Gravitational waves from spinning neutron stars:

Observational results and modelling

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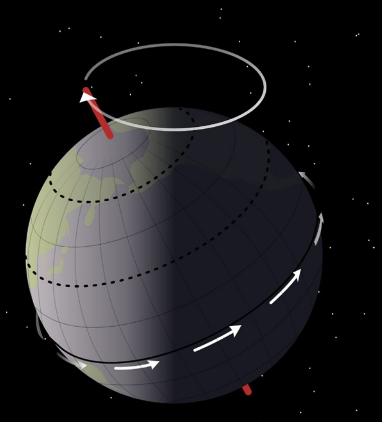
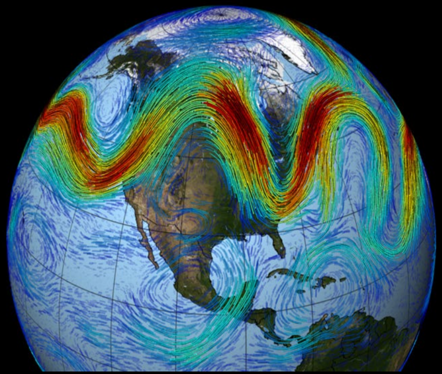
Three gravitational wave emission mechanisms

"Mountains" – non-axisymmetric deformation



"Wobble" - free precession
Most general motion of rigid body



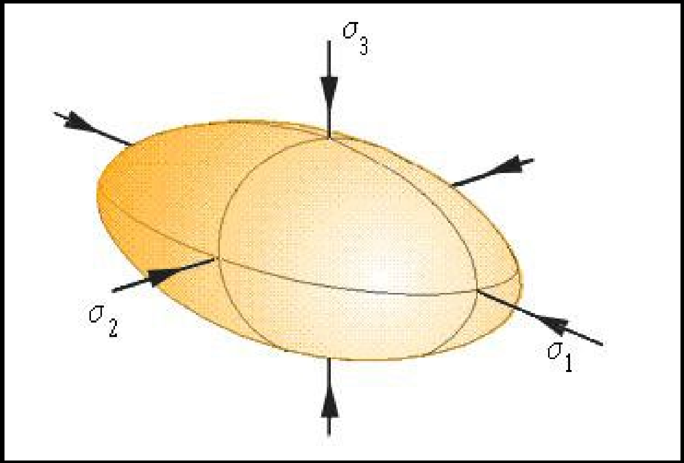
Fluid oscillations – many possible sorts.

Geophysics is a useful guide

Phenomenon	Earth	Neutron Star
Deformed shape	✓	Not yet observed
Quakes	✓ Earthquakes	✓(but more complicated)
Oscillations	✓ Mainly elasto-gravitational	✓ (can be elasto-magneto-gravitational)
Rossby waves	✓	Not yet observed
Free precession	✓ ("Chandler wobble")	May have been observed

Gravitational waves from mountains

A *triaxial* neutron star, rotating steadily, emits gravitational waves:



$$h = 3 \times 10^{-28} \left(\frac{\epsilon}{10^{-6}}\right) \left(\frac{f_{\rm spin}}{10\,{\rm Hz}}\right)^2 \left(\frac{1\,{\rm kpc}}{r}\right)$$

$$\epsilon = \frac{I_{yy} - I_{xx}}{I_{zz}}$$

Dimensionless asymmetry in moment of inertia tensor

Spin frequency

Distance to source

Gravitational wave searches

Data taking started 2002. First publications 2004. First LSC, then LVC, now LVK.

Approx 238 published papers, approx. 90 detections, all CBC. Mainly BBH.

~50 publications on continuous GWs, all upper limits.

Gravitational wave searches

Three main types of continuous wave search:

1. Targeted searches
2. Directed searches
3. All-sky searches

In all cases, can get upper bounds on GW amplitude from energy balance.

For mountains:

$$\frac{d}{dt}\left(\frac{1}{2}I\Omega^2\right) = -\frac{32}{5}\Omega^6(I\epsilon)^2$$

If distance known, can translate into upper bound on GW amplitude.

Targeted searches

Look for GWs from known pulsars with *known timing solutions*.

Can use measured spin period and period derivate to give spindown upper limit

$$\epsilon_{\rm spindown} = \left[\frac{5\dot{P}P^3}{32(2\pi)^4 I_{zz}}\right]^{1/2}$$

e.g. for Crab pulsar, $\varepsilon_{\rm spindown} \approx 7.6 \times 10^{-4}$

GW upper limits: spin-down and direct

Spindown upper limit beaten for 23 pulsars.

Example: for Crab, $\varepsilon < 6.5 \times 10^{-6}$.

Implies no more than 0.009% of spin-down energy is going into the GW channel.

Is this interesting?

What does theory have to say about ellipticity?

