Flow and hyperon polarization from 3-fluid dynamical model `MUFFIN`

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Jakub Cimerman, IK, Boris Tomasik, Pasi Huovinen, Phys.Rev. C **107** (2023) 4, 044902 [2301.11894 [nucl-th]]
plus some new results I've generated last week

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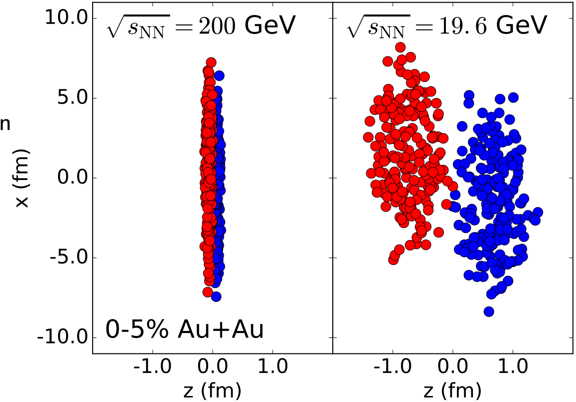
Motivation

1) When simulating heavy-ion collisions at lower energies, the paradigm of "thin pancakes" gradually loses its applicability.

- Initial state: thick pancakes
  - boost ivariance is not a good approximation → need for 3 dimensional initial state
  - previous-gen IP-Glasma, EKRT are formulated for mid-rapidity (but there is development of 3D IP-Glasma and EKRT)
- Nonzero baryon and electric charge densities

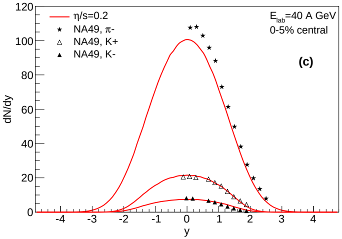


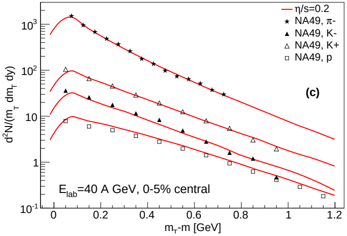
picture credit: C. Shen, B. Schenke, Phys. Rev. C 97, 024907 (2018)

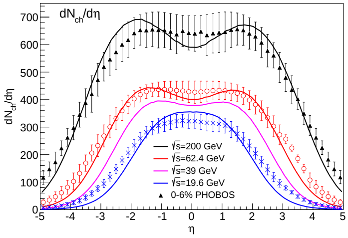
Our first shot at RHIC BES: `UrQMD + vHLLE + UrQMD`

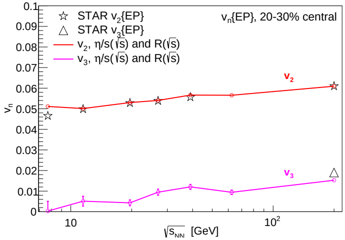
IK, Huovinen, Petersen, Bleicher, Phys.Rev. C91 (2015) no.6, 064901

- UrQMD initial state:
  - 3D, non-boost-invariant
  - baryon + electric charges
- 3D viscous fluid dynamics (vHLLE)
- Monte Carlo Cooper-Frye at fixed $\varepsilon = 0.5$ GeV/fm^3
- UrQMD final state
- $\Rightarrow$ decent agreement with the mix of RHIC BES + NA49 + PHOBOS data





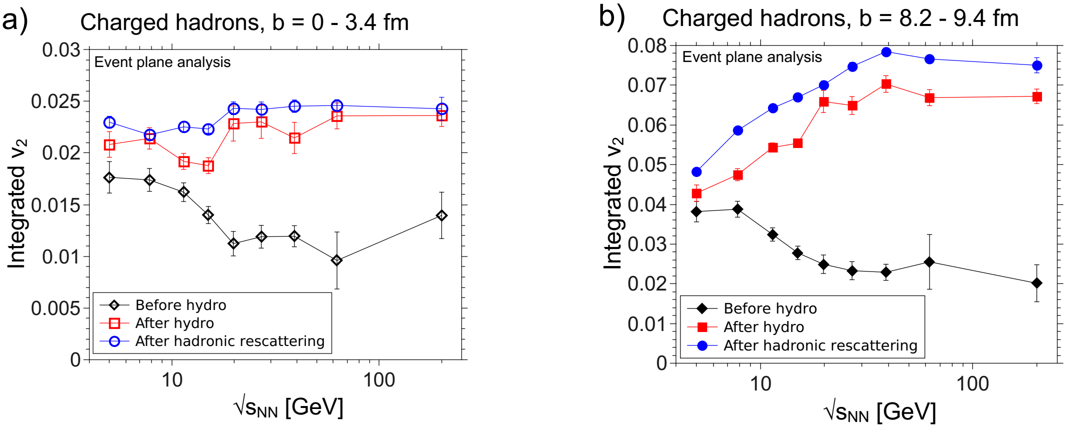




From a parallel development of hybrid UrQMD by Jussi Auvinen:

2) A lot of evolution is happening before the nuclei have completely passed through each other

UrQMD IS + ideal hydro + UrQMD afterburner, J. Auvinen, H. Petersen, Phys.Rev.C 88:064908,2013



One must start hydro description early!

