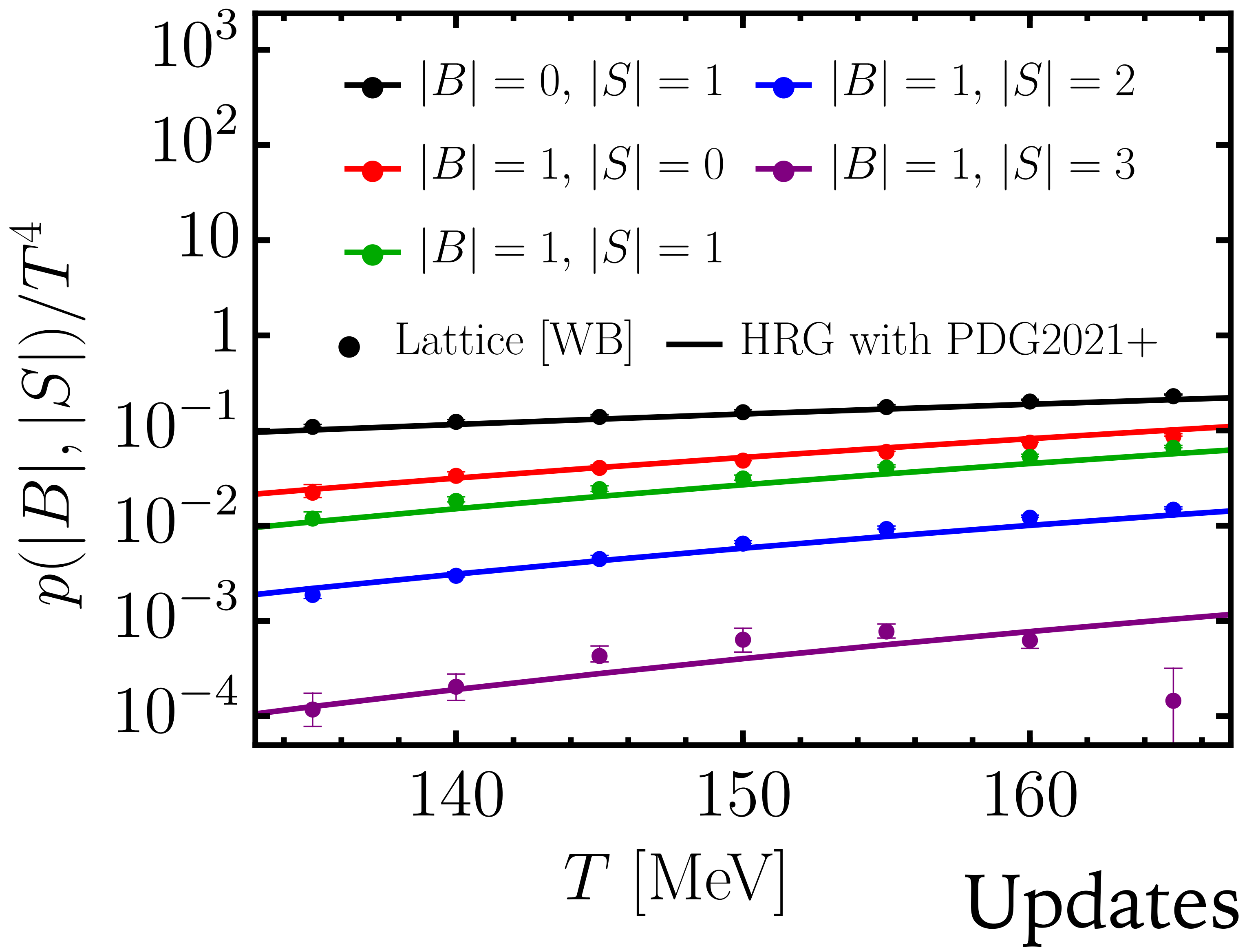


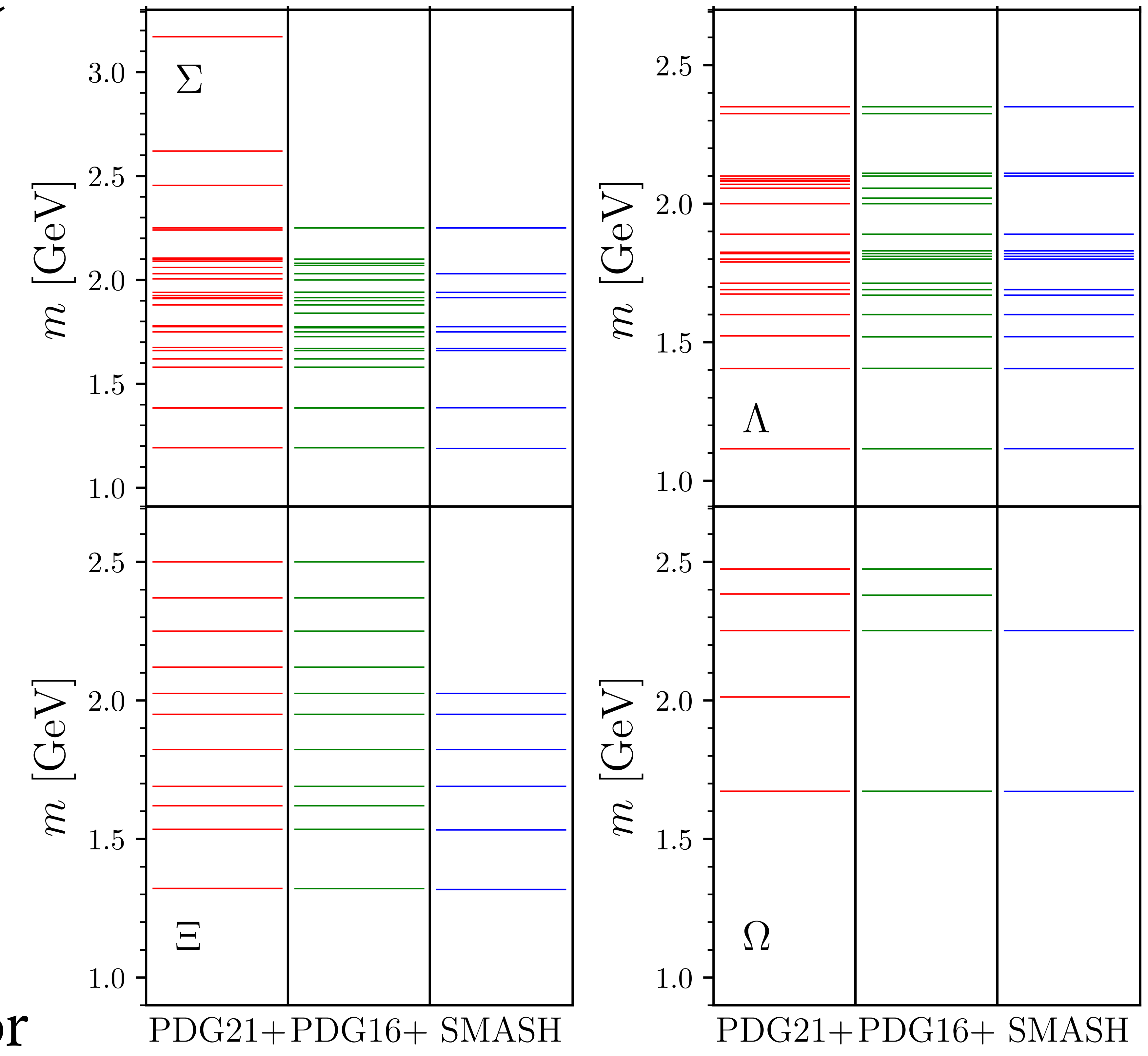


A. Andronic, P. Braun-Munzinger, K. Redlich, J. Stachel, Nature (2018)
*J. Salinas San Martin, R. Hirayama, J. Hammelmann, **J.M. Karthein** et al, arXiv:2309.01737*
F. Flor, G. Olinger, R. Bellwied, PLB (2022)

Hadronic Composition

- Partial pressure from lattice QCD help determine hadronic spectrum
  - Improved agreement with lattice when including more states: PDG2021+
  - Decays compatible with SMASH hadronic transport





Updates mainly to strange sector including newly measured Ω baryon

J. Salinas San Martin, R. Hirayama, J. Hammelmann, ***J.M. Karthein*** *et al, arXiv:2309.01737*

Historical Theoretical View of the QCD CP

- Expectations for a proper (first order) phase transition
  - Cabibbo & Parisi interpreted Hagedorn temperature as evidence for a change in degrees of freedom: hadrons to quarks

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EXPONENTIAL HADRONIC SPECTRUM AND QUARK LIBERATION

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G. PARISI
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Received 9 June 1975

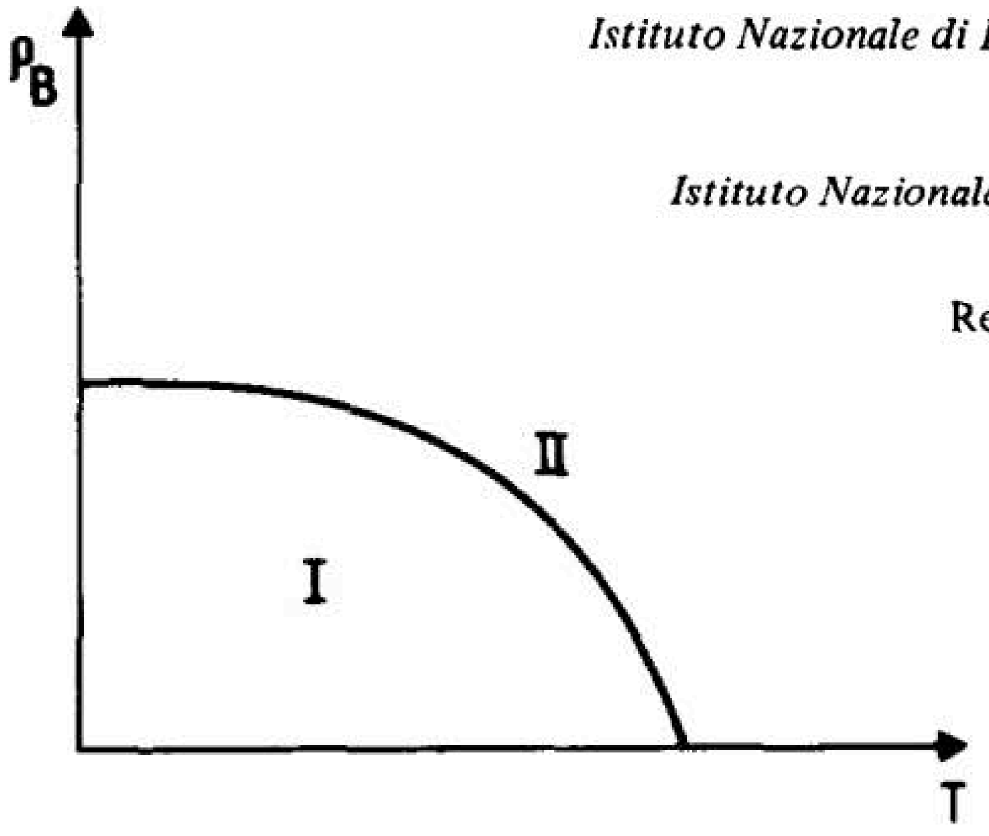


Fig. 1. Schematic phase diagram of hadronic matter. ρ_B is the density of baryonic number. Quarks are confined in phase I and unconfined in phase II.

Historical Theoretical View of the QCD CP

➤ Theoretical efforts on the nature of the QCD phase transition

➤ Early efforts

[10] Robert D. Pisarski and Frank Wilczek. Remarks on the chiral phase transition in chromodynamics. *Phys. Rev.*, D29:338–341, 1984.