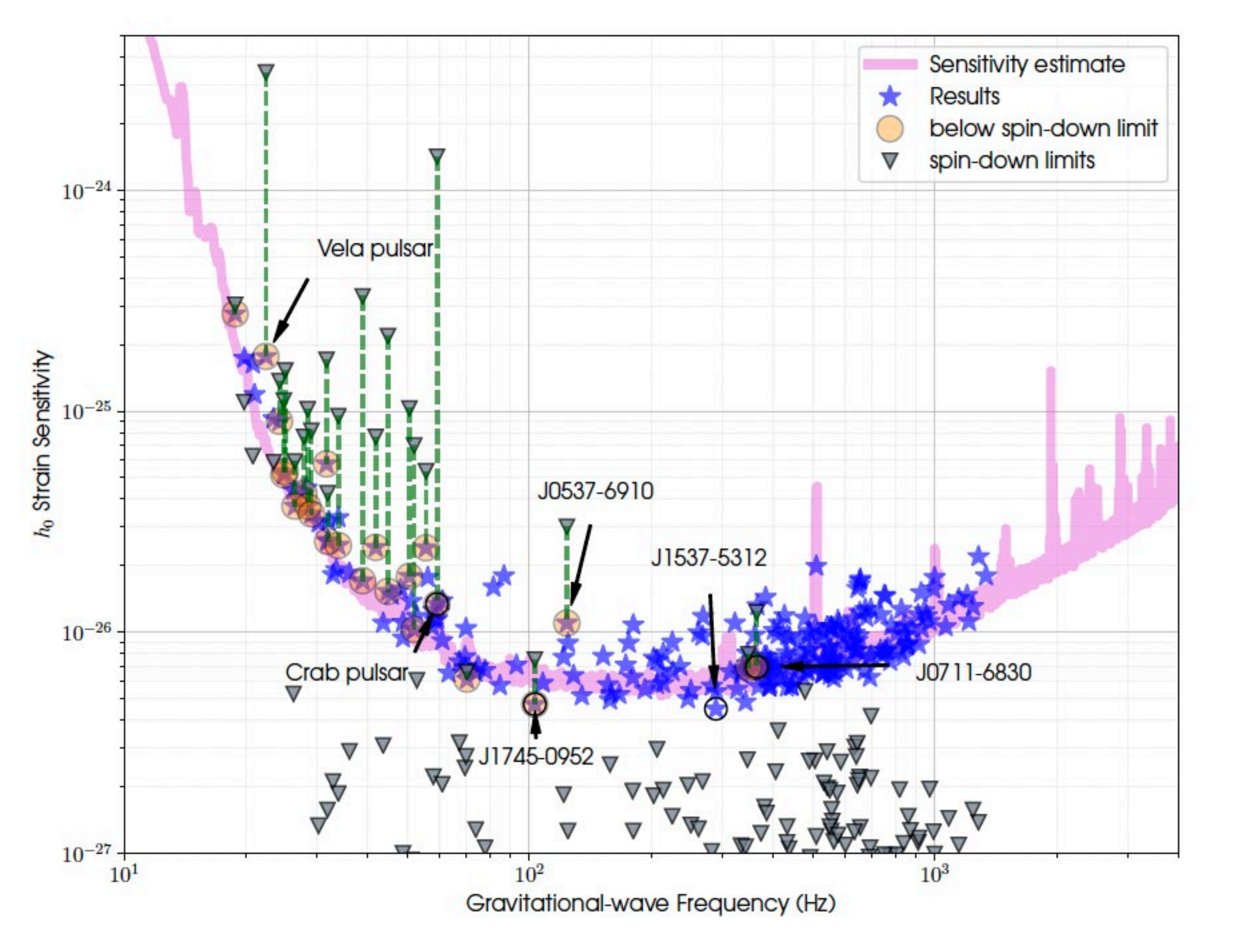


Figure: Aasi+ 2021; O2-O3 data, arXiv:2111.13106

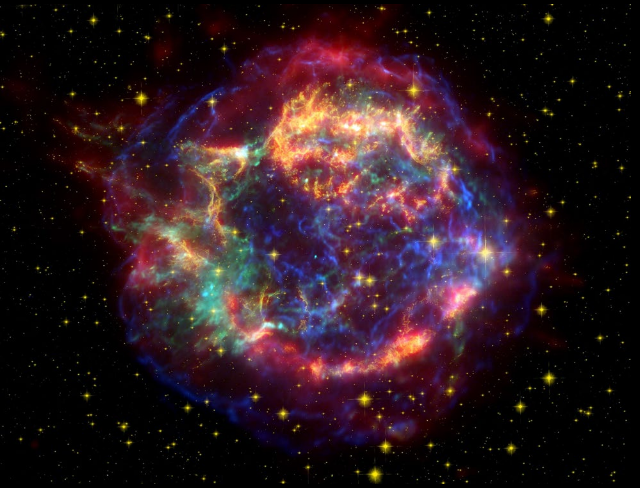
Directed searches

Searches over small sky regions, e.g.

- Supernova remnants
- Globular clusters
- Galactic centre

Timing solution not available: assume a Taylor series.

Of moderate computational cost.



