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Ab initio calculations of muon capture in light nuclei, and connections to neutrinoless double-beta decay matrix elements

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TRIUMF, Theory Department

INT Program 24-1
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Arthur B. McDonald
Canadian Astroparticle Physics Research Institute

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Outline

Introduction to double-beta decay

Corrections to $0\nu\beta\beta$-decay nuclear matrix elements

Muon capture as a probe of $0\nu\beta\beta$ decay

Summary and Outlook

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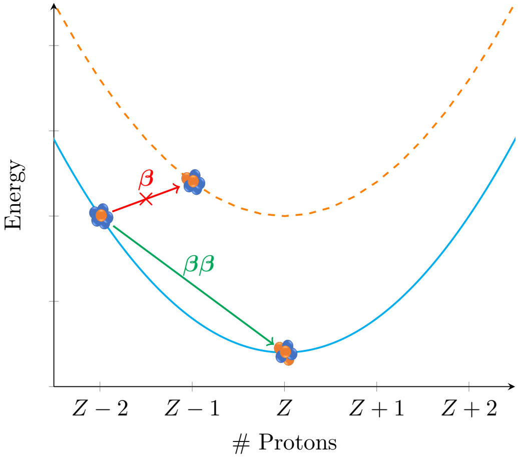
Double-Beta Decay

$$\beta^- : \mathrm{n} \rightarrow \mathrm{p} + \mathrm{e}^- + \bar{\nu}_\mathrm{e}$$
$$\beta^-\beta^- : 2\mathrm{n} \rightarrow 2\mathrm{p} + 2\mathrm{e}^- (+2\bar{\nu}_\mathrm{e})$$

- May happen, when **β-decay is not allowed / suppressed**



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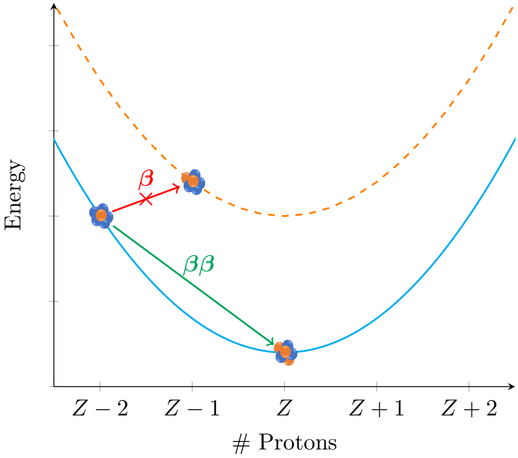

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Double-Beta Decay

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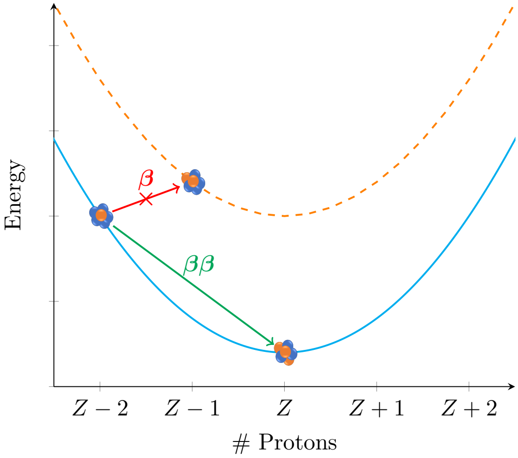
  - Standard two-neutrino $\beta\beta$ decay ($2\nu\beta\beta$)



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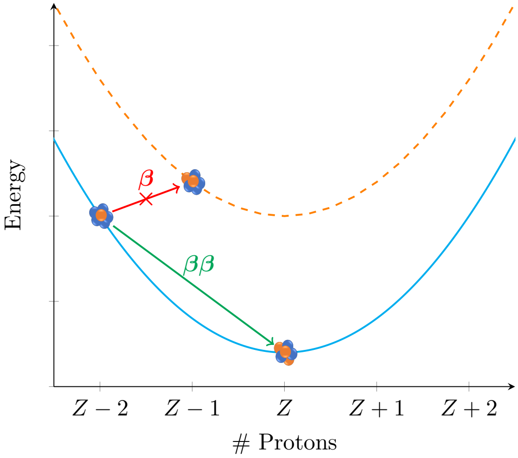
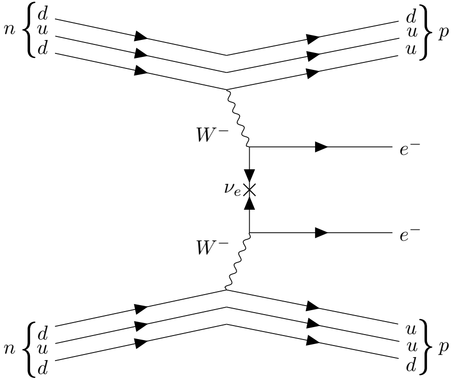
  - Hypothetical **neutrinoless $\beta\beta$ ($0\nu\beta\beta$) decay**



Neutrinoless double-beta decay via light neutrino exhange

$$\frac{1}{t_{1/2}^{0\nu}} = g_A^4 G^{0\nu} |\boldsymbol{M^{0\nu}}|^2 \left(\frac{m_{\beta\beta}}{m_e}\right)^2$$

- Violates lepton-number conservation



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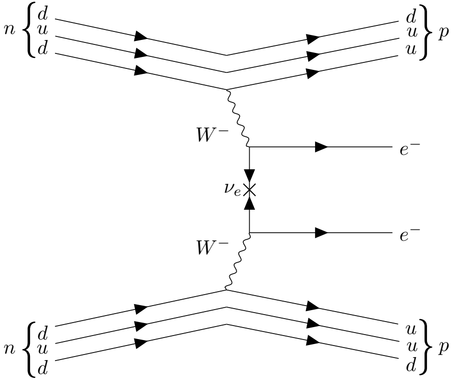

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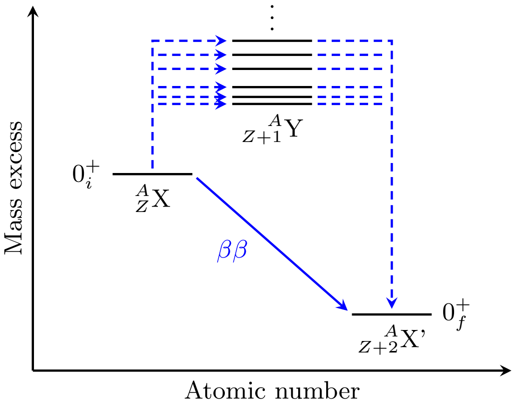

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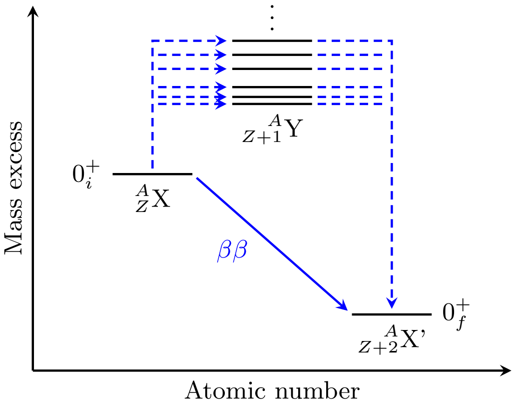

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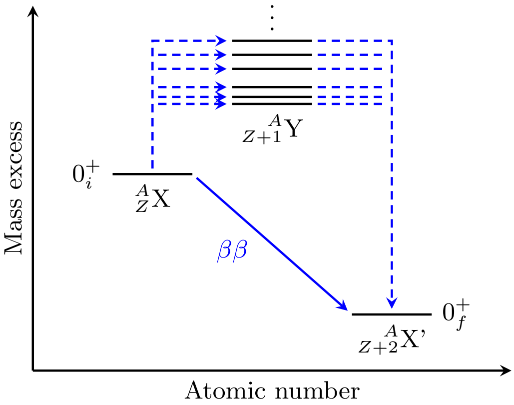

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Half-life of $0\nu\beta\beta$ Decay

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New physics

$m_{\beta\beta} = \sum_k (U_{ek})^2 m_k$

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Half-life of $0\nu\beta\beta$ Decay

What would be measured

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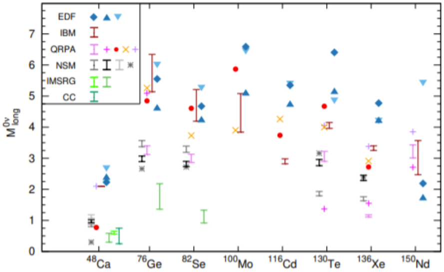

M. Agostini et al., Rev. Mod. Phys. ***95****, 025002 (2023)*

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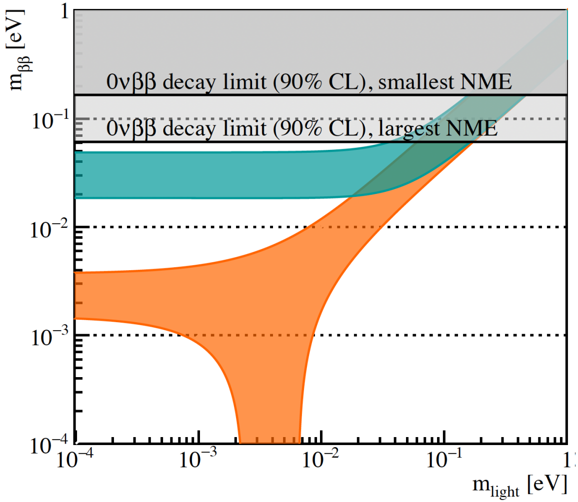
Current reach of the experiments



