A Feynman-Hellman approach to four quark matrix elements

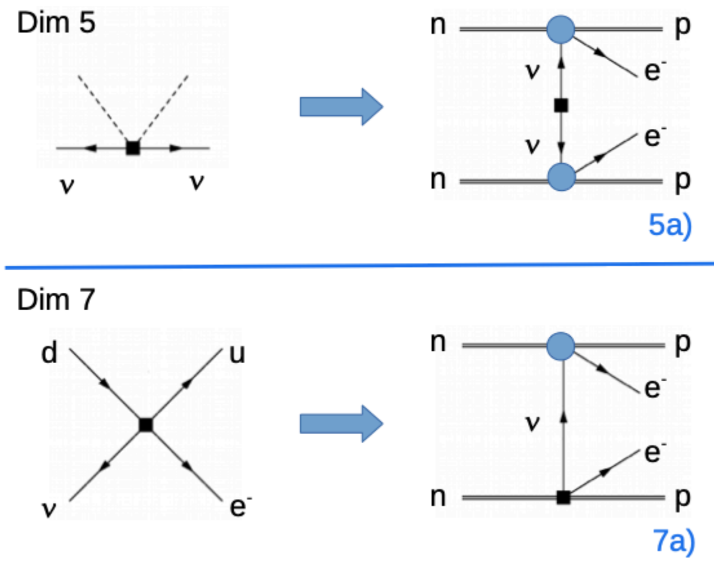
Lattice methods applied to QCD contributions to neutrinoless double beta decay

Charles Farley Kacir; July 3, 2026

Neutrinoless double beta decay (0νββ)

Why do we care about it?

- Requires lepton-number violation (LNV)
- Low energy discriminant between theories beyond the Standard Model (BSM)