[11] T. Celik, J. Engels, and H. Satz. The order of the deconfinement transition in su(3) yang- mills theory. *Phys. Lett.*, B125:411–414, 1983.

[12] John B. Kogut et al. Deconfinement and chiral symmetry restoration at finite temperatures in su(2) and su(3) gauge theories. *Phys. Rev. Lett.*, 50:393–396, 1983.

[13] Steven A. Gottlieb et al. The deconfining phase transition and the continuum limit of lattice quantum chromodynamics. *Phys. Rev. Lett.*, 55:1958–1961, 1985.

[14] F. R. Brown, N. H. Christ, Y. F. Deng, M. S. Gao, and T. J. Woch. Nature of the deconfining phase transition in su(3) lattice gauge theory. *Phys. Rev. Lett.*, 61:2058–2061, 1988.

[15] M. Fukugita, M. Okawa, and A. Ukawa. Order of the deconfining phase transition in su(3) lattice gauge theory. *Phys. Rev. Lett.*, 63:1768–1771, 1989.

[16] M. A. Halasz, A. D. Jackson, R. E. Shrock, Misha A. Stephanov, and J. J. M. Verbaarschot. On the phase diagram of QCD. *Phys. Rev.*, D58:096007 [11 pages], 1998.

[17] Jurgen Berges and Krishna Rajagopal. Color superconductivity and chiral symmetry restoration at nonzero baryon density and temperature. *Nucl. Phys.*, B538:215–232, 1999.

[18] Bernd-Jochen Schaefer and Jochen Wambach. The phase diagram of the quark meson model. *Nucl. Phys.*, A757:479–492, 2005.

[19] T. Herpay, A. Patkos, Zs. Szep, and P. Szepfalusy. Mapping the boundary of the first order finite temperature restoration of chiral symmetry in the (m(pi) - m(k))-plane with a linear sigma model. *Phys. Rev.*, D71:125017 [15 pages], 2005.

➤ Physical point: Aoki et al (2006)

Rapid crossover! $T_c \sim 155$ MeV

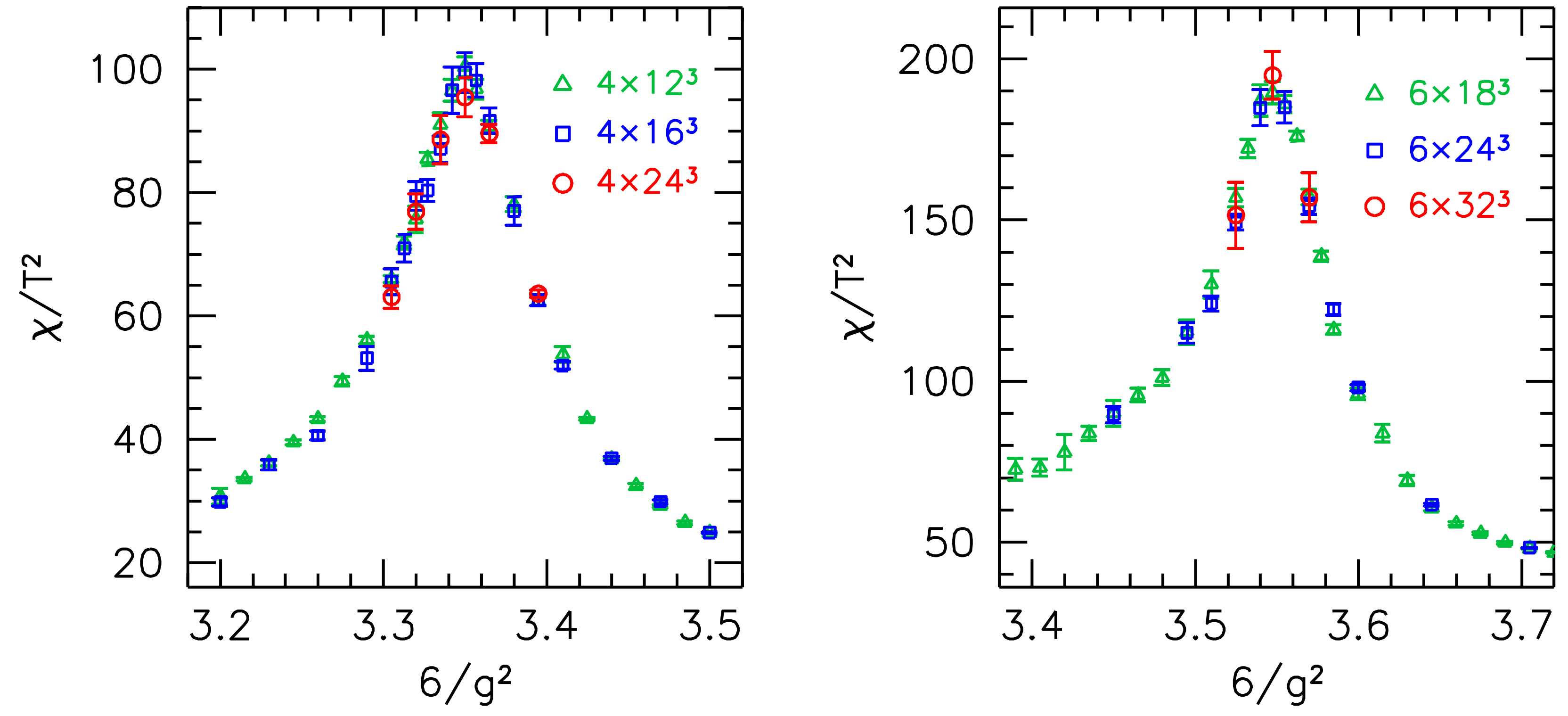


Figure 1: Susceptibilities for the light quarks for N_t=4 (left panel) and for N_t=6 (right panel) as a function of $6/g^2$, where g is the gauge coupling (T grows with $6/g^2$). The largest volume is eight times bigger than the smallest one, so a first-order phase transition would predict a susceptibility peak that is eight times higher (for a second-order phase transition the increase would be somewhat less, but still dramatic). Instead of such a significant change we do not observe any volume dependence. Error bars are s.e.m.

What about finite density?

Historical Theoretical View of the QCD CP

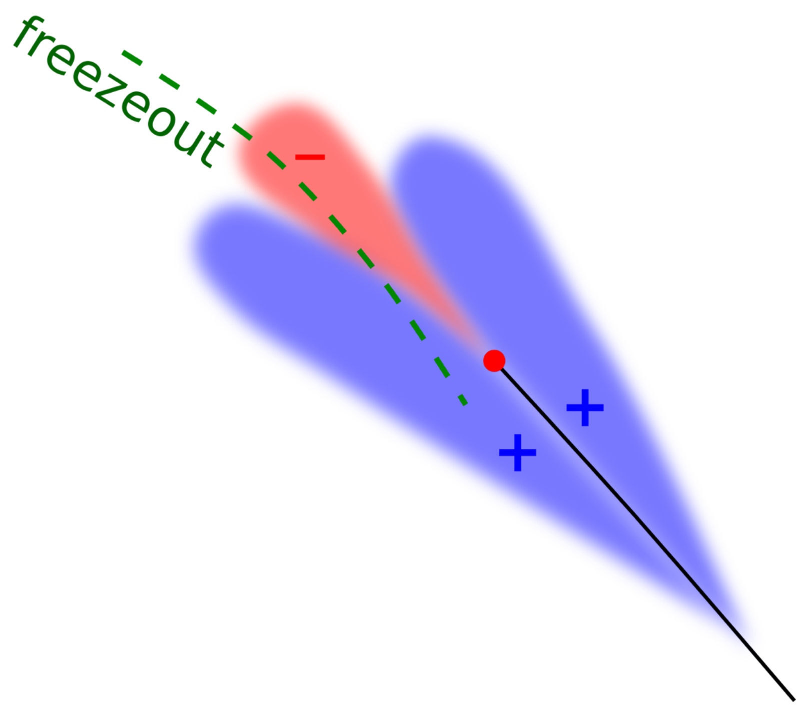
- Change in the order of the transition → critical point: enter universality classes
  - Static: 3D Ising - Rajagopal & Wilczek, Nucl.Phys.B (1993)
  - Dynamic: Model H - Son & Stephanov, Phys.Rev.D (2004)
  - Scaling equation of state of 3D Ising model - Guida & Zinn-Justin, Nucl.Phys.B 489 (1997) based Josephson-Schofield (1969) parametric equation of state

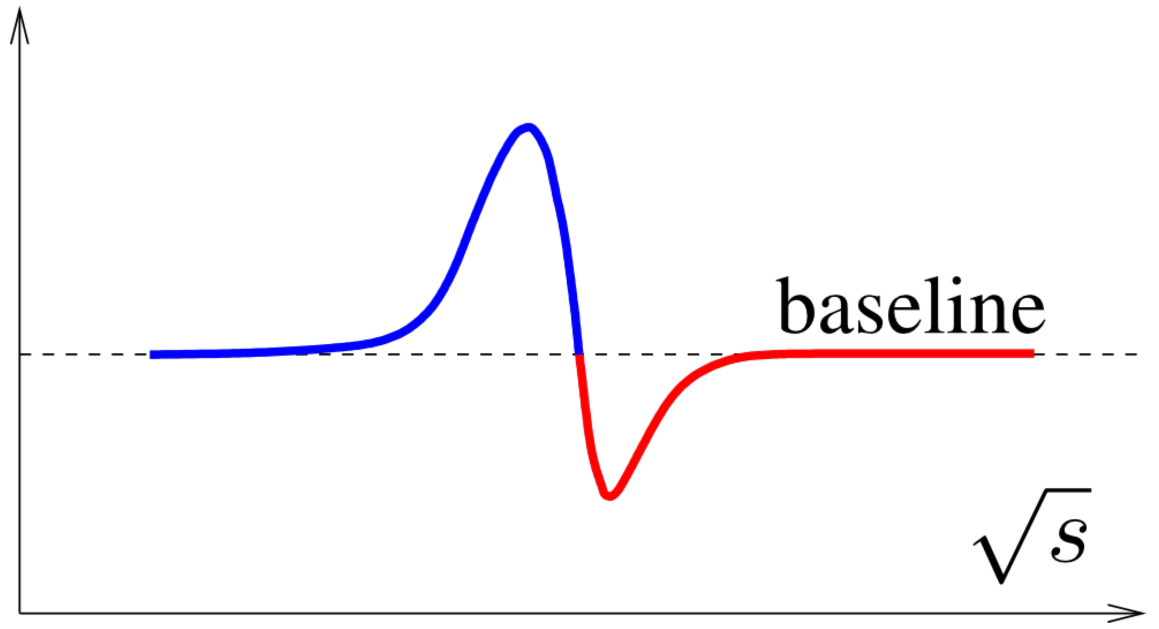
Exponent	Definition
α	$C \propto (T - T_c)^{-\alpha}$
β	$M \propto (T_c - T)^{\beta}$
γ	$\chi \propto (T - T_c)^{-\gamma}$
δ	$M \propto h^{1/\delta}$
ν	$\xi \propto (T - T_c)^{-\nu}$
η	$\Gamma(n) \propto \|n\|^{2-d-\eta}$

