Path from lattice QCD to neutrinoless double-beta decay amplitude

Saurabh Kadam

with

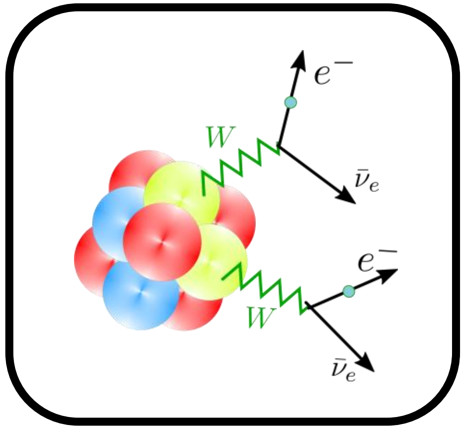
Zohreh Davoudi

PRD 102, 114521 (2020)
PRL 126, 152003 (2021)
PRD 105, 094502 (2022)

Double β Decays

Two neutrino double beta decay ($2\nu\beta\beta$)

Eugene Wigner, Goeppert-Mayer (1935)



- Standard model (SM) process
- Extremely rare and has been observed

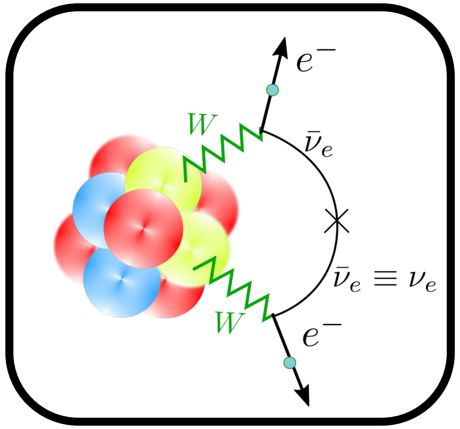
$$T_{1/2}(2\nu\beta\beta) \sim 10^{20} y$$

- Dominant background for $0\nu\beta\beta$ search experiments
- $2\nu\beta\beta$ can also probe potential BSM scenarios

Deppisch, Graf, and Šimkovic
PhysRevLett.125.171801

Neutrinoless double beta decay ($0\nu\beta\beta$)

Racah (1937)
Furry (1939)



- Total lepton number is violated
- Beyond the standard model (BSM) process
- Searches for it are ongoing
- Neutrinos are their own anti-particles

Avignone, Elliott and Engel
Reviews of Modern Physics, 80 (2008)

What can possibly be responsible for a $0\nu\beta\beta$ decay?

- Minimal deviation from the SM:

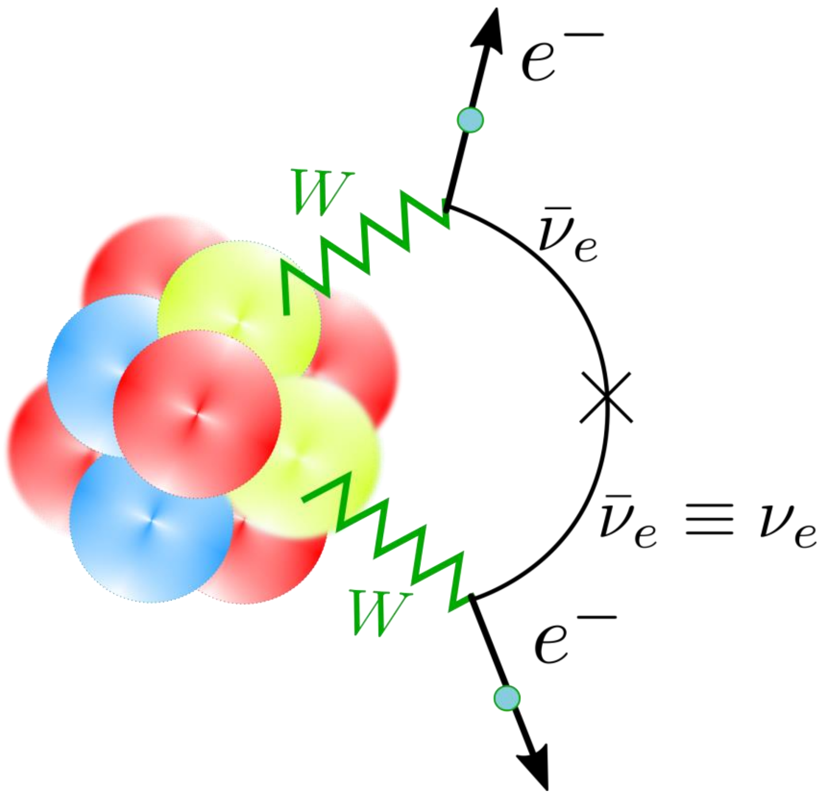
  Light neutrino exchange scenario

- SM neutrinos are promoted to Majorana neutrinos

- Effective Majorana mass

$$m_{\beta\beta} = \sum_k m_k U_{ek}^2$$

Needs accurate constraints



Obtaining Effective Majorana Mass from Experiments

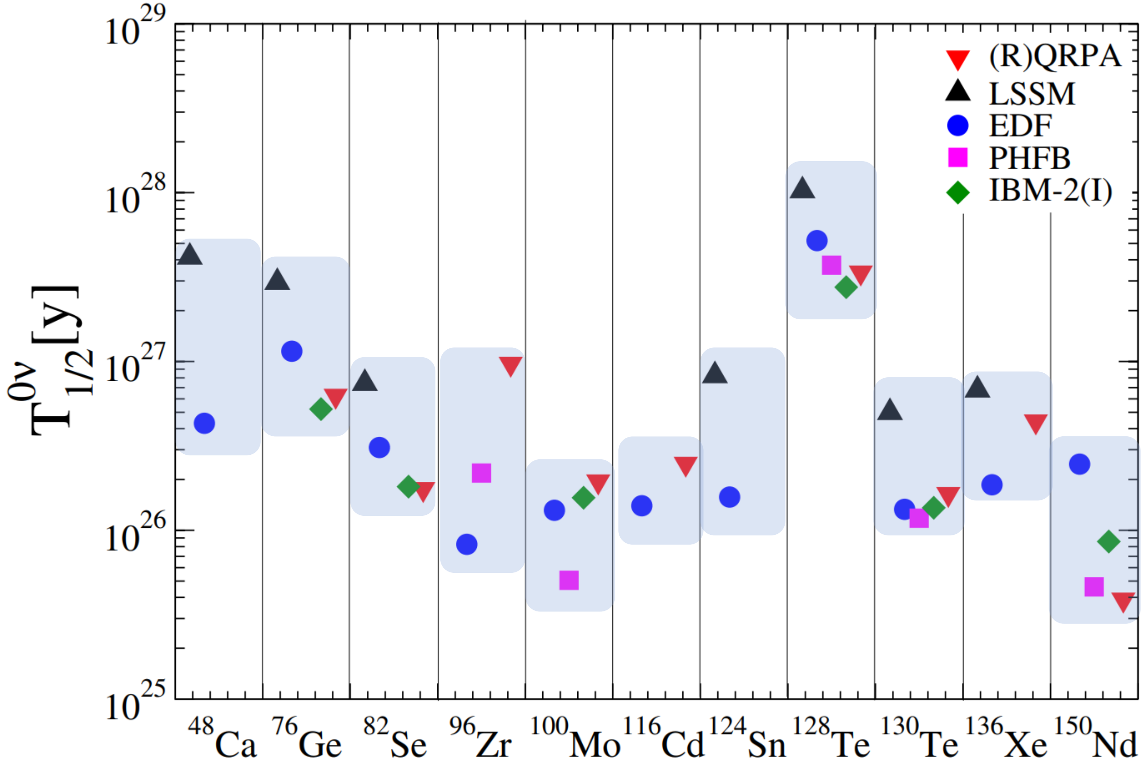
Phase space factor

$$(T^{0\nu}_{1/2})^{-1} = G_{0\nu}(Q_{\beta\beta}, Z)\, |M_{0\nu}|^2\, \langle m_{\beta\beta}\rangle^2$$

Nuclear Matrix Element (NME) in Light Neutrino Exchange Scenario

Half-lives from different methods for NME calculations

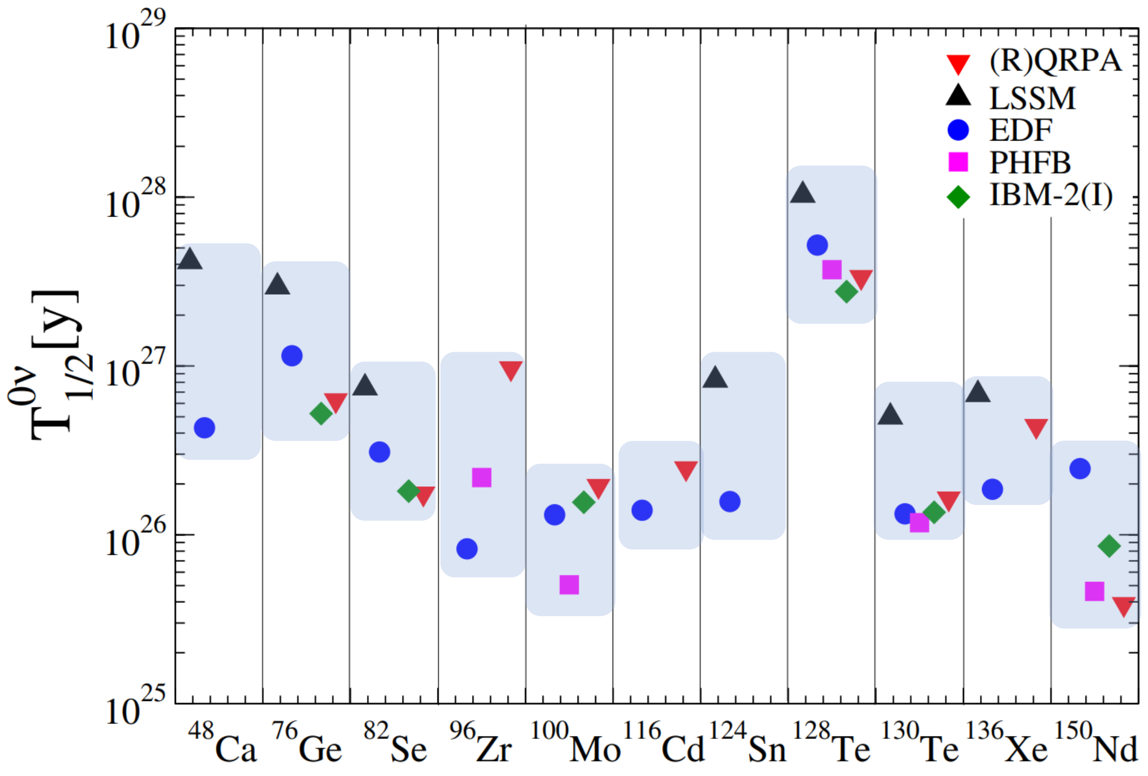
$$\langle m_{\beta\beta}\rangle = 0.05 \text{ eV}$$



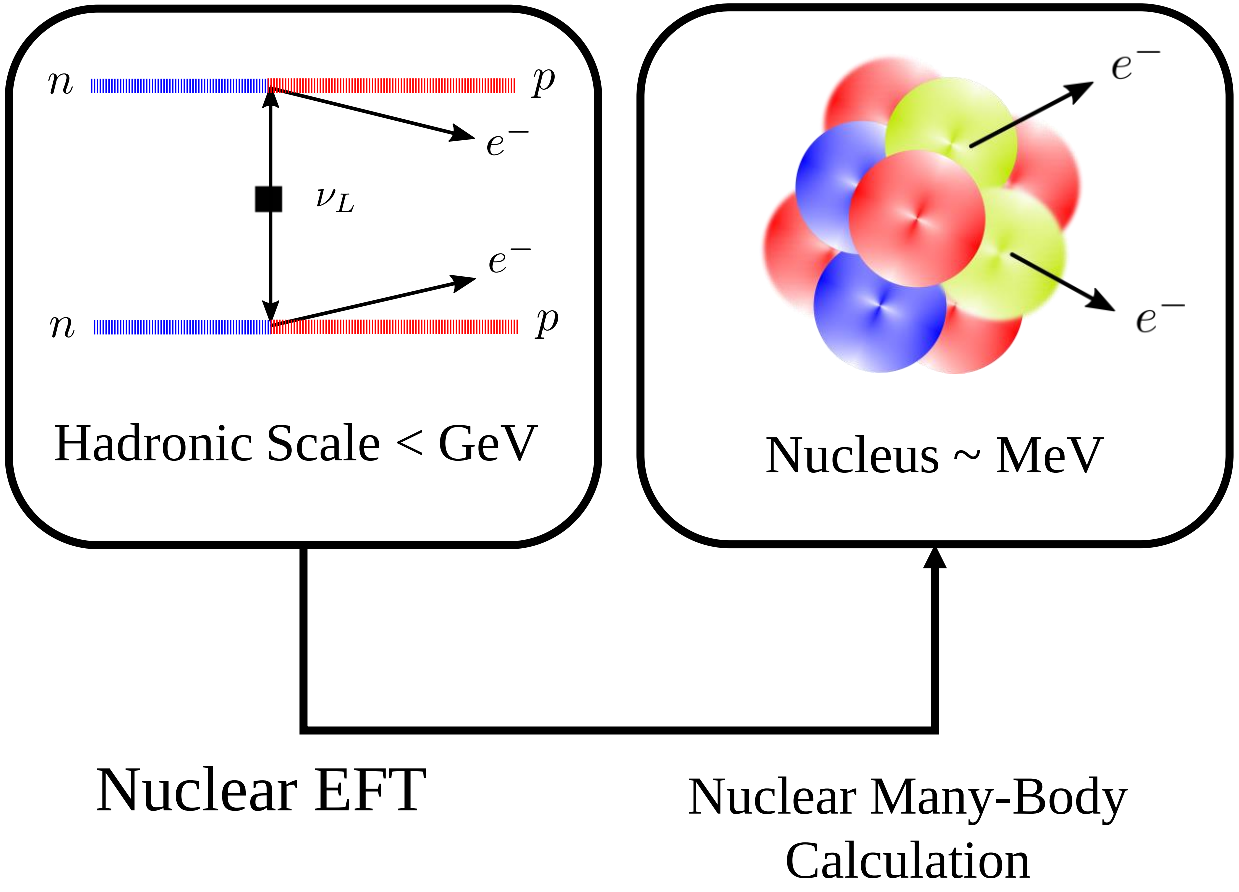
Vergados, Ejiri and Simkovic, Rep. Prog. Phys. 75 106301 (2012)

See a more recent plot of NMEs Agostini, Benato, Detwiler, Men´endez, and Vissani arXiv:2202.01787

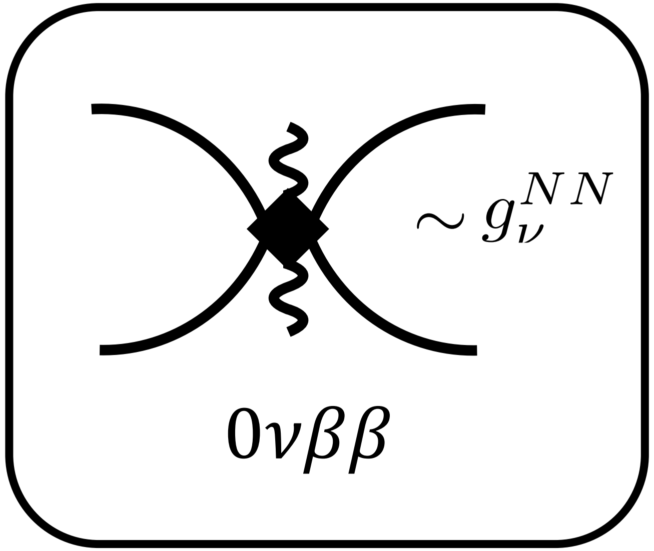
What is the source of these uncertainties in NME?

