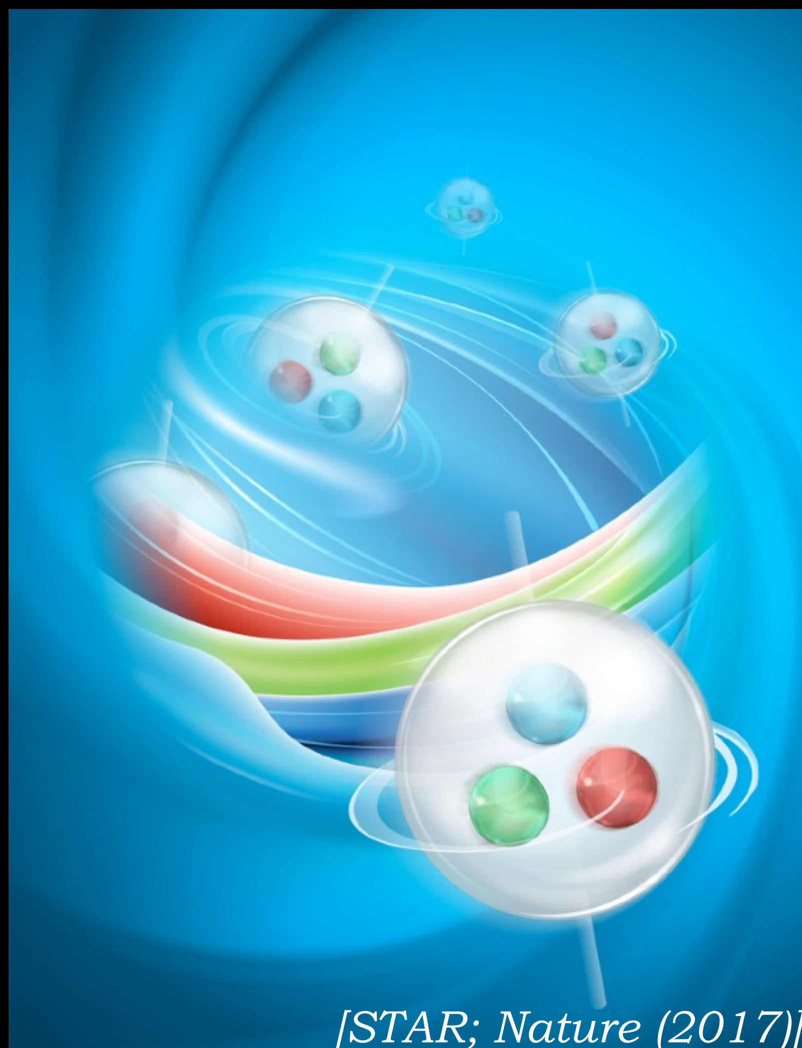
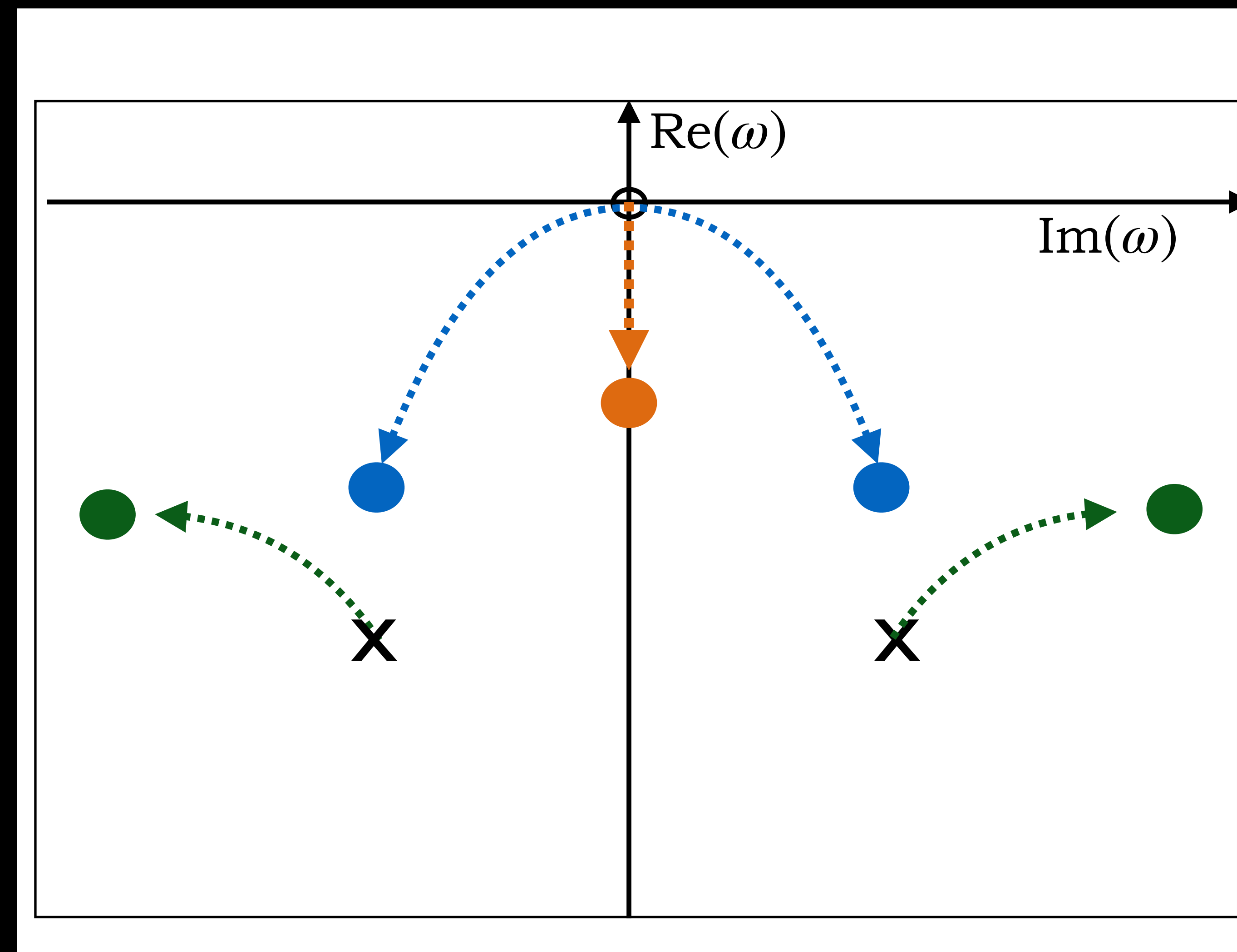
Convergence of hydrodynamics in rapidly spinning strongly coupled plasma

Chirality and Criticality: Novel Phenomena in Heavy-Ion Collisions, INT, University of Washington, Seattle

August 24th, 2023



[STAR; Nature (2017)]



[Garbiso-Amano,Kaminski; JHEP (2020]
[Cartwright,Garbiso-Amano;Kaminski,Noronha,Speranza; PRD (2023)]
[Cartwright,Garbiso-Amano;Kaminski,Wu; arXiv:2308.11686]

related: [Hongo,Huang,Kaminski,Stephanov,Yee; JHEP (2021)]

Matthias Kaminski
University of Alabama

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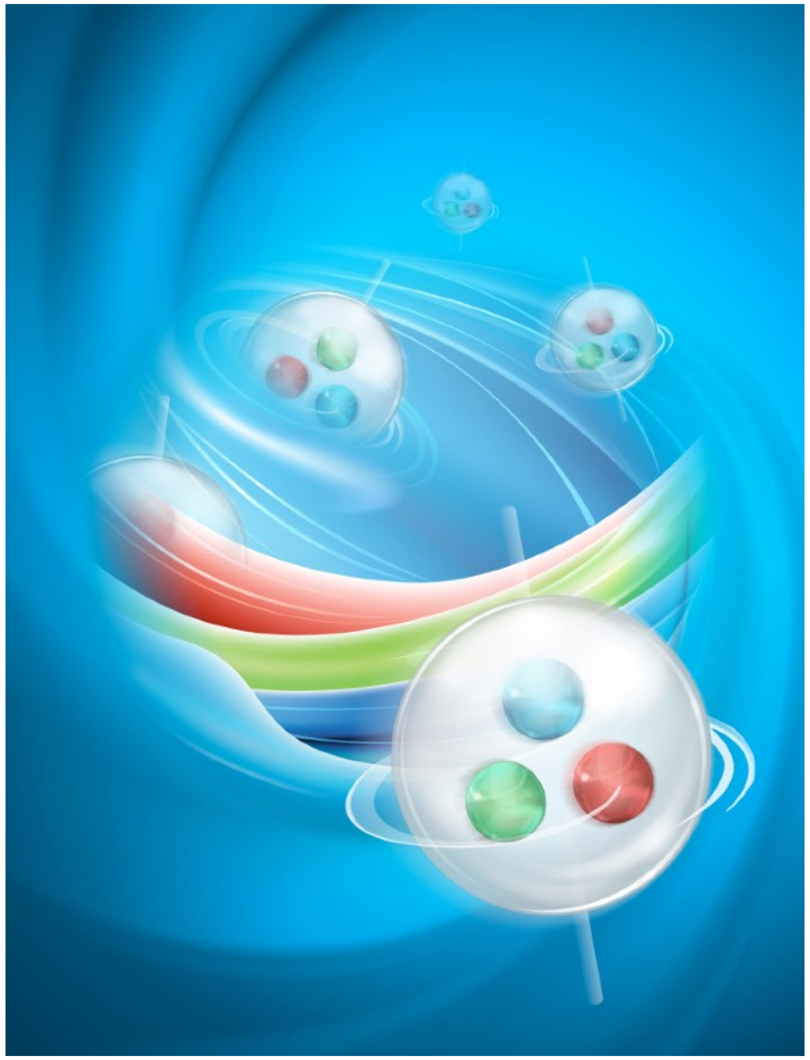
U.S. DEPARTMENT OF ENERGY

Experimental: The most vortical fluid



[STAR; Nature (2017)]

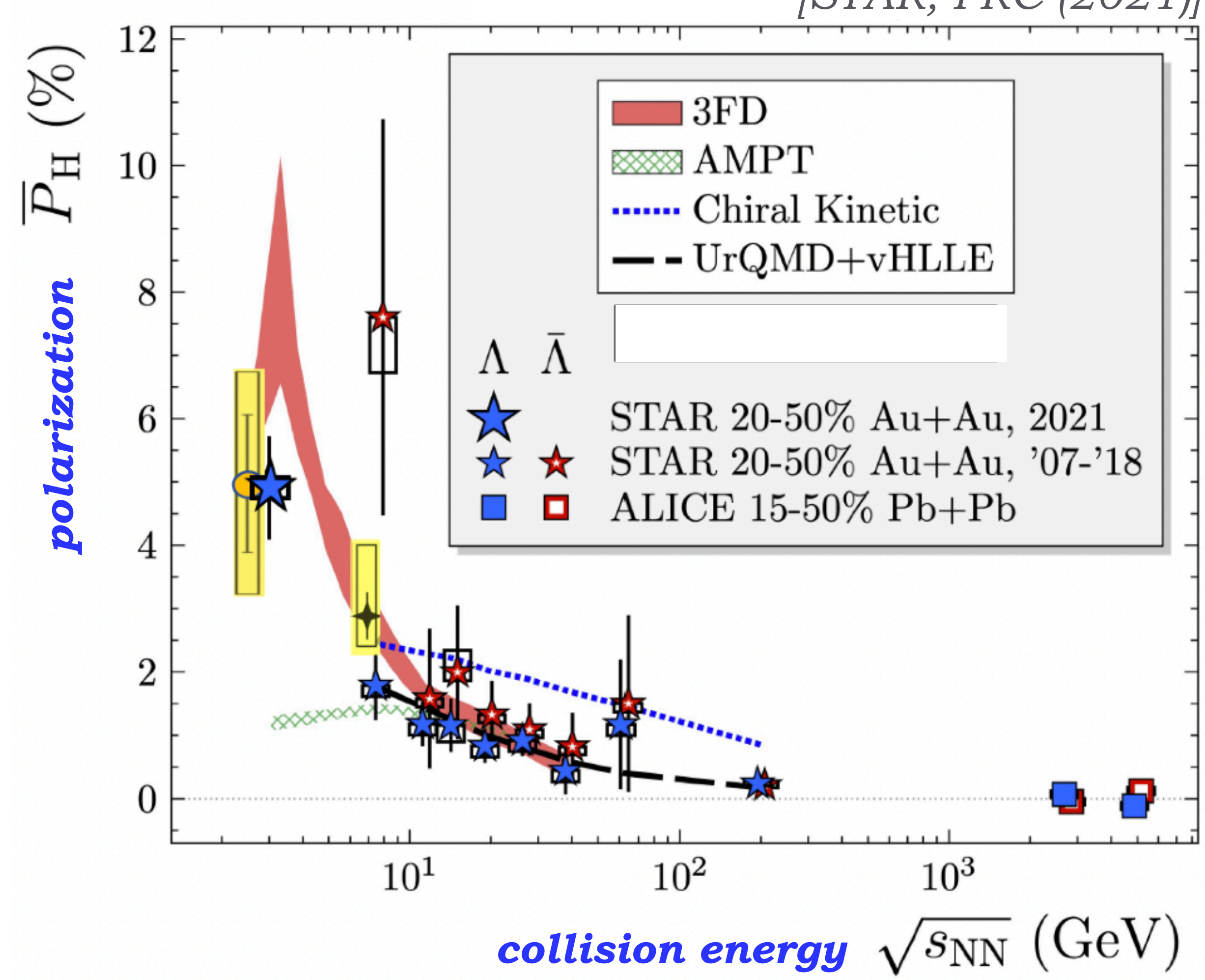
➡ Hydrodynamic description including vorticity and spin

[Becattini, …]
[Florkowski, Ryblewski, …]
[Rischke, Speranza, Weickgenannt, …]
[Hongo, Huang, Kaminski, Stephanov, Yee; JHEP (2021)]
…

→ ***Talks by Buzzegoli, Lin, Singh***

- Λ hyperon polarization measured
- highly vortical quark-gluon-plasma

[STAR; PRC (2021)]



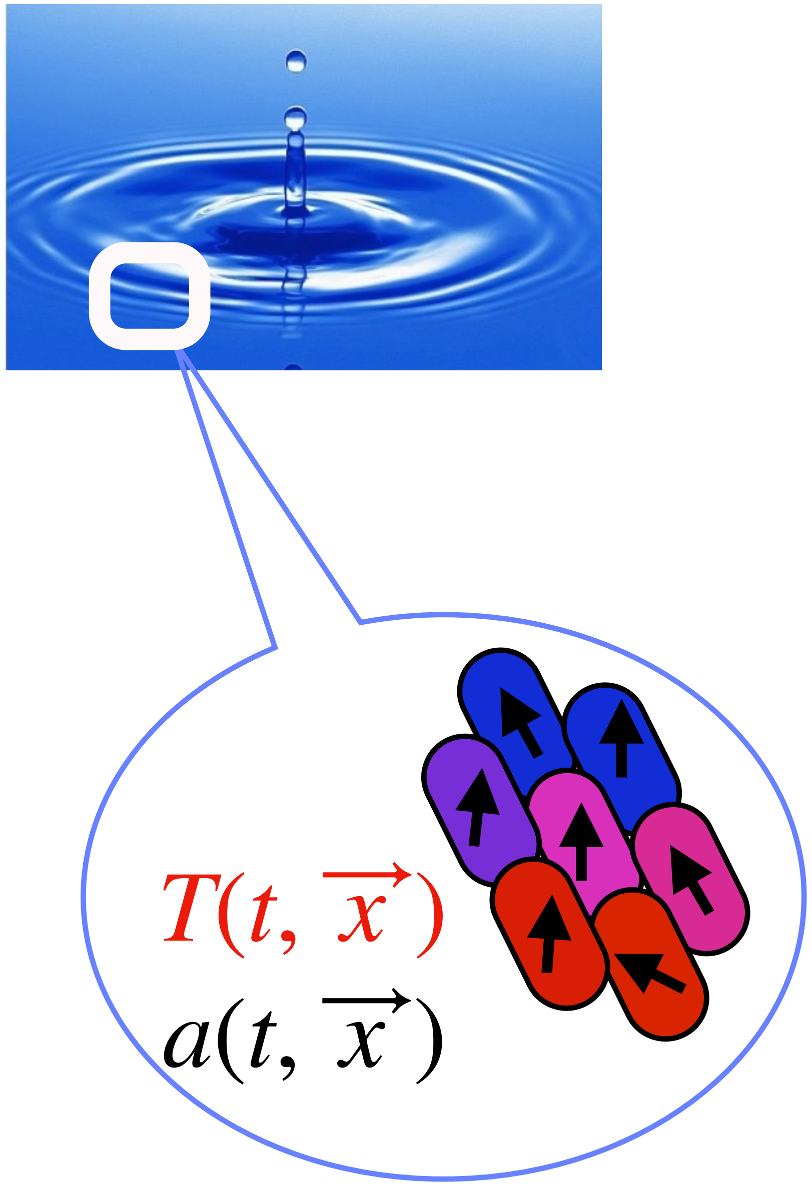
[from slides by Michael Lisa at the 6th International Conference on Chirality, Vorticity and Magnetic Field in Heavy Ion Collisions (Nov 1-5, 2021)]

Physical questions about rotating plasma

1. Can it be in global/local thermal equilibrium?

— Yes.

- **rotating black hole is in equilibrium**
- **dual fluid flow is highly vortical**



fluid cells with distinct temperatures T, and distinct angular momentum eigenvalues a

Physical questions about rotating plasma

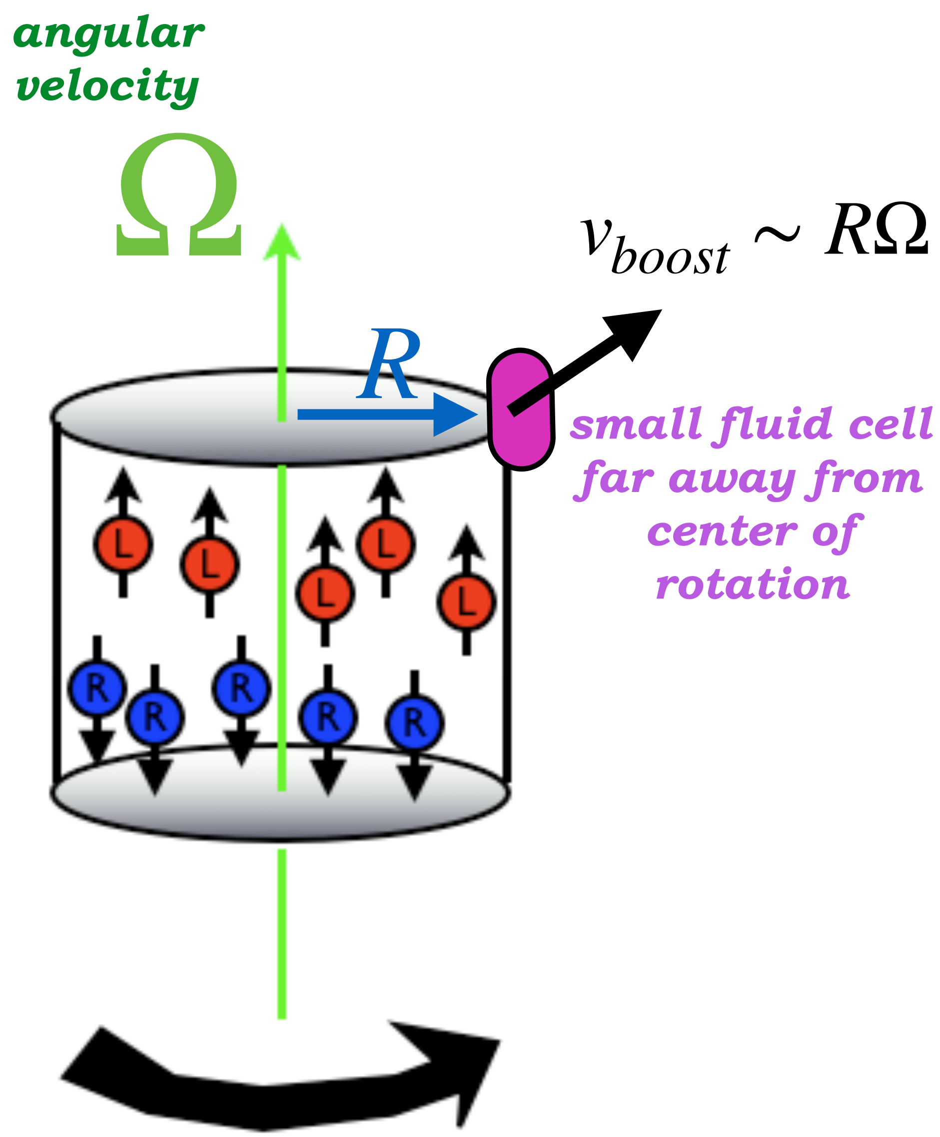
1. Can it be in global/local thermal equilibrium?

— Yes.

2. Can it be approximated by a boosted fluid?

— Yes, BUT … .

- **hydrodynamic fluctuations around hot rotating holographic fluid <u>see</u> boosted fluid**
- **<u>radius of convergence</u> of hydrodynamics mostly unaffected by rotation**



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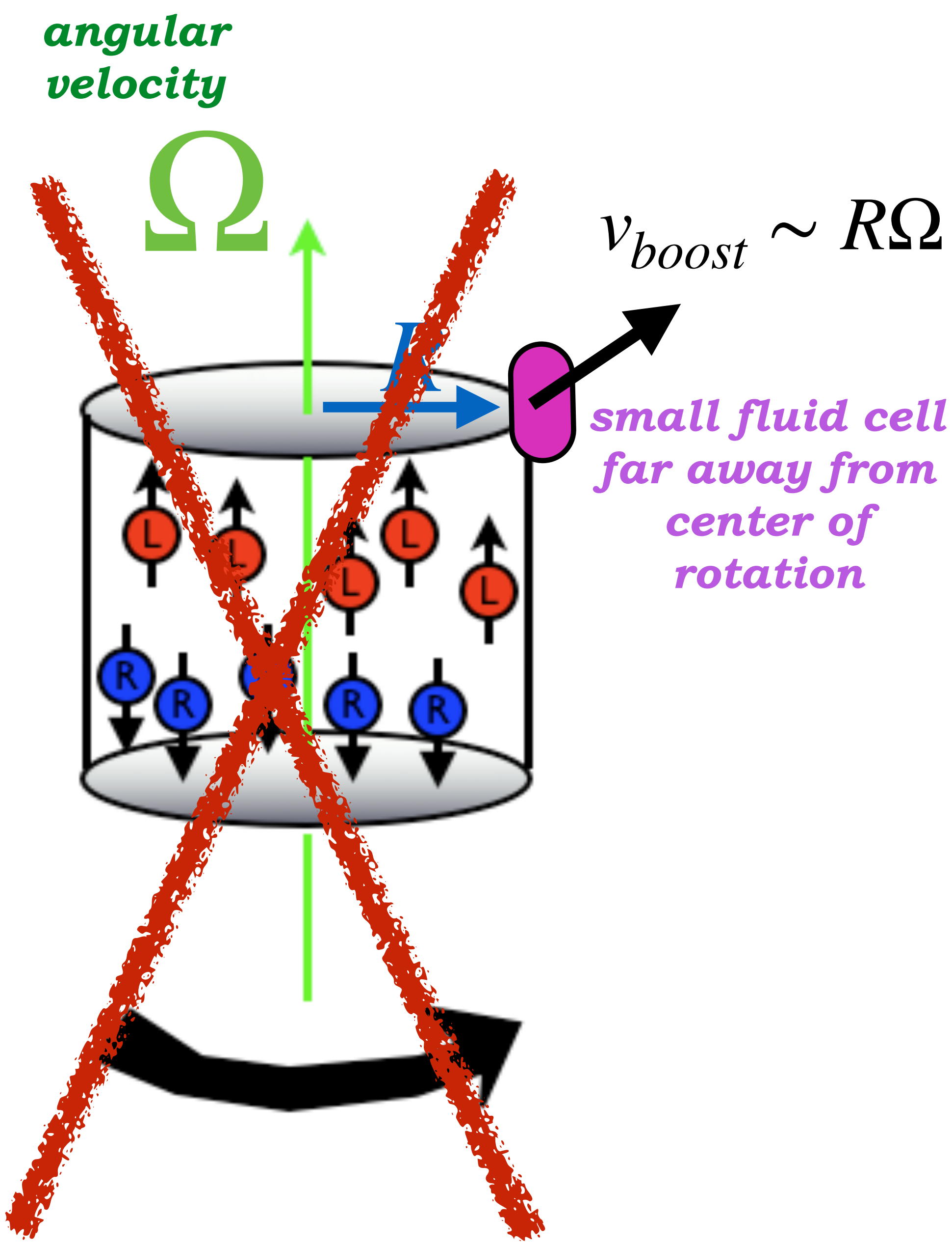


Physical questions about rotating plasma

1. Can it be in global/local thermal equilibrium?

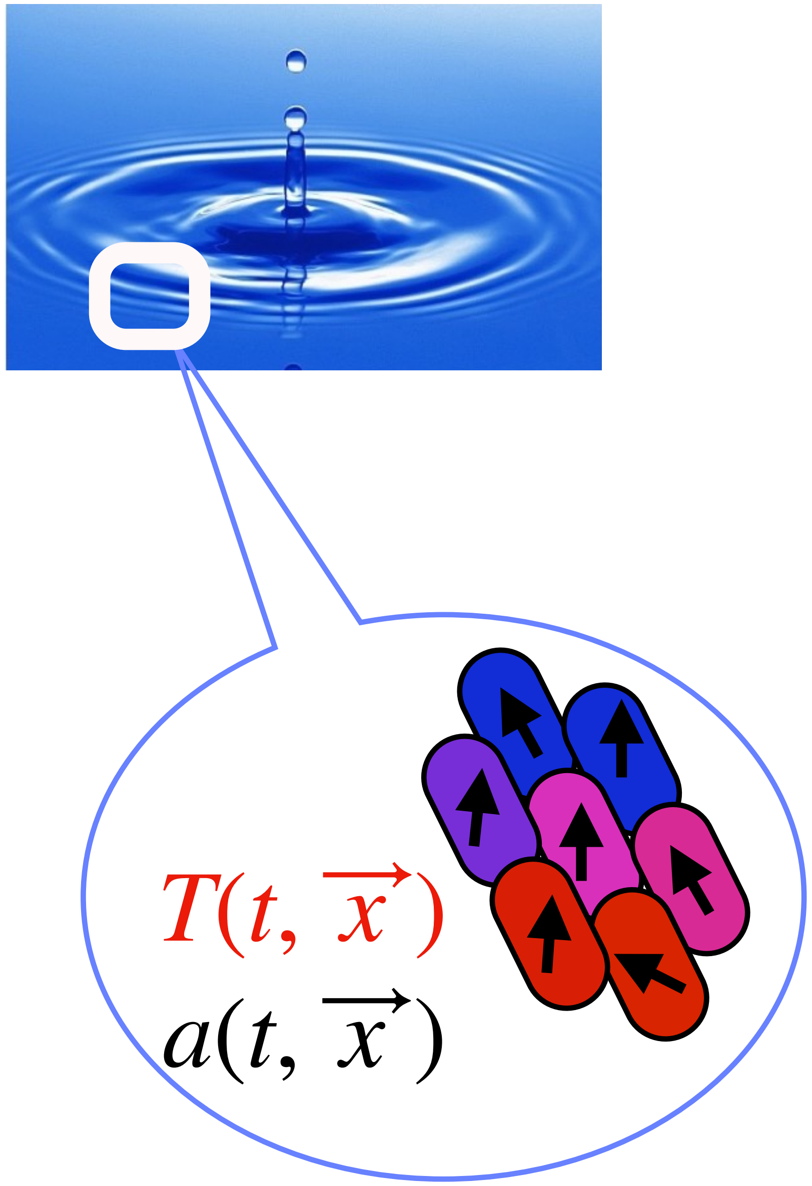
   — Yes.

2. Can it be approximated by a boosted fluid?

   — Yes, BUT … .

