Incorporating Thermal Mesons into CMF for a Realistic EoS

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Based on : R. Kumar et al., *Phys. Rev. D* **111**, 074029 (2025)

muses

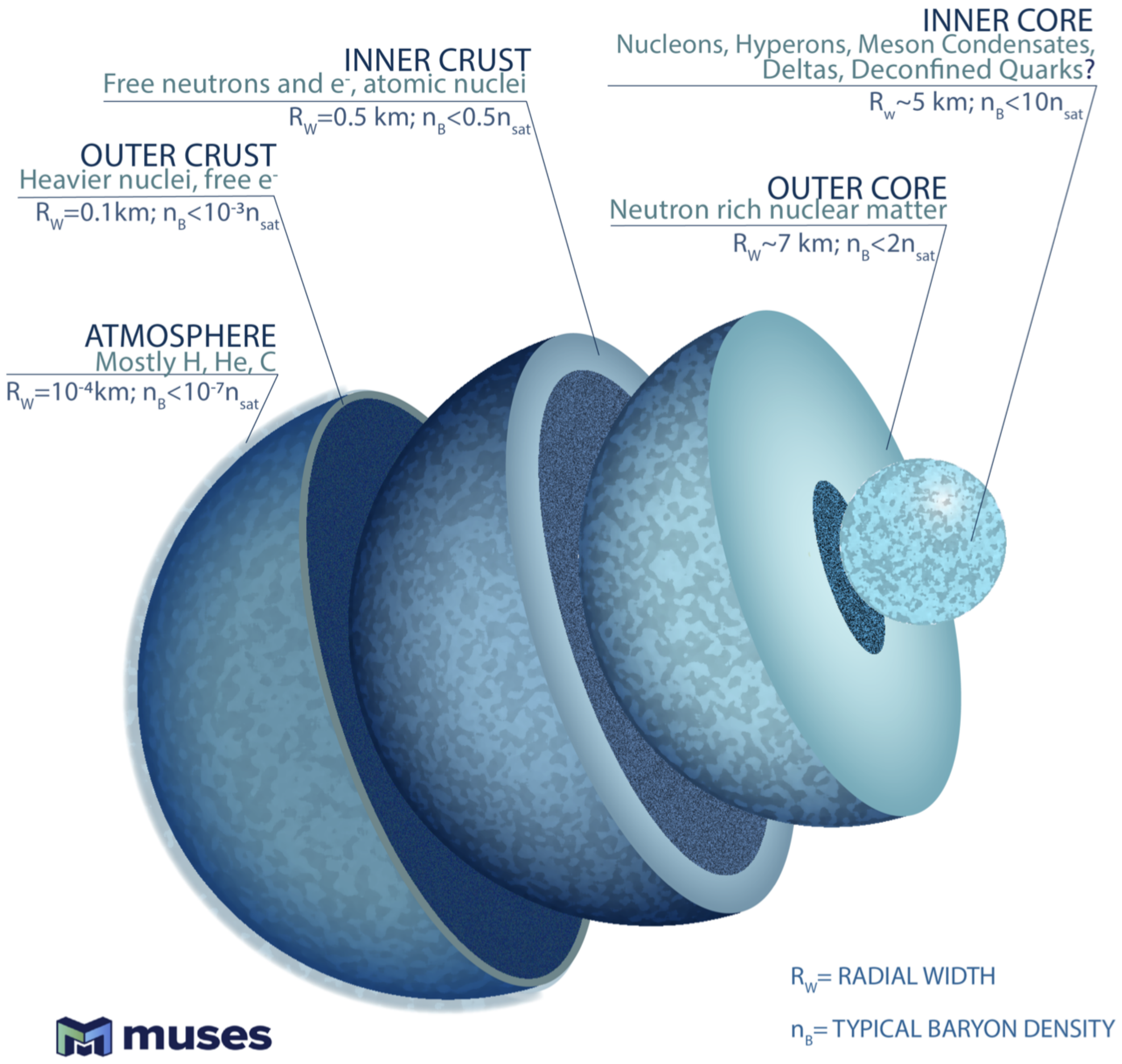
Finite-Temperature Effects in Multi-Messenger Astrophysics

INT Workshop, Seattle WA

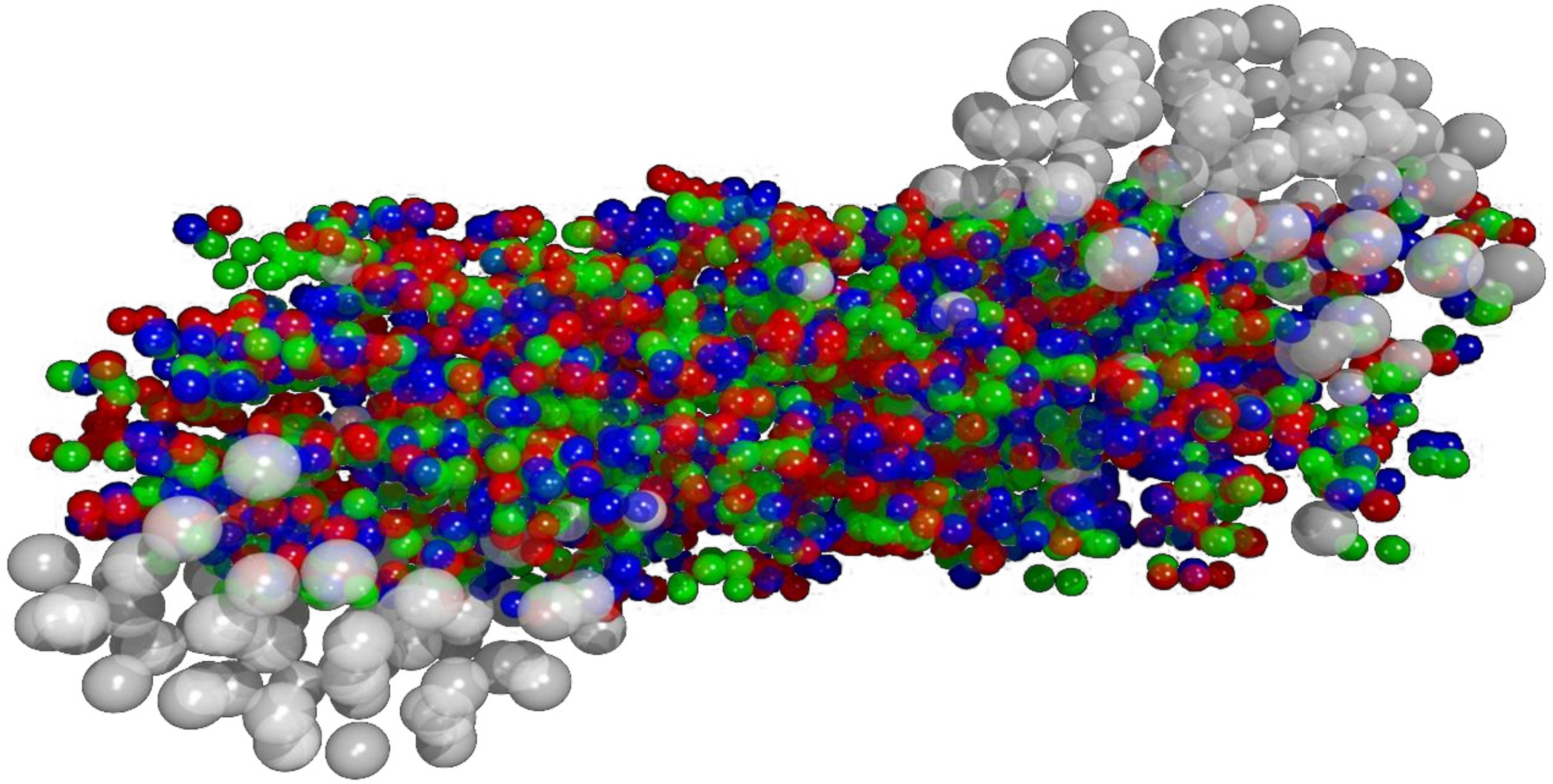
June, 15-19, 2026

Center for Nuclear Research

Neutron Star



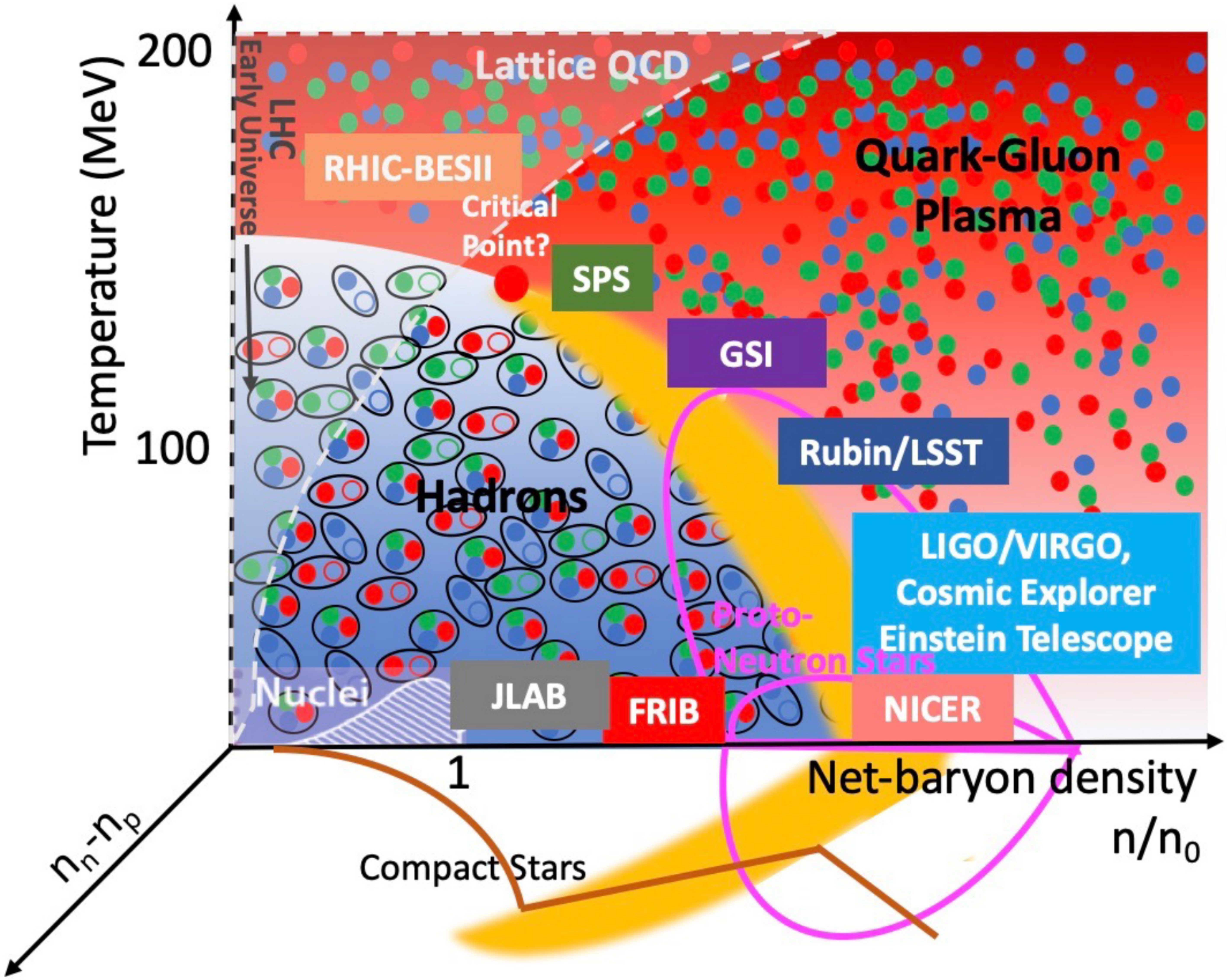
Heavy-Ion physics



https://physics.fjfi.cvut.cz/index.php/en/research2/research-fields/theory-and-phenomenology-of-heavy-ion-collisions-physics