Multi-fluid dynamics

Hydrodynamic description starts from the very beginning of the collision.

Difficulty: reasonability of fluid description at the very start of heavy ion collision?

Dynamical fluidization (1 fluid)

Regions of fluid phase are created dynamically, where (and when) the density is large enough.

Difficulty: how to treat non-fluid and fluid phase together (in the intial state)?

Equations of motion in multi-fluid dynamics

The incoming nuclei are represented by two blobs of cold baryon-rich fluids: projectile (p) and target (t) fluids. As the fluids inter-penetrate each other, local friction forces start to develop. The kinetic energy lost to friction is channeled into creation of a third fluid (f). The third, or fireball, fluid vaguely correspond to mesons and baryons+anti-baryons produced in the reaction.

$$\begin{aligned}
\partial_\mu T^{\mu\nu}_{\mathrm{p}}(x) &= -F^{\nu}_{\mathrm{p}}(x) + F^{\nu}_{\mathrm{fp}}(x), \\
\partial_\mu T^{\mu\nu}_{\mathrm{t}}(x) &= -F^{\nu}_{\mathrm{t}}(x) + F^{\nu}_{\mathrm{ft}}(x), \\
\partial_\mu T^{\mu\nu}_{\mathrm{f}}(x) &= F^{\nu}_{\mathrm{p}}(x) + F^{\nu}_{\mathrm{t}}(x) - F^{\nu}_{\mathrm{fp}}(x) - F^{\nu}_{\mathrm{ft}}(x),
\end{aligned}$$

The total energy of all 3 fluids is conserved:

$$\partial_\mu \left[T^{\mu\nu}_{p}(x) + T^{\mu\nu}_{t}(x) + T^{\mu\nu}_{f}(x) \right] = 0.$$

the friction terms are F^{μ}_{p} and F^{μ}_{t} for projectile-target friction acting on p- and t-fuids, respectively, and F^{μ}_{fp}, F^{μ}_{ft} for projectile-fireball and target-fireball friction.

Following an assumption from the reference(s) on the next slide, **there is no transfer of conserved charge between the fluids**.

Friction terms

Projectile-target friction [Ivanov, Russkikh, Toneev, Phys.Rev.C 73 (2006) 044904]:
Derived based on average energy-momentum transfer in NN scattering [L.M. Satarov, Sov. J. Nucl. Phys. 52, 264 (1990)]

$$F_\alpha^\nu = \vartheta^2 \rho_p^\xi \rho_t^\xi m_N V_{\mathrm{rel}}^{pt} [(u_\alpha^\nu - u_{\bar\alpha}^\nu)\sigma_{\mathrm{P}}(s_{pt}) + (u_p^\nu + u_t^\nu)\sigma_{\mathrm{E}}(s_{pt})]$$

where:

- ϑ^2 is a unification factor - which suppresses the friction further when the fluids slow down with respect to each other,
- ρ_p^ξ, ρ_t^ξ are generalised densities of constituents in the projectile and target fluids,
- V_{rel}^{pt} is a relative velocity of the $p-$ and $t-$ fluid cells,
- m_N is nucleon mass,
- u_α , $\alpha = p, t$, $\bar\alpha = t, p$ are 4-velocities of the fluid cells,
- $\sigma_{\mathrm{P}}, \sigma_{\mathrm{E}}$ are cross-sections for momentum and energy transfer, respectively.

Friction terms (2)

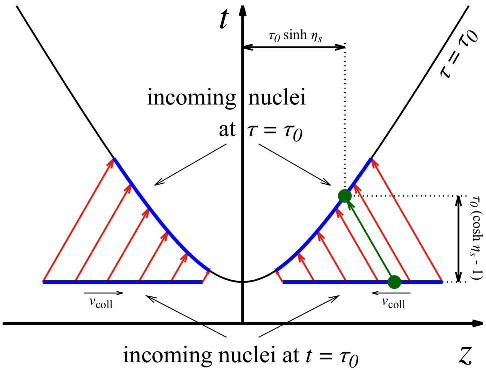
Fireball-projectile/target friction [same reference]:

$$F^{\nu}_{f\alpha} = \rho^b_\alpha \xi_{f\alpha}(s_{f\alpha}) V^{f\alpha}_{\mathrm{rel}} \frac{T^{0\nu}_{f(eq)}}{u^0_f} \sigma^{N\pi\to R}_{\mathrm{tot}}(s_{f\alpha}),$$

where:

- ρ^b_p, ρ^b_t are baryon densities of of the projectile and target fluids,
- $V^{f\alpha}_{\mathrm{rel}}$ is a relative velocity of the fireball and baryon-rich fluid cells,
- $T^{0\nu}_{f(eq)}$ is energy-momentum tensor of the fireball fluid,
- $\sigma^{N\pi\to R}_{\mathrm{tot}}$ is a pion-nucleon cross-section.
- $\xi_{f\alpha}$ is a "K-factor" (a fitting factor) for the friction term, which is intended to compensate for all the missing/incorrect physics therein, and to lead to better agreement with the data

