What do

Simulations

tell us?

H. Elfner, I. Karpenko, V. Kuznietsov, D. Mroczek, J. Noronha-Hostler

Simulations = dynamical simulations

= hydro or transport (hadronic cascades)

Progress on Applying Hydro at Lower Energy

Hybrid models for RHIC BES have been constructed and reproduce most of the basic observables in the region. The densest part of the fireball is still dense enough for hydro to make sense. [Nonaka&Bass showed it as early as 2006].

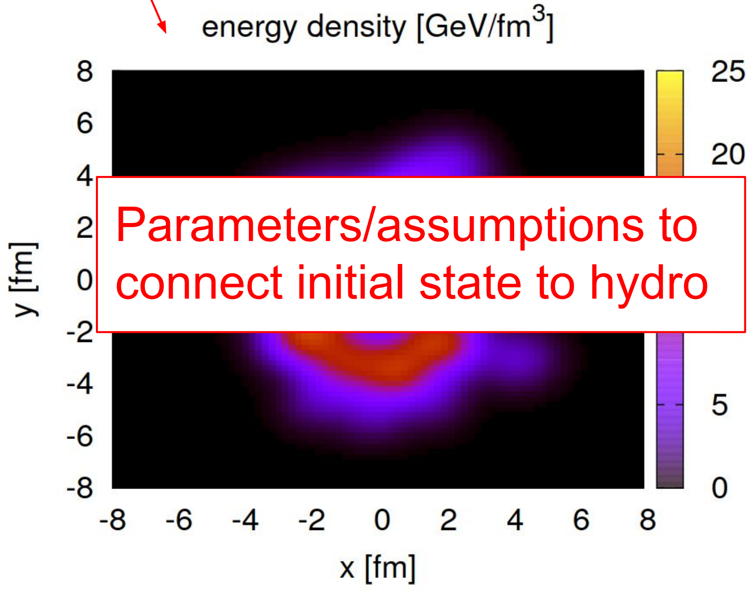
Hydro-based models for RHIC BES region include: UrQMD+vHLLE (2015), SMASH+vHLLE (2022), EPOS4 2024), 3D Glauber+MUSIC (2018?), SMASH+MUSIC (a.k.a. JETSCAPE, 2025), CCAKE+SMASH (a.k.a. MUSES, 2025)

The recipe is mostly imported from higher energies:

- 3D initial state: from transport or parametrized
- 3D hydro with finite viscosity (not a challenge)
  and conserved charges (not a challenge either)
- EoS at finite muB (e.g. Chiral model EoS)
- Same or slightly adapted Cooper-Frye
- final-state hadronic cascade

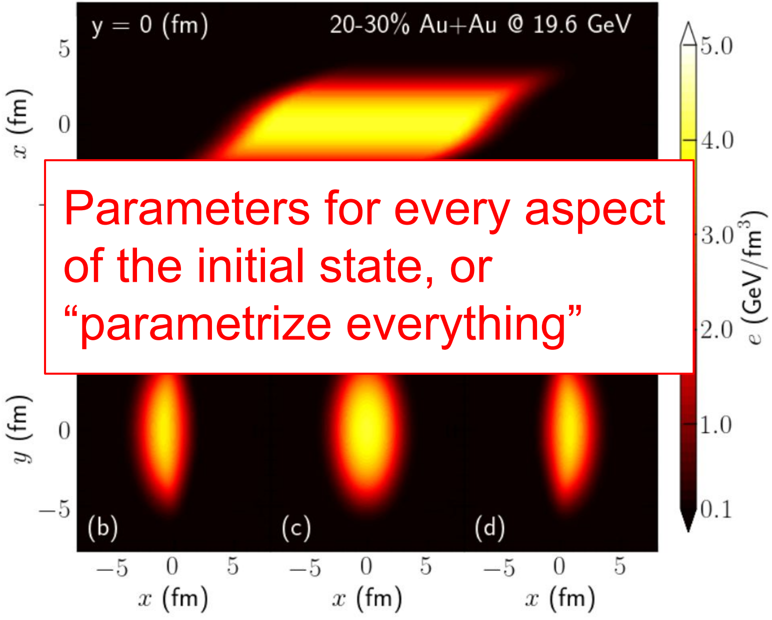
✅ 3D initial state with finite n_B, n_Q, n_S

Either taken from transport (UrQMD, SMASH, EPOS, JAM)



Karpenko, Huovinen, Petersen, Bleicher, Phys.Rev.C 91 (2015) 6, 064901

Or parametrized



Shen, Alzhrani, Phys.Rev.C 102 (2020) 1, 014909

\+ follow-up by Lipei du et al

✅ **3D fluid-dynamic evolution** (no boost invariance)

Temperature- and μ_B-dependent η/s

A working horse is Israel-Stewart-type hydro equations with higher order terms varying from code to code

$$\partial_\mu T^{\mu\nu} = 0$$

$$\tau_\Pi \Pi + \dot{\Pi} = -\zeta\theta - \delta_{\Pi\Pi}\Pi\theta + \lambda_{\Pi\pi}\pi^{\mu\nu}\sigma_{\mu\nu},$$

$$\tau_\pi \dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} - \delta_{\pi\pi}\pi^{\mu\nu}\theta + \phi_7 \pi_\alpha^{\langle\mu}\pi^{\nu\rangle\alpha} - \tau_{\pi\pi}\pi_\alpha^{\langle\mu}\sigma^{\nu\rangle\alpha} + \lambda_{\pi\Pi}\Pi\sigma^{\mu\nu}.$$

a.k.a. 14-momentum approximation

$$\tau_\pi \dot{\pi}^{\mu\nu} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} + \frac{\tau_\pi \pi^{\mu\nu}}{2}\theta - \frac{\tau_\pi}{2\beta_\pi}\dot{\beta}_\pi \pi^{\mu\nu} - \frac{2\eta}{\beta}\left(\gamma_1^q \nabla^{\langle\mu} n_q^{\nu\rangle} + \frac{1}{2} n_q^{\langle\mu}\nabla^{\nu\rangle}\gamma_1^q\right)$$

$$\tau_\Pi \dot{\Pi} + \Pi = -(\zeta + \frac{\tau_\Pi}{2}\Pi)\theta - \frac{\tau_\Pi}{2\beta_\Pi}\dot{\beta}_\Pi \Pi - \frac{\zeta}{\beta}\left(\gamma_0^q D_\mu n_q^\mu + \frac{1}{2} n_q^\mu \nabla_\mu \gamma_0^q\right)$$

$$\tau_{qq'}\dot{n}_{q'}^\mu + n_q^\mu = -\kappa_{qq'}\nabla^\mu \alpha_{q'} + \frac{\tau_{qq'} n_{q'}^\mu}{2}\theta - \frac{\tau_{qq'}}{2\beta_{qq'}}\dot{\beta}_{qq'} n_{qq'}^\mu - \frac{\kappa_{qq'}}{\beta}\left(\gamma_0^{qq'}\nabla^\mu \Pi - \frac{\Pi}{2}\nabla^\mu \gamma_0^{qq'}\right) - \frac{\kappa_{qq'}}{\beta}\left(\gamma_1^{qq'}\nabla_\nu \pi^{\mu\nu} + \frac{\pi^{\mu\nu}}{2}\nabla_\nu \gamma_1^{qq'}\right.$$

Max. entropy

Almaalol, Dore, Noronha-Hostler, PRD 111, 014020

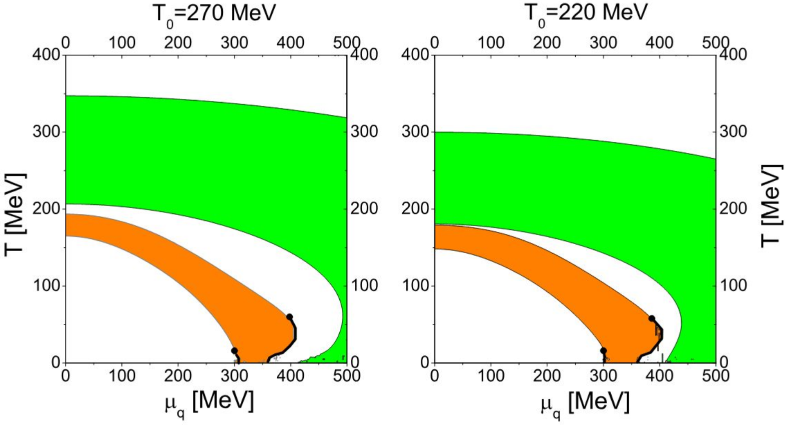
General direction: better re-summation / more non-eq corrections, wider range of applicability.

✅ EoS at finite n_B (optionally n_Q, n_S)

EoS can include first-order PT but only Maxwell construction

EoS has to be known/tabulated down to very low T and high mub → limits the candidate EoS to use.

Older choices: Chiral model EoS

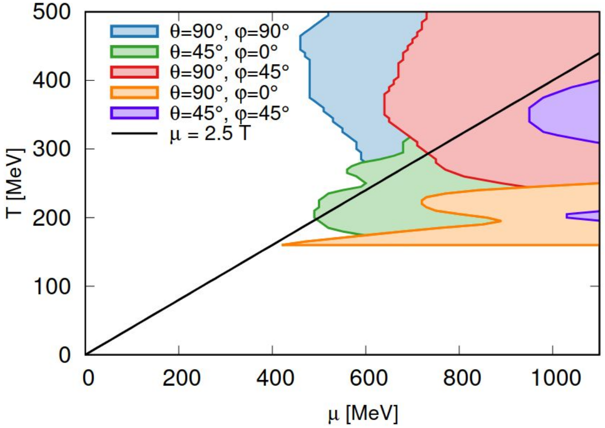


Phase diagram of the EoS

Steinheimer, Schramm, J.Phys.G 38 (2011) 035001

- Energy and baryon density (2D)
- Compatible with latticeQCD at μ_B=0
- Reproduces ground state of nucleus (?!)
- Crossover transition at all μ_B

State-of-the-art: **MUSES**



White region is where the used expansion works

Ahmed Abuali et al (MUSES), Phys.Rev.D 112 (2025) 5, 054502

- 4D (T, mub, muq, mus)
- latticeQCD at μ_B=0
- CP location parameterized

✅ **Freeze-out / particlization** at finite μ_B (μ_Q , μ_S)

Cooper-Frye = local grand-canonical ensemble

(energy-momentum and charge conservation only on average)