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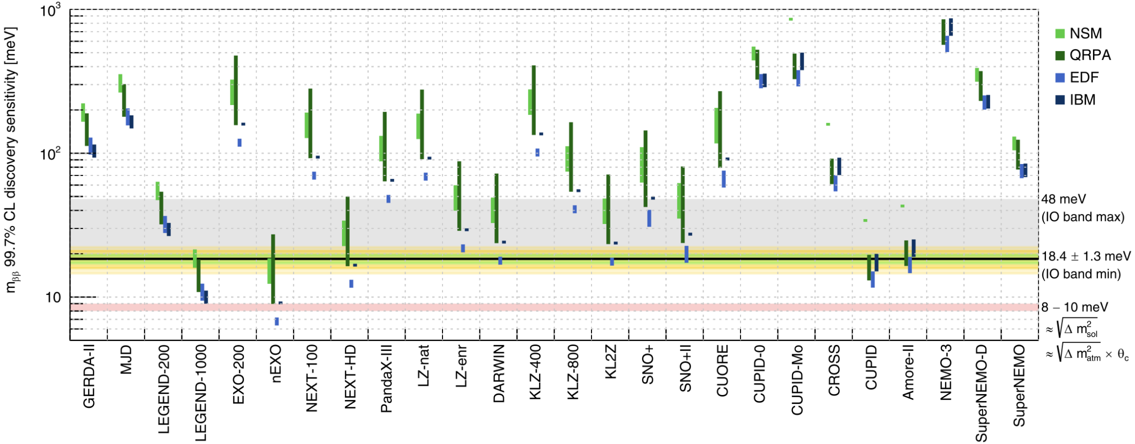
Next generation experiments



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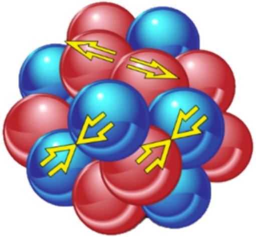

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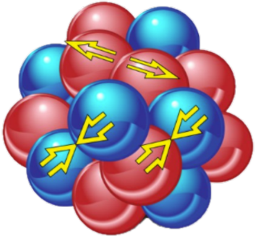

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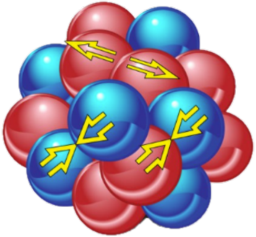

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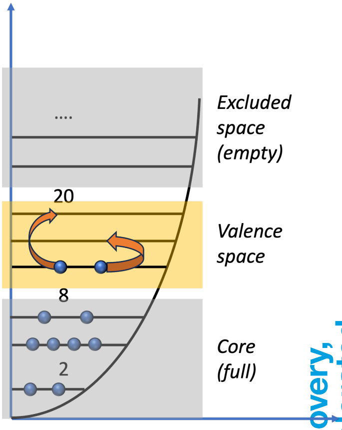

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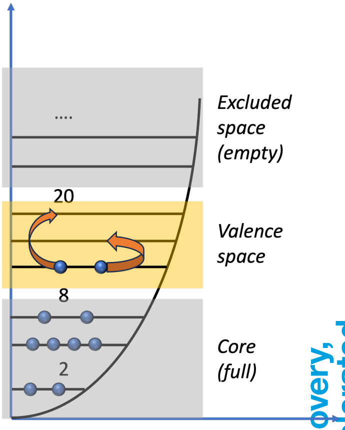

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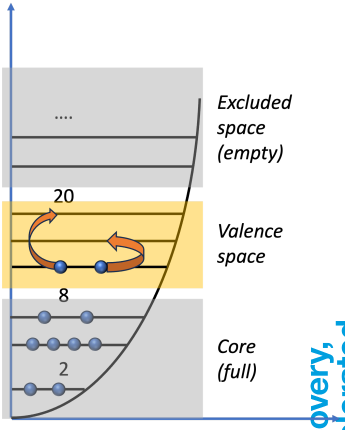

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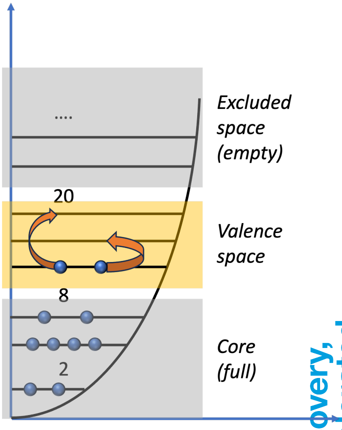

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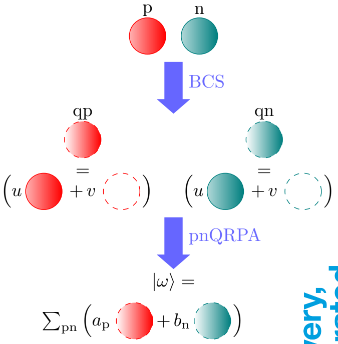

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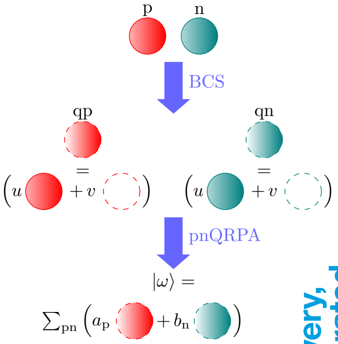

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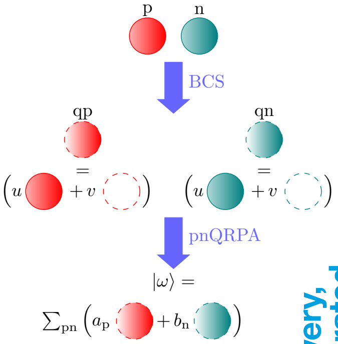

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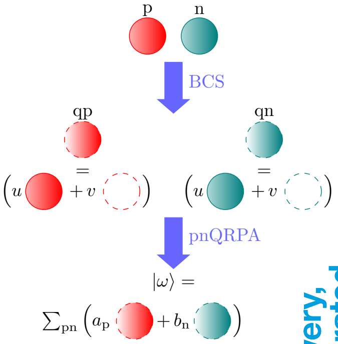

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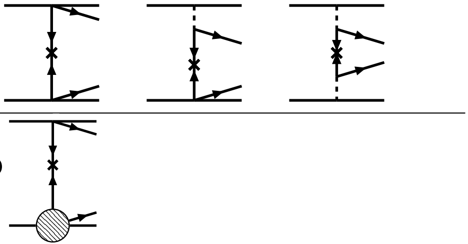
Effective field theory corrections to $0\nu\beta\beta$ decay

$$\frac{1}{t_{1/2}^{0\nu}} = g_A^4 G^{0\nu} |M_L^{0\nu}|^2 \left(\frac{m_{\beta\beta}}{m_e}\right)^2$$

V. Cirigliano et al., Phys. Rev. C 97, 065501 (2018), Phys. Rev. Lett. 120, 202001 (2018), Phys. Rev. C 100, 055504 (2019)

LO

N^{2}LO



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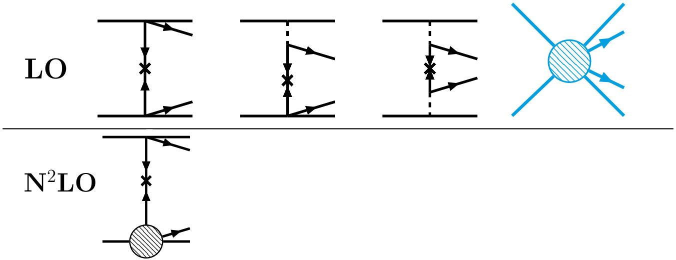

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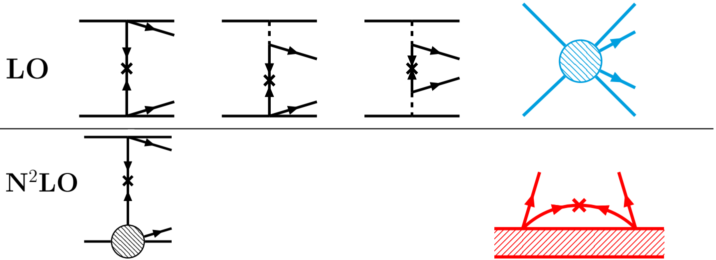

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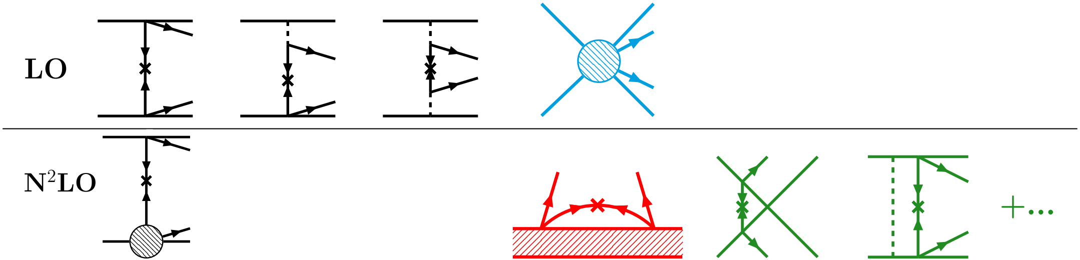

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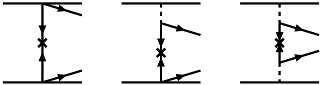

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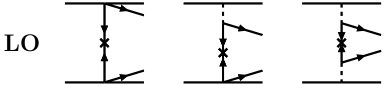

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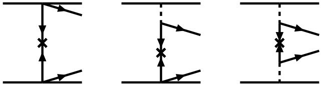

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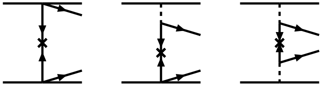

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Discovery, accelerated

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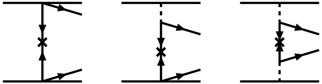