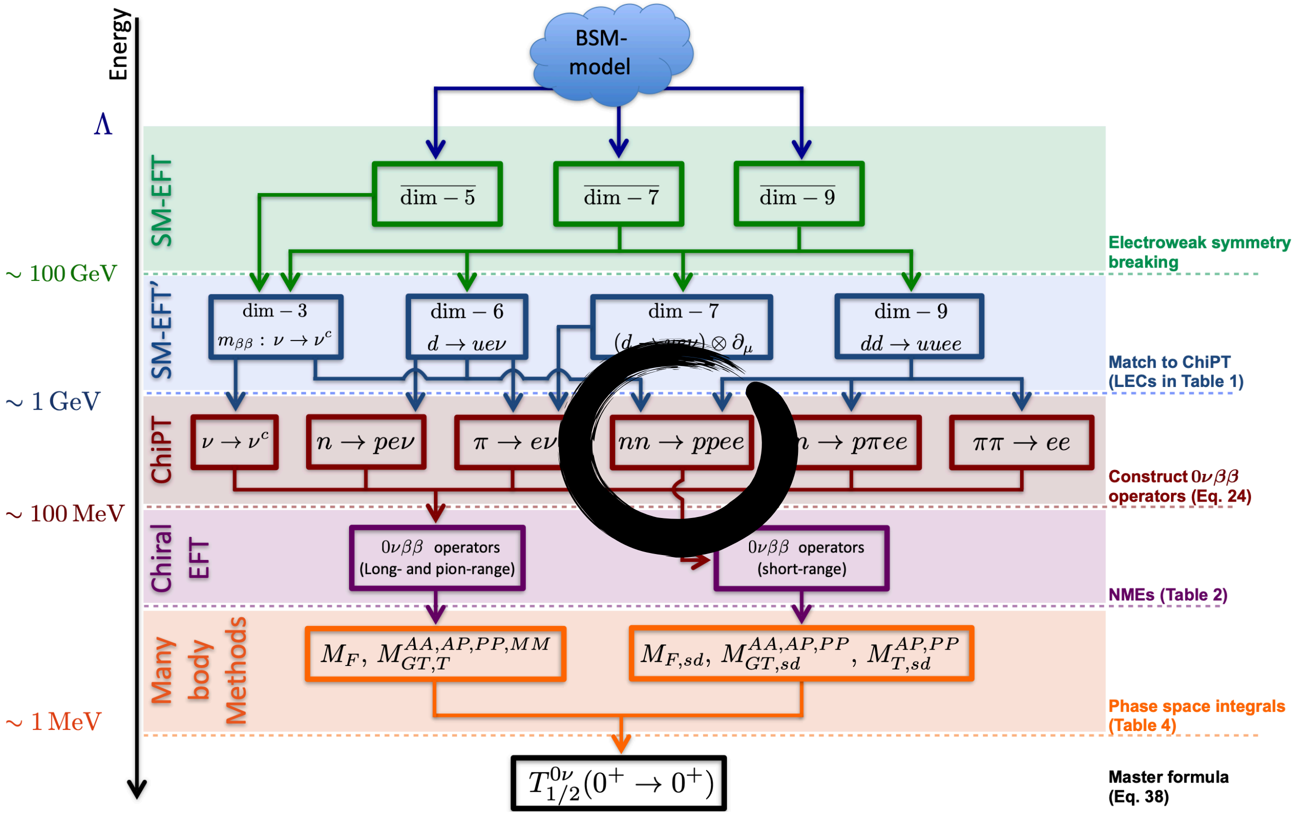


V. Cirigliano et al. (submitted to Snowmass 2021)

Neutrinoless double beta decay (0vββ)

How are calculations done?

- Effective field theories generically extend the Standard Model
- Effective theories combine individual contributions from QCD, QED, BSM model, etc
- Note the short-range operators



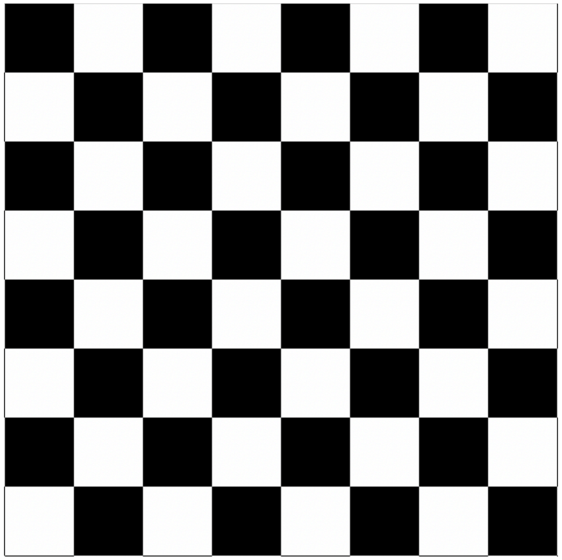
V. Cirigliano, W. Dekens, J. de Vries, M.L. Graesser, and E. Mereghetti (https://doi.org/10.1007/JHEP12(2018)097)

Field theory on the lattice

The lattice as a regulator

- Put fields in a finite lattice with periodic boundary conditions (PBC)
- Increase lattice size and look for convergence in continuum limit
- Integrals become sums over all lattice points
- Wick rotate into Euclidean time
  - Enables stochastic evaluation of path integral

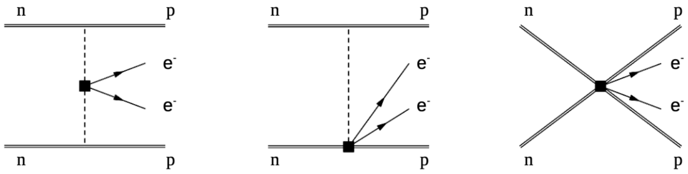
$$\int_E \mathcal{D}^n\phi \; e^{-S}$$



Four quark matrix elements for 0vββ

Heavy physics contact interaction

- At QCD level, these are four quark operators
- Start by testing those tools against previously calculated π– → π+