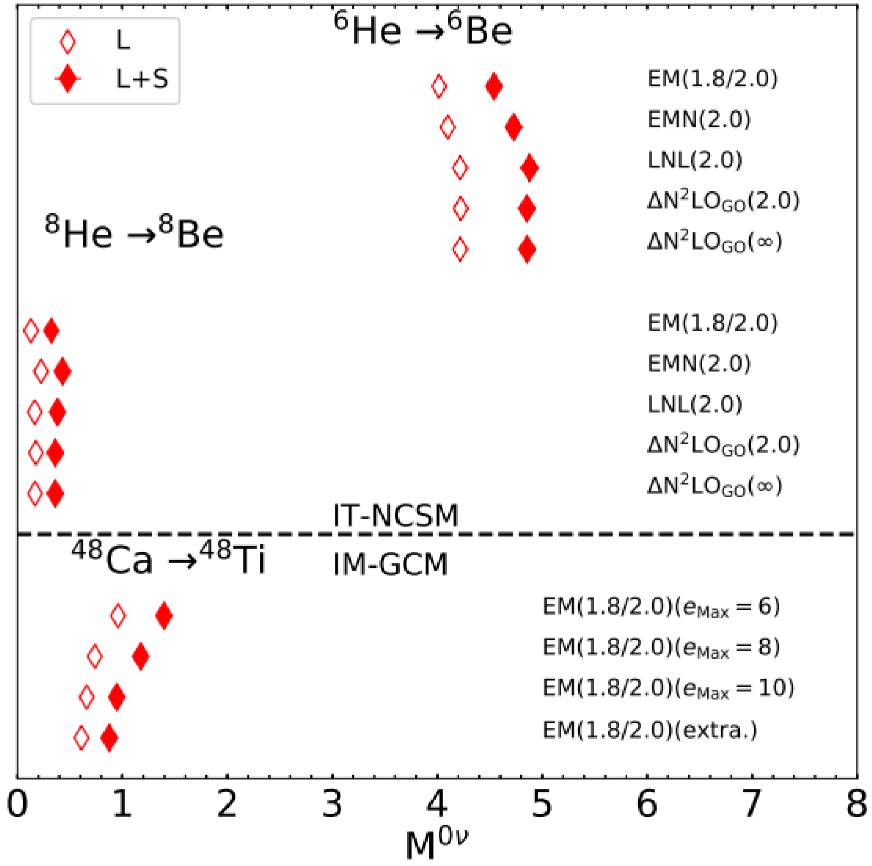


Nuclear Matrix Elements



Low Energy Constants (LECs)





Wirth, Yao, Hergert
PRL 127, 242502 (2021)

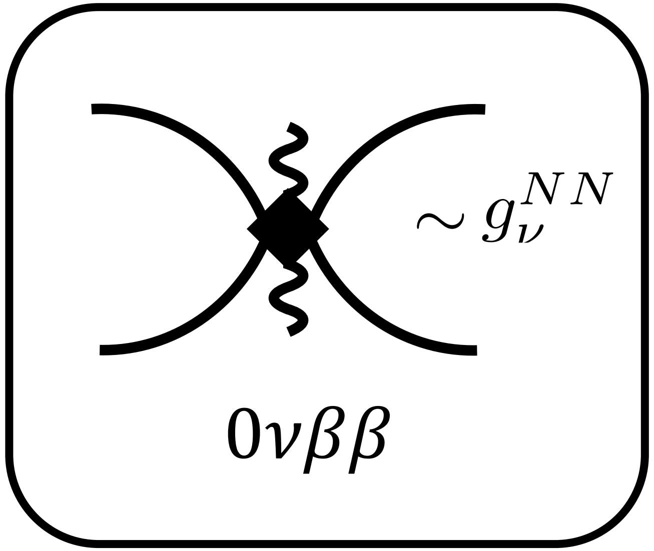
- Undertermined LEC g_ν^{NN} in pionless EFT
- LO $0\nu\beta\beta$ amplitude remains unknown
- An indirect estimate of g_ν^{NN} suggests an enhancement of $\sim$ 40% in NME for Ca nucleus

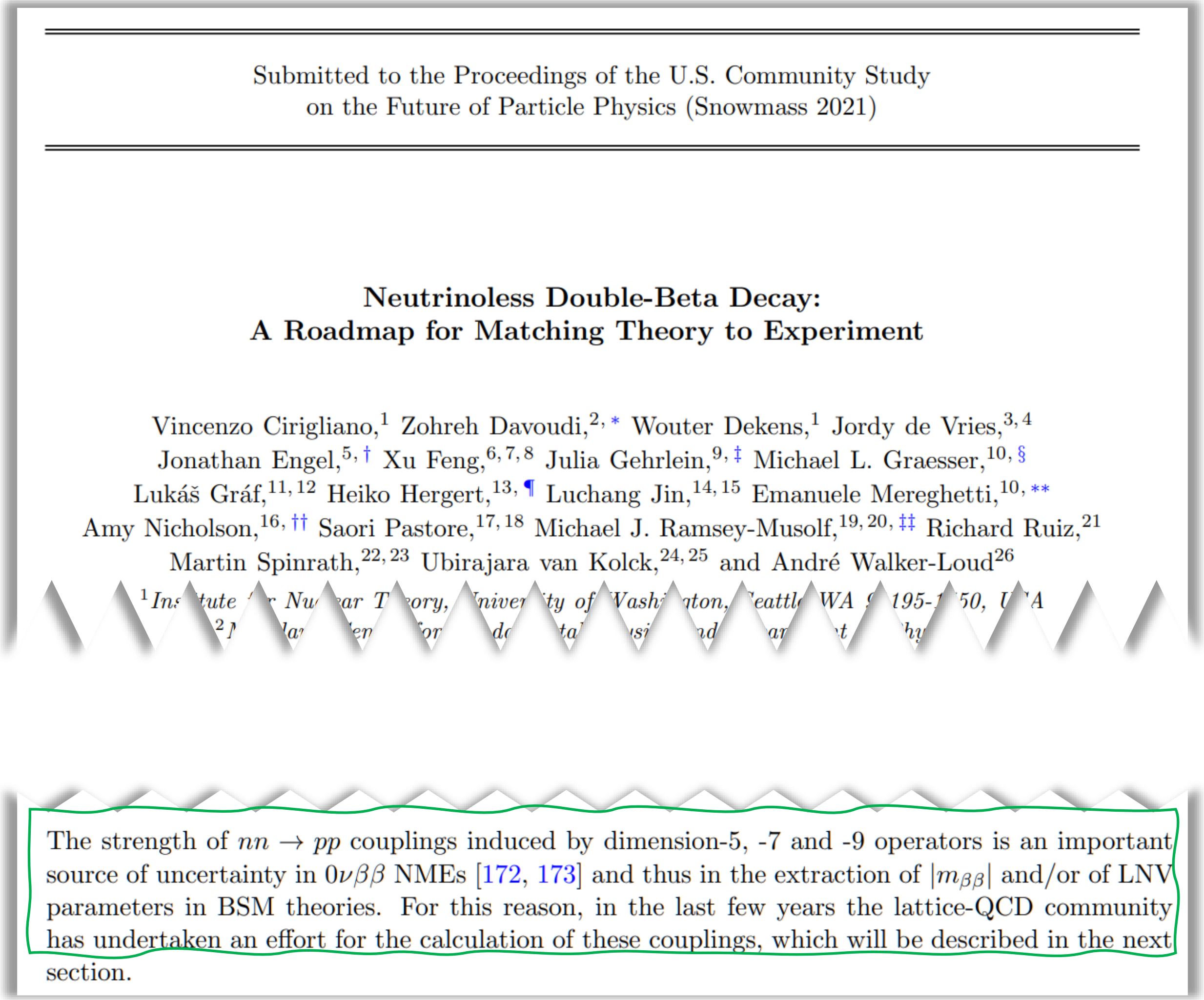
Jokiniemi, Soriano, Menendez, j.physletb.2021.136720 (2021)

Weiss, Soriano, Lovato, Menendez, Wiringa,
Phys Rev C 106 **6**, 065501 (2022)

Cirigliano, Dekens, de Vries, Hoferichter, Mereghetti
PRL 126, 172002
JHEP 2021, 289 (2021)

Low Energy Constants (LECs)




Submitted to the Proceedings of the U.S. Community Study
on the Future of Particle Physics (Snowmass 2021)

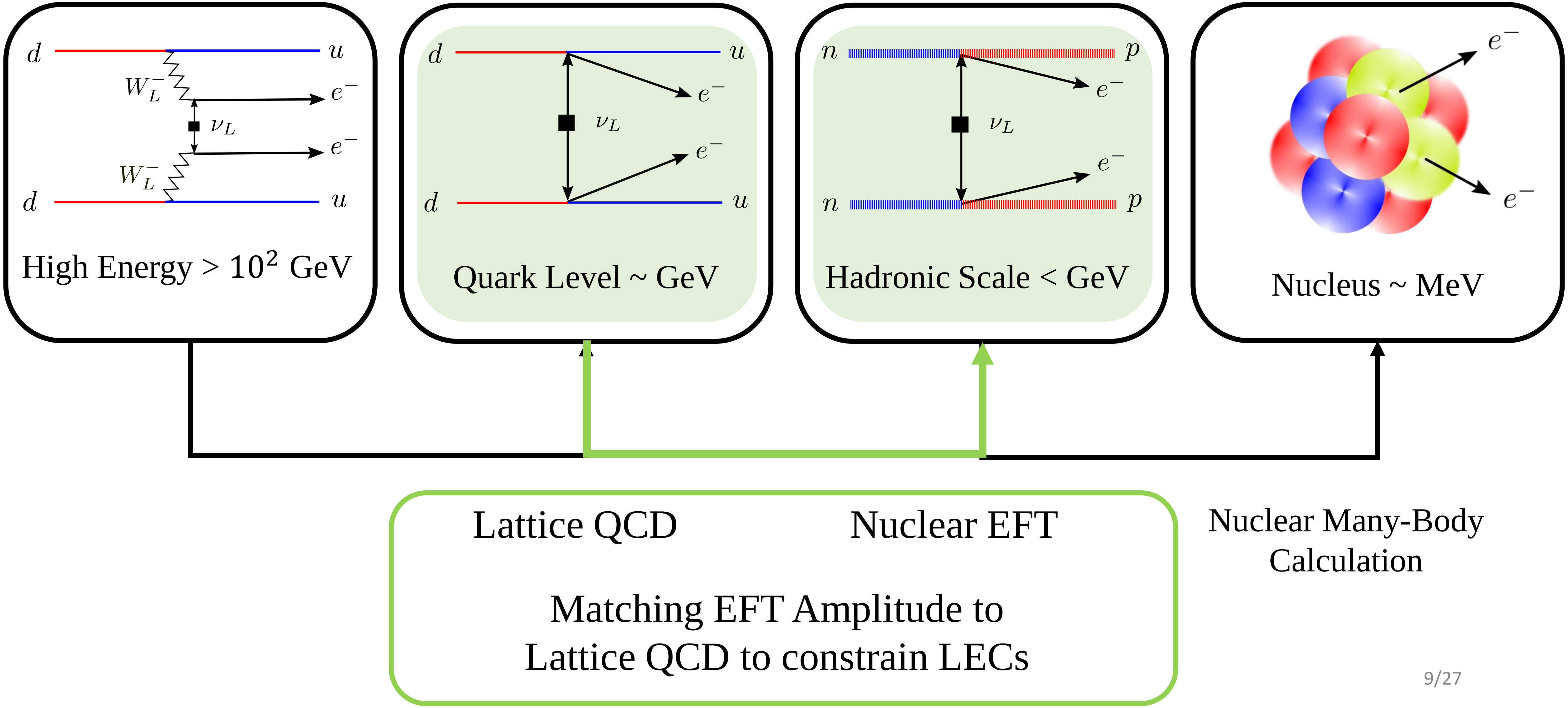
Neutrinoless Double-Beta Decay:
A Roadmap for Matching Theory to Experiment

Vincenzo Cirigliano,[1] Zohreh Davoudi,[2,*] Wouter Dekens,[1] Jordy de Vries,[3,4] Jonathan Engel,[5,†] Xu Feng,[6,7,8] Julia Gehrlein,[9,‡] Michael L. Graesser,[10,§] Lukáš Gráf,[11,12] Heiko Hergert,[13,¶] Luchang Jin,[14,15] Emanuele Mereghetti,[10,**] Amy Nicholson,[16,††] Saori Pastore,[17,18] Michael J. Ramsey-Musolf,[19,20,‡‡] Richard Ruiz,[21] Martin Spinrath,[22,23] Ubirajara van Kolck,[24,25] and André Walker-Loud[26]

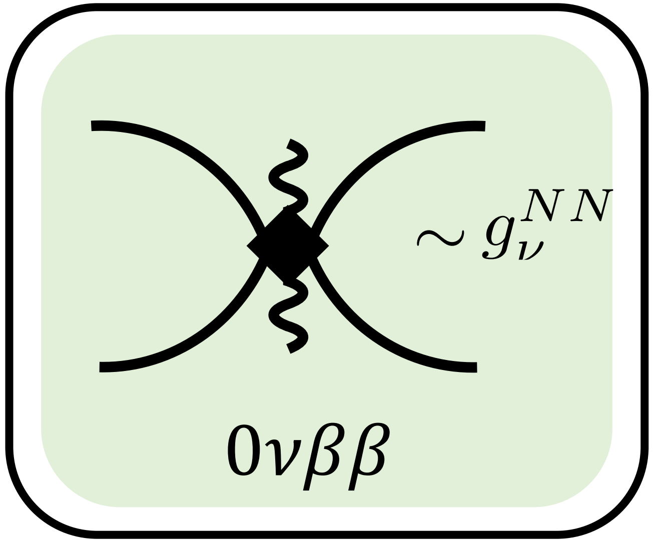
The strength of $nn \to pp$ couplings induced by dimension-5, -7 and -9 operators is an important source of uncertainty in $0\nu\beta\beta$ NMEs [172, 173] and thus in the extraction of $|m_{\beta\beta}|$ and/or of LNV parameters in BSM theories. For this reason, in the last few years the lattice-QCD community has undertaken an effort for the calculation of these couplings, which will be described in the next section.

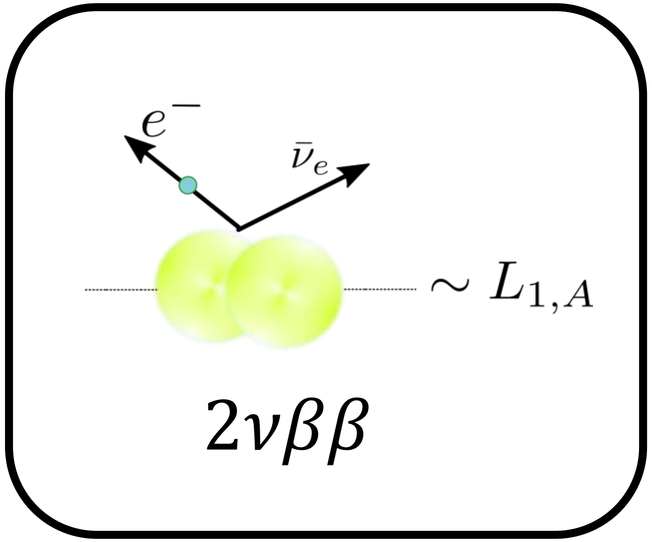
- Undertermined LEC g_ν^{NN} in pionless EFT
- LO $0\nu\beta\beta$ amplitude remains unknown
- An indirect estimate of g_ν^{NN} suggests an enhancement of ~ 40% in NME for Ca nucleus

Nuclear Matrix Elements



Constraining LECs from Lattice QCD





Davoudi and Kadam
Phys. Rev. Lett. 126, 152003 (2021)