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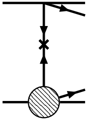

TRIUMF

Traditional nuclear matrix elements of neutrinoless double-beta decay

$$M^{0\nu} = \frac{R}{g_A^2} \int \frac{\mathrm{d}\mathbf{k}}{2\pi^2} \frac{e^{i\mathbf{k}(\mathbf{x}-\mathbf{y})}}{E_\nu} \sum_n \frac{\langle f|\, J_\mu(\mathbf{x})\, |n\rangle \langle n|\, J^\mu(\mathbf{y})\, |i\rangle}{E_\nu + E_n - \frac{1}{2}(E_i + E_f) - \frac{1}{2}(E_1 - E_2)}$$

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Discovery, accelerated

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TRIUMF

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Discovery, accelerated

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Discovery, accelerated

Leading-order short-range contribution to $0\nu\beta\beta$ decay

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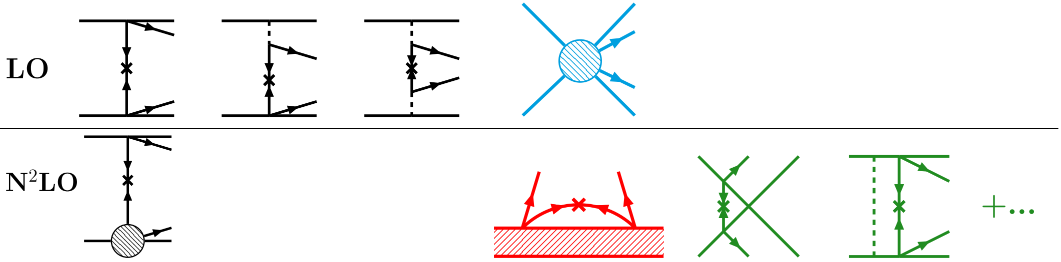
V. Cirigliano et al., Phys. Rev. C 97, 065501 (2018), Phys. Rev. Lett. 120, 202001 (2018), Phys. Rev. C 100, 055504 (2019)



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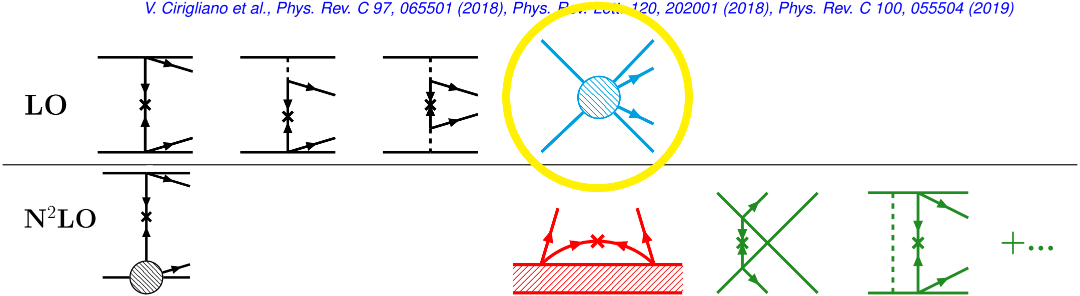

Contact Term in pnQRPA and NSM

$$M_{\mathrm{S}}^{0\nu} = \frac{2R}{\pi g_{\mathrm{A}}^2} \langle 0_f^+ | \sum_{m,n} \tau_m^- \tau_n^- \int j_0(qr) \boldsymbol{h_{\mathrm{S}}(q^2)}\, q^2 \mathrm{d}q | 0_i^+ \rangle$$

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Discovery, accelerated

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Couplings (g_ν^{NN}) and scales (Λ) of the Gaussian regulator [1].

$g_\nu^{\mathrm{NN}} (\mathrm{fm}^2)$	Λ (MeV)
-0.67	450
-1.01	550
-1.44	465
-0.91	465
-1.44	349
-1.03	349

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Discovery, accelerated

Contact Term in pnQRPA and NSM

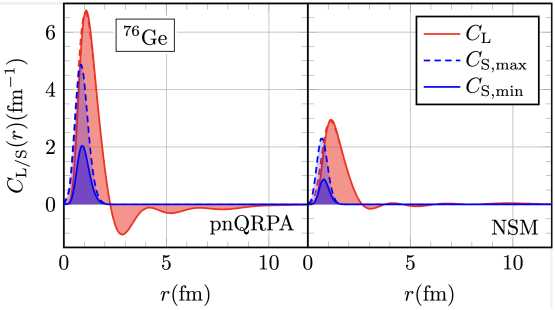
$$\int C_{\mathrm{L/S}}(r)\mathrm{d}r = M^{0\nu}_{\mathrm{L/S}}$$

In pnQRPA:

$M_{\mathrm{S}}/M_{\mathrm{L}} \approx 30\% - 80\%$

In NSM:

$M_{\mathrm{S}}/M_{\mathrm{L}} \approx 15\% - 50\%$



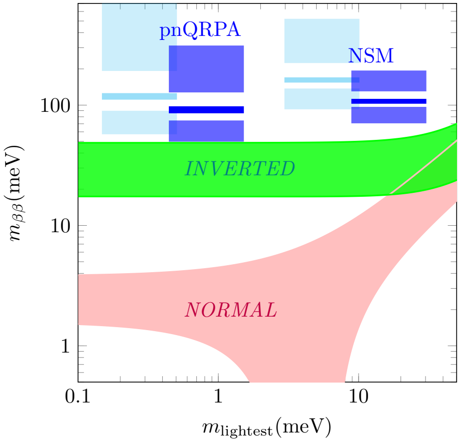
*LJ, P. Soriano and J. Menéndez, Phys. Lett. B **823**, 136720 (2021)*

Effective Neutrino Masses

- Effective neutrino masses combining the likelihood functions of GERDA (^{76}Ge), CUORE (^{130}Te), EXO-200 (^{136}Xe) and KamLAND-Zen (^{136}Xe)

*S. D. Biller, Phys. Rev. D **104**, 012002 (2021)*

- Middle bands: $M_{\mathbf{L}}^{(0\nu)}$
  Lower bands: $M_{\mathbf{L}}^{(0\nu)} + M_{\mathbf{S}}^{(0\nu)}$
  Upper bands: $M_{\mathrm{L}}^{(0\nu)} - M_{\mathrm{S}}^{(0\nu)}$



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TRIUMF

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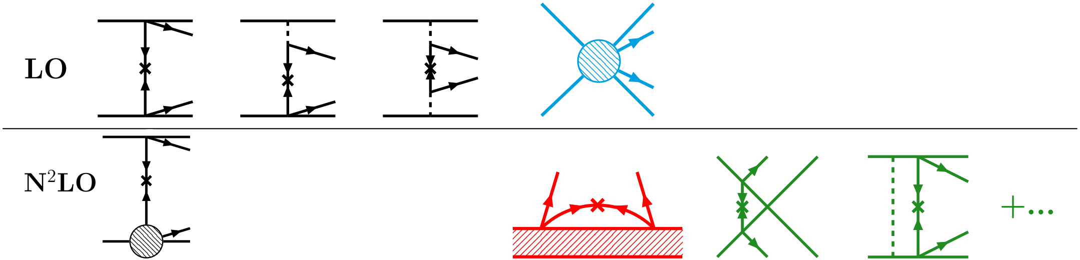

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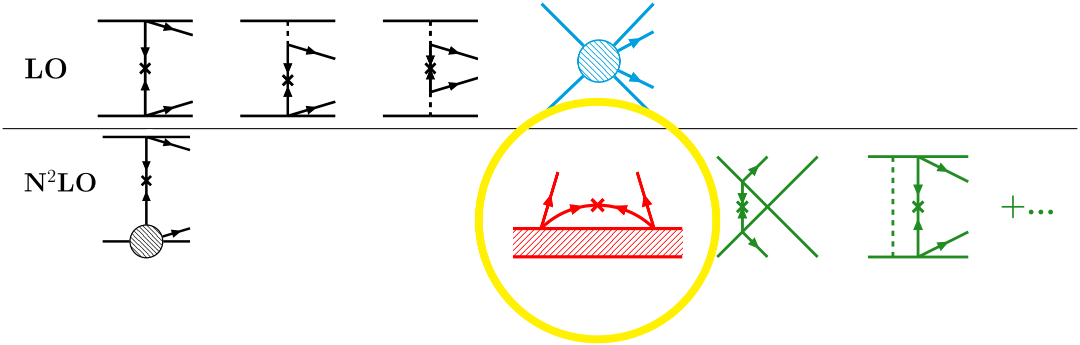

TRIUMF

Contribution of ultrasoft neutrinos

- Contribution of ultrasoft neutrinos ($|\mathbf{k}| << k_{\mathrm{F}} \approx 100$ **MeV**) to $0\nu\beta\beta$ decay:

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$$M^{0\nu}_{\mathrm{usoft}} = \frac{\pi R}{g_{\mathrm{A}}^2} \sum_n \frac{\mathrm{d}^{d-1}k}{(2\pi)^{d-1}} \frac{1}{|\mathbf{k}|} \left[\frac{\langle f| J_\mu |n\rangle \langle n| J^\mu |i\rangle}{|\mathbf{k}| + E_2 + E_n - E_i - i\eta} + \frac{\langle f| J_\mu |n\rangle \langle n| J^\mu |i\rangle}{|\mathbf{k}| + E_1 + E_n - E_i - i\eta} \right]$$