Cas A

Globular cluster 47 Tuc

Galactic centre

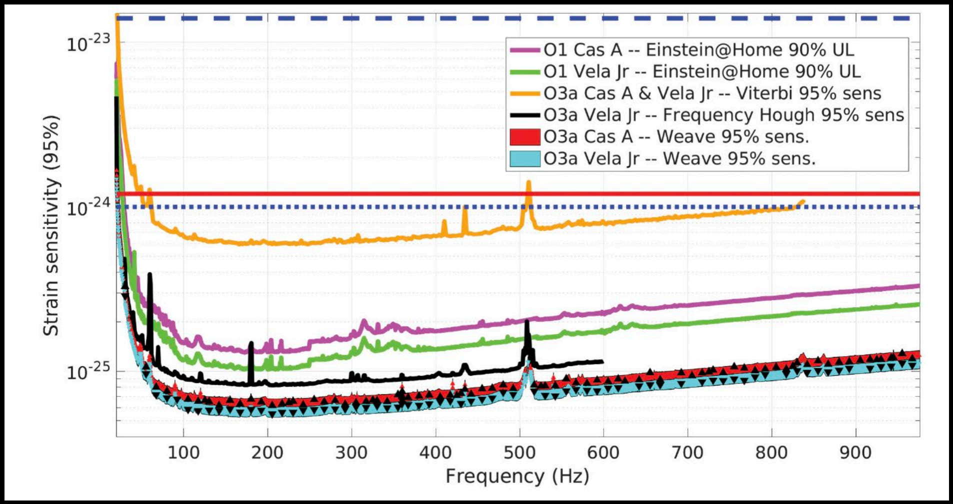
Directed searches cont…

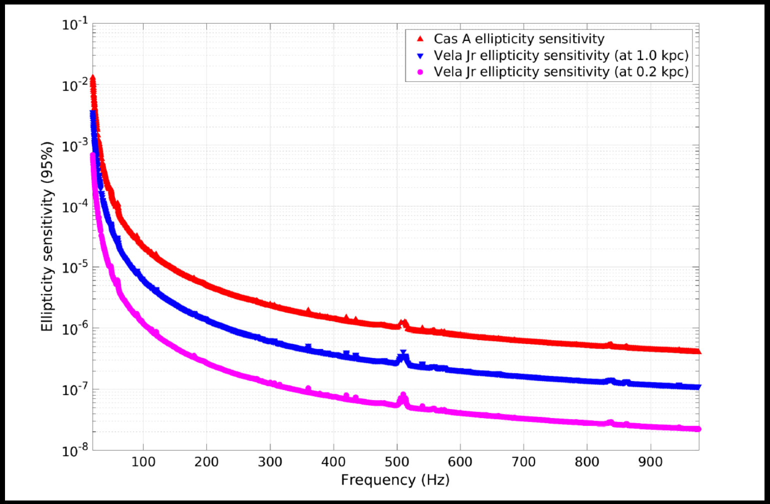
If distance and age of source known, can obtain energy-based indirect upper limit, using assumption star has spun down a lot since birth:

$$h_0(t) \approx \left(\frac{5G}{8c^3}\right)^{1/2} I^{1/2} \frac{1}{r} \frac{1}{t^{1/2}}$$

Note the cancellation: no dependence on birth spin or ellipticity!

E.g. for the supernova remnants Cas A and Vela Jr. (Abbott+ PRD 105 082005 (2022)):



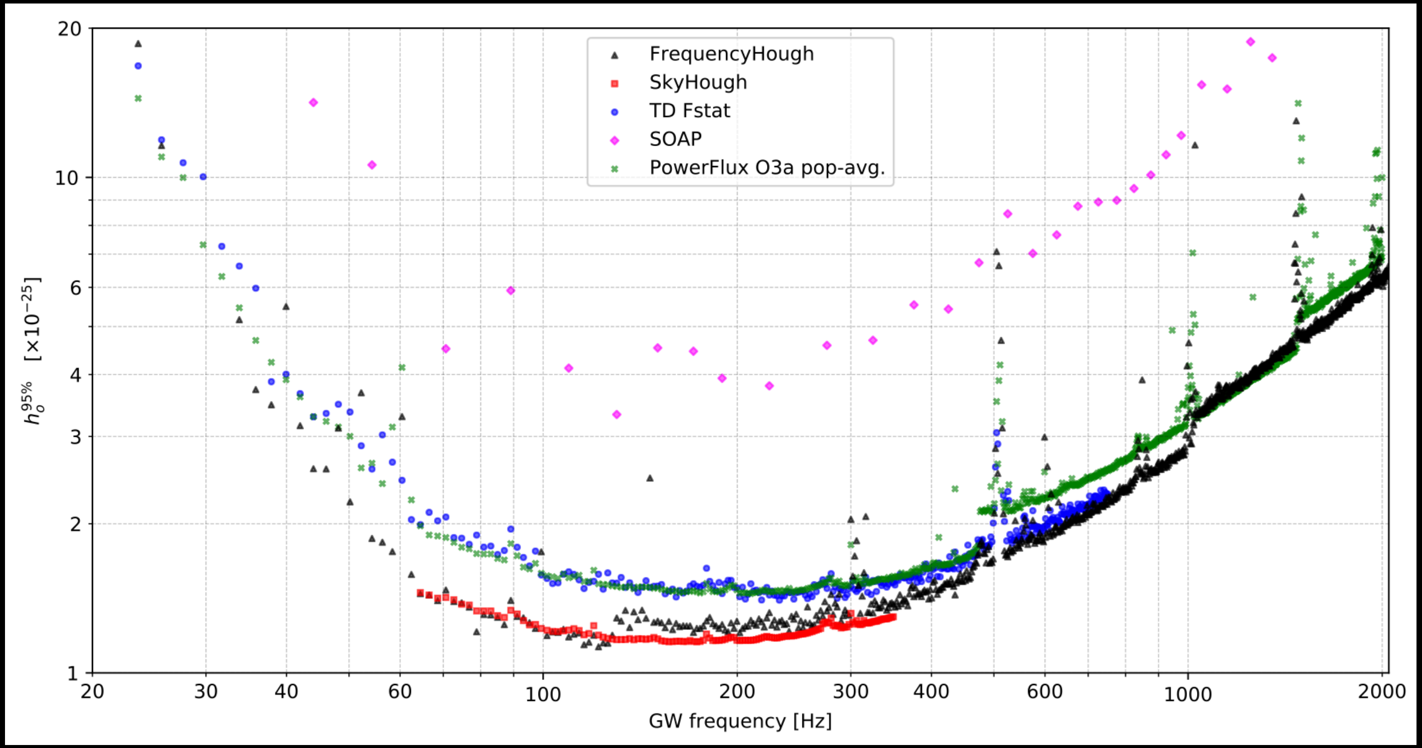


All-sky searches

Timing solution not available: assume a Taylor series.

Need to search over:
- All sky directions
- Ranges in spin parameters

Extremely computationally expensive



Abbott+ (2022), arXiv:2201.00697

So what?