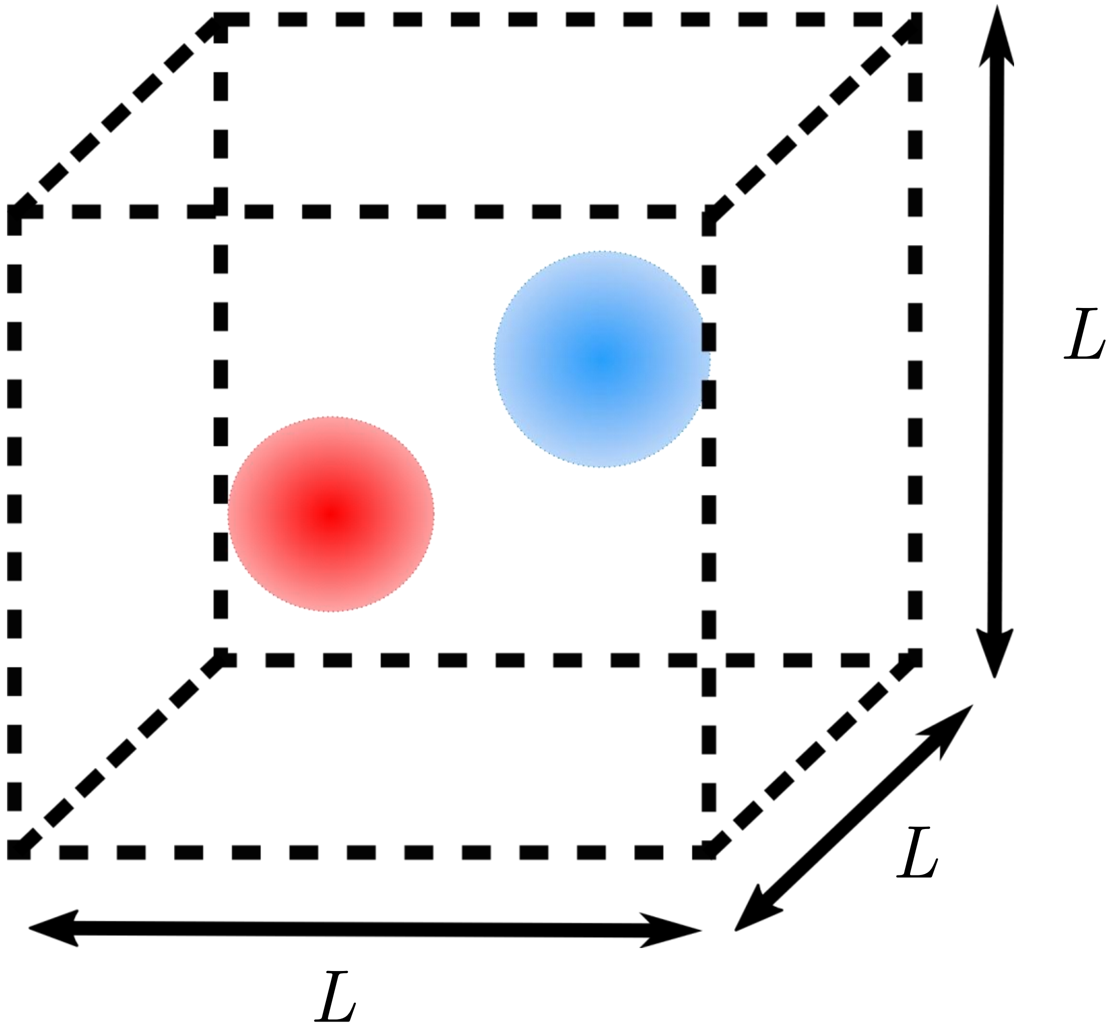
Davoudi and Kadam
Phys. Rev. D 102, 114521 (2020)

Already discussed in previous talks

Lattice QCD

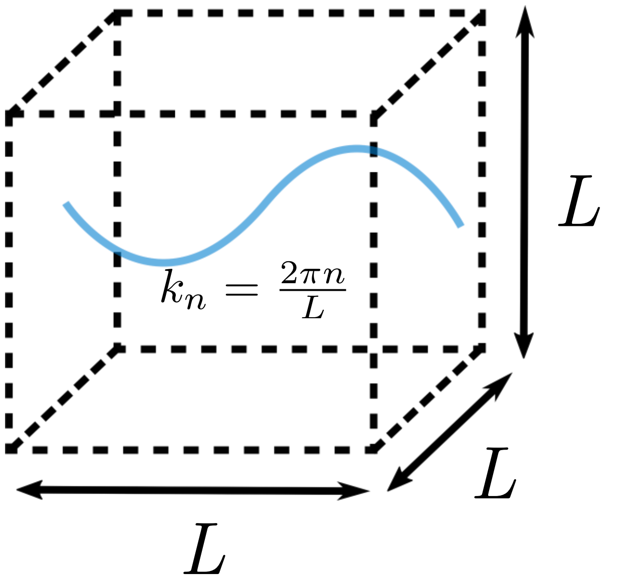
QCD Formulated on

- Discrete Euclidean Spacetime Grid
- Lattice spacing a
- Finite Volume L^3
- Monte-Carlo Sampling



Finite vs. Infinite Volume Physics

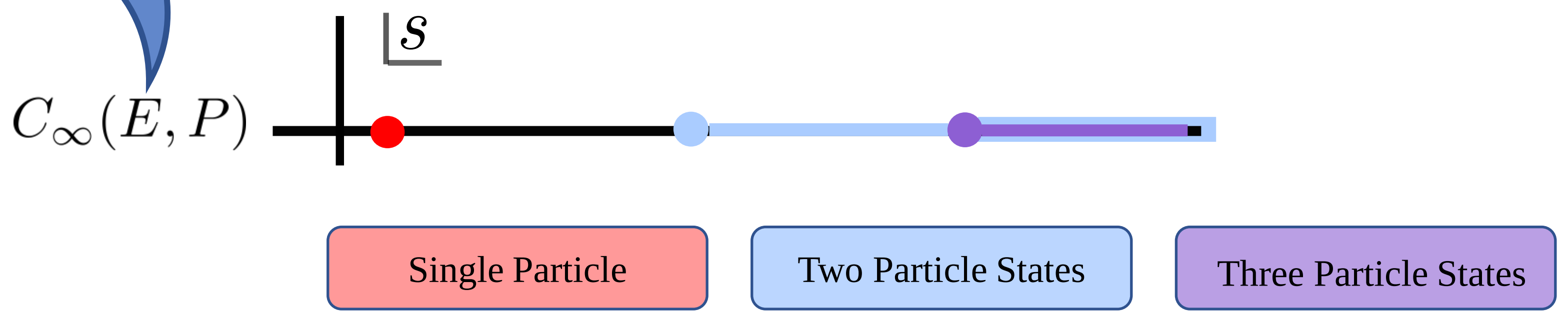
Finite Volume (FV)



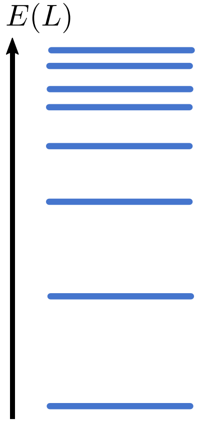
Periodic Boundary Condition

Quantization of Momenta

Particle Spectrum



- Branch cuts are replaced with poles
- Corresponding energies give the FV spectrum
- Position of poles is related to the scattering amplitude



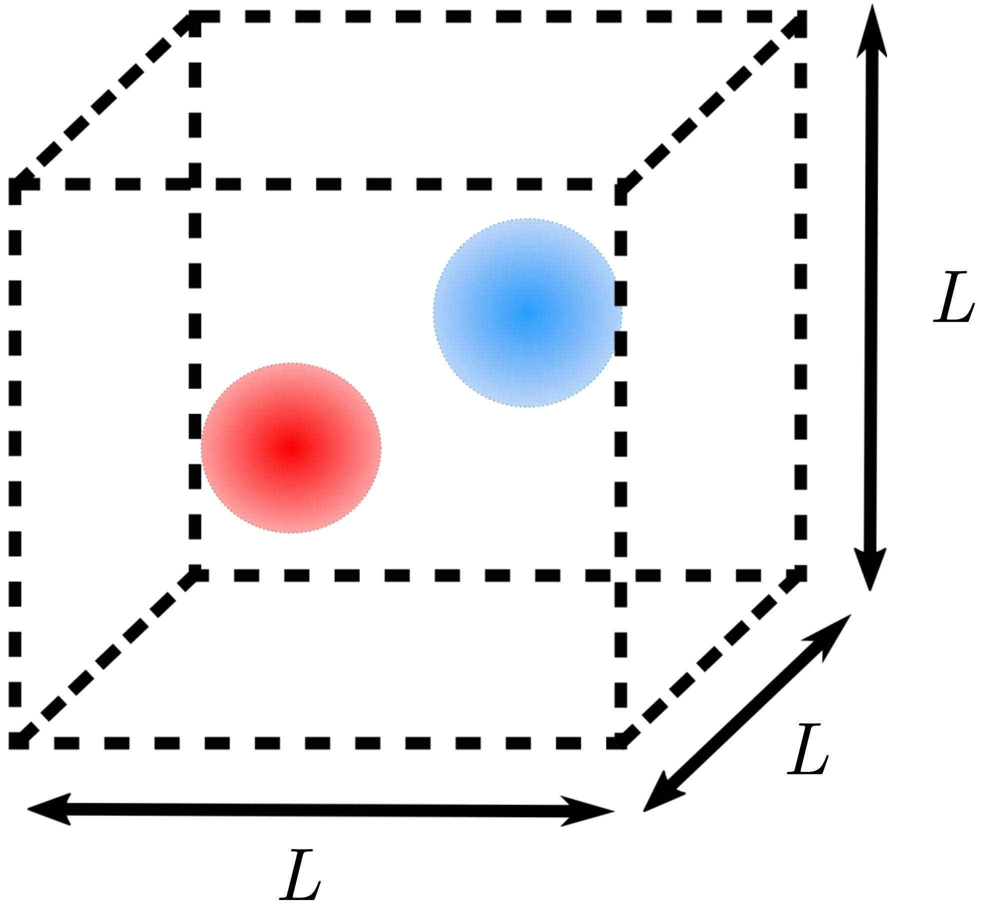
Discrete Finite Volume Spectrum

Luscher (1986a), Commun. Math. Phys. 104, 177

Lellouch, and Luscher (LL) (2001), Commun. Math. Phys. 219,

Kim, Sachrajda, and Sharpe (2005), Nucl. Phys. B727

Assumptions

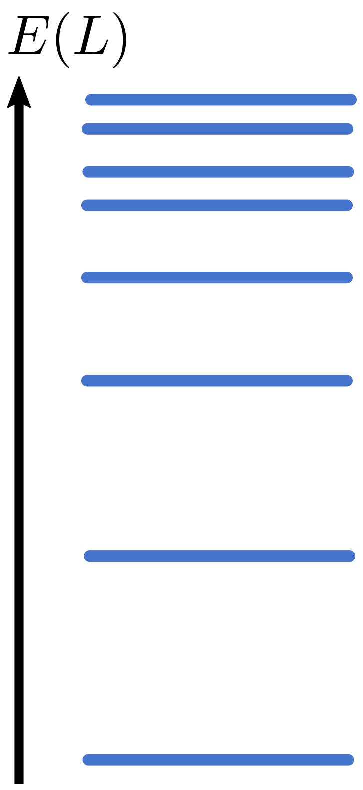


- Lattice spacing $a \to 0$
- Ignore discretization effects

- Infinite temporal extent
- Energy is a continuous variable

Two Nucleon Scattering Amplitude

?

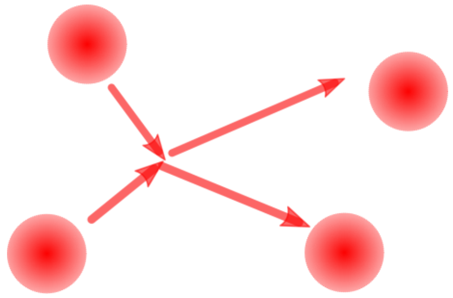


Discrete Finite Volume Spectrum

Evaluate

$$C_L(E) = \int_L d^3x \int dx_0 \, e^{iEx_0} \left[\langle 0 | \, T[B(x) B^\dagger(0)] \, | 0 \rangle \right]_L$$

Two Nucleon Source-Sink Interpolating Operators



$i\mathcal{M}_{NN \to NN}$

Infinite Volume Scattering Amplitude

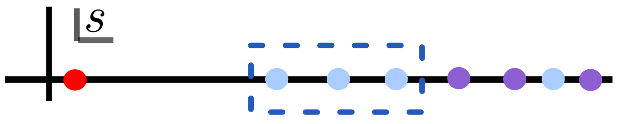
Goal: Identify poles from the FV effects

Source of Power Law FV effects

Which diagrams give the FV effects ?

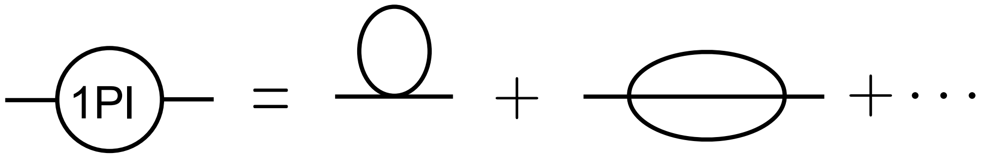
If particles in summed loops can go on-shell

- In COM frame $P = (E, \mathbf{0})$
- Energy below three particle threshold

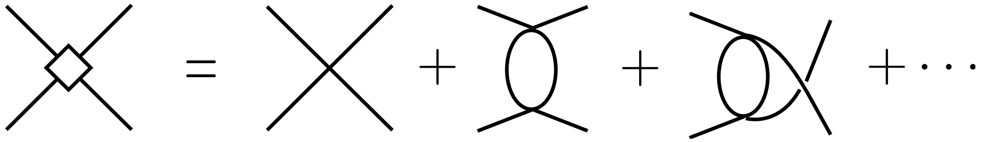
$C_L(E)$ 

Same as Infinite Volume

One particle irreducible diagrams

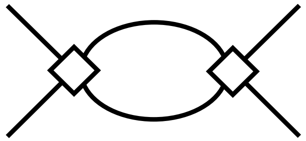


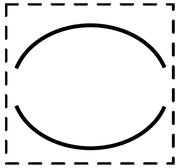
$2 \to 2$ Bethe-Salpeter Kernel



Power Law Difference

Two particle loop in s - channel



 $\sim \; I_0^V(E) = I_0^\infty(E) + F_0(E)$

Poles in $C_L(E)$ are identified using $F_0(E)$

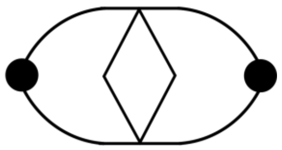
FV method

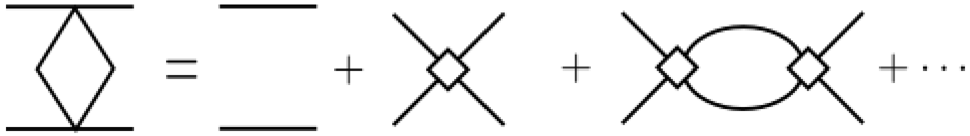
Lellouch, and Luscher (LL) (2001), Commun. Math. Phys. 219,

Kim, Sachrajda, and Sharpe (2005), Nucl. Phys. B727

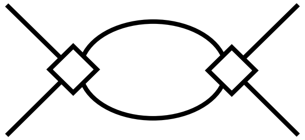
$$C_L(E) = \int_L d^3x \int dx_0\, e^{iEx_0} \left[\langle 0 | \, T[B(x)B^\dagger(0)] \, | 0 \rangle \right]_L$$

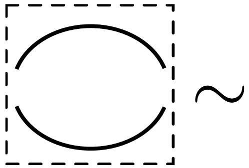
Evaluate the correlation function non-perturbatively

$C_L =$ 



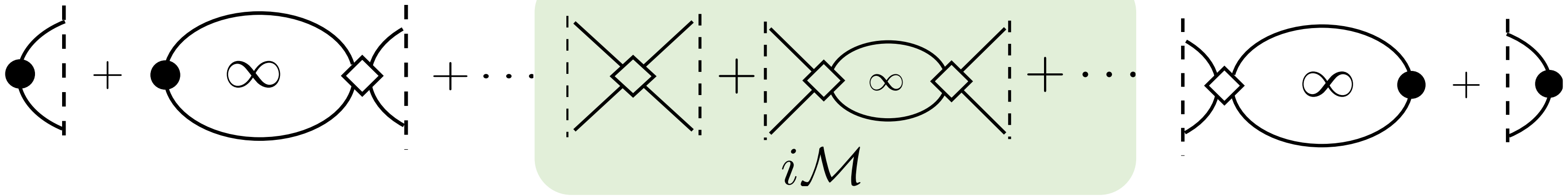
Identify and isolate FV corrections





$$I_0^V(E) = I_0^\infty(E) + F_0(E)$$

Rearrange to get Infinite volume 2 → 2 amplitude

$C_L =$ 

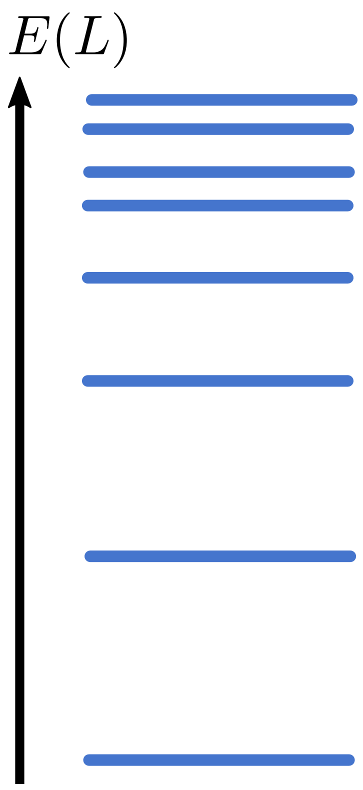
Geometric sum over F_0 terms

$$C_L \sim \frac{1}{F_0^{-1} + \mathcal{M}}$$

Poles of $C_L(E)$ are identified by

$$\det \left[F_0^{-1}(E_n) + \mathcal{M}(E_n) \right] = 0$$

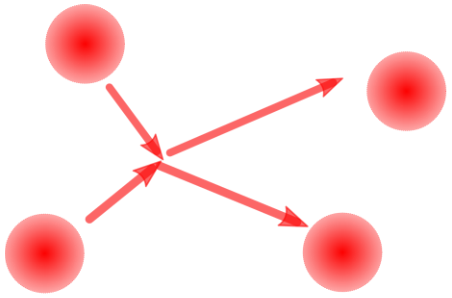
Two Nucleon Scattering Amplitude



Finite Volume Two Particle Spectrum

Luscher's Quantization Condition

$$\det\left[F_0^{-1}(E_n) + \mathcal{M}(E_n)\right] = 0$$



$i\mathcal{M}_{NN\to NN}$

Infinite Volume Scattering Amplitude

Luscher Commun. Math. Phys. 104, 177 (1986)
Luscher Commun. Math. Phys. 105, 153 (1986)
Kim, Sachrajda, and Sharpe, Nucl. Phys. B727 (2005)

FV Formalism

Review: Davoudi, Detmold, Shanahan, Orginos, Parreno, Savage, Wagman physrep.2020.10.004

Extended towards electro-weak current ($\mathcal{J}$) interactions:

- Formalism for generalized $0 + \mathcal{J} \to 2$ and $1 + \mathcal{J} \to 2$ processes. Briceno, Hansen, and Walker-Loud (2015) Phys. Rev. D 91, 034501; Briceno and Hansen (2015),Phys. Rev. D92 (7), 074509

Briceño and Davoudi (2013) Phys. Rev. D 88, 094507 Briceno and Hansen (2016) Phys. Rev. D94 (1), 013008

- Formalism for $2 + \mathcal{J} \to 2$ processes.