Notable exceptions:Cooper-Frye + conservation

Oliinychenko and Koch (sampling in patches)

Vovchenko

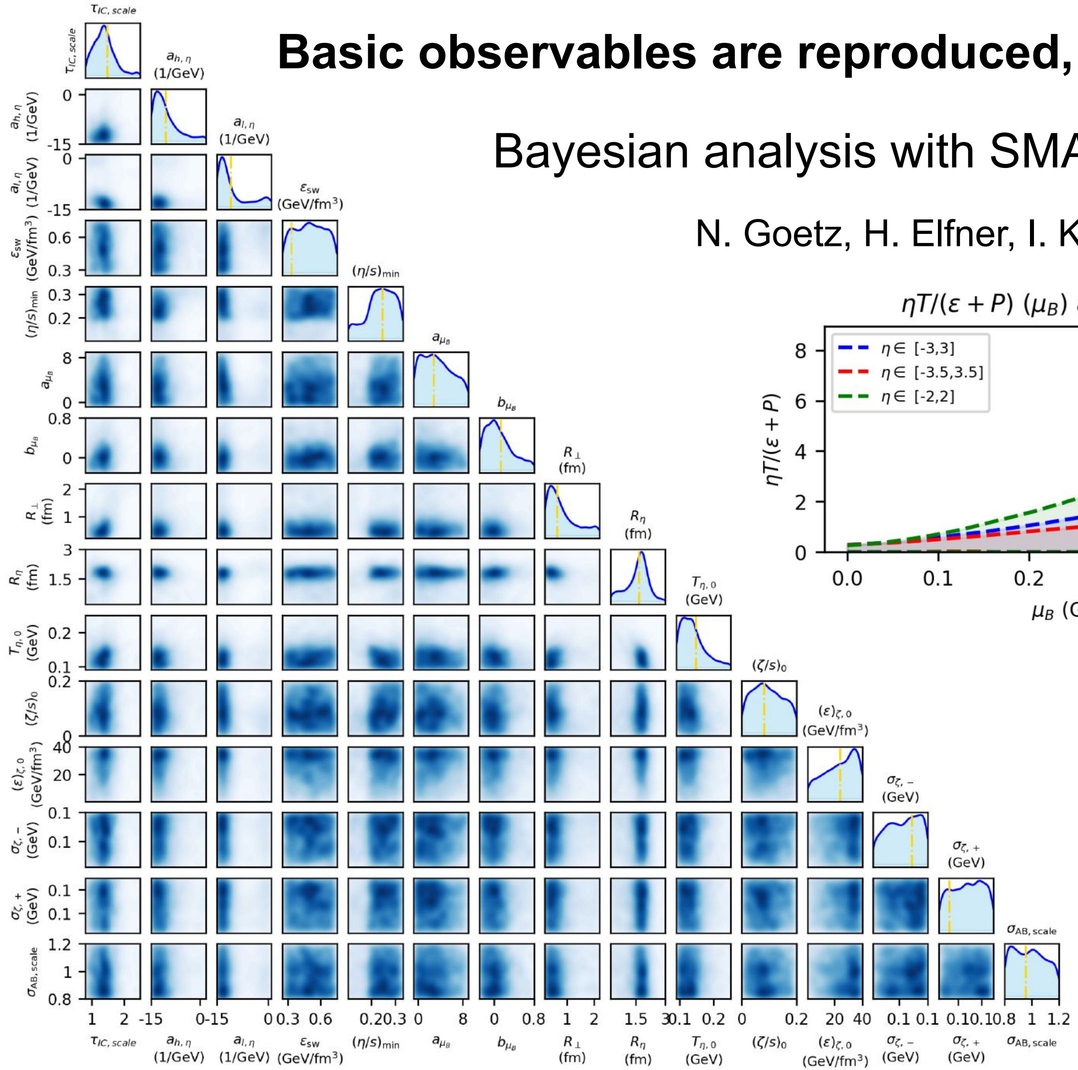
✅ Final-state hadronic cascade

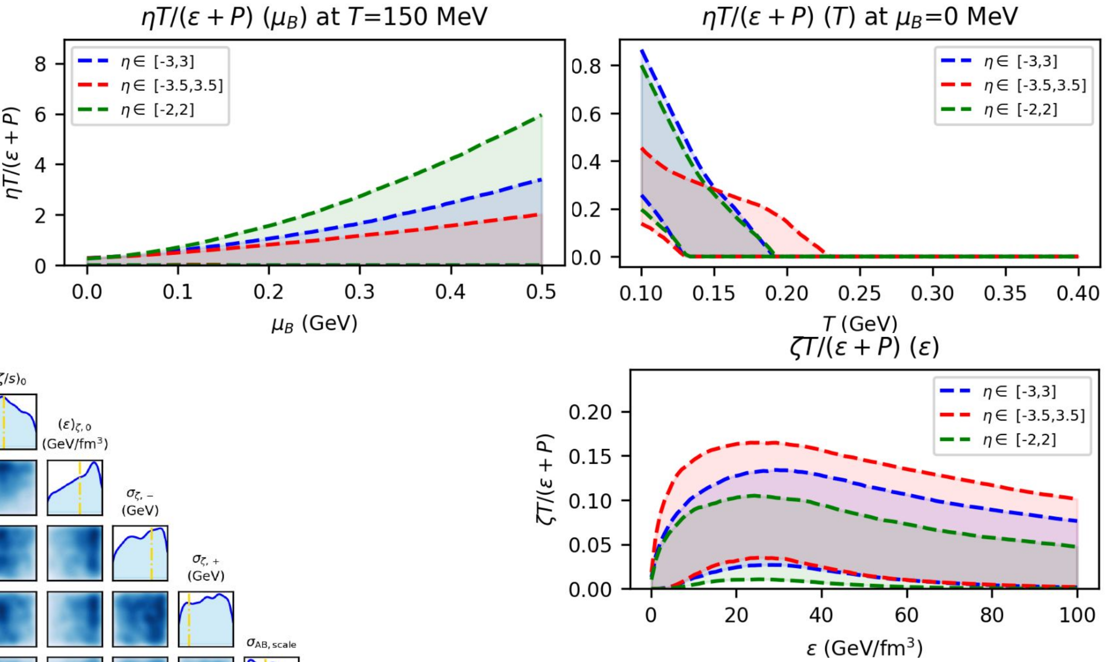
(UrQMD, SMASH, JAM)

Basic observables are reproduced, models constrained via BA

Bayesian analysis with SMASH+vHLLE, sqrt(s)=7.7…39 GeV

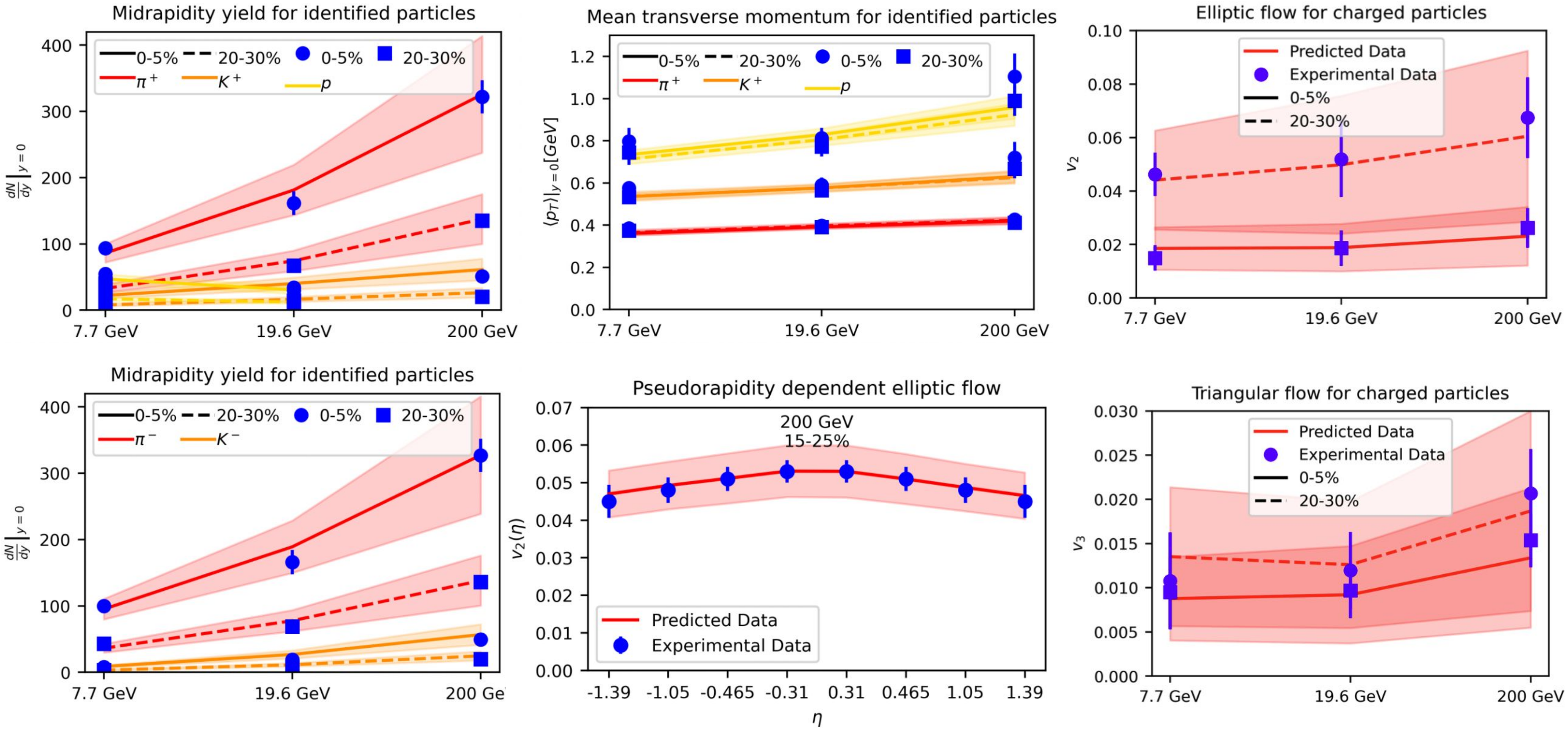
N. Goetz, H. Elfner, I. Karpenko, Phys.Rev.C 112 (2025) 1, 014910





Parameter space is complex, Bayesian analysis is challenging.