3. Is the hydrodynamic description valid?

   — Yes, with modifications.
   - **transport coefficients change**
   - **constitutive equations change**



Momentum diffusion mode

$$\omega(k) = -\, iDk^2 + \mathcal{O}(3)$$

➡ diffusion coefficient *D* is function of state variables: *T*, *a*

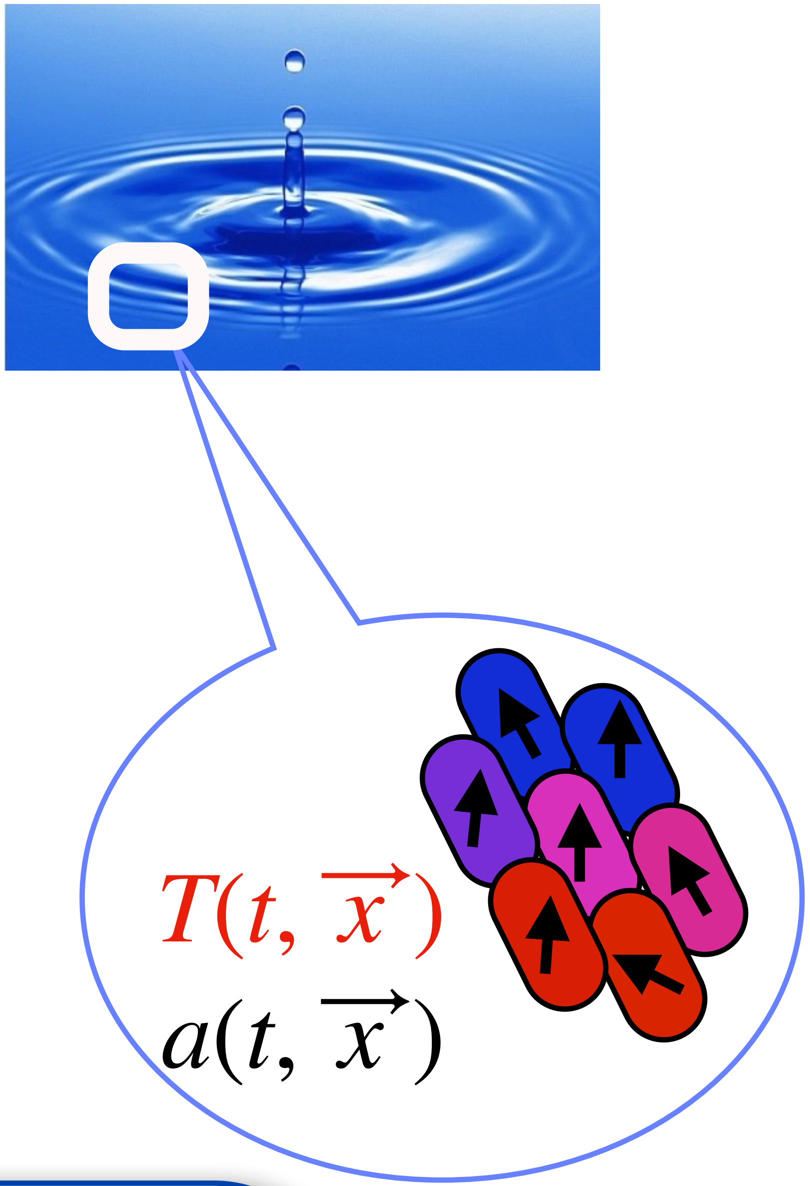
Outline

0. Definitions: Hydrodynamics, Holography

1. Rotating fluid state (thermodynamics)

2. Large T limit: boosted fluid hydrodynamics

3. Rotating fluid hydrodynamics valid at smaller T



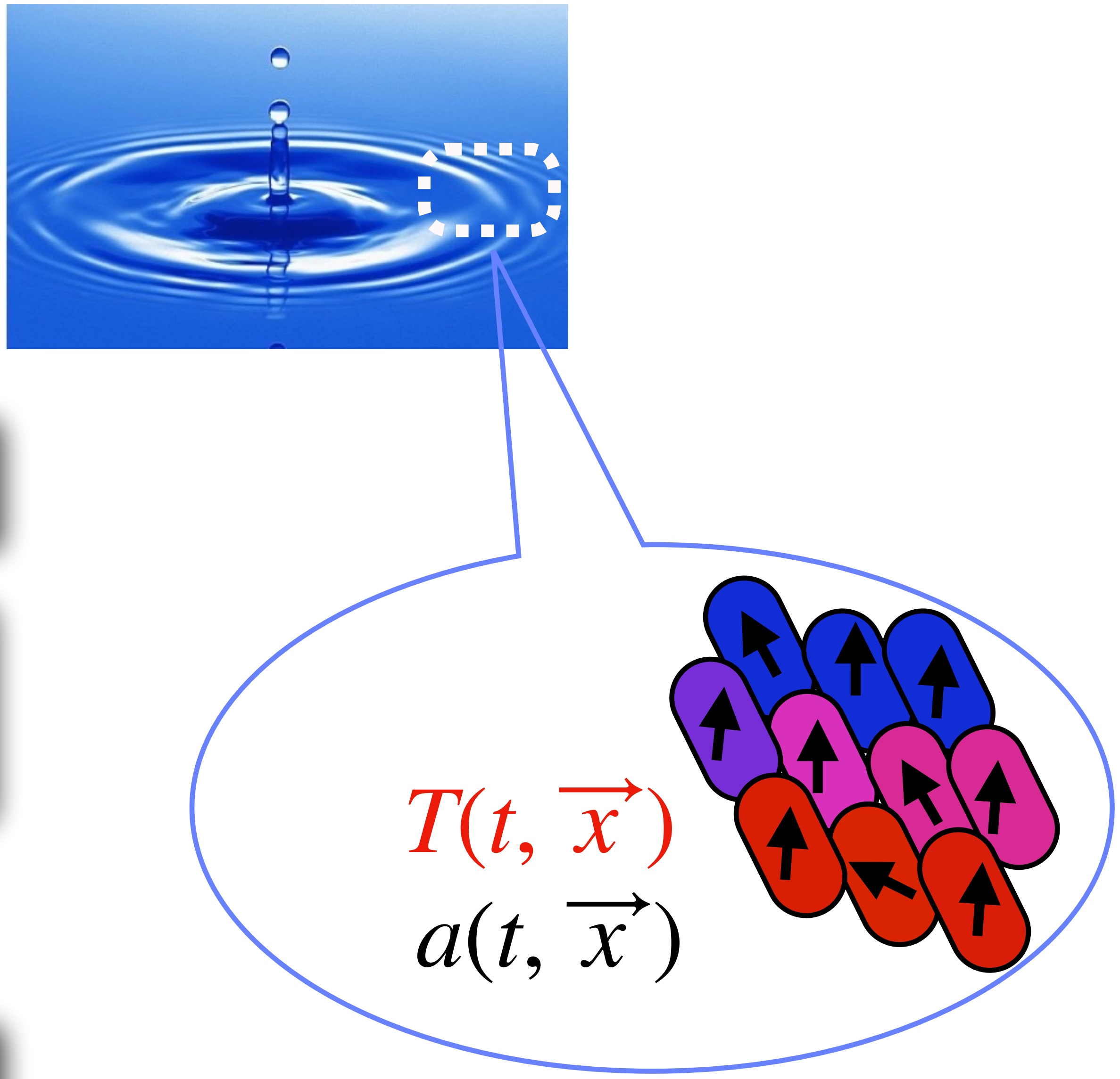
What is the range of applicability (convergence radius) of the linear hydrodynamic description of a rapidly rotating strongly coupled $\mathcal{N}$=4 SYM plasma?

0. Definitions

What is hydrodynamics?

Hydrodynamics

- effective description of systems at late times and large distances
- small gradients $\partial_t \, e^{-i\omega t} = -i\omega \, e^{-i\omega t}$
- large temperature $\frac{\omega}{T} \ll 1, \quad \frac{|\vec{k}|}{T} \ll 1$
- conserved quantities survive $n(t, \vec{x}) \propto e^{-i\omega t + i \vec{k} \cdot \vec{x}_3} \, n(\omega, \vec{k})$
- can express expectation values and **Green's functions** of the energy-momentum tensor



Example: rotation-invariant fluid

Momentum diffusion mode

$\omega(k) = -\,iDk^2 + \mathcal{O}(3)$

Sound modes

$\omega(k) = \pm v_s k - i\Gamma k^2 + \mathcal{O}(3)$

What is holography?

Holography

[Kovtun/Starinets; JHEP (2005)]

- consider **Einstein gravity** which is dual to $\mathcal{N}$=4 SYM theory and derive Einstein equations
- metric of a **rotating asymptotically AdS5 black hole** (solution to Einstein equations) is dual to a rotating thermal SYM state
- **black hole thermodynamics** "determines" thermodynamic properties of the dual SYM state
- poles of the SYM Green's functions are dual to quasi normal mode (QNM) frequencies of black holes: **QNMs encode SYM dispersion relations**

➡Compute the QNM frequencies around rotating black hole as function of momentum.

What is holography?

Holography

Example: rotation-invariant fluid from QNMs of metric fluctuations *[Kovtun/Starinets; JHEP (2005)]*

Momentum diffusion mode

$\omega(k) = -iDk^2 + \mathcal{O}(3)$ $\longleftrightarrow$ $\delta g_{tx}, \delta g_{zx}, \ldots$ **(vector)**

Sound modes

$\omega(k) = \pm v_s k - i\Gamma k^2 + \mathcal{O}(3)$ $\longleftrightarrow$ $\delta g_{tt}, \delta g_{tz}, \delta g_{zz}$ **(scalar)**

- poles of the SYM Green's functions are dual to quasi normal mode (QNM) frequencies of black holes: **QNMs encode SYM dispersion relations**

➡Compute the QNM frequencies around rotating black hole as function of momentum.

1. Rotating fluid state (thermodynamics)

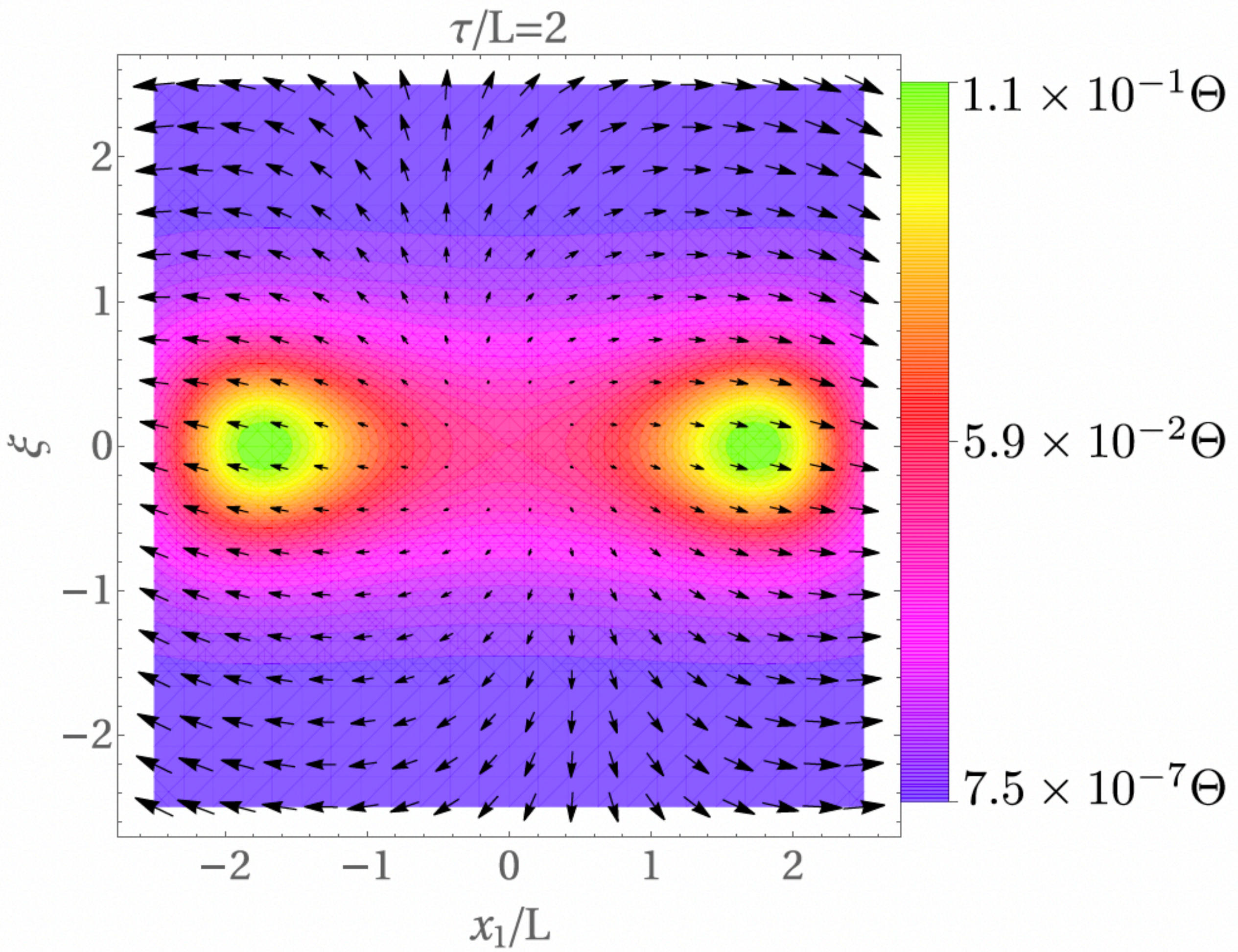
Thermal vortical equilibrium state

Rotating AdS5 black hole

$$ds^2 = -\left(1+\frac{r^2}{L^2}\right)dt^2 + \frac{dr^2}{G(r)} + \frac{r^2}{4}\left((\sigma^1)^2 + (\sigma^2)^2 + (\sigma^3)^2\right) + \frac{2\mu}{r^2}\left(dt + \frac{a}{2}\sigma^3\right)^2$$

$$G(r) = 1 + \frac{r^2}{L^2} - \frac{2\mu(1-a^2/L^2)}{r^2} + \frac{2\mu a^2}{r^4},$$

$$\mu = \frac{r_+^4\left(L^2+r_+^2\right)}{2L^2r_+^2 - 2a^2\left(L^2+r_+^2\right)},$$



Rotating thermal SYM state

$$u^\tau = \lambda\left[\cosh\xi\left(L^2+\tau^2+x_\perp^2\right) + 2\Omega(Lx_1\sinh\xi + \tau x_2)\right]$$

$$u^1 = \lambda\left[2(L\tau\Omega\sinh\xi + \tau x_1\cosh\xi + x_1x_2\Omega)\right],$$

$$u^2 = \lambda\left[\Omega\left(L^2+\tau^2-x_1^2+x_2^2\right) + 2\tau x_2\cosh\xi\right],$$

$$u^\xi = -\tau^{-1}\lambda\left[-\sinh\xi\left(L^2-\tau^2+x_\perp^2\right) - 2Lx_1\Omega\cosh\xi\right.$$

$$\epsilon = \left(16L^8\Theta^4\right)\left(1-\Omega^2\right)^{-2} \times \left(2L^2\tau^2\cosh 2\xi + \left(L^2+x_\perp^2\right)^2 + \tau^4 - 2\tau^2x_\perp^2\right)^{-2},$$

$$\lambda = \left(\frac{\epsilon}{16L^8\Theta^4}\right)^{1/4}, \quad \Theta = \left(\frac{3(1-\Omega^2)\mu}{8\pi G_5 L^3}\right)^{1/4},$$

$$r_+ \to \alpha r_+, \quad r \to \alpha r, \quad \alpha \to \infty$$

[Bantilan, Ishii, Romatschke; PLB (2018)]

Milne coordinates $(\tau, x_1, x_2, \xi; r)$

$$\xi = \tfrac{1}{2}\ln[(t+x_3)/(t-x_3)] \qquad \tau = \sqrt{t^2 - x_3^2}$$

Thermal vortical equilibrium state

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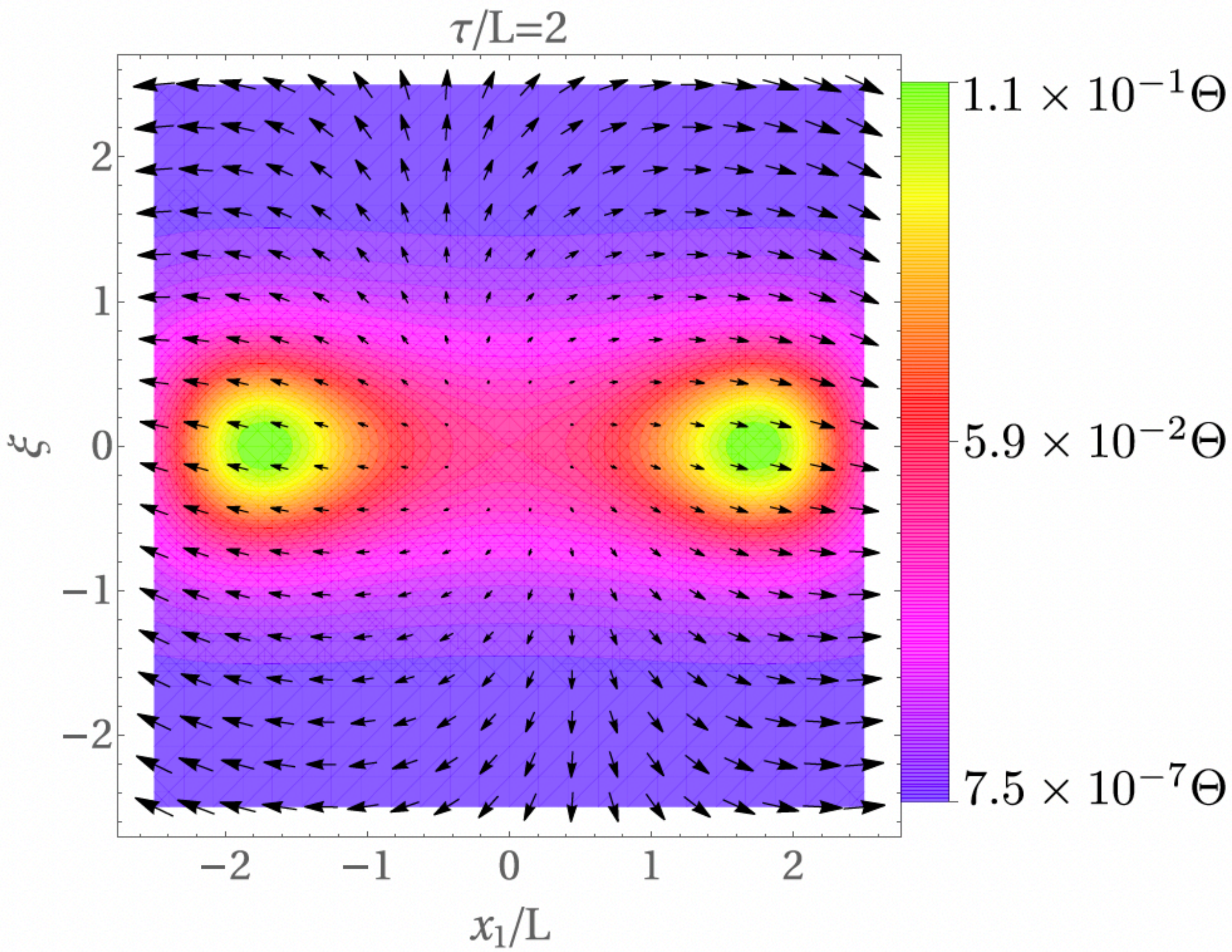

Rotating thermal SYM state

analytic fluid flow (cf. Gubser flow)

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Large black holes: large T

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Result 1: Rotating fluid flow (cf. Gubser flow)

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— Yes.

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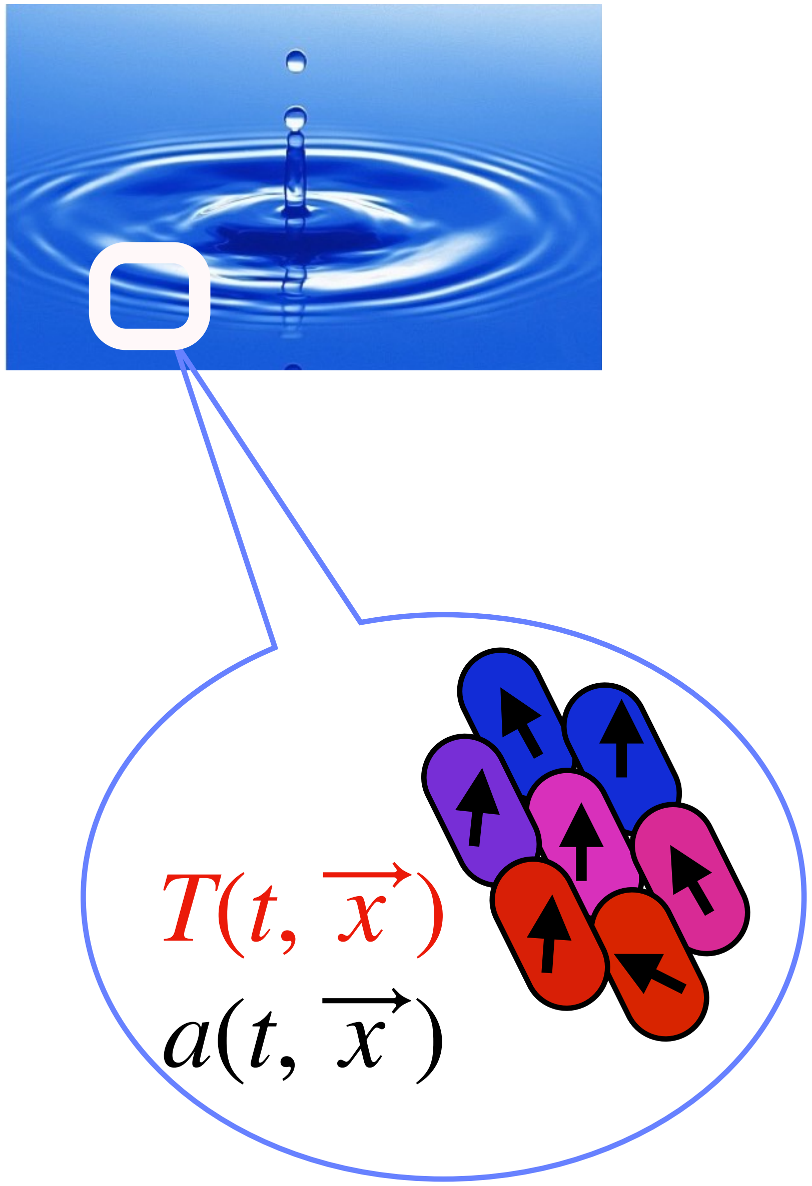
- **dual analytic fluid flow is highly vortical**



fluid cells with distinct temperatures T, and distinct angular momentum eigenvalues a

2. Large T limit: boosted fluid hydrodynamics

Hydrodynamic modes

Interacting many-body systems at large temperature T have collective excitations, damped **eigenmodes**, with specific dispersion relations :

(assuming rotation invariance: $k \equiv |\vec{k}|$)

Sound modes

$$\omega(k) = \pm v_s k - i\Gamma k^2 + \mathcal{O}(3)$$

Momentum diffusion mode

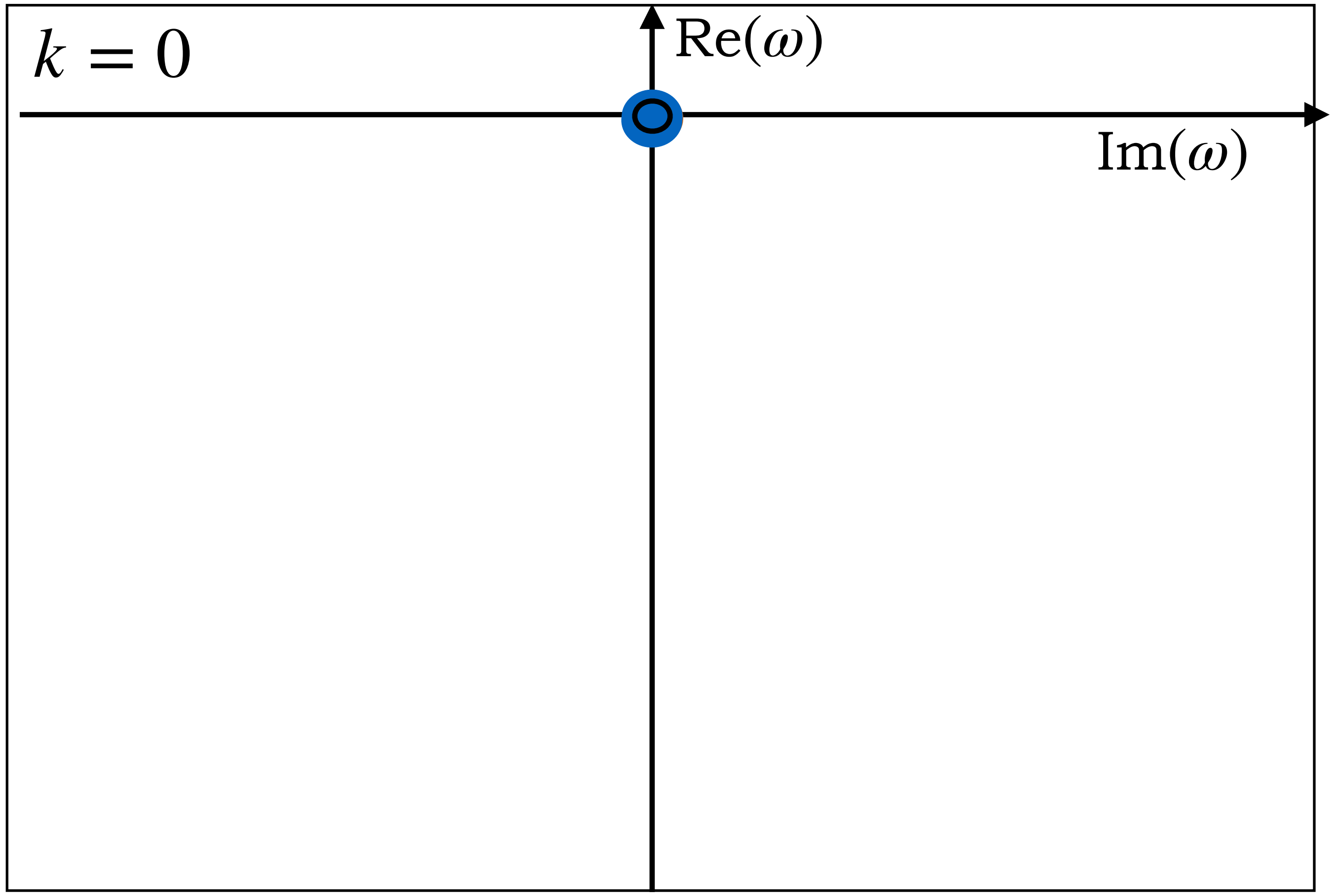
$$\omega(k) = -iDk^2 + \mathcal{O}(3)$$

$$\mathcal{P}\,\phi = 0$$

linear equation of motion for conserved quantity

$$\mathcal{P}\,G^R = \delta$$

$$G^R_{diffusion} \propto \mathcal{P}^{-1}_{diffusion} \propto \frac{1}{\partial_t - D\partial_x^2 + \mathcal{O}(3)} \propto \frac{1}{\omega + iDk^2 + \mathcal{O}(3)}$$



Complex frequency plane

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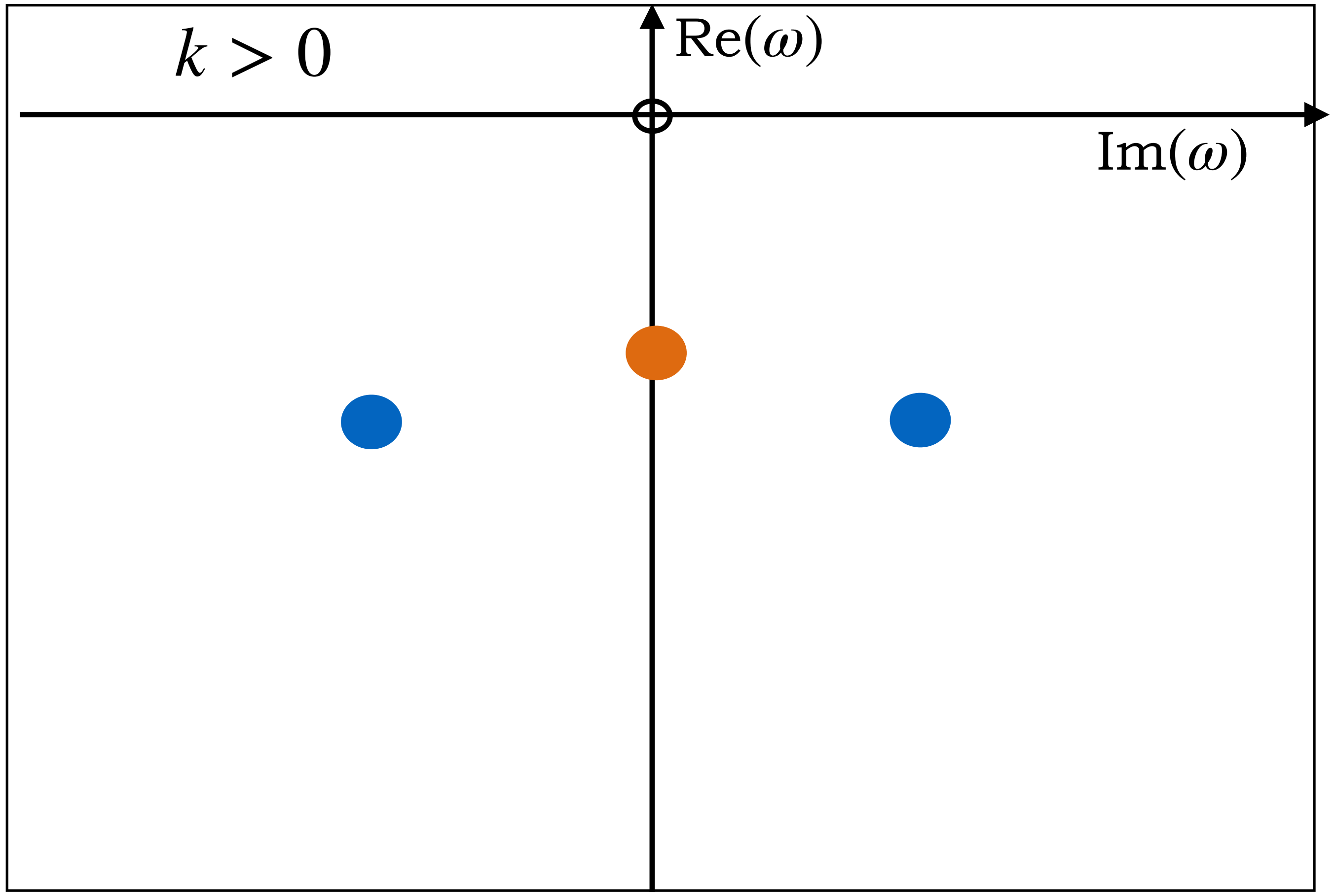

