

Neutrinoless double-beta decay and muon capture as a probe

Lotta Jokiniemi
TRIUMF, Theory Department
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Arthur B. McDonald
Canadian Astroparticle Physics Research Institute



Introduction

Corrections to $0\nu\beta\beta$ -decay nuclear matrix elements

The contact term

Contribution of ultrasoft neutrinos

Muon capture as a probe of $0\nu\beta\beta$ decay

VS-IMSRG Study on Muon Capture on ^{24}Mg

No-Core Shell-Model Studies on Muon Capture on Light Nuclei

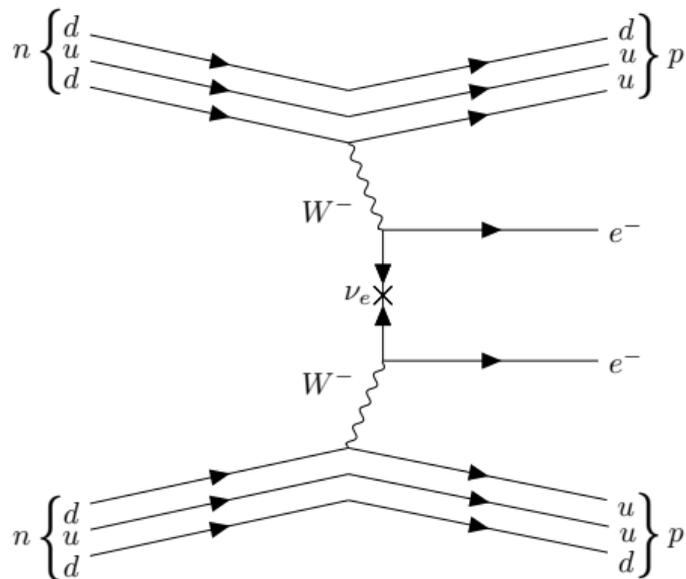
Phenomenological study on muon capture on ^{136}Ba

Summary and Outlook

Neutrinoless double-beta decay

$$\frac{1}{t_{1/2}^{0\nu}} = g_A^4 G^{0\nu} |M_L^{0\nu}|^2 \left(\frac{m_{\beta\beta}}{m_e} \right)^2$$

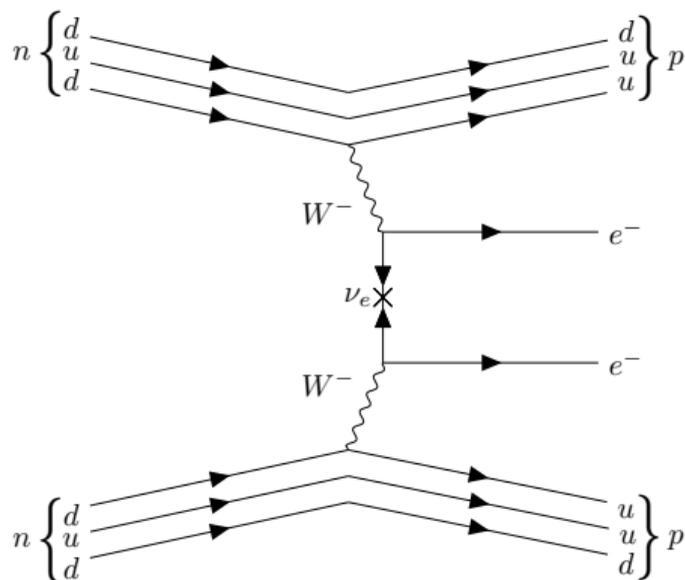
- Violates lepton-number conservation



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