What experimental data is reproduced in (being used for) the Bayesian analysis:

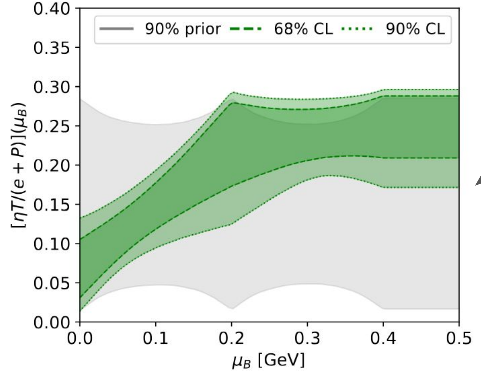


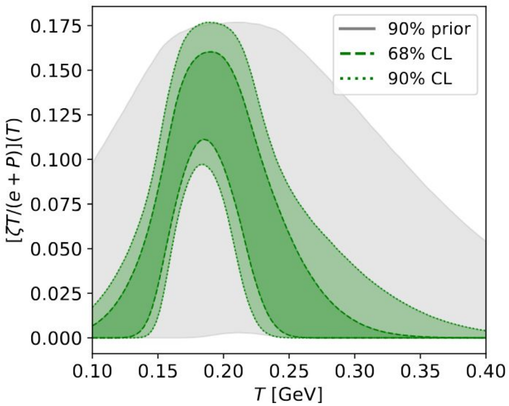
N. Goetz, H. Elfner, I. Karpenko, Phys.Rev.C 112 (2025) 1, 014910

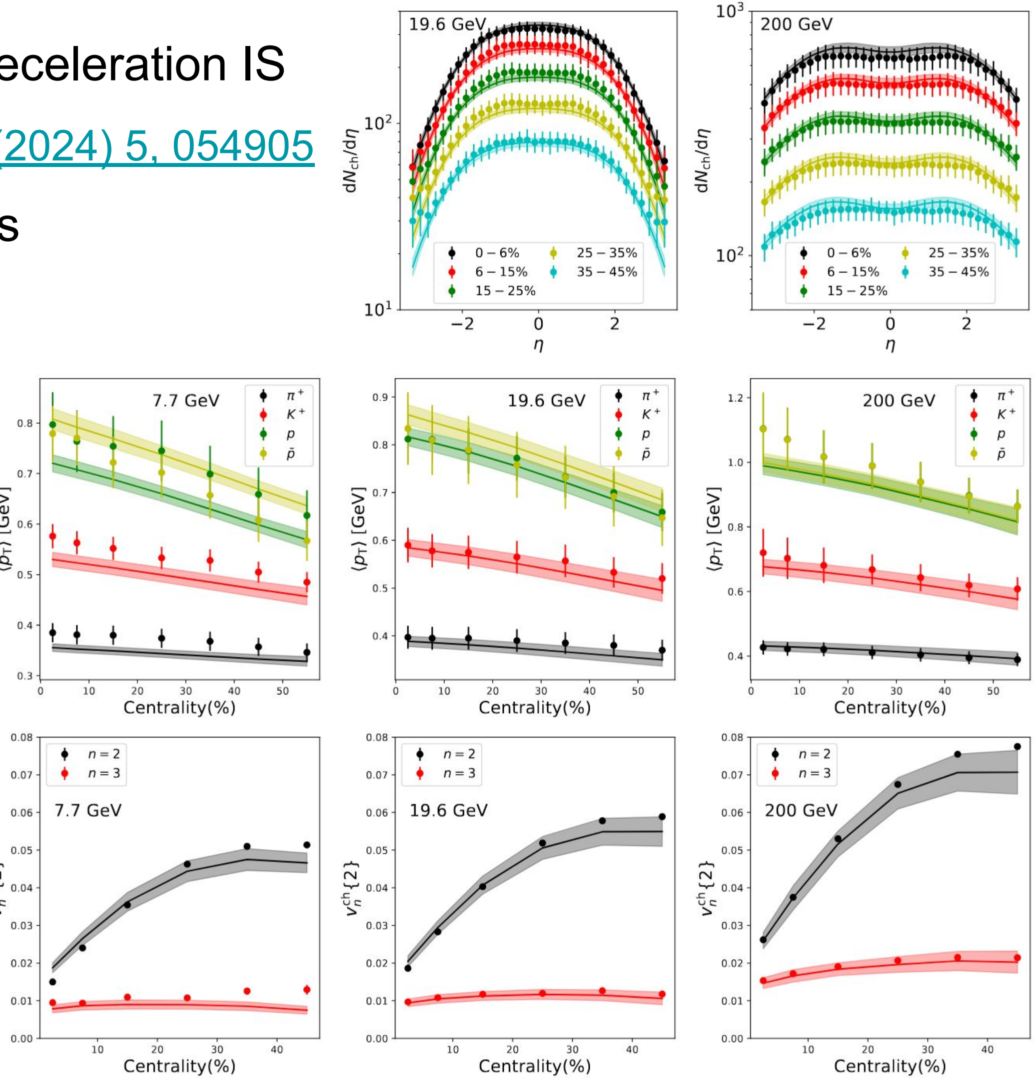
Another Bayesian analysis: string deceleration IS

Jahan, Roch, Shen, Phys.Rev.C 110 (2024) 5, 054905

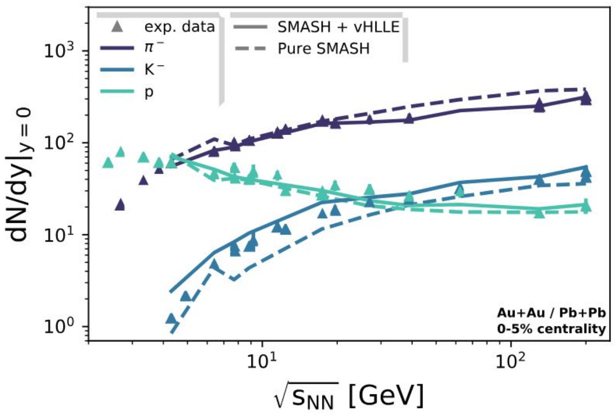
Constraints







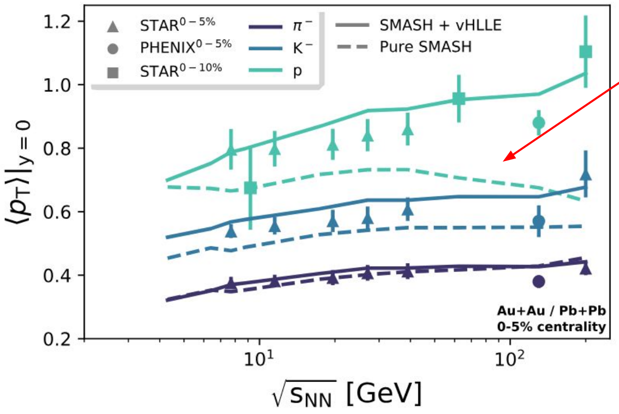
Aggregate conclusions from the hydro modelling

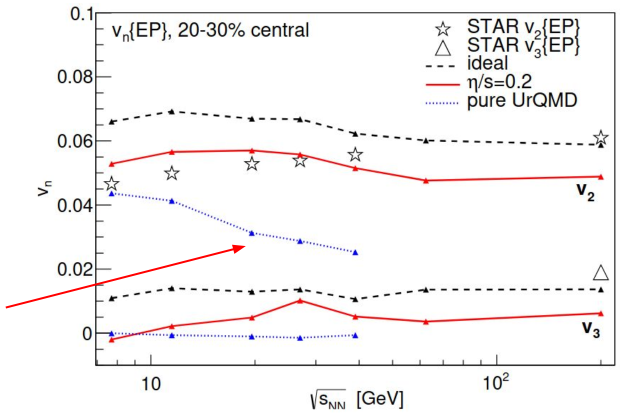


❗ Fluid-dynamic phase improves agreement with the data; pure cascade fails to describe flow and mean pT above √s~7 GeV

Schäfer, Karpenko, Wu, Hammelmann, Elfner,
[Eur.Phys.J.A 58 (2022) 11, 230]

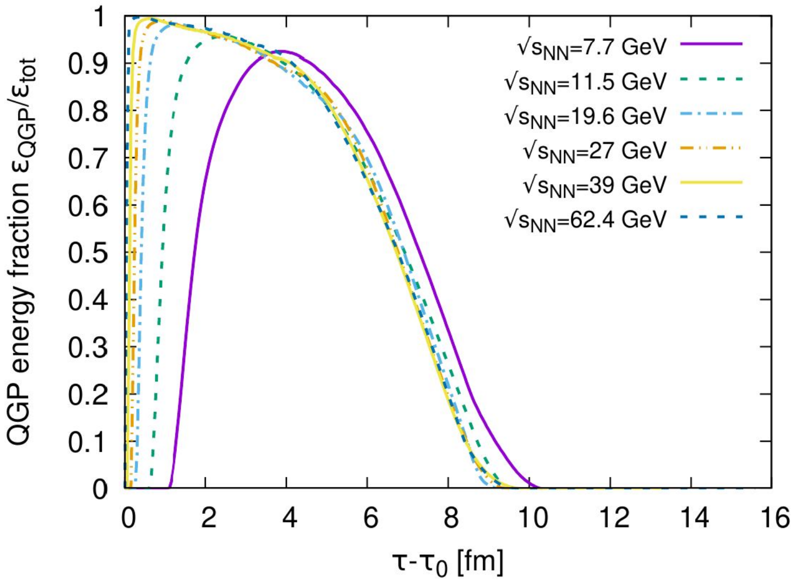
Karpenko, Huovinen, Petersen, Bleicher,
[Phys.Rev.C 91 (2015) 6, 064901]