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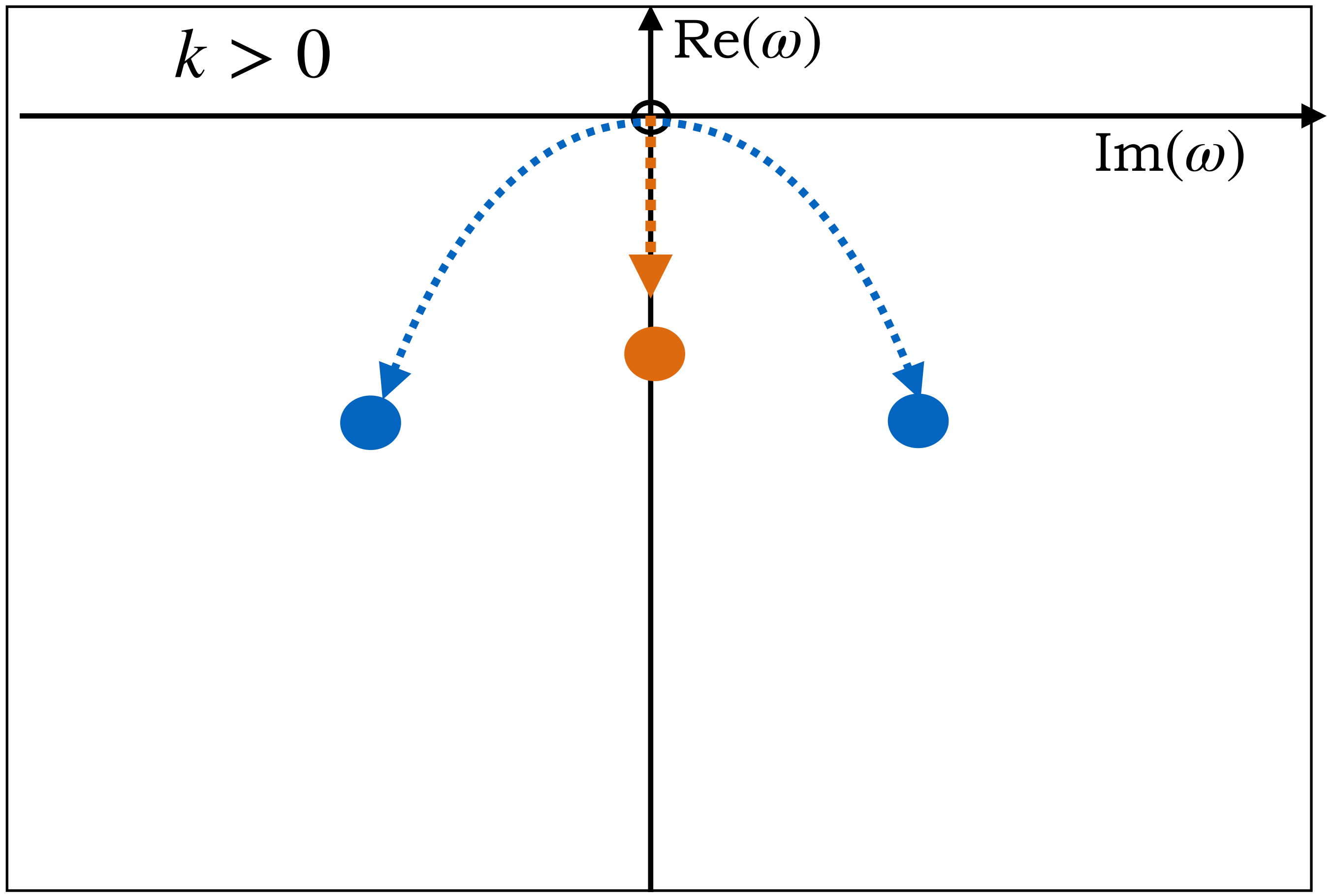

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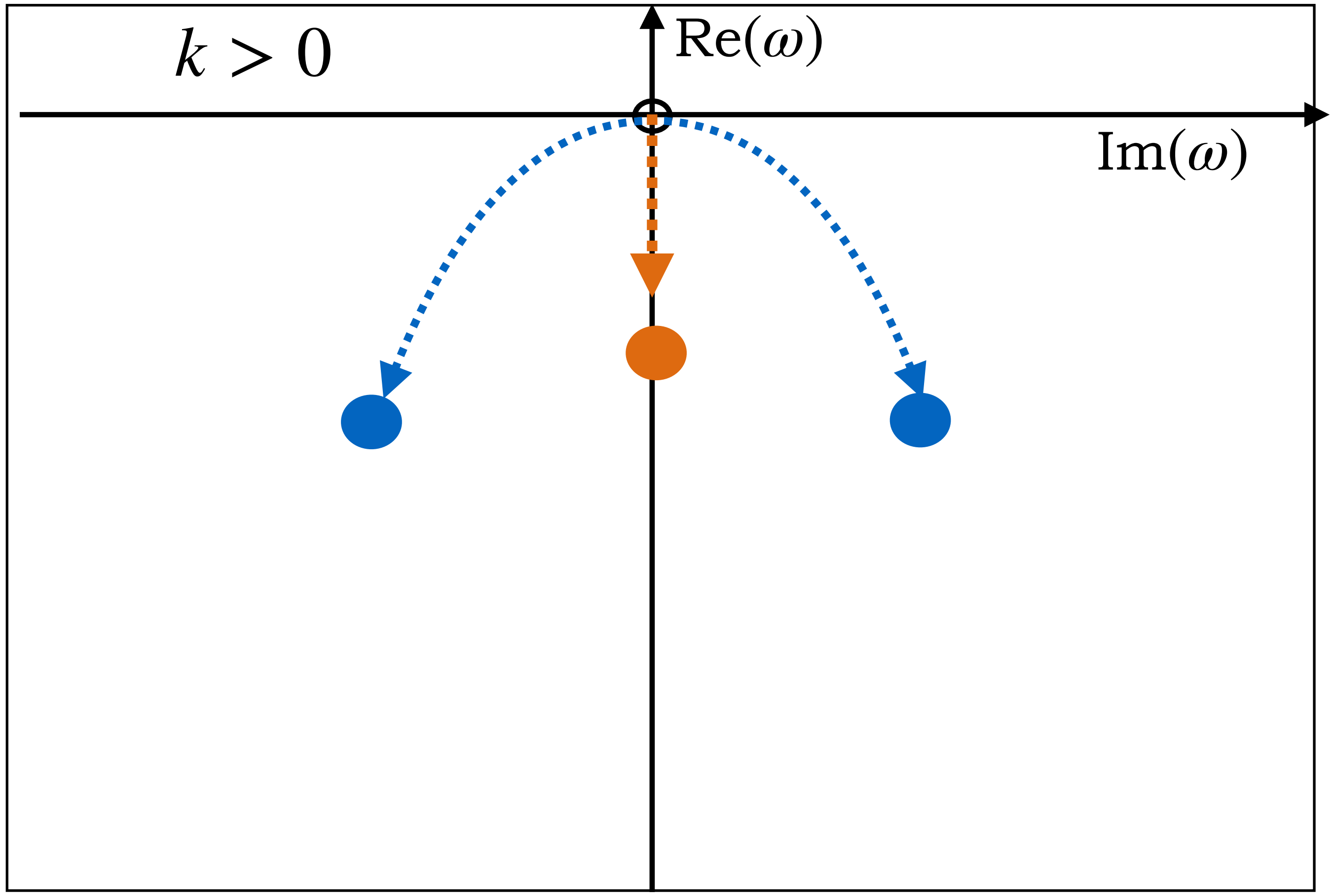

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➡**Compute** $\mathscr{P}(\omega, k) = 0$ **from holography:** $\mathscr{P} \sim |\delta g_{\mu\nu}|_{\textbf{boundary}}$

High temperature: dispersion relations look like boosted fluid

[Cartwright,Garbiso-Amano;Kaminski,Noronha,Speranza; PRD (2023)]
[Garbiso-Amano, Kaminski; JHEP (2019)]
cf. [Hoult,Kovtun (2020)] [Kovtun (2019)]

$$v_{||} = a, \qquad v_{s,\pm} = v_{s,0}\frac{\sqrt{3}a \pm 1}{1 \pm \frac{a}{\sqrt{3}}},$$

$$\mathcal{D}_{||} = \mathcal{D}_0(1-a^2)^{3/2}, \qquad \Gamma_{s,\pm} = \Gamma_0\frac{\left(1-a^2\right)^{3/2}}{\left(1 \pm \frac{a}{\sqrt{3}}\right)^3},$$

$$\mathcal{D}_{||}(a) = 2\pi T_0 \frac{\eta_{||}(a)}{\epsilon(a) + P_\perp(a)},$$

$$\Gamma_\pm(a) = \frac{2\eta_{||}(a)}{3(\epsilon(a) + P_\perp(a))}\frac{1}{(1 \pm a/\sqrt{3})^3}.$$

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Dispersion relations:

$$\nu(j) = -aj - i\frac{1}{2}(1-a^2)^{3/2} j^2 + \mathcal{O}(j^3)$$

$$\nu(j) = \frac{\pm 1 - \sqrt{3}a}{\sqrt{3} \mp a} j - i\sqrt{3}\frac{(1-a^2)^{3/2}}{(\sqrt{3}-a)^3} j^2 + \mathcal{O}(j^3)$$

Boost transformation:

$$\mathfrak{q}^2 = \frac{(a\nu + j)^2}{1-a^2}, \qquad \mathfrak{w}^2 = \frac{(\nu + aj)^2}{1-a^2}$$

"Speeds of diffusion":

$$v_{||} = a,$$

Speeds of sound:

$$v_{s,\pm} = v_{s,0}\frac{\sqrt{3}a \pm 1}{1 \pm \frac{a}{\sqrt{3}}},$$

Corresponding damping:

$$\mathcal{D}_{||} = \mathcal{D}_0(1-a^2)^{3/2}, \quad \Gamma_{s,\pm} = \Gamma_0\frac{\left(1-a^2\right)^{3/2}}{\left(1 \pm \frac{a}{\sqrt{3}}\right)^3},$$

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Shear viscosities:

$$\eta_\perp(a) = \eta_0\frac{1}{\sqrt{1-a^2}}, \quad \eta_{||}(a) = \eta_0\sqrt{1-a^2},$$

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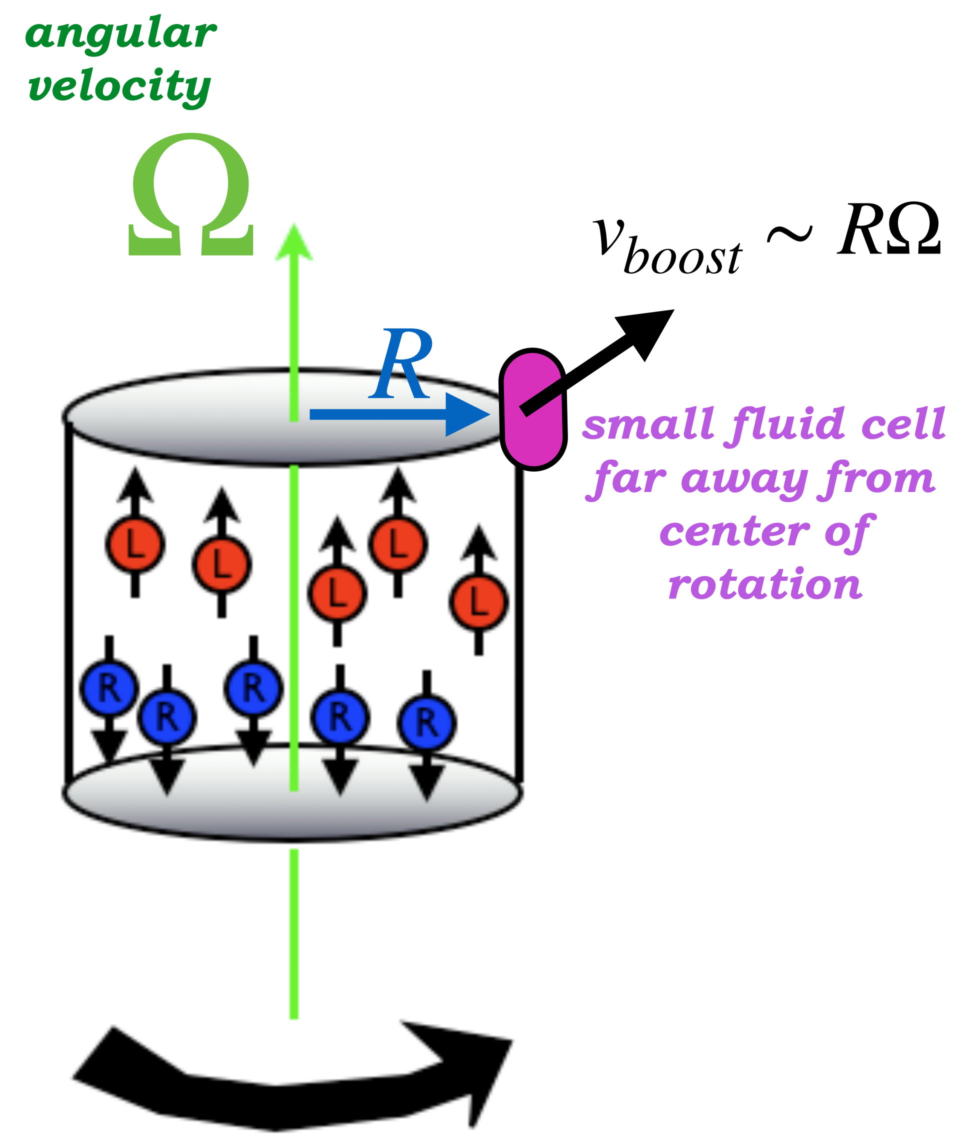
➡If transport coefficients known at rest, then they are known in high T rotating fluid (boosted fluid).

Result 2: Hydrodynamic fluctuations see boosted fluid

2. Can it be approximated by a boosted fluid?

— **Yes, BUT … .**

Hydrodynamic fluctuations at large temperatures perceive the rotating holographic fluid as if it was a boosted fluid.



Gravity side:

- **metric is *not* a boosted black brane (that means the fluid is not a boosted fluid but a rotating one)**
- **metric fluctuation equations in the limit of small frequencies and momenta (hydrodynamic limit) *<u>see</u> effectively a boosted metric***

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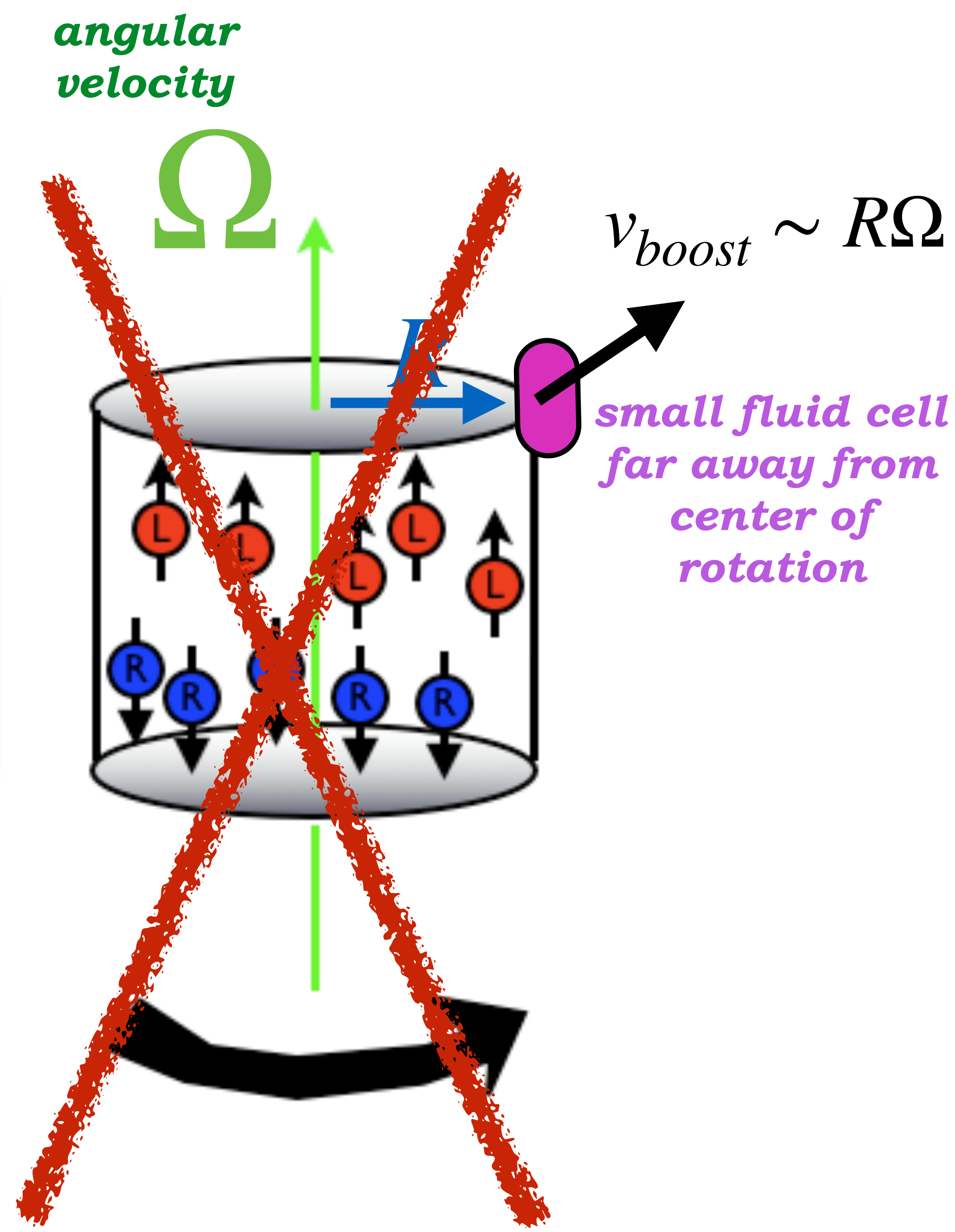

3. Rotating fluid hydrodynamics valid at smaller T?

- **validity of the of the derivative expansion: convergence radius of**

$$\omega(k) = \sum c_n k^n$$

- **validity of the constitutive relations and transport coefficients**

Convergence radius: Hydrodynamic modes

Interacting many-body systems at large temperature T have collective excitations, damped eigenmodes, with specific dispersion relations :

(assuming rotation invariance: $k \equiv |\vec{k}|$)

Sound modes

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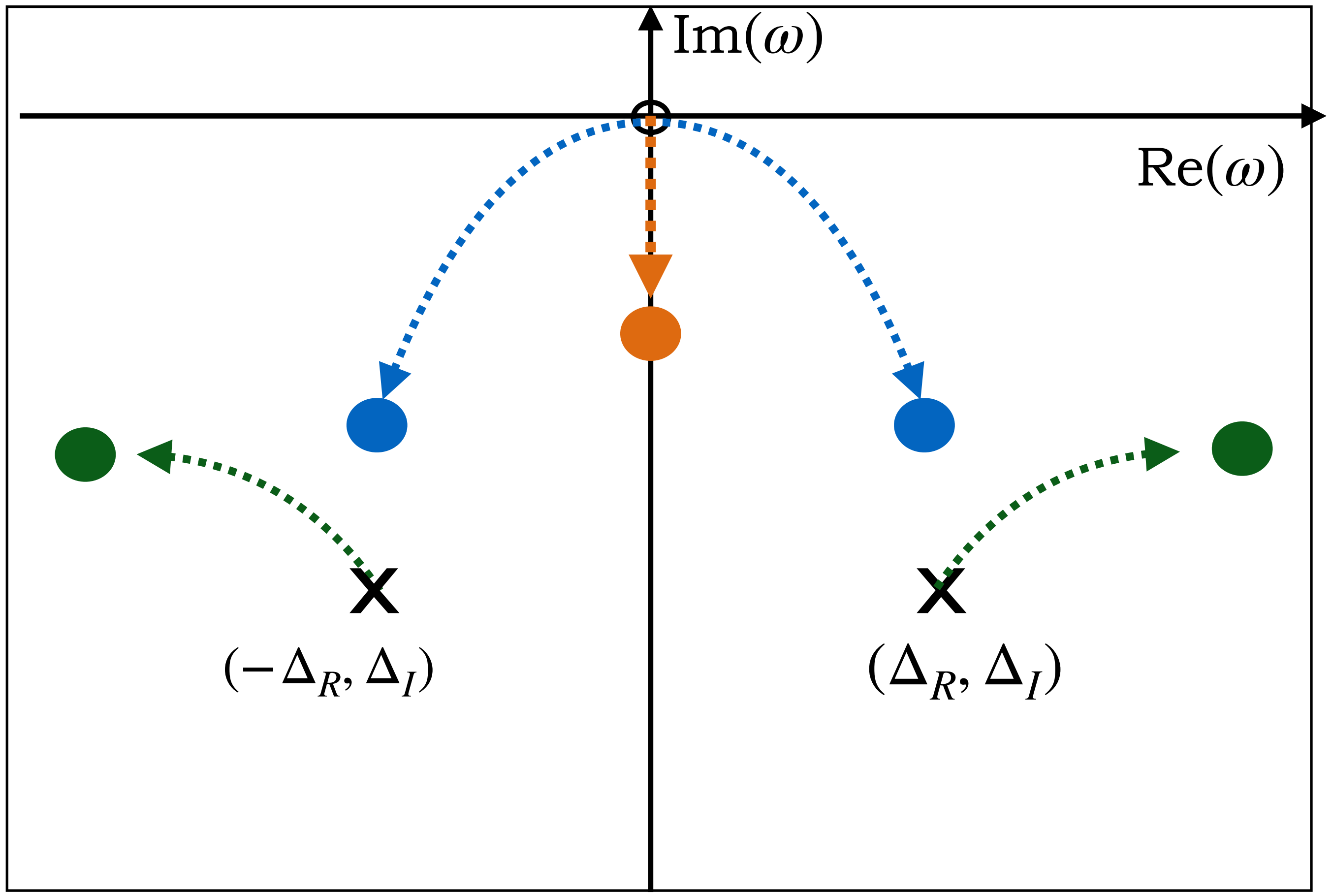
Momentum diffusion mode

$$\omega(k) = -iDk^2 + \mathcal{O}(3)$$

Non-hydrodynamic modes

$$\omega(k) = \underbrace{\pm\Delta_\mathrm{R} - i\Delta_\mathrm{I}}_{\text{gap}} + v_0 k - i\Gamma_0 k^2 + \mathcal{O}(3)$$



Complex frequency plane

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(assuming rotation invariance: $k \equiv |\vec{k}|$)

Sound modes

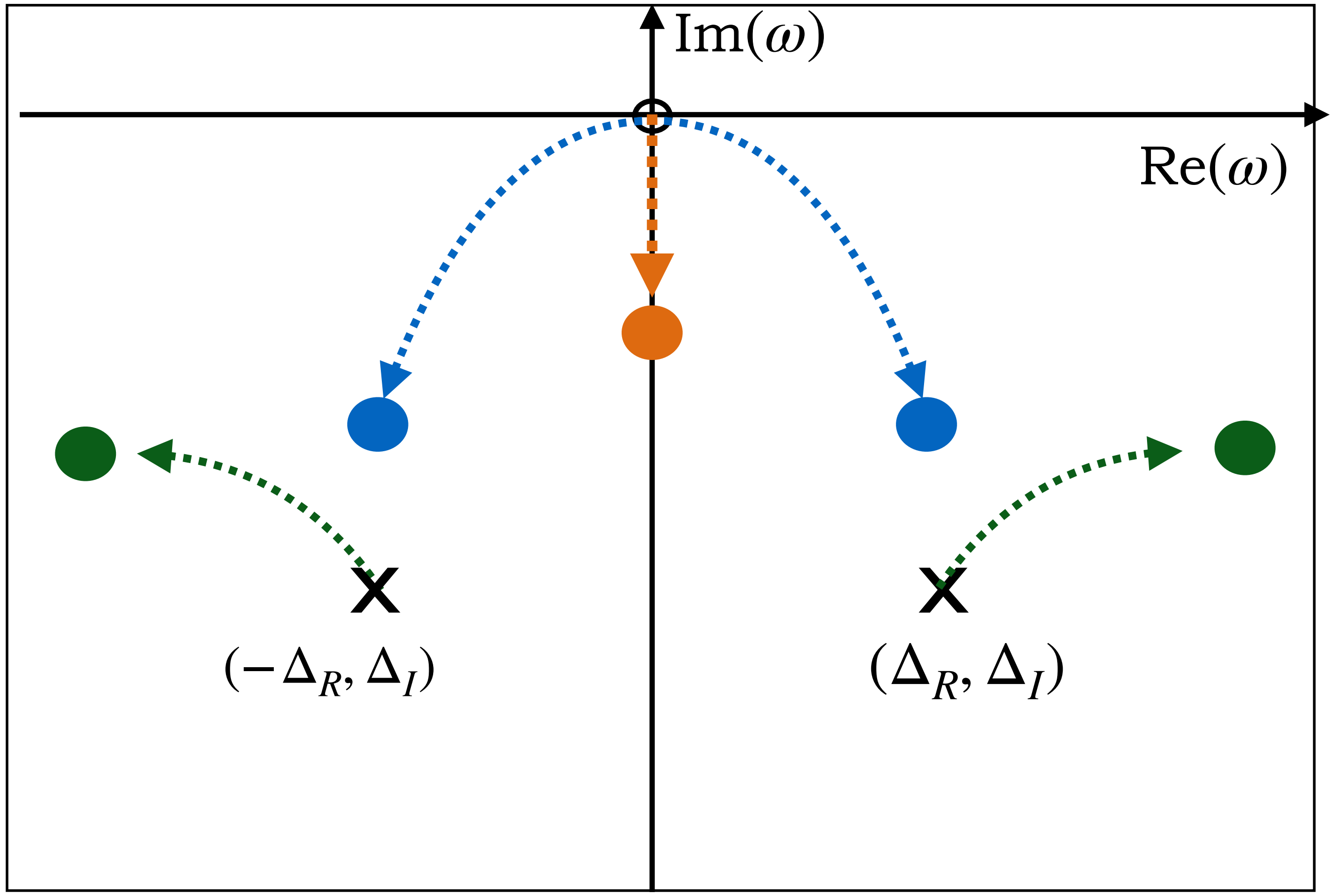
$$\omega(k) = \pm v_s k - i\Gamma k^2 + \mathcal{O}(3)$$

Momentum diffusion mode

$$\omega(k) = -iDk^2 + \mathcal{O}(3)$$

Non-hydrodynamic modes

$$\omega(k) = \underbrace{\pm\Delta_R - i\Delta_I}_{\text{gap}} + v_0 k - i\Gamma_0 k^2 + \mathcal{O}(3)$$



Complex frequency plane

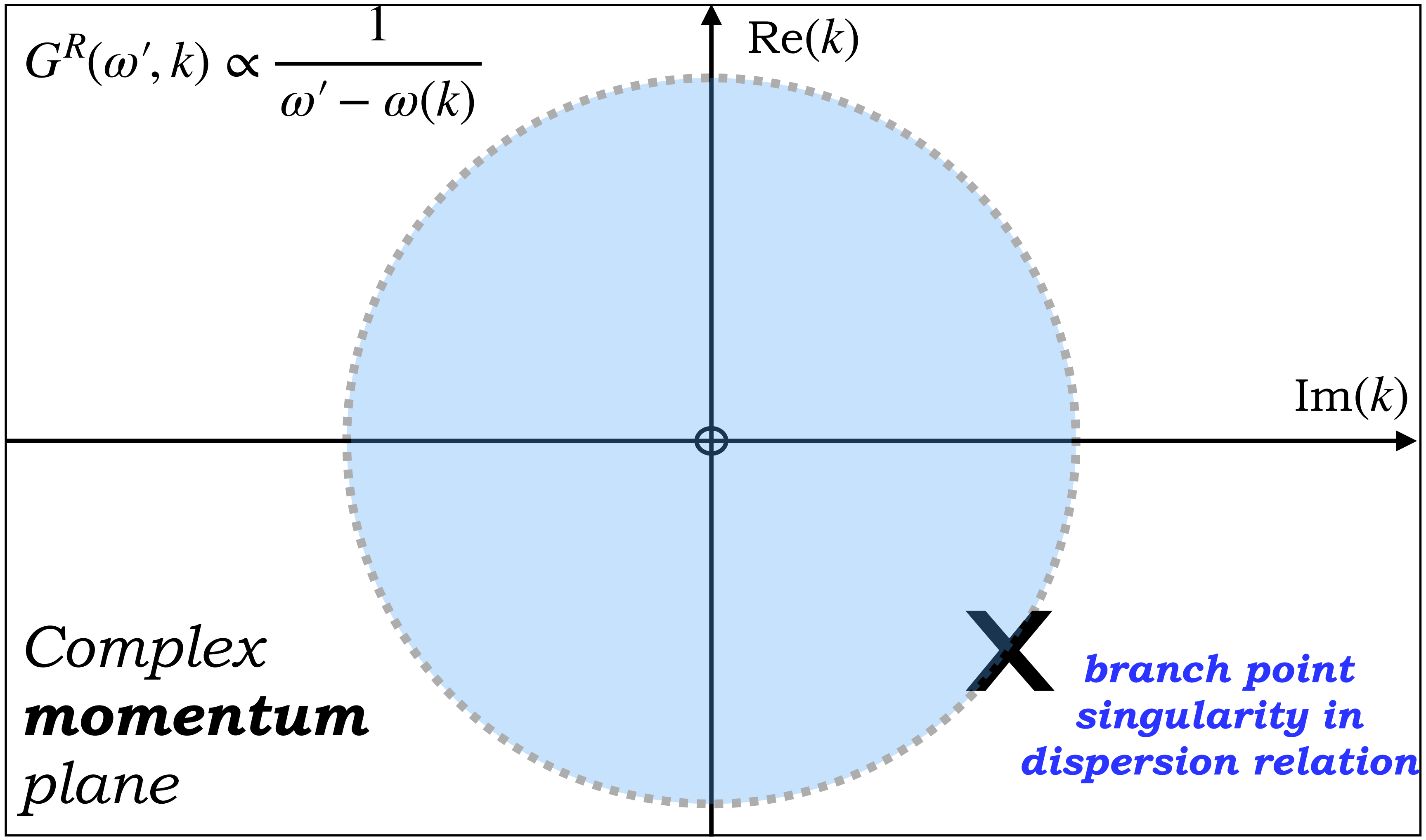
➡ **these modes are the locations of poles in the retarded Green's function of conserved operators in this theory**

$$G^R(\omega', k) \propto \frac{1}{\omega' - \omega(k)}$$

Singularities in the dispersion relations

Poles can collide in **complex momentum direction**, leading to branch singularities (critical points) in dispersion relations limiting <u>convergence radius</u> of hydrodynamics