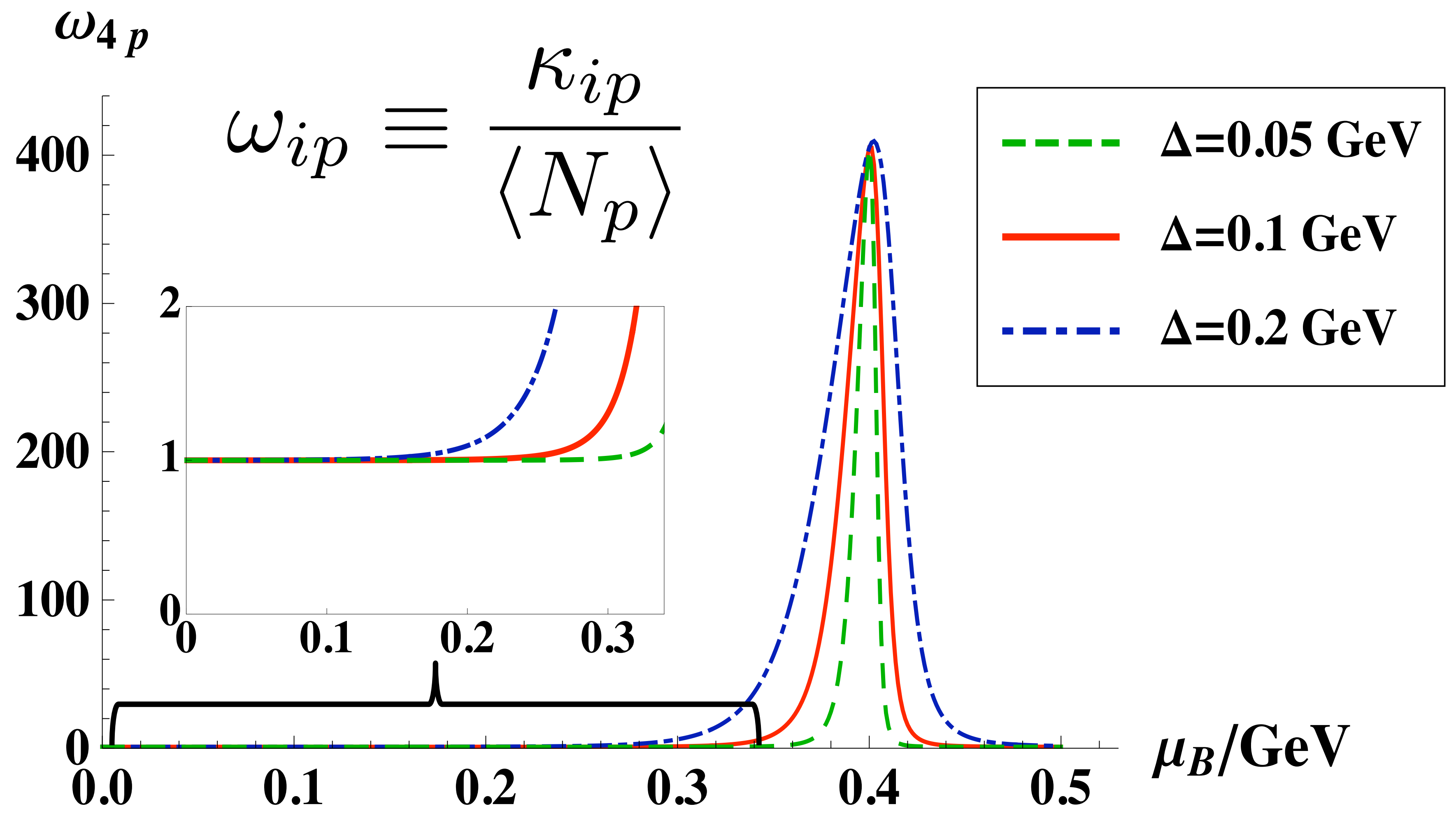
How to make use of universal EoS?

Historical Theoretical View of the QCD CP

- Fluctuations serve as critical signal (diverging ξ):
  - Higher order susceptibilities diverge with higher power of the correlation length, $\kappa_4 \propto \xi^7$
  - Susceptibilities are derivatives of EoS: $$\chi_n^B \equiv \frac{\partial^n (p/T^4)}{\partial(\mu_B/T)^n}$$
  - Relate baryon fluctuations to experimentally observable proton fluctuations





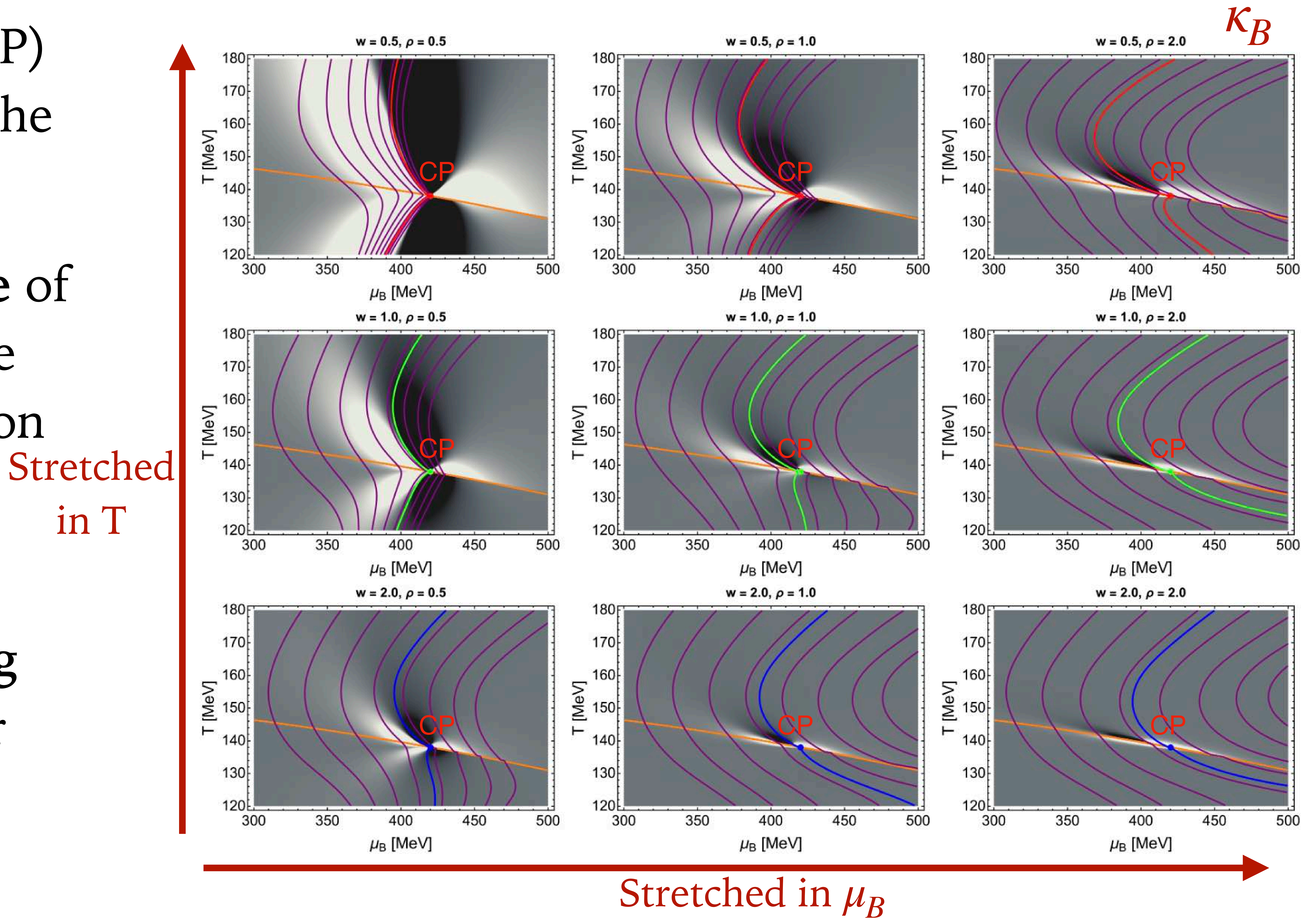


Realistic estimates from universal input?

M. Stephanov, K. Rajagopal and E. Shuryak, PRD (1999)
M. Stephanov, PRL (2009) & PRL (2011)
C. Athanasiou, K. Rajagopal, M. Stephanov, PRD (2010)

Kurtosis and Critical Lensing in Phase Diagram

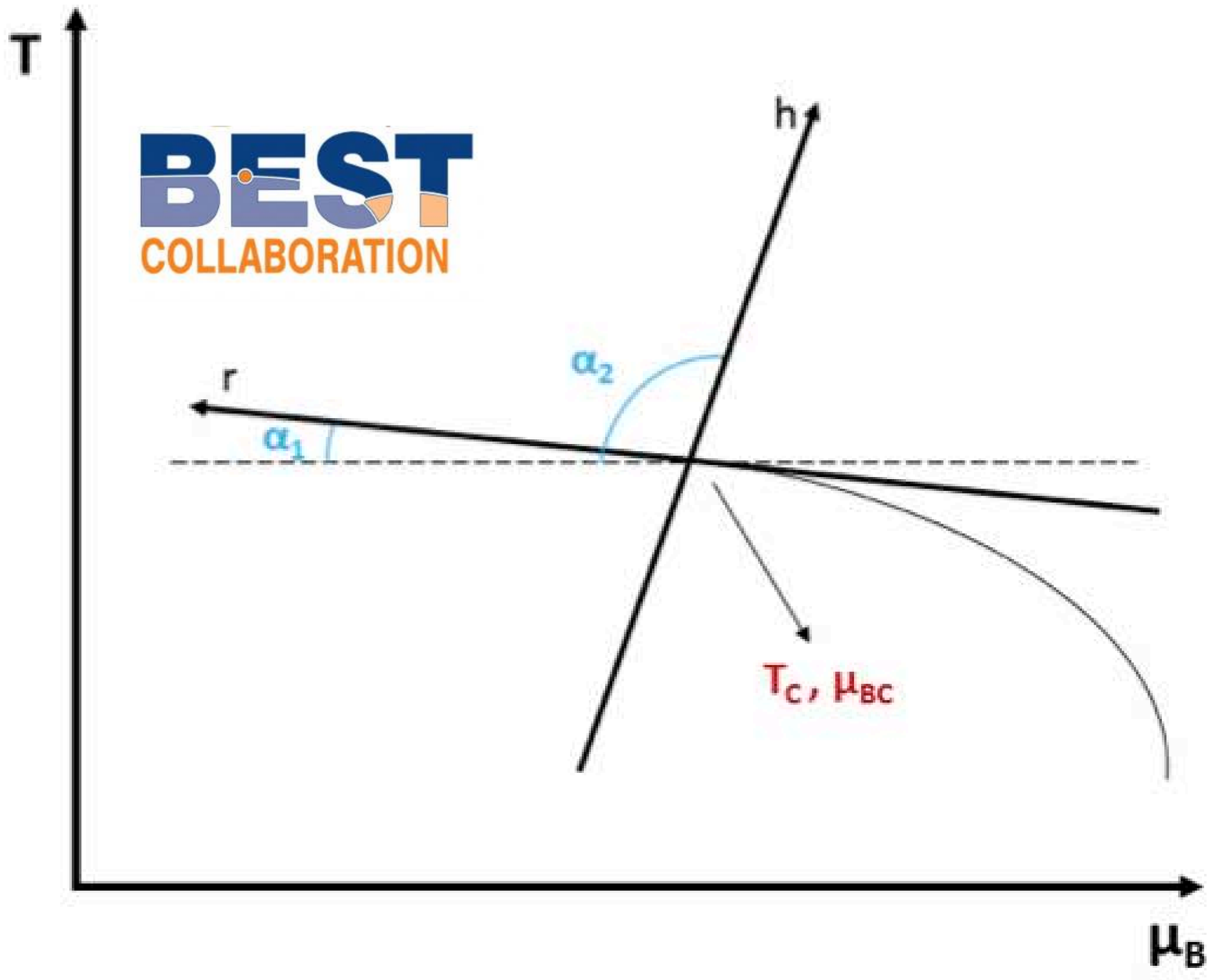
- Critical lensing: critical point (CP) is an attractor of trajectories in the QCD phase diagram
  - Study how the **size** and **shape** of the critical region affects these trajectories within the Equation of State with a critical point from the BEST COLLABORATION
- Critical regions **extending along the T-direction** show a stronger lensing effect