QCD Phase diagram

- Lattice QCD is reliable at zero baryon density
- Extrapolation limited to $\mu_B/T \sim 3.5$
- Finite density in the regime of neutron star and mergers we use effective models



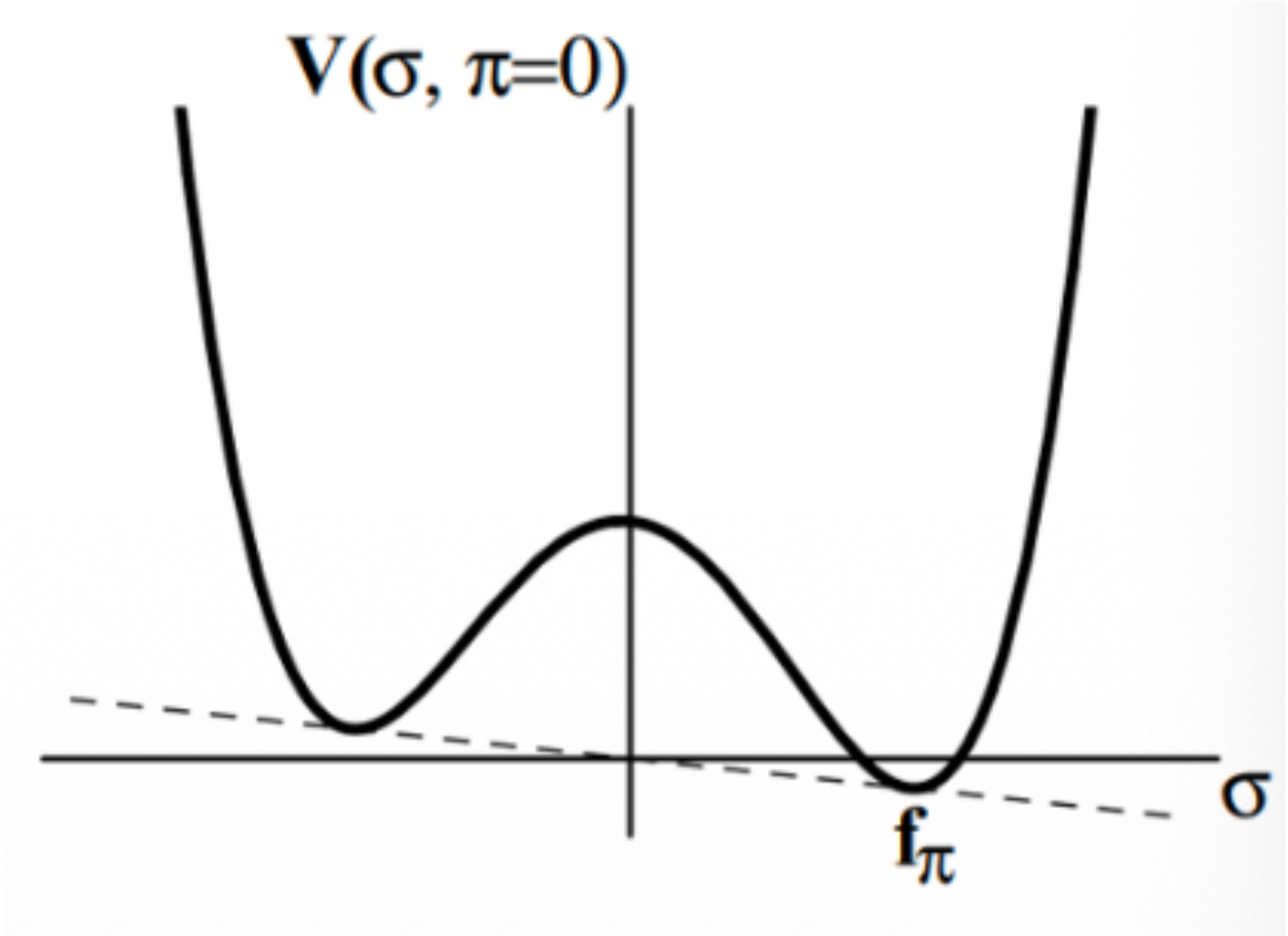
Effective Models for Nuclear Matter

We need a simplified theoretical framework that describes QCD in the desired energy range

Interpret data ⟷ **make predictions**

Requirements:

- Chiral symmetry
- Broken scale invariance
- Nuclear matter degrees of freedom and interactions
- Constrained by first-principles results and/or experiments/observations



Chiral Mean Field (CMF) Model

The chiral mean-field Lagrangian is written as

$$\mathscr{L}_{CMF} = \mathscr{L}_{kin} + \mathscr{L}_{int} + \mathscr{L}_{scal} + \mathscr{L}_{vec} + \mathscr{L}_{SB} - U_\Phi$$

Non-linear realization of $SU(3)$ linear sigma model

$$\mathscr{L}_{int} = -\sum_i \bar{\psi}\left[g_{i\omega}\gamma_0\omega + g_{i\phi}\gamma_0\phi + g_{i\rho}\gamma_0\tau_3\rho + m_i^*\right]\psi$$

$$m_i^* = g_{i\sigma}\sigma + g_{i\delta}\tau_3\delta + g_{i\zeta}\zeta + \delta m$$

$$\mathscr{L}_{vec} = -\frac{1}{2}\left(m_\omega^2\omega^2 + m_\rho^2\rho^2 + m_\phi^2\phi^2\right)\frac{\chi^2}{\chi_0^2} - g_4\left(w^4 + \frac{\phi^4}{4} + 3\omega^2\phi^2 + \frac{4\omega^3\phi}{\sqrt{2}} + \frac{4\omega\phi^3}{\sqrt{2}}\right)$$

Can be written with different forms of vector interaction (C1-C4)

$$\mathscr{L}_{scal} = -\frac{1}{2}\kappa_0\chi^2\left(\sigma^2 + \zeta^2 + \delta^2\right) - \kappa_1\left(\sigma^2 + \zeta^2 + \delta^2\right)^2 - \kappa_2\left(\frac{\sigma^4}{2} + \frac{\delta^4}{2} + 3\sigma^2\delta^2 + \zeta^4\right)$$
$$-\kappa_3\chi(\sigma^2 - \delta^2)\zeta + \kappa_4\chi^4 + \frac{1}{4}\chi^4\ln\frac{\chi^4}{\chi_0^4} - \epsilon\chi^4\ln\frac{\sigma^2 - \delta^2}{\sigma_0^2\zeta_0}$$

$$\mathscr{L}_{SB} = \left(\frac{\chi}{\chi_0}\left[m_\pi^2 f_\pi\sigma + \left(\sqrt{2}m_k^2 f_k - \frac{1}{\sqrt{2}}m_\pi^2 f_\pi\right)\zeta\right]\right)$$

$$U_\Phi = \left(a_0T^4 + a_1\mu_B^4 + a_2T^2\mu_B^2\right)\Phi + a_3T_0^4\ln\left(1 - 6\Phi^2 + 8\Phi^3 - 3\Phi^4\right)$$

Frozen limit

$$\chi = \chi_0$$

Self-Consistent EoS

N. Cruz-Camacho, et al., PRD 111 (2025) 9, 094030

Chiral Mean Field (CMF) Model

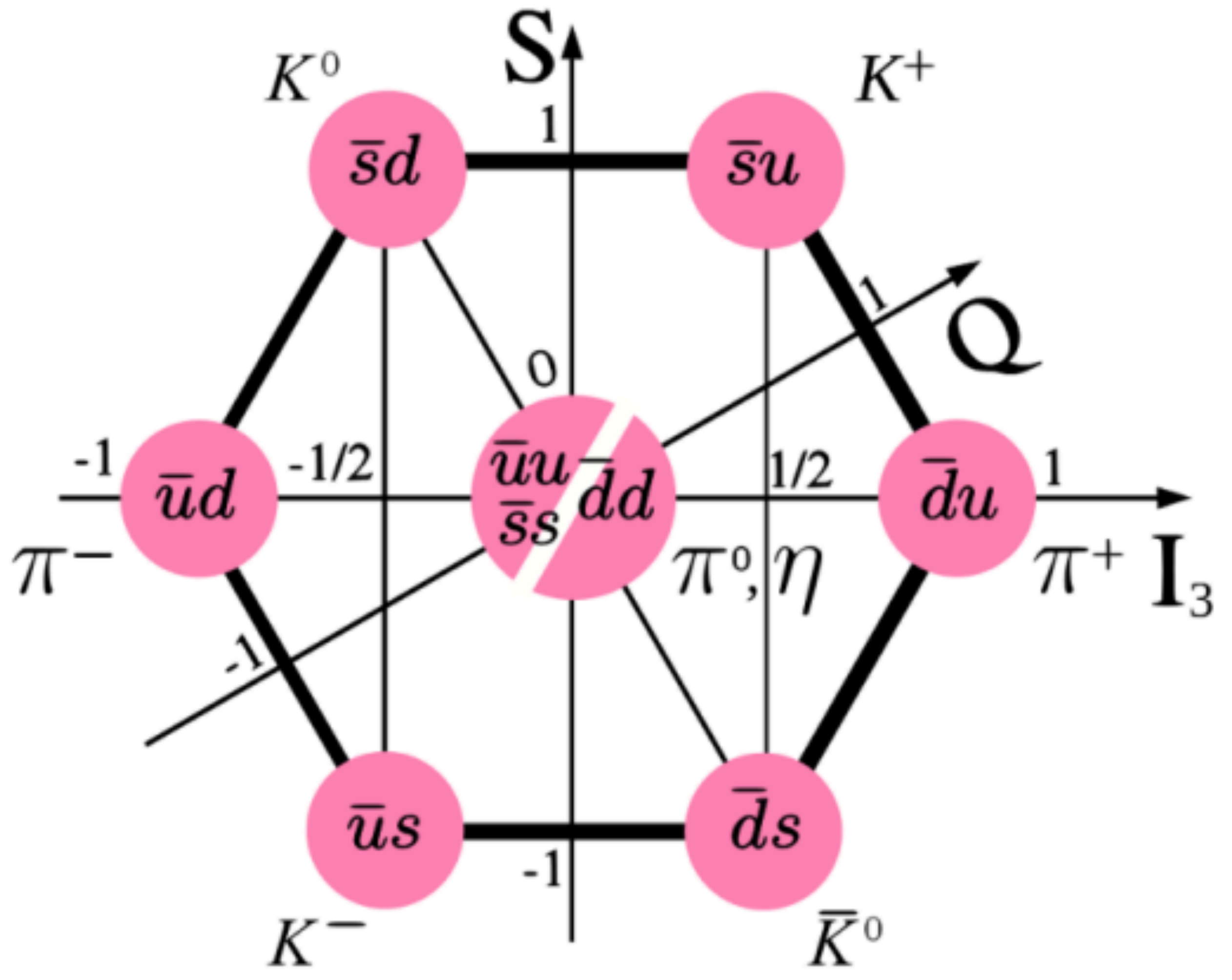
The mean field approximation (MFA)