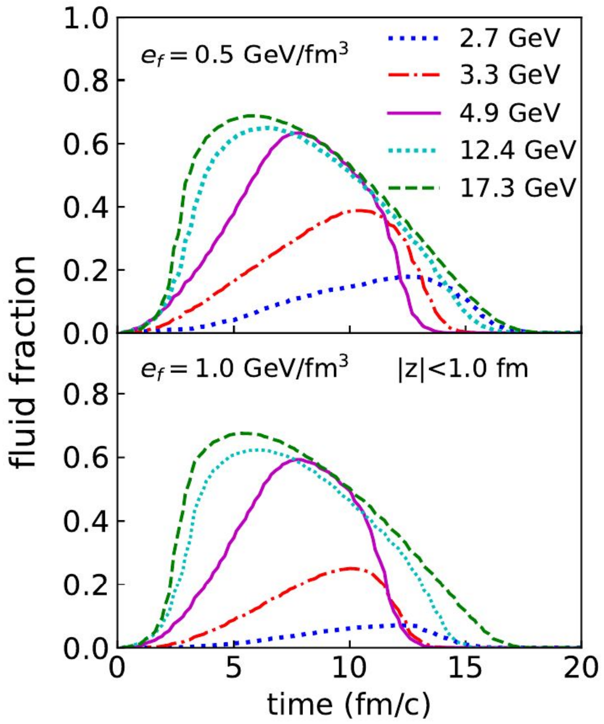


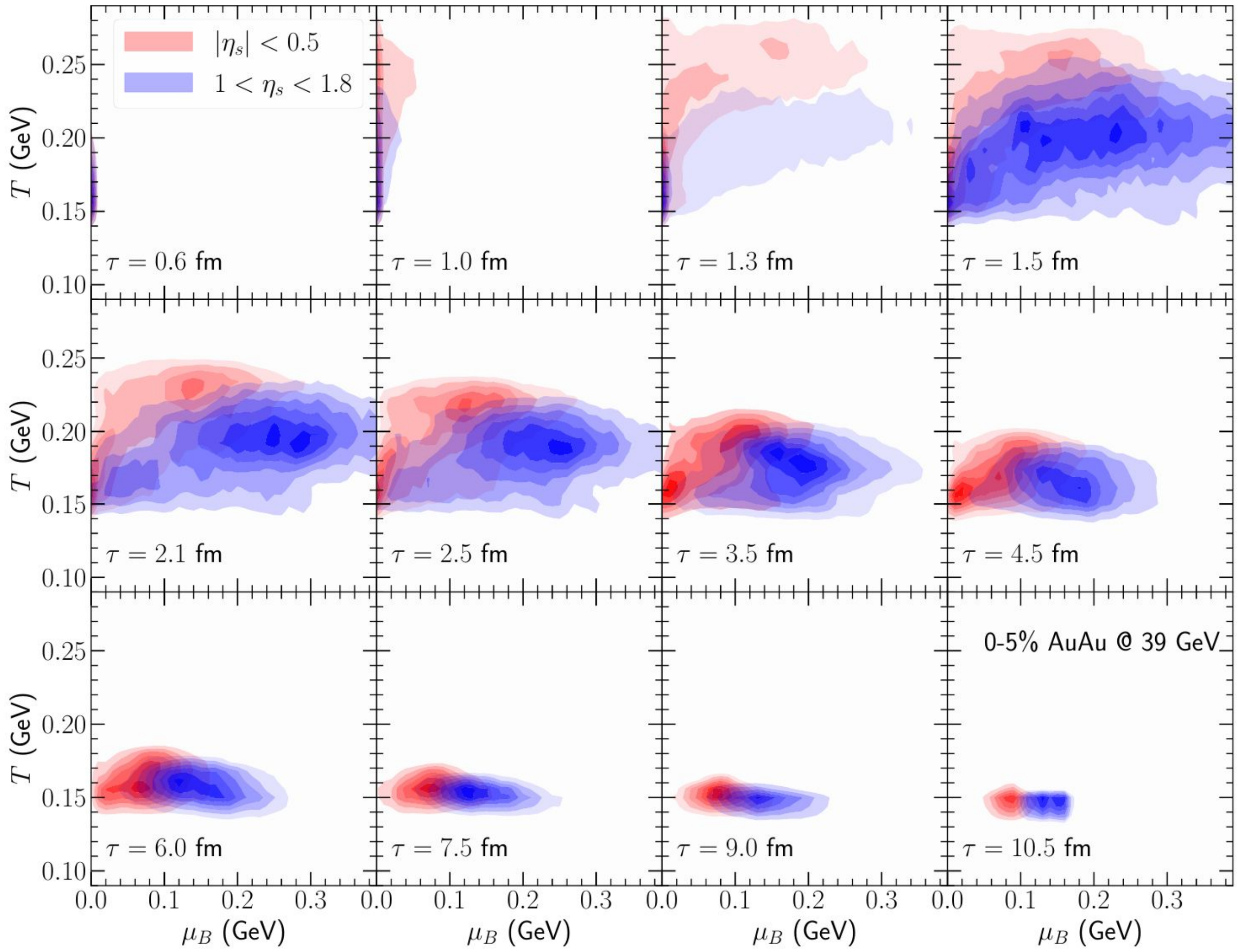
❗ High densities are reached at low collision energies

MUFFIN 1.0: Cimerman, Karpenko, Tomasik, Huovinen, [Phys.Rev.C 107 (2023) 4]



JAM+hydro: Yasushi Nara et al





Evolution movies: https://smash-transport.github.io/movies-hybrid.html

❗ The fireball is quite inhomogeneous

A √s point does not map to a (T, μ_B) point.

This plot is taken from Chun Shen, 2108.04987

Other hybrids show a similar picture

For particle number ratios, the whole system is integrated out, which results in one point at the phase diagram

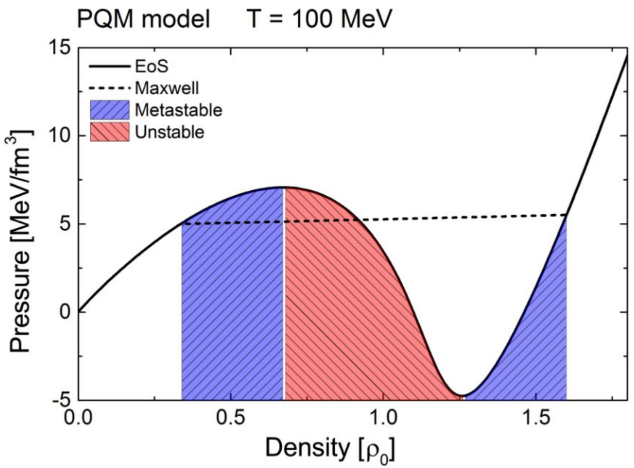
EoS sensitivity / constraints?

There is no simple answer for that.

- As it was discussed last week, EoS sensitivity in the models can be obscured by other sensitivities (just like at high energies).
- E.g. EoS constraining via directed flow does not seem to work.
- State-of-the-art EoS with CP location as a free parameter are applied in *some* fluid-dynamic models (MUSIC code, NEOS).
- But there is no BA constraining the EoS at non-zero baryon density yet.

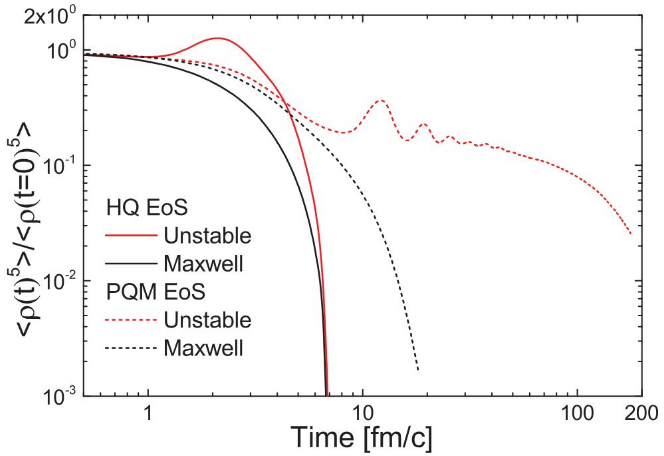
Fluid dynamics with spinodal decomposition in the EoS

Steinheimer, Randrup, Koch, Phys. Rev. C 89, 034901 (2014),



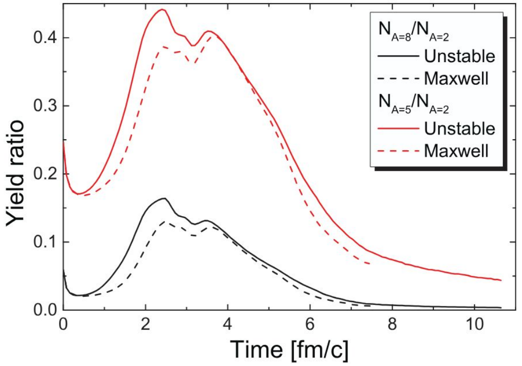
EoS w/ Mechanically unstable region (phase coexistence with hadronic med.)

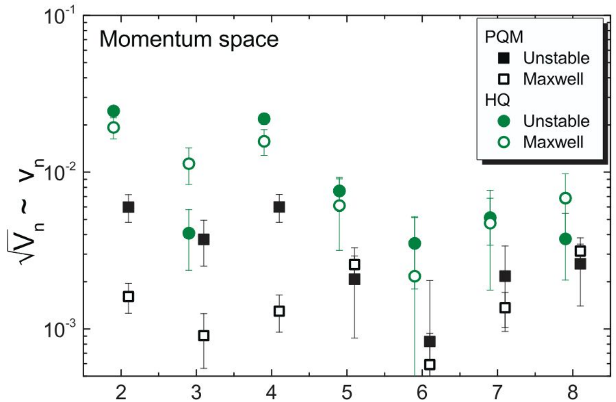
translates into clumping in coordinate space



But intuitively sensitive observables show little sensitivity.