*T. Dore, **J.M. Karthein**, D. Mroczek et al, PRD (2022)*

Equation of State for QCD with a Critical Point

- Incorporate universal critical features into the QCD phase diagram from the 3D Ising Model equation of state via BEST framework



$$(\mathbf{r}, \mathbf{h}) \longleftrightarrow (\mathbf{T}, \mu_\mathbf{B}) : \quad \frac{T - \mathbf{T_C}}{\mathbf{T_C}} = \mathbf{w}\,(r\rho\,\sin\alpha_1 + h\,\sin\alpha_2)$$

$$\frac{\mu_B - \mu_\mathbf{BC}}{\mathbf{T_C}} = \mathbf{w}\,(-r\rho\,\cos\alpha_1 - h\,\cos\alpha_2)$$

- Reconstruct the pressure via Taylor expansion coefficients from Lattice QCD

$$T^4 c_n^{\mathrm{LAT}}(T) = T^4 c_n^{\mathrm{Non-Ising}}(T) + c_n^{\mathrm{Ising}}(T)$$

$$P(T, \mu_B) = T^4 \sum_n c_n^{\mathrm{Non-Ising}}(T) \left(\frac{\mu_B}{T}\right)^n + P_{\mathrm{crit}}^{\mathrm{QCD}}(T, \mu_B)$$

- Reduce free parameters by imposing constraints from Lattice

$$T = T_0 + \kappa T_0 \left(\frac{\mu_B}{T_0}\right)^2 + O(\mu_B^4), \qquad \alpha_1 = \tan^{-1}\left(2\frac{\kappa}{T_0}\mu_{BC}\right)$$

P. Parotto et al, PRC (2020),
***J.M. Karthein** et al, EPJ+ (2021)*

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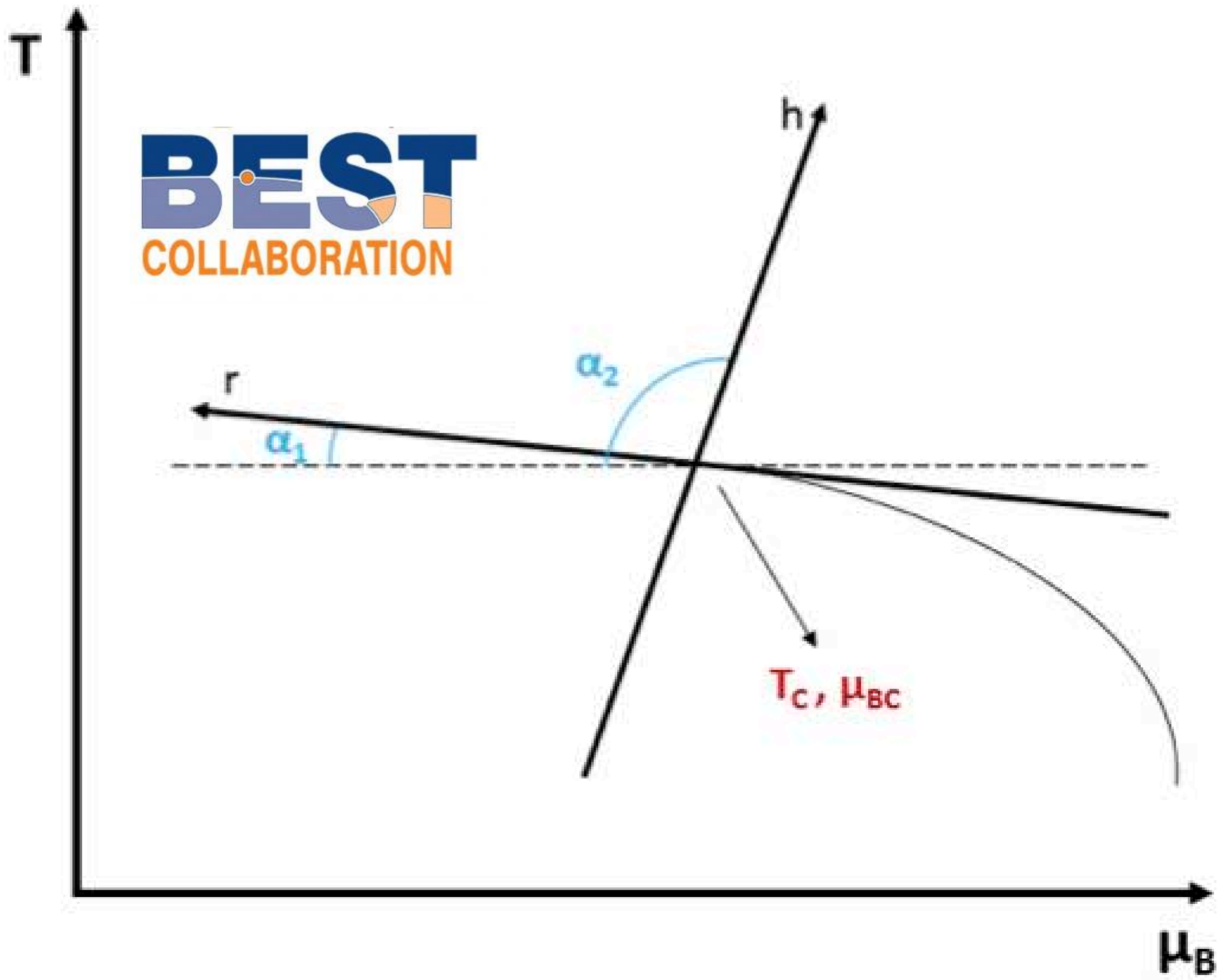

Utilize critical EoS mapped quadratically to QCD

$$(\mathbf{r},\mathbf{h}) \longleftrightarrow (\mathbf{T},\mu_\mathbf{B}): \quad h(\mu,T) = -\frac{\Delta T' \cos\alpha_1}{T_c w \, \sin(\alpha_1-\alpha_2)}$$

$$r(\mu,T) = -\frac{\mu^2-\mu_c^2}{2\mu_c T_c \, \rho w \, \cos\alpha_1} + \frac{\Delta T' \cos\alpha_2}{T_c \, \rho w \, \sin(\alpha_1-\alpha_2)}$$

M. Kahangirwe et al, PRD (2024)

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***J.M. Karthein** et al, EPJ+ (2021)*

Quantifying Fluctuation Signatures

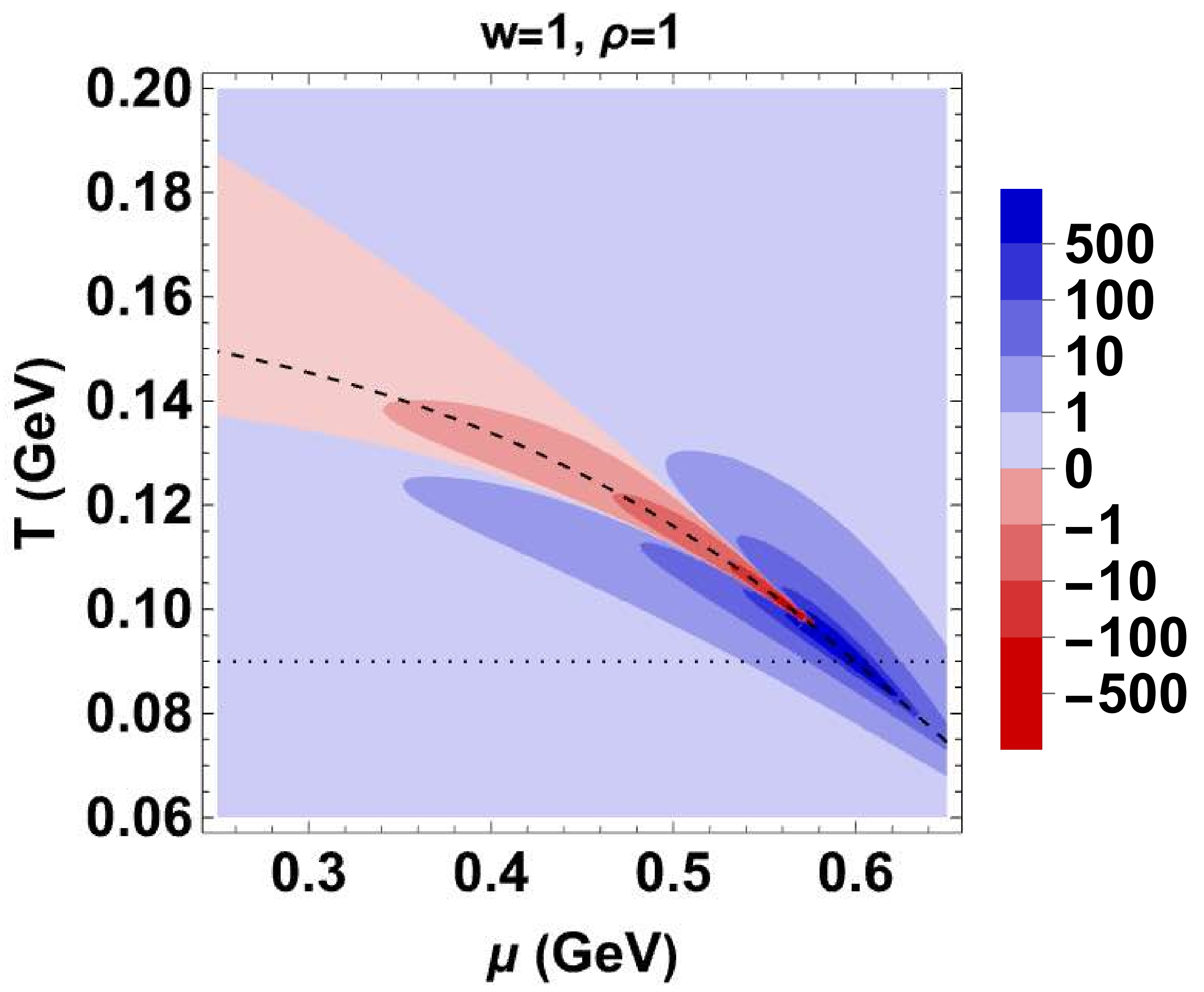
- Utilize the maximum entropy freeze-out procedure to calculate proton fluctuations due to critical point & study the influence of the unknown EoS parameters
  - Determine the particle fluctuations (G) from only the input of the EoS and by matching to the hydrodynamic description (H)

EoS input (here k=4): $\Delta H_{kn} \equiv \langle \delta n^k \rangle$

$$\widehat{\Delta} G_{A_1 \ldots A_k} = \widehat{\Delta} H_{a_1 a_2 \ldots a_n} P^{a_1}_{A_1} P^{a_2}_{A_2} \ldots P^{a_n}_{A_k}$$

$\widehat{\Delta}$: critical contribution (subtract background HRG EoS) to irreducible relative cumulants (subtract lower order cumulants)

$P^{a_n}_{A_k}$: contribution of particle A to conserved quantity a in the hydrodynamic cell



$\mu_c = 600\,\text{MeV}, \alpha_2 = 0^{\text{o}}\,(\alpha_1 = 16.6^{\text{o}}, \text{T}_\text{c} = 89\,\text{MeV})$