- Meson fields are replaced by their expectation value
- We assume a dense, spatially isotropic, and rotationally invariant system
- Mesons are not dynamical degrees of freedom
- Negative parity states (π, K, η) are neglected.
- Only mesons with $I_z^3 = 0$ survive

$$\sigma \rightarrow < \sigma > \equiv \sigma_0$$

$$V^\mu \rightarrow < V^\mu > \equiv < V_0, 0 >$$

$$< \pi_i > = 0$$



Consequence

- Standard CMF includes only **baryons** and **quarks**
- Degenerate vector mesons masses for Vector nonet

Dexheimer et al. PRC.82, 035803

Field Redefined Chiral Mean-Field Model

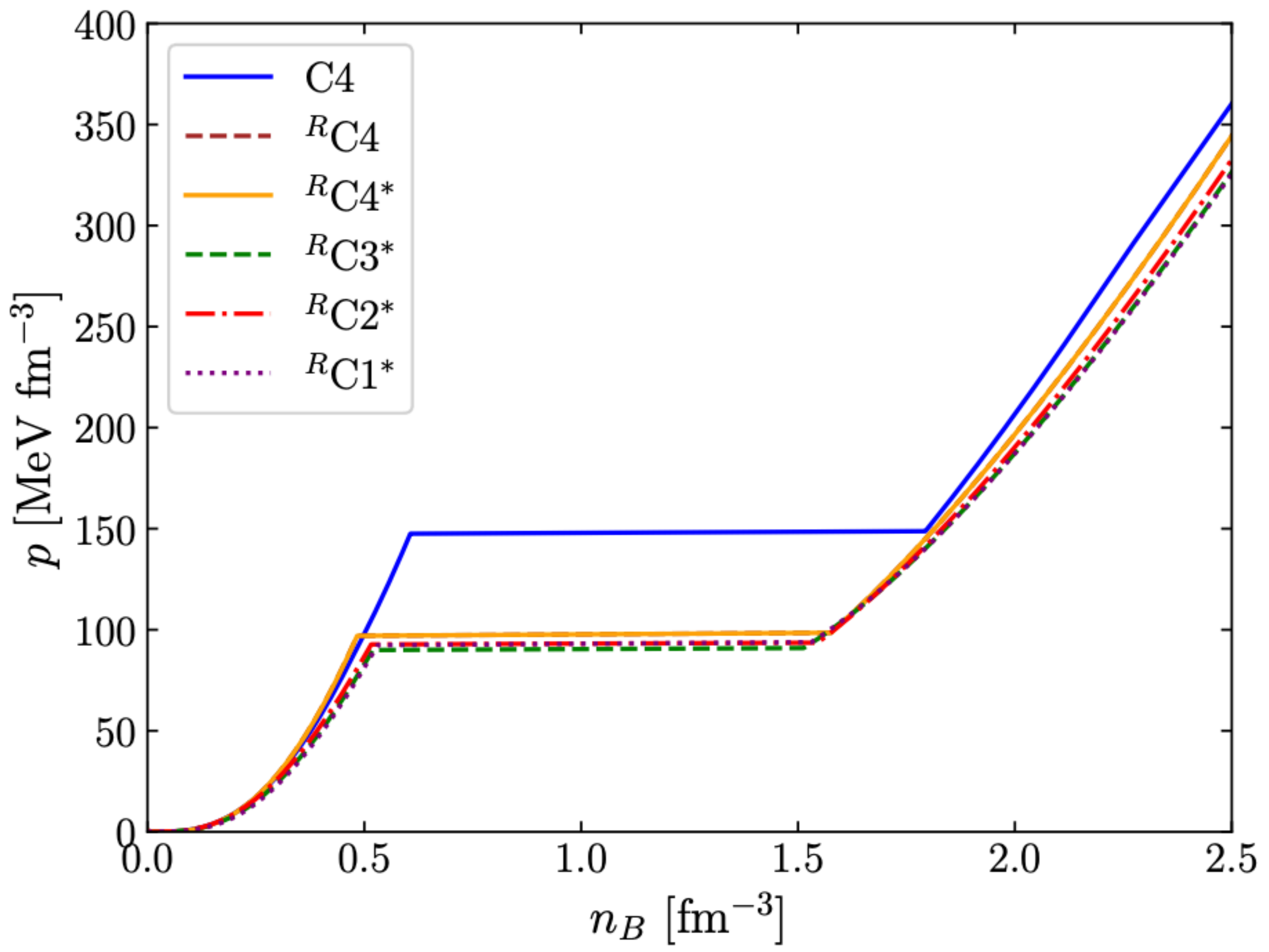
- Vector nonet masses redefined to break mass degeneracy

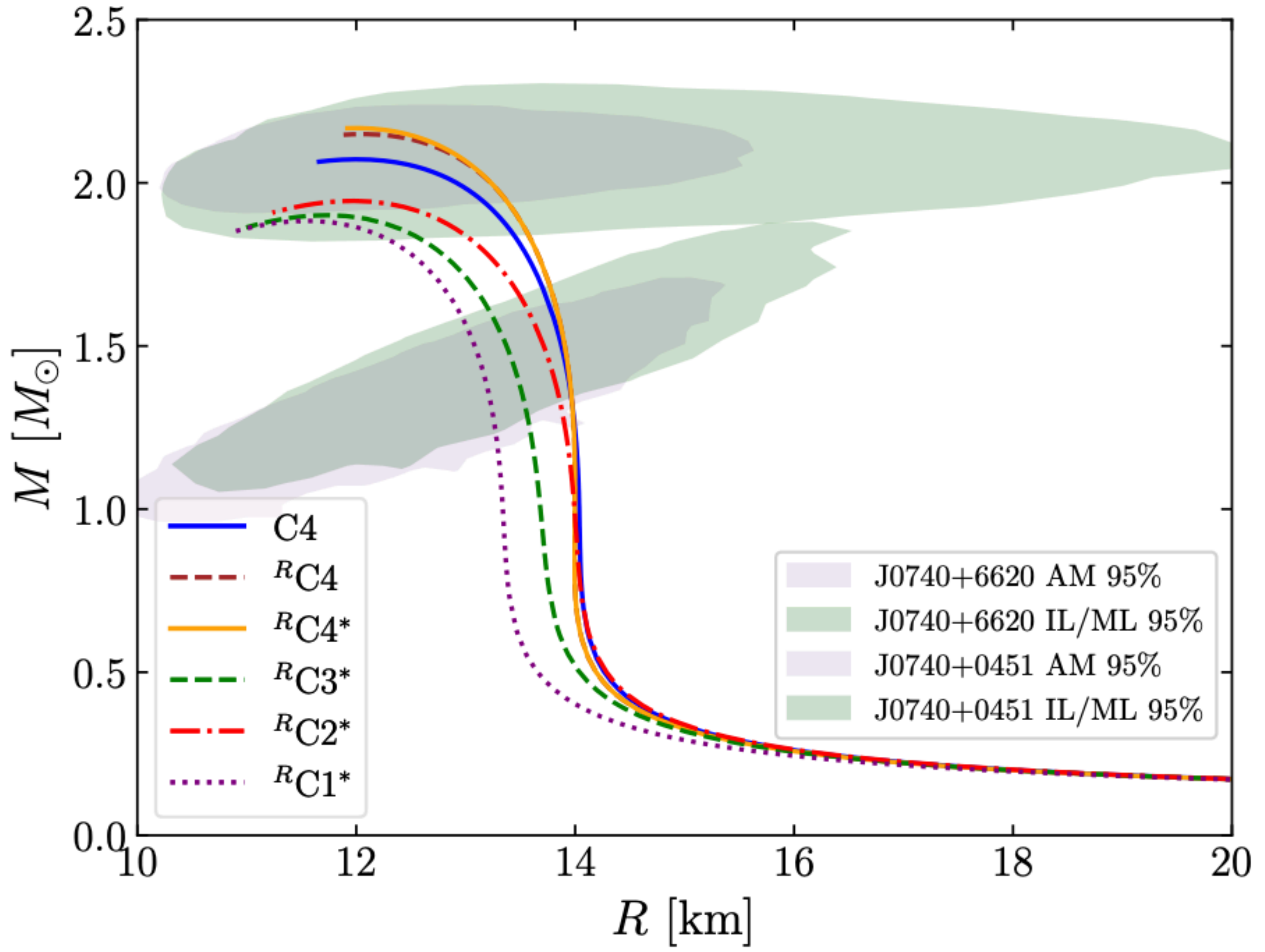
$$\mathscr{L}^{CI}_{vec} = \frac{1}{4}\mu Tr\left[\tilde{V}_{\mu\nu}\tilde{V}^{\mu\nu} < X >^2\right]$$

$$m_K^2 = Z_{K*} m_V^2 \qquad m_{\omega/\rho}^2 = Z_{\omega/\rho} m_V^2 \qquad m_\phi^2 = Z_\phi m_V^2$$