Coordinate frame and initial state



- Nucleons from the incoming nuclei are sampled at $t = t_0$ surface (fixed Cartesian time).
- The nucleons are then propagated according to free-flying trajectories onto $\tau = \tau_0$ hypersurface.
- The nucleons are then melted into the fluids: their energies and momenta are distributed to nearby fluid cells using a smearing kernel:

$$T^{0\mu}(x_{\rm cell}, y_{\rm cell}, \eta_{\rm cell}) = \sum_{i \in \rm nucleons} p_i^\mu K(\Delta x, \Delta y, \Delta\eta_s)$$

$$N^0_{b,q}(x_{\rm cell}, y_{\rm cell}, \eta_{\rm cell}) = \sum_{i \in \rm nucleons} \{B_i, Q_i\} K(\Delta x, \Delta y, \Delta\eta_s)$$

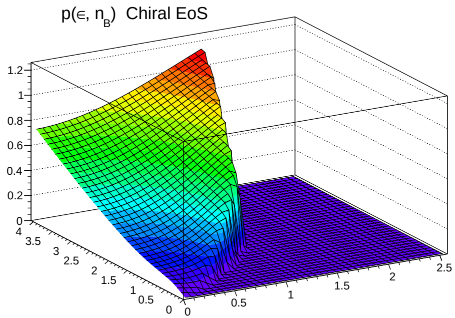
with a smearing kernel: $K(\Delta x, \Delta y, \Delta\eta_s) = A \exp\left(-\frac{\Delta x^2 + \Delta y^2 + \Delta\eta_s^2 \tau^2 \cosh^2\eta_s \cosh^2 y}{2\sigma^2}\right)$

Equations of state in the fluid stage

Chiral model

J. Steinheimer, et al, J. Phys. G 38, 035001 (2011)

- good agreement with lattice QCD at $\mu_B = 0$
- crossover type PT between confined and deconfined phases at all μ_B

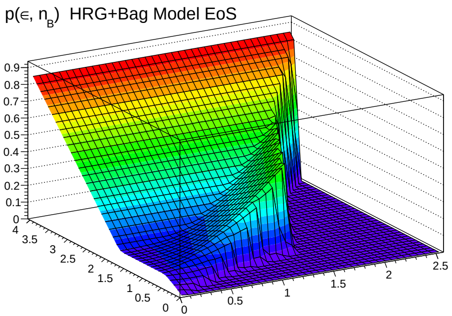


Hadron resonance gas + Bag Model

P.F. Kolb, et al, Phys.Rev. C 62, 054909 (2000)

(a.k.a. `EoS Q`)

- hadron resonance gas made of u, d quarks including repulsive meanfield
- Maxwell construction resulting in 1^{st} order PT



The core part: `vHLLE`

`https://github.com/yukarpenko/vhlle`
Comput. Phys. Commun. 185 (2014), 3016 [arXiv:1312.4160]
(this reference paper is outdated!)

✓ shear and bulk viscosity in "Israel-Stewart" with cross-terms

✓ $\tau - \eta$ (hyperbolic), as well as Cartesian coordinate frames (separate branches of the code)

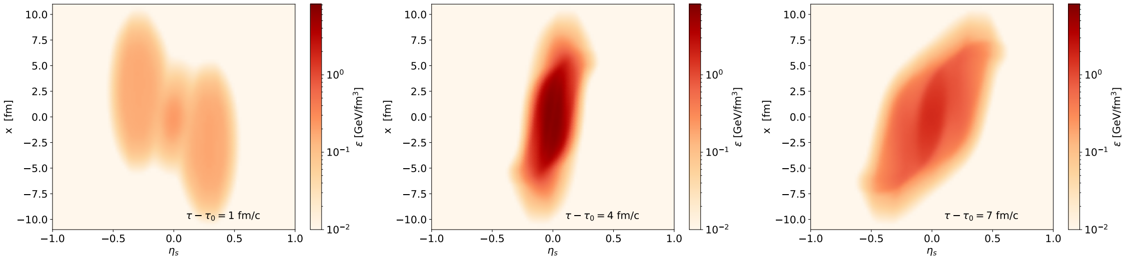
✓ grid resize to optimize CPU time

✓ several initial state, EoS modules. All realized via classes ⇒ easy to plug in new IS/EoS

✓ multi-fluid evolution added with very little overhead ⇒ see a fork by Jakub Cimerman

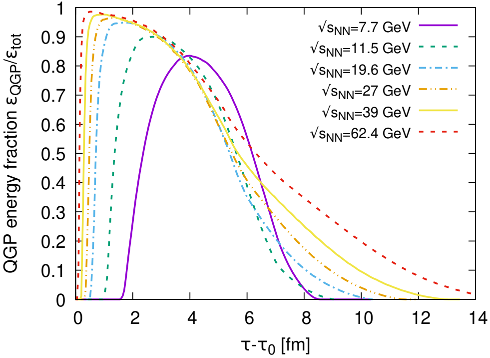
✓ using vHLLE as a library: possible (WIP)