J.M. Karthein, K. Rajagopal, M. Pradeep, M. Stephanov, Y. Yin, arXiv:2508.19237

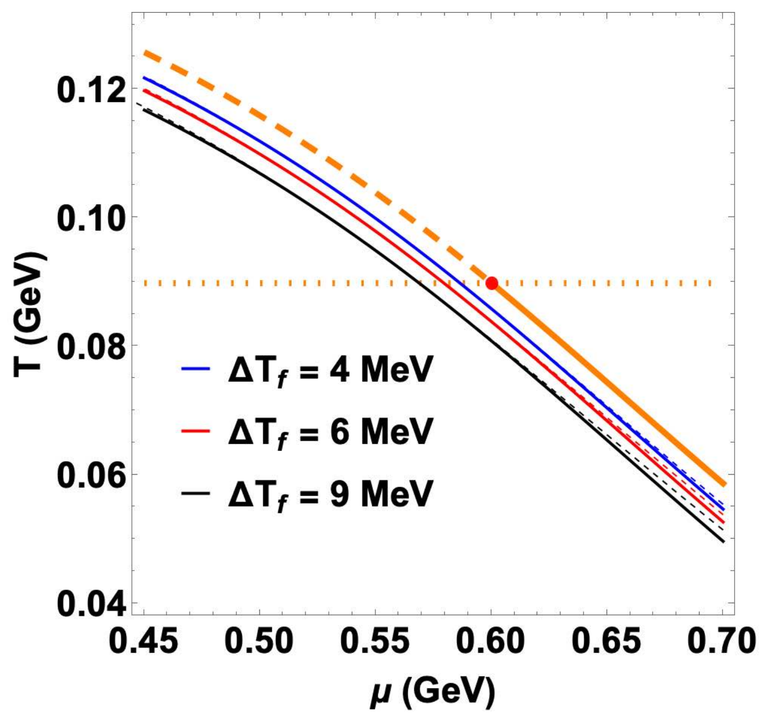
Quantifying Fluctuation Signatures

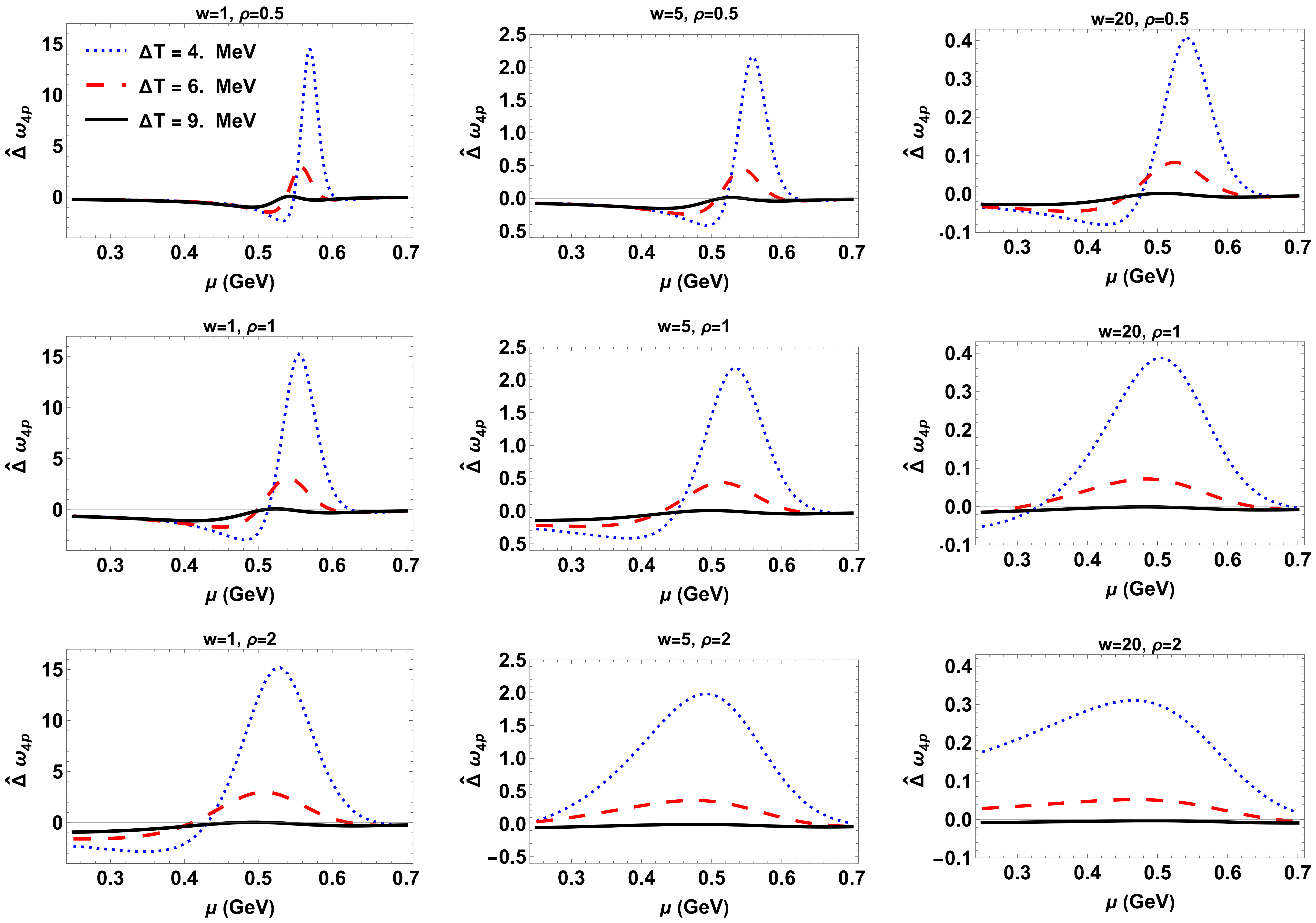
- Utilize the maximum entropy freeze-out procedure to calculate proton fluctuations due to critical point & study the influence of the unknown EoS parameters

Normalized proton factorial cumulants:

$$\widehat{\Delta}\omega_{kp} = \frac{\widehat{\Delta}H_{a_1\ldots a_k}P_p^{a_1}\ldots P_p^{a_k}}{\langle N_p\rangle} = \frac{\kappa_{4p}}{\kappa_{1p}}$$

Along freeze-out curves:





Increasing ρ increases peak width

J.M. Karthein, K. Rajagopal, M. Pradeep, M. Stephanov, Y. Yin, arXiv:2508.19237

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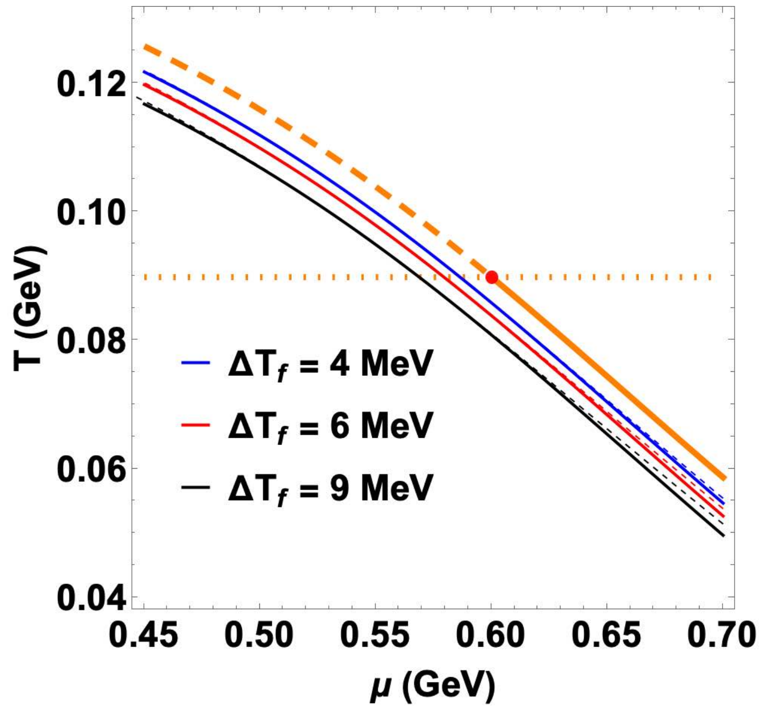
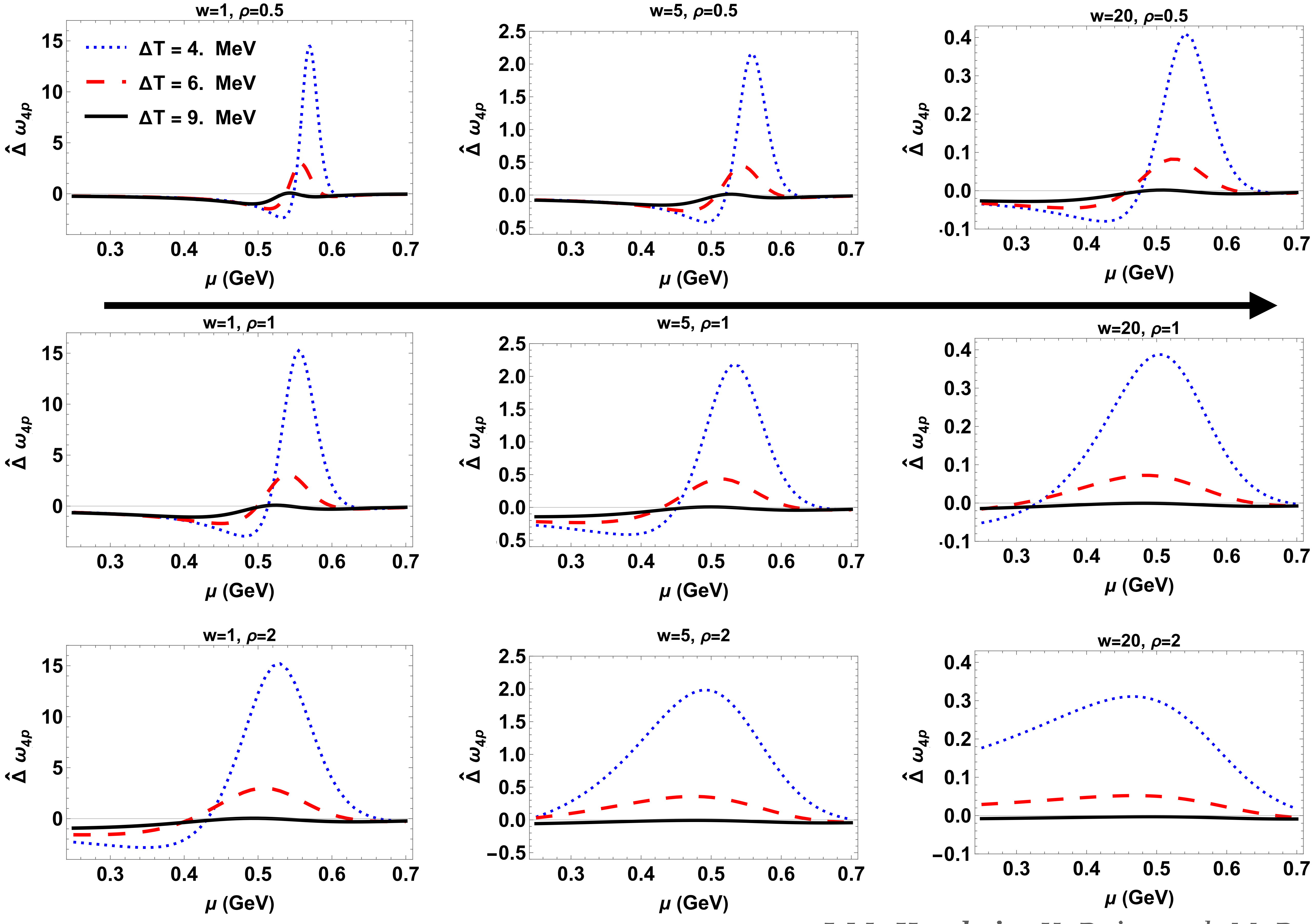