$$m_\xi^2 = Z_\xi^{1/2} \xi \qquad \xi = \rho, \omega, K^*, \phi$$

- Refit Polyakov-loop potential

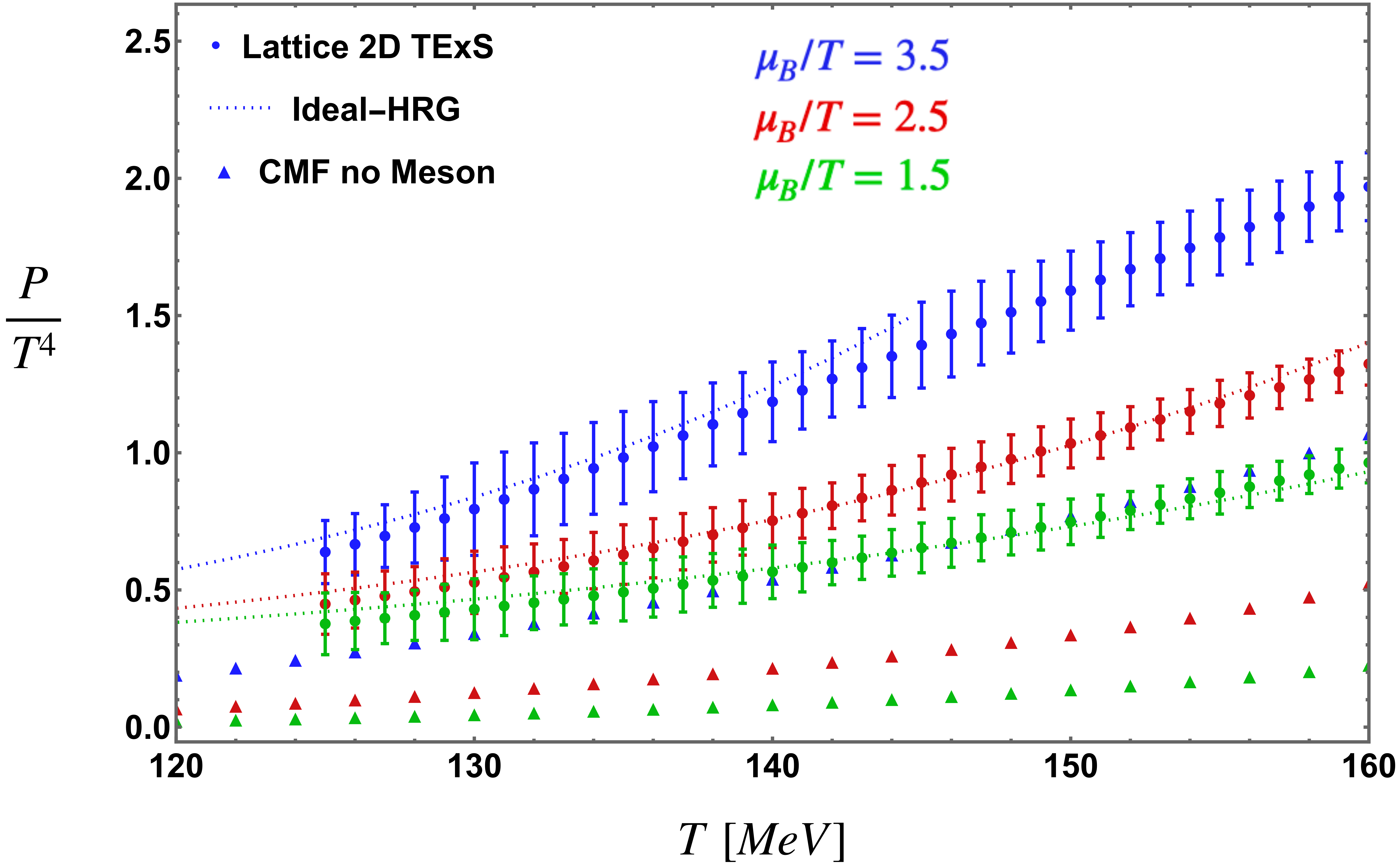




R.Kumar, et al., Phys.Rev.D 109 (2024) 7, 074008

Still no vector mesons (ρ, ω, ϕ) and pseudoscalar mesons (π, K, η)

Pressure

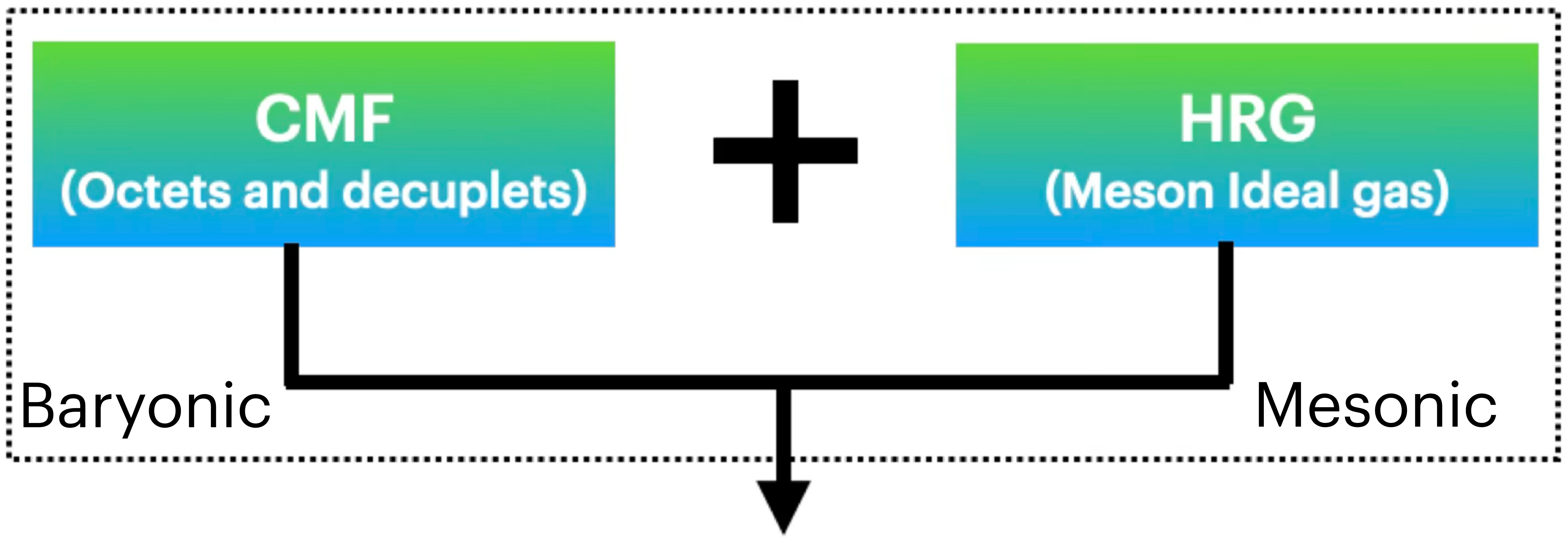


Lattice 2D TExS
Ideal–HRG
CMF no Meson
$\mu_B/T = 3.5$
$\mu_B/T = 2.5$
$\mu_B/T = 1.5$
$\frac{P}{T^4}$
$T\ [MeV]$
0.0
0.5
1.0
1.5
2.0
2.5
120
130
140
150
160