Snapshots of multi-fluid evolution in x-η_s plane, Au-Au collision at $\sqrt{s_{\rm NN}} = 7.7$ GeV

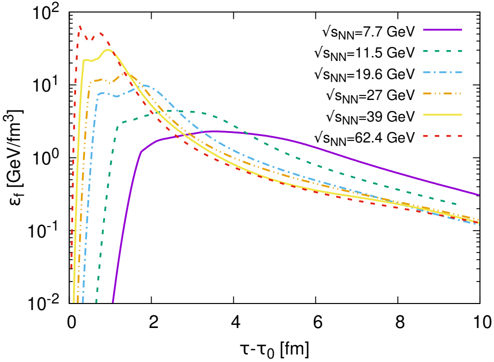


When do we witness QGP creation in MUFFIN?

QGP fraction as a function of time at different $\sqrt{s_{\mathrm{NN}}}$:



Energy density in central cell of fireball fluid as a function of time at different $\sqrt{s_{\mathrm{NN}}}$:

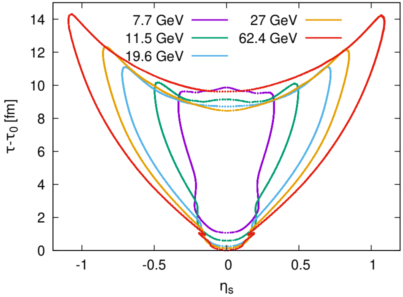


Significant fraction of medium in QGP phase exists down to $\sqrt{s_{\mathrm{NN}}} = 7.7$ GeV.

Fluid-to-particle transition (particlization) part 1

- Diagonalize $T_p^{\mu\nu}(x) + T_t^{\mu\nu}(x) + T_f^{\mu\nu}(x)$

  $\Rightarrow$ extract energy density $\varepsilon_{\rm sw}$
- construct a hypersurface of fixed $\varepsilon_{\rm sw} = 0.5$ GeV/fm^3 using `CORNELIUS`.



- On such hypersurface, $\int d\Sigma_\mu T^{0\mu} = 0$ (Gauss theorem)

  $\Rightarrow$ we use it to check the accuracy of the simulations

Particlization part 2

- Exclude parts of hypersurface which corresponds to matter flowing in:

$$\mathrm{d}\Sigma^\mu \mathrm{d}\Sigma_\mu > 0 \quad \text{and} \quad \mathrm{d}\Sigma_0 < 0,$$

$$\mathrm{d}\Sigma^\mu \mathrm{d}\Sigma_\mu < 0 \quad \text{and} \quad \mathrm{d}\Sigma_\mu T^{\mu 0} < 0$$

- distribution function on the particlization surface:

$$f(x,p) = f_p(x,p) + f_t(x,p) + f_f(x,p)$$

- Hadron sampling according to Cooper-Frye, using `SMASH-hadron-sampler`:

$$N = \int \frac{\mathrm{d}^3 p}{E_p} \int \mathrm{d}\Sigma_\mu(x) p^\mu f(p, T(x), \mu_i(x))$$

(grand canonical sampling with $T(x)$, $\mu_i(x)$)

- Sampled hadrons +spectator nucleons

$$\Downarrow$$

`SMASH` for rescatterings and resonance decays

Centrality determination in MUFFIN vs. "Monte Carlo Glauber"

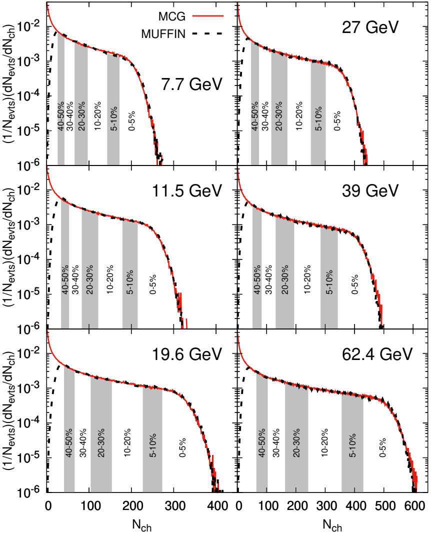
We make a comparison between:

- a semi-minimum-bias MUFFIN simulation $(0 < b < 12$ fm) and
- a two-component model for particle production, where $N_{\rm part}$ and $N_{\rm coll}$ come from a Monte Carlo Glauber sampling:

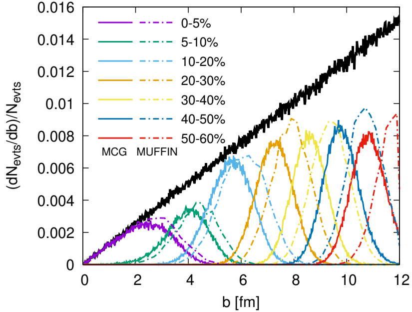
$$\frac{dN_{\rm ch}}{d\eta} = n_{pp}\left[(1-x)\frac{\langle N_{\rm part}\rangle}{2} + x\langle N_{\rm coll}\rangle\right]$$

$$P_{\rm NBD}(n_{pp},k;n) = \frac{\Gamma(n+k)}{\Gamma(n+1)\Gamma(k)}\frac{(n_{pp}/k)^n}{(n_{pp}/k+1)^{n+k}}$$