Elastic mountain: Back-of-the-envelope

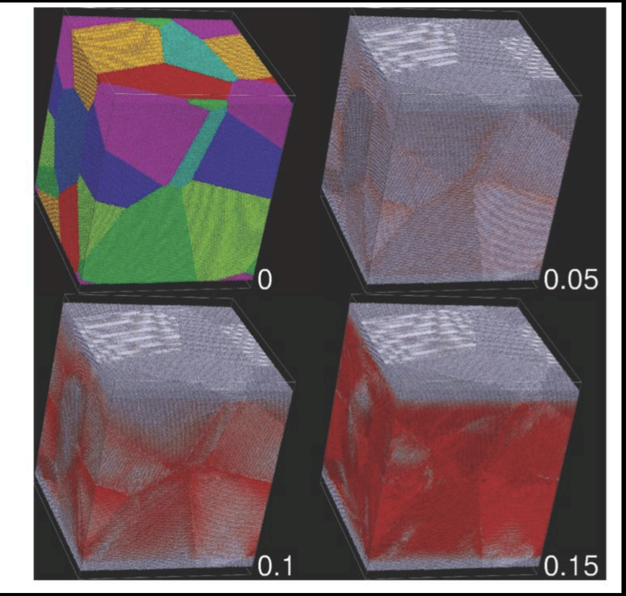
Maximum elastic mountain size determined by balance between gravitational and elastic forces:

$$\epsilon \approx \frac{\mu V_{\rm crust}}{GM^2/R} \times u_{\rm break} \approx 10^{-6} \left(\frac{u_{\rm break}}{10^{-1}} \right)$$

Shear modulus μ, ~ 10^{29} erg cm^{-3} for *crust*

Breaking strain u_{break} less well understood

Molecular dynamics of Horowitz & Kadau (2009) indicate high breaking strain, ~ 0.1 (see Figure).



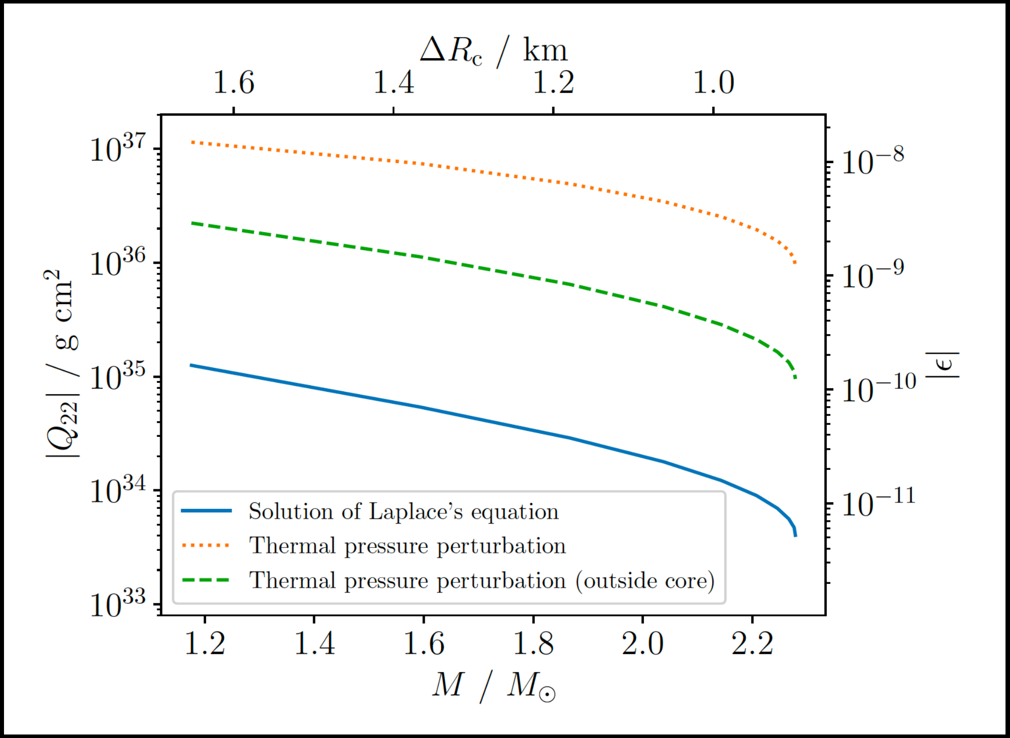
Plastic flow may relax crust on longer timescales (Chugunov & Horowitz 2010)

Elastic mountains: numerical calculations

Accuracy of rough estimate verified by numerical calculations of Ushomirsky, Cutler & Bildsten (2000).

Several follow up calculations since, using different set-upas and making different assummptions.

E.g. Gittins, Andersson & DIJ (2021), Gittins & Andersson (2021):



"Exotic" elastic mountains

Crust may not be the only solid phase

$\varepsilon_{max} \sim 10^{-1}$ possible for solid quark stars, 10^{-3} for hybrid stars (Johnson-McDaniel & Owen 2013).

Crystalline colour superconducting quark matter also relevant (Mannarelli et al 2007) leading to similarly large maximum ellipticities (Haskell et al 2007 and Lin 2007).