  Value of $L_{1,A}$ from LQCD via studying pp fusion $pp \to de^+\nu$ process. Savage, Shanahan, Tiburzi, Wagman, Winter, Beane, Change, Davoudi, Detmold, OrginosNPLQCD Collaboration Phys. Rev. Lett. 119 (6) (2017) 62002.

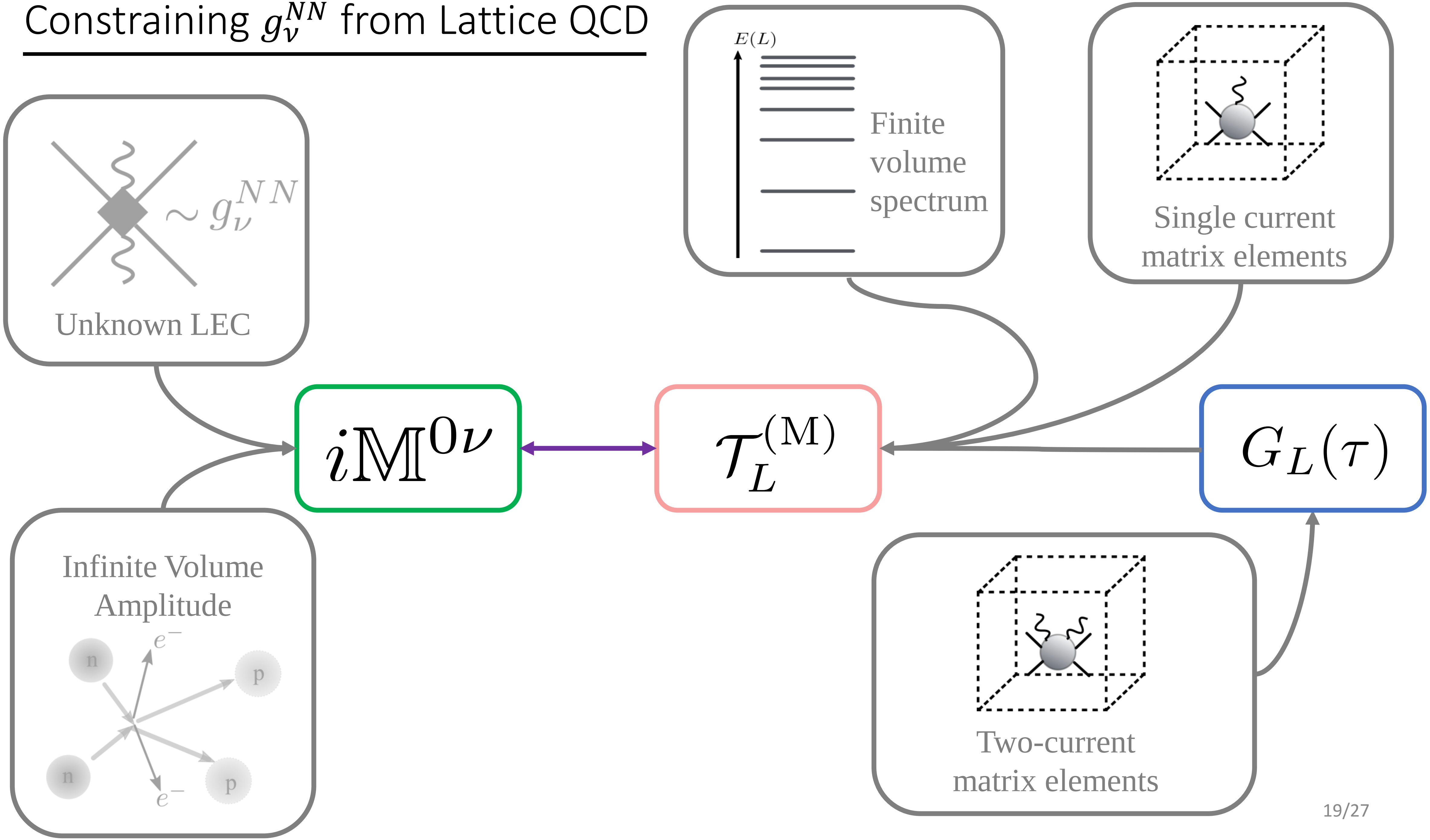
- Formalism for $1 + 2\,\mathcal{J} \to 1$ processes. Briceño, Davoudi, Hansen, Schindler and Baroni Phys. Rev. D 101, 014509

  $2\nu\beta\beta$ matrix elements (MEs) at $m_\pi \sim 800$ MeV

  Tiburzi, Wagman, Winter, Chang, Davoudi, Detmold, Orginos, Savage, Shanahan (NPLQCD) Collaboration Phys. Rev. D 96, 054505.

- Formalism for $1 + 2\,\mathcal{J} \to 1$ processes with massless leptonic propagators Christ, Feng, Jin, and Sachrajda Phys. Rev. D 103, 014507 (2021); Feng, Jin, Wang, and Zhang Phys. Rev. D 103, 034508 (2021)
- Formalism for $2 + 2\,\mathcal{J} \to 2$ processes for $2\nu\beta\beta$ and $0\nu\beta\beta$. Davoudi and Kadam Phys. Rev. D 102, 114521 (2020); Davoudi and Kadam Phys. Rev. Lett. 126, 152003 (2021)
- $\pi^- \to \pi^+$ from LQCD at m_π in 130-310 MeV Nicholson et al. (CalLat Collaboration) Phys. Rev. Lett. 121, 172501 (2018)
- Light sterile neutrino contribution to $\pi^- \to \pi^+ e^- e^-$ from LQCD at the physical pion mass Tuo, Feng, and Jin Phys.Rev.D 106 (2022) 7, 074510
- $\pi^- \to \pi^+ e^- e^-$ from LQCD at m_π in 300-430 MeV Detmold and Murphy arXiv:2004.07404; Detmold, Jay, Murphy, Oare, and Shanahan arXiv:2208.05322

Constraining g_ν^{NN} from Lattice QCD



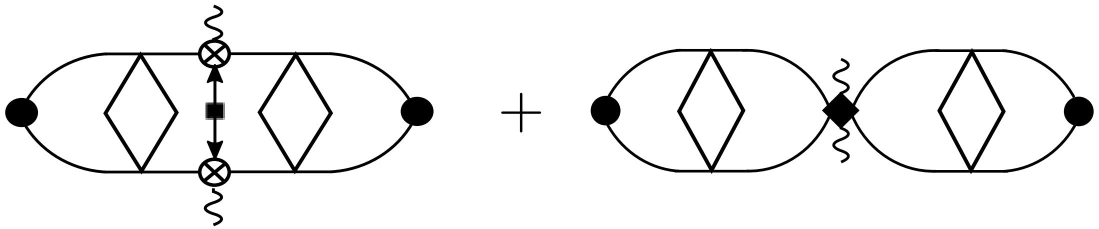
$0\nu\beta\beta$ Decay

Finite Volume

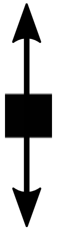
For details see:
Davoudi and Kadam Phys. Rev. Lett. 126, 152003 (2021)

$$\mathcal{T}_L^{(M)} = \int_L d^3z \int dz_0\, e^{iE_1 z_0} \left[\langle E_{n_f}|\, T^{(M)}[\mathcal{J}(z)\, S_\nu(z)\, \mathcal{J}(0)]\, |E_{n_i}\rangle\right]_L$$

Evaluate the correlation function in EFT non-perturbatively

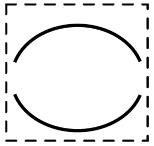
$C_L^{0\nu} =$ 

Sum over quantized momenta

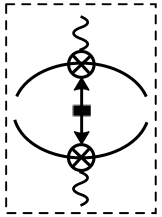
With neutrino propagator  $\sim \frac{1}{L^3} \sum_{\mathbf{q}\neq\mathbf{0}} \frac{m_{\beta\beta}}{|\mathbf{q}|^2}$

IR regulated by removing zero mode

Identify and isolate FV corrections



F_0



δJ^{V}

Constraining g_ν^{NN} from Lattice QCD

For details see:
Davoudi and Kadam Phys. Rev. Lett. 126, 152003 (2021)

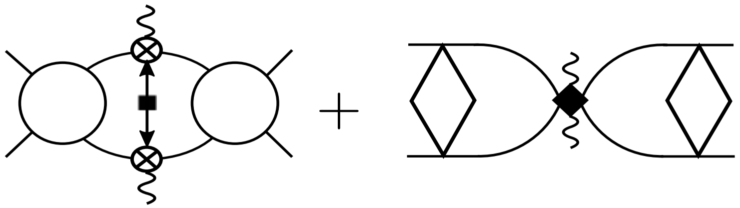
$$L^6 \left|\mathcal{T}_L^{(\mathrm{M})}\right|^2 = \left|\mathcal{R}^*(E_{n_f})\right| \left|i\mathbb{M}^{0\nu}\right|^2 \left|\mathcal{R}^*(E_{n_i})\right|$$

Lellouch Luscher residue matrix

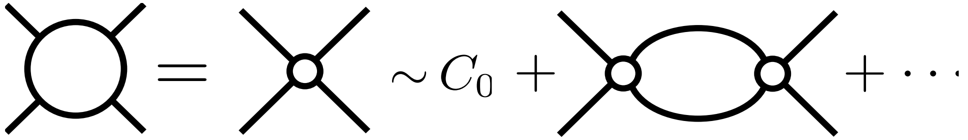
$$\mathcal{R}(E_n) = \lim_{E\to E_n} \frac{(E-E_n)}{F_0^{-1}+\mathcal{M}}$$

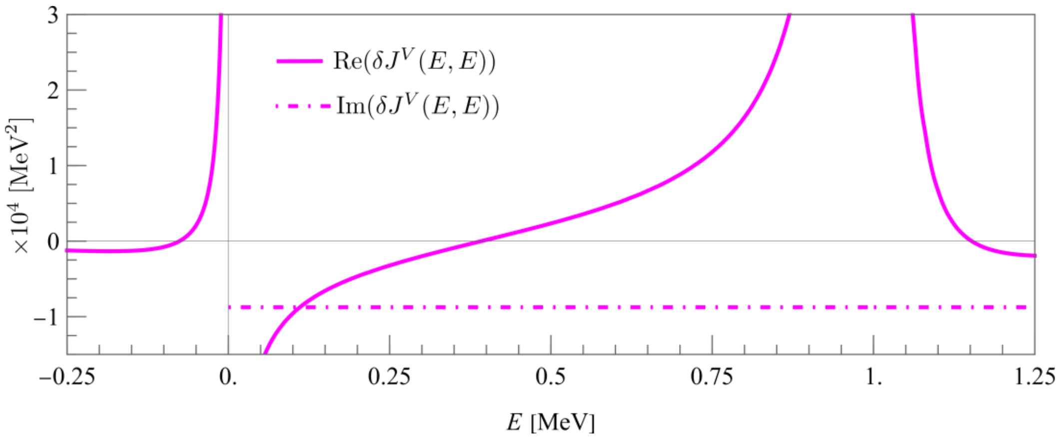
$$i\mathbb{M}^{0\nu} = i\mathcal{M}^{(\mathrm{Int.})}_{nn\to pp} - m_{\beta\beta}(1+3g_A^2)\mathcal{M}_{nn}\,\delta J^V\,\mathcal{M}_{pp}$$

$i\mathcal{M}^{(\mathrm{Int.})}_{nn\to pp} =$

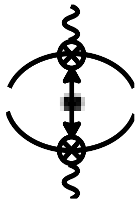


Physical LO two-nucleon amplitude



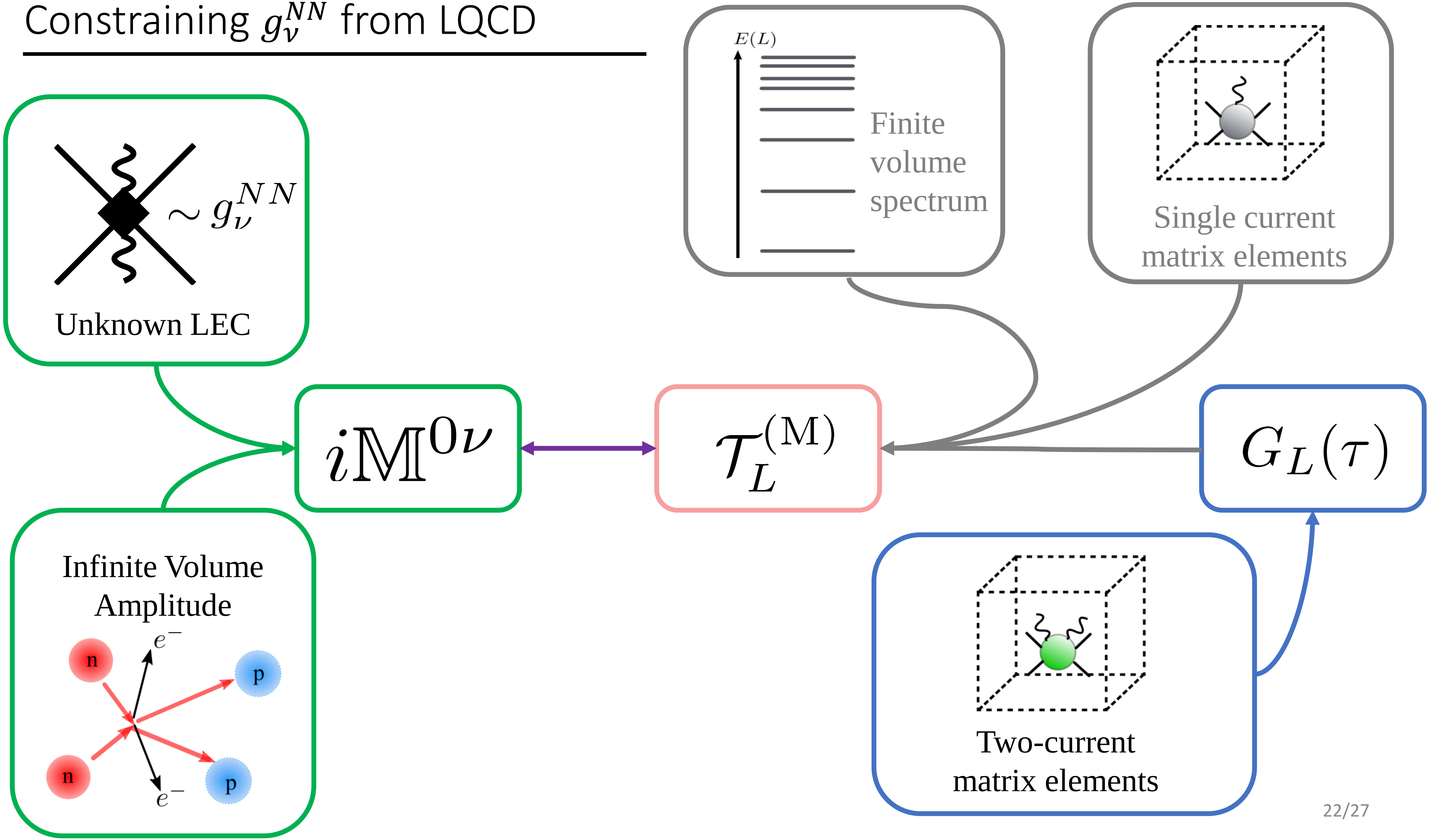


Finite volume corrections



$$\sum_{\vec{k}} = \int_{\vec{k}} + \delta J^V$$

Constraining g_ν^{NN} from LQCD



$\mathcal{T}_L^{(\mathrm{M})}$

Minkowski Signature Correlation Function

$$\mathcal{T}_L^{(M)} = \int_L d^3z \int dz_0\, e^{iEz_0} \left[\langle E_{n_f}|\, T^{(M)}[\mathcal{J}(z)\, S_\nu(z)\, \mathcal{J}(0)]\, |E_{n_i}\rangle\right]_L$$

?