[Grozdanov,Kovtun,Starinets,Tadic; JHEP (2019)]
[Grozdanov,Kovtun,Starinets,Tadic; PRL (2019)]



Simpler example: geometric series $\sum_{n=0}^{\infty} s^n = \frac{1}{1-s} \xrightarrow{s=-x^2} \sum_{n=0}^{\infty} (-x^2)^n = \frac{1}{1+x^2}$

Talk by Skokov

singularities at $x = \pm i$

Singularities in the dispersion relations

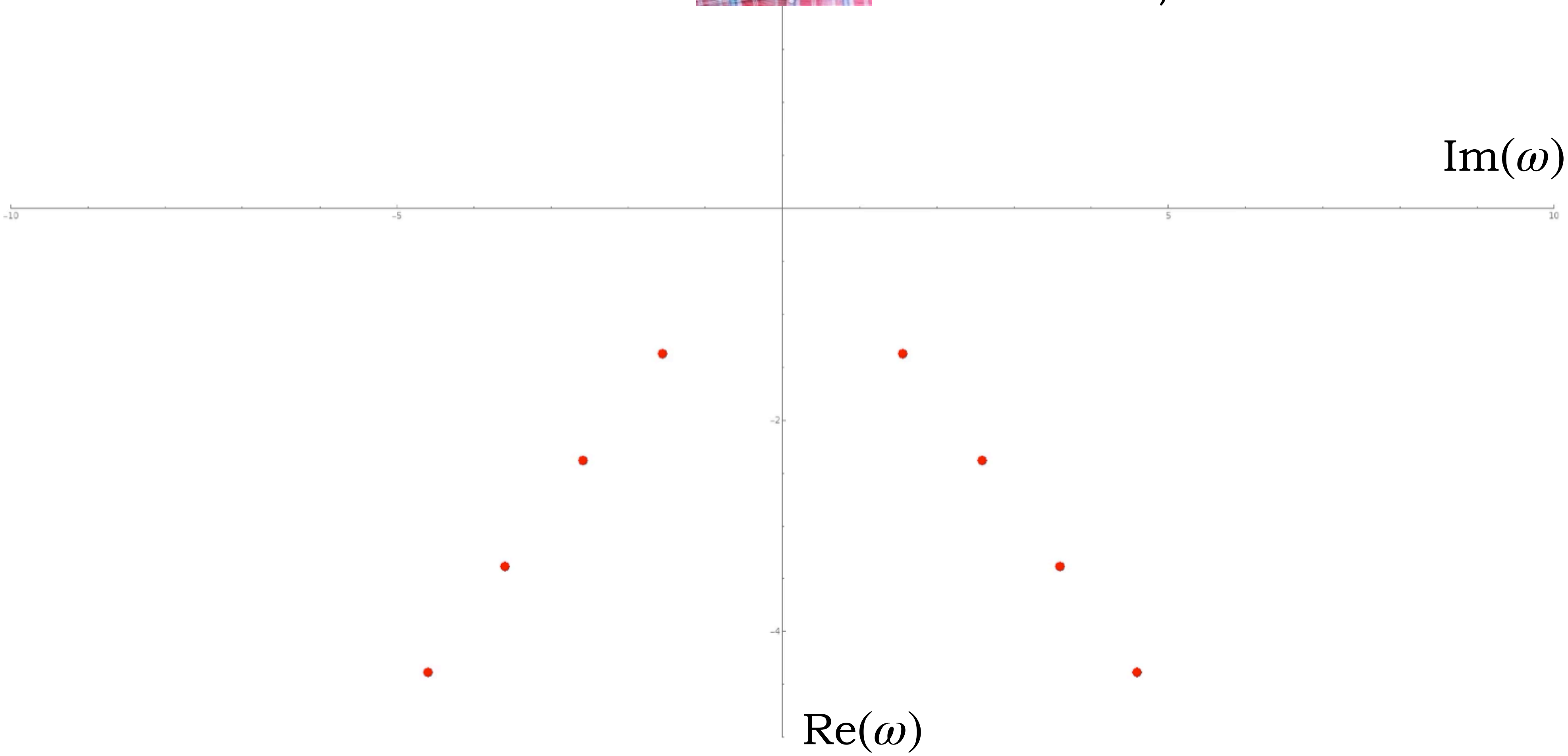
Poles can collide in **imaginary momentum direction**, leading to branch singularities, **critical points,** in dispersion relations limit convergence radius of hydrodynamics

[animation by Markus Amano (Garbiso); (2021)]

$$k = |k| e^{i\alpha}$$

Dr. Markus Garbiso (now postdoc at Henan U.)

|k^2| = 0



*Complex **frequency** plane*

Singularities in the dispersion relations

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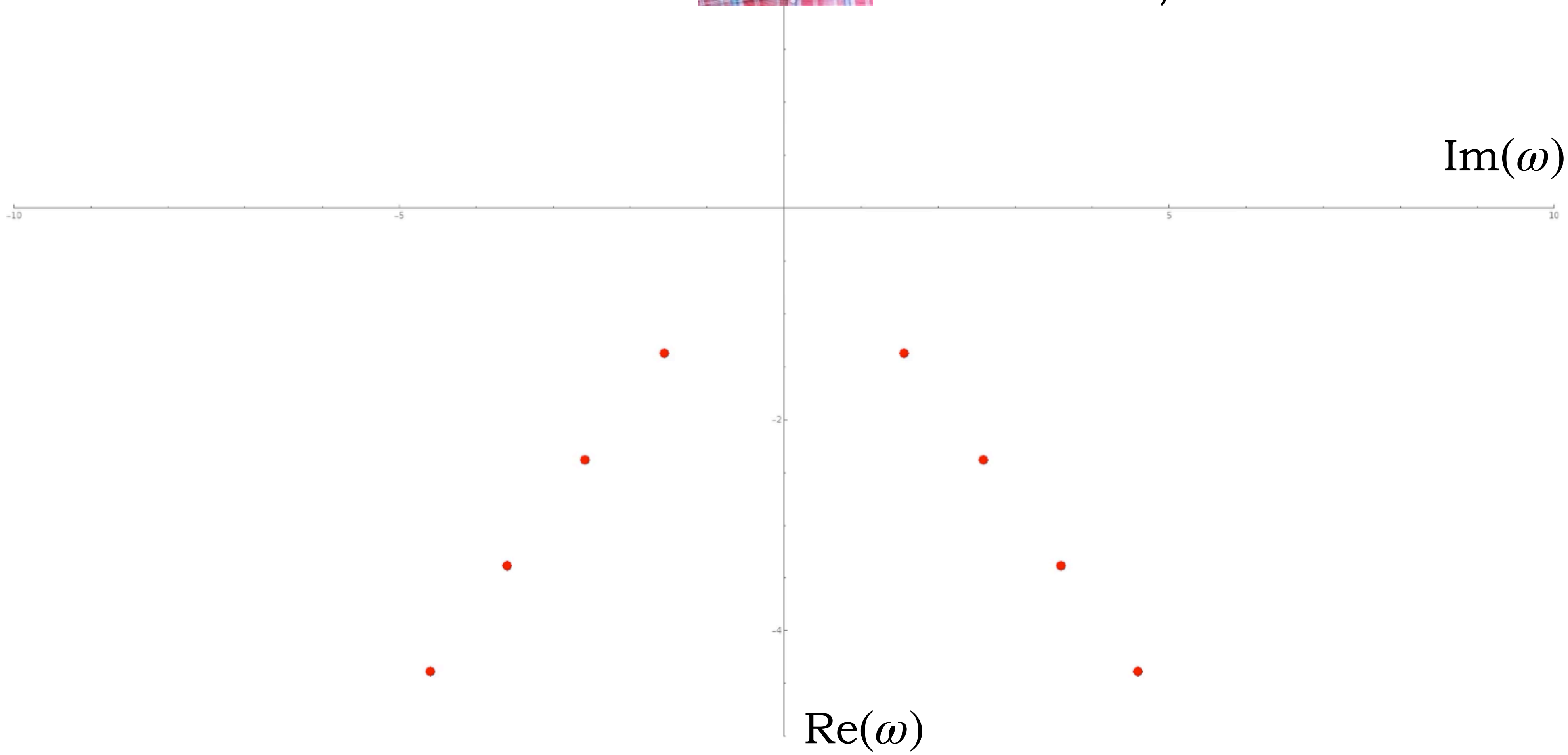


Complex ***frequency*** *plane*

Singular points of plane curves

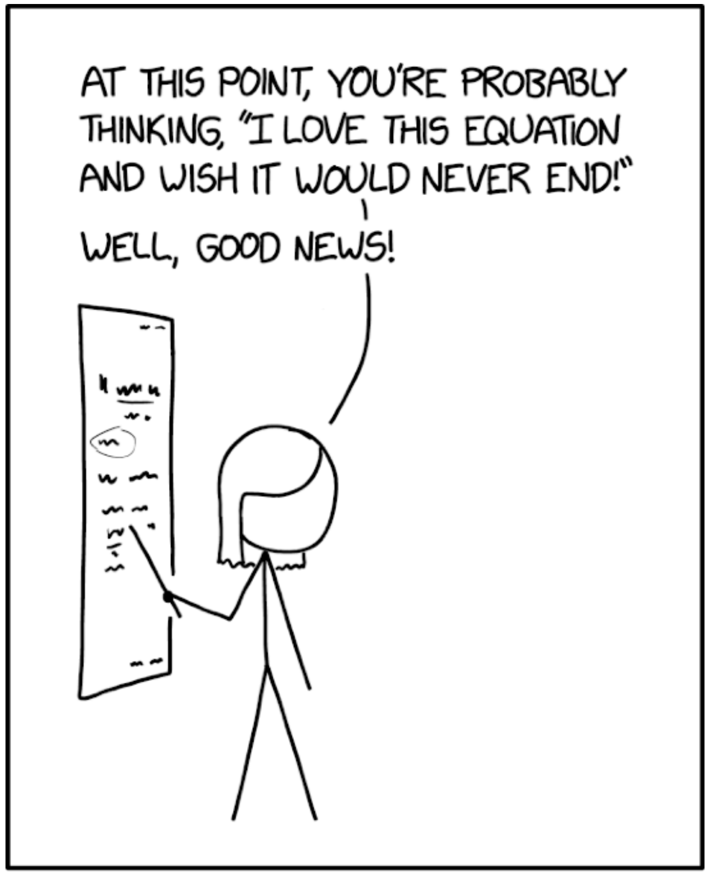
[C.T.C. Wall (2004)]

Puiseux theorem:

Any equation f(x, y) = 0, where f is a polynomial with f(O) = 0 or more generally f ∈ C[[x, y]] with zero constant term, admits at least one solution in formal power series of the form

$$x = t^n, \quad y = \sum_{1}^{\infty} a_r t^r$$

(some n ∈ N).
Thus, y can be expressed as power series in fractional powers of x.



TAYLOR SERIES EXPANSION IS THE WORST.

[https://xkcd.com/2605/]

Example: hydrodynamics

$$x = k, \quad y = \omega, \quad f(x,y) = \mathscr{P}(\omega, k)$$

$$\mathscr{P}\,\phi = 0 \Rightarrow \mathscr{P} = \omega + iDk^2 + \mathscr{O} = 0$$

Singular points of plane curves

[C.T.C. Wall (2004)]

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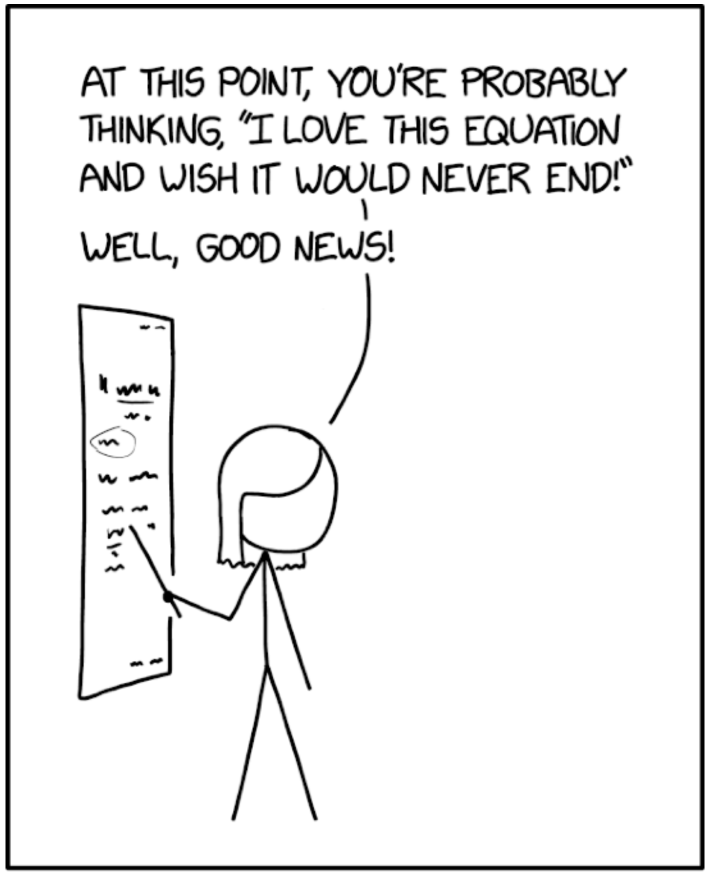

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$$\mathscr{P}\phi = 0 \Rightarrow \mathscr{P} = \omega + iDk^2 + \mathscr{O} = 0$$

➡There exists convergent hydrodynamic expansion. Critical points limit the radius of convergence in complex *k*.

Computing critical points

[Grozdanov,Kovtun,Starinets,Tadic; JHEP (2019)]
[Grozdanov,Kovtun,Starinets,Tadic; PRL (2019)]
[Heller,Serantes,Spalinski,Svensson,Withers; PRD (2020)]

Spectral curve encodes dispersion
Example: momentum diffusion mode

$$P(\omega, k^2) = \omega + iDk^2 + \mathcal{O}(3) = 0$$

Spectral curve yields critical point

$$P(\mathfrak{w}, \mathfrak{q})|_{(\mathfrak{w}_c, \mathfrak{q}_c)} = 0\,, \quad \partial_{\mathfrak{w}} P(\mathfrak{w}, \mathfrak{q})|_{(\mathfrak{w}_c, \mathfrak{q}_c)} = 0\,,$$

Holographically:

$$P(\omega, k) = \phi(\omega, k; u = u_{bdy})$$

Gravitational fluctuation (e.g. metric fluctuation)

RECALL: What is holography?

Holography

[Kovtun/Starinets; JHEP (2005)]

- consider **Einstein gravity** which is dual to $\mathcal{N}=4$ SYM theory and derive Einstein equations
- metric of a **rotating asymptotically AdS5 black hole** (solution to Einstein equations) is dual to a rotating thermal SYM state
- **black hole thermodynamics** "determines" thermodynamic properties of the dual SYM state
- poles of the SYM Green's functions are dual to quasi normal mode (QNM) frequencies of black holes: **QNMs encode SYM dispersion relations**

➡Compute the QNM frequencies around rotating black hole as function of momentum.

Holography

Example: rotation-invariant fluid from QNMs of metric fluctuations *[Kovtun/Starinets; JHEP (2005)]*

Momentum diffusion mode

$$\omega(k) = -iDk^2 + \mathcal{O}(3) \longleftrightarrow \delta g_{tx}, \delta g_{zx}, \ldots \textbf{(vector)}$$

Sound modes

$$\omega(k) = \pm v_s k - i\Gamma k^2 + \mathcal{O}(3) \longleftrightarrow \delta g_{tt}, \delta g_{tz}, \delta g_{zz} \textbf{(scalar)}$$

- poles of the SYM Green's functions are dual to quasi normal mode (QNM) frequencies of black holes: **QNMs encode SYM dispersion relations**

➡Compute the QNM frequencies around rotating black hole as function of momentum.

Metric fluctuations in rotating AdS5 black holes are complicated

Rotating AdS5 black hole

$$ds^2 = -\left(1+\frac{r^2}{L^2}\right)dt^2 + \frac{dr^2}{G(r)} + \frac{r^2}{4}\left((\sigma^1)^2+(\sigma^2)^2 + (\sigma^3)^2\right) + \frac{2\mu}{r^2}\left(dt+\frac{a}{2}\sigma^3\right)^2$$

$$G(r) = 1+\frac{r^2}{L^2} - \frac{2\mu(1-a^2/L^2)}{r^2} + \frac{2\mu a^2}{r^4}\,,$$

$$\mu = \frac{r_+^4\left(L^2+r_+^2\right)}{2L^2r_+^2 - 2a^2\left(L^2+r_+^2\right)}\,,$$

Wigner-D functions are basis on S^3

$$h^V_{\mu\nu} \equiv e^{-i\omega\tau} r^2 \Big(h_{++}(r)\sigma^+_\mu\sigma^+_\nu D^{\mathcal{J}}_{(\mathcal{J}-1)\mathcal{M}} + 2\big(h_{+r}(r)\sigma^+_{(\mu}\sigma^r_{\nu)} + h_{+t}(r)\sigma^+_{(\mu}\sigma^t_{\nu)} + h_{+3}(r)\sigma^+_{(\mu}\sigma^3_{\nu)}\big)D^{\mathcal{J}}_{\mathcal{J}\mathcal{M}}\Big)\,,$$

$$Z_1(u) \equiv \mathfrak{q}u h_{tx}/(\pi T L)^2 + \mathfrak{w}u h_{zx}/(\pi T L)^2.$$

$\Rightarrow$

[Cartwright,Garbiso-Amano;Kaminski,Noronha,Speranza; PRD (2023)]

Select J=K (transverse): momentum diffusion

three dynamical equations,

$$\begin{aligned}
0 =& h''_{t+}(r) + \frac{L^2\left(2a^2\mu - 10\mu r^2 + 5r^4\right) + 2a^2\mu r^2 + 5r^6}{L^2\left(2a^2\mu r - 2\mu r^3 + r^5\right) + 2a^2\mu r^3 + r^7} h'_{t+}(r) + \\
& \frac{8a\mu\left(L^2+2r^2\right)}{L^2\left(2a^2\mu r - 2\mu r^3 + r^5\right) + 2a^2\mu r^3 + r^7} h'_{3+}(r) + \\
& \frac{L^2 h_{t+}(r)}{\left(L^2\left(2a^2\mu r - 2\mu r^3 + r^5\right) + 2a^2\mu r^3 + r^7\right)^2}(-4L^2(4a^4\mu^2 - 2a^2\mu^2 r^2(a\omega - 2\mathcal{J}) \\
& + \mathcal{J}\mu r^6(a\omega - 2\mathcal{J} - 4) + \mathcal{J}(\mathcal{J}+2)r^8) - 16a^4\mu^2r^2 - 4\mathcal{J}(\mathcal{J}+2)r^{10}) - \\
& \frac{2i\sqrt{2}\sqrt{\mathcal{J}}L^2r^2\omega}{L^2\left(2a^2\mu - 2\mu r^2 + r^4\right) + 2a^2\mu r^2 + r^6} h_{++}(r) - \\
& \frac{4L^2 h_{3+}(r)}{\left(L^2\left(2a^2\mu r - 2\mu r^3 + r^5\right) + 2a^2\mu r^3 + r^7\right)^2}\left(-8a^3\mu^2\left(L^2+r^2\right) - 2a^2\mu r^2\omega\left(L^2\left(2\mu + r^2\right) + r^4\right) + \right. \\
& \left. a\mu r^2\left(L^2r^4\omega^2 - 4(\mathcal{J}+2)\left(L^2\left(r^2-2\mu\right)+r^4\right)\right) + \mathcal{J}r^6\omega\left(L^2\left(r^2-2\mu\right)+r^4\right)\right), \\
0 =& h''_{++}(r) + \frac{L^2\left(3r^4 - 2\mu\left(a^2+r^2\right)\right) + 2a^2\mu r^2 + 5r^6}{L^2\left(2a^2\mu r - 2\mu r^3 + r^5\right) + 2a^2\mu r^3 + r^7} h'_{++}(r) - \\
& \frac{2i\sqrt{2}\sqrt{\mathcal{J}}L^4r^4\left(2a\mu(a\omega - 2\mathcal{J} - 2) + r^4\omega\right)}{\left(L^2\left(2a^2\mu - 2\mu r^2 + r^4\right) + 2a^2\mu r^2 + r^6\right)^2} h_{t+}(r) + \\
& \frac{h_{++}(r)}{\left(L^2\left(2a^2\mu - 2\mu r^2 + r^4\right) + 2a^2\mu r^2 + r^6\right)^2}(-4(\mathcal{J}+1)L^2r^2(\mathcal{J}r^4\left(L^2+r^2\right) - \\
& 2\mu\left(a^2\left(L^2+r^2\right) + \mathcal{J}L^2r^2\right)) + L^4r^4\omega^2\left(2a^2\mu + r^4\right) - 8a(\mathcal{J}+1)\mu L^4r^4\omega) + \\
& \frac{8i\sqrt{2}\sqrt{\mathcal{J}}L^2r^4\left(a\mu L^2\omega + (\mathcal{J}+1)\left(L^2\left(r^2-2\mu\right)+r^4\right)\right)}{\left(L^2\left(2a^2\mu - 2\mu r^2 + r^4\right) + 2a^2\mu r^2 + r^6\right)^2} h_{3+}, \\
0 =& h''_{3+}(r) - \frac{4a\mu L^2 r}{L^2\left(2a^2\mu - 2\mu r^2 + r^4\right) + 2a^2\mu r^2 + r^6} h'_{t+}(r) + \\
& \frac{L^2\left(6a^2\mu - 2\mu r^2 + 3r^4\right) + 5r^2\left(2a^2\mu + r^4\right)}{L^2\left(2a^2\mu r - 2\mu r^3 + r^5\right) + 2a^2\mu r^3 + r^7} h'_{3+}(r) - \\
& \frac{L^4\left(\mathcal{J}r^4 - 2a^2\mu\right)\left(2a\mu(a\omega - 2\mathcal{J} - 2) + r^4\omega\right)}{\left(L^2\left(2a^2\mu - 2\mu r^2 + r^4\right) + 2a^2\mu r^2 + r^6\right)^2} h_{t+}(r) - \\
& \frac{2i\sqrt{2}\sqrt{\mathcal{J}}(\mathcal{J}+1)L^2r^2}{L^2\left(2a^2\mu - 2\mu r^2 + r^4\right) + 2a^2\mu r^2 + r^6} h_{++}(r) + \\
& \frac{h_{+3}(r)}{\left(L^2\left(2a^2\mu r - 2\mu r^3 + r^5\right) + 2a^2\mu r^3 + r^7\right)^2}(L^4(-32a^4\mu^2 - 8a^2\mu^2r^2(a\omega - 2\mathcal{J} - 6) - \\
& 16a^2(\mathcal{J}+2)\mu r^4 + 2\mu r^6(-2\mathcal{J}(a\omega - 4) + a\omega(a\omega - 4) + 8) \\
& - 8(\mathcal{J}+1)r^8 + r^{10}\omega^2) - 8L^2r^2\left(2a^2\mu + r^4\right)\left(2a^2\mu + (\mathcal{J}+1)r^4\right)),
\end{aligned} \tag{23}$$

and one first order constraint equation,

$$\begin{aligned}
0 =& h'_{3+}(r) - \frac{i\sqrt{\mathcal{J}}\left(L^2\left(2a^2\mu - 2\mu r^2 + r^4\right) + 2a^2\mu r^2 + r^6\right)}{\sqrt{2}\left(L^2\left(-2a^2\mu + \mu r^2(a\omega - 2\mathcal{J}) + \mathcal{J}r^4\right) - 2a^2\mu r^2 + \mathcal{J}r^6\right)} h'_{++}(r) - \\
& \frac{L^2r^2\left(2a\mu(a\omega - 2\mathcal{J} - 2) + r^4\omega\right)}{4\left(L^2\left(-2a^2\mu + \mu r^2(a\omega - 2\mathcal{J}) + \mathcal{J}r^4\right) - 2a^2\mu r^2 + \mathcal{J}r^6\right)} h'_{t+}(r) + \\
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& \frac{8a^2\mu\left(L^2+r^2\right)}{L^2\left(-2a^2\mu r + \mu r^3(a\omega - 2\mathcal{J}) + \mathcal{J}r^5\right) - 2a^2\mu r^3 + \mathcal{J}r^7} h_{3+}(r)\,.
\end{aligned} \tag{24}$$

Metric fluctuations in rotating AdS5 black holes are complicated

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$$ds^2 = -\left(1+\frac{r^2}{L^2}\right)dt^2 + \frac{dr^2}{G(r)} + \frac{r^2}{4}\left((\sigma^1)^2+(\sigma^2)^2\right.$$
$$\left.+(\sigma^3)^2\right) + \frac{2\mu}{r^2}\left(dt+\frac{a}{2}\sigma^3\right)^2$$
$$G(r) = 1+\frac{r^2}{L^2} - \frac{2\mu(1-a^2/L^2)}{r^2} + \frac{2\mu a^2}{r^4}\,,$$
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$$h^V_{\mu\nu} \equiv e^{-i\omega\tau} r^2 (h_{++}(r)\sigma^+_\mu\sigma^+_\nu D^{\mathcal{J}}_{(\mathcal{J}-1)\mathcal{M}} +$$
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[Cartwright,Garbiso-Amano;Kaminski,Noronha,Speranza; PRD (2023)]

Select J=K (transverse): momentum diffusion

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& - 8(\mathcal{J}+1)r^8 + r^{10}\omega^2) - 8L^2r^2\left(2a^2\mu + r^4\right)\left(2a^2\mu + (\mathcal{J}+1)r^4\right)),
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\end{aligned} \tag{24}$$

$\Rightarrow$

$\Leftrightarrow$

Equivalent to momentum diffusion at rest:

$$Z_1'' + \frac{(\mathfrak{w}^2 - \mathfrak{q}^2 f)f - u\mathfrak{w}^2 f'}{uf(\mathfrak{q}^2 f - \mathfrak{w}^2)} Z_1' + \frac{\mathfrak{w}^2 - \mathfrak{q}^2 f}{uf^2} Z_1 = 0\,,$$

Master field: $Z_1(u) \equiv \mathfrak{q}uh_{tx}/(\pi T L)^2 + \mathfrak{w}uh_{zx}/(\pi T L)^2.$

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$$ds^2 = -\left(1+\frac{r^2}{L^2}\right)dt^2 + \frac{dr^2}{G(r)} + \frac{r^2}{4}\left((\sigma^1)^2 + (\sigma^2)^2 + (\sigma^3)^2\right) + \frac{2\mu}{r^2}\left(dt + \frac{a}{2}\sigma^3\right)^2$$

$$G(r) = 1 + \frac{r^2}{L^2} - \frac{2\mu(1-a^2/L^2)}{r^2} + \frac{2\mu a^2}{r^4}\,,$$

$$\mu = \frac{r_+^4\left(L^2+r_+^2\right)}{2L^2 r_+^2 - 2a^2\left(L^2+r_+^2\right)}\,,$$

Wigner-D functions are basis on S^3

$$h^V_{\mu\nu} \equiv e^{-i\omega\tau} r^2 \big(h_{++}(r)\sigma^+_\mu\sigma^+_\nu D^{\mathcal{J}}_{(\mathcal{J}-1)\mathcal{M}} + 2(h_{+r}(r)\sigma^+_{(\mu}\sigma^r_{\nu)} + h_{+t}(r)\sigma^+_{(\mu}\sigma^t_{\nu)} + h_{+3}(r)\sigma^+_{(\mu}\sigma^3_{\nu)})D^{\mathcal{J}}_{\mathcal{J}\mathcal{M}}\big)\,,$$

$\Rightarrow$

[Cartwright,Garbiso-Amano;Kaminski,Noronha,Speranza; PRD (2023)]