Workflow

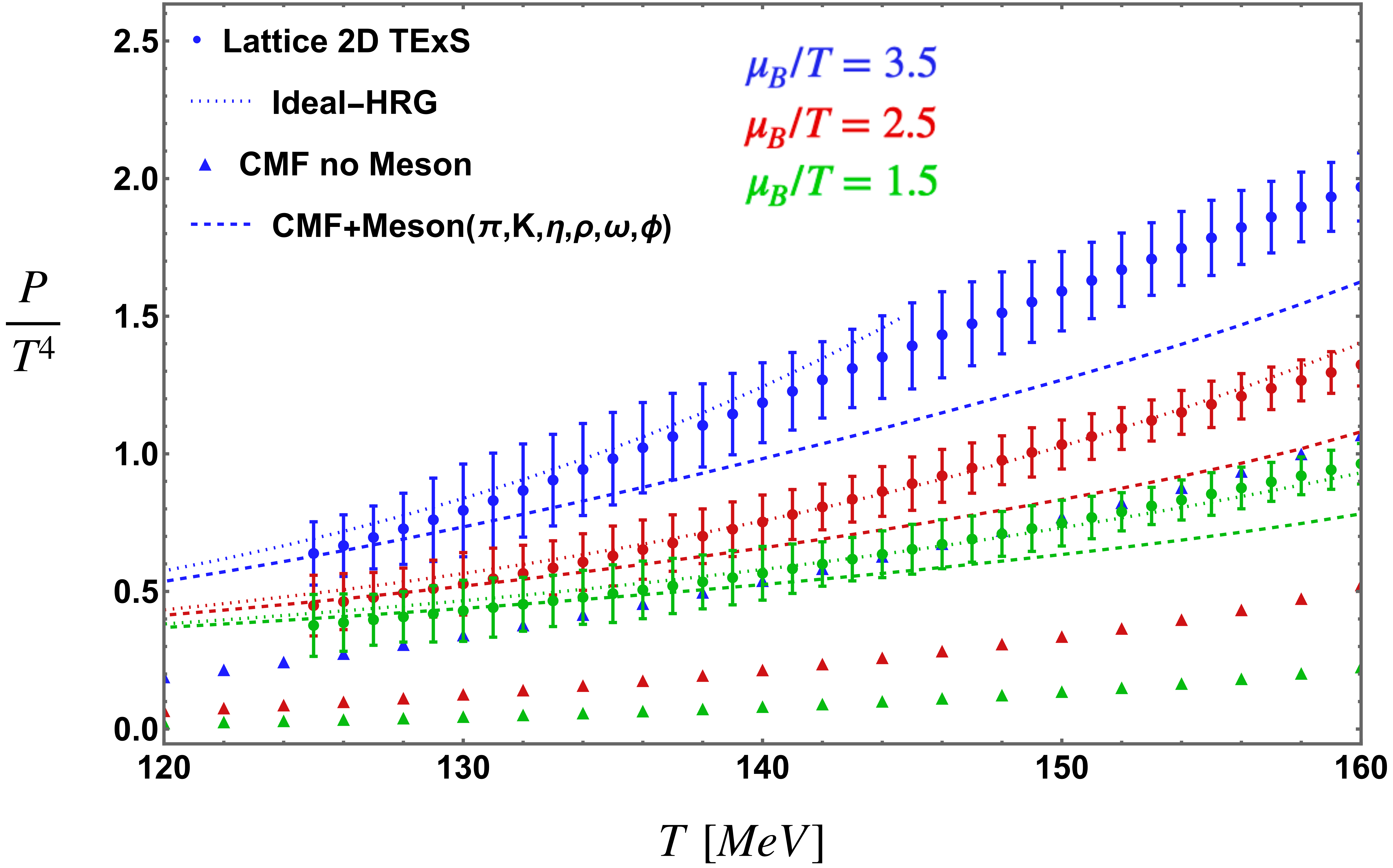
muses

CMF++
Chiral Mean Field



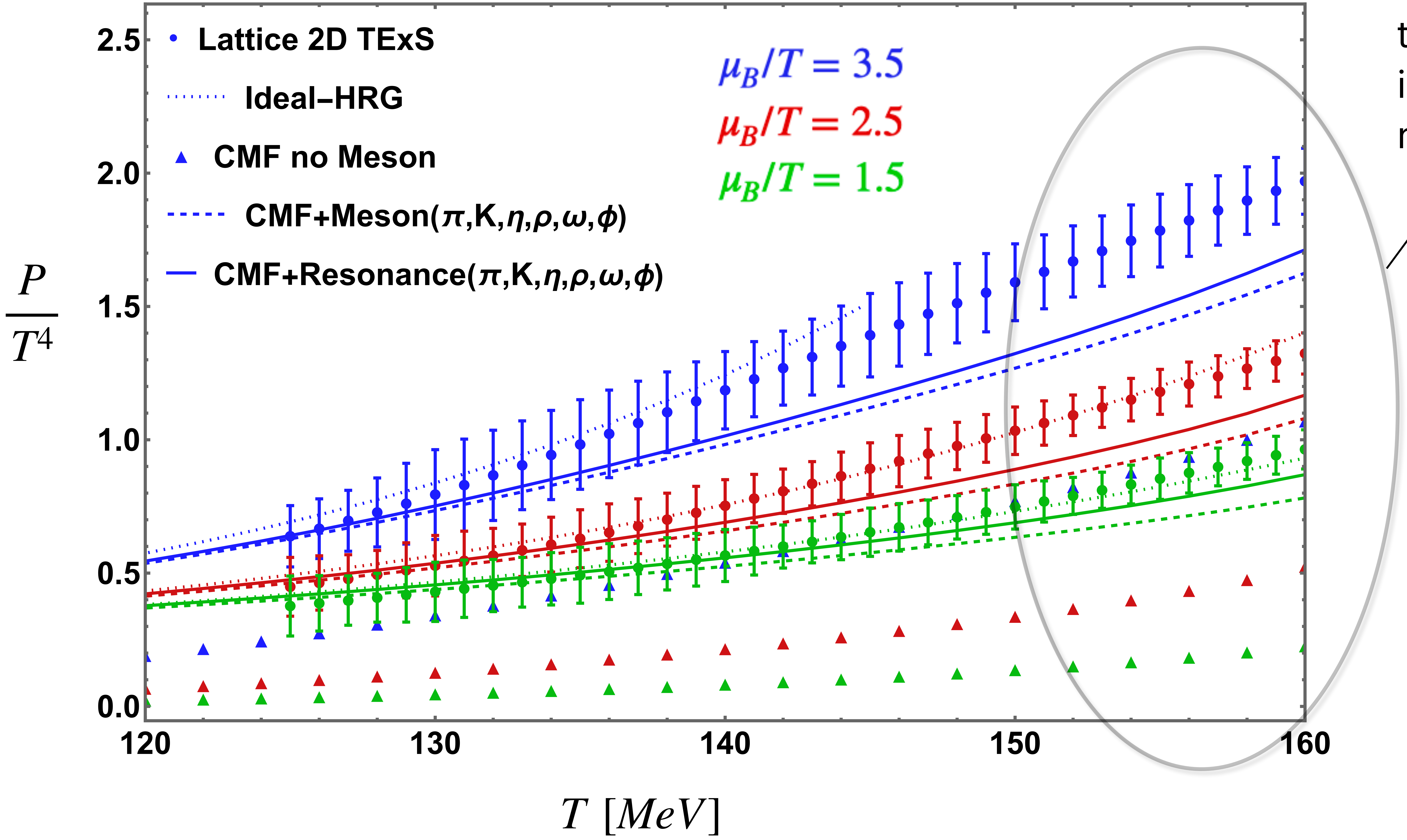
$$P_H(T, \mu_B) = P_{CMF}(T, \mu_B) + P^{Ideal}_{meson}(T, \mu_B)$$

Pressure



Lattice 2D TExS
Ideal–HRG
CMF no Meson
CMF+Meson(π,K,η,ρ,ω,ϕ)
$\mu_B/T = 3.5$
$\mu_B/T = 2.5$
$\mu_B/T = 1.5$
$\frac{P}{T^4}$
2.5
2.0
1.5
1.0
0.5
0.0
120
130
140
150
160
T [MeV]

Pressure



Need to include thermal interacting mesons

Role of Interacting Mesons in the QCD Equation of State

- Interacting mesons are important at finite temperature because of their large thermal population

- Medium-dependent meson properties are crucial for a realistic QCD equation of state

- Mesons can condense at finite isospin density, therefore important for neutron star physics

In-medium meson mass description (mCMF model)

Hadronic (H) grand canonical potential

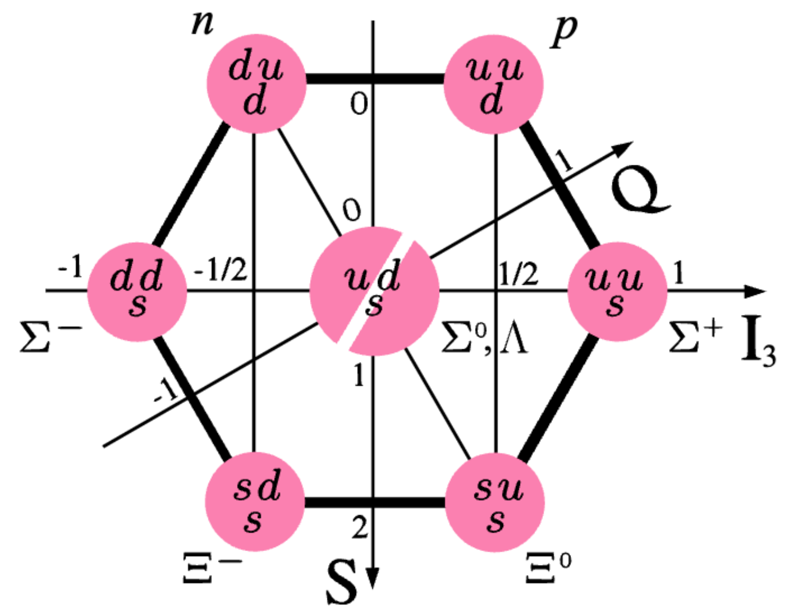
- For a thermal model with baryons and mesons

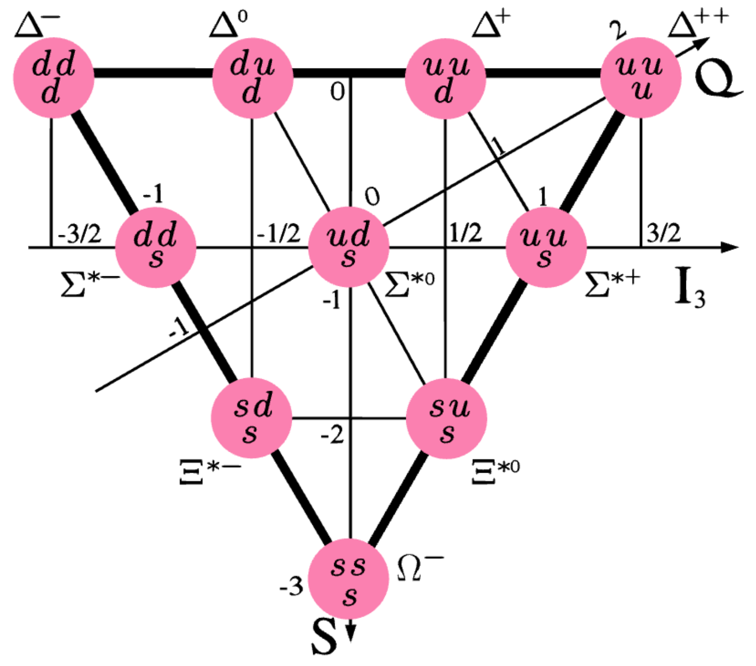
$$\frac{\Omega^H}{V} = U + \frac{\Omega^B_{th}}{V} + \frac{\Omega^M_{th}}{V}$$

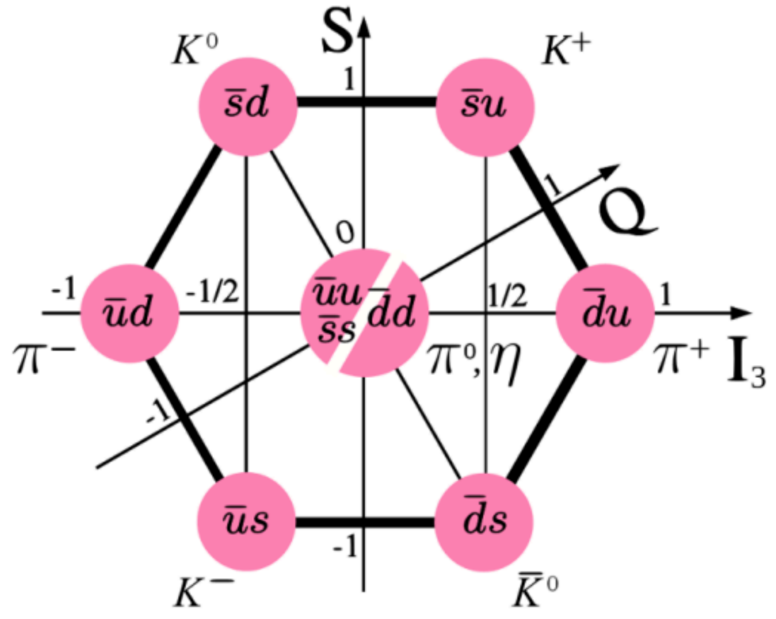
$$\frac{\Omega^B_{th}}{V} = -T\sum_{i\in B}\frac{\gamma_i}{2\pi^2}\int dk k^2\left(\ln\left[1+e^{-\frac{1}{T}(E_i^*(k)-\mu_i^*)}\right]+\ln\left[1+e^{-\frac{1}{T}(E_i^*(k)+\mu_i^*)}\right]\right)$$

$$\frac{\Omega^M_{th}}{V} = T\sum_{i\in M}\frac{\gamma_i}{2\pi^2}\int dk k^2\left(\ln\left[1-e^{-\frac{1}{T}(E_i^*(k)-\mu_i^*)}\right]\right)$$

$$\mu_i^* = \mu_i - g_{\omega_i}\omega_0 - g_{\rho_i}\rho_0 - g_{\phi_i}\phi_0 \qquad E_i^* = \sqrt{k^2 + m_i^{*2}}$$







Before applying mean field approximation:

- Compute the pseudoscalar meson mass
- Compute the vector meson mass

Chiral Mean Field (CMF) Model

- The inclusion of interacting mesons is essential for constructing a realistic dense-matter EoS at finite temperature, relevant for neutron stars and neutron star merger simulations

The feedback in CMF equation of motion

$$\frac{\partial\left(\Omega^{H}/V\right)}{\partial\vartheta}=\frac{\partial\left(\Omega^{orig}/V\right)}{\partial\vartheta}+\sum_{i\in M}n_s^M\frac{\partial m_i^*}{\partial\vartheta}$$

In mCMF, the $M = M(\sigma, \zeta, \delta, \omega, \rho, \phi)$ and the fields depend on T and μ_B