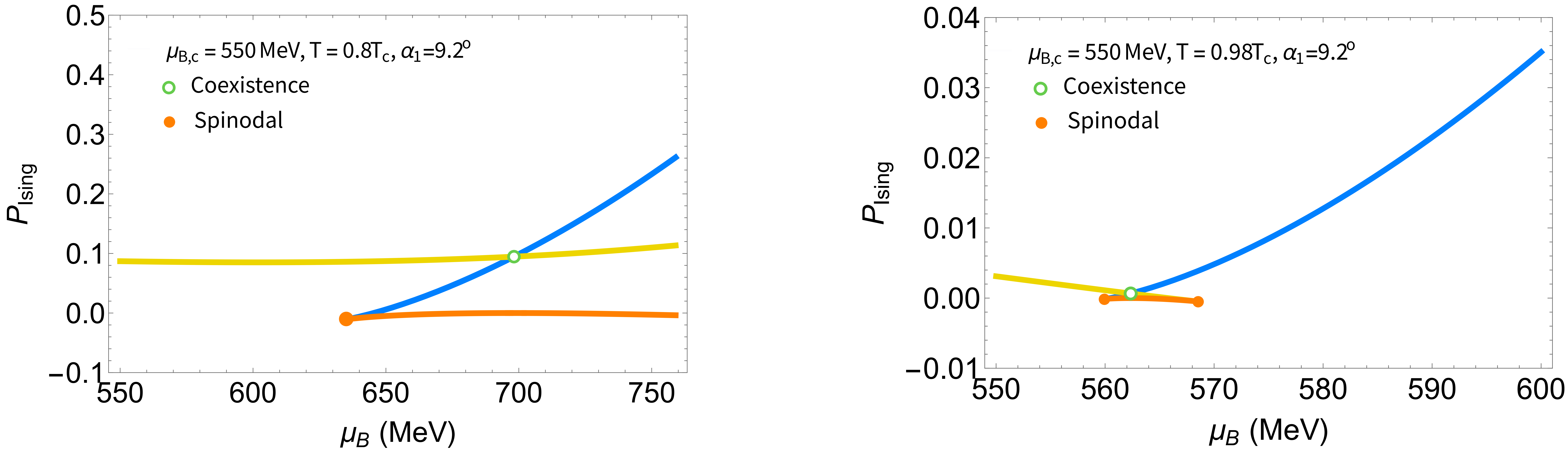


Increasing w reduces peak height

J.M. Karthein, K. Rajagopal, M. Pradeep, M. Stephanov, Y. Yin, arXiv:2508.19237

EoS for First Order Regime

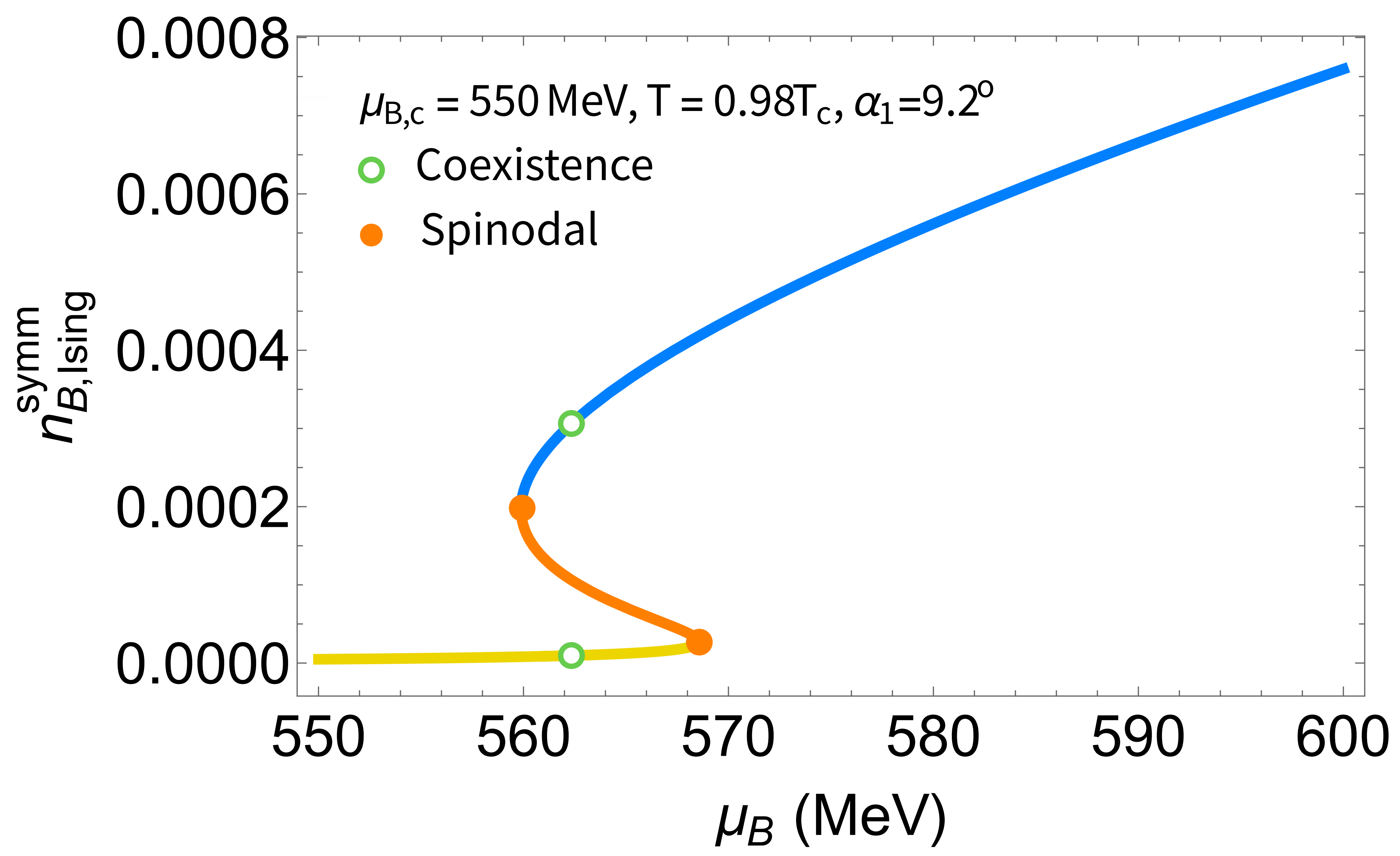
- Considering a mean field Ising model mapped to QCD, we can implement first order features in the phase diagram: isotherms show coexistence and spinodal points

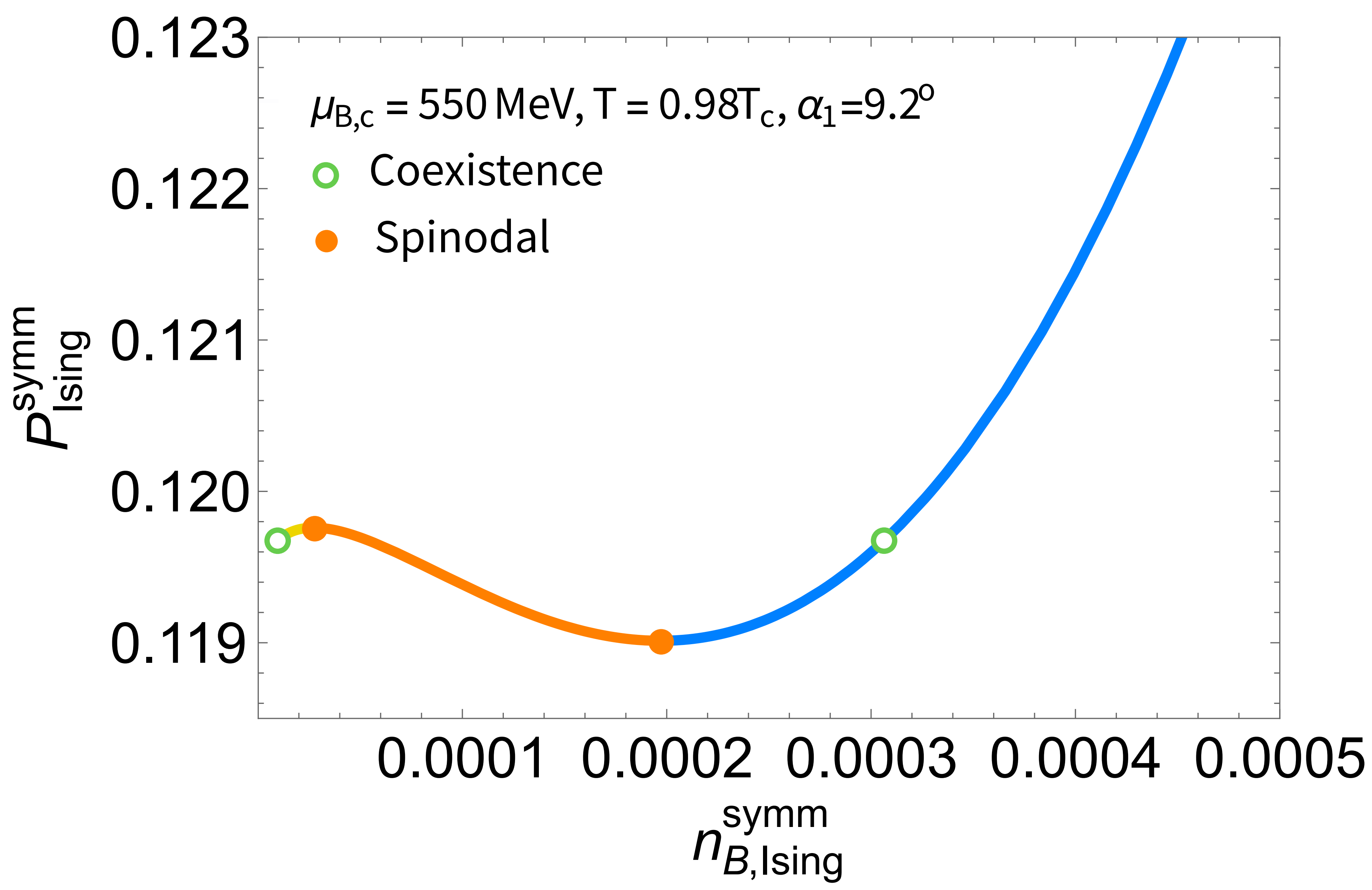


***J.M. Karthein**, V. Koch, C. Ratti, PRD (2025)*

EoS for First Order Regime

- With Landau theory, we find expected spinodal features in n_B isotherms unlike with 3D Ising where spinodals are Lee-Yang edge singularities in complex plane
- Mapping parameters including α_1, w, ρ control the shape of critical region and first order features