- "MCG" fits the $N_{\rm ch}$ distribution from a semi-minbias MUFFIN simulation with $b = 0-12$ fm



- we bin the generated events in centrality classes based on $dN_{\rm ch}/d\eta$ at mid-rapidity:



- For each centrality class, the mean impact parameter in MUFFIN has a larger value as compared to the “Monte Carlo Glauber”

Similar findings: arXiv:2303.07919 by Kuttan, Steinheimer, Zhou, Bleicher and Stoecker

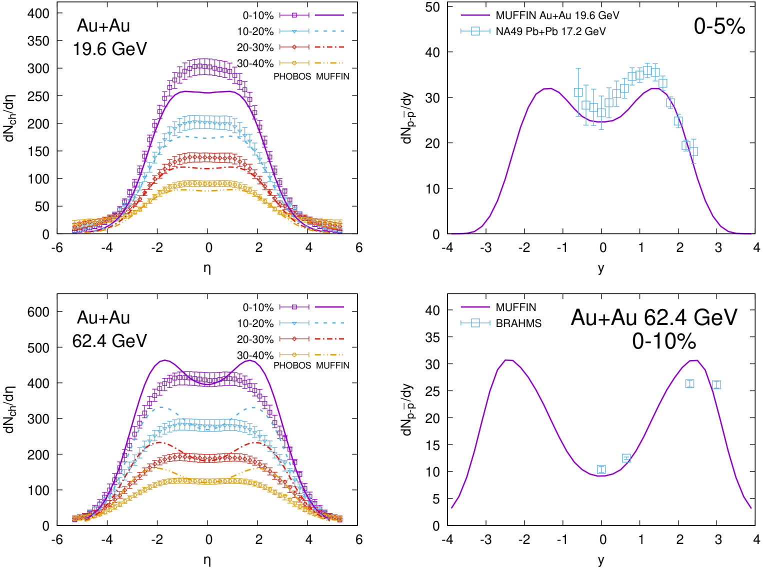
Basic observables vs. the data: $dN/d\eta$, net protons

Fitting parameters in the model:

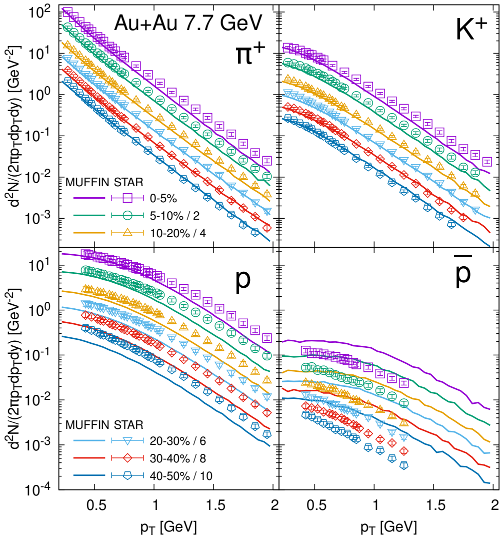
Friction Terms

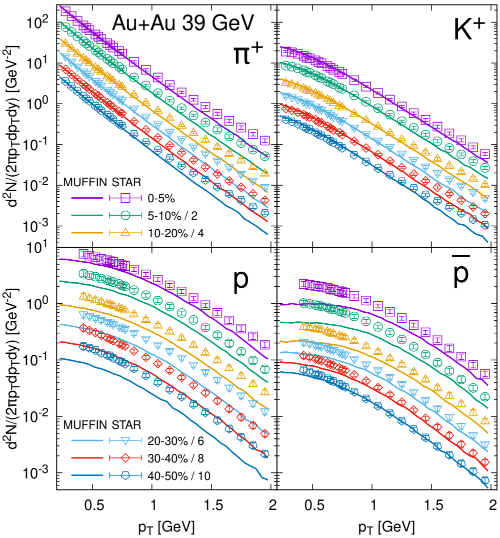
(both their functional form and the amplitudes)

We fix the functional form and vary the $\xi_{pt}(\sqrt{s_{\rm NN}})$, $\xi_{f\alpha}(\sqrt{s_{\rm NN}})$ to get overall agreement with the data $\Rightarrow$