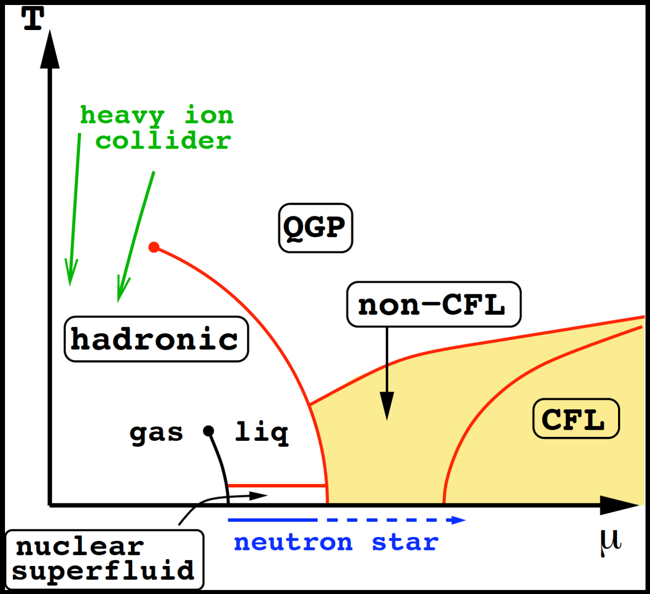


Figure: Alford et al

Lack of detection of such a large mountain *does not* rule out such exotic states of matter…

. . . need estimates of *likely* ellipticities, not just upper bounds!

Mountain building

Bildsten (1998) proposed building mountain via temperature/composition asymmetry in accreting stars.

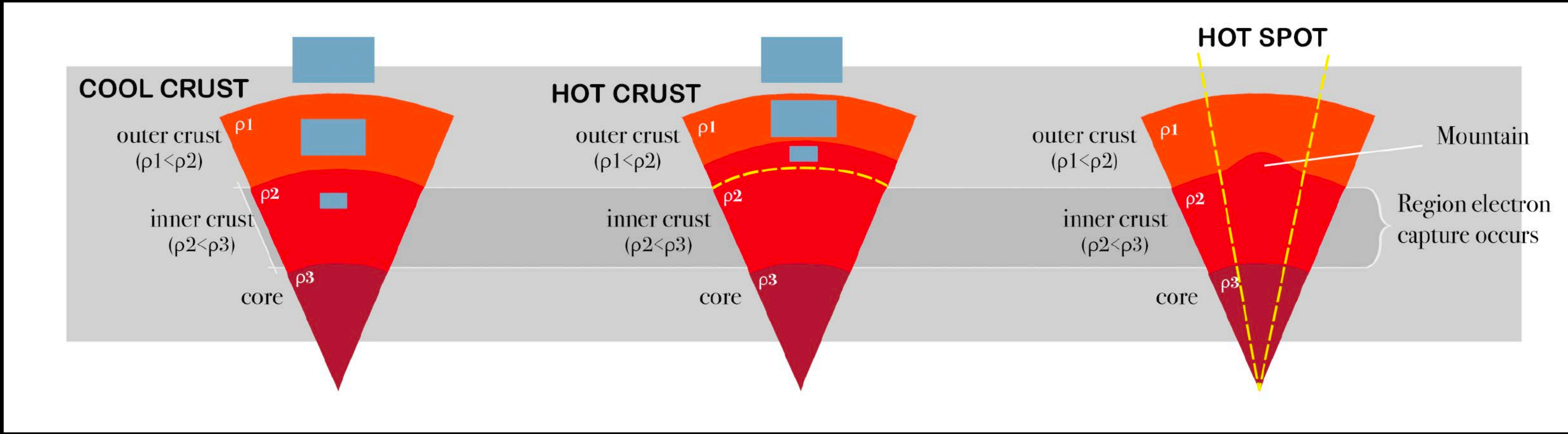


Figure credit: Emma Osborne (2020)

Viability of mechanism confirmed by Ushomirsky, Cutler & Bildsten (2000).

Possible evidence for mechanism at work proposed recently (Haskell & Patruno 2017).

But key unknown is likely level of temperature/composition asymmetry.

See Osborne & DIJ (2020) and Tom Hutchins' talk for concrete proposal: anisotropic heat conduction in magnetic field.

Magnetic mountains: back-of-the-envelope

Magnetic field lines have an effective tension, and deform star; roughly:

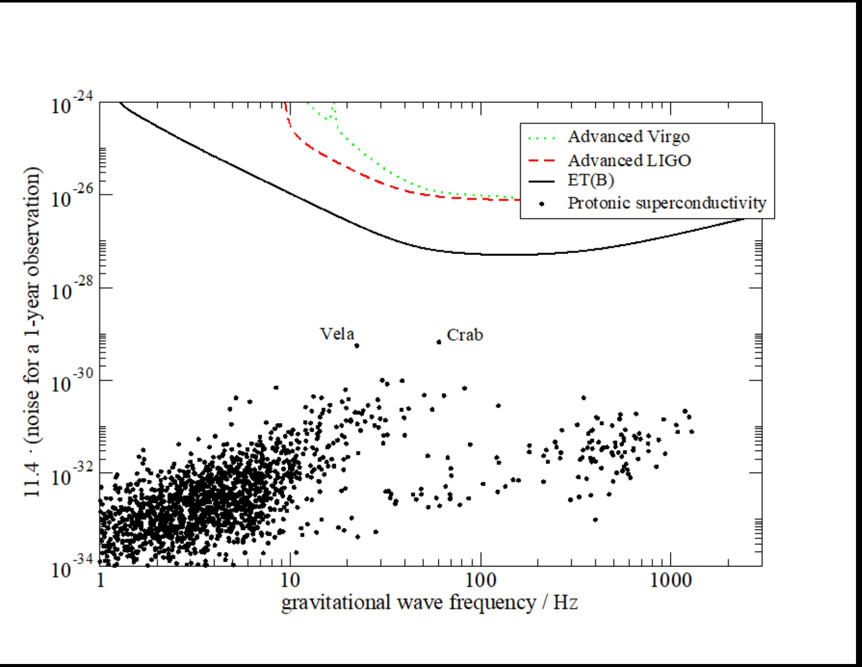
$$\epsilon \sim \frac{\int B^2\, dV}{GM^2/R} \sim 10^{-12} \left(\frac{B}{10^{12}\,\mathrm{G}} \right)^2$$

If protons form type II superconductor, magnetic field confirmed to fluxtubes. Effect of this is to increase tension by a factor of H_c/B, where $H_c \sim 10^{15}$ G, increasing ellipticity:

$$\epsilon \sim 10^{-9} \frac{B}{10^{12}\,\mathrm{G}}$$

But even then, the GW emission from known pulsars is not detectable.