For non-interacting mesons

m_i^* – constant $\qquad \dfrac{\partial m_i^*}{\partial\vartheta} = 0$

R. Kumar, et al PRD 111(2025)7, 074029

In-medium meson mass (mCMF)

Fluctuations of the interaction potential around the vacuum give rise to the in-medium meson masses

$$m^{*2}_{\varphi_{ij}} = \lim_{\varphi \to <\varphi>} \frac{\partial^2}{\partial \varphi_i \partial \varphi_j} U$$

with $\varphi_i = \pi, \eta, \eta', K, \omega, \rho, K^*, \phi$, and the vacuum expectation for the mesons is $< \varphi > = 0.$

Pseudoscalar mesons

The explicit chiral symmetry-breaking Lagrangian

$$\mathscr{L}^u_{esb} = \left(-\frac{1}{2}m^2_{\eta^0}\mathrm{Tr}Y^2 - \frac{1}{2}\mathrm{Tr}\left[A_p\left(u(X+iY)u + u^\dagger(X-iY)u^\dagger\right)\right]\right)$$

$$m^{*2}_{\pi^0/\pi^+/\pi^-} = m^2_\pi \frac{\sigma}{\sigma_0}$$

$$m^{*2}_{K^+/K^-} = \frac{0.5m^2_K\left(2\zeta + \sqrt{2}\left(\delta+\sigma\right)\right)\left(\sqrt{2}\sigma_0 + 2\zeta_0\right)}{\left(\sigma_0 + \sqrt{2}\zeta_0\right)^2}$$

$$m^{*2}_{K^0/\bar{K}^0} = \frac{0.5m^2_K\left(2\zeta + \sqrt{2}\left(-\delta+\sigma\right)\right)\left(\sqrt{2}\sigma_0 + 2\zeta_0\right)}{\left(\sigma_0 + \sqrt{2}\zeta_0\right)^2}$$

$$m^{*2}_{\eta^8} = \frac{m^2_\pi\sigma\sigma_0 + \sqrt{2}\zeta\left(\sqrt{2}m^2_K\left(\sqrt{2}\sigma_0 + 2\zeta_0\right) - 2m^2_\pi\sigma_0\right)}{\sigma_0^2 + 4\zeta_0^2}$$

Vector mesons

Vector self interacting term

$$\mathscr{L}_{vec} = \mathscr{L}^m_{vec} + \mathscr{L}^{SI}_{vec}$$

$$m^{*2}_\omega = m^2_\omega + 6g_4\left(\frac{Z_\phi}{Z_\omega}\right)\phi^2$$

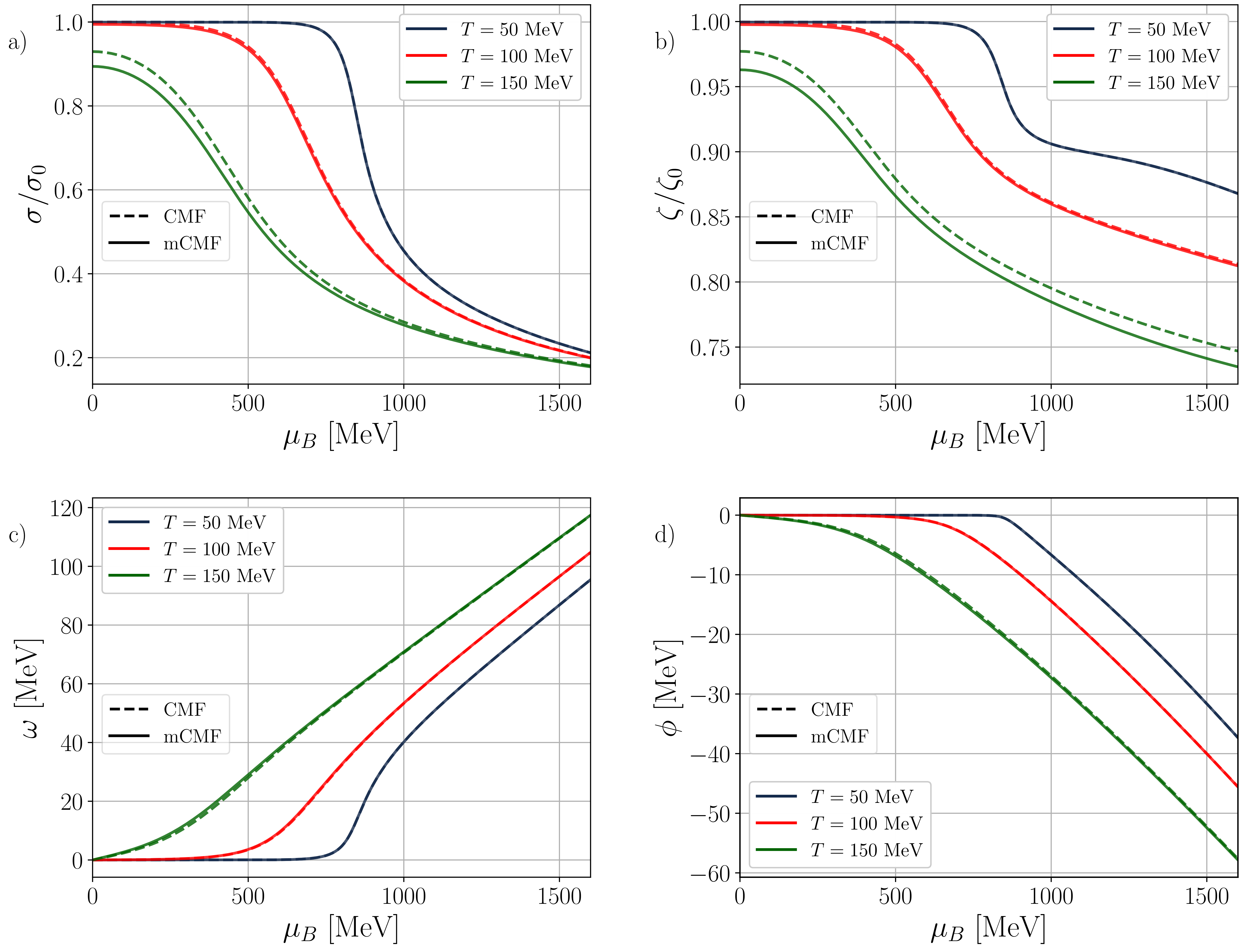
$$m^{*2}_\phi = m^2_\phi + 6g_4\left(\frac{Z_\phi}{Z_\omega}\right)\omega^2$$

For interacting mesons, the masses are medium dependent and vary through the mean fields as functions of T and μ_B

$$m^*_i = m^*_i(\varphi(T,\rho_B) = m^*_i(\varphi(T,\mu_B) = m^*_i(\sigma,\zeta,\delta,\omega,\rho,\phi)$$