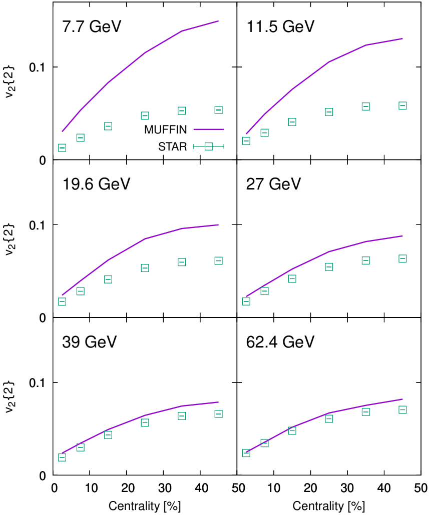
Basic observables vs. the data: dN/dp_T



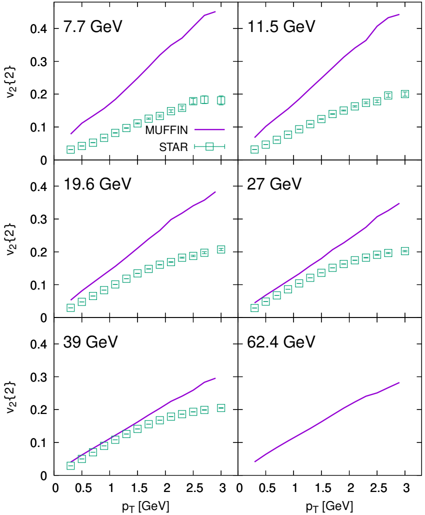


Basic observables vs. the data: v_2

v_2 as a function of centrality



v_2 as a function of p_T



Here a probable culprit is ideal fluid evolution: we haven't switched the viscosity on yet.

Directed flow: origins

Rischke, Puersuen, Maruhn, Stoecker, Greiner,

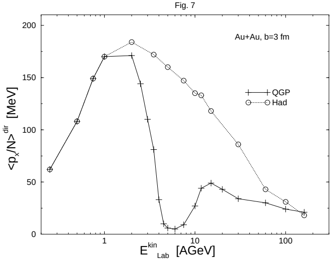
Acta Physica Hungarica: 1, 309–322 (1995)

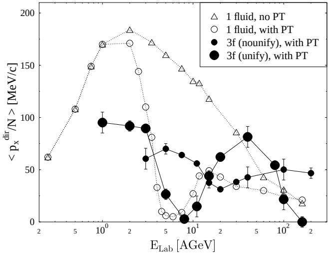
[nucl-th/9505014]

Brachmann, Soff, Dumitru, Stöcker, Maruhn, Greiner, Rischke,

Phys. Rev. C61 (2000) 024909

[nucl-th/9908010]



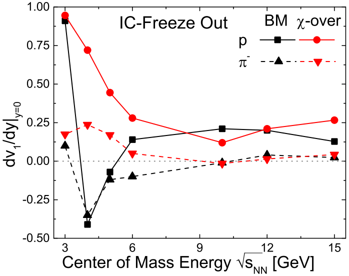


The conclusion was clear: non-monotonic dependence of $v_1 \rightarrow$ phase transition.

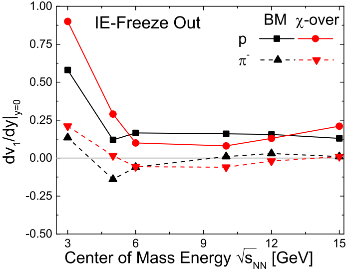
Directed flow: further developments circa 2014

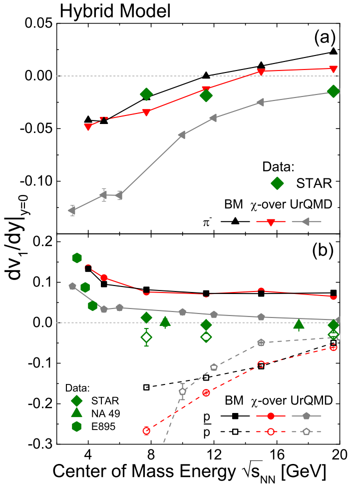
J. Steinheimer, J. Auvinen, H. Petersen, M. Bleicher, H. Stöcker, Phys. Rev. C 89 (2014) 054913, arXiv:1402.7236

- 1-fluid model with iso-time freezeoout:
  sign change of dv_1/dy with 1^{st}-order PT EoS



- 1-fluid model with iso-T freezeoout:
  NO sign change of dv_1/dy with 1^{st}-order PT EoS





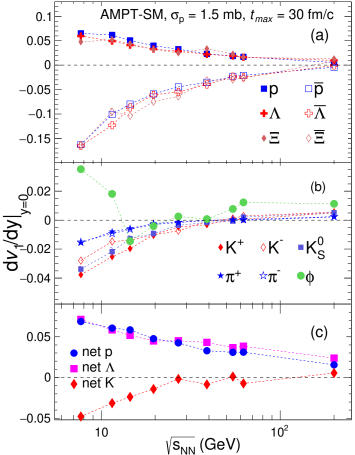
- Full hybrid model: no sign change of dv_1/dy, weak EoS dependence and no agreement with the data

Full-fledged models generally struggle to reproduce the v_1

AMPT model:

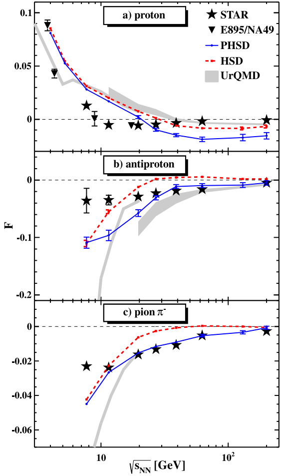
K. Nayak, S. Shi, Nu Xu, Zi-Wei Lin,

Phys. Rev. C 100, 054903 (2019)