Can get stronger emission from local field burial in accreting systems (Haskell et al 2015).



“Exotic” magnetic mountains

If CFL or 2SC phases occur in neutron star cores, can get *colour-magnetic flux tubes* (Iida & Baym 2002, Iida 2005, Alford & Sedrakian 2010).

This leads to flux tube tension $\sim 10^3$ larger than in protonic superconductivity case. Glampedakis, DIJ & Samuelsson (2012) estimate ellipticity:

$$\epsilon_{\mathrm{CFL}} \sim 10^{-7} \left(\frac{f_{\mathrm{vol}}}{1/2}\right) \left(\frac{B_{\mathrm{int}}}{10^{12}\,\mathrm{G}}\right) \left(\frac{\mu_{\mathrm{q}}}{400\,\mathrm{MeV}}\right)^2$$

Fraction of stellar volume in CFL/2SC state

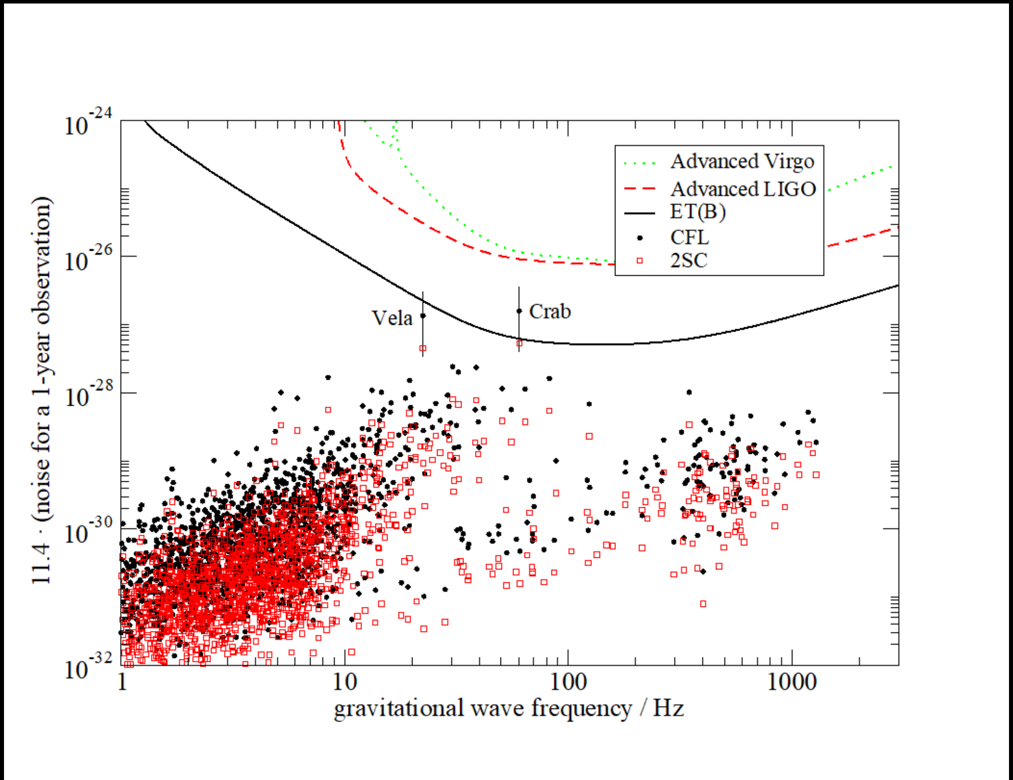
Internal field strength; set $B_{\mathrm{int}} = \alpha B_{\mathrm{ext}}$

Quark chemical potential

“Exotic” magnetic mountains: detectability

For given stellar parameters f_{vol}, α and μ_q can then balance observed spin-down of pulsars against combined GW & EM torque to estimate B_{int} and hence h.

GW amplitudes scale as $h \sim f_{vol}\, \alpha\, \mu_q^2$; for sensible values ($f_{vol}$ = 0.5, α = 2, μ_q = 400 MeV) obtain:



An intriguing possibility

During April 2018 INT workshop, Graham Woan showed a pulsar “P - P-dot” plot.

Was asked about empty bottom left hand corner.

Woan et al (2018) investigated. May be evidence for a minimum ellipticity in MSPs.

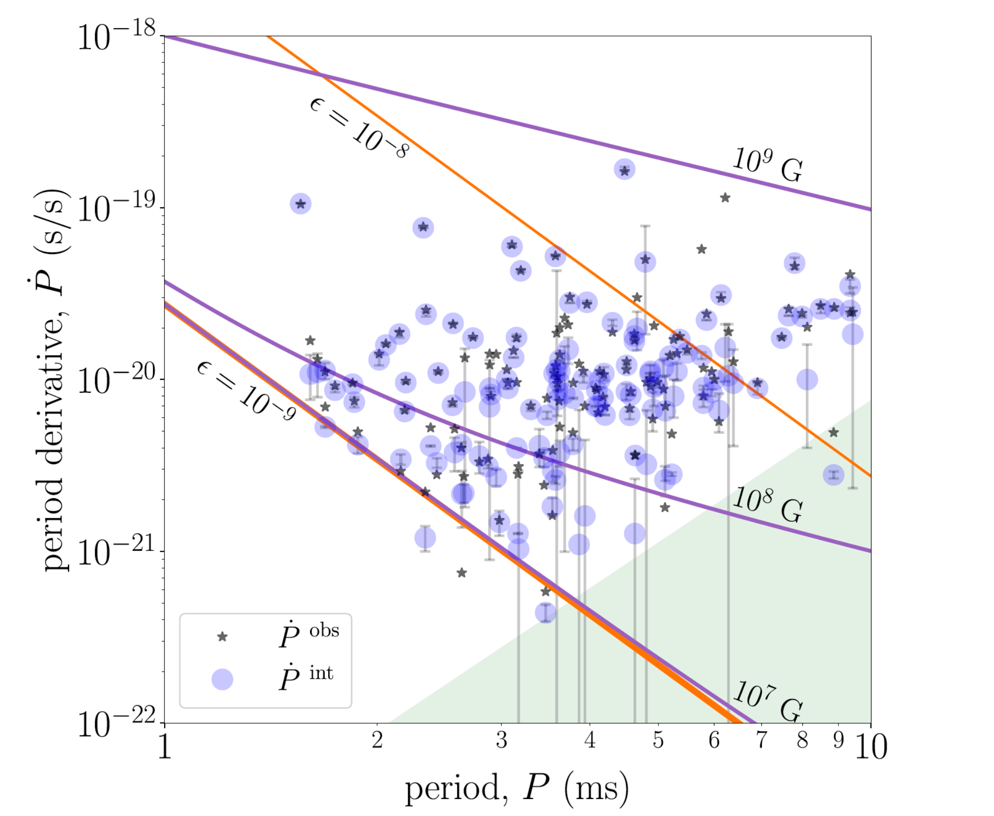


Figure: Woan et al (2018)

Fluid oscillations

Neutron stars have a complex set of normal modes.

Frequencies depend upon mass, and many details of internal structure/microphysics

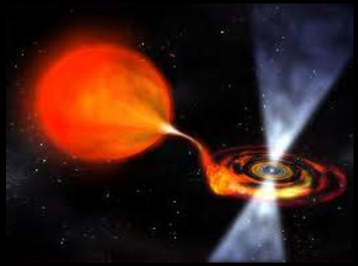
Can be excited in two different ways:

1. Impulsive excitation
2. Instability

Impulsive excitation

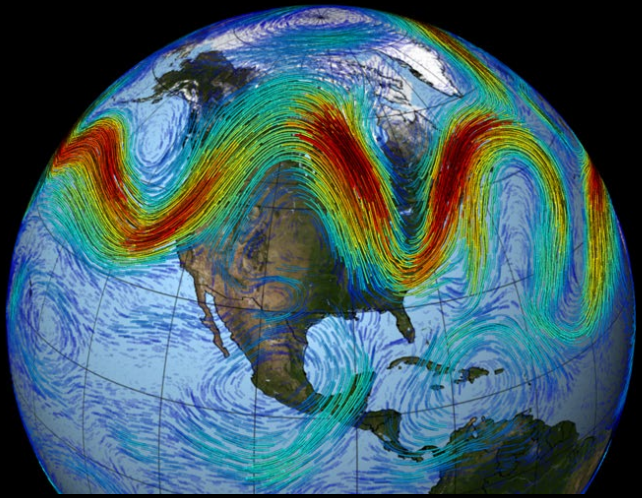
Pulsars *glitch*
Abadie+ 2011: Vela glitch, $E_{GW} < 10^{45}$ erg

Magnetars *flare*
Abbott+ 2019: E_{GW} $3.4 < 10^{44}$ erg
for SGR 1806-20.



Low-mass X-ray binaries burst

Rossby waves



Class of oscillations connected with rotation of a fluid ball.

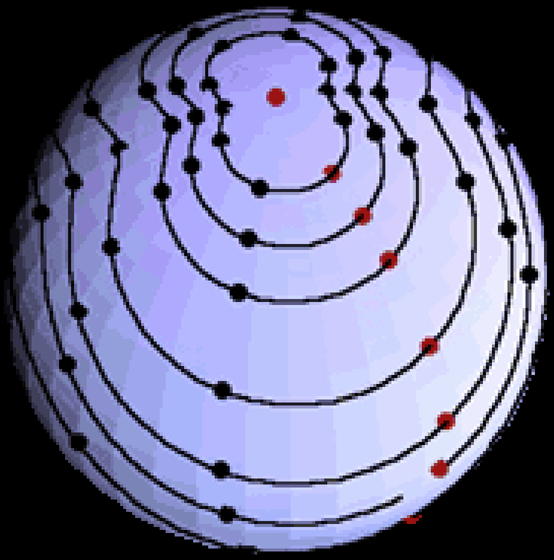
Named after Carl-Gustaf Rossby, who identified them in atmosphere in 1939.

Also found in the Earth's oceans.

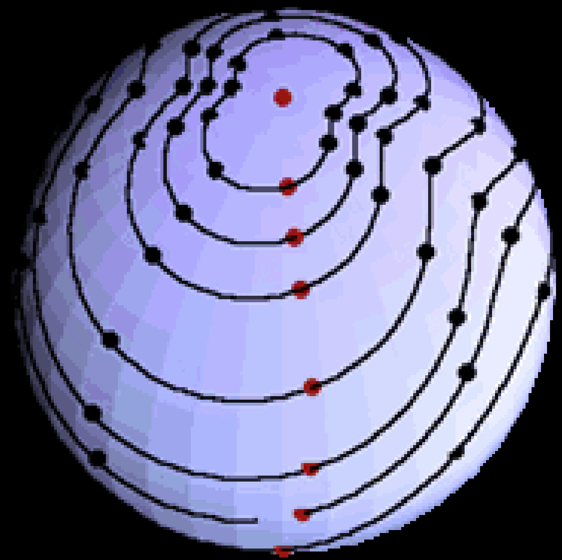
Restoring force is the Coriolis force.

Instability: the CFS mechanism

GW emission can drive the r-mode unstable*, a two-stream instability*.



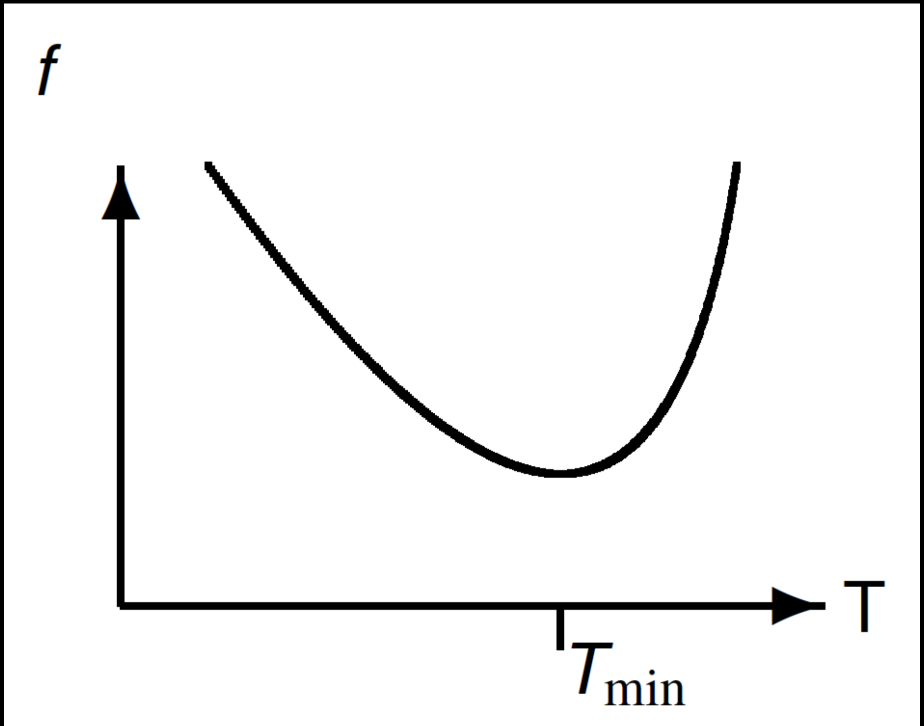
View from the inertial frame.



View from the corotating frame

Movie credit: Hanna & Owen

R-mode instability curve



Not all rotating fluid bodies are unstable…

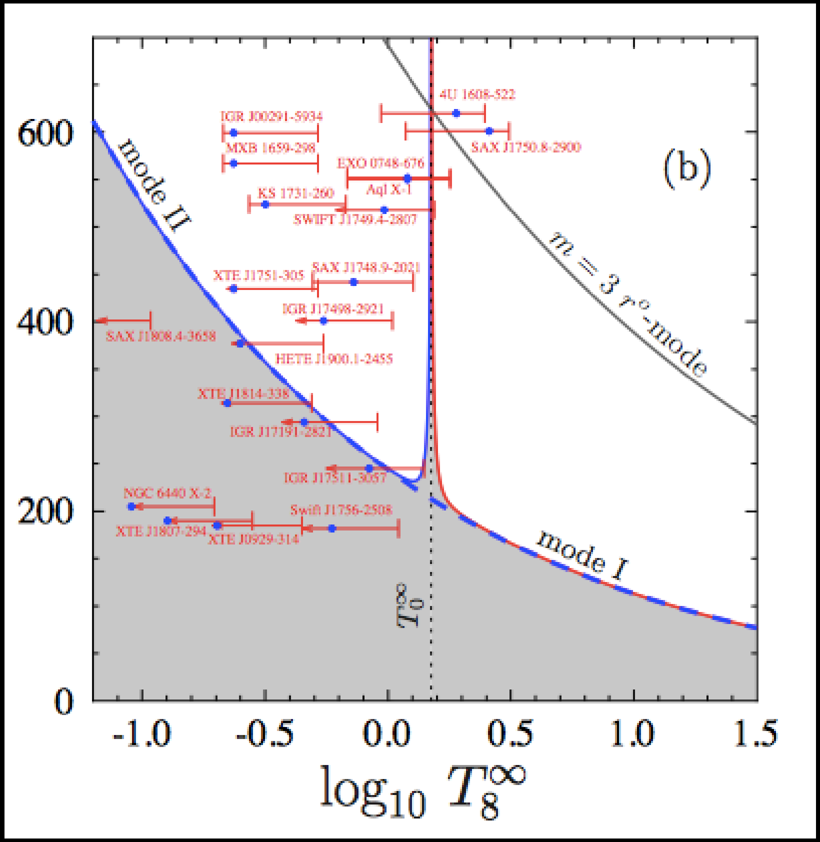
… gravitational wave instability must compete with stabalizing effects of *dissipation*.

Dissipation rate temperature-dependent, giving rise to an *instability window*.

Active area of research: relating instability curve to stellar model.

Realistic instability curves

Need to take many pieces of physics into account, including elastic crust, magnetic field, superfluidity, …



Example: Gusakov, Chugunov & Kantor(2014) consider "superfluid" modes, finding "stability spikes":

Summary

- LIGO-Virgo observations beginning to probe regimes of astrophysical interest, but detection likely requires:
  - Elastic mountains close to maximally strained, or…
  - ... elastic mountains from exotic phases, or …
  - … exotic magnetic field configuration, or …
  - … excitation of oscillation mode, probably via instability.
- Amplitude and frequency encode interesting information.
- Not clear when first detections will be made.