Select J=K (transverse): momentum diffusion

three dynamical equations,

$$\begin{aligned}
0 =\,& h''_{t+}(r) + \frac{L^2\left(2a^2\mu - 10\mu r^2 + 5r^4\right) + 2a^2\mu r^2 + 5r^6}{L^2\left(2a^2\mu r - 2\mu r^3 + r^5\right) + 2a^2\mu r^3 + r^7} h'_{t+}(r) + \\
& \frac{8a\mu\left(L^2+2r^2\right)}{L^2\left(2a^2\mu r - 2\mu r^3 + r^5\right) + 2a^2\mu r^3 + r^7} h'_{3+}(r) + \\
& \frac{L^2 h_{t+}(r)}{\left(L^2\left(2a^2\mu r - 2\mu r^3 + r^5\right) + 2a^2\mu r^3 + r^7\right)^2}(-4L^2(4a^4\mu^2 - 2a^2\mu^2 r^2(a\omega - 2\mathcal{J}) \\
& + \mathcal{J}\mu r^6(a\omega - 2\mathcal{J} - 4) + \mathcal{J}(\mathcal{J}+2)r^8) - 16a^4\mu^2 r^2 - 4\mathcal{J}(\mathcal{J}+2)r^{10}) - \\
& \frac{2i\sqrt{2}\sqrt{\mathcal{J}}L^2 r^2\omega}{L^2\left(2a^2\mu - 2\mu r^2 + r^4\right) + 2a^2\mu r^2 + r^6} h_{++}(r) - \\
& \frac{4L^2 h_{3+}(r)}{\left(L^2\left(2a^2\mu r - 2\mu r^3 + r^5\right) + 2a^2\mu r^3 + r^7\right)^2}(-8a^3\mu^2\left(L^2+r^2\right) - 2a^2\mu r^2\omega\left(L^2\left(2\mu + r^2\right) + r^4\right) + \\
& a\mu r^2\left(L^2 r^4\omega^2 - 4(\mathcal{J}+2)\left(L^2\left(r^2 - 2\mu\right) + r^4\right)\right) + \mathcal{J}r^6\omega\left(L^2\left(r^2-2\mu\right) + r^4\right)), \\
0 =\,& h''_{++}(r) + \frac{L^2\left(3r^4 - 2\mu\left(a^2+r^2\right)\right) + 2a^2\mu r^2 + 5r^6}{L^2\left(2a^2\mu r - 2\mu r^3 + r^5\right) + 2a^2\mu r^3 + r^7} h'_{++}(r) - \\
& \frac{2i\sqrt{2}\sqrt{\mathcal{J}}L^4 r^4\left(2a\mu(a\omega - 2\mathcal{J} - 2) + r^4\omega\right)}{\left(L^2\left(2a^2\mu - 2\mu r^2 + r^4\right) + 2a^2\mu r^2 + r^6\right)^2} h_{t+}(r) + \\
& \frac{h_{++}(r)}{\left(L^2\left(2a^2\mu - 2\mu r^2 + r^4\right) + 2a^2\mu r^2 + r^6\right)^2}(-4(\mathcal{J}+1)L^2 r^2(\mathcal{J}r^4\left(L^2+r^2\right) - \\
& 2\mu\left(a^2\left(L^2+r^2\right) + \mathcal{J}L^2 r^2\right)) + L^4 r^4\omega^2\left(2a^2\mu + r^4\right) - 8a(\mathcal{J}+1)\mu L^4 r^4\omega) + \\
& \frac{8i\sqrt{2}\sqrt{\mathcal{J}}L^2 r^4\left(a\mu L^2\omega + (\mathcal{J}+1)\left(L^2\left(r^2-2\mu\right) + r^4\right)\right)}{\left(L^2\left(2a^2\mu - 2\mu r^2 + r^4\right) + 2a^2\mu r^2 + r^6\right)^2} h_{3+}, \\
0 =\,& h''_{3+}(r) - \frac{4a\mu L^2 r}{L^2\left(2a^2\mu - 2\mu r^2 + r^4\right) + 2a^2\mu r^2 + r^6} h'_{t+}(r) + \\
& \frac{L^2\left(6a^2\mu - 2\mu r^2 + 3r^4\right) + 5r^2\left(2a^2\mu + r^4\right)}{L^2\left(2a^2\mu r - 2\mu r^3 + r^5\right) + 2a^2\mu r^3 + r^7} h'_{3+}(r) - \\
& \frac{L^4\left(\mathcal{J}r^4 - 2a^2\mu\right)\left(2a\mu(a\omega - 2\mathcal{J} - 2) + r^4\omega\right)}{\left(L^2\left(2a^2\mu - 2\mu r^2 + r^4\right) + 2a^2\mu r^2 + r^6\right)^2} h_{t+}(r) - \\
& \frac{2i\sqrt{2}\sqrt{\mathcal{J}}(\mathcal{J}+1)L^2 r^2}{L^2\left(2a^2\mu - 2\mu r^2 + r^4\right) + 2a^2\mu r^2 + r^6} h_{++}(r) + \\
& \frac{h_{+3}(r)}{\left(L^2\left(2a^2\mu r - 2\mu r^3 + r^5\right) + 2a^2\mu r^3 + r^7\right)^2}(L^4(-32a^4\mu^2 - 8a^2\mu^2 r^2(a\omega - 2\mathcal{J} - 6) - \\
& 16a^2(\mathcal{J}+2)\mu r^4 + 2\mu r^6(-2\mathcal{J}(a\omega - 4) + a\omega(a\omega - 4) + 8)
\end{aligned} \quad (23)$$

Large black hole limit:

$$r_+ \to \alpha r_+\,, \quad r \to \alpha r\,, \quad \alpha \to \infty$$

$$\omega \to 2\alpha\nu r_+/L\,, \quad \mathcal{J} \to \alpha j r_+/L\,, \quad \alpha \to \infty\,,$$

Equivalent to momentum diffusion at rest:

[Kovtun/Starinets; JHEP (2005)]

$$Z_1'' + \frac{(\mathfrak{w}^2 - \mathfrak{q}^2 f)f - u\mathfrak{w}^2 f'}{uf(\mathfrak{q}^2 f - \mathfrak{w}^2)} Z_1' + \frac{\mathfrak{w}^2 - \mathfrak{q}^2 f}{uf^2} Z_1 = 0$$

$\Leftrightarrow$

By use of boost transformation:

$$\mathfrak{q}^2 = \frac{(a\nu + j)^2}{1-a^2}\,, \qquad \mathfrak{w}^2 = \frac{(\nu + aj)^2}{1-a^2}$$

Master field: $Z_1(u) \equiv \mathfrak{q}u h_{tx}/(\pi T L)^2 + \mathfrak{w}u h_{zx}/(\pi T L)^2.$

Spectral curve encodes dispersion
Example: ideal sound mode

poles = quasinormal modes

ideal sound dispersion

$$P(\mathfrak{w},\mathfrak{q}^2) = v_s^2\mathfrak{w}^2 - \mathfrak{q}^2 = 0$$

$$\mathfrak{w} = \pm v_s \mathfrak{q}$$

Spectral curve yields critical point

$$P(\mathfrak{w},\mathfrak{q})|_{(\mathfrak{w}_c,\mathfrak{q}_c)} = 0\,, \quad \partial_{\mathfrak{w}} P(\mathfrak{w},\mathfrak{q})|_{(\mathfrak{w}_c,\mathfrak{q}_c)} = 0$$

Critical points of *N*=4 SYM at vanishing rotation
(dual to AdS Schwarzschild black hole)

[Grozdanov,Kovtun,Starinets,Tadic; PRL (2019)]
[Heller,Serantes,Spalinski,Svensson,Withers; PRD (2020)]

$$\mathfrak{w}_c \approx \pm 1 - i,\ \mathfrak{q}_c^2 \approx \pm 2i \quad \text{(sound)}\,,$$

$$\mathfrak{w}_c \approx \pm 1.4436414 - 1.0692250i,$$

$$\mathfrak{q}_c^2 \approx 1.8906469 \pm 1.1711505i \quad \text{(shear diffusion)}$$

$$\mathfrak{w}(\mathfrak{q}) = -i\mathfrak{q}^2/2 + \mathcal{O}(\mathfrak{q}^3)$$

[Cartwright,Garbiso-Amano;Kaminski,Noronha,Speranza; PRD (2023)]

Computation through gauge/gravity correspondence & boost

Spectral curve encodes dispersion

Example: ideal sound mode

poles = quasinormal modes

ideal sound dispersion

$$P(\mathfrak{w},\mathfrak{q}^2) = v_s^2\mathfrak{w}^2 - \mathfrak{q}^2 = 0 \qquad \mathfrak{w} = \pm v_s \mathfrak{q}$$

Spectral curve yields critical point

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Consider rotating AdS black hole:

[Hawking,Hunter,Taylor; PRD (1998)]
[Hawking,Reall; PRD (1999)]
[Cartwright,Garbiso-Amano;Kaminski,Noronha,Speranza; PRD (2023)]

Boost symmetry for metric fluctuations around large rotating black holes

$$\mathfrak{q}^2 = \frac{(a\nu + j)^2}{1-a^2}\,, \qquad \mathfrak{w}^2 = \frac{(\nu + aj)^2}{1-a^2}$$

relates modes in rotating to those in non-rotating state

$$\nu(j) = -aj - i\frac{1}{2}(1-a^2)^{3/2}j^2 + \mathcal{O}(j^3)$$

shear diffusion mode

➡Direct calculation of poles in rotating black hole agrees with semi-analytic boost-symmetry inferred result

$$\nu(j) = \frac{\pm 1 - \sqrt{3}a}{\sqrt{3} \mp a} j - i\sqrt{3}\frac{(1-a^2)^{3/2}}{(\sqrt{3}-a)^3} j^2 + \mathcal{O}(j^3)$$

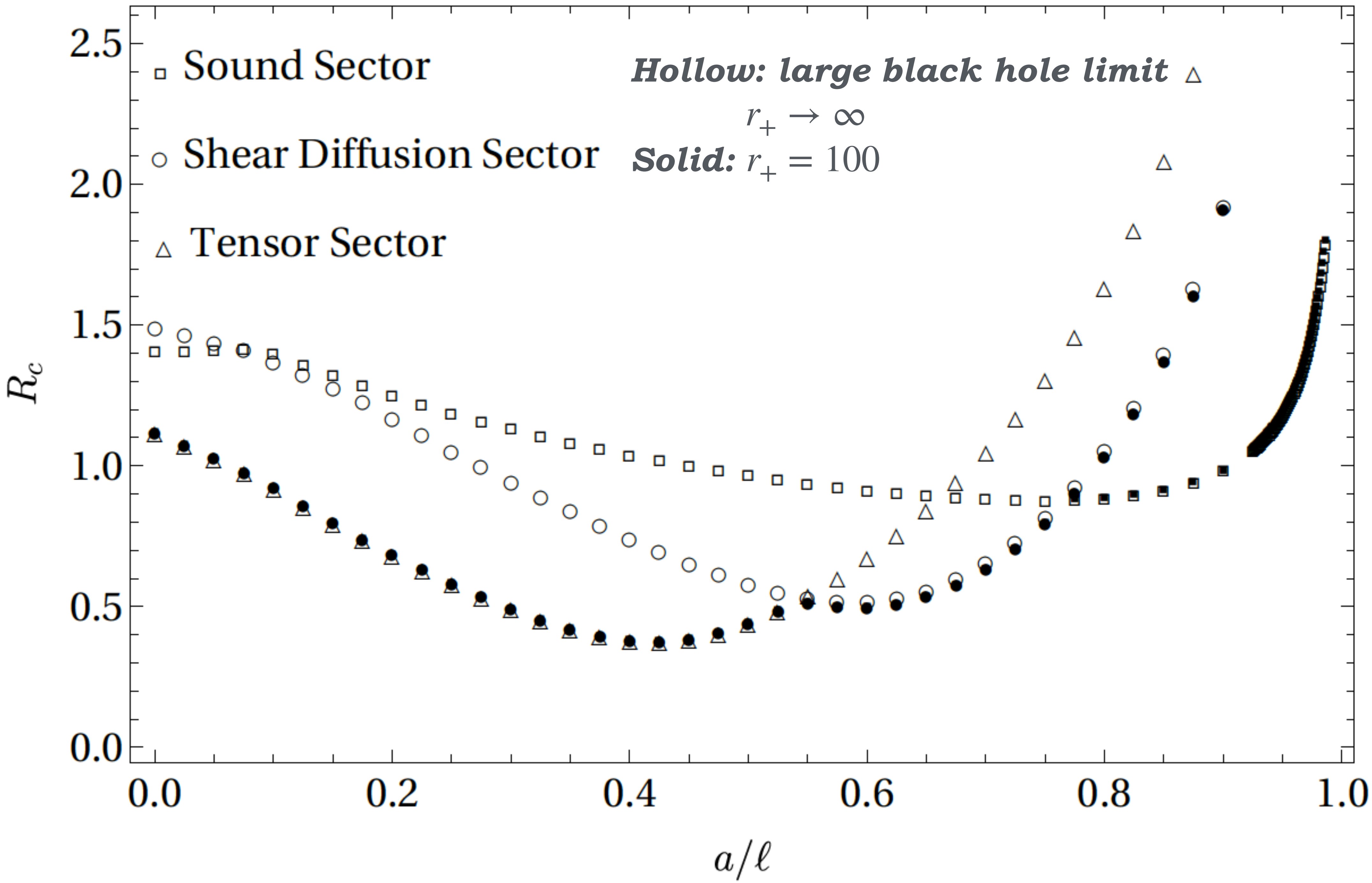
sound modes

Convergence radius of hydrodynamic description

of $\mathcal{N}$=4 Super-Yang-Mills theory in a rotating thermal state

[Cartwright,Garbiso-Amano;Kaminski,Noronha,Speranza; PRD (2023)]

convergence radius

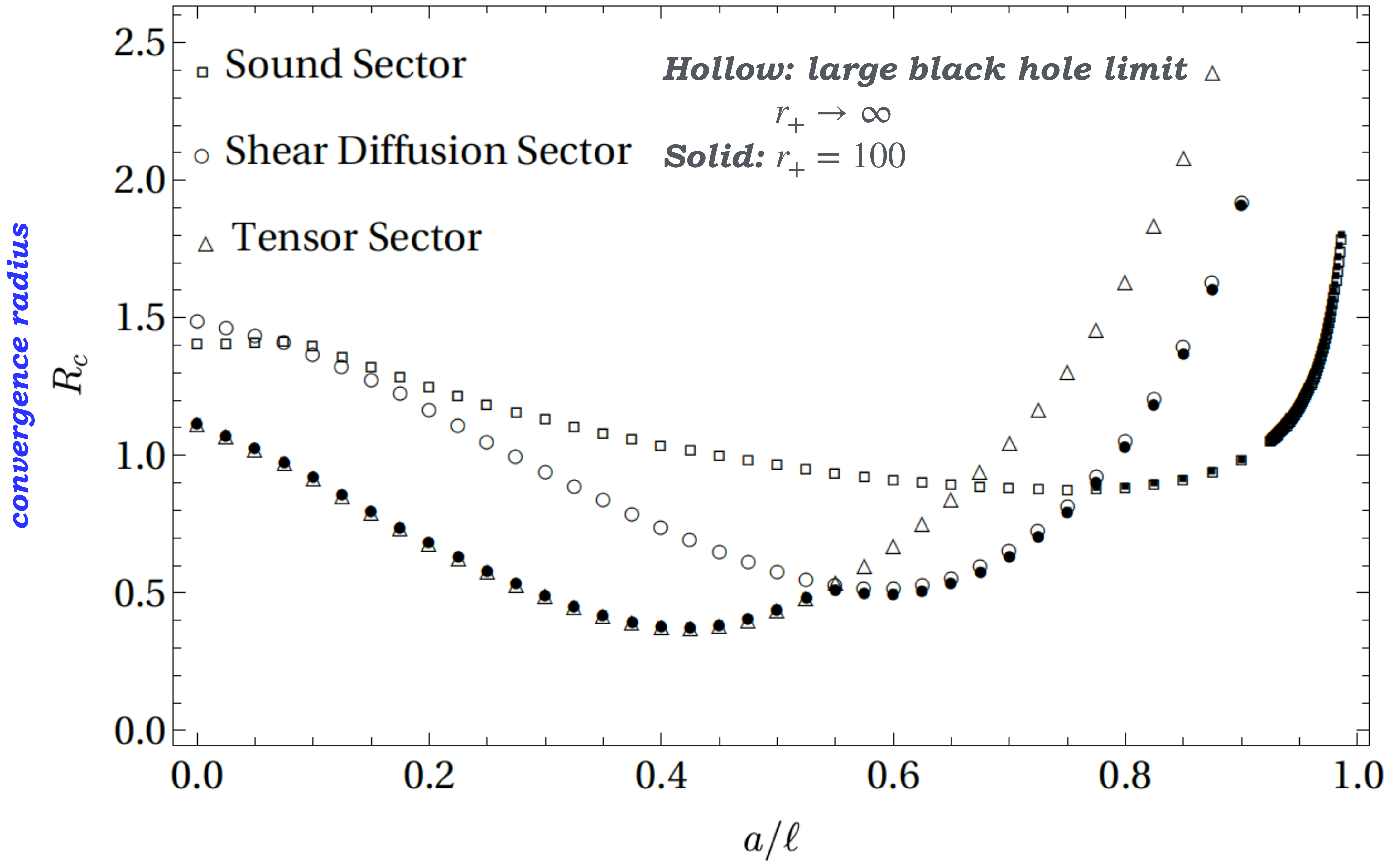


angular momentum of plasma

Convergence radius of hydrodynamic description

of $\mathcal{N}$=4 Super-Yang-Mills theory in a rotating thermal state

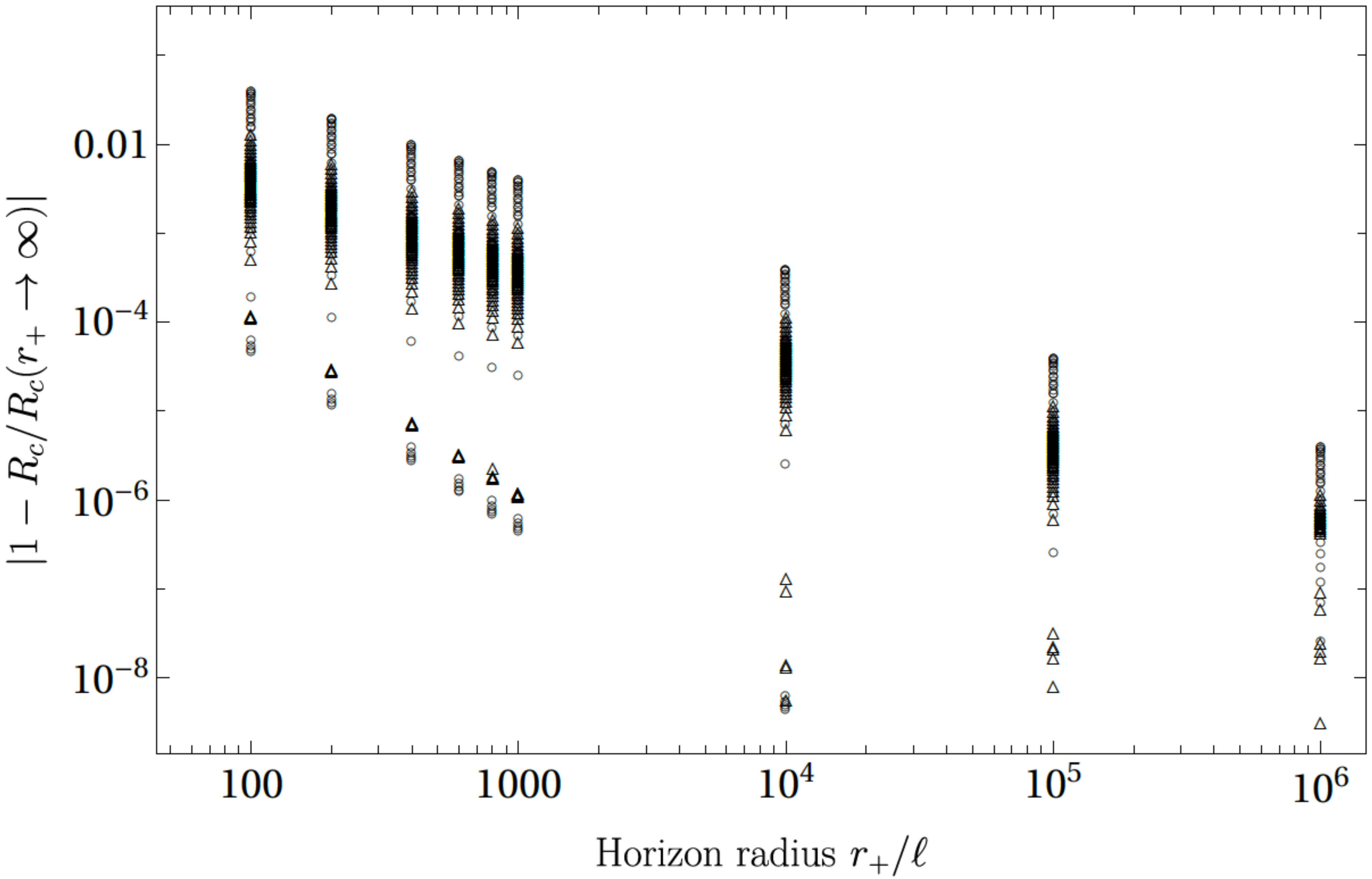
[Cartwright,Garbiso-Amano;Kaminski,Noronha,Speranza; PRD (2023)]



➡radius of convergence decreases at most by 60%, increases at fast rotation

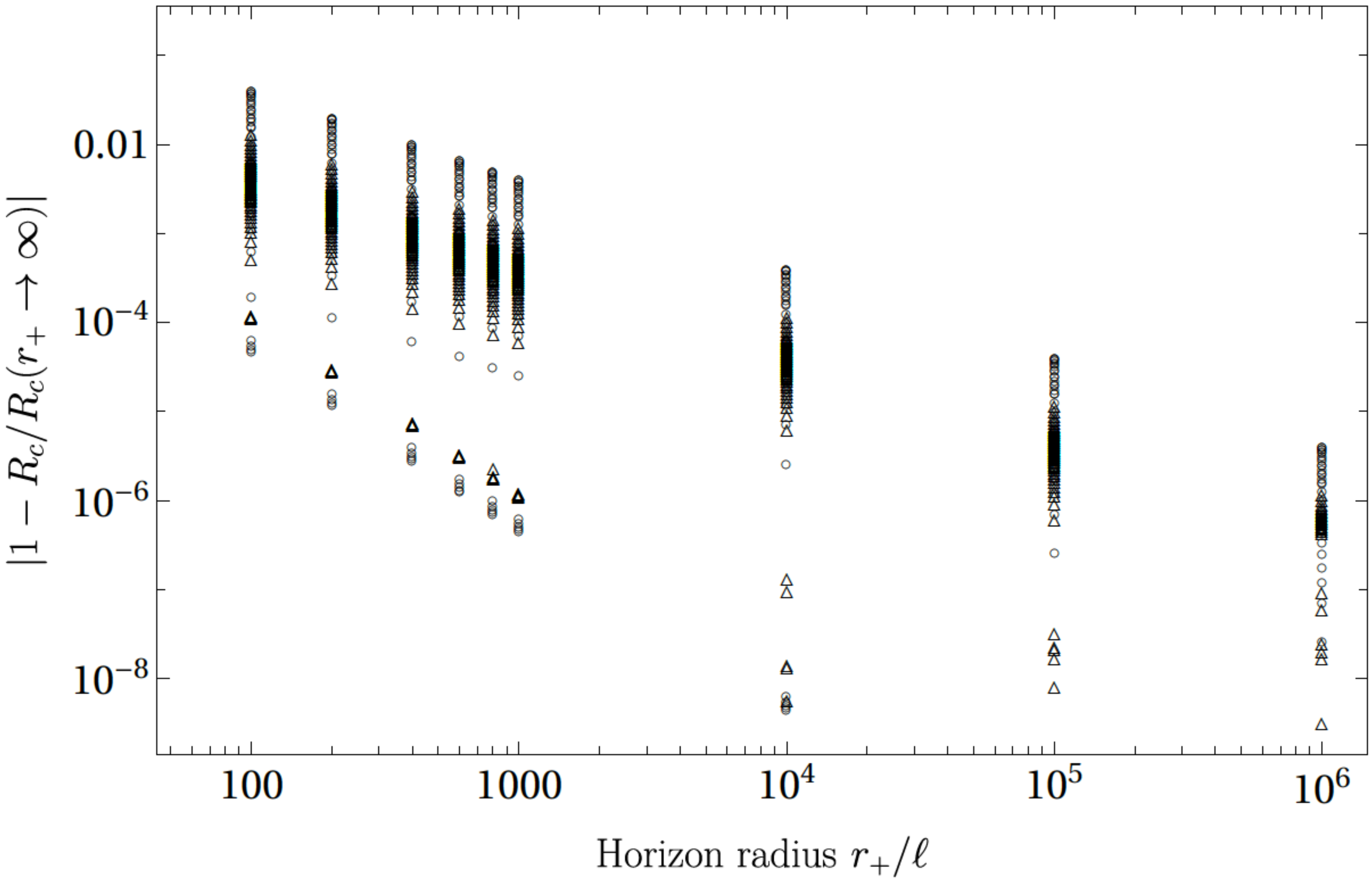
Temperature / horizon dependence of convergence

[Cartwright,Garbiso-Amano;Kaminski,Noronha,Speranza; PRD (2023)]



Temperature / horizon dependence of convergence

[Cartwright,Garbiso-Amano;Kaminski,Noronha,Speranza; PRD (2023)]



➡**approximately no temperature dependence down to $r_+/\ell = 10^4$, greatest change at $r_+/\ell = 10^2$**

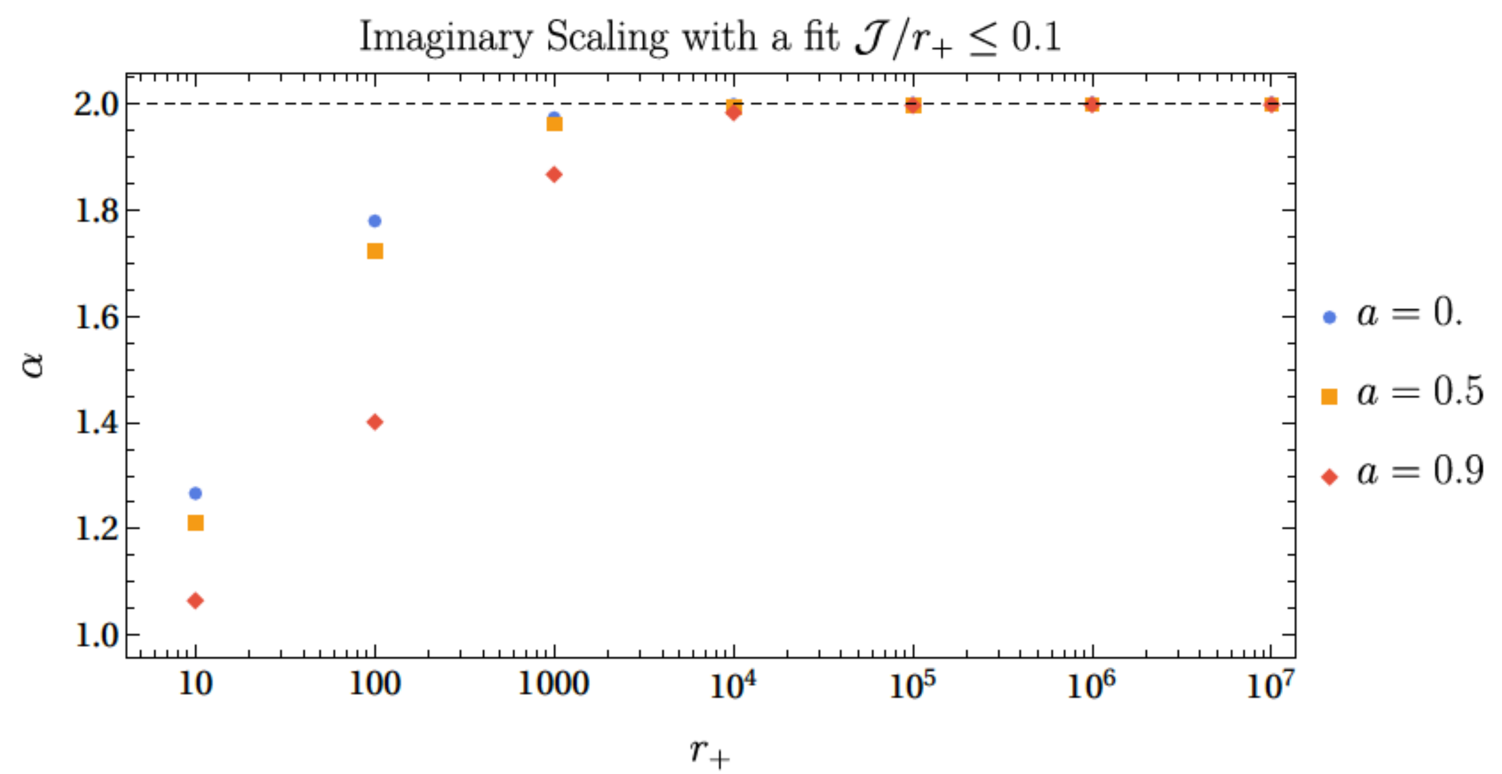
Is hydrodynamics valid? - Scaling

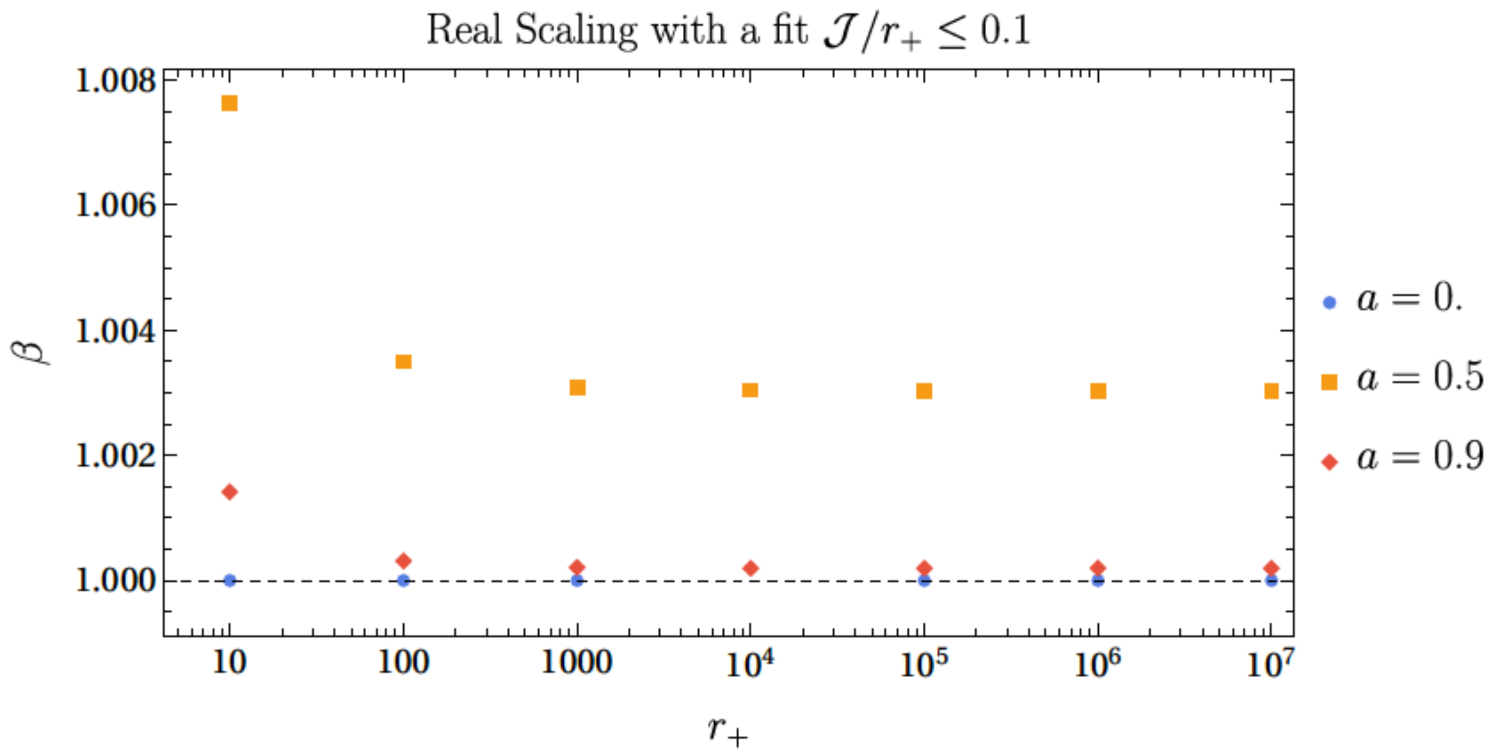
[Cartwright,Garbiso-Amano;Kaminski,Wu; arXiv:2308.11686]