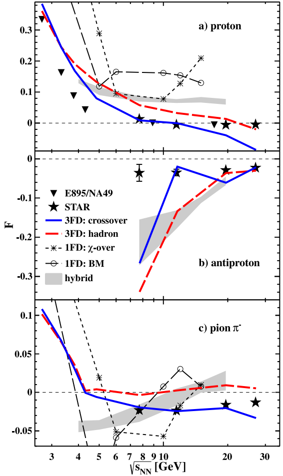
PHSD/HSD/UrQMD models:

Konchakovski, Cassing, Ivanov, Toneev,



3-fluid/1-fluid models:

Phys. Rev. C 90, 014903 (2014)



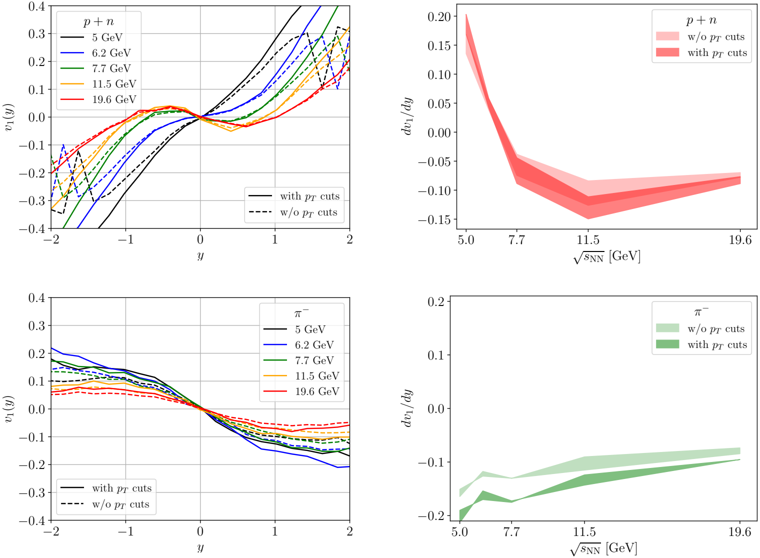
Directed flow in MUFFIN

p_T cut dependence

- MUFFIN simulation: averaged initial state for $b \in [4.5, 9.2]$ fm $\approx$ 20-50% centrality

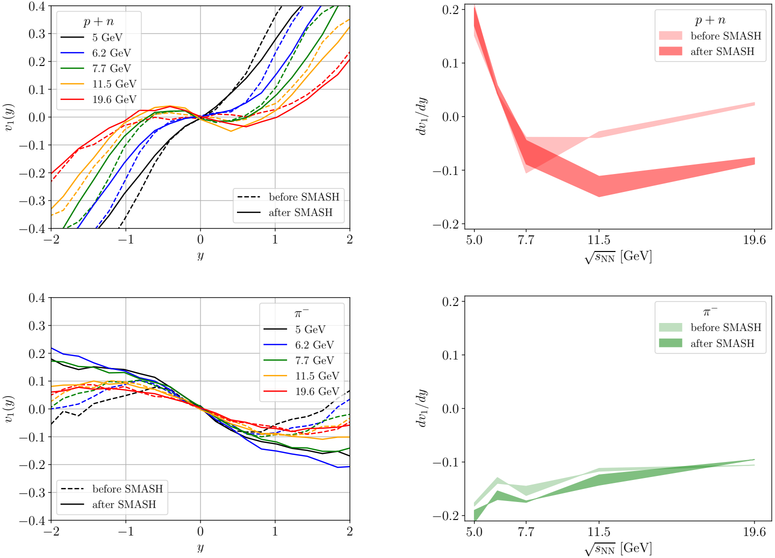
- p_T cuts: STAR [1401.3043]
  pions: $0.2 < p_T < 1.6$ GeV
  protons: $0.4 < p_T < 2.0$ GeV

- v_1 is computed as
  $v_1 = \langle cos(\phi_p) \rangle$

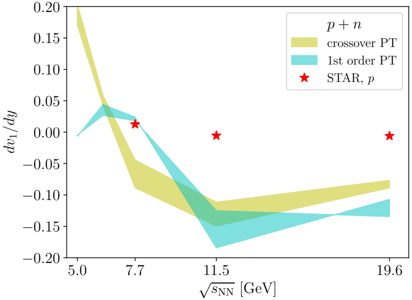


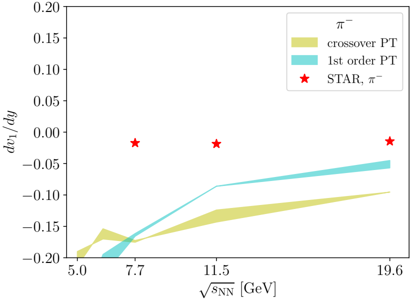
Directed flow in MUFFIN

effects of hadronic cascade



Directed flow in MUFFIN: EoS dependence

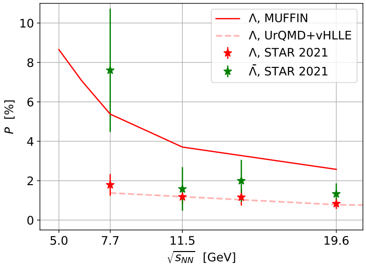




- The directed flow is **much stronger** than what STAR measured
- There is no clear trend in the EoS dependece.