Discovery, accelerated

TRIUMF

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- Keeping only $\mathbf{k} = \mathbf{0}$ term in the current:

$$\begin{aligned} M^{0\nu}_{\mathrm{usoft}}(\mu_{\mathrm{us}}) = &- \frac{R}{2\pi} \sum_n \langle f| \sum_a \boldsymbol{\sigma}_a \tau_a^+ |n\rangle \langle n| \sum_b \boldsymbol{\sigma}_b \tau_b^+ |i\rangle \\ &\times \left[(E_1 + E_n - E_i) \left(\ln \frac{\mu_{\mathrm{us}}}{2(E_1 + E_n - E_i)} + 1 \right) \right. \\ &\left. + (E_2 + E_n - E_i) \left(\ln \frac{\mu_{\mathrm{us}}}{2(E_2 + E_n - E_i)} + 1 \right) \right] \end{aligned}$$

Discovery, accelerated

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- Contribution of ultrasoft neutrinos ($|\mathbf{k}| << k_{\mathrm{F}} \approx 100$ **MeV**) to $0\nu\beta\beta$ decay:

V. Cirigliano et al., Phys. Rev. C 97, 065501 (2018)

$$M^{0\nu}_{\mathrm{usoft}} = \frac{\pi R}{g_{\mathrm{A}}^2} \sum_n \frac{\mathrm{d}^{d-1}k}{(2\pi)^{d-1}} \frac{1}{|\mathbf{k}|} \left[\frac{\langle f| J_\mu |n\rangle \langle n| J^\mu |i\rangle}{|\mathbf{k}| + E_2 + E_n - E_i - i\eta} + \frac{\langle f| J_\mu |n\rangle \langle n| J^\mu |i\rangle}{|\mathbf{k}| + E_1 + E_n - E_i - i\eta} \right]$$

- Keeping only $\mathbf{k} = 0$ term in the current:

$$M^{0\nu}_{\mathrm{usoft}}(\mu_{\mathrm{us}}) = -\frac{R}{2\pi} \sum_n \langle f| \sum_a \boldsymbol{\sigma}_a \tau_a^+ |n\rangle \langle n| \sum_b \boldsymbol{\sigma}_b \tau_b^+ |i\rangle$$
$$\times \Big[(E_1 + E_n - E_i) \left(\ln \frac{\mu_{\mathrm{us}}}{2(E_1 + E_n - E_i)} + 1 \right) + (E_2 + E_n - E_i) \left(\ln \frac{\mu_{\mathrm{us}}}{2(E_2 + E_n - E_i)} + 1 \right) \Big]$$

Are we missing a factor of 2?

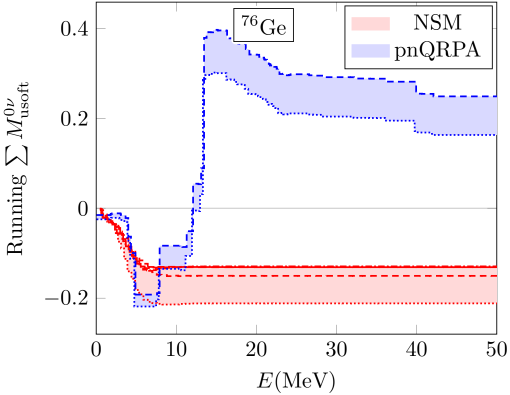
PRELIMINARY Ultrasoft neutrinos in pnQRPA and nuclear shell model

In pnQRPA:

$|M^{0\nu}_{\mathrm{usoft}}/M^{0\nu}_{\mathrm{L}}| \leq 15\%$

In NSM:

$|M^{0\nu}_{\mathrm{usoft}}/M^{0\nu}_{\mathrm{L}}| \leq 5\%$



LJ, D. Castillo,P. Soriano, J Menéndez, work in progress

Discovery, accelerated

Ultrasoft neutrinos as correction of the closure approximation

$$\frac{1}{t_{1/2}^{0\nu}} = g_A^4 G^{0\nu} \left| \boldsymbol{M_L^{0\nu}} + M_S^{0\nu} + \boldsymbol{M_{usoft}^{0\nu}} + M_{N2LO}^{0\nu} \right|^2 \left(\frac{m_{\beta\beta}}{m_e} \right)^2$$

Discovery, accelerated

TRIUMF

Ultrasoft neutrinos as correction of the closure approximation

$$\frac{1}{t_{1/2}^{0\nu}} = g_A^4 G^{0\nu} |\boldsymbol{M_L^{0\nu}} + M_S^{0\nu} + \boldsymbol{M_{\text{usoft}}^{0\nu}} + M_{\text{N2LO}}^{0\nu}|^2 \left(\frac{m_{\beta\beta}}{m_e}\right)^2$$

In EFT:

$$M_L^{0\nu} \propto \frac{\langle f | \, J_\mu(\mathbf{x}) J^\mu(\mathbf{y}) \, | i \rangle}{|\mathbf{k}|}$$

Discovery, accelerated

Ultrasoft neutrinos as correction of the closure approximation

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$\rightarrow M_{cl}^{0\nu}$ with $< E >= 0$

Discovery, accelerated

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In EFT:

$$M_L^{0\nu} \propto \frac{\langle f | J_\mu(\mathbf{x}) J^\mu(\mathbf{y}) | i \rangle}{|\mathbf{k}|}$$

$\rightarrow M_{cl}^{0\nu}$ with $< E >= 0$

$$M_{usoft}^{0\nu} \propto \sum_n \langle f | \sum_a \boldsymbol{\sigma}_a \tau_a^+ | n \rangle \langle n | \sum_b \boldsymbol{\sigma}_b \tau_b^+ | i \rangle \times f(E_n)$$

PRELIMINARY Ultrasoft neutrinos vs closure approximation in NSM

Nucleus	Interaction	$M^{0\nu}$	$M^{0\nu 2}_{\rm cl}$	$M^{0\nu} - M^{0\nu}_{\rm cl}$	$M^{0\nu}_{\rm usoft}$
^{48}Ca	KB3G	0.92	0.96	-0.04	-0.01
	GXPF1.a42	0.78	0.78	0.00	0.02
^{76}Ge	JUN45	3.37	3.61	-0.24	-0.13
^{82}Se	JUN45	3.16	3.39	-0.23	-0.11

LJ, D. Castillo,P. Soriano, J Menéndez, work in progress

[2] R. A. Sen'kov, M. Horoi, , PRC **90**, 051301(R) (2014)

PRELIMINARY Ultrasoft neutrinos vs closure approximation in NSM

Nucleus	Interaction	$M^{0\nu}$	$M^{0\nu}_{\rm cl}$[2]	$M^{0\nu} - M^{0\nu}_{\rm cl}$	$M^{0\nu}_{\rm usoft}$
^{48}Ca	KB3G	0.92	0.96	-0.04	-0.01
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LJ, D. Castillo,P. Soriano, J Menéndez, work in progress

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TRIUMF

PRELIMINARY Ultrasoft neutrinos vs closure approximation in pnQRPA

Nucleus	$M^{0\nu}$	$M^{0\nu}_{\rm cl}$	$M^{0\nu} - M^{0\nu}_{\rm cl}$	$M^{0\nu}_{\rm usoft}$
^{76}Ge	4.83	4.68	0.15	0.25
^{82}Se	4.30	4.20	0.10	0.18
^{96}Zr	4.29	4.04	0.25	0.25
^{100}Mo	3.52	2.71	0.81	0.65
^{116}Cd	4.31	4.47	-0.16	-0.03
^{124}Sn	5.12	4.88	0.24	0.29
^{128}Te	3.99	3.76	0.23	0.27
^{130}Te	3.52	3.36	0.16	0.22
^{136}Xe	2.60	2.71	-0.11	0.06

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Discovery, accelerated

TRIUMF

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LJ, D. Castillo, P. Soriano, J Menéndez, work in progress

TRIUMF

PRELIMINARY Ultrasoft neutrinos vs closure approximation in pnQRPA

Nucleus	$M^{0\nu}$	$M^{0\nu}_{\rm cl}$	$M^{0\nu} - M^{0\nu}_{\rm cl}$	$M^{0\nu}(1^+) - M^{0\nu}_{\rm cl}(1^+)$	$M^{0\nu}_{\rm usoft}$
^{76}Ge	4.83	4.68	0.15	**0.26**	0.25
^{82}Se	4.30	4.20	0.10	**0.18**	0.18
^{96}Zr	4.29	4.04	0.25	**0.26**	0.25
^{100}Mo	3.52	2.71	0.81	**0.75**	0.65
^{116}Cd	4.31	4.47	-0.16	**-0.06**	-0.03
^{124}Sn	5.12	4.88	0.24	**0.31**	0.29
^{128}Te	3.99	3.76	0.23	**0.26**	0.27
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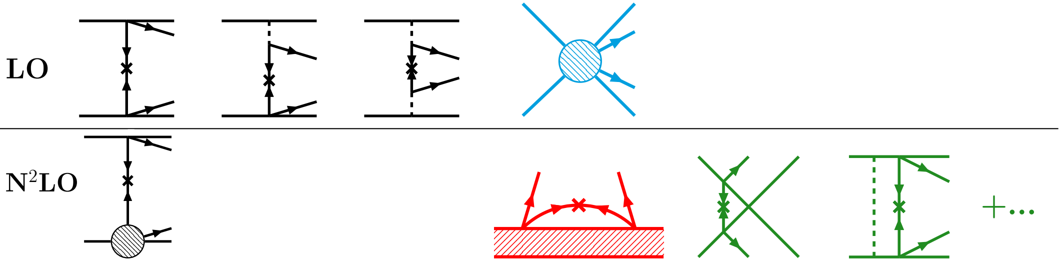
Discovery, accelerated

TRIUMF

Genuine $\mathrm{N^2LO}$ corrections to $0\nu\beta\beta$ decay

$$\frac{1}{t_{1/2}^{0\nu}} = g_A^4 G^{0\nu} \left| M_L^{0\nu} + M_S^{0\nu} + M_{\mathrm{usoft}}^{0\nu} + M_{\mathrm{N^2LO}}^{0\nu} \right|^2 \left(\frac{m_{\beta\beta}}{m_e} \right)^2$$