- **validity of the <u>constitutive relations and transport coefficients</u>**

Momentum diffusion

$$\omega = v\mathcal{J}^{\beta} - iD\mathcal{J}^{\alpha}$$

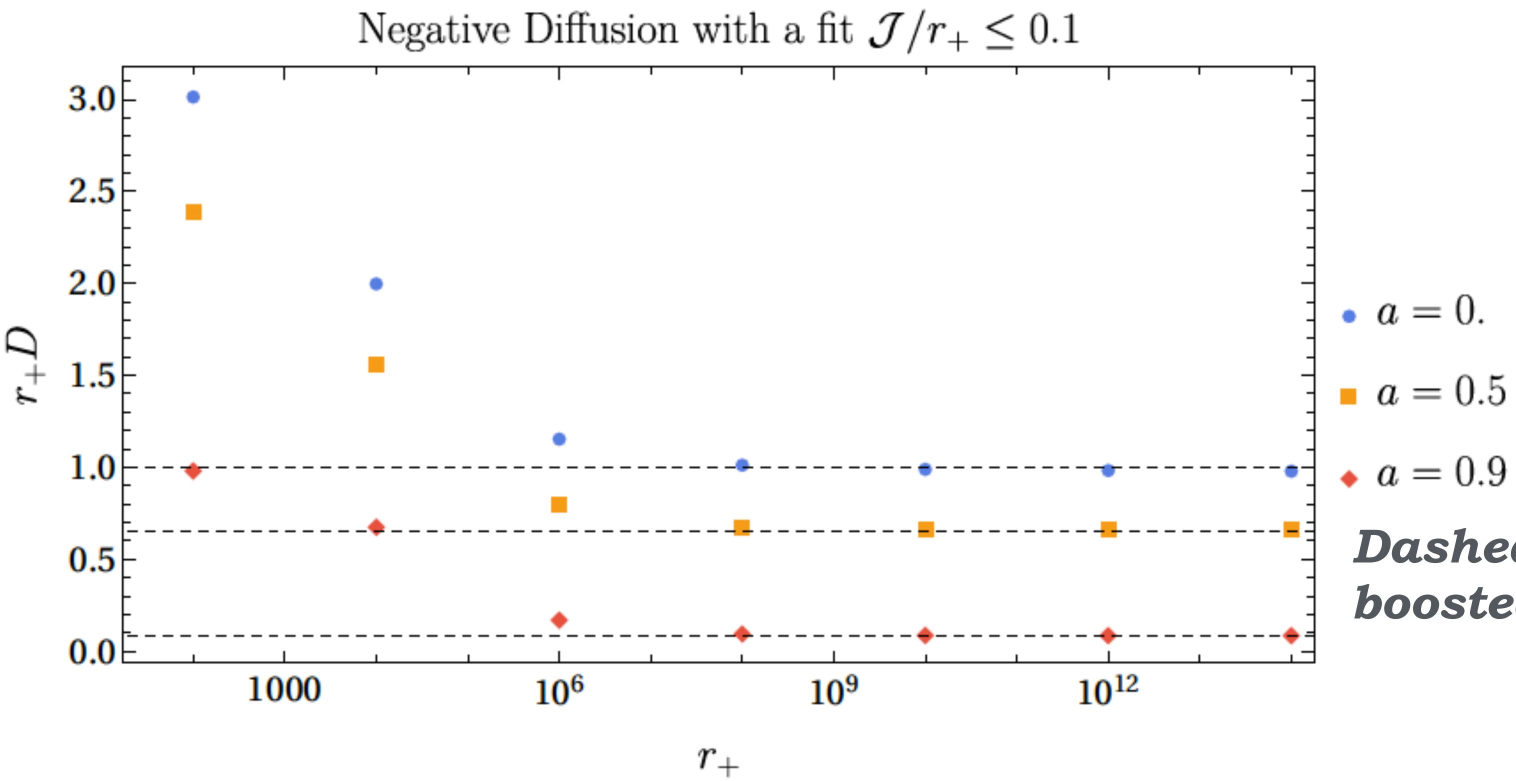




Is hydrodynamics valid? - Transport coefficients

[Cartwright,Garbiso-Amano;Kaminski,Wu; arXiv:2308.11686]

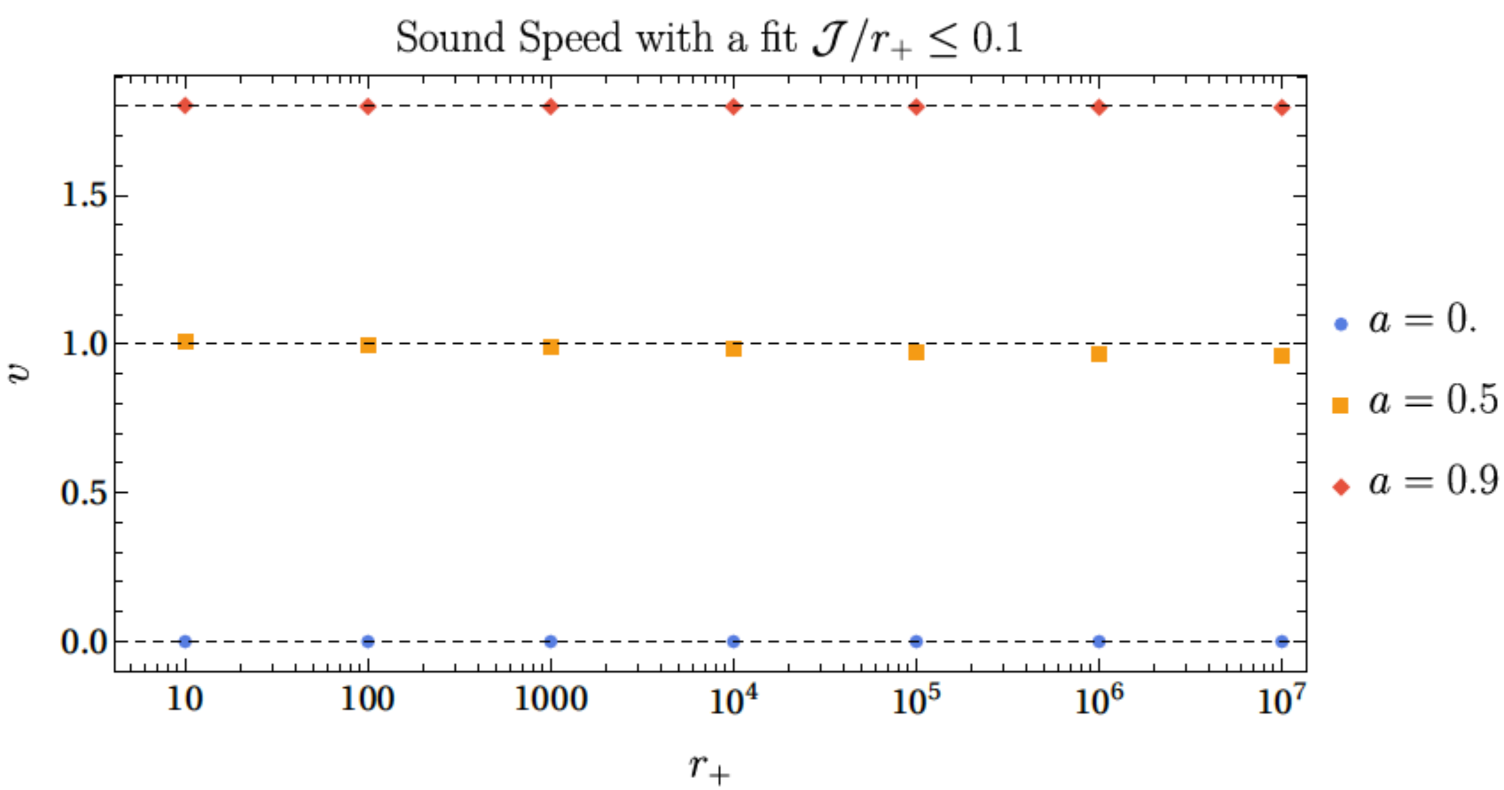
- **validity of the <u>constitutive relations and transport coefficients</u>**



Momentum diffusion

$$\omega = v\mathcal{J}^{\beta} - iD\mathcal{J}^{\alpha}$$

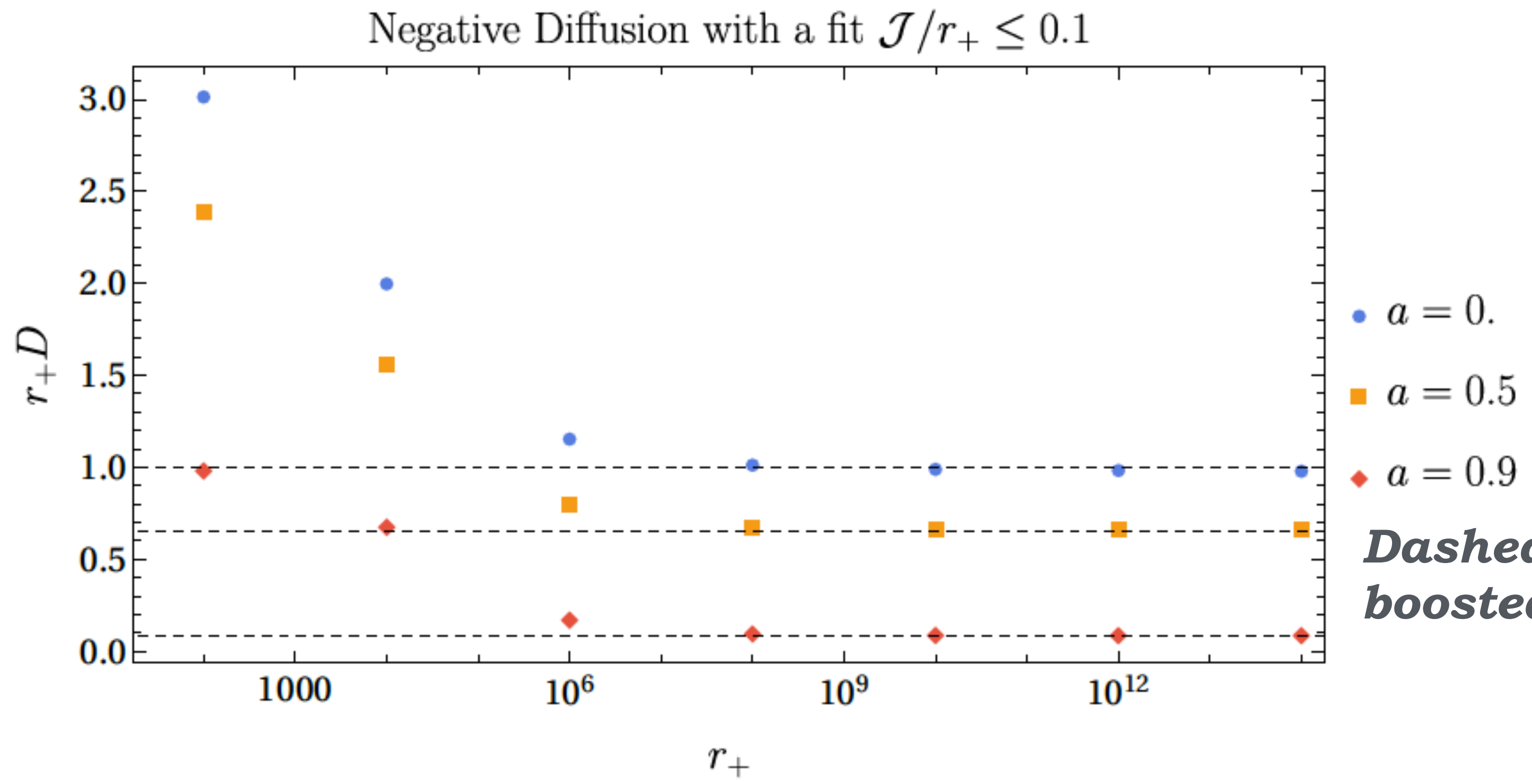
Dashed horizontal lines: boosted fluid values



Is hydrodynamics valid? - Transport coefficients

[Cartwright,Garbiso-Amano;Kaminski,Wu; arXiv:2308.11686]

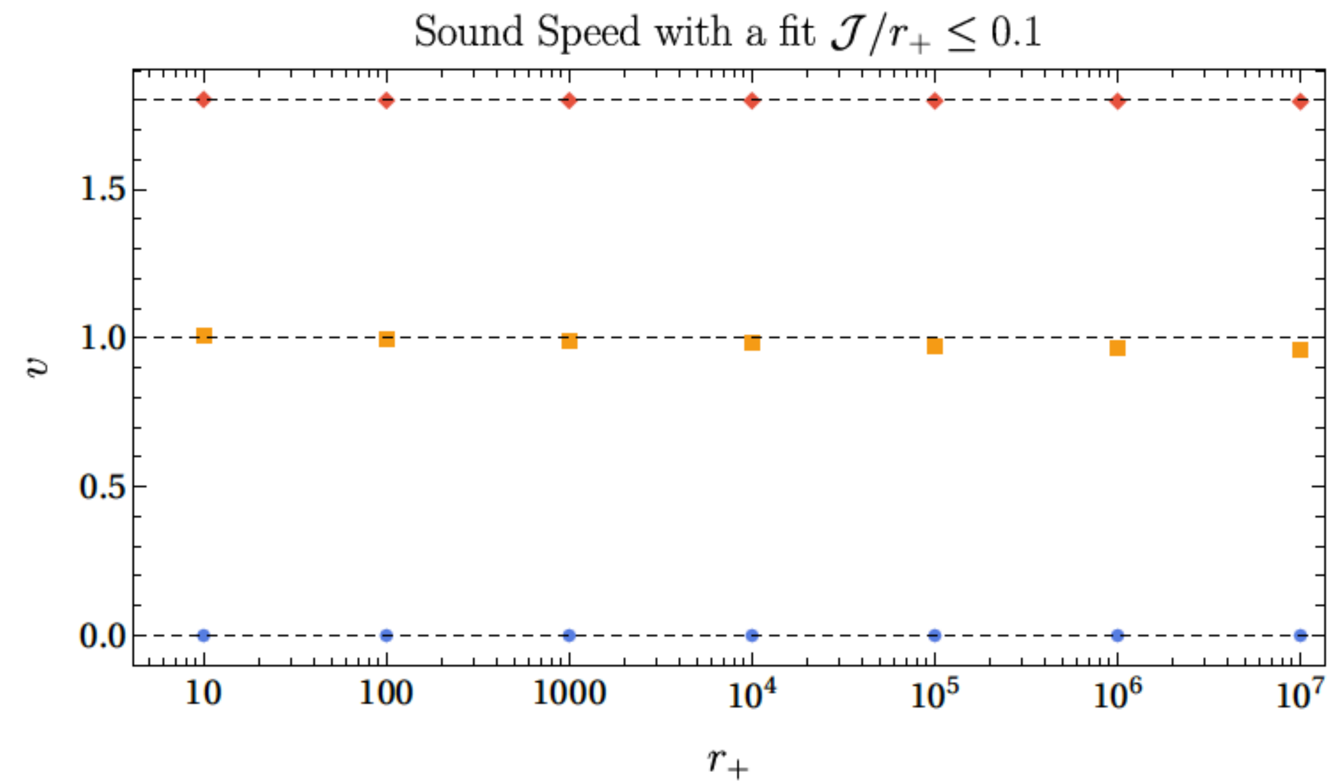
- **validity of the constitutive relations and transport coefficients**



Momentum diffusion

$$\omega = v\mathcal{J}^\beta - iD\mathcal{J}^\alpha$$

Dashed horizontal lines: boosted fluid values



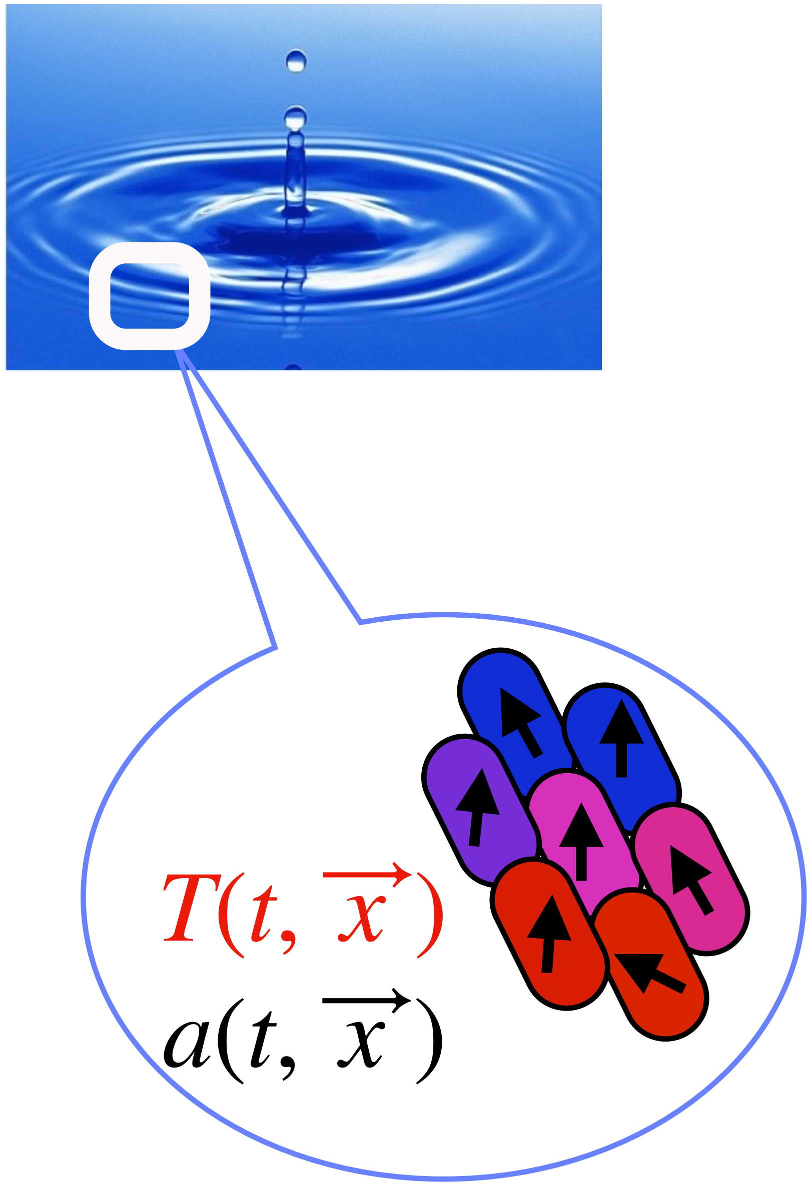
➡window of horizon values 1,000 < r+ < 10^7: hydrodynamic behavior distinct from a boosted fluid

Result 3: Hydrodynamics is valid

3. Is the hydrodynamic description valid?

— Yes, with modifications.

- **radius of convergence only changes by 60%**
- **transport coefficients change**
- **constitutive equations change**



Momentum diffusion mode

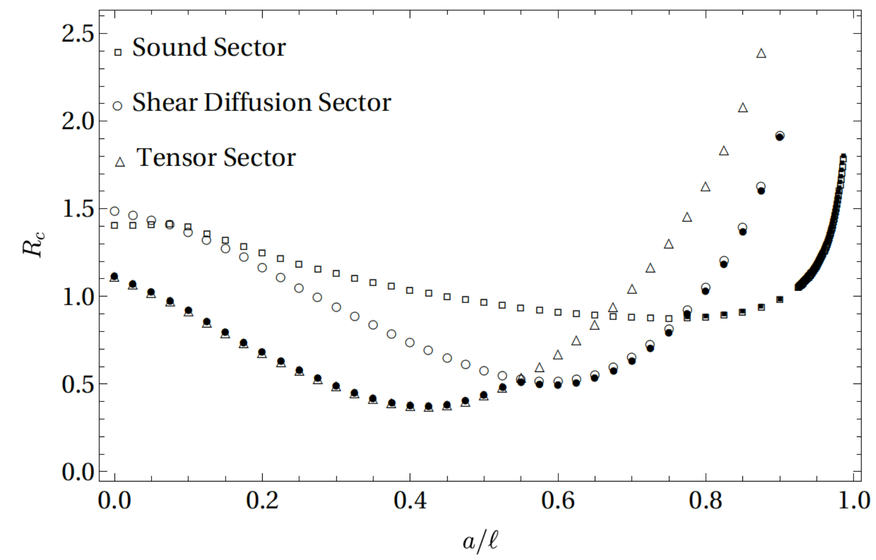
$$\omega(k) = -iDq^2 + \mathcal{O}(3)$$

➡ diffusion coefficient *D* is function of state variables: *T*, *a*

Summary

- derived convergence radius of hydrodynamics in rotating *N*=4 Super-Yang-Mills theory
- hydrodynamic expansion in momentum space is convergent for angular momenta
- hydrodynamics sees boosted fluid at high *T*: transport coefficients and their (Einstein) relations like boosted fluid
- analytic vortical plasma flow (cf. Gubser flow)
- different hydro needed at lower (transport coefficients & constitutive eq.)

Convergence radius



Boosted fluid transport coefficients

$$v_{||} = a\,,$$

$$\mathcal{D}_{||} = \mathcal{D}_0(1-a^2)^{3/2}\,,$$

$$v_{s,\pm} = v_{s,0}\frac{\sqrt{3}a \pm 1}{1 \pm \frac{a}{\sqrt{3}}}\,,$$

$$\Gamma_{s,\pm} = \Gamma_0\frac{\left(1-a^2\right)^{3/2}}{\left(1 \pm \frac{a}{\sqrt{3}}\right)^3}\,,$$

Boosted fluid Einstein relations

$$\mathcal{D}_{||}(a) = 2\pi T_0\frac{\eta_{||}(a)}{\epsilon(a)+P_\perp(a)}\,,$$

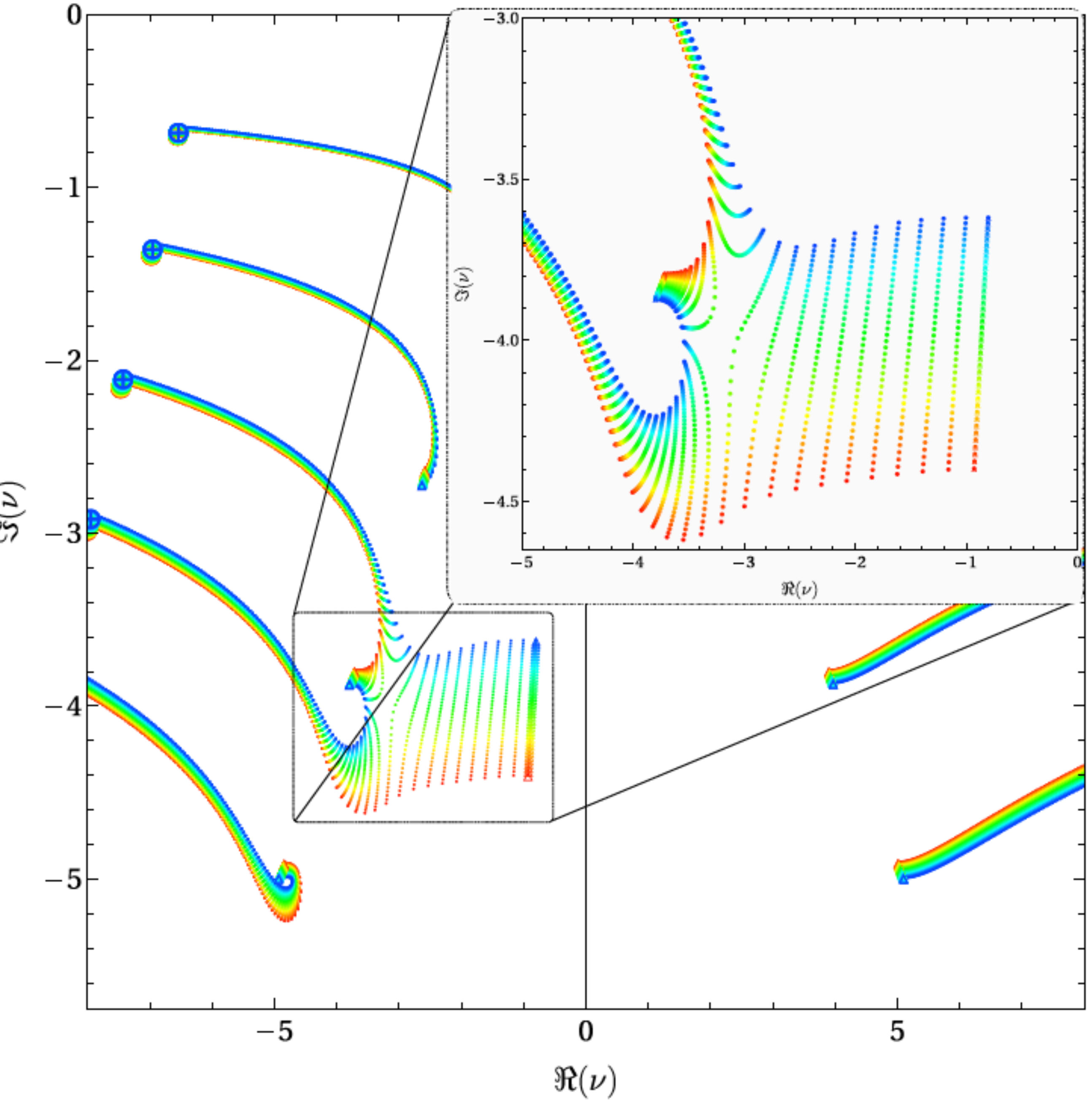
$$\Gamma_\pm(a) = \frac{2\eta_{||}(a)}{3(\epsilon(a)+P_\perp(a))}\frac{1}{(1\pm a/\sqrt{3})^3}$$