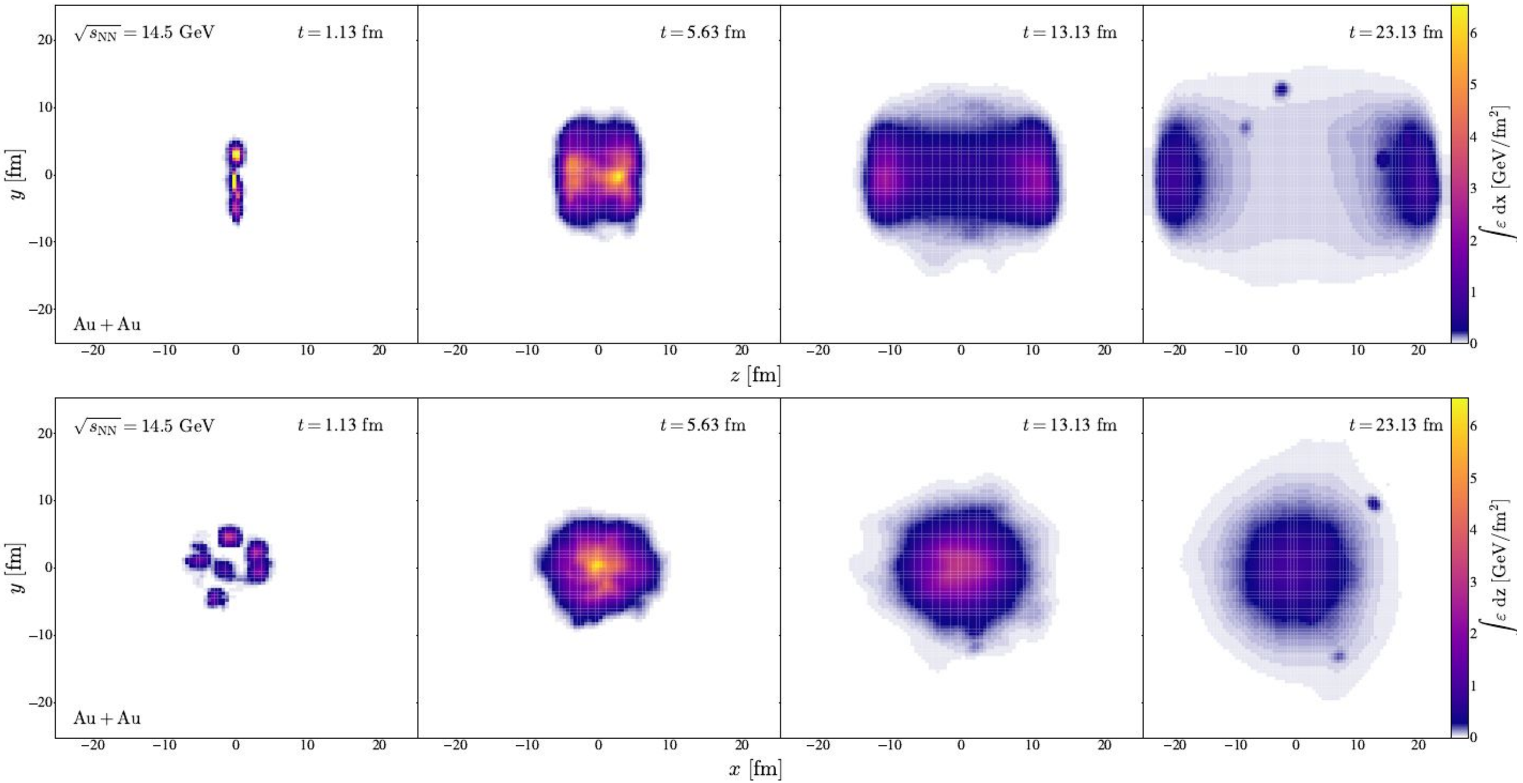
A follow-up JHEP 12, 122 using machine learning





Making hydro useful at even lower collision energies: dynamical fluidization

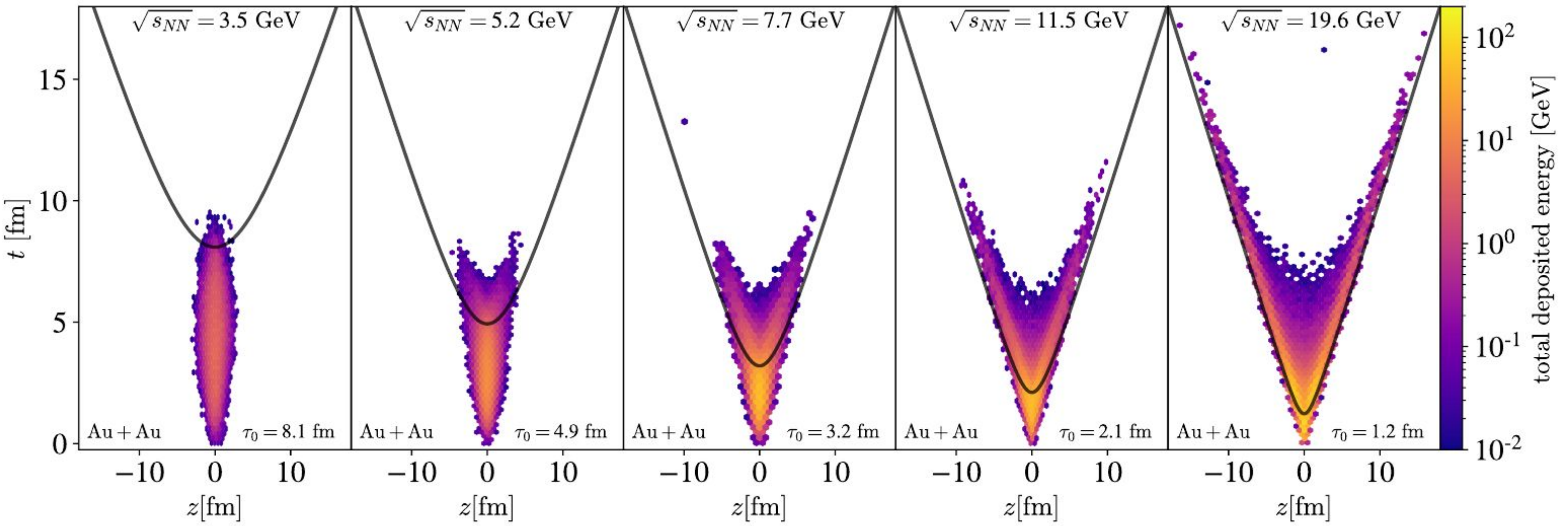
The idea: once the energy density in LRF is high enough *locally*, fluidize this part.



Góes-Hirayama, Egger, Paulínyová, Karpenko, Elfner, [2507.19389](2507.19389)

As a result, the simulation does not wait for the two nuclei to completely pass through each other, which is a long time at low √s

Fluid approximation still applies for the most dense part of the evolution



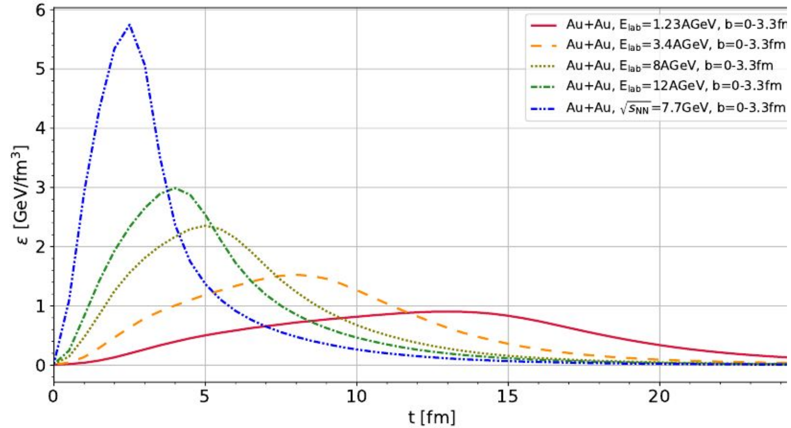
Góes-Hirayama, Egger, Paulínyová, Karpenko, Elfner, [2507.19389]

Does the medium look like a fluid when it is fluidized?

In case of event averaged initial state, in some cases the answer is yes.

Gabriele Inghirami, Hannah Elfner, Eur. Phys. J. C 82, 796 (2022)

From $E_{\text{lab}} = 1.23$ GeV up to $\sqrt{s} = 7.7$ GeV

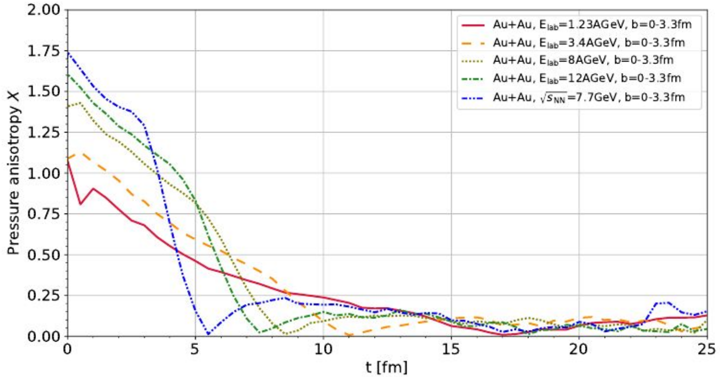


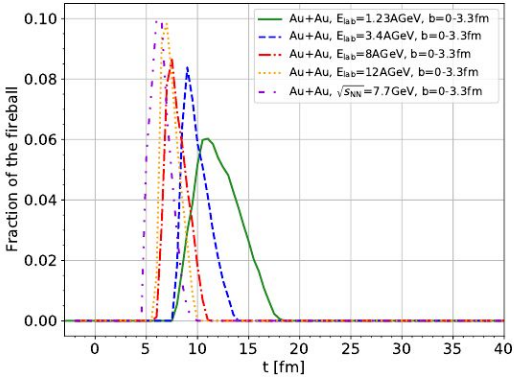
Examining the pressure anisotropy:

$$T^{\mu\nu} = \sum_i \frac{p_i^\mu p_i^\nu}{p_i^0} K(\mathbf{r} - \mathbf{r}_i, \mathbf{p_i})$$

$$X \equiv \frac{|\langle T_L^{11}\rangle - \langle T_L^{22}\rangle| + |\langle T_L^{22}\rangle - \langle T_L^{33}\rangle| + |\langle T_L^{33}\rangle - \langle T_L^{11}\rangle|}{\langle T_L^{11}\rangle + \langle T_L^{22}\rangle + \langle T_L^{33}\rangle},$$

$$Y \equiv \frac{3(|\langle T_L^{12}\rangle| + |\langle T_L^{23}\rangle| + |\langle T_L^{13}\rangle|)}{\langle T_L^{11}\rangle + \langle T_L^{22}\rangle + \langle T_L^{33}\rangle}$$