V. Cirigliano et al., Phys. Rev. C 97, 065501 (2018), Phys. Rev. Lett. 120, 202001 (2018), Phys. Rev. C 100, 055504 (2019)



TRIUMF

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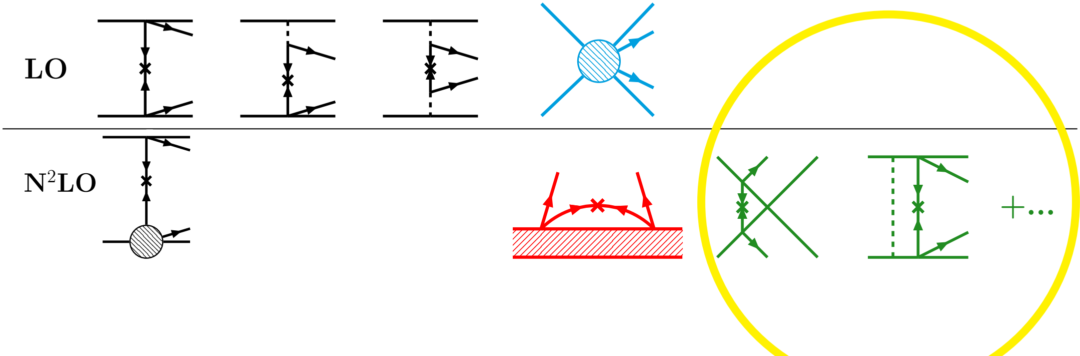

Discovery, accelerated

TRIUMF

PRELIMINARY Genuine N^2LO Corrections in pnQRPA

Nucleus	$M_{\mathrm{L}}^{0\nu}$	$M_{\mathrm{N^2LO}}^{0\nu}$	$\lvert M_{\mathrm{N^2LO}}^{0\nu}/M_{\mathrm{L}}^{0\nu}\rvert$
^{76}Ge	4.83	-0.04–0.53	$\lesssim 10\%$
^{82}Se	4.30	0.28–0.44	$6\% - 10\%$
^{96}Zr	4.29	-0.04–0.42	$\lesssim 10\%$
^{100}Mo	3.52	-0.05–0.62	$\lesssim 18\%$
^{116}Cd	4.31	-0.02–0.29	$\lesssim 7\%$
^{124}Sn	5.12	-0.04–0.66	$\lesssim 13\%$
^{128}Te	3.99	-0.04–0.55	$\lesssim 14\%$
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Discovery, accelerated

TRIUMF

PRELIMINARY Genuine N^2LO Corrections in pnQRPA

Nucleus	$M^{0\nu}_{\rm L}$	$M^{0\nu}_{\rm N^2LO}$	$\lvert M^{0\nu}_{\rm N^2LO}/M^{0\nu}_{\rm L}\rvert$
^{76}Ge	4.83	-0.04–0.53	$\lesssim 10\%$
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Caveats:

Discovery, accelerated

TRIUMF

PRELIMINARY Genuine N^2LO Corrections in pnQRPA

Nucleus	$M^{0\nu}_{L}$	$M^{0\nu}_{N^2LO}$	$\lvert M^{0\nu}_{N^2LO}/M^{0\nu}_{L}\rvert$
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Caveats:

- Unknown parameters

Discovery, accelerated

TRIUMF

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LJ, D. Castillo,P. Soriano, J Menéndez, work in progress

Caveats:

- Unknown parameters
- Scale dependence

Discovery, accelerated

PRELIMINARY Genuine N^2LO Corrections in pnQRPA

Nucleus	$M_{\rm L}^{0\nu}$	$M_{\rm N^2LO}^{0\nu}$	$\lvert M_{\rm N^2LO}^{0\nu}/M_{\rm L}^{0\nu}\rvert$
^{76}Ge	4.83	-0.04–0.53	$\lesssim 10\%$
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^{136}Xe	2.60	-0.02–0.07	$\lesssim 3\%$

LJ, D. Castillo,P. Soriano, J Menéndez, work in progress

Caveats:

- Unknown parameters
- Scale dependence
- Regulator dependence

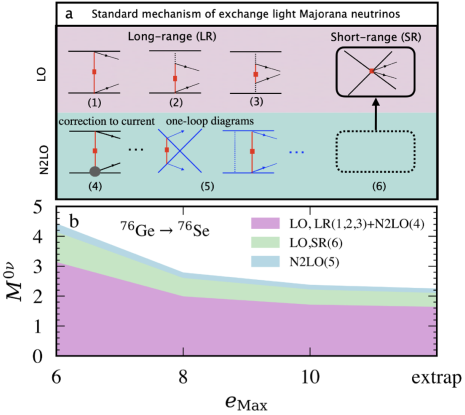
Similar effects found in *ab initio* studies

- In ^{76}Ge:

$$M^{0\nu}_{\rm S}/M^{0\nu}_{\rm L} \sim 40\%\ ,$$

$$M^{0\nu}_{\rm N^2LO}/M^{0\nu}_{\rm L} \sim 5\%$$

A. Belley et al. arXiv:2308.15634 (2023)



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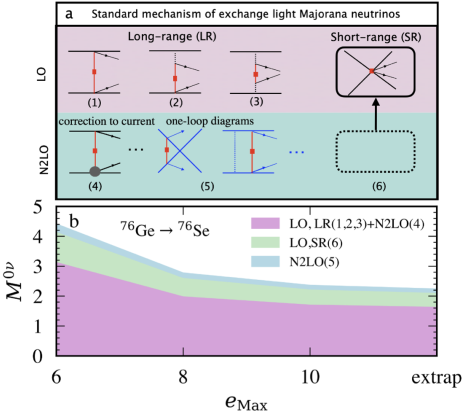
A. Belley et al. arXiv:2308.15634 (2023)

- In ^{130}Te and ^{136}Xe:

$$M^{0\nu}_{\mathrm{S}} / M^{0\nu}_{\mathrm{L}} \sim 20\% - 120\%$$

A. Belley et al. arXiv:2307.15156 (2023)



A. Belley et al. arXiv:2308.15634 (2023)

Discovery, accelerated

Outline

Introduction to double-beta decay

Corrections to $0\nu\beta\beta$-decay nuclear matrix elements

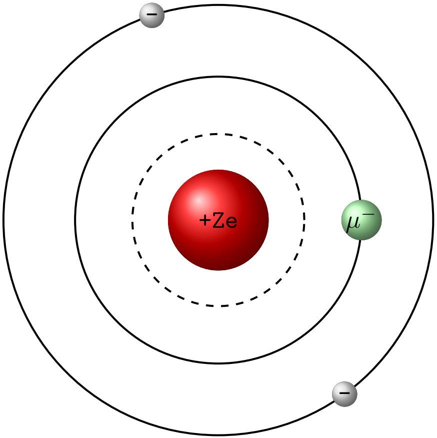
Muon capture as a probe of $0\nu\beta\beta$ decay

Summary and Outlook

TRIUMF

Ordinary Muon Capture (OMC)

- A muon can replace an electron in an atom, forming a *muonic atom*



Discovery, accelerated

TRIUMF

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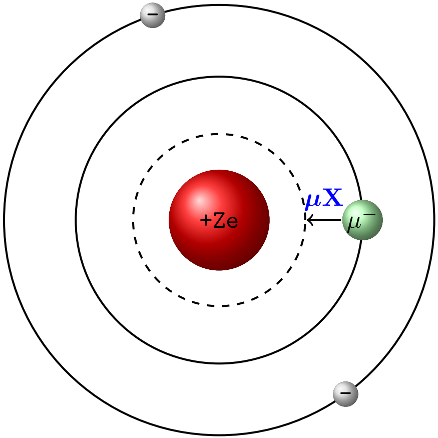
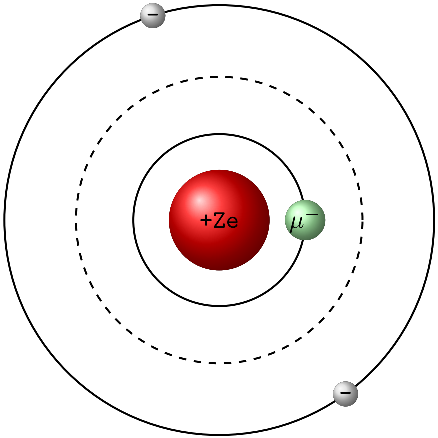


Discovery, accelerated

TRIUMF

Ordinary Muon Capture (OMC)

- A muon can replace an electron in an atom, forming a *muonic atom*
  - Eventually bound on **the $1s_{1/2}$ orbit**

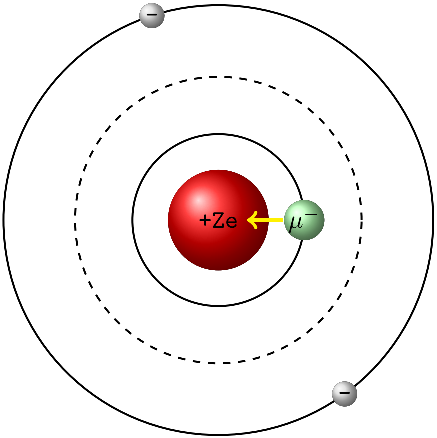


Discovery, accelerated

Ordinary Muon Capture (OMC)

- A muon can replace an electron in an atom, forming a *muonic atom*
  - Eventually bound on **the $1s_{1/2}$ orbit**
- The *muon can then be captured* by the nucleus

$$\mu^- + {}^{A}_{Z}\mathrm{X}(J_i^{\pi_i}) \rightarrow \nu_\mu + {}^{A}_{Z-1}\mathrm{Y}(J_f^{\pi_f})$$