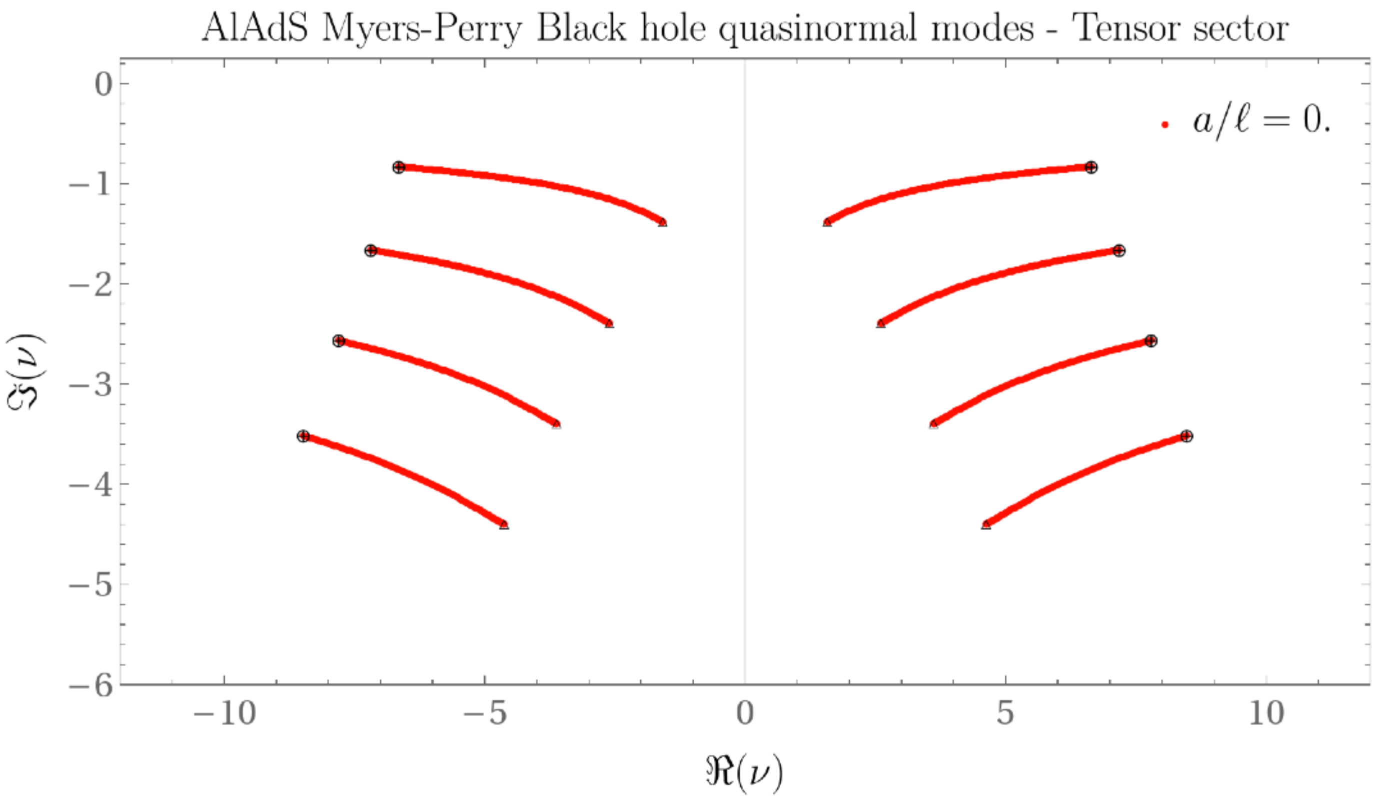
Outlook

- construct hydrodynamics around rotating state, then make rotation local (vorticity); fluid/gravity for rotating black holes
  [Erdmenger,Haack,Kaminski,Yarom; JHEP (2008)]
- holographic tests of existing hydrodynamic descriptions including rotation *[Cartwright,Garbiso-Amano;Kaminski,Wu; arXiv:2308.11686]*
- include into numerical hydrodynamic codes used for data analysis at RHIC and LHC
- include spin and torsion *[Hongo,Huang,Kaminski,Stephanov,Yee; JHEP (2021)]*
  → ***Talks by Buzzegoli, Lin, Singh***
- chiral vortical effect / chiral magnetic effect
  [Cartwright, Kaminski, Schenke; PRC (2022)]
- include hydrodynamic fluctuations, leading to long time tails etc.
  → ***Talk by Schaefer*** *[Abbasi, Kaminski, Tavakol; arXiv:2212.11499]*

Level crossings between modes

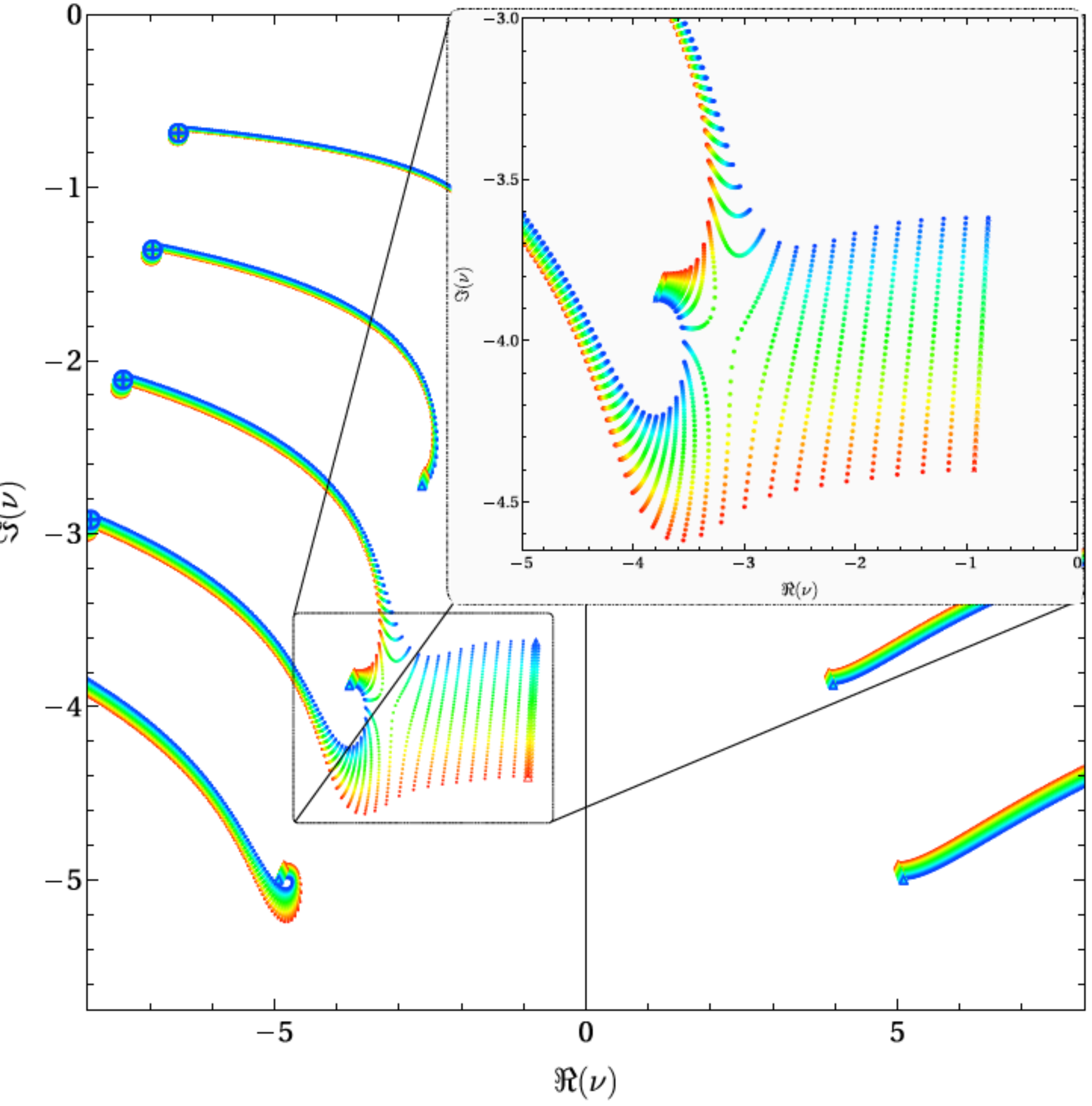
Multiple level crossings occur between distinct non-hydrodynamic modes

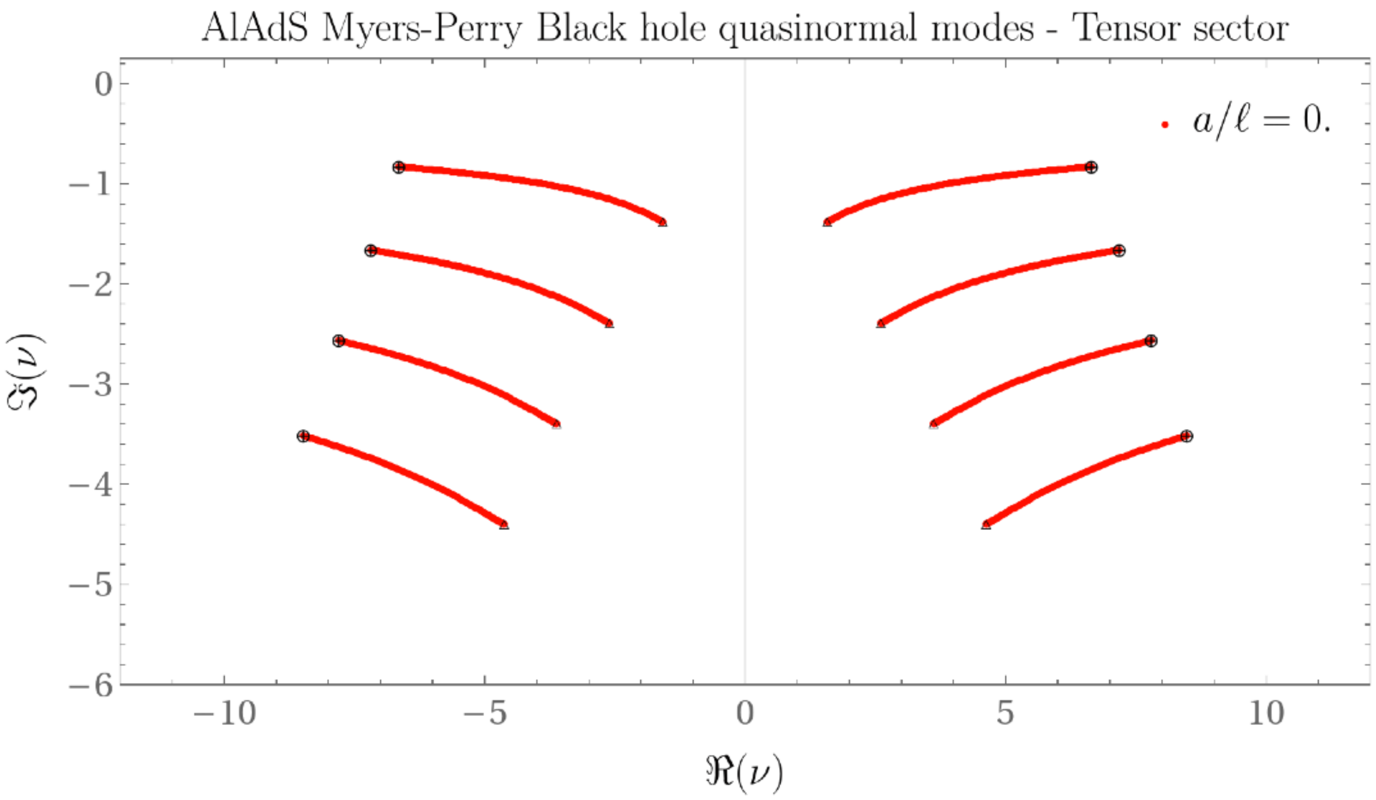




Level crossings between modes

Multiple level crossings occur between distinct non-hydrodynamic modes





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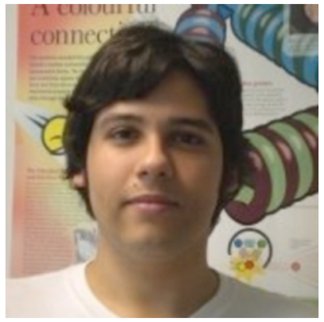
University of Illinois Urbana-Champaign



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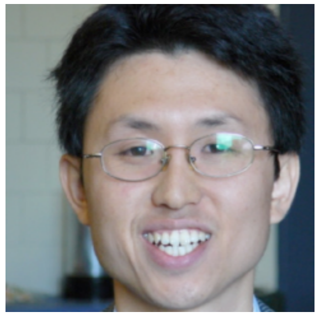
University of Illinois Chicago

Prof. Dr. Misha Stephanov

Dr. Masaru Hongo (now Niigata U)

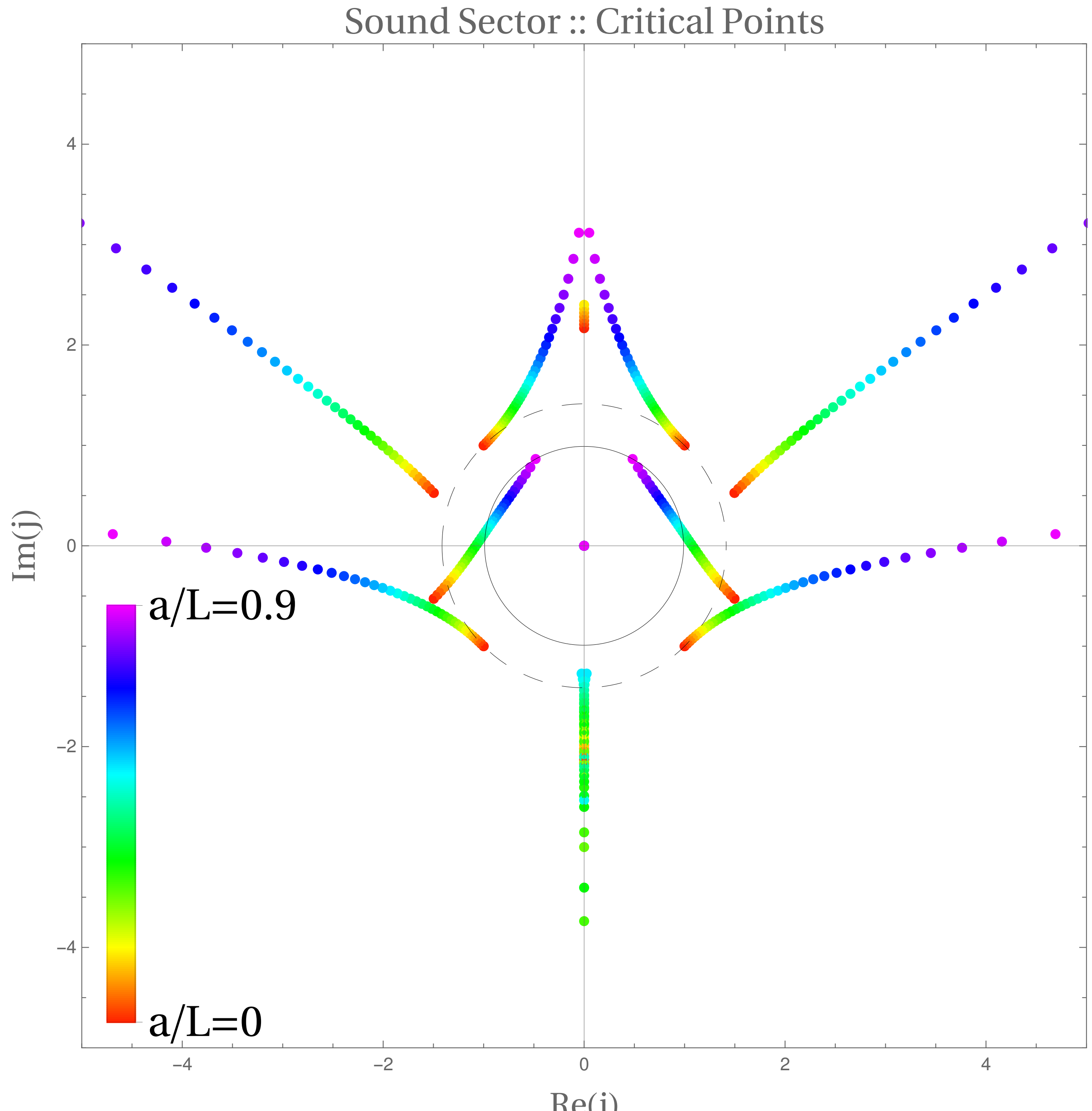


Prof. Dr. Ho-Ung Yee

APPENDIX

Singularities in the dispersion relations

Branch singularities (critical points) in dispersion relations of $\mathcal{N}$=4 Super-Yang-Mills theory in a rotating state with angular momentum a/L.



Complex ***momentum*** *plane*

[Cartwright,Garbiso-Amano;Kaminski,Noronha,Speranza; PRD (2023)]

Vision: Quantum fluids far from equilibrium

Hydrodynamics

- far from equilibrium

[Romatschke; PRL (2018)]

[Jensen, Kaminski, Kovtun,Meyer,Ritz,Yarom.; PRL (2012)]
[Banerjee et al. JHEP (2012)]
[Glorioso,Liu]
[Haehl,Loganayagam,Rangamani]

- quantum chaos

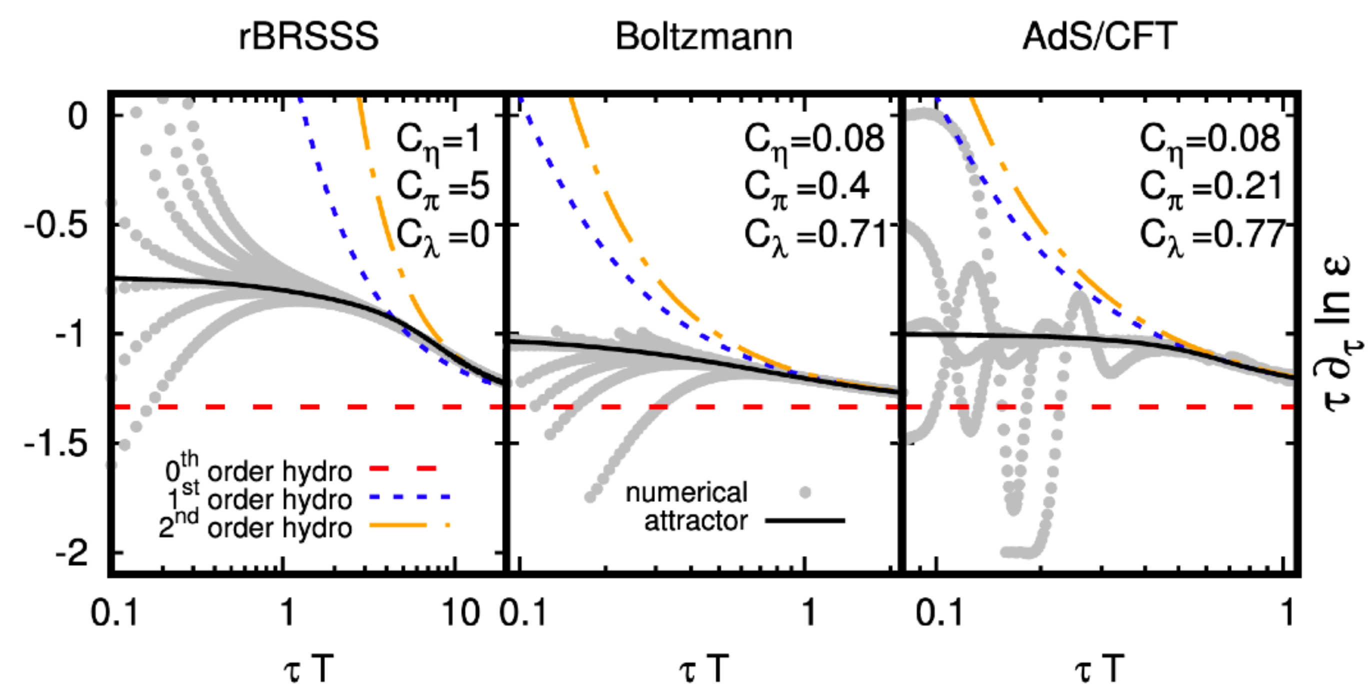
[Blake, Lee, Liu; JHEP (2018)]
[Grozdanov et al. (2019)]

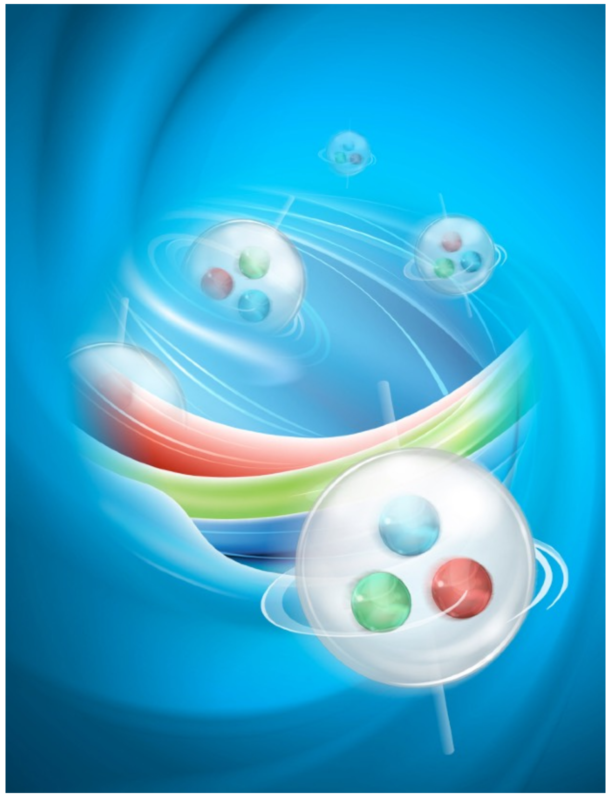
- convergence & stability

[Kovtun; JHEP (2019)]
[Grozdanov, Kovtun, Starinets, Tadic; PRL (2019)]
[Withers; JHEP (2018)]
[Heller, Janik, Witaszczyk; PRL (2013)]
[Heller, Spalinski; PRL (2018)]

- most vortical fluid

[Garbiso, Kaminski; JHEP (2019)]
[Cartwright,Garbiso-Amano;Kaminski,Noronha,Speranza;arXiv:2112.10781]

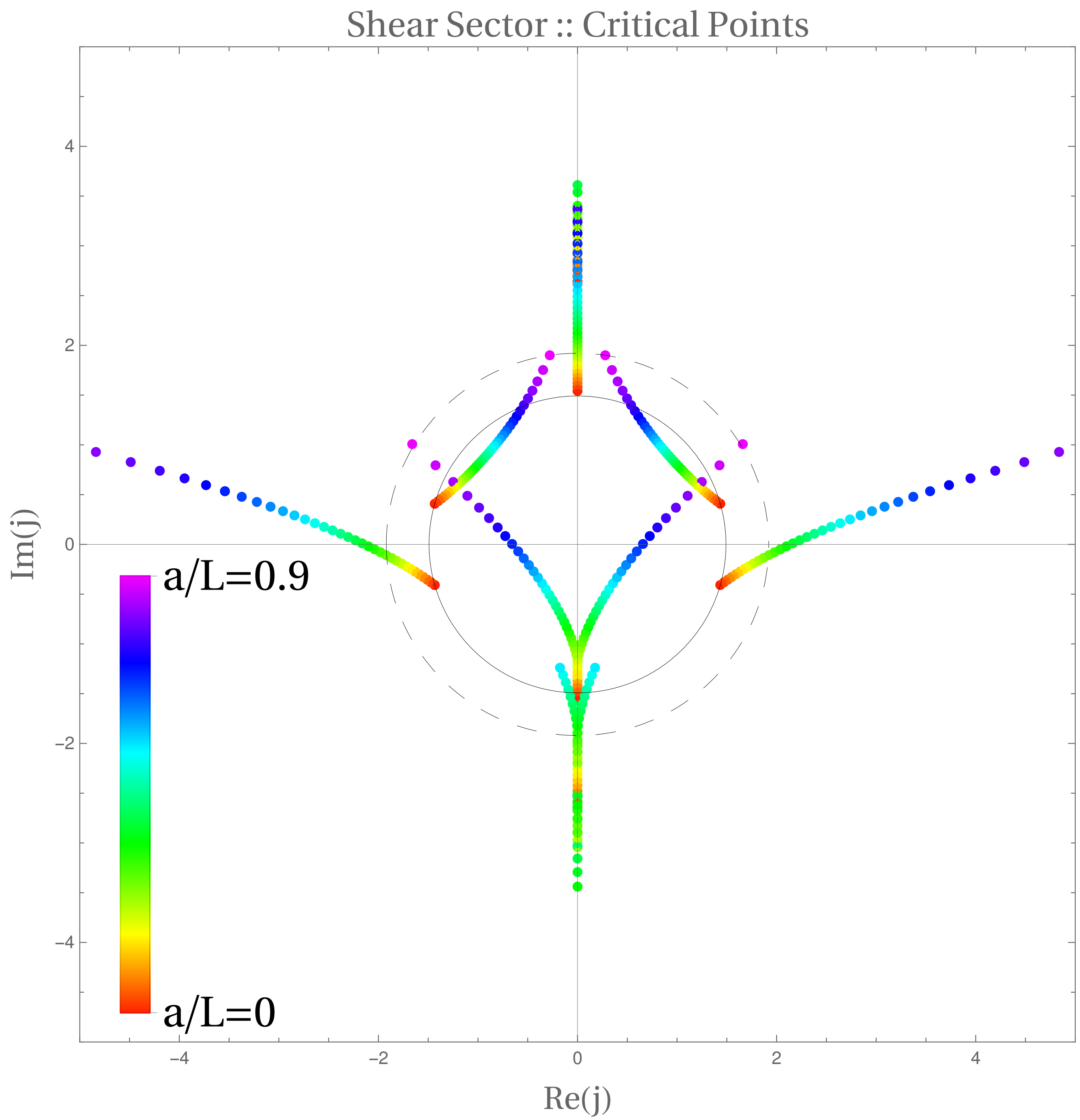




[STAR; Nature (2017)]

Singularities in the dispersion relations

Branch singularities (critical points) in dispersion relations of N=4 Super-Yang-Mills theory in a rotating state with angular momentum a/L.



*Complex **momentum** plane*

[Cartwright,Garbiso-Amano;Kaminski,Noronha,Speranza; PRD (2023)]

AdS5 Schwarzschild pole-skipping:

$$\mathfrak{w} = i, \quad \mathfrak{q} = \pm\sqrt{\frac{3}{2}}\, i\,.$$

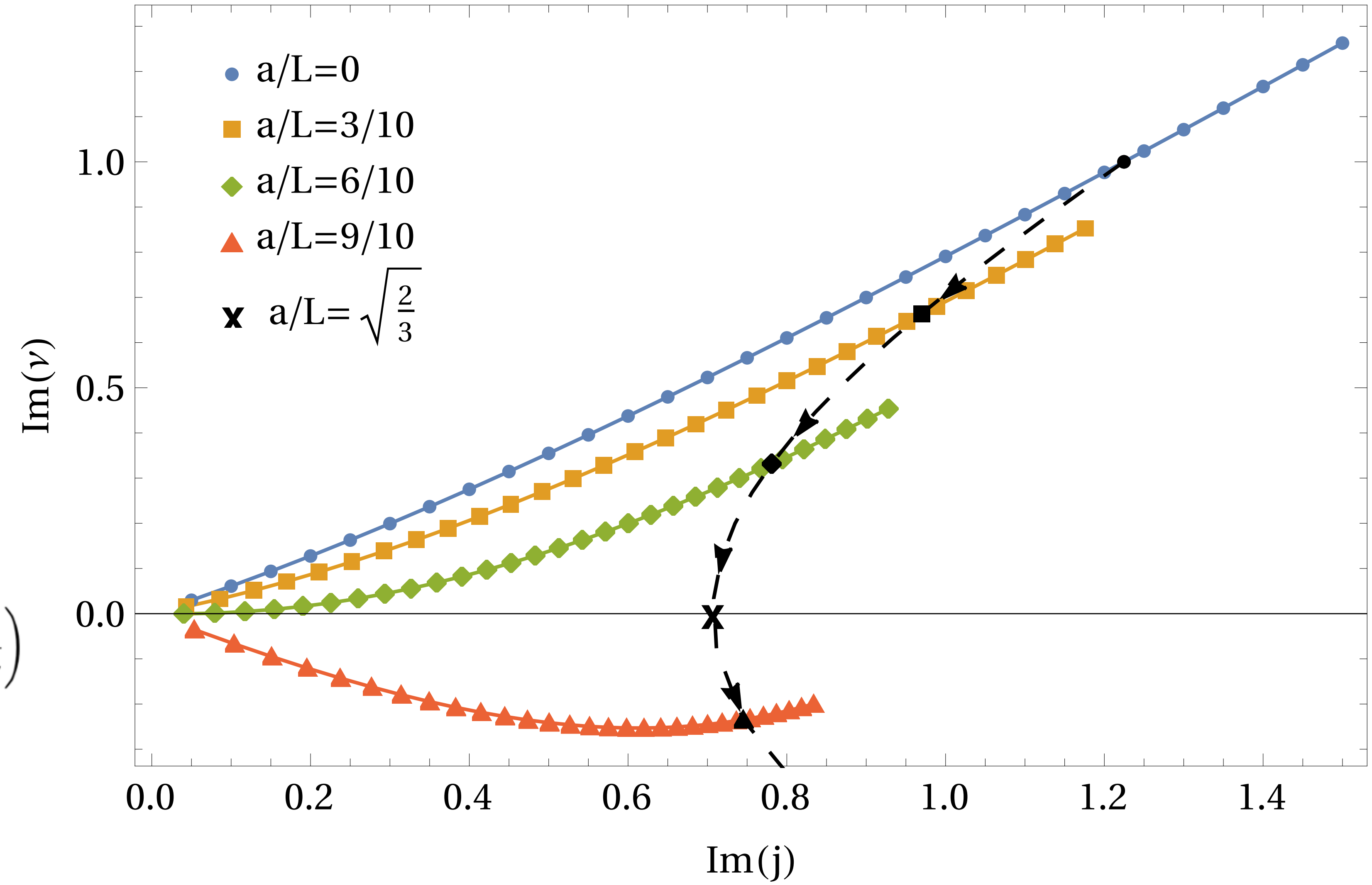
Apply transformation:

$$\mathfrak{q} = \frac{a\nu + j}{\sqrt{1-a^2}}, \quad \mathfrak{w} = \frac{aj + \nu}{\sqrt{1-a^2}}$$

Shifted pole-skipping points:

$$\nu_{scalar} = \frac{i}{\sqrt{1-a^2/L^2}}\left(1 \mp \frac{\sqrt{3}}{\sqrt{2}}\frac{a}{L}\right), \quad j_{scalar} = \frac{i}{\sqrt{1-a^2/L^2}}\left(\pm\frac{\sqrt{3}}{\sqrt{2}} - \frac{a}{L}\right)$$

[Amano(Garbiso),Blake,Cartwright,Kaminski,Thompson; (2022)]



$$\lambda_L = 2\pi T\left(1 - \sqrt{\frac{3}{2}}\frac{|a|}{L}\right) = 2\pi T\left(1 - |v|/v_B^{(0)}\right)$$

$$v_B^{\pm} = \frac{\sqrt{\frac{2}{3}} \mp \frac{a}{L}}{1 \mp \sqrt{\frac{2}{3}}\frac{a}{L}}$$

Agrees with shock-wave computation and with near-horizon expansion method.