R. Kumar et al., PRD 111, 074029 (2025)

Pseudoscalar meson mass and particle population

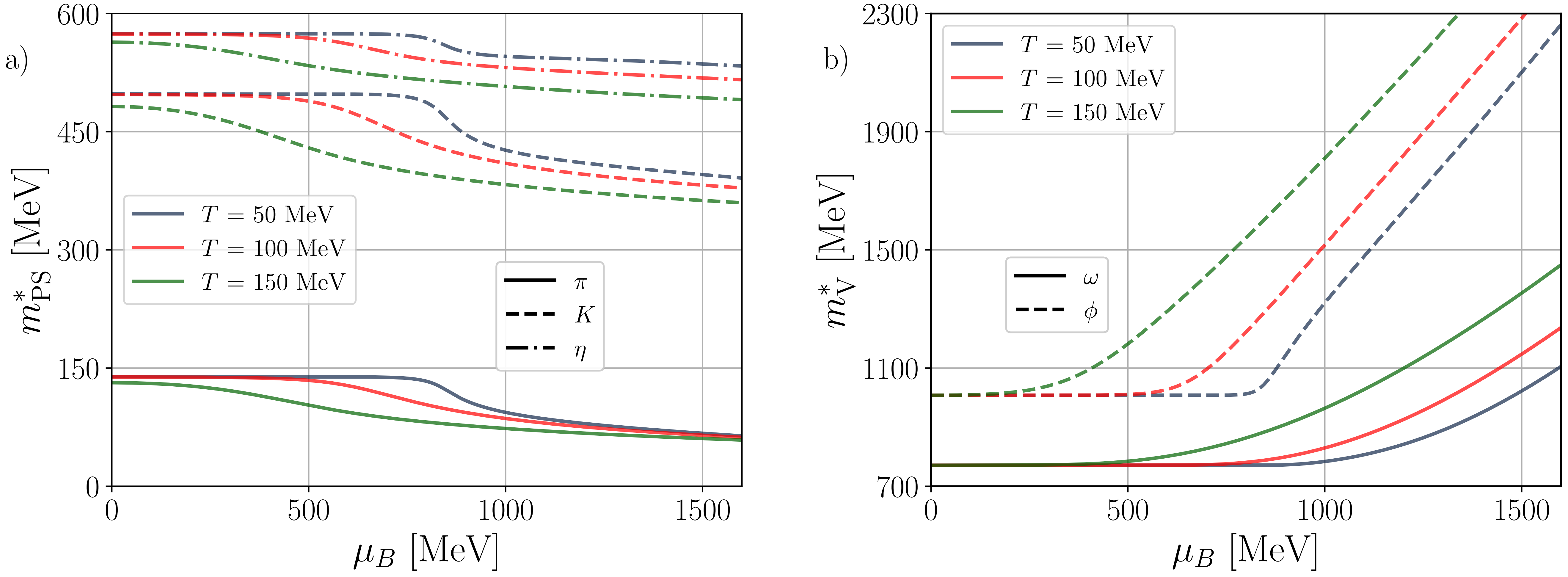


- Increasing T or μ_B modifies mean fields

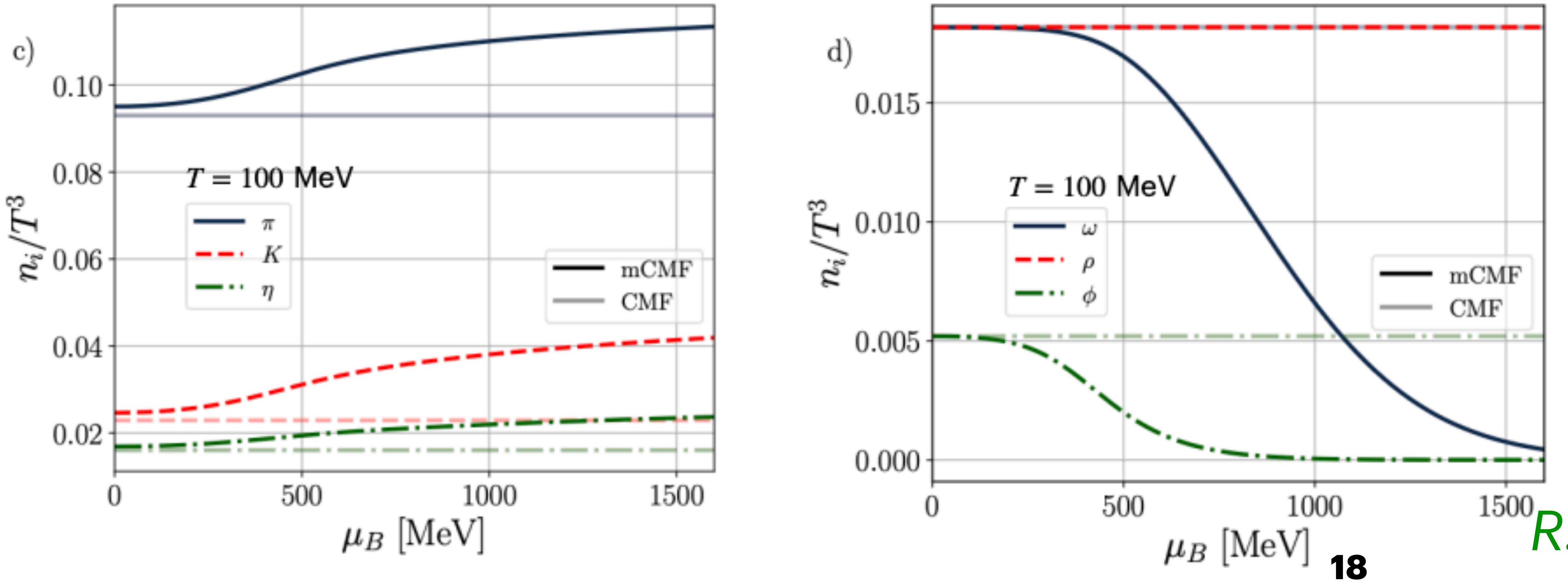
R. Kumar et al., PRD 111, 074029 (2025)

Pseudoscalar meson mass and particle population

In medium masses



Particle population



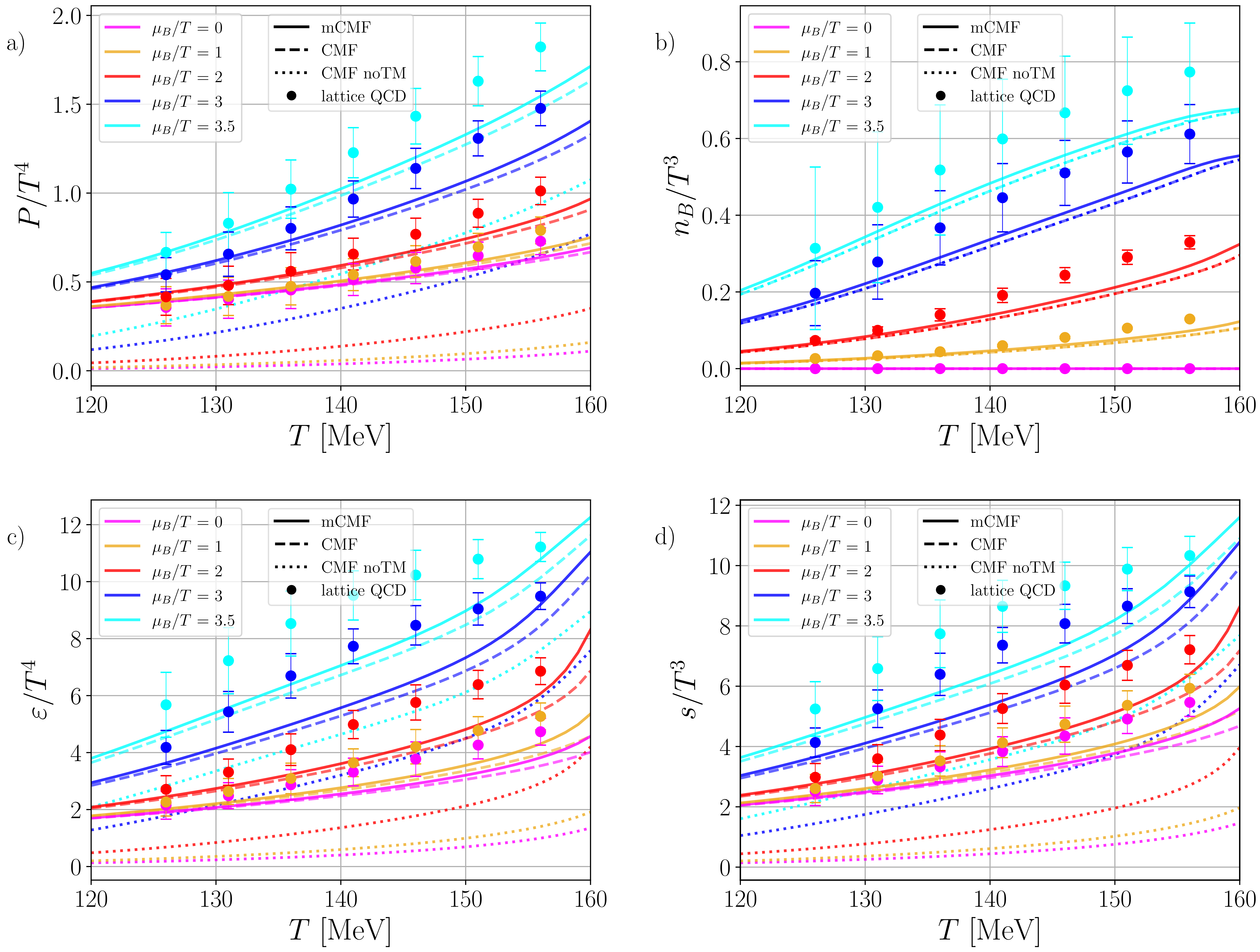
- Increasing T or μ_B modifies mean fields
- Mean fields modify meson masses
- Meson masses reveal the onset of chiral symmetry restoration

$$m^{*2}_{\pi^0/\pi^+/\pi^-} = m_\pi^2 \frac{\sigma}{\sigma_0}$$

- Modified masses change particle populations.

R. Kumar et al., PRD 111, 074029 (2025)

Thermodynamics

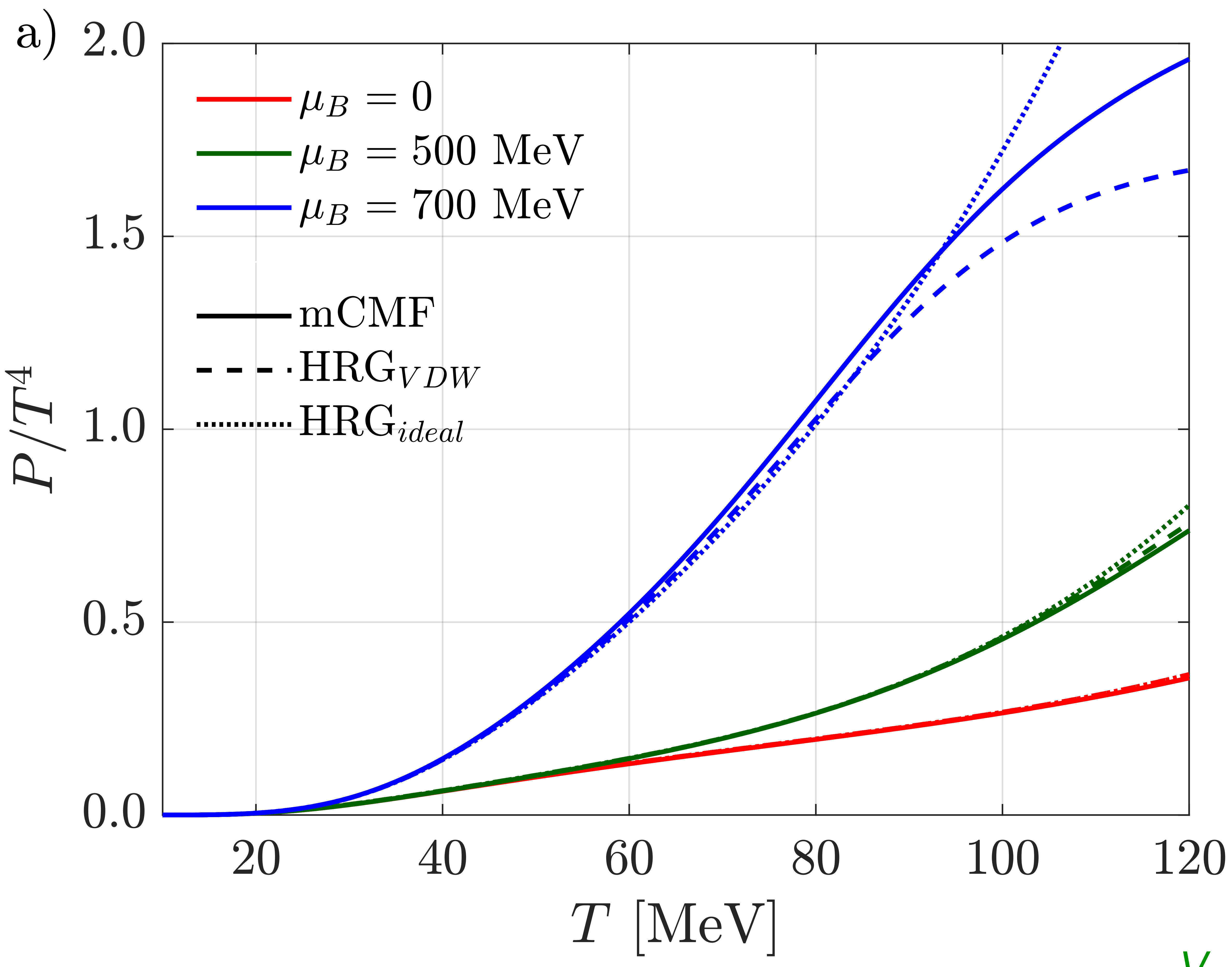


- Improved agreement with lattice data for:
  - Pressure
  - Baryon density
  - Energy density
  - Entropy density
- Valid up to T ≈ 160 MeV