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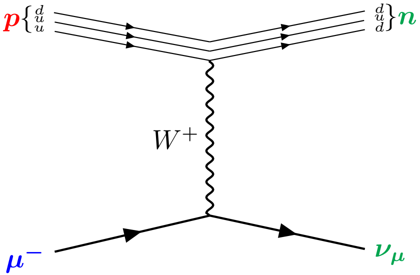

Ordinary Muon Capture (OMC)

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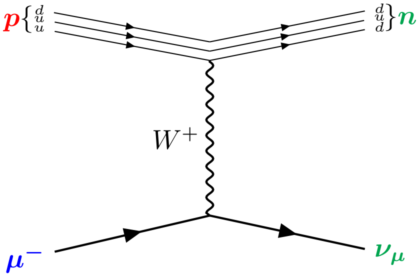
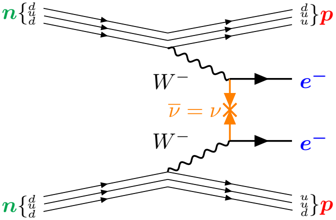


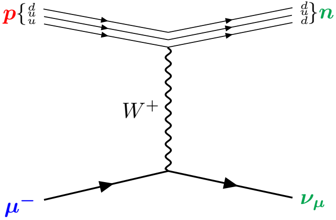
Ordinary = non-radiative

$$\left(\begin{array}{c} \text{Radiative muon capture (RMC):} \\ \mu^- + {}^{A}_{Z}\mathrm{X}(J_i^{\pi_i}) \rightarrow \nu_\mu + {}^{A}_{Z-1}\mathrm{Y}(J_f^{\pi_f}) + \gamma \end{array} \right)$$

$0\nu\beta\beta$ Decay vs. Muon Capture

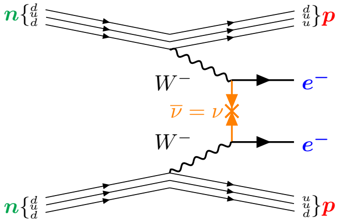


$$^{A}_{Z}\mathrm{X}(J_i^{\pi_i}) \rightarrow \ {}^{A}_{Z+2}\mathrm{X}'(J_f^{\pi_f}) + 2e^-$$

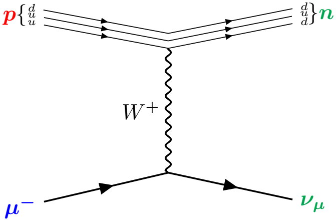


$$\mu^- + {}^{A}_{Z}\mathrm{X}(J_i^{\pi_i}) \rightarrow \nu_\mu + {}^{A}_{Z-1}\mathrm{Y}(J_f^{\pi_f})$$

$0\nu\beta\beta$ Decay vs. Muon Capture



$$ {}^{A}_{Z}\mathrm{X}(J_i^{\pi_i}) \rightarrow {}^{\quad A}_{Z+2}\mathrm{X}'(J_f^{\pi_f}) + 2e^- $$



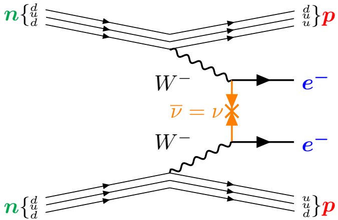
$$ \mu^- + {}^{A}_{Z}\,\mathrm{X}(J_i^{\pi_i}) \rightarrow \nu_\mu + {}^{\quad A}_{Z-1}\,\mathrm{Y}(J_f^{\pi_f}) $$

Both involve hadronic current:

$$ \langle \boldsymbol{p}|\, j^{\alpha\dagger}\, |\boldsymbol{p}\rangle = \bar{\Psi}\left[g_{\mathrm{V}}(q^2)\gamma^\alpha - g_{\mathrm{A}}(q^2)\gamma^\alpha\gamma_5 - g_{\mathrm{P}}(q^2)q^\alpha\gamma_5 + i g_{\mathrm{M}}(q^2)\frac{\sigma^{\alpha\beta}}{2m_p}q_\beta \right]\tau^\pm\Psi $$

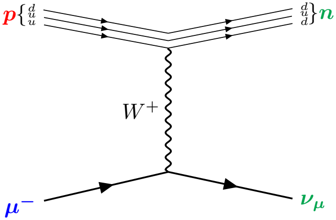
TRIUMF

$0\nu\beta\beta$ Decay vs. Muon Capture



$$^{A}_{Z}\mathrm{X}(J_i^{\pi_i}) \rightarrow \ _{Z+2}^{\ \ A}\mathrm{X}'(J_f^{\pi_f}) + 2e^-$$

- $q \approx 1/|\mathbf{r_1} - \mathbf{r_2}| \approx 100 - 200$ **MeV**



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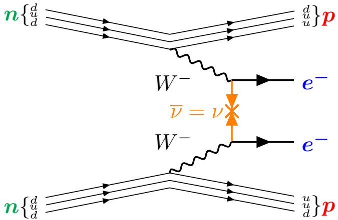
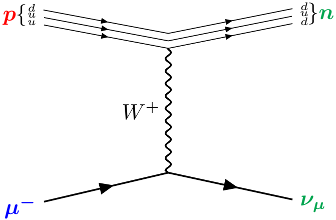
$$\langle \boldsymbol{p}|\, j^{\alpha\dagger}\, |\boldsymbol{p}\rangle = \bar{\Psi}\left[g_{\mathrm{V}}(q^2)\gamma^\alpha - g_{\mathrm{A}}(q^2)\gamma^\alpha\gamma_5 - g_{\mathrm{P}}(q^2)q^\alpha\gamma_5 + i g_{\mathrm{M}}(q^2)\frac{\sigma^{\alpha\beta}}{2m_p}q_\beta\right]\tau^\pm\Psi$$

Discovery, accelerated

TRIUMF

$0\nu\beta\beta$ Decay vs. Muon Capture





$$^A_Z\mathrm{X}(J_i^{\pi_i}) \rightarrow \; _{Z+2}^{\;\;A}\mathrm{X}'(J_f^{\pi_f}) + 2e^-$$

$$\mu^- + ^A_Z\mathrm{X}(J_i^{\pi_i}) \rightarrow \nu_\mu + _{Z-1}^{\;\;A}\mathrm{Y}(J_f^{\pi_f})$$

- $\boldsymbol{q} \approx 1/|\mathbf{r_1} - \mathbf{r_2}| \approx 100 - 200$ **MeV**
- $\boldsymbol{q} \approx m_\mu + M_i - M_f - m_e - E_X \approx 100$ **MeV**

Both involve hadronic current:

$$\langle \boldsymbol{p}| \, j^{\alpha\dagger} \, |\boldsymbol{p}\rangle = \bar{\Psi}\left[g_{\mathrm{V}}(q^2)\gamma^\alpha - g_{\mathrm{A}}(q^2)\gamma^\alpha\gamma_5 - g_{\mathrm{P}}(q^2)q^\alpha\gamma_5 + ig_{\mathrm{M}}(q^2)\frac{\sigma^{\alpha\beta}}{2m_p}q_\beta\right]\tau^\pm\Psi$$

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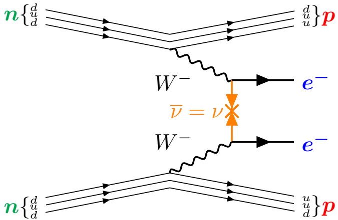

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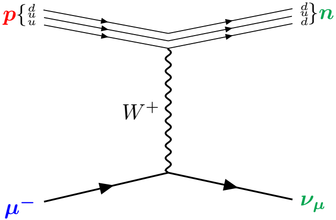
- **Yet hypothetical**



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Discovery, accelerated

TRIUMF

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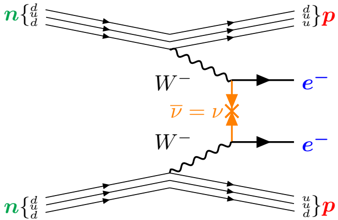

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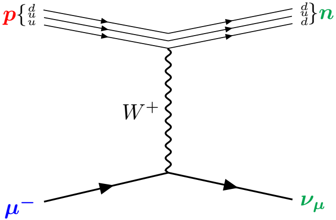

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- **Has been measured!**

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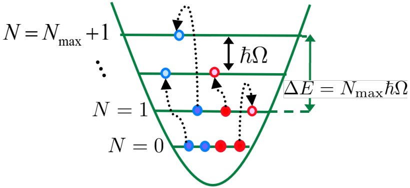
Discovery, accelerated

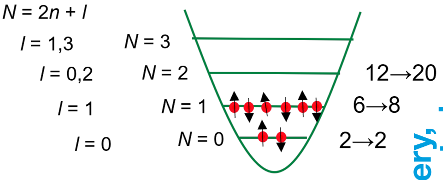
TRIUMF

Ab initio No-Core Shell Model (NCSM)

- Solve nuclear many-body problem

$$H^{(A)}\Psi^{(A)}(\mathbf{r}_1, \mathbf{r}_2, ..., \mathbf{r}_A) = E^{(A)}\Psi^{(A)}(\mathbf{r}_1, \mathbf{r}_2, ..., \mathbf{r}_A)$$





$$E = (2n + l + \tfrac{3}{2})\hbar\Omega$$

Figure courtesy of P. Navrátil

Discovery, accelerated

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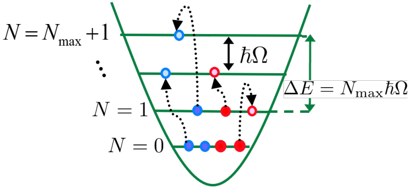