$G_L(\tau)$

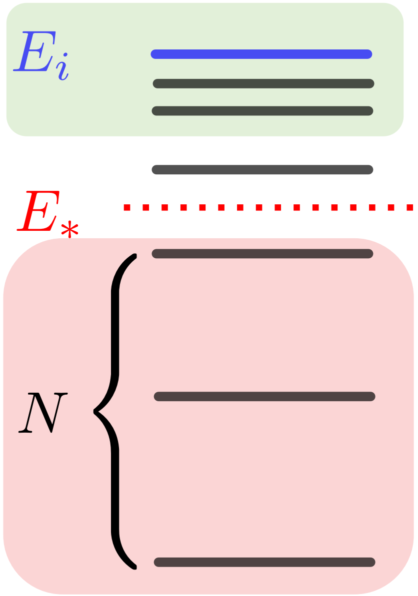
Euclidean Time Four-point Correlation Function from LQCD

$$G_L(\tau) = \int_L d^3z \left[\langle E_f, L|\, T^{(\mathrm{E})}[\mathcal{J}^{(\mathrm{E})}(\tau,z)\, S_\nu^E(\tau,z) \mathcal{J}^{(\mathrm{E})}(0)]\, |E_i, L\rangle\right]_L ,$$

Plugging back the missing time integral

$$\mathcal{T}_L^{(\mathrm{E})} \stackrel{?}{=} \int d\tau e^{E\tau} G_L(\tau)$$

Diverges for intermediate states that can go on-shell



Need to remove these divergences for analytic continuation!!

For details see:
Davoudi and Kadam Phys. Rev. Lett. 126, 152003 (2021)

$\mathcal{T}_L^{(\mathrm{M})}$

Minkowski Signature Correlation Function

$G_L(\tau)$

Euclidean Time Four-point Correlation Function from LQCD

$$\mathcal{T}_L^{(M)} = \int_L d^3z \int dz_0\, e^{iEz_0} \left[\langle E_{n_f}|\, T^{(M)}[\mathcal{J}(z)\, S_\nu(z)\, \mathcal{J}(0)]\, |E_{n_i}\rangle\right]_L$$

?

$$G_L(\tau) = \int_L d^3z \left[\langle E_f, L|\, T^{(\mathrm{E})}[\mathcal{J}^{(\mathrm{E})}(\tau, z)\, S_\nu^E(\tau, z) \mathcal{J}^{(\mathrm{E})}(0)]\, |E_i, L\rangle\right]_L ,$$

Plugging back the missing time integral

$$\mathcal{T}_L^{(\mathrm{E})} \stackrel{?}{=} \int d\tau e^{E\tau} G_L(\tau)$$

Diverges for intermediate states that can go on-shell

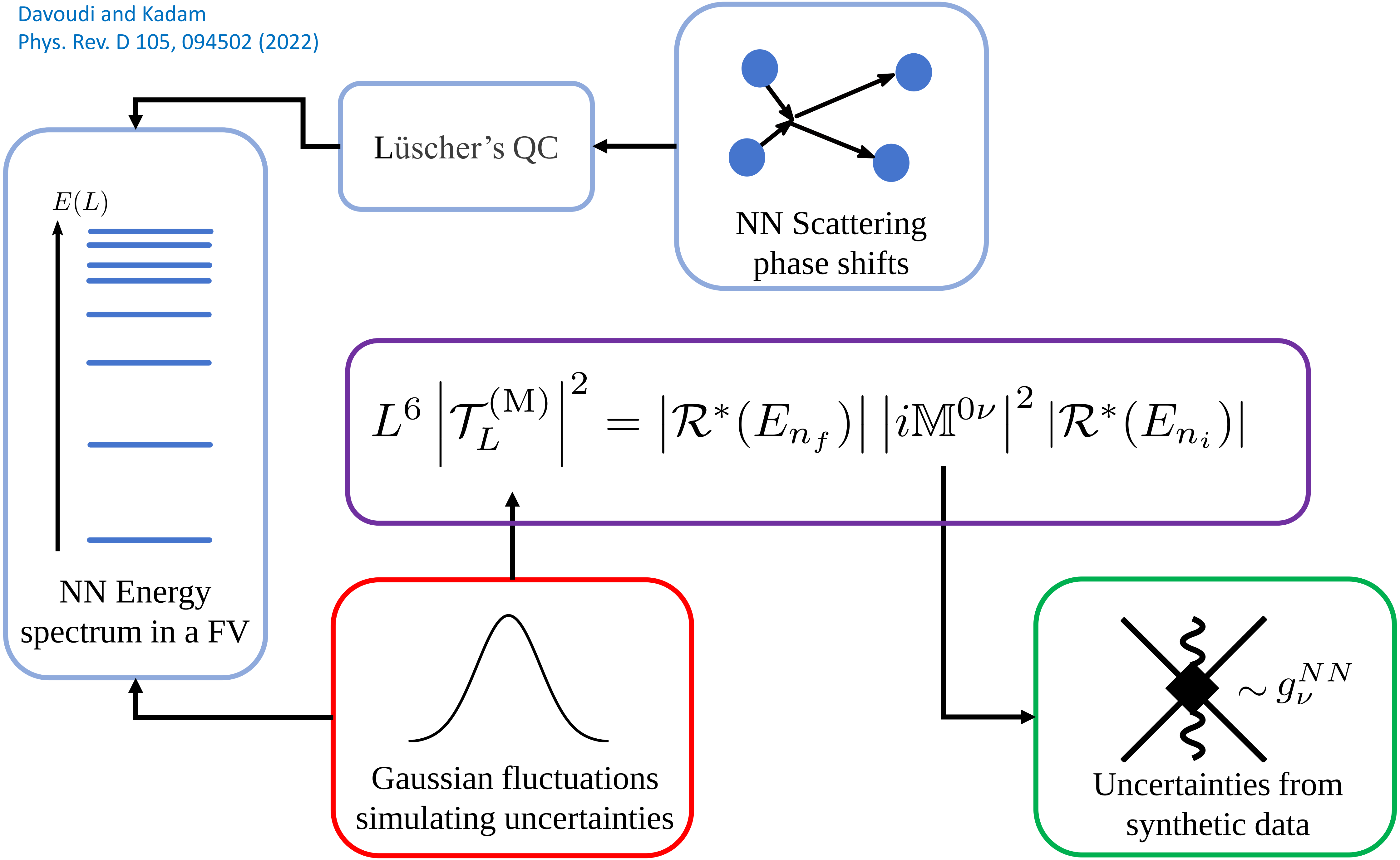
NN finite volume spectrum

For $L = 8$ fm

$$E_{^1S_0} = -2.728,\ 19.043, \cdots \ \text{MeV}$$
$$E_{*m} = -5.579,\ 13.688, \cdots \ \text{MeV}$$
$$|\mathbf{P}_*| = 2\pi/L \approx 155 \ \text{MeV}$$

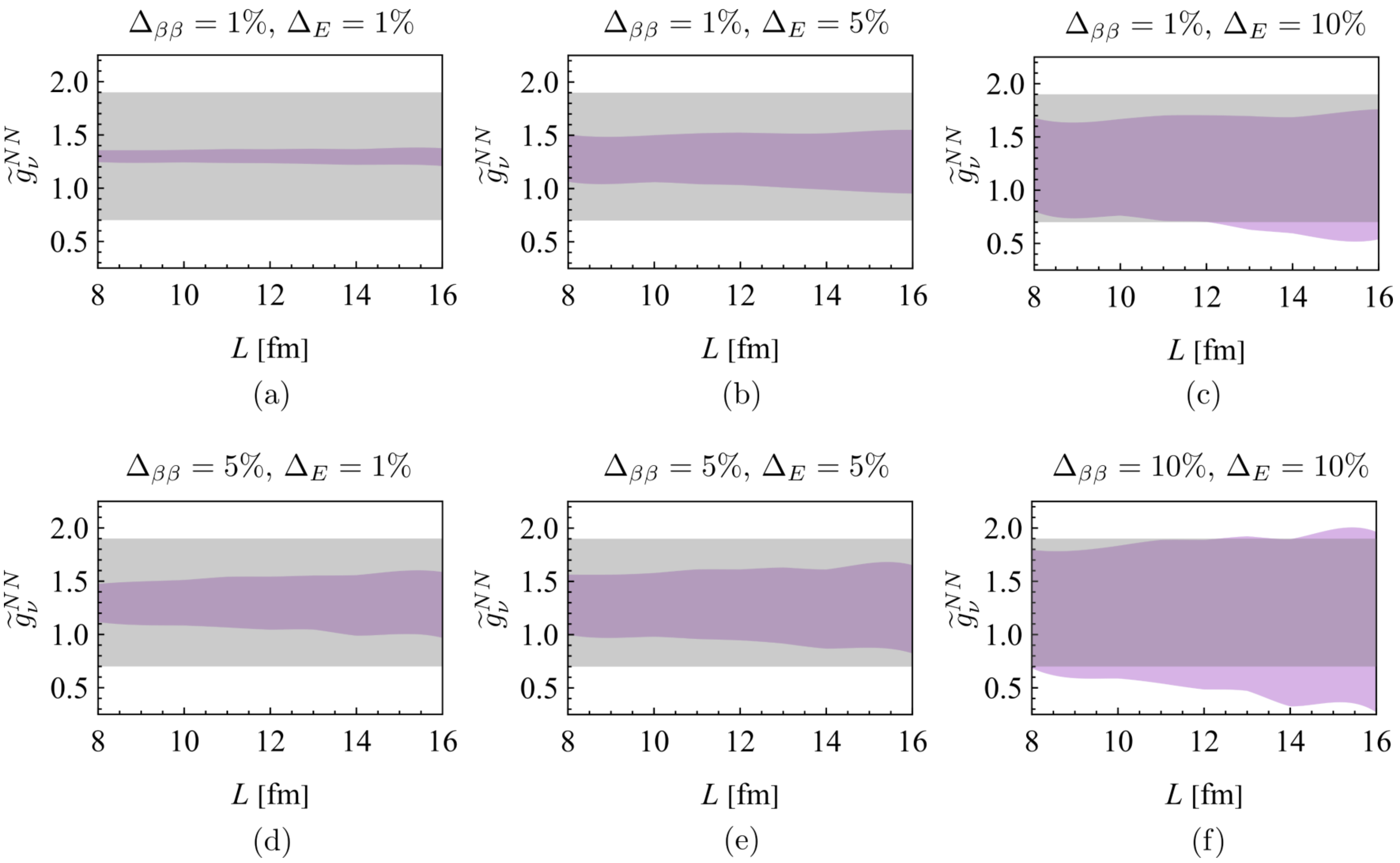
$$i\mathcal{T}_L^{(\mathrm{E})} = i \int d\tau\, e^{E_1 \tau} G_L^{(\mathrm{E})}(\tau)$$

Davoudi and Kadam
Phys. Rev. D 105, 094502 (2022)

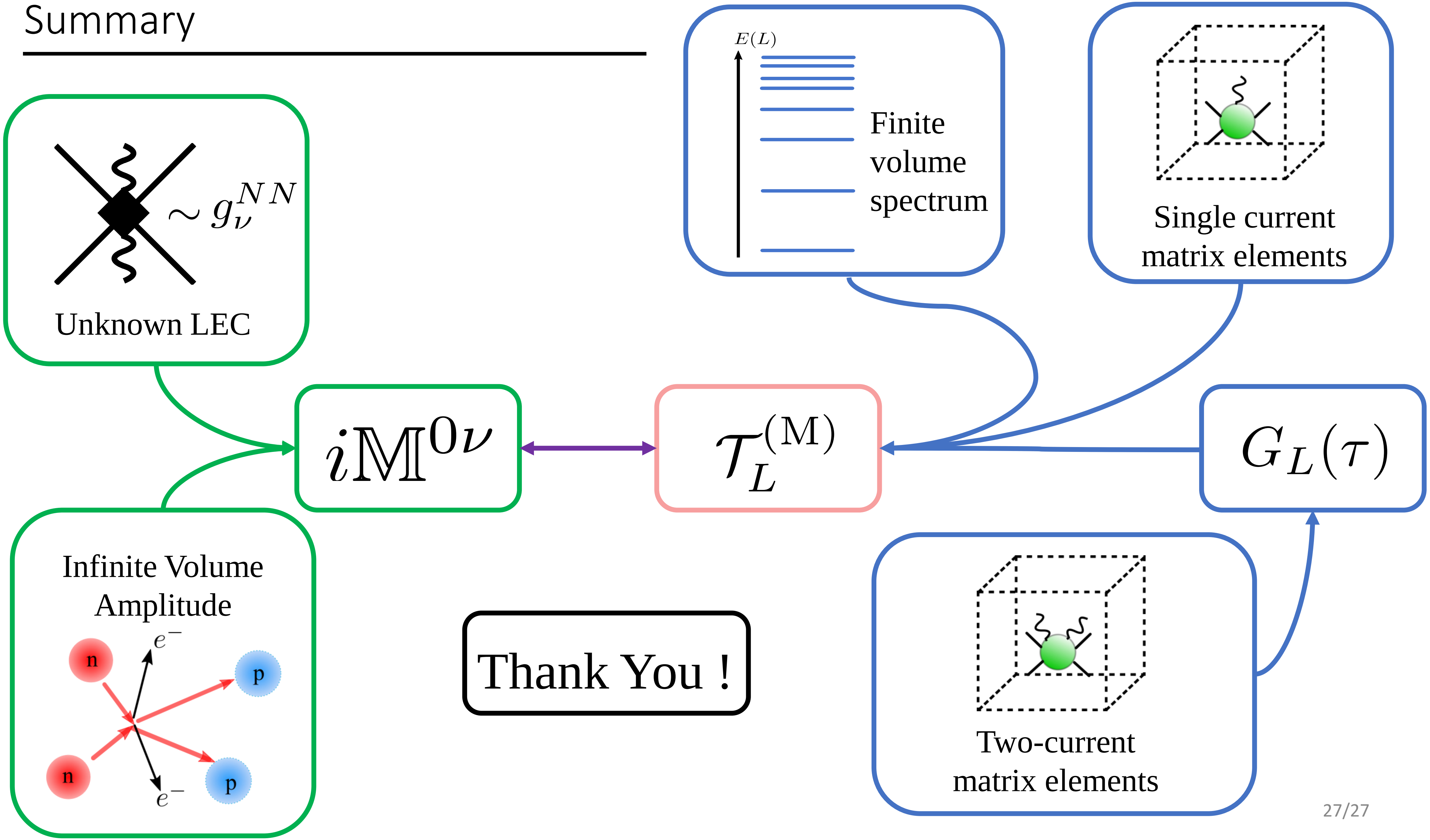


Davoudi and Kadam
Phys. Rev. D 105, 094502 (2022)

$\Delta_{\beta\beta}$: Uncertainty in four-point function
Δ_E : Uncertainty in NN energy eigenvalues



Summary


$E(L)$
Finite volume spectrum
Single current matrix elements
$\sim g_\nu^{NN}$
Unknown LEC
$i\mathbb{M}^{0\nu}$
$\mathcal{T}_L^{(\mathrm{M})}$
$G_L(\tau)$
Infinite Volume Amplitude
n
n
p
p
e^-
e^-
Thank You !
Two-current matrix elements