Pole-skipping points in rotating black holes in AdS4: *[Blake, Davison; JHEP (2021)]*

Quantum chaos in (large) rotating AdS5 black holes

AdS5 Schwarzschild pole-skipping:

$$\mathfrak{w} = i, \quad \mathfrak{q} = \pm\sqrt{\frac{3}{2}}i\,.$$

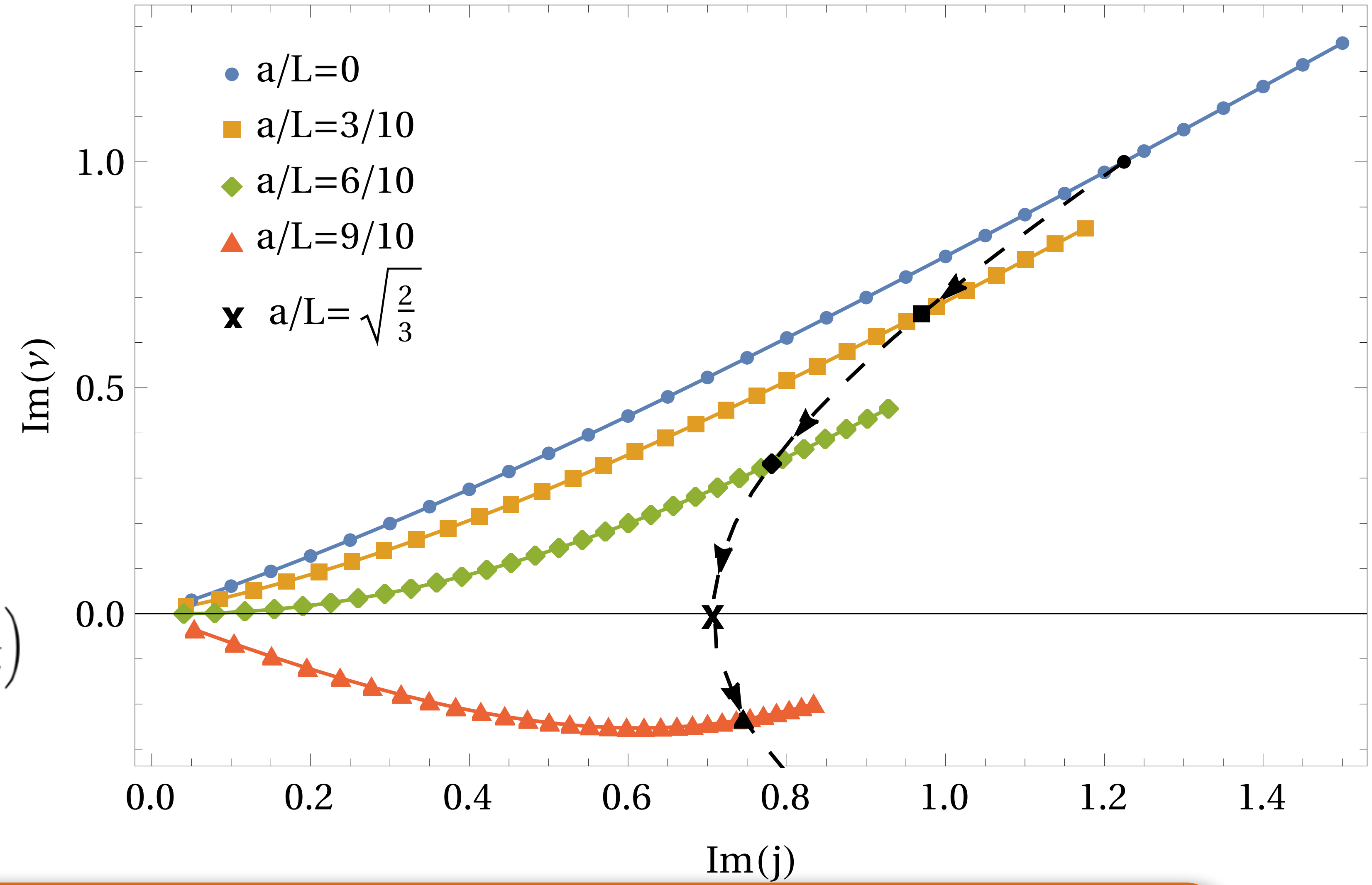
Apply transformation:

$$\mathfrak{q} = \frac{a\nu + j}{\sqrt{1-a^2}}, \quad \mathfrak{w} = \frac{aj + \nu}{\sqrt{1-a^2}}$$

Shifted pole-skipping points:

$$\nu_{scalar} = \frac{i}{\sqrt{1-a^2/L^2}}\left(1 \mp \frac{\sqrt{3}}{\sqrt{2}}\frac{a}{L}\right), \quad j_{scalar} = \frac{i}{\sqrt{1-a^2/L^2}}\left(\pm\frac{\sqrt{3}}{\sqrt{2}} - \frac{a}{L}\right)$$

[Amano(Garbiso),Blake,Cartwright,Kaminski,Thompson; (2022)]



$$\lambda_L = 2\pi T\left(1 - \sqrt{\frac{3}{2}}\frac{|a|}{L}\right) = 2\pi T\left(1 - |v|/v_B^{(0)}\right)$$

quantum Lyapunov exponent

$$v_B^{\pm} = \frac{\sqrt{\frac{2}{3}} \mp \frac{a}{L}}{1 \mp \sqrt{\frac{2}{3}}\frac{a}{L}}$$

butterfly velocity

Agrees with shock-wave computation and with near-horizon expansion method.

Pole-skipping points in rotating black holes in AdS4: *[Blake, Davison; JHEP (2021)]*

$$h^V_{\mu\nu} \equiv e^{-i\omega\tau} r^2 (h_{++}(r)\sigma^+_\mu \sigma^+_\nu D^{\mathcal{J}}_{(\mathcal{J}-1)\mathcal{M}} + 2(h_{+r}(r)\sigma^+_{(\mu}\sigma^r_{\nu)} + h_{+t}(r)\sigma^+_{(\mu}\sigma^t_{\nu)} + h_{+3}(r)\sigma^+_{(\mu}\sigma^3_{\nu)}) D^{\mathcal{J}}_{\mathcal{J}\mathcal{M}}) \,,$$

$$\partial_+ D^{\mathcal{J}}_{\mathcal{K}\mathcal{M}} = \sqrt{(\mathcal{J}+\mathcal{K})(\mathcal{J}-\mathcal{K}+1)} D^{\mathcal{J}}_{\mathcal{K}-1\ \mathcal{M}} \,,$$

$$\partial_- D^{\mathcal{J}}_{\mathcal{K}\mathcal{M}} = -\sqrt{(\mathcal{J}-\mathcal{K})(\mathcal{J}+\mathcal{K}+1)} D^{\mathcal{J}}_{\mathcal{K}+1\ \mathcal{M}} \,,$$

$$\partial_3 D^{\mathcal{J}}_{\mathcal{K}\mathcal{M}} = -i\mathcal{K} D^{\mathcal{J}}_{\mathcal{K}\mathcal{M}} \,,$$

Hydrodynamics as far-from-equilibrium description?

Far-from-equilibrium HYDRODYNAMICS ?

HYDRODYNAMICS & THERMODYNAMICS

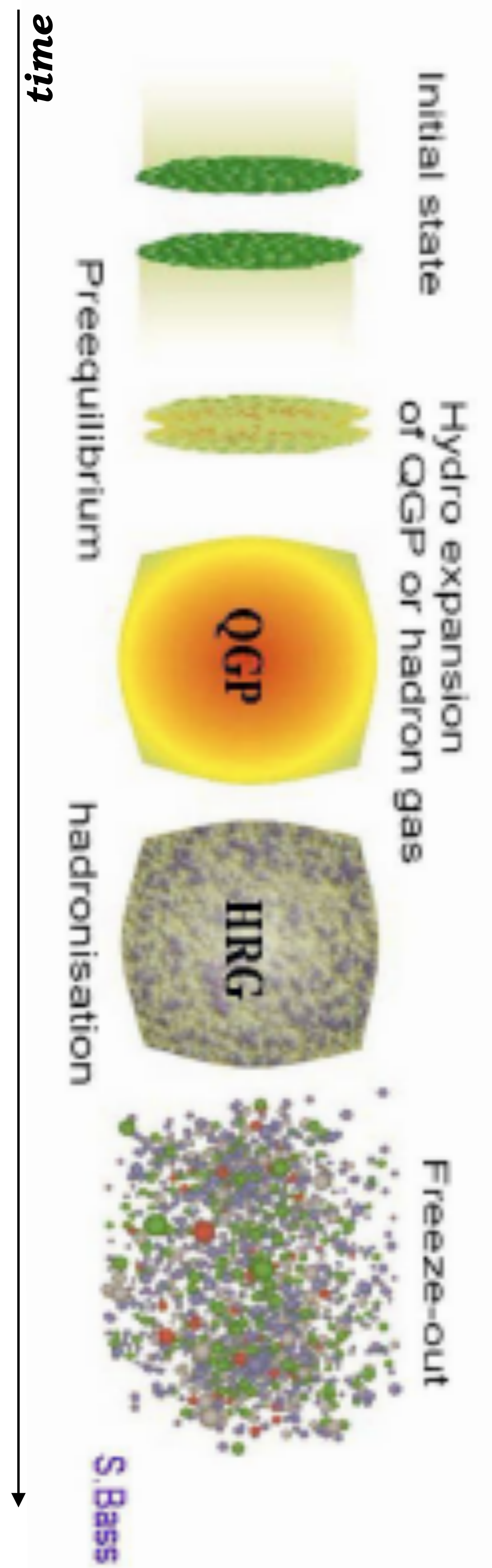
Chiral Magnetic Effect (CME) from chiral anomaly

[Kharzeev; PRC (2004)]
[Son,Surowka; PRL (2009)]
[Neiman,Oz; JHEP (2010)]

$$J^\mu_A = \xi_B B$$

$\xi_B \sim C\mu_A$ $\qquad$ $\nabla_\mu J^\mu_A = C\, E\cdot B$

?

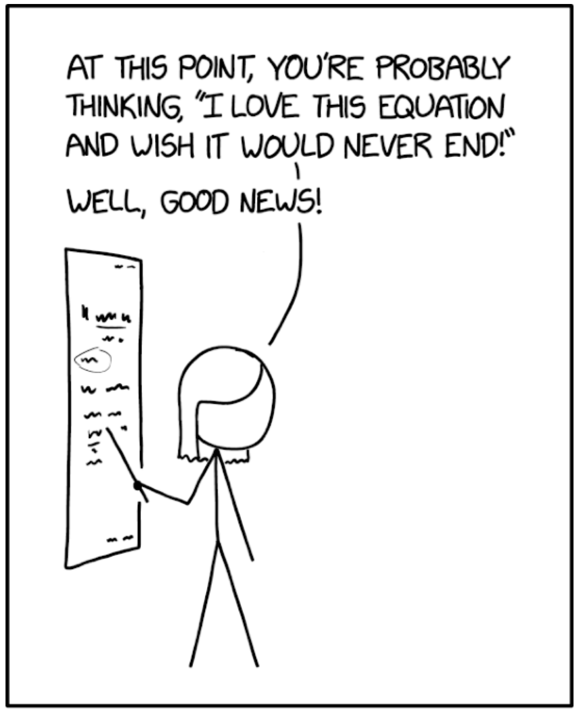


HOLOGRAPHY

Invitation: Hydrodynamic expansion is asymptotic

Hydrodynamic expansion of dispersion relations
around far-from-equilibrium state

- asymptotic expansion: *coefficients* $\sim n!$
- attractors *[Heller, Spalinski; PRL (2015)]* *[Heller et al; PRL (2021)]*
- resurgence
- far-from-equilibrium holography *[Kurkela et al; PRL (2019)]* *[Janik, Jankowski, Soltanpanahi; PRL (2017)]*
- far-from-equilibrium fluid dynamics *[Romatschke; PRL (2017)]*

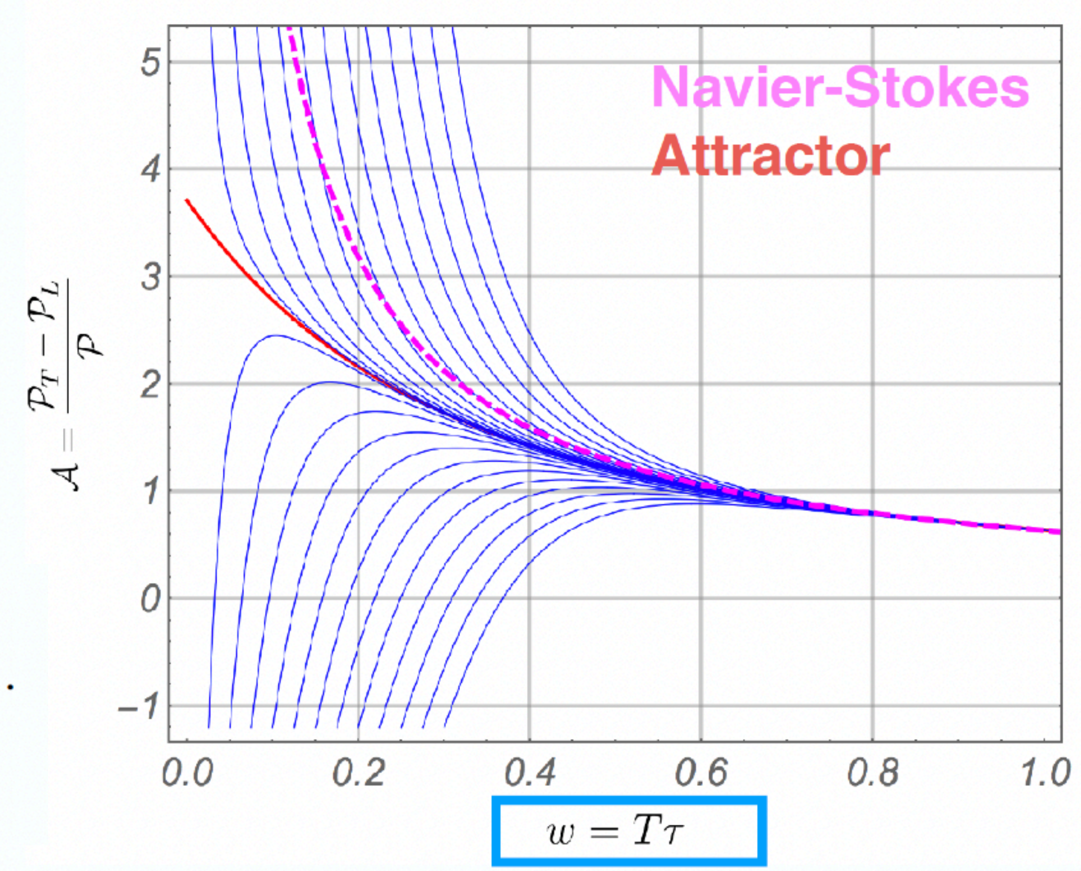


TAYLOR SERIES EXPANSION IS THE WORST.

➡**asymptotic is worse**

Pressure anisotropy in *N*=4 SYM:

$$\mathcal{A} = \underbrace{\frac{8C_\eta}{w}}_{\text{Navier–Stokes}} + \underbrace{\frac{16C_\eta C_\tau}{3w^2}}_{\text{2nd order}} + \ldots = \underbrace{\sum_{n>0}\frac{a_n^{(0)}}{w^n}}_{\text{gradient expansion}} + \underbrace{\left(\sigma\, w^{\frac{C_\eta}{C_\tau}} e^{-\frac{3}{2C_\tau}w}\right)\sum_{n\geq 0}\frac{a_n^{(1)}}{w^n}}_{\text{transseries sectors}} + \ldots$$



[from Talk by Spalinski at QuarkMatter22]

Successful example: Discovery of new transport effect

phenomenological hydrodynamics → **formal hydrodynamics**

Chiral Vortical Effect

[Erdmenger,Haack,Kaminski,Yarom; JHEP (2008)]
[Banerjee et al.; JHEP (2011)]
[Son,Surowka; PRL (2009)]
phenomenological: Vilenkin; (1979)]

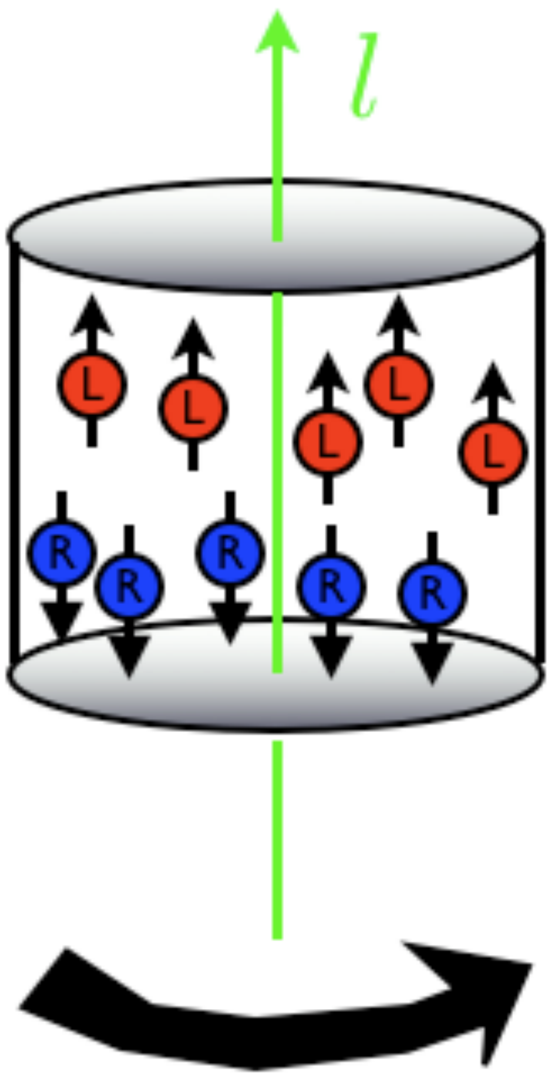
$$J^\mu_A = \xi \, \Omega^\mu$$

vorticity

Chiral vortical conductivity:

$$\xi = C \mu_A{}^2 + c_g T^2$$

Quantum transport effect originating from chiral anomaly



Hydrodynamics - formalism

Universal **effective field theory (EFT)**

- expansion in gradients of fields
- systematic construction
- historically: "phenomenological" start from constitutive equations *[Landau, Lifshitz]*
- modern: "formal" generating functional

[Jensen, Kaminski, Kovtun, Meyer, et al.; PRL (2012)]
[Banerjee et al. JHEP (2012)]

Hydrodynamic limit $\frac{\omega}{T} \ll 1, \quad \frac{|\vec{k}|}{T} \ll 1$

- fields $T(x)$, $n(x)$, $u^\alpha(x)$
  temperature — *charge density* — *fluid velocity*
- constitutive equations

$$\langle j^\alpha \rangle = n\, u^\alpha + \nu^\alpha$$

ideal hydro — *derivative corrections*

- conservation equations

$$\nabla_\alpha \langle j^\alpha \rangle = 0$$

e.g. continuity: $\partial_t n + \vec{\nabla} \cdot \vec{j} = 0$

- sources *[Luttinger]* $g_{\mu\nu}$ — *metric*; $A_\alpha(x)$ — *gauge field*

Fluid/gravity correspondence

Conservation equations from gravity

5-dimensional Einstein-Maxwell-Chern-Simons equations of motion :

$$R_{MN} + 4g_{MN} = \frac{1}{2}F_{MK}F_N{}^K - \frac{1}{12}g_{MN}F^2$$

$$\partial_N(\sqrt{-g}F^{NM}) = \frac{1}{4\sqrt{3}}\epsilon^{MNOPQ}F_{NO}F_{PQ}$$

dual to anomaly

$$\xi_N = dr$$

Constraint equations arise from contraction with one-form dr (normal to boundary) :

$$(\text{contraints})_M = \xi^N(\text{Einstein equations})_{MN} \quad \Rightarrow \quad \nabla_\mu T^{\mu\nu} = F^{\nu\lambda}j_\lambda$$

$$(\text{contraint}) = \xi^N(\text{Maxwell} - \text{Chern} - \text{Simons equations})_N \quad \Rightarrow \quad \nabla_\mu j^\mu = CE^\mu B_\mu$$

Constitutive equations from gravity

Example: no matter content, vanishing gauge fields :

$$\langle T_{\mu\nu}\rangle = \lim_{r\to\infty}\left[\frac{r^{(D-3)}}{\kappa_D^2}(K_{\mu\nu} - K\gamma_{\mu\nu} - (D-2)\gamma_{\mu\nu})\right]$$

with extrinsic curvature $K_{\mu\nu} = -\frac{1}{2n}(\partial_r\gamma_{\mu\nu} - \nabla_\mu n_\nu - \nabla_\nu n_\mu)$

$$ds^2 = n^2dr^2 + \gamma_{\mu\nu}(dx^\mu + n^\mu dr)(dx^\nu + n^\nu dr)$$

Example: R-charged solution

[Erdmenger, Haack, MK, Yarom; JHEP (2009)]

Gravity dual: 5-dimensional Einstein-Maxwell-Chern-Simons action

$$S = -\frac{1}{2{\kappa_5}^2}\int\left[\sqrt{-g}\left(R+12-\frac{1}{4}F^2\right)-\frac{1}{12\sqrt{3}}\epsilon^{MNOPQ}A_M F_{NO}F_{PQ}\right]d^4x\,dr$$

CS-term dual to chiral anomaly

Black hole with R-charge (in boosted Eddington-Finkelstein coordinates):

$$ds^2 = -r^2 f(r)u_\mu u_\nu dx^\mu dx^\nu + r^2\Delta_{\mu\nu}dx^\mu dx^\nu - 2u_\mu dx^\mu dr$$

solution with constant parameters $Q,\ b,\ u^\mu$.

$$f(r) = 1+\frac{Q^2}{r^6}-\frac{1}{b^4r^4} \qquad A_r = 0\,,\quad A_\mu = -\frac{\sqrt{3}Q}{r^2}u_\mu \qquad \Delta_{\mu\nu} = \eta_{\mu\nu}+u_\mu u_\nu$$

Make parameters boundary-coordinate-dependent: *dual to hydrodynamic fields*

$$b\to b(x)\,,\quad Q\to Q(x),\quad u^\mu\to u^\mu(x)$$

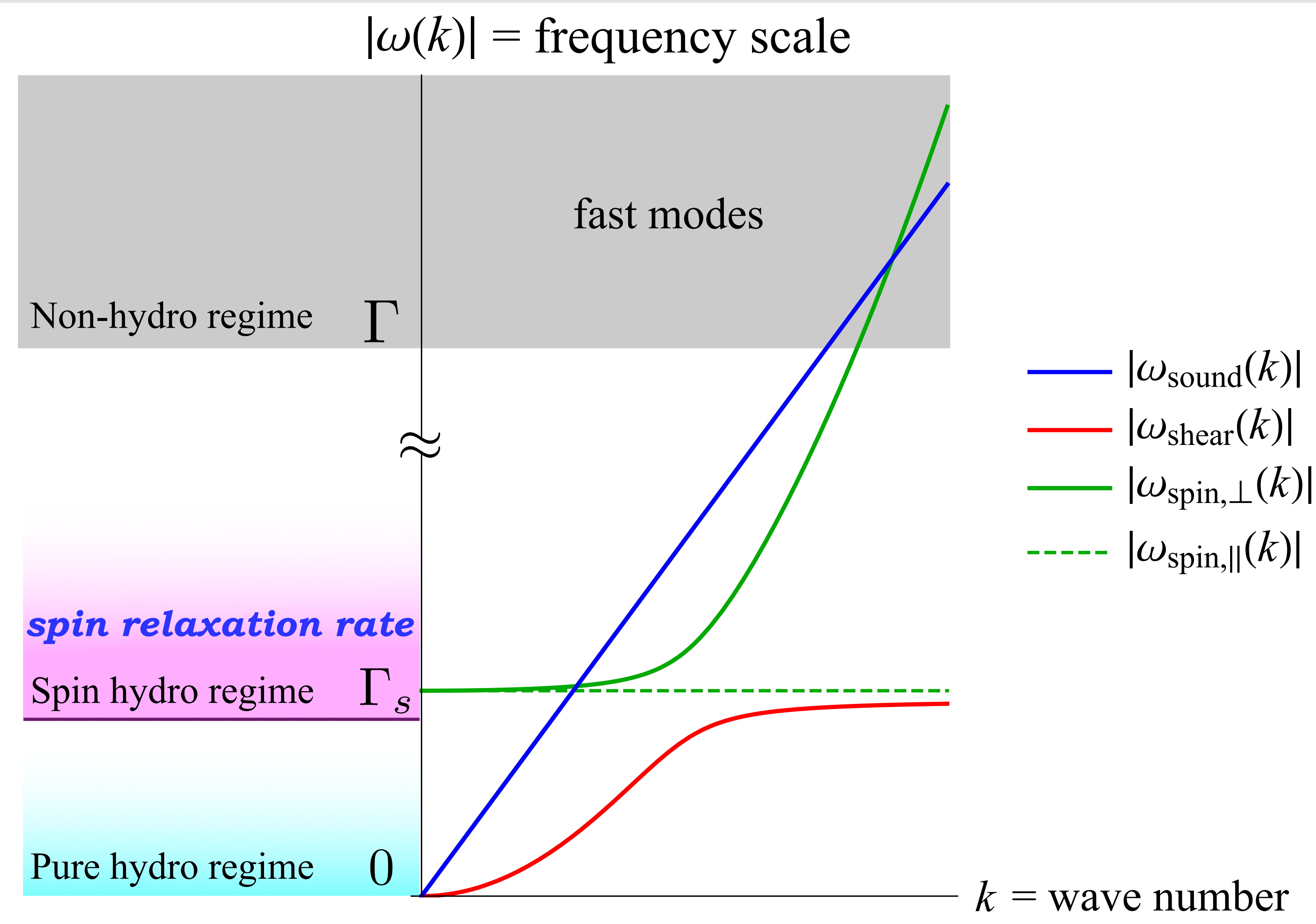
- expand in gradients of b, Q and u
  dual to hydrodynamic expansion in the field theory
- new analytical solutions to Einstein equations
  give values of transport coefficients in field theory

Basic ideas - spin is not conserved but slow

[Hongo,Huang,Kaminski,Stephanov,Yee; JHEP (2021)]

Consider spin relaxation as slow enough to survive long time

non-conserved quantities disappear fast



conserved charges diffuse slowly

➡ **consider spin together with *conserved* slow quantities.**

Basic ideas - why torsion?

[Hongo,Huang,Kaminski,Stephanov,Yee; JHEP (2021)]

Consider geometric definitions in curved metric with nonzero torsion

Energy-momentum tensor

- Variation of generating functional with respect to metric

$$\underset{\text{symmetric}}{T^{\mu\nu}} = \frac{2}{\sqrt{-g}} \frac{\delta\Gamma}{\delta g_{\mu\nu}}\bigg|_{g_{\mu\nu}=\eta_{\mu\nu}}$$

Spin current

- Variation with respect to **spin connection**

$$\Sigma^{\mu}_{\hat{a}\hat{b}} = -\frac{2}{e}\frac{\delta\Gamma}{\delta\omega_{\mu}^{\hat{a}\hat{b}}}\bigg|_{\text{torsion}=0}$$

First Cartan equation $De^{\hat{a}} = de^{\hat{a}} + e^{\hat{a}} \wedge \omega^{\hat{a}}_{\hat{b}} = T^{\hat{a}}$

- spin connection slaved to metric at zero torsion

$$g_{\mu\nu} = e_{\mu}^{\ \hat{a}} e_{\nu}^{\ \hat{b}} \eta_{\hat{a}\hat{b}}$$

➡ Consider nonzero torsion, promoting spin connection to be an *independent* source, *uniquely* defining spin current.

Future topics for hydrodynamics

Hydrodynamics

- applications, e.g topological insulators?
- 2D hydrodynamics
  [Jensen, Kaminski, Kovtun, Meyer, et al.; PRL (2012)]
  [JHEP (2011)]
- non-relativistic
  [Kaminski, Moroz; PRB (2014)]
  [Davison, Grozdanov, Janiszewski, Kaminski; JHEP (2016)] *[Garbiso, Kaminski; JHEP (2019)]*
- surface states of 3D hydro
- far from equilibrium
  [Cartwright, Kaminski; JHEP (2019)]
  [Wondrak, Kaminski, Bleicher; PRB (2020)]
- quantum chaos
  [Blake, Lee, Liu; JHEP (2018)]
  [Grozdanov et al. (2019)]
- convergence

Examples:

QCP in 2D topological / band insulator
[Amaricci, Budich, Capone, Trauzettel, Sangiovanni; PRL (2015)]

Turbulent hydrodynamics in strongly correlated Kagome metals
[Di Sante, Erdmenger, Greiter, Matthaiakakis, Meyer, Fernandez, Thomale, van Loon, Wehling; Nature Commun. (2020)]

Surface States in Holographic Weyl Semimetals
[Ammon, et al.; PRL (2017)]

Thank you for listening!