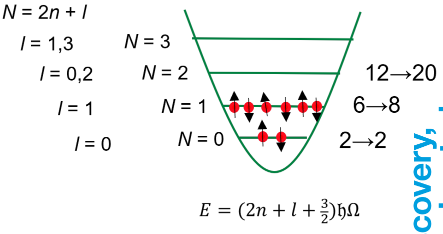

Figure courtesy of P. Navrátil

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- A-nucleon wave functions expanded in harmonic oscillator (HO) basis

$$\Psi^{(A)} = \sum_{N=0}^{N_{\max}} \sum_{j} c_{Nj} \Phi_{Nj}^{\mathrm{HO}}(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_A)$$

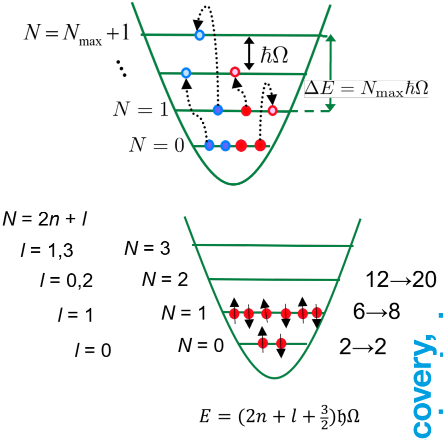

Figure courtesy of P. Navrátil

Dependency on the Harmonic-Oscillator Frequency

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TRIUMF

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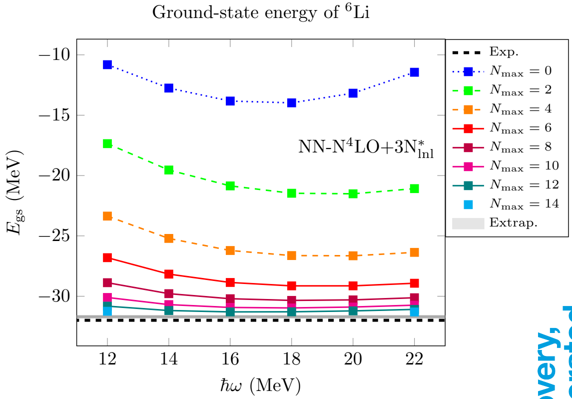
  - **Increasing $N_{\max}$ leads towards convergenced results**

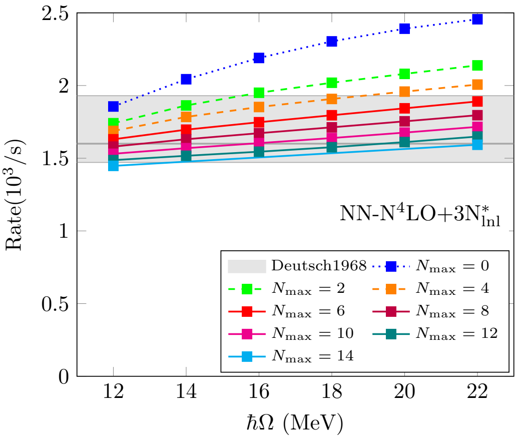


LJ, Navrátil, Kotila, Kravvaris, arXiv:2403.XXXX

Discovery, accelerated

Harmonic-Oscillator Frequency Dependence of Muon Capture

$^{6}\mathrm{Li}(1^{+}_{\mathrm{gs}}) + \mu^{-} \rightarrow {}^{6}\mathrm{He}(0^{+}_{\mathrm{gs}}) + \nu_{\mu}$



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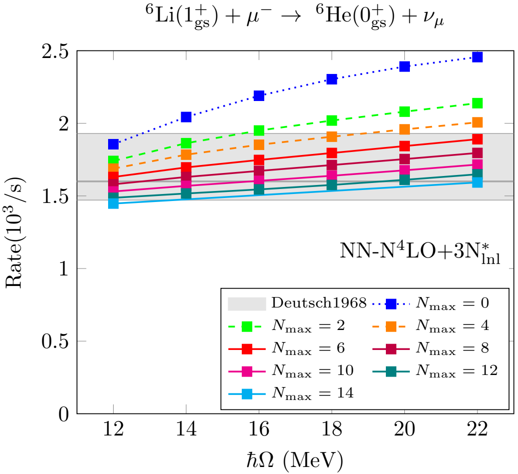

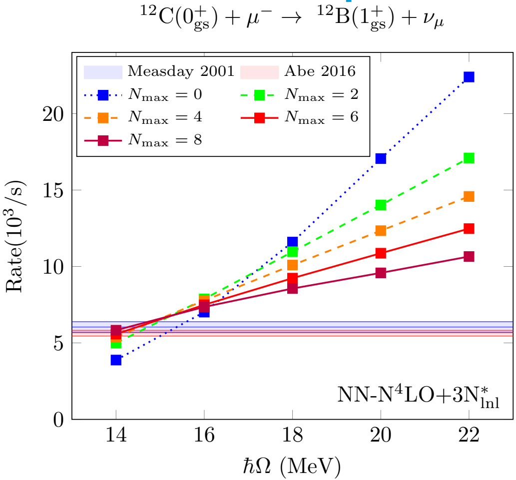

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Muon Capture on ^{6}Li

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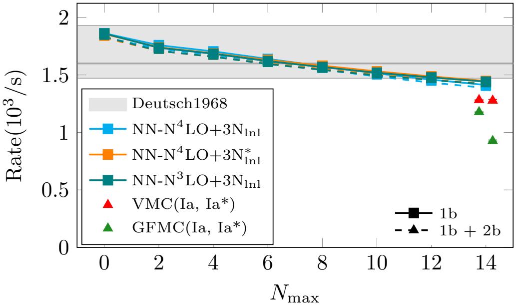

LJ, Navrátil, Kotila, Kravvaris, arXiv:2403.XXXX

TRIUMF

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King *et al.*, Phys. Rev. C **105**, L042501 (2022)

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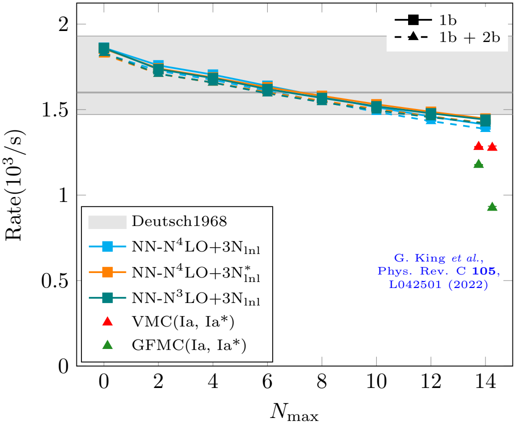

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Discovery, accelerated

TRIUMF

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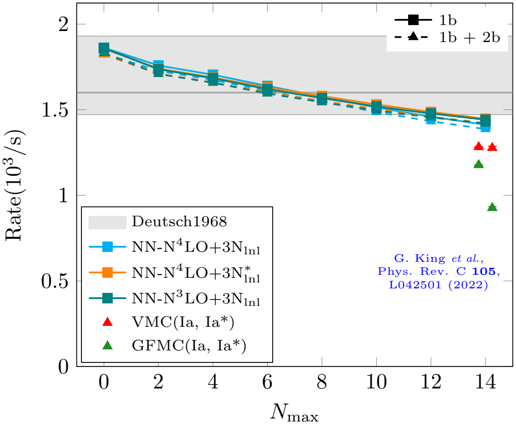

LJ, Navrátil, Kotila, Kravvaris, arXiv:2403.XXXX

Discovery, accelerated

TRIUMF

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- Slow convergence due to cluster-structure?
  - **NCSM with continuum (NCSMC) might give better results?**

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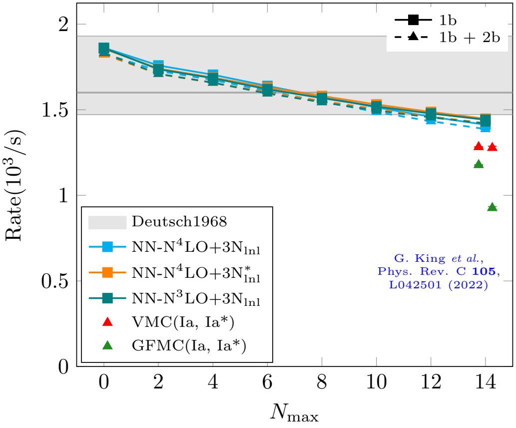


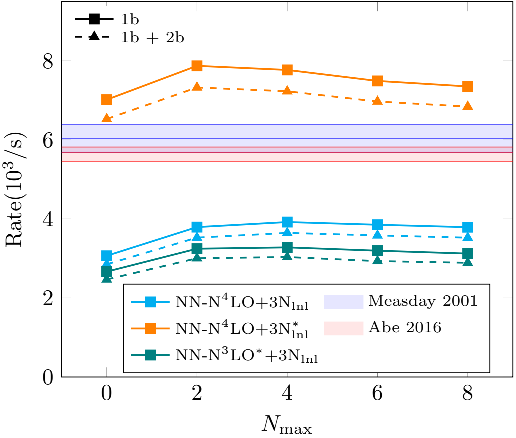
LJ, Navrátil, Kotila, Kravvaris, arXiv:2403.XXXX

Discovery, accelerated

Muon capture on ^{12}C

$$^{12}\mathrm{C}(0^+_{\mathrm{gs}}) + \mu^- \rightarrow \, ^{12}\mathrm{B}(1^+_{\mathrm{gs}}) + \nu_\mu$$

- The **NN-N^{4}LO+3N$^*_{\mathrm{lnl}}$** interaction with the additional spin-orbit 3N-force term most consistent with experiment