Appendix

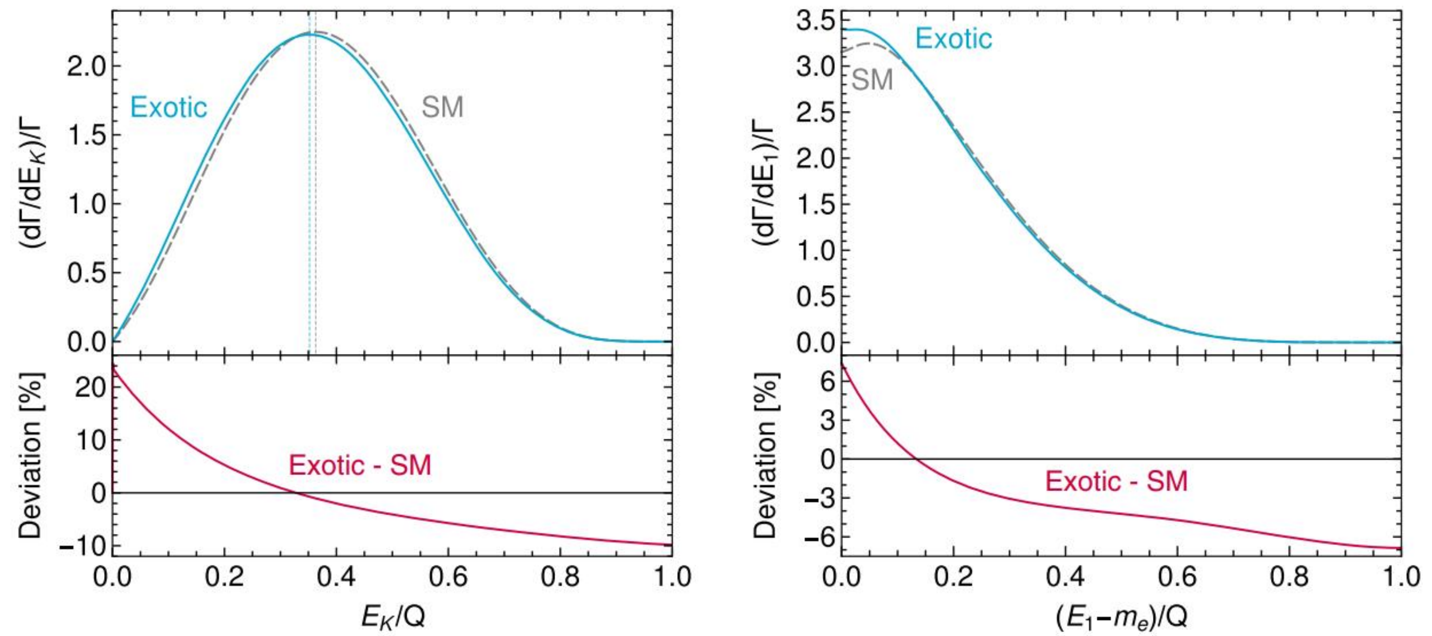


FIG. 2. Left: normalized $2\nu\beta\beta$ decay distributions with respect to the total kinetic energy $E_K = E_{e_1} + E_{e_2} - 2m_e$ of the emitted electrons for standard $2\nu\beta\beta$ decay through SM $V - A$ currents (dashed) and a pure RH lepton current (solid). Right: normalized $2\nu\beta\beta$ decay distributions with respect to the energy of a single electron in the same scenarios. Both plots are for the isotope ^{100}Mo and the energies are normalized to the Q value. The bottom panels show the relative deviation of the exotic distribution from the SM case.

Agostini, Benato, Detwiler, Men´endez, and Vissani
arXiv:2202.01787

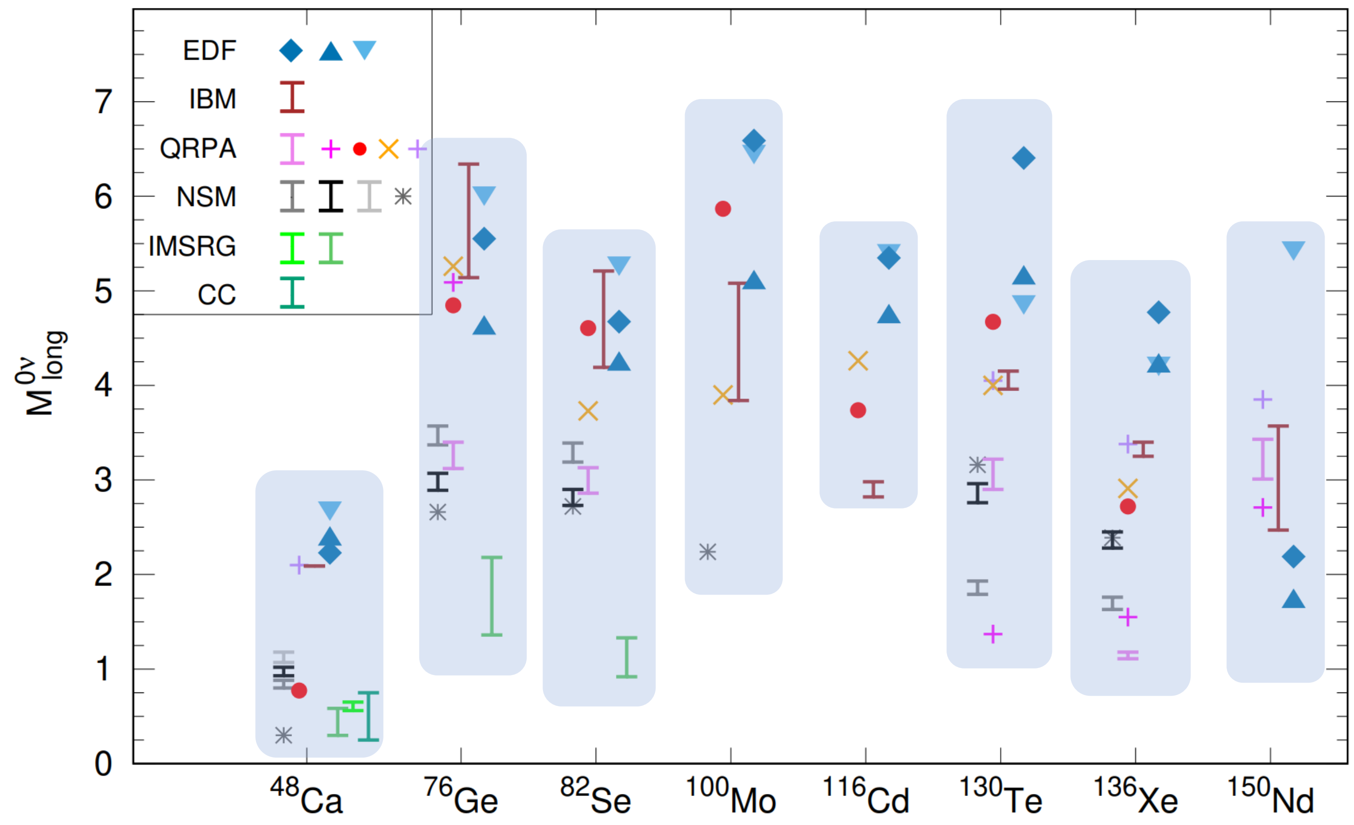


FIG. 9 Nuclear matrix elements $M^{0\nu}$ for light-neutrino exchange from different many-body methods. NSM: black (Menéndez, 2018), grey (Horoi and Neacsu, 2016b), light-grey (Iwata *et al.*, 2016) bars and grey stars (Coraggio *et al.*, 2020, 2022); QRPA: deformed in violet bars (Fang *et al.*, 2018)), and spherical in magenta (Mustonen and Engel, 2013) and purple (Terasaki, 2015, 2020; Terasaki and Iwata, 2019) crosses, red circles (Šimkovic *et al.*, 2018b), and orange multiplication signs (Hyvarinen and Suhonen, 2015); IBM: brown bars (Barea *et al.*, 2015a; Deppisch *et al.*, 2020a); EDF theory: nonrelativistic in blue diamonds (Rodriguez and Martinez-Pinedo, 2010) and blue up-triangles (López Vaquero *et al.*, 2013)), and relativistic in light-blue down-triangles (Song *et al.*, 2017); IMSRG: IM-GCM in the light green ^{48}Ca bar (Yao *et al.*, 2020), and valence space in green bars (Belley *et al.*, 2021); and CC theory: dark green ^{48}Ca bar (Novario *et al.*, 2021).

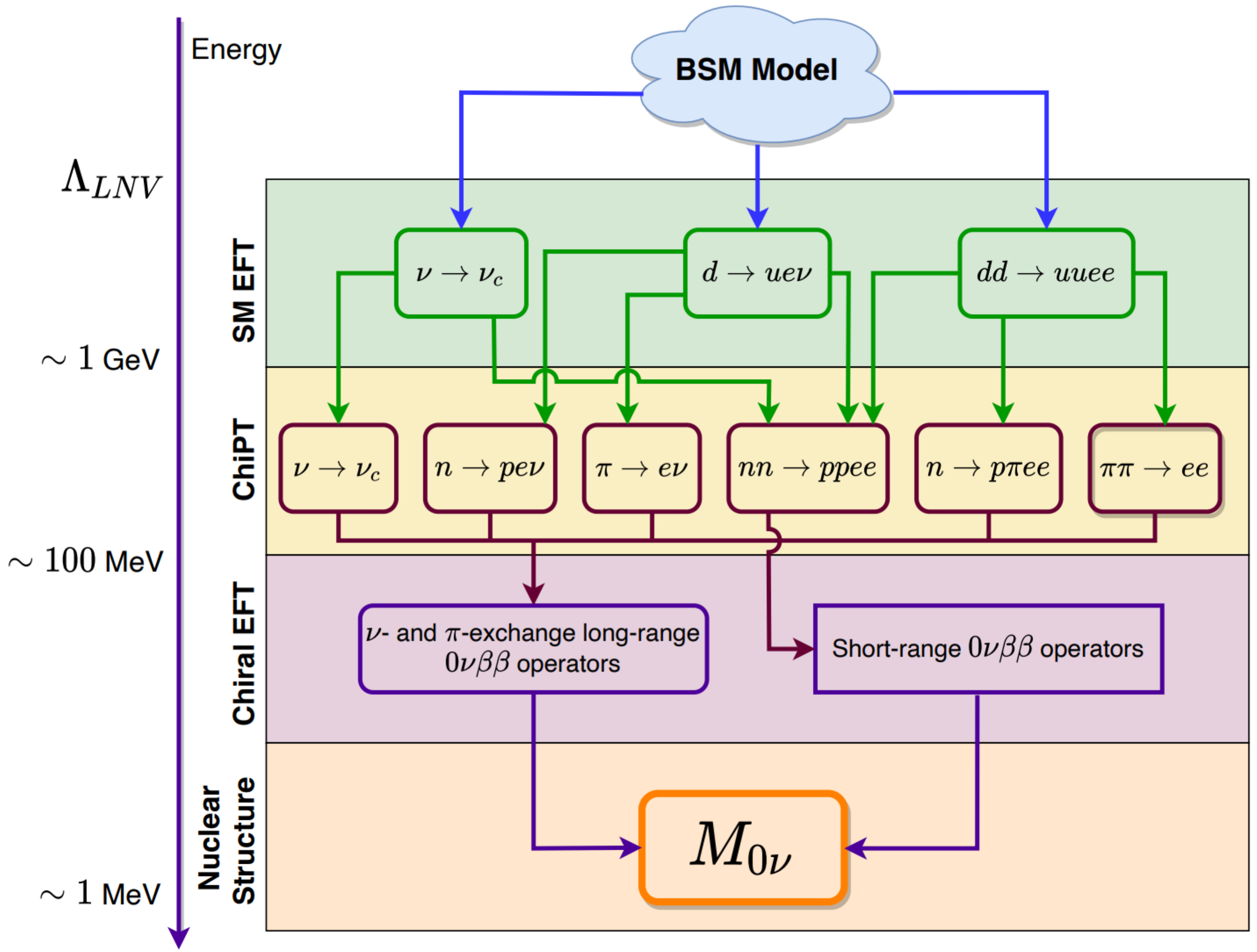


FIG. 1. A "tower of theories" leading to the computation of the nuclear matrix elements $M_{0\nu}$ that control the rate of $0\nu\beta\beta$ decay. At the highest level, above the cutoff Λ_{LNV} for all effective theories, is the ultimate BSM LNV. It manifests itself at the quark and gluon level through Standard-Model EFT, at the nucleon and pion level through chiral perturbation theory (ChiPT), at the nucleon-only level (i.e., with pions no longer part of the Hilbert space, but instead accounted for in multi-nucleon operators) through chiral EFT, and at the nuclear level through the techniques of nuclear-structure theory. Figure adapted from Ref. [35].

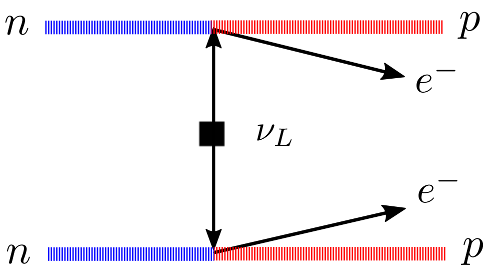
$0\nu\beta\beta$ Decay

Infinite Volume

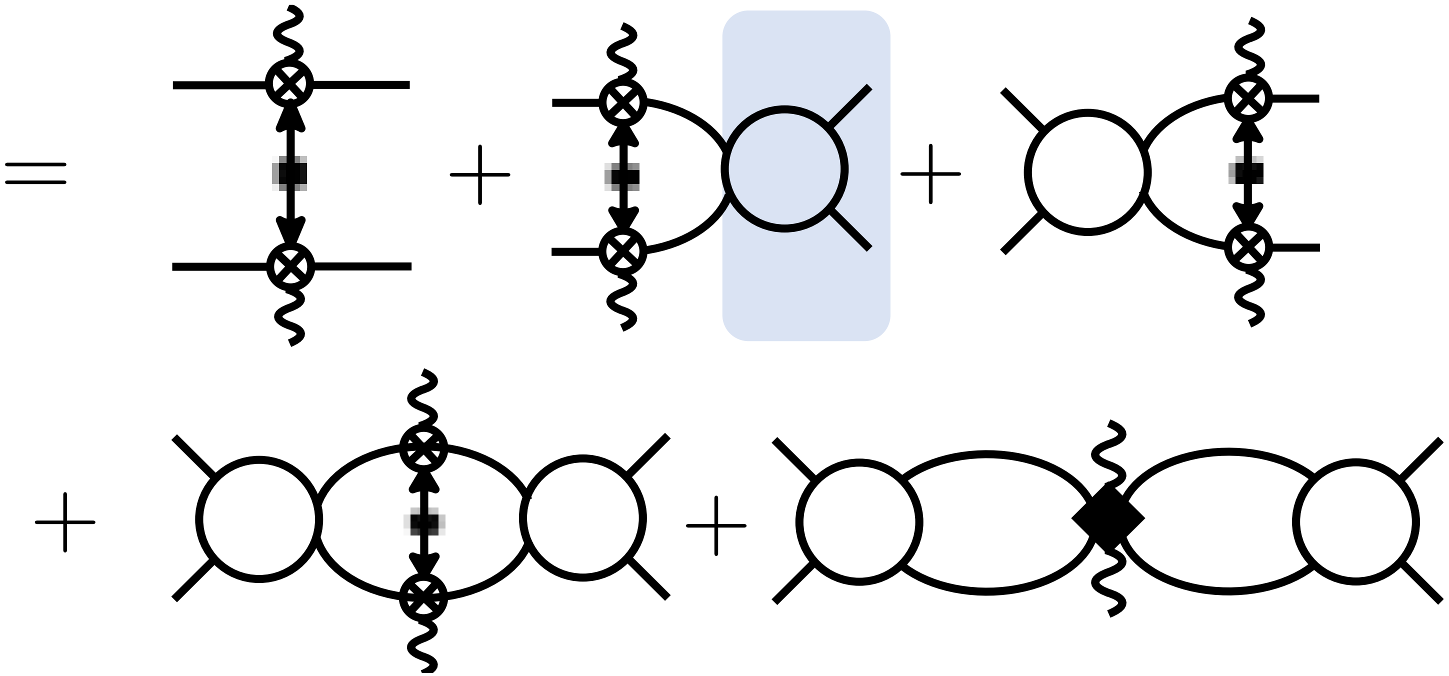
At LO in Pionless EFT

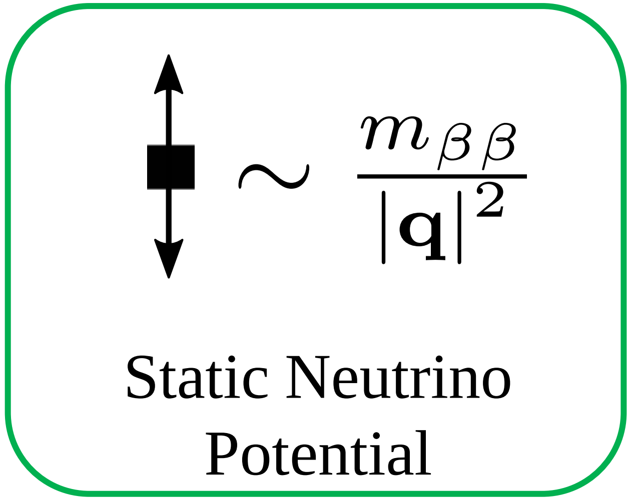
Cirigliano, Dekens, de Vries, Graesser, Mereghetti, Pastore, Piarulli, Van Kolck and Wiringa Phys. Rev. C 100, 055504 (2019)