S. Borsányi, et al. PRL 126 (2021)

R. Kumar et al., PRD 111, 074029 (2025)

Comparison with HRG thermodynamics



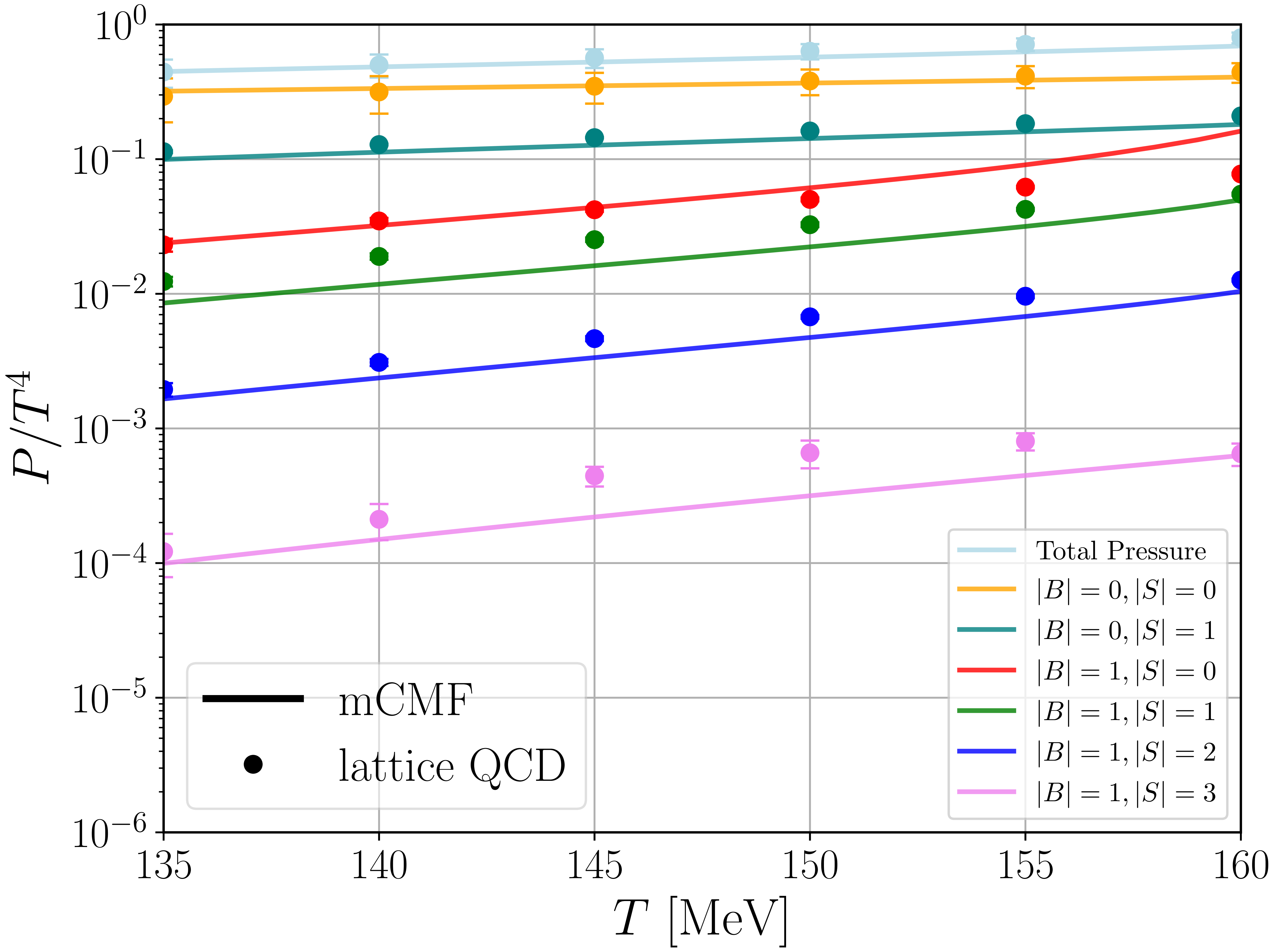
HRG model

- Reproduces HRG behavior at low to intermediate temperature T
- The agreement can be further improved by including quark degree of freedom

V. Vovchenko, et al. Comp. Phys. Com. 244, 295 (2019)

R. Kumar et al., PRD 111, 074029 (2025)

Comparison with Partial Pressures



Partial Pressures

- Validates hadronic content of CMF
- Good agreement across sectors:
  - Baryonic
  - Strange

P. Alba, et al. PRD 96 (2017)

R. Kumar et al., PRD 111, 074029 (2025)

Summary

- Extended the CMF model by including pseudoscalar and vector meson interactions.
- We achieved an unprecedented level of agreement with extrapolated lattice QCD and HRG thermodynamics that could be improved by adding quark degrees of freedom
- Our goal is to obtain a realistic equation of state for dense matter at finite temperature, which will be useful in neutron stars and neutron star merger simulations.

Ongoing work without details

- Extending mCMF framework to quark sector

Thank you for your attention!

APPENDIX

Multiplets

- Baryon Octet

$$B = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda_0}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & \frac{-\Sigma^0}{\sqrt{2}} + \frac{\Lambda^0}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -2\frac{\Lambda^0}{\sqrt{6}} \end{pmatrix}$$

- Scalar Matrix: Mean-Fields

$$X = \begin{pmatrix} \frac{\delta^0+\sigma}{\sqrt{2}} & 0 & 0 \\ 0 & \frac{-\delta^0+\sigma}{\sqrt{2}} & 0 \\ 0 & 0 & \zeta \end{pmatrix}$$

- A_p matrix

$$A_p = \frac{1}{\sqrt{2}} \begin{pmatrix} m_\pi^2 f_\pi & 0 & 0 \\ 0 & m_\pi^2 f_\pi & 0 \\ 0 & 0 & 2m_K^2 f_K - m_\pi^2 f_\pi \end{pmatrix}$$

Multiplets

- Pseudoscalar Nonet

$$B = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda_0}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & \frac{-\Sigma^0}{\sqrt{2}} + \frac{\Lambda^0}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -2\frac{\Lambda^0}{\sqrt{6}} \end{pmatrix}$$

- Vector Meson Nonet : Mean Fields (ω,ρ and ϕ)

$$V = \begin{pmatrix} \frac{\rho^0+\omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & \frac{-\rho^0+\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & K^{*0} & \phi \end{pmatrix}$$

- Pseudoscalar Singlet

$$Y = \sqrt{\frac{1}{3}}\eta_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

The CMF Lagrangian

$$\mathcal{L}_{\text{CMF}} = \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{int}} + \mathcal{L}_{\text{scal}} + \mathcal{L}_{\text{vec}} + \mathcal{L}_{\text{esb}}$$

$$\mathcal{L}_{\text{kin}} = i \operatorname{Tr}\left(\bar{B}\gamma_\mu D^\mu B\right) = i \sum_{i\in B}\left(\bar{\psi}_i \gamma_\mu \partial^\mu \psi_i\right)$$

$$\mathcal{L}_{\text{scal}} = -\frac{1}{2} k_0 \chi_0^2 \left(\sigma^2 + \zeta^2 + \delta^2\right) + k_1 \left(\sigma^2 + \zeta^2 + \delta^2\right)^2 + k_2 \left[\frac{\sigma^4 + \delta^4}{2} + \zeta^4 + 3(\sigma\delta)^2\right]$$
$$+ k_3 \chi_0 \left(\sigma^2 - \delta^2\right)\zeta + k_{3N}\chi_0 \left(\frac{\sigma^3}{\sqrt{2}} + \frac{3}{\sqrt{2}}\sigma\delta^2 + \zeta^3\right) - k_4 \chi_0^4 + \frac{\epsilon}{3}\chi_0^4 \ln\left[\frac{\left(\sigma^2 - \delta^2\right)\zeta}{\sigma_0^2 \zeta_0}\right]$$

$$\mathcal{L}_{\text{vec}} = \frac{1}{2}\left(m_\omega^2 \omega^2 + m_\phi^2 \phi^2 + m_\rho^2 \rho^2\right) + \mathcal{L}_{\text{vec}}^{\text{SI}}$$

$$\mathcal{L}_{\text{esb}}^u = -\left[m_\pi^2 f_\pi \sigma + \left(\sqrt{2} m_K^2 f_K - \frac{1}{\sqrt{2}} m_\pi^2 f_\pi\right)\zeta\right]$$

$$\mathcal{L}_{\text{int}} = -\sum_{i\in B} \bar{\psi}_i \left[\gamma_0 \left(g_{i\omega}\omega + g_{i\rho}\rho + g_{i\phi}\phi\right) + g_{i\sigma}\sigma + g_{i\zeta}\zeta + g_{i\delta}\delta\right]\psi_i$$

Formalism: CMF thermodynamic potential

$$\frac{\Omega^H}{V} = \frac{\Omega}{V} + \frac{\Omega^M_{\text{th}}}{V} \,,$$
$$= U_M + \frac{\Omega^B_{\text{th}}}{V} + \frac{\Omega^M_{\text{th}}}{V} \,, \tag{1}$$

$$U_M = \mathcal{L}_{\text{vec}} - \mathcal{L}_{\text{scal}} - \mathcal{L}_{\text{esb}} + \mathcal{L}_{\text{vac}} \,, \tag{2}$$

$$\frac{\Omega^B_{\text{th}}}{V} = -T \sum_{i \in \text{baryons}} \frac{\gamma_i}{2\pi^2} \int dk k^2 \left(\ln\left[1 + e^{-\frac{1}{T}\left(E^*_i(k) - \mu^*_i\right)}\right] + \ln\left[1 + e^{-\frac{1}{T}\left(E^*_i(k) + \mu^*_i\right)}\right] \right) \,, \tag{3}$$

$$\frac{\Omega^M_{\text{th}}}{V} = T \sum_{i \in \text{mesons}} \frac{\gamma_i}{2\pi^2} \int dk k^2 \ln\left[1 - e^{-\frac{1}{T}\left(E^*_i(k) - \mu^*_i\right)}\right] \,. \tag{4}$$

$$m^{*2}_{\varphi_{ij}} = \lim_{\varphi \to \langle\varphi\rangle} \frac{\partial^2}{\partial\varphi_i \partial\varphi_j} U \,,$$

$${m^*}^2_{\pi^0/\pi^+/\pi^-} = m_\pi^2 \frac{\sigma}{\sigma_0}$$

$${m^*}^2_{K^+/K^-} = \frac{0.5 m_K^2 \left(2\zeta + \sqrt{2}\,(\delta + \sigma)\right) \left(\sqrt{2}\sigma_0 + 2\zeta_0\right)}{\left(\sigma_0 + \sqrt{2}\zeta_0\right)^2}$$

$${m^*}^2_{K^0/\bar{K}^0} = \frac{0.5 m_K^2 \left(2\zeta + \sqrt{2}\,(-\delta + \sigma)\right) \left(\sqrt{2}\sigma_0 + 2\zeta_0\right)}{\left(\sigma_0 + \sqrt{2}\zeta_0\right)^2}$$

$${m^*}^2_{\eta^8} = \frac{m_\pi^2 \sigma\sigma_0 + \sqrt{2}\zeta \left(\sqrt{2} m_K^2 \left(\sqrt{2}\sigma_0 + 2\zeta_0\right) - 2 m_\pi^2 \sigma_0\right)}{\sigma_0^2 + 4\zeta_0^2}$$

$$m^{*2}_{\omega} = m_\omega^2 + 6 g_4 \left(\frac{Z_\phi}{Z_\omega}\right) \phi^2$$

$$m^{*2}_{\phi} = m_\phi^2 + 6 g_4 \left(\frac{Z_\phi}{Z_\omega}\right) \omega^2$$