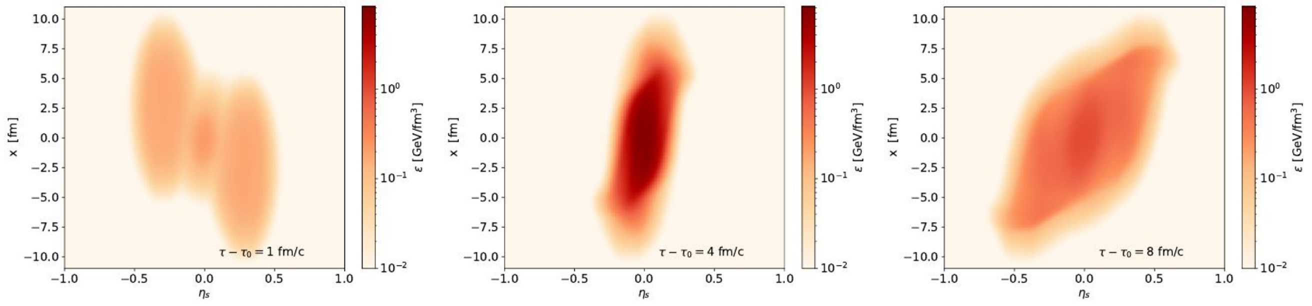


Making hydro useful at even lower collision energies: *multi-fluid approach*

- Incoming nuclei = two fluids labelled as projectile and target
- Interaction of the fluids (slowing down) via "friction" terms
- Friction transports energy and momentum into the third fluid labelled as fireba
- It is a minimal setup to reproduce baryon stopping at low $\sqrt{s}$ and baryon transparency at high $\sqrt{s}$.

$$\partial_\mu T_{\mathrm{p}}^{\mu\nu}(x) = -F_{\mathrm{p}}^{\nu}(x) + F_{\mathrm{fp}}^{\nu}(x),$$
$$\partial_\mu T_{\mathrm{t}}^{\mu\nu}(x) = -F_{\mathrm{t}}^{\nu}(x) + F_{\mathrm{ft}}^{\nu}(x),$$
$$\partial_\mu T_{\mathrm{f}}^{\mu\nu}(x) = F_{\mathrm{p}}^{\nu}(x) + F_{\mathrm{t}}^{\nu}(x) - F_{\mathrm{fp}}^{\nu}(x) - F_{\mathrm{ft}}^{\nu}(x),$$

Cimerman, Karpenko, Tomasik, Huovinen, Phys.Rev.C 107 (2023) 4, 4



Snapshots of multi-fluid evolution in x-η_s plane, Au-Au collision at $\sqrt{s_{\mathrm{NN}}} = 7.7$ GeV

(Hadronic) Transport

✅ natively 3D

✅ EoS can be emulated with potentials

✅ Exact energy-momentum, charge conservation

✅ no interfaces (to/from hydro)

Caveats:

⚠️ No change in the degrees of freedom (UrQMD, SMASH, HSD, JAM)

Unless: partonic phase + coalescence to hadrons (AMPT, PHSD, PHQMD(?))

⚠️ No good agreement with the data above sqrt(s)>10 GeV (except for JAM!)

Unless: …

Cumulant calculations (non-critical baseline)

Relations between cumulants in experiment and theory: uneasy situation.

Experiment:

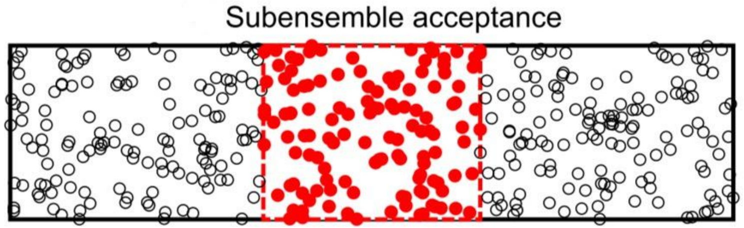
- Volume fluctuations
- Finite time, finite observation range
- Non-conserved charges
- Momentum space observations
- Inhomogeneous

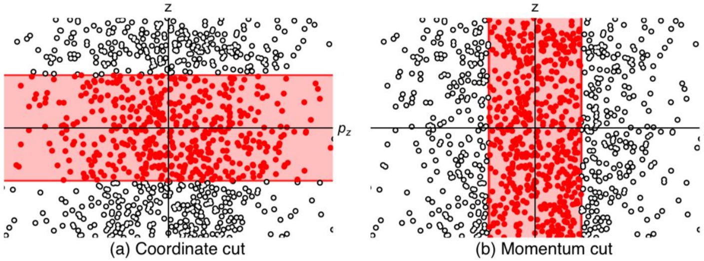
Theory:

- Coordinate and/or momentum space
- Conserved charges
- Uniform
- Fixed volume
- No CP in transport models

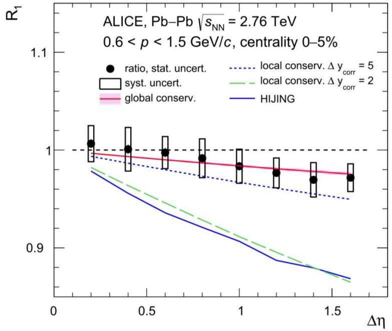
Simulations: not an answer, but a method to face and study these issues at low cost.

V. Vovchenko et al., [PLB 811 (2020) 135868]





S. Acharya et al. (ALICE), [PLB 807, 2020, 135564]



Cumulant calculations (non-critical baseline)

“Old” calculations in UrQMD:
Haussler, Scherer, Stoecker, Bleicher, 2005 and 2007

Hydro:
Vovchenko, Koch, Shen, 2021
Monte Carlo version: Vovchenko, 2022
Hirayama et al, 2023 (net-Q, net-p)

UrQMD: Zhang, Zhang, Xu (2025)

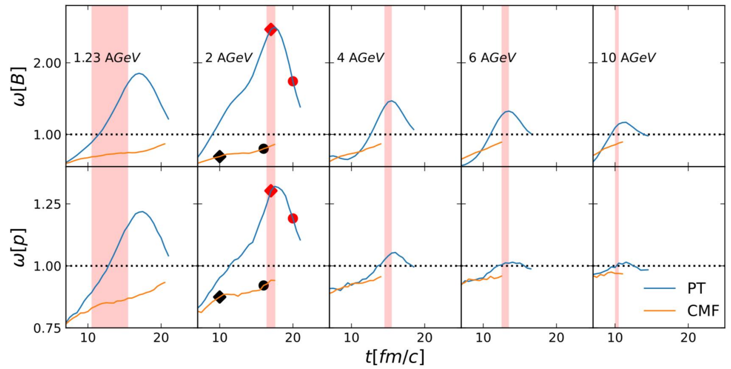
Cumulant calculations (with criticality)

Alternatively to the approaches without CP in EOS, one can try to directly introduce it into the transport model to observe a manifestation of the critical fluctuations.

However, now this is complicated to do it in a full quantum, relativistic and self-consistent approach. That's why common tactic is using of the toy models, which representing features that we wish to study and connect to an actual experimental data.

Despite giving oversimplified picture for QCD matter, these results are important in a context of understanding finite-size effects, correlations between momentum and coordinate space, and interplay between proton and baryon cumulants with charge conservation.

O. Savchuk et al., PhysRevC.107.024913

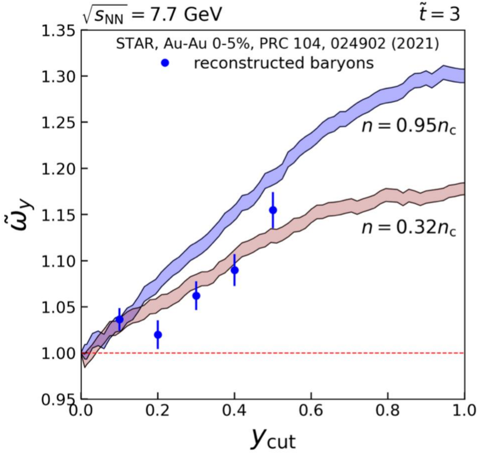


V. Kuznietsov et al., PhysRevC.105.044903

$$\omega = \frac{\kappa_2}{\kappa_1} = \frac{\langle(\Delta N)^2\rangle}{\langle N\rangle}$$

$$\tilde{\omega} = \frac{\omega}{1-\alpha}$$

(SAM correction)

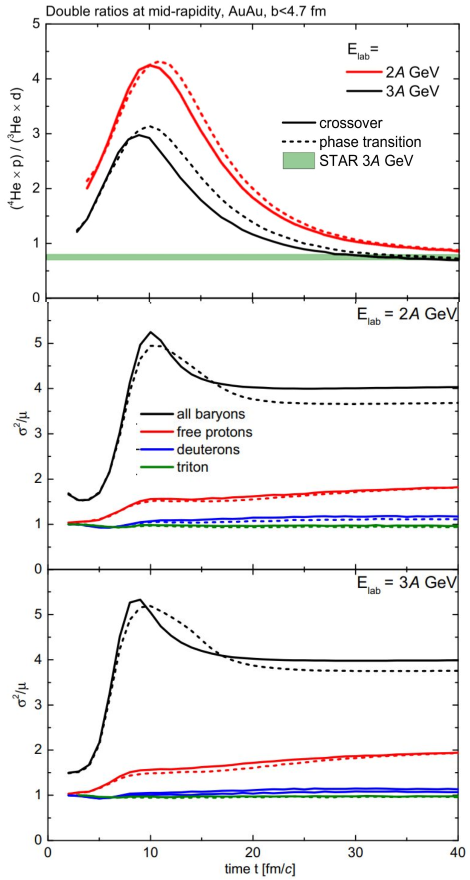


Cumulant calculations (metastable state)

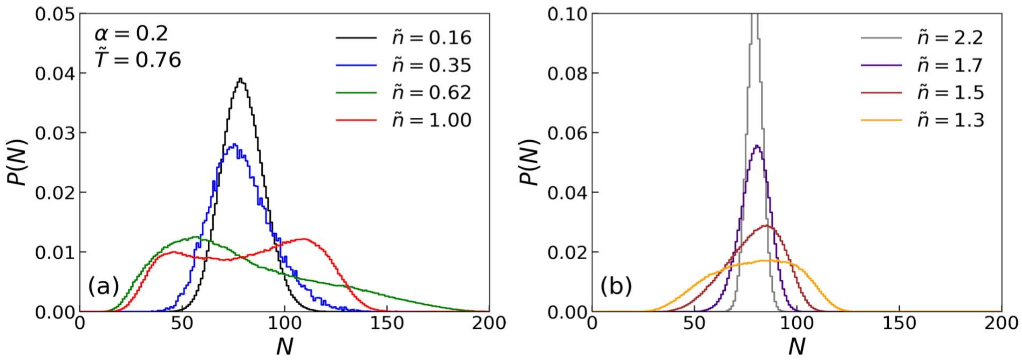
T. Bumnedpan et al., PhysRevC.111.034910

Another possibility: simulations of the metastable (and unstable) finial size systems.
Nucleation and spinodal decomposition even in finite systems leads to the enhancement of fluctuations. However, two issues exist:

- Consistent description of the finite size unstable region requires a model which would include time evolution, t/d instability (hydro?) and conservation effects
- To compare with cluster production in experiment more data at low energy needed



V. Kuznietsov et al. PhysRevC.107.055206

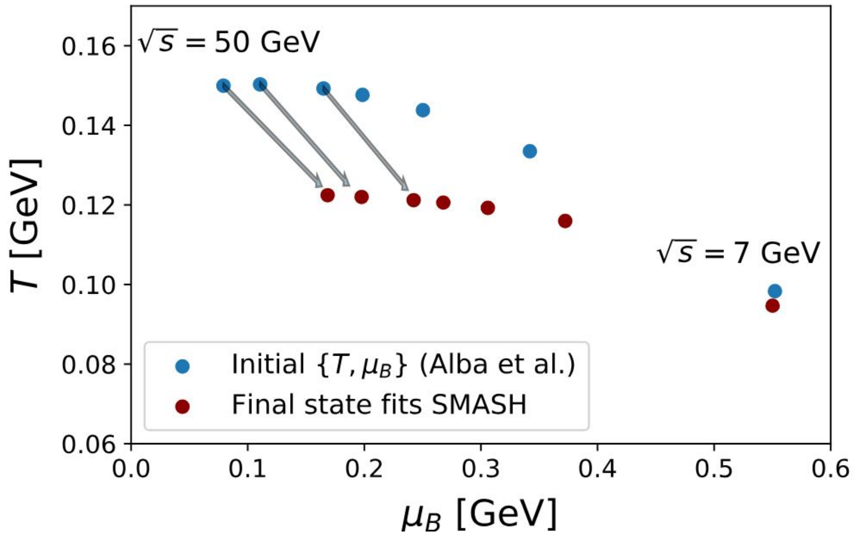


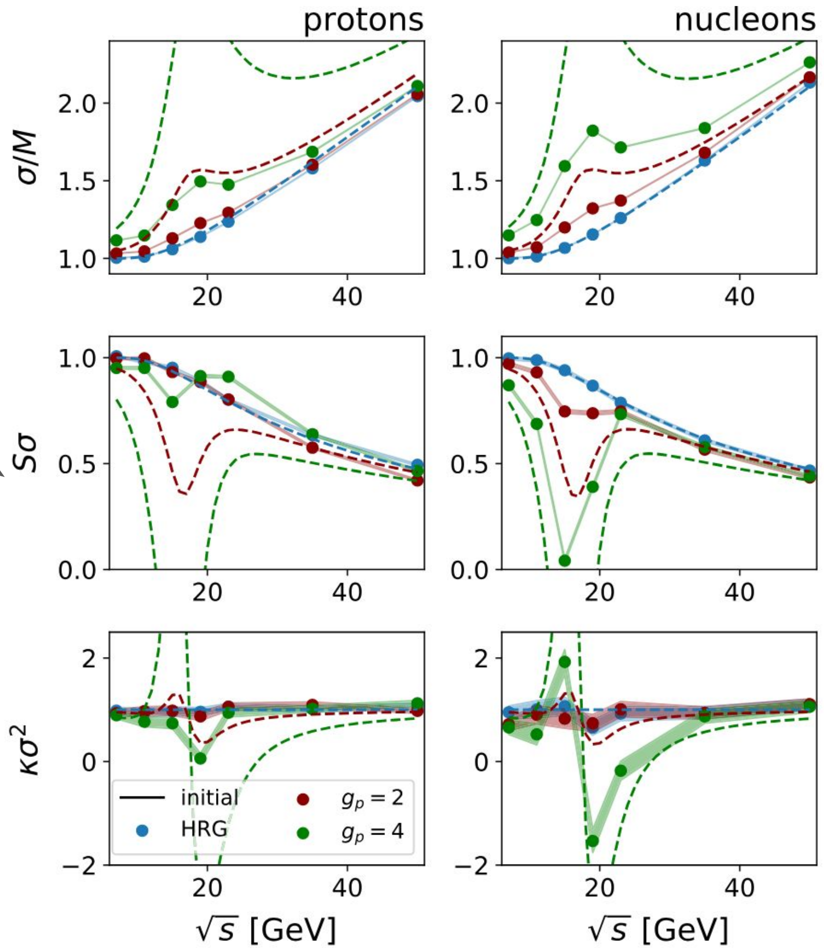
Fate of critical fluctuations in an interacting hadronic medium

Hammelmann,Bluhm,Nahrgang,Elfner,
Phys.Rev.C 110 (2024) 5, 054910

Expanding system cools down further

Isospin randomization processes greatly impacts the evolution of the cumulants

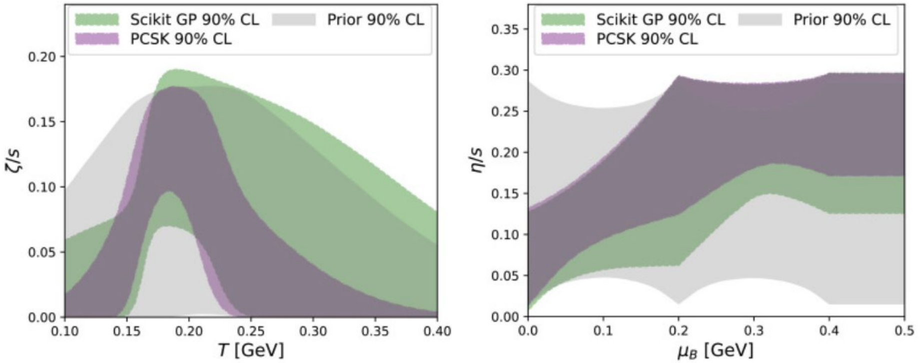




(3+1)D Bayesian analysis of RHIC BES data

Syed Afrid Jahan, Hendrik Roch, Chun Shen

Fig. 1 presents the 90 % confidence intervals (CI) for the prior and posteriors of the QGP shear and bulk viscosity as functions of net baryon chemical potential and temperature, respectively.



Download: Download high-res image (317KB)

Download: Download full-size image

Fig. 1. The net baryon chemical potential dependent QGP shear viscosity and temperature dependent bulk viscosity. The gray band represents the 90 % confidence intervals (CI) of their prior distribution, the green and pink band represents the 90 % CI of the posterior distributions obtained from the Scikit and PCSK emulators, respectively. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article).

Uncertainty quantification and computational challenges

- Theoretical uncertainties on the location of the CP, size and shape of the critical region (a.k.a strength of the critical point), curvature of the transition line.
- Theoretical uncertainties in transport coefficients and how they scale near the CP.
- Systematic uncertainties in the dynamical treatment of the collision -- different approaches need to be tested (hybrid and transport). How robust are the dynamics of the collision to different transport approaches? Same for different treatments of charge diffusion, critical fluctuations.
- Systematic uncertainties in treatment of finite-size, volume fluctuation, and charge conservation effects.
- What data goes into a Bayesian analysis? How do we define the likelihoods?
- Computational cost of sampling + simulating across theory/modeling parameter space. ML very useful :)

Summary (dynamics)

- There is a zoo of models for RHIC BES energies, covering a region of √s=5-200 GeV.
- There is WIP to extend the picture further down to a few GeV, with either dynamical fluidisation or multi-fluid dynamics.
- Most of models cover most of basic observables: rapidity distributions, net protons, radial flow, elliptic flow, (triangular flow). HBT seems to be not much off from the data.
- There has been a few Bayesian analyses that
- The studies above are done mostly with crossover-type EoS. There has been no constraints on the EoS yet.
- Hadronic cascades allow to emulate EoS softening (~1PT EoS) but fundamentally their applicability stops (gradually) at around sqrt(s)=10 GeV.
- Effects of spinodal instabilities in the EoS: clumping in coordinate space, weak effects in momentum space.
- As we have seen in the WG1 (flow), modelling of the early stage probably differs between the models, which presumably translates in different directed flow v_1.
- However, it may not introduce big uncertainties in the signals of critical behaviour, as the latter happens close to the phase bonudary.

A path forward

- Figure out discrepancies between the models (wherever possible)
  E.g. parametrized initial state vs. initial state from transport

- Bayesian analysis to provide constraints on the EoS
  For that, one needs an EoS with parametrized location of CP

- Get more consistent experimental data (does not depend on us)

Introducing critical behaviour in transport & hydro

(should be discussed in the corresponding WP)

- Vector density functional in SMASH: A. Sorensen, V. Koch, 2020
  Skyrme potential; 4-term polynomial in vector density omega.
- Savchuk, Poberezhnyuk, Motornenko, Steinheimer, Gorenstein, Vovchenko, 2022
- **[Critical dynamics through the transport coefficients**: Bulk viscosity near a critical point: Monnai et al, 2017; Shear+Bulk near a critical point Dore et al, 2020; + Diffusion Du et al, 2021]
- Lipei Du, Xin An, Ulrich Heinz, Phys. Rev. C 104, 064904
  critical scaling of the relaxation time for the baryon diffusion current ($\tau_{pi} \sim \xi^2$); critical scaling for kappa (baryon diffusion); 1D, BEShydro by Lipei Du.
  Israel-Stewart type equation for the baryon diffusion current plays the role of a single-mode Hydro+ equation for a vector slow mode $q \sim \xi^{-1}$
- hydro+CMF dynamics. Nahrgang, et al, Phys. Rev. C 84, 024912 (2011)
- Hydro+ (Misha Stephanov)
- Stochastic hydro with noise enhanced around CP

The BEST framework for the search for the QCD critical point and the chiral magnetic effect

BEST Collaboration, 2108.13867

of the RHIC BES, BESII. Second, and this is at the heart of the effort we are reporting on here, to definitively claim or rule out the presence of a QCD critical point or anomalous transport requires a comprehensive framework for modeling the salient features of heavy ion collisions at BES energies which allows for a *quantitative* description of the data. A crucial aspect of this effort is the need to embed equilibrium quantities like the critical equation of state and anomalous conservation laws into a dynamical scheme. This framework correlates different observables, predicts the magnitude of the expected effects, includes "conventional" backgrounds, and relates a possible discovery at a given beam energy, nuclear species and impact parameter to the existence of a phase boundary or a critical point at a location (μ_B, T) in the phase diagram.

+ accounts for statistical and systematic uncertainties

(3+1)D Bayesian analysis of RHIC BES data

Syed Afrid Jahan, Hendrik Roch, Chun Shen

Fig. 1 presents the 90 % confidence intervals (CI) for the prior and posteriors of the QGP shear and bulk viscosity as functions of net baryon chemical potential and temperature, respectively.