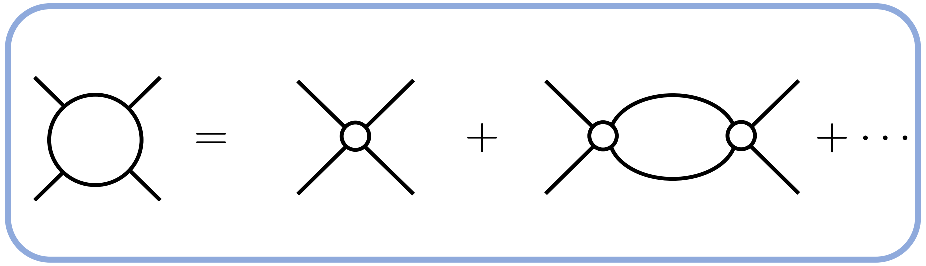
Cirigliano, Dekens, de Vries, Graesses, Mereghetti, Pastore and van Klock Phys. Rev. Lett. 120, 202001

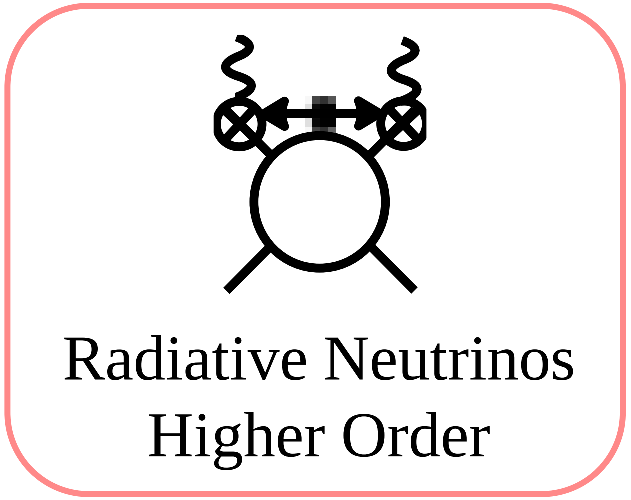


$i\mathcal{M}_{0\nu} =$









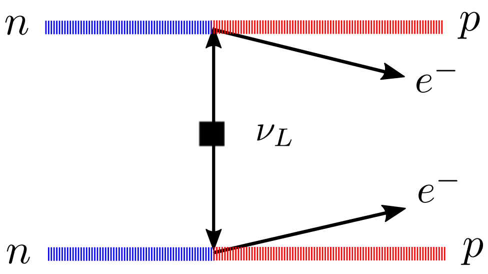
$0\nu\beta\beta$ Decay

Infinite Volume

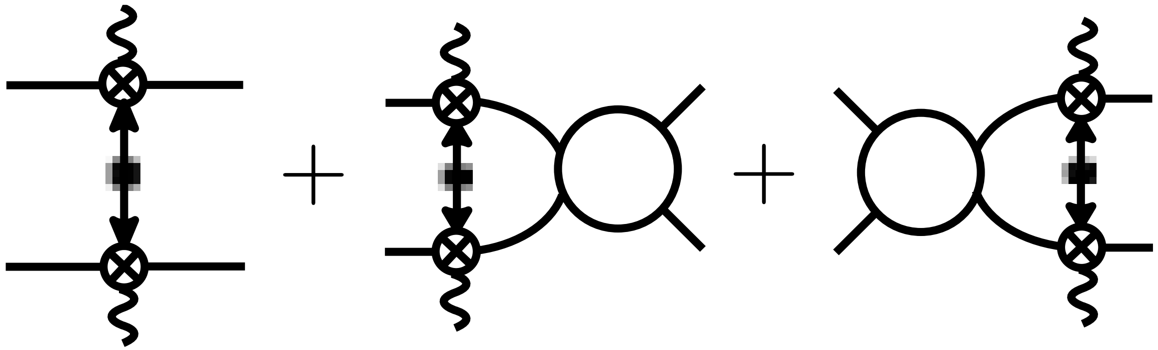
At LO in Pionless EFT

Cirigliano, Dekens, de Vries, Graesser, Mereghetti, Pastore, Piarulli, Van Kolck and Wiringa Phys. Rev. C 100, 055504 (2019)

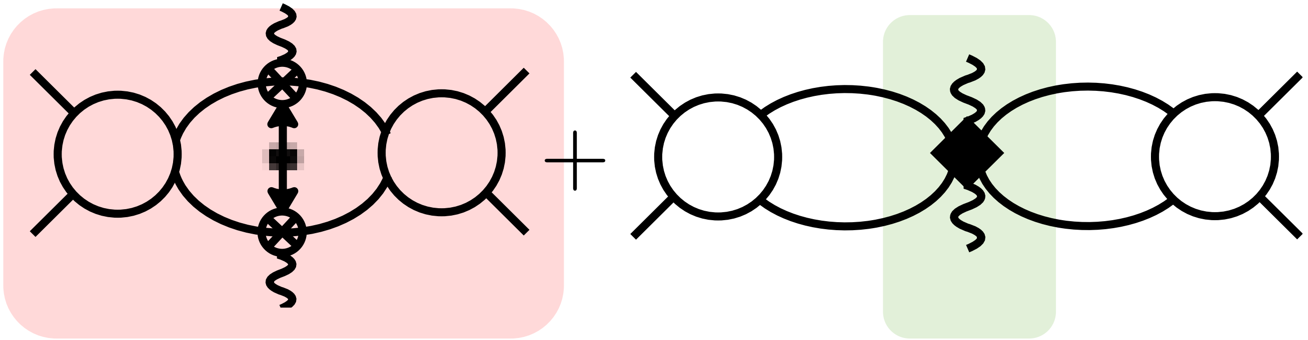
Cirigliano, Dekens, de Vries, Graesses, Mereghetti, Pastore and van Klock Phys. Rev. Lett. 120, 202001



$$i\mathcal{M}_{0\nu} =$$

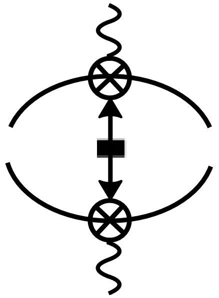


Divergent $+$

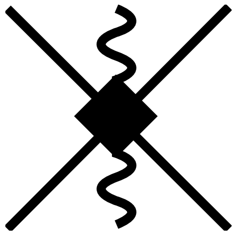


Scale dependent LEC

$$I^{\infty}_{0\nu}(E_i, E_f) \sim$$



New LEC



$$\sim g_\nu^{NN}$$

$$\mu \frac{d}{d\mu}\left(\frac{g_\nu^{NN}}{C_0^2}\right) = \frac{1+3g_A^2}{2}\,\frac{M_N^2}{16\pi^2}$$

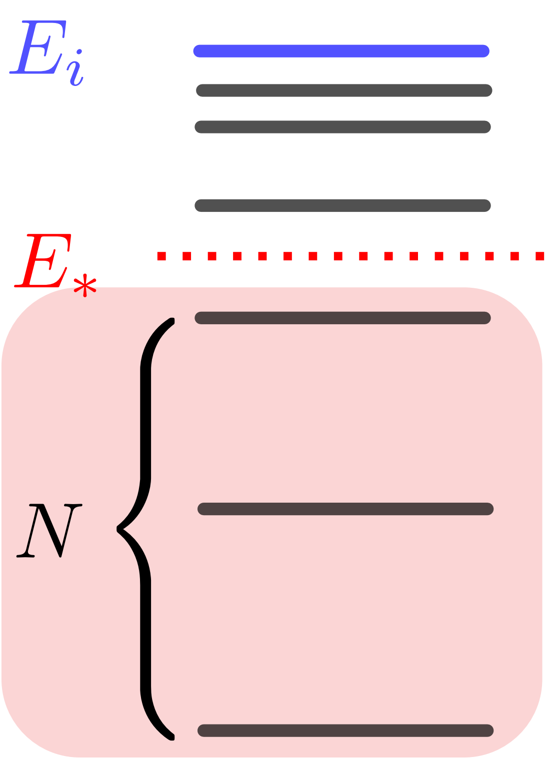
Removing Divergences from $G_L(\tau)$

Davoudi and Kadam
Phys. Rev. Lett. 126, 152003 (2021)

Where is the divergence coming from?

Two particle FV states which can go on-shell

$$|\mathbf{P}_{*m}| + E_{*m} < E_*$$



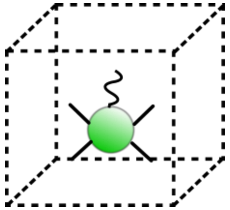
Finite Volume Two Particle Spectrum

Constructing divergent contributions

Spectral representation by integrating over time

$$\mathcal{T}_L^{(\mathrm{M})} = i \sum_{m=0}^{\infty} \frac{c_m}{E_* - E_{*m} - |\mathbf{P}_{*m}| + i\epsilon}$$

Two-body spectrum to identify N low-lying states

$c_m \sim$  Finite volume matrix elements of single hadronic current between the initial (final) and intermediate states

$$G_L^{<}(\tau) \equiv \sum_{m=0}^{N-1} c_m\, \theta(\tau)\, e^{-(|\mathbf{P}_{*m}| + E_{*m} - E_f)|\tau|}$$

Can be analytically continued to Minkowski space

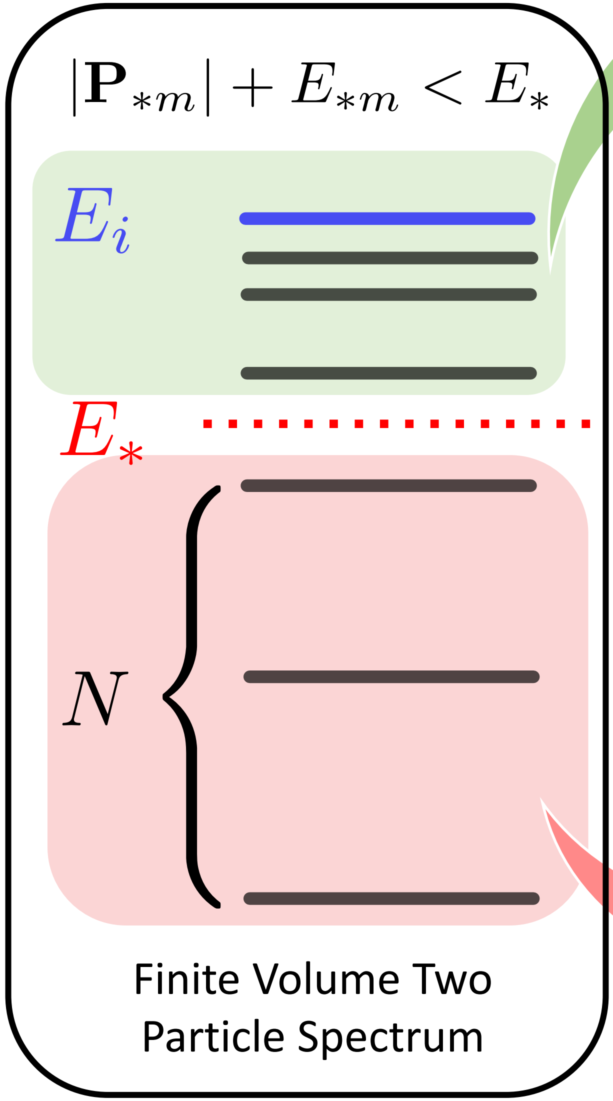
$G_L(\tau)$

Four-point Function From LQCD

$$\mathcal{T}_L^{(\mathrm{E})\geq} \equiv \int d\tau\, e^{E\tau} \left[G_L(\tau) - G_L^{<}(\tau)\right]$$

What do we want?

$$\mathcal{T}_L^{(\mathrm{M})} = i\sum_{m=0}^{\infty}\frac{c_m}{E_*-E_{*m}-|\mathbf{P}_{*m}|+i\epsilon}$$



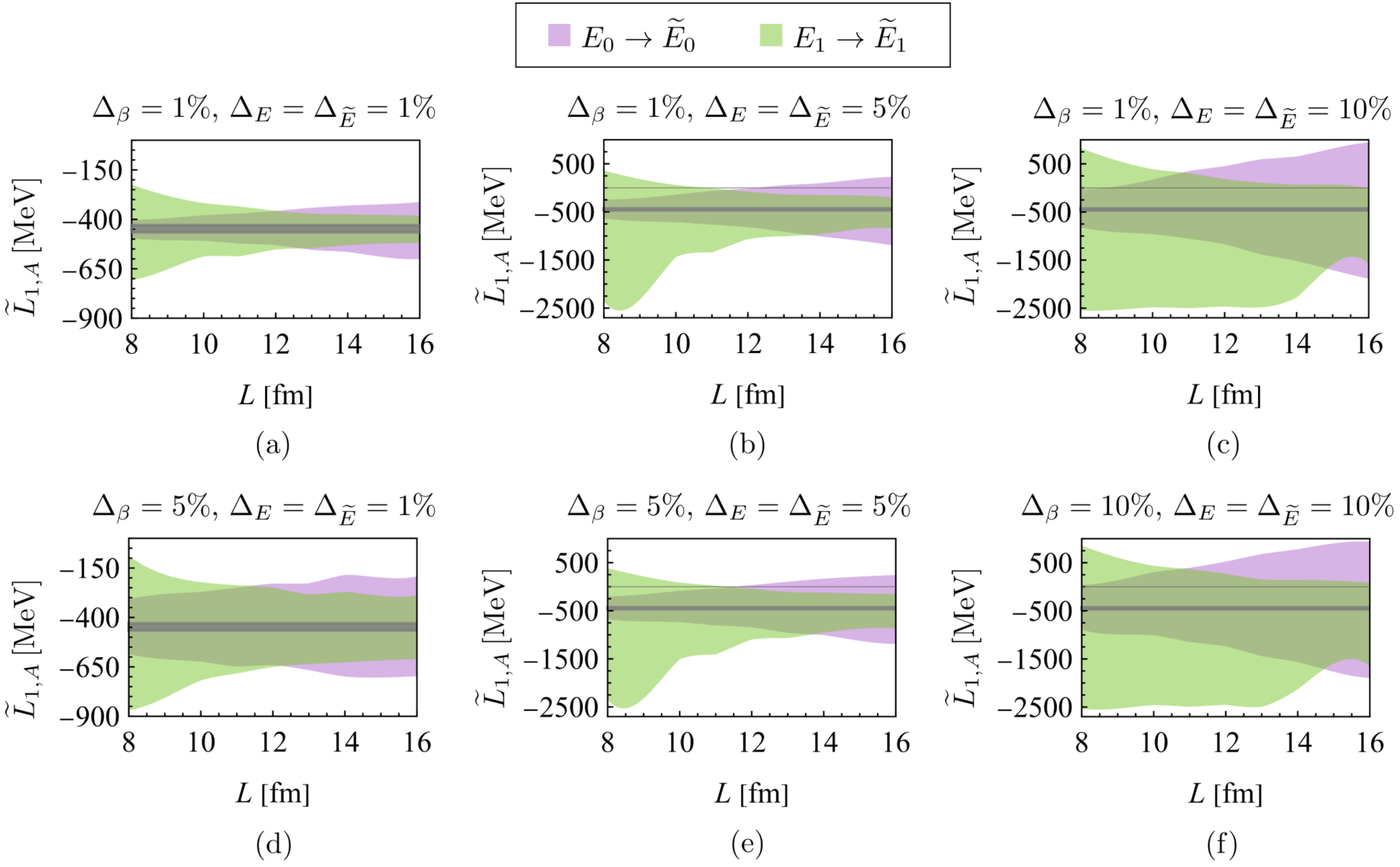
So far, we have

$$\mathcal{T}_L^{(\mathrm{E})\geq} \equiv \int d\tau\, e^{E\tau}\left[G_L(\tau)-G_L^{<}(\tau)\right]$$

Missing Piece

$$\mathcal{T}_L^{(\mathrm{E})<} \equiv \sum_{m=0}^{N-1}\frac{c_m}{E_*-E_{*m}-|\mathbf{P}_{*m}|}$$

$$\mathcal{T}_L^{(\mathrm{M})} = i\mathcal{T}_L^{(\mathrm{E})<} + i\mathcal{T}_L^{(\mathrm{E})\geq}$$



$E_0 \to \widetilde{E}_0$
$E_1 \to \widetilde{E}_1$
$\Delta_\beta = 1\%,\ \Delta_E = \Delta_{\widetilde{E}} = 1\%$
$\Delta_\beta = 1\%,\ \Delta_E = \Delta_{\widetilde{E}} = 5\%$
$\Delta_\beta = 1\%,\ \Delta_E = \Delta_{\widetilde{E}} = 10\%$
$\Delta_\beta = 5\%,\ \Delta_E = \Delta_{\widetilde{E}} = 1\%$
$\Delta_\beta = 5\%,\ \Delta_E = \Delta_{\widetilde{E}} = 5\%$
$\Delta_\beta = 10\%,\ \Delta_E = \Delta_{\widetilde{E}} = 10\%$
$\widetilde{L}_{1,A}$ [MeV]
L [fm]
(a)
(b)
(c)
(d)
(e)
(f)

TABLE I. Lower bounds achievable for $m_{\beta\beta}$ by some $0\nu\beta\beta$ experiments, depending on their reached sensitivities (upper group) or sensitivity goals (lower group). The different results correspond to the different quenching of g_A, according to the definitions in Eq. (9). The 1σ uncertainties on $m_{\beta\beta}$ are calculated by assuming uncertainties both on the matrix elements and phase space factors, according to [1] and [8], respectively.