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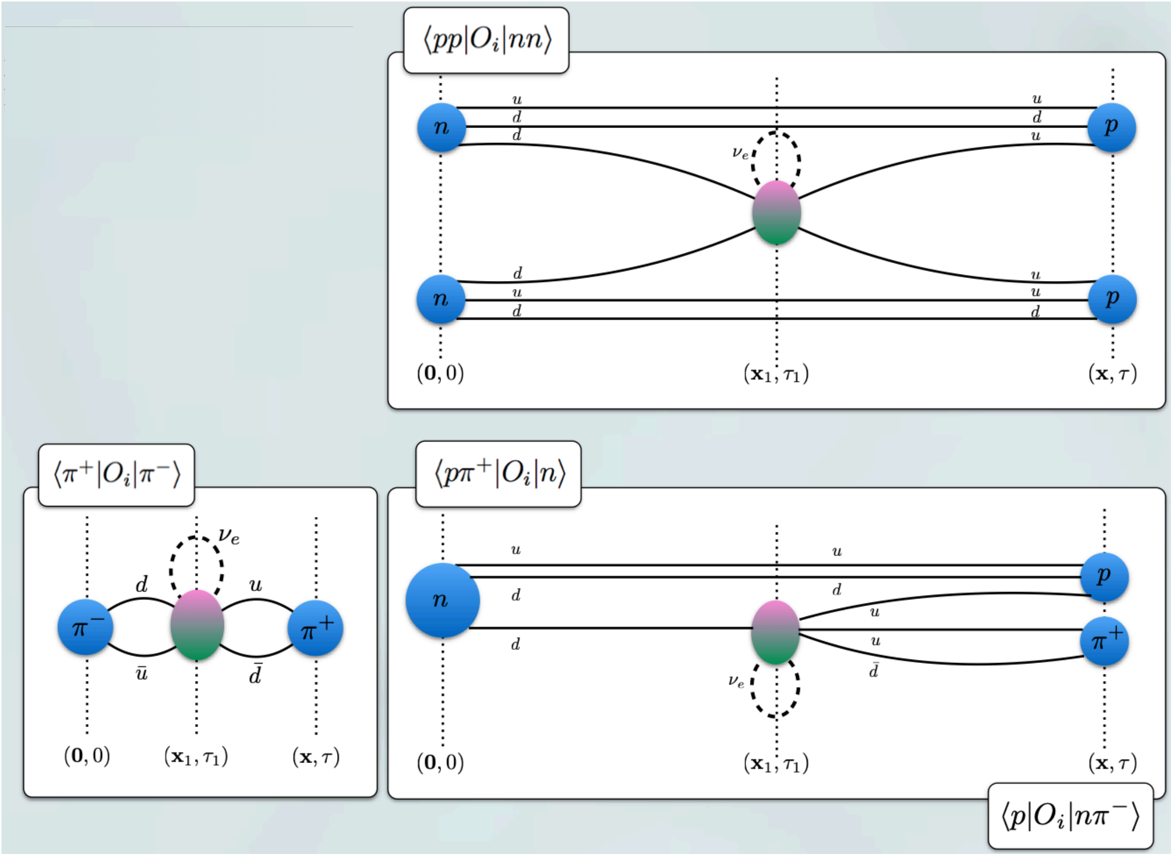


Z. Davoudi (Presentation, July 2019)

Four quark matrix elements for 0vββ

What do current insertions usually look like on the lattice?

- Invert Dirac operator (D below) for propagator (P below)
- In practice, inversion is too expensive, so we solve for one-to-all propagators
- Tie propagators together with weak currents into 3 point function
- Repeat for every gluon configuration (:

$$S_F = -\bar{\psi} D \psi \qquad\qquad DP = \mathbb{I}$$

$$D_f(n|m)_{\substack{\alpha\beta \\ ab}} = \left(m_f + \frac{4}{a}\right) \delta_{\alpha\beta}\delta_{ab}\delta_{nm} - \frac{1}{2a} \sum_{\mu=\pm 1}^{\pm 4} (\mathbb{I} - \gamma_\mu)_{\alpha\beta} U_\mu(n)_{ab} \delta_{\{n+\hat{\mu}\}m}$$

$$F^i(z,y,x)_{\substack{\beta\alpha \\ ba}} = P^u(z,y)_{\substack{\beta\beta' \\ bc}} \Gamma^i_{\beta'\alpha'} P^d(y,x)_{\substack{\alpha'\alpha \\ ca}}$$

Four quark matrix elements for 0νββ

Proposal: FH method extended to two current insertions

- Feynman-Hellmann (FH) methods can boost the signal of the current matrix element.
- FH propagators (from partial derivative of C with respect to λ) include sum over all intermediate points, so they allow analysis of current insertion with two point functions instead of three point functions if one can find them

$$\frac{\partial E_n}{\partial \lambda} = \left\langle n \left| \frac{\partial H_\lambda}{\partial \lambda} \right| n \right\rangle \rightarrow \left.\frac{\partial m_{eff}(\lambda)}{\partial \lambda}\right|_{\lambda=0} = \frac{\langle 0|J|0\rangle}{2E_0}$$

$$\left.\frac{\partial m_\lambda^{eff}(t,\tau)}{\partial \lambda}\right|_{\lambda=0} = \frac{\partial_\lambda C_\lambda(t+\tau)}{C(t+\tau)\tau} - \frac{\partial_\lambda C_\lambda(t)}{C(t)\tau}$$