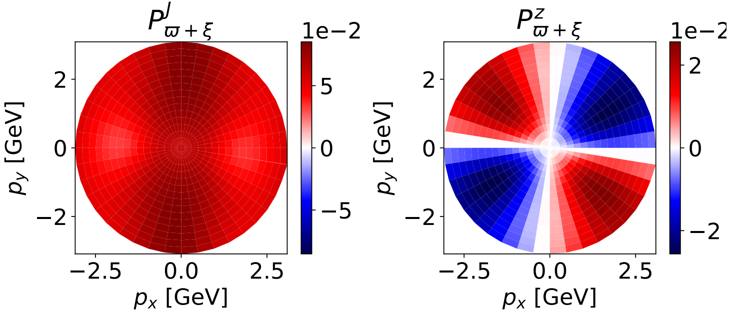
Hyperon polarization

- global polarization in 20-50% central Au-Au



Mean hyperon polarization is much stronger in MUFFIN as compared to STAR data

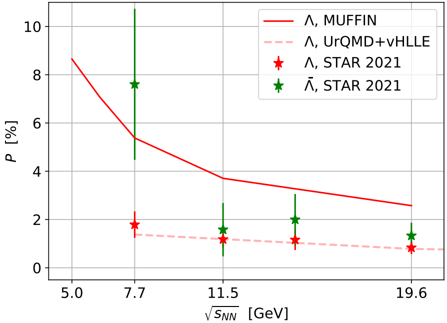
- local polarization in 20-50% central Au-Au at $\sqrt{s_{\rm NN}} = 7.7$ GeV



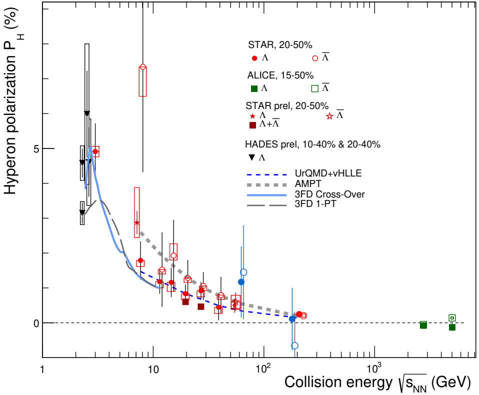
Local polarization: same patterns as observed at high energies

Hyperon polarization

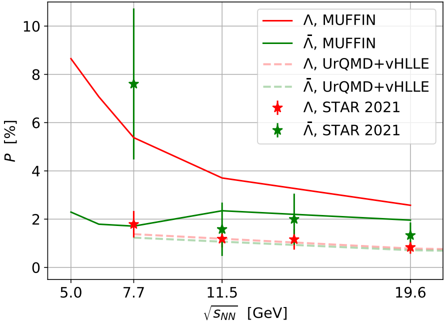
MUFFIN compared to other models



Compilation by Subhash Singha @ SQM 2022



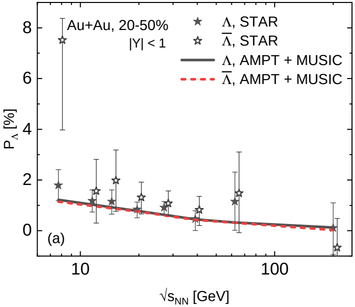
Polarization of $\bar{\Lambda}$ vs. Λ



- MUFFIN produces strong $\Lambda - \bar{\Lambda}$ splitting but with a wrong sign!
- There was a similar but much weaker trend with UrQMD+vHLLE

Same trend in AMPT+MUSIC:

Baochi Fu, Kai Xu, Xu-Guang Huang, Huichao Song, Phys. Rev. C 103, 024903 (2021) [arXiv:2011.03740]



Conclusions

- We present the next incarnation of 3-fluid model for relativistic heavy-ion collisions at RHIC BES/FAIR/... energies.
- Different from the existing model by Ivanov,Toneev,Soldatov, there is fluctuating initial state, shear and bulk viscosities (implemented but not enabled yet), Monte Carlo hadron sampling and hadronic afterburner (SMASH). Equation of state can be easily swapped.
- We fit the dN/dy and p_T distributions of hadrons from RHIC BES.
- v_2 is overestimated, which presumably happens due to ideal hydro evolution
- Directed flow is much stronger than the data (same as in other models), and there is no clear EoS trend
- Global polarization is stronger than the data; splitting between $\bar{\Lambda}$ and Λ is strong but has a wrong sign.
- **Outlook:** construct different friction terms based on different underlying assumptions; explore viscous fluid evolution,
- plug in different equations of state to explore sensitivity to the EoS
  (currently used EoS are outdated).