Experiment	Isotope	$t^{1/2}$(90% C.L.)(10^{25} yr)	Lower bound for $m_{\beta\beta}$(eV)		
			g_{nucleon}	g_{quark}	$g_{\mathrm{phen.}}$
IGEX [9]	^{76}Ge	1.57	0.31 ± 0.03	0.49 ± 0.05	1.44 ± 0.16
HEIDELBERG-MOSCOW [10]	^{76}Ge	1.9	0.28 ± 0.03	0.44 ± 0.05	1.31 ± 0.14
GERDA-I [11]	^{76}Ge	2.1	0.26 ± 0.03	0.42 ± 0.05	1.25 ± 0.14
KamLAND-Zen-I [12]	^{136}Xe	1.9	0.18 ± 0.02	0.29 ± 0.03	1.06 ± 0.12
KamLAND-Zen-II [13]	^{136}Xe	1.3	0.22 ± 0.02	0.35 ± 0.04	1.28 ± 0.14
EXO-200 [14]	^{136}Xe	1.1	0.24 ± 0.03	0.38 ± 0.04	1.39 ± 0.15
Combined Ge [11]	^{76}Ge	3.0	0.22 ± 0.02	0.35 ± 0.04	1.05 ± 0.11
Combined Xe	^{136}Xe	2.6	0.15 ± 0.02	0.25 ± 0.03	0.91 ± 0.10
Combined Ge + Xe	^{76}Ge/^{136}Xe		0.15 ± 0.01	0.24 ± 0.02	0.81 ± 0.07
CUORE [15]	^{130}Te	9.5	0.07 ± 0.01	0.11 ± 0.01	0.39 ± 0.04
GERDA-II [16]	^{76}Ge	15	0.10 ± 0.01	0.16 ± 0.02	0.47 ± 0.05
SuperNEMO [17]	^{82}Se	10	0.07 ± 0.01	0.12 ± 0.01	0.36 ± 0.04

TABLE II. Sensitivity and exposure necessary to discriminate between $\mathcal{NH}$ and $\mathcal{IH}$: the goal is $m_{\beta\beta} = 8$ meV. The two cases refer to the unquenched value of $g_A = g_{\mathrm{nucleon}}$ (mega) and $g_A = g_{\mathrm{phen.}}$ (ultimate). The calculations are performed assuming *zero background* experiments with 100% detection efficiency and no fiducial volume cuts. The last column shows the maximum value of the product $B \cdot \Delta$ in order to actually comply with the zero background condition.

Experiment	Isotope	$t^{1/2}$(yr)	Exposure (estimate) $M \cdot T$ (ton · yr)	$B \cdot \Delta_{(\mathrm{zero\ bkg})}$(counts/kg/yr)
Mega Te	^{130}Te	6.8×10^{27}	2.1	4.7×10^{-4}
Mega Ge	^{76}Ge	2.3×10^{28}	4.1	2.4×10^{-4}
Mega Xe	^{136}Xe	9.7×10^{27}	3.2	3.2×10^{-4}
Ultimate Te	^{130}Te	2.3×10^{29}	71	1.4×10^{-5}
Ultimate Ge	^{76}Ge	5.1×10^{29}	93	1.1×10^{-5}
Ultimate Xe	^{136}Xe	3.3×10^{29}	109	9.2×10^{-6}

Table 1. Limits on neutrinoless DBDs $T_{1/2}^{0\nu-\text{exp}}$ (claim for evidence is denoted in [42]). $Q_{\beta\beta}$: Q-value for the $0^+ \to 0^+$ ground-state transition. $G^{0\nu}$: kinematical (phase space volume) factor ($g_A = 1.25$ and $R = 1.2$ fm $A^{1/3}$). $\langle m_\nu \rangle$: the upper limit on the effective Majorana neutrino mass, deduced from $T_{1/2}^{0\nu-\text{exp}}$ by assuming the ISM [236] ($g_A^{\text{eff}} = 1.25$, UCOM src), the EDF [131] ($g_A^{\text{eff}} = 1.25$, UCOM src), the (R)QRPA ($1.00 \leqslant g_A^{\text{eff}} \leqslant 1.25$, the modern self-consistent treatment of src), and the IBM-2 [130] ($1.00 \leqslant g_A^{\text{eff}} \leqslant 1.25$, Miller–Spencer src), NMEs (see section 10). src means short-range correlations.

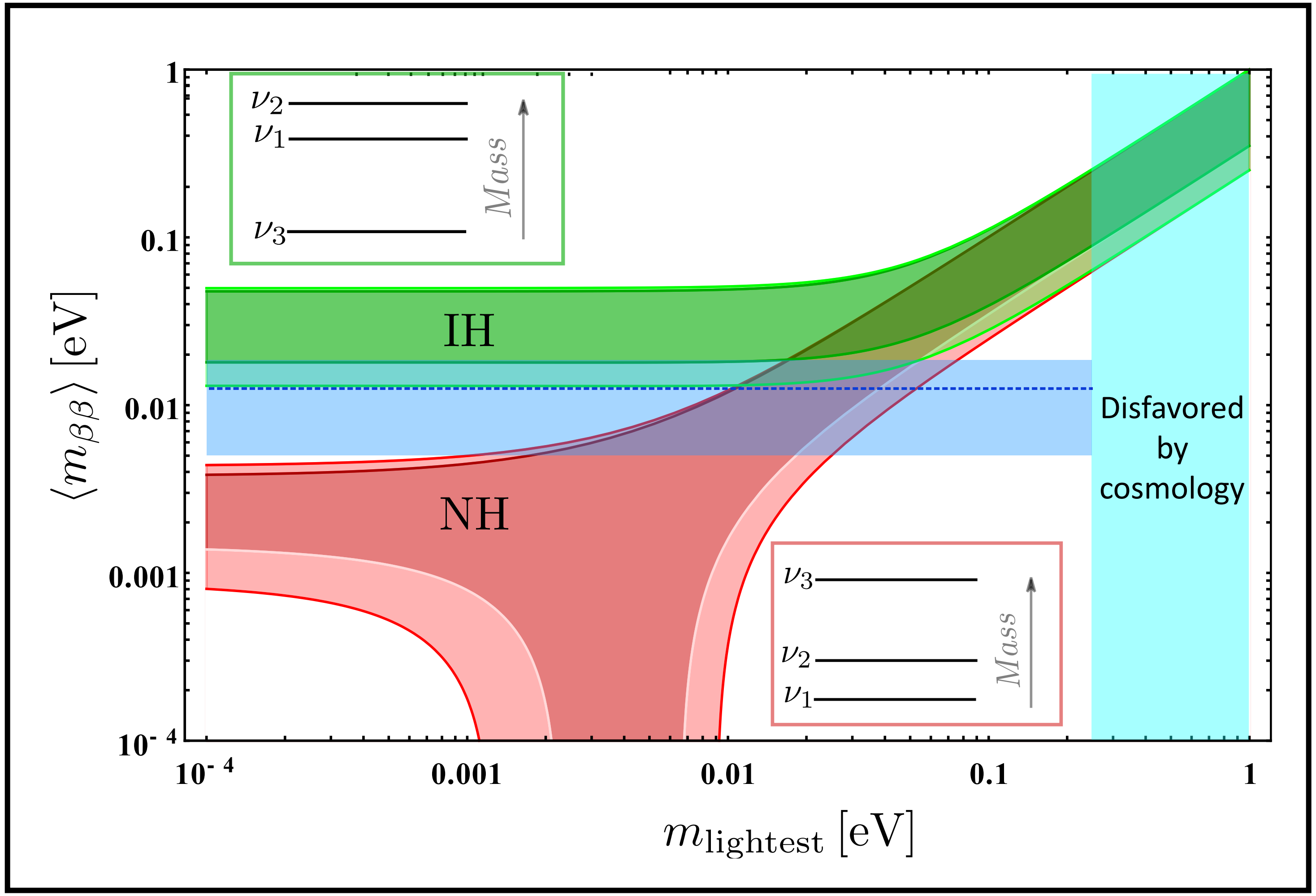
Isotope	A (%)	$Q_{\beta\beta}$ (MeV)	$G^{0\nu}$ (10^{-14} y)	$T_{1/2}^{0\nu-\text{exp}}$ (10^{24} y)	NME	$\lvert\langle m_\nu \rangle\rvert$ eV (eV)	Future experiments
^{48}Ca	0.19	4.276	7.15	0.014 [237]	ISM	19.1	CANDLES
					EDF	7.0	
^{76}Ge	7.8	2.039	0.71	19 [36, 227, 228]	ISM, EDF	0.51, 0.31	GERDA
					(R)QRPA	(0.20, 0.32)	
					EDF	(0.26, 0.35)	
	7.8	2.039	0.71	22 [42]	ISM, EDF	0.47, 0.29	—
					(R)QRPA	(0.18, 0.30)	
					EDF	(0.24, 0.32)	
	7.8	2.039	0.71	16 [229, 230]	ISM, EDF	0.55, 0.34	MAJORANA
					(R)QRPA	(0.22, 0.35)	
					EDF	(0.28, 0.38)	
^{82}Se	9.2	2.992	3.11	0.36 [38, 234, 235]	ISM, EDF	1.88, 1.17	SuperNEMO
					(R)QRPA	(0.76, 1.28)	MOON
					EDF	(1.12, 1.49)	
^{100}Mo	9.6	3.034	5.03	1.0 [38, 234]	EDF	0.46	MOON
					(R)QRPA	(0.38, 0.73)	AMoRE
					EDF	(0.62, 1.06)	
^{116}Cd	7.5	2.804	5.44	0.17 [238]	EDF	1.15	COBRA
					(R)QRPA	(1.20, 2.16)	$CdWO_4$
^{130}Te	34.5	2.529	4.89	3.0 [231, 232, 239]	ISM, EDF	0.52, 0.27	CUORE
					(R)QRPA	(0.25, 0.43)	
					EDF	(0.33, 0.46)	
^{136}Xe	8.9	2.467	5.13	5.7 [40]	ISM, EDF	0.44, 0.23	EXO, NEXT
					(R)QRPA	(0.17, 0.30)	KamLAND-Zen
^{150}Nd	5.6	3.368	23.2	0.018 [38, 240]	EDF	4.68	SuperNEMO
					(R)QRPA	(2.13, 2.88)	SNO+ DCBA

Including Uncertainties in NMEs for $\langle m_{\beta\beta} \rangle = 0.05$ eV

Avignone, Elliott and Engel
Reviews of Modern Physics, 80 (2008)

Dell'Oro, Marcocci, Viel and Vissani
Advances in High Energy Physics (2016)

Vergados, Ejiri and Simkovic,
Rep. Prog. Phys. 75 106301 (2012)



Need to reduce uncertainties in $0\nu\beta\beta$ decay NMEs !!!

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \mathrm{diag}\{e^{i\alpha_1/2}, e^{i\alpha_2/2}, 1\},$$

TABLE I. Neutrino mixing parameters as summarized by the Particle Data Book [Yao *et al.* (2006)] based on the individual experimental reference reporting. The limit on $\langle m_\beta \rangle$ and Σ are based on the references given. The $\langle m_{\beta\beta} \rangle$ limit comes from the Ge experiments. The parameter values would be slightly different if determined by a global fit to all oscillation data (Fogli *et al.*, 2006).

Parameter	Value	Confidence level	Reference
$\sin^2(2\theta_{12})$	$0.86^{+0.03}_{-0.04}$	68%	Aharmin *et al.* (2005)
$\sin^2(2\theta_{23})$	>0.92	90%	Ashie *et al.* (2005)
$\sin^2(2\theta_{13})$	<0.19	90%	Apollonio *et al.* (1999)
Δm^2_{21}	$8.0^{+0.4}_{-0.3} \times 10^{-5}$ eV^2	68%	Aharmin *et al.* (2005)
$\lvert\Delta m^2_{32}\rvert$	$2.4^{+0.6}_{-0.5} \times 10^{-3}$ eV^2	90%	Ashie *et al.* (2004)
$\langle m_\beta \rangle$	<2 eV	95%	Lobashev *et al.* (1999); Kraus *et al.* (2005)
$\langle m_{\beta\beta} \rangle$	<0.7 eV[a]	90%	Klapdor-Kleingrothaus *et al.* (2001a); Aalseth *et al.* (2002a)
Σ	<2 eV	95%	Elgaroy and Lahov (2003)

[a]Using the matrix element of Rodin *et al.* (2006).

Table 2. The $2\nu\beta\beta$ matrix elements $|M^{2\nu}|$ deduced from the measured half-life $T^{2\nu}_{1/2}$ [40, 241]. $g_A = 1.269$ is assumed.

Nucleus	$T^{2\nu}_{1/2}$ years	$\vert M^{2\nu}\vert$ (MeV)$^{-1}$
^{48}Ca	$4.4^{+0.6}_{-0.5} \times 10^{19}$	$0.046^{+0.003}_{-0.003}$
^{76}Ge	$1.5^{+0.1}_{-0.1} \times 10^{21}$	$0.137^{+0.005}_{-0.004}$
^{82}Se	$9.2^{+0.7}_{-0.7} \times 10^{19}$	$0.095^{+0.004}_{-0.003}$
^{96}Zr	$2.3^{+0.2}_{-0.2} \times 10^{19}$	$0.091^{+0.004}_{-0.004}$
^{100}Mo	$7.1^{+0.4}_{-0.4} \times 10^{18}$	$0.234^{+0.007}_{-0.006}$
^{100}Mo*	$5.9^{+0.8}_{-0.6} \times 10^{20}$	$0.189^{+0.010}_{-0.012}$
^{116}Cd	$2.8^{+0.2}_{-0.2} \times 10^{19}$	$0.128^{+0.005}_{-0.004}$
^{128}Te	$1.9^{+0.4}_{-0.4} \times 10^{24}$	$0.047^{+0.007}_{-0.003}$
^{130}Te	$6.8^{+1.2}_{-1.1} \times 10^{20}$	$0.034^{+0.003}_{-0.003}$
^{136}Xe	$2.38^{+0.14}_{-0.14} \times 10^{21}$	$0.018^{+0.003}_{-0.001}$
^{150}Nd	$8.2^{+0.9}_{-0.9} \times 10^{18}$	$0.061^{+0.004}_{-0.003}$
^{150}Nd*	$1.33^{+0.45}_{-0.26} \times 10^{20}$	$0.045^{+0.005}_{-0.006}$

Identifying FV Effects

$$C_L(E) = \int_L d^3x \int dx_0\, e^{iEx_0} \left[\langle 0|\, T[B(x)B^\dagger(0)]\, |0\rangle\right]_L$$

- Feynman diagrammatic expansion in given EFT
- Calculate to all orders in perturbation theory

- Replace the loop integrals with sums

$$\int \frac{d^4k}{(2\pi)^4} \to \frac{1}{L^3}\sum_{\vec{k}} \int \frac{dk^0}{2\pi}$$

Perform k^0 integrals → quantity dependent on sums over quantized $\vec{k}$

Poisson Summation Formula

- $f(\vec{k})$ smooth in $\vec{k}$

$$\sum_{\vec{k}} f(\vec{k}) = \int_{\vec{k}} f(\vec{k}) + \mathcal{O}(e^{-m_\pi L})$$

Ignored by taking sufficiently large L

- Power-law FV corrections from singularities in $\vec{k}$

$$\sum_{\vec{k}} \frac{f(\vec{k})}{k^2-p^2} = \int_{\vec{k}} \frac{f(\vec{k})}{k^2-p^2+i\epsilon} + F(p)$$

Captures FV effects