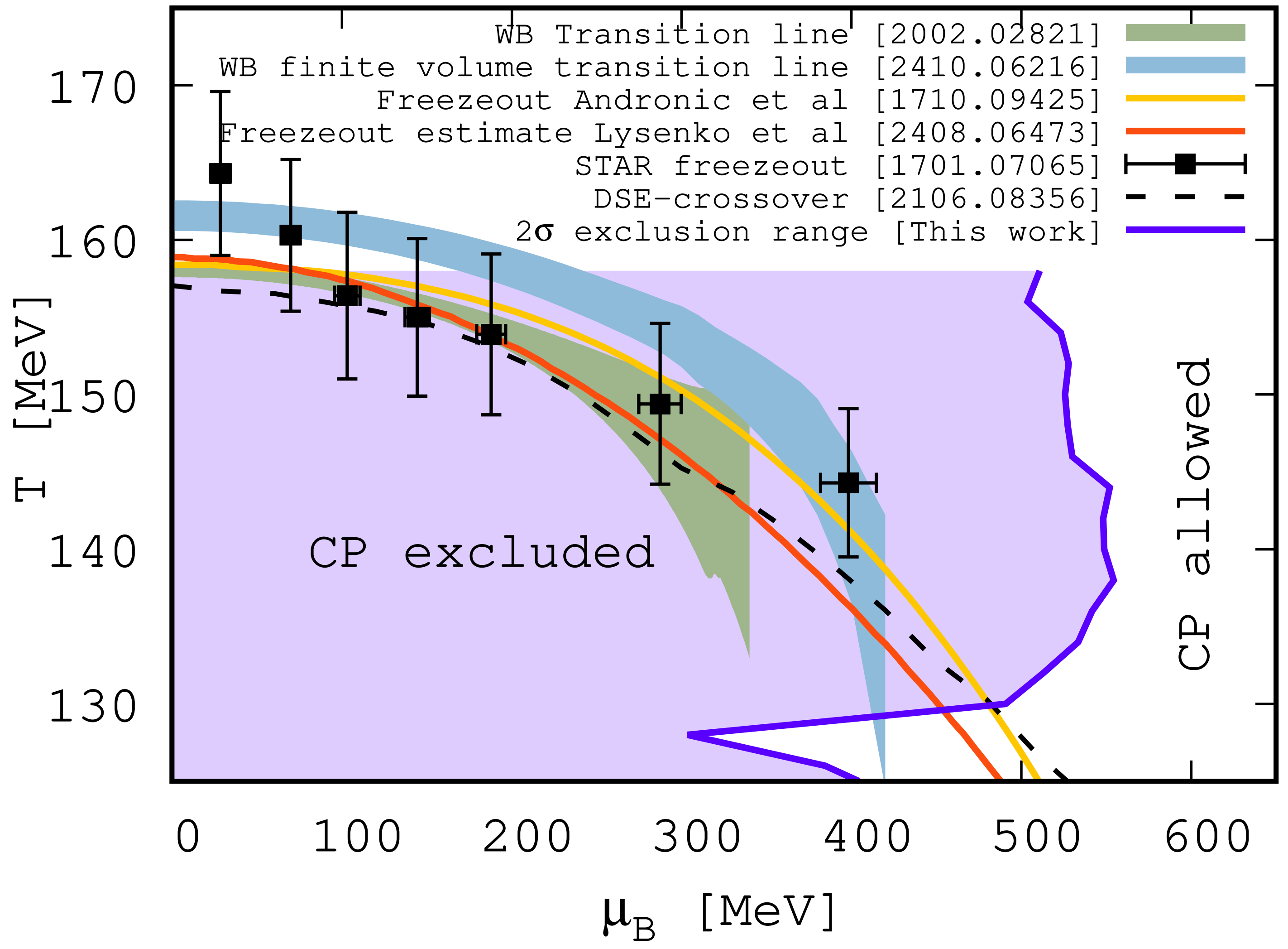


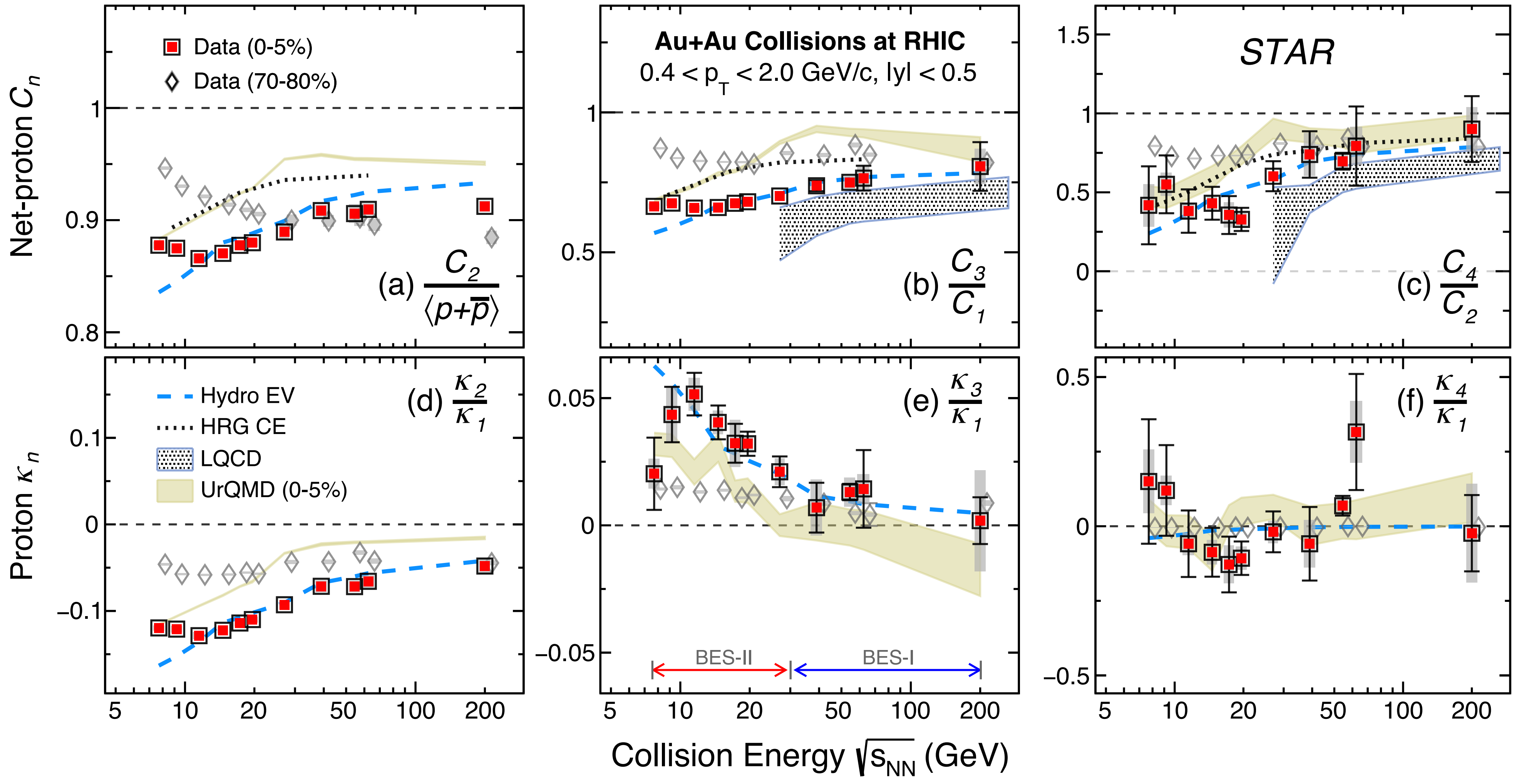


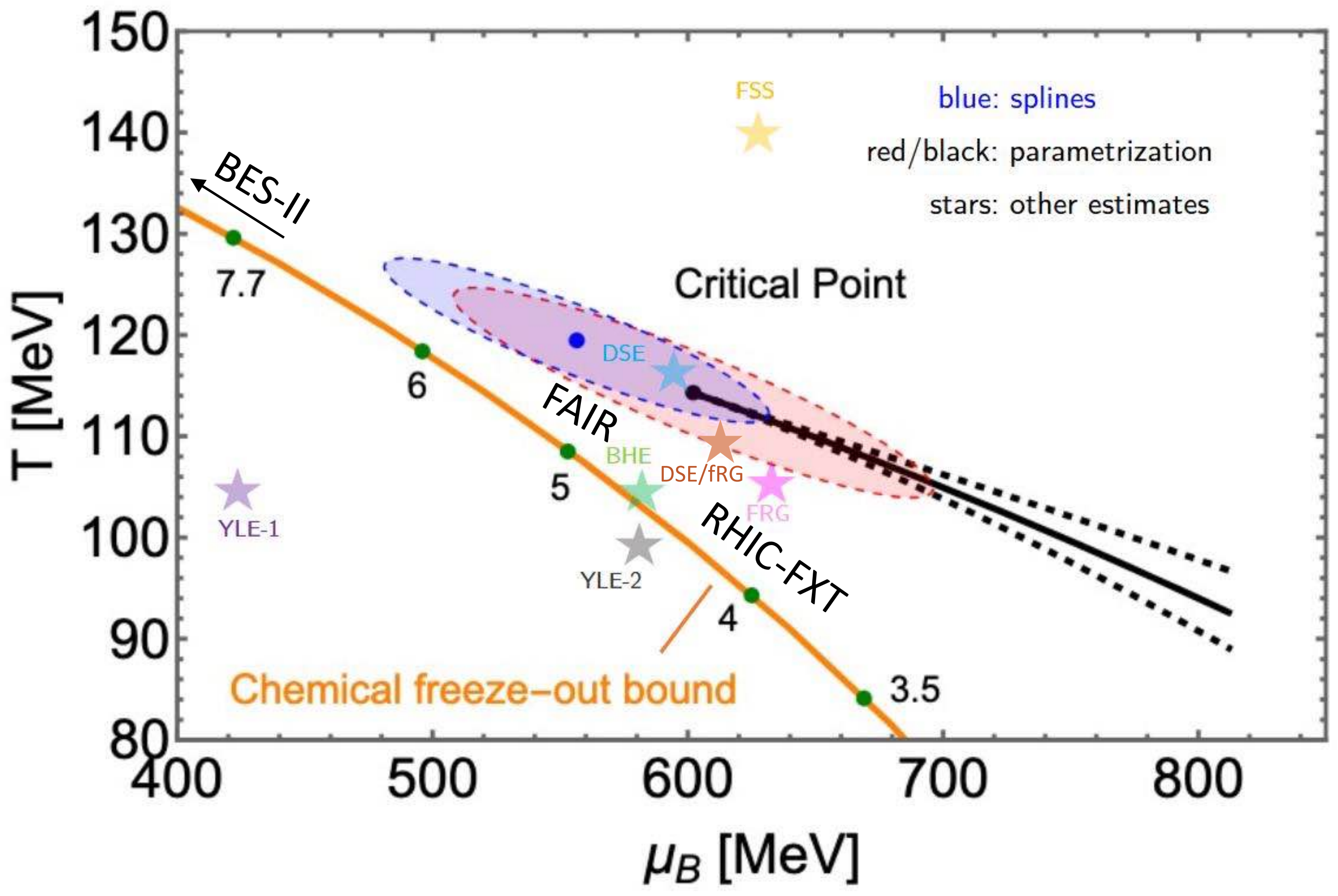
J.M. Karthein, V. Koch, C. Ratti, PRD (2025)

State of the Critical Point

- Lattice QCD limits ($\mu_B \gtrsim 450$ MeV) agree well with experiment ($\mu_B \gtrsim 420$ MeV)
- Theory estimates: $T_c \sim 100$ MeV and $\mu_B \sim 600$ MeV







YLE-1: D.A. Clarke et al, arXiv:2405.10196

YLE-2: G. Basar, PRC 110, 015203 (2024)

BHE: M. Hippert et al., PRD 110, 094006 (2024)

FRG: W-J. Fu et al., PRD 101, 054032 (2020)

DSE: P.J. Gunkel et al., PRD 104, 052202 (2021)

DSE/fRG: Gao, Pawlowski., PLB 820, 136584 (2021)

FSS: A. Sorensen et al., arXiv:2405.10278

Many groups/methods converging on expectations for critical point location!