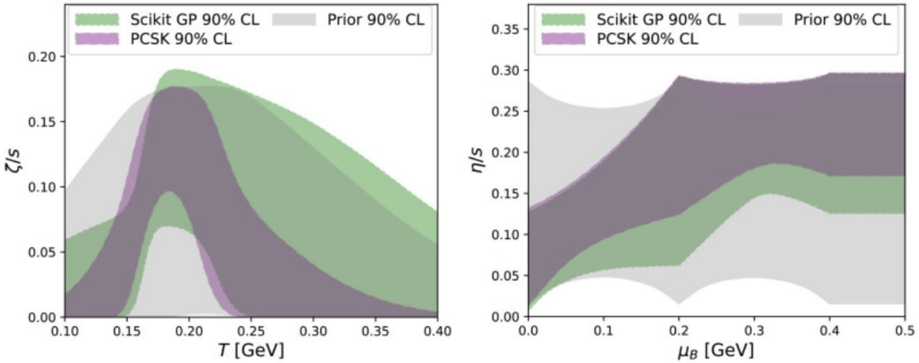


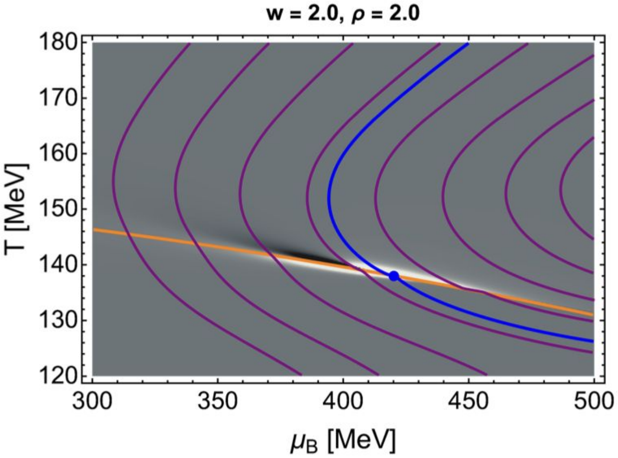
Fig. 1. The net baryon chemical potential dependent QGP shear viscosity and temperature dependent bulk viscosity. The gray band represents the 90 % confidence intervals (CI) of their prior distribution, the green and pink band represents the 90 % CI of the posterior distributions obtained from the Scikit and PCSK emulators, respectively. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article).

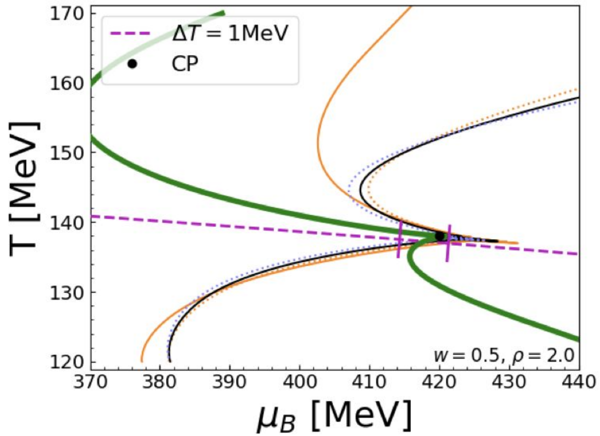
Uncertainty quantification and computational challenges

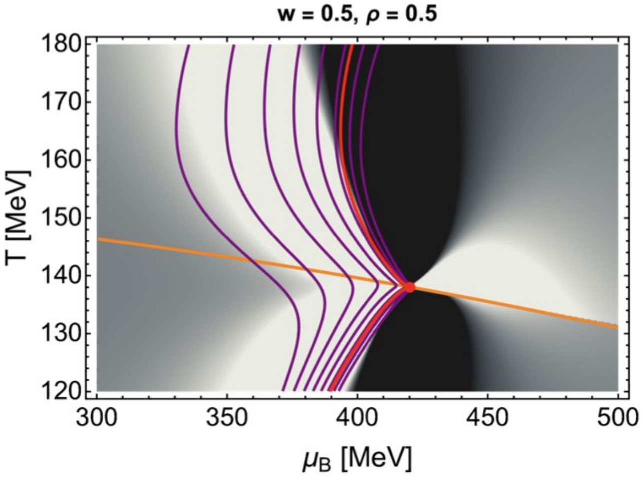
- Theoretical uncertainties on the location of the CP, size and shape of the critical region (a.k.a strength of the critical point), curvature of the transition line.
- Theoretical uncertainties in transport coefficients and how they scale near the CP.
- Systematic uncertainties in the dynamical treatment of the collision -- different approaches need to be tested (hybrid and transport). How robust are the dynamics of the collision to different transport approaches? Same for different treatments of charge diffusion, critical fluctuations.
- Systematic uncertainties in treatment of finite-size, volume fluctuation, and charge conservation effects.
- What data goes into a Bayesian analysis? How do we define the likelihoods?
- Computational cost of sampling + simulating across theory/modeling parameter space. ML very useful :)

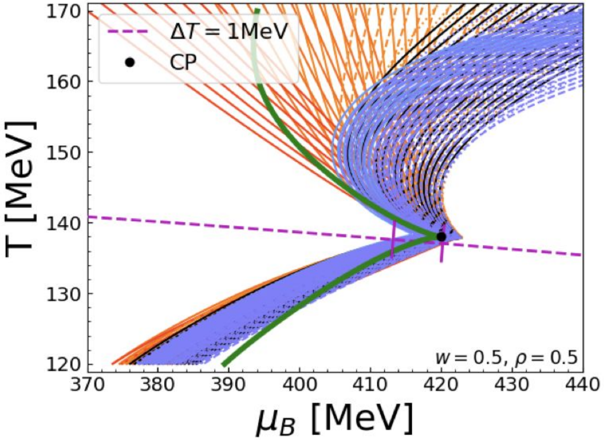
T. Dore,J. M. Karthein, I. Long D. Mroczek, et al. PRD (2022)

ideal vs. viscous









STASH

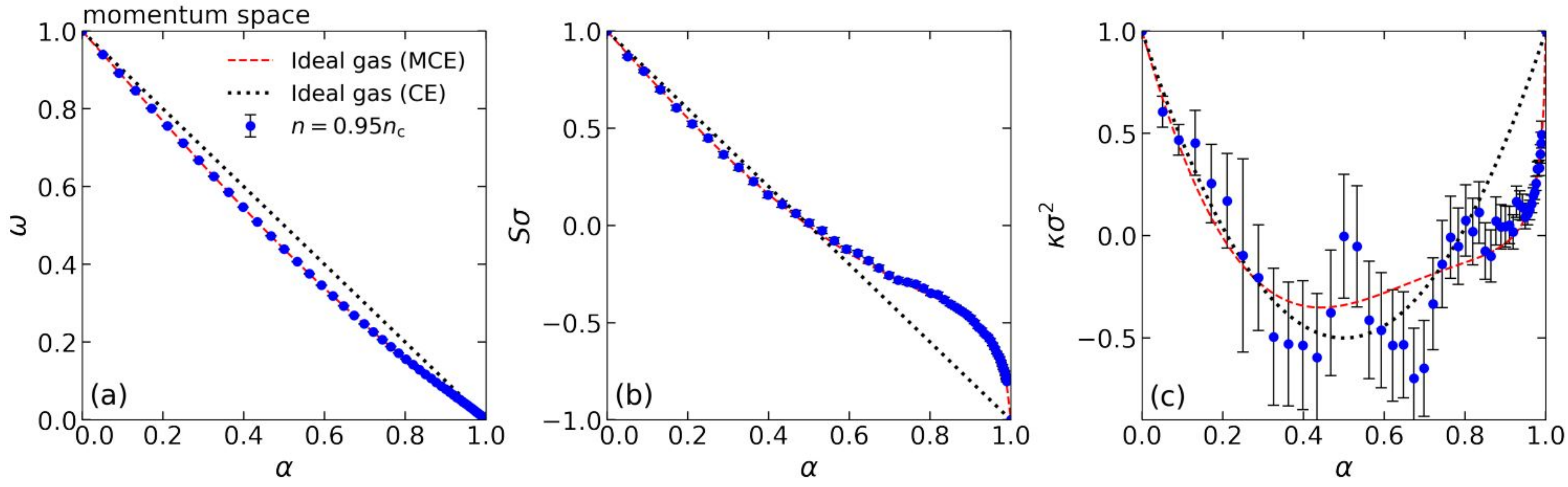
Backup for cumulants (Ideal gas)

$$\Delta\omega^{\mathrm{id}}(\alpha) = -\frac{2e^{-2\mathrm{erf}^{-1}(\alpha)^2}(\mathrm{erf}^{-1}(\alpha))^2}{3\pi\alpha},$$

V. Kuznietsov et al. arXiv:2511.00755v1

$$\Delta(S\sigma)^{\mathrm{id}}(\alpha) = -\frac{2e^{-\mathrm{erf}^{-1}(\alpha)^2}\mathrm{erf}^{-1}(\alpha)^2(6\ e^{\mathrm{erf}^{-1}(\alpha)^2}\sqrt{\pi}\ (2\alpha-1)+\mathrm{erf}^{-1}(\alpha)-6\ \mathrm{erf}^{-1}(\alpha)^3)}{3\sqrt{\pi}(3\pi\ e^{2\mathrm{erf}^{-1}(\alpha)^2}(\alpha-1)\alpha+2\ \mathrm{erf}^{-1}(\alpha)^2)}$$

$$\Delta(\kappa\sigma^2)^{\mathrm{id}}(\alpha) = \frac{4\ e^{-2\mathrm{erf}^{-1}(\alpha)^2}\mathrm{erf}^{-1}(\alpha)^2(27e^{2\mathrm{erf}^{-1}(\alpha)^2}\pi(1-5\alpha+5\alpha^2)+9\ e^{\mathrm{erf}^{-1}(\alpha)^2}\sqrt{\pi}(2\alpha-1)\mathrm{erf}^{-1}(\alpha)^2+36\ \mathrm{erf}^{-1}(\alpha)^2)}{9\pi(3e^{2\mathrm{erf}^{-1}(\alpha)^2}\pi(\alpha-1)\alpha+2\ \mathrm{erf}^{-1}(\alpha)^2)}$$



Backup for cumulants (Lennard-Jones)

V. Kuznietsov et al. arXiv:2511.00755v1