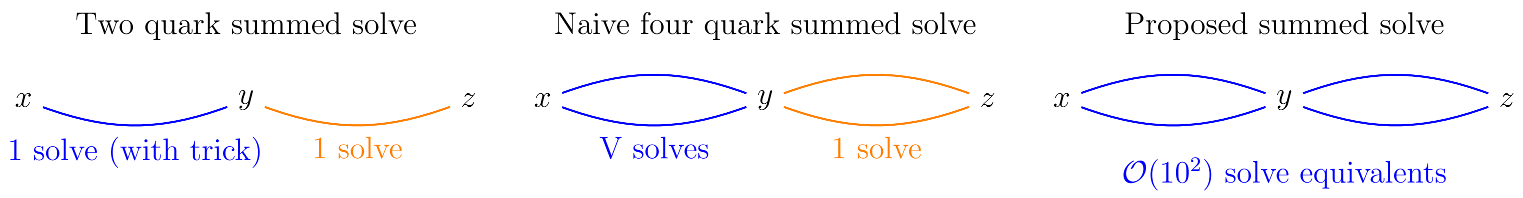
$$\partial_\lambda C_\lambda(t) = \int dt' \left\langle T \left(\underset{\substack{\uparrow \\ \text{Quark}}}{\mathcal{O}(t)} \int d^3x \; \underset{\substack{\uparrow \\ \text{Current}}}{j(t',\vec{x})} \underset{\substack{\uparrow \\ \text{Quark}}}{\mathcal{O}(0)} \right) \right\rangle$$

Four quark matrix elements for 0νββ

Proposal: summed sequential block solver

- We need the summed sequential propagator below, but it is very expensive to build with standard one-to-all propagators. We want to build a custom solver which performs a partial inversion of a combination of two Dirac operators at once. Adapting existing tools to do that turned out to be nontrivial.

- Two quark FH propagators have been done, we need to develop entirely new machinery for the four quark operator

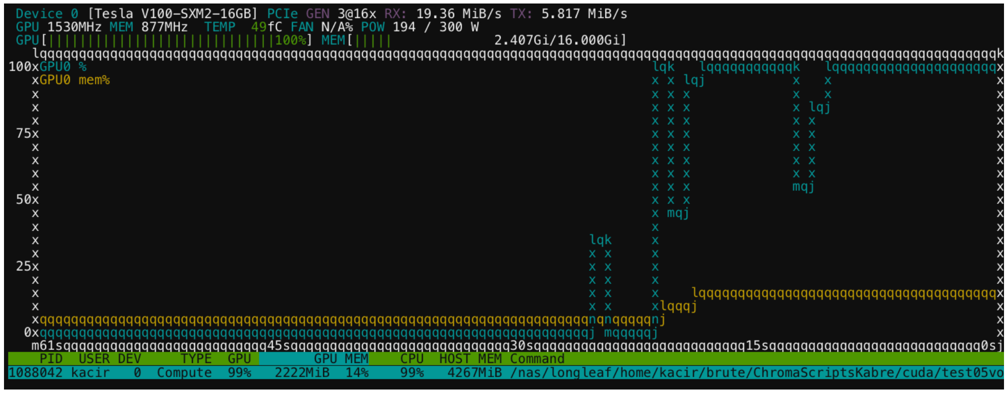


$$F^i(z,y,x)_{\substack{\beta\alpha\\ba}} = P^u(z,y)_{\substack{\beta\beta'\\bc}}\Gamma^i_{\beta'\alpha'}P^d(y,x)_{\substack{\alpha'\alpha\\ca}} \qquad B_\Sigma(z,x_0)_{\substack{\beta\alpha\\\nu\mu\\ba\\dc}} = \sum_{y'} F^2(z,y',x_0)_{\substack{\nu\mu\\dc}}F^1(z,y',x_0)_{\substack{\beta\alpha\\ba}}$$

Four quark matrix elements for 0νββ

Next steps

- Naive sum on small lattice
  - Test out analysis pipeline (index contraction, fitting, etc.)
  - Make sure that signal is actually boosted
- Code an efficient double current insertion solver which exploits short range nature and linearity of Dirac operator



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 GPU[||||||||||||||||||||||||||||100%] MEM[|||||              2.407Gi/16.000Gi]
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