STAR collaboration, arXiv:2504.00817
S. Borsanyi et al, arXiv:2502.10267
H. Shah et al, arXiv:2410.16206

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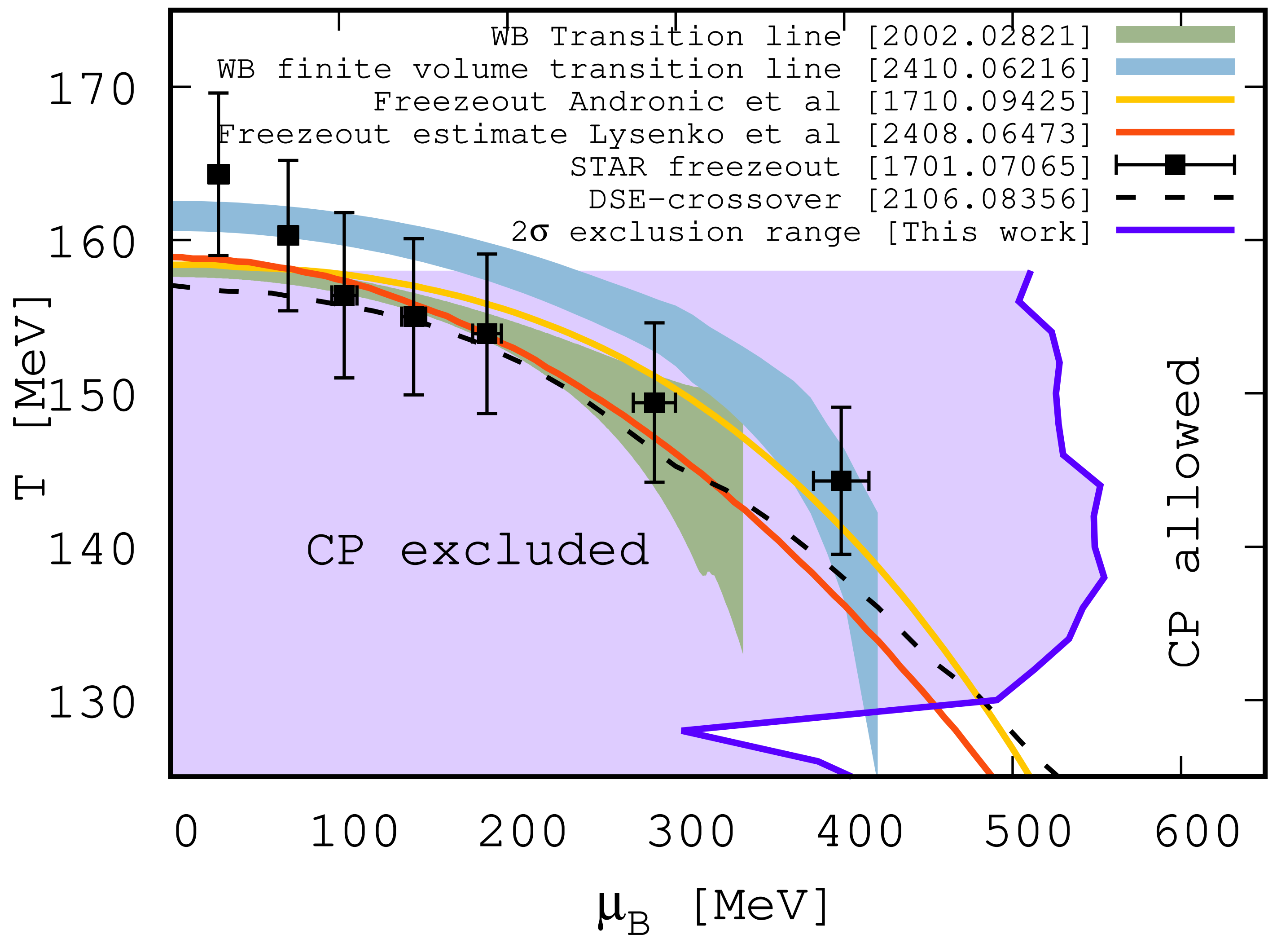

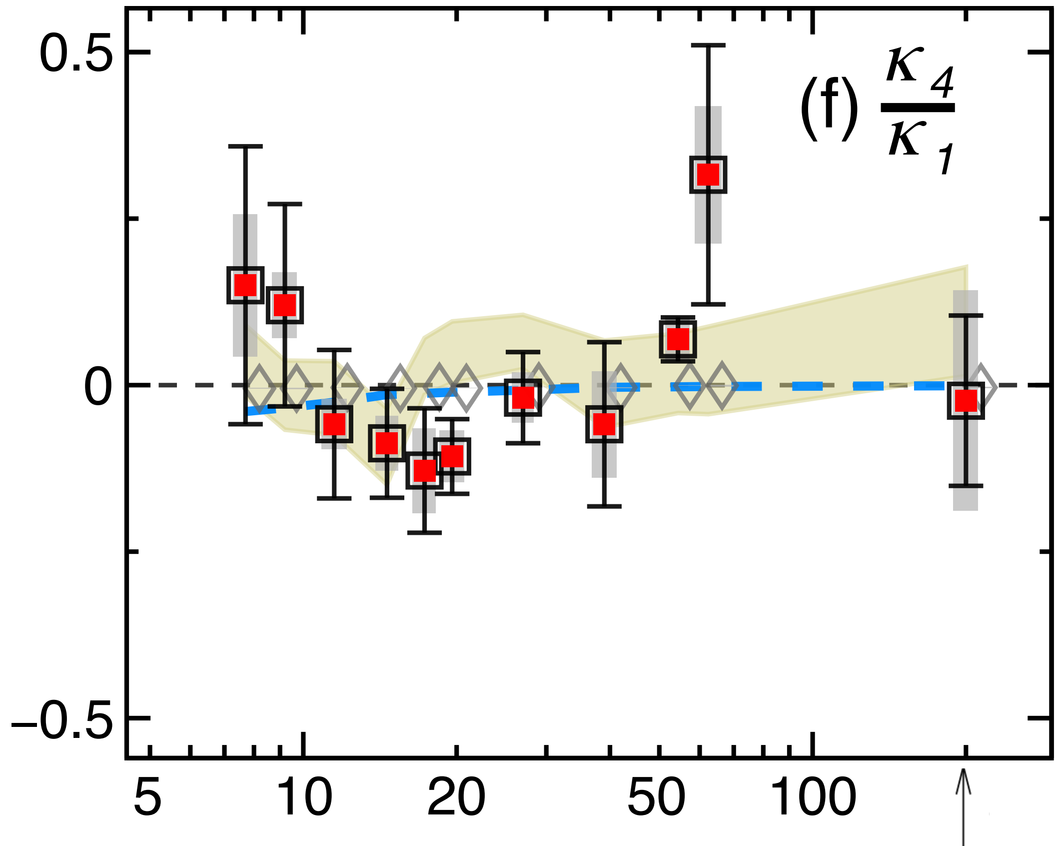

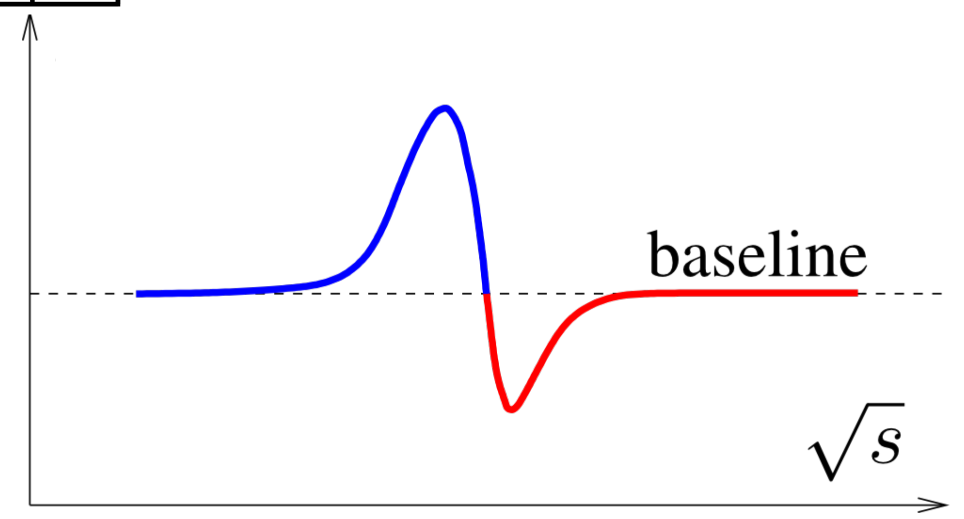

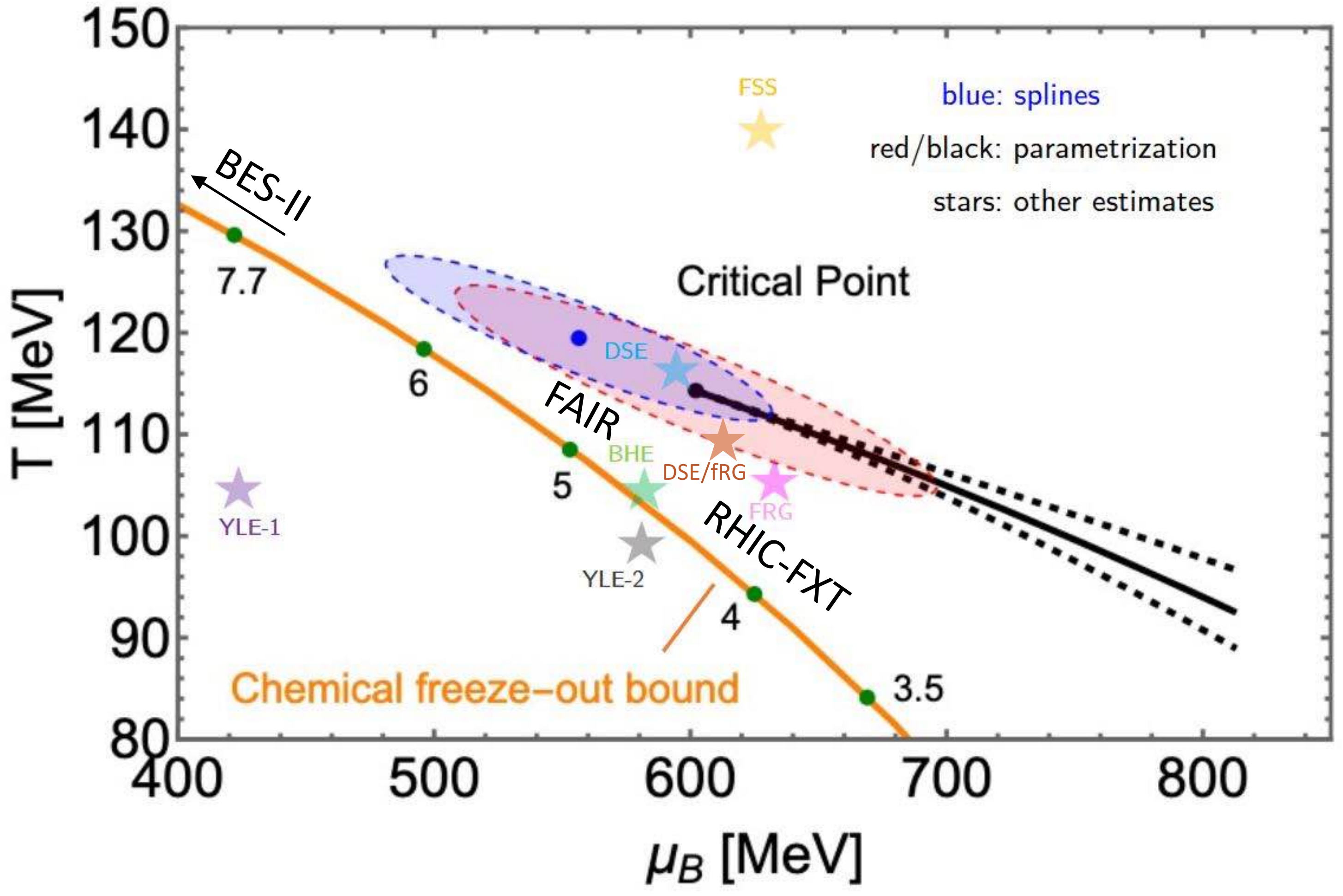

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