LJ, Navrátil, Kotila, Kravvaris, arXiv:2403.XXXX

TRIUMF

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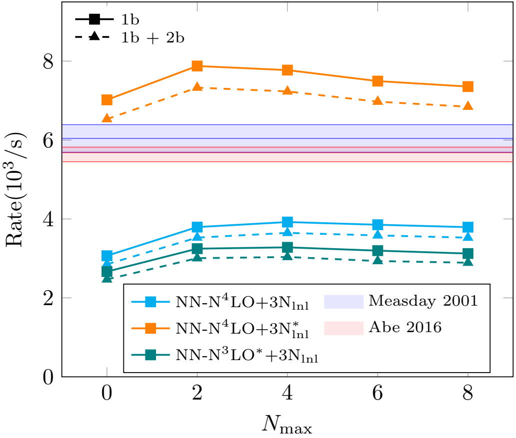

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Discovery, accelerated

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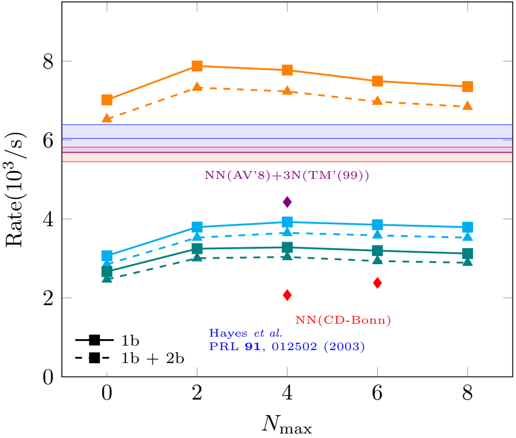
Hayes *et al.*, *Phys. Rev. Lett.* **91**, 012502 (2003)



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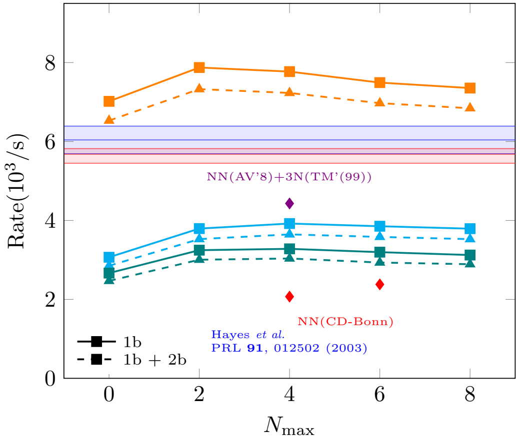
- 3N-forces essential to reproduce the measured rate



LJ, Navrátil, Kotila, Kravvaris, arXiv:2403.XXXX

Discovery, accelerated

TRIUMF

Muon capture on ^{16}O

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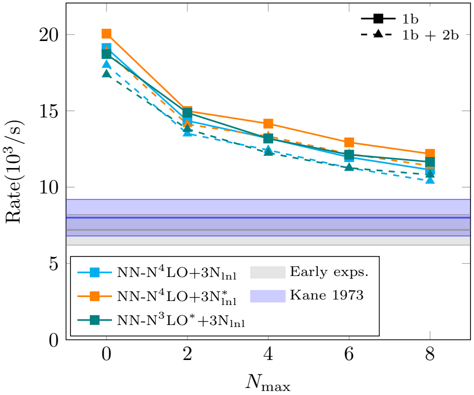

LJ, Navrátil, Kotila, Kravvaris, arXiv:2403.XXXX

Discovery, accelerated

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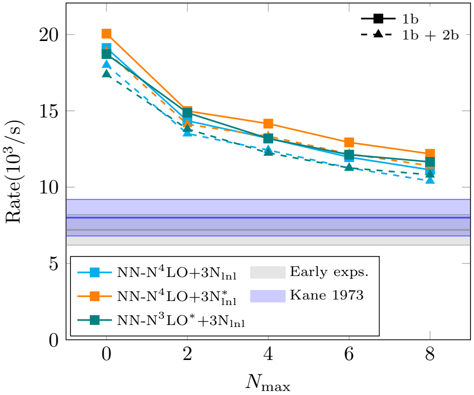

LJ, Navrátil, Kotila, Kravvaris, arXiv:2403.XXXX

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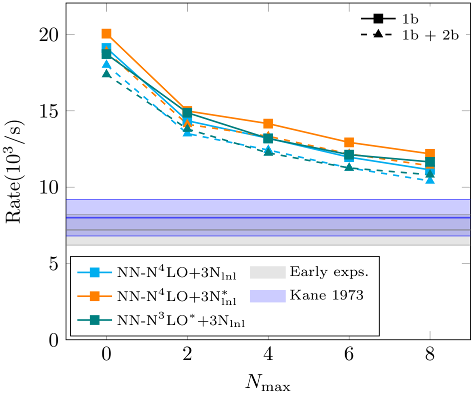
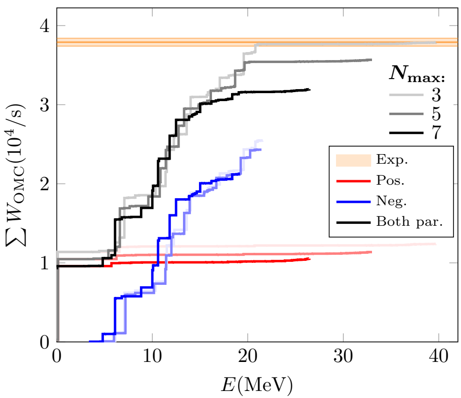


LJ, Navrátil, Kotila, Kravvaris, arXiv:2403.XXXX

Total Muon-Capture Rates

- Rates obtained summing over ~ 50 final states of each parity

$$\mu^- + {}^{12}\text{C}(0^+_{\text{gs}}) \to \nu_\mu + {}^{12}\text{B}(J^\pi_k)$$



LJ, Navrátil, Kotila, Kravvaris, arXiv:2403.XXXX

Discovery, accelerated

TRIUMF

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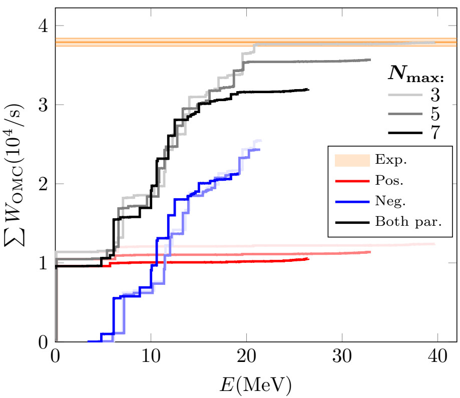

LJ, Navrátil, Kotila, Kravvaris, arXiv:2403.XXXX

Discovery, accelerated

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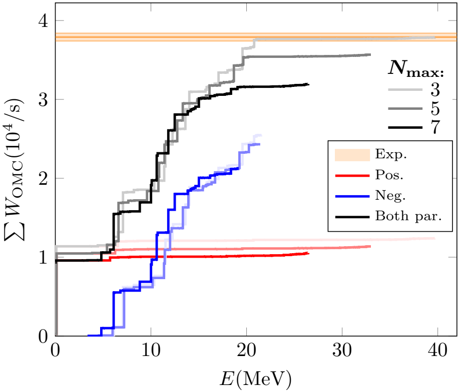
- Better estimation with the Lanczos strength function method underway



LJ, Navrátil, Kotila, Kravvaris, arXiv:2403.XXXX

Discovery, accelerated

Outline

Introduction to double-beta decay

Corrections to $0\nu\beta\beta$-decay nuclear matrix elements

Muon capture as a probe of $0\nu\beta\beta$ decay

Summary and Outlook

Discovery, accelerated

TRIUMF

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Discovery, accelerated

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TRIUMF

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- Ab initio muon-capture studies could shed light on g_{A} quenching at finite momentum exchange regime relevant for $0\nu\beta\beta$ decay

Discovery, accelerated

Outlook

- Study the effect of **vector two-body currents** (one-pion-exchange & pion-in-flight) on OMC rates

TRIUMF

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Discovery, accelerated

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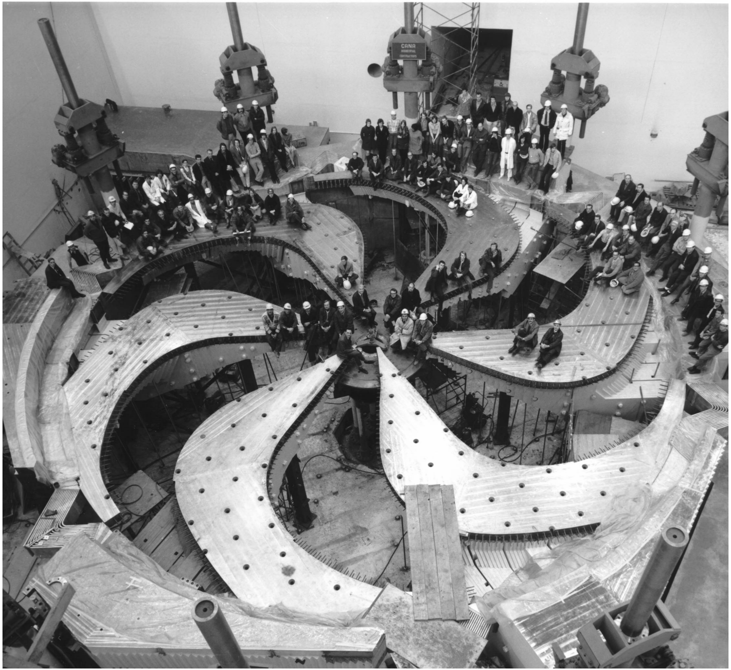
- Extend the NCSM studies to other processes
  - **^{16}N** potential candidate for **forbidden β-decay** studies (**ongoing**)
  - **^{12}C and ^{16}O** are both of interest in **neutrino-scattering experiments**

$$\left(\nu_\mu + {}^{12}\mathrm{C} \rightarrow \mu^- + {}^{12}\mathrm{N} \right)$$

TRIUMF

Thank you
Merci



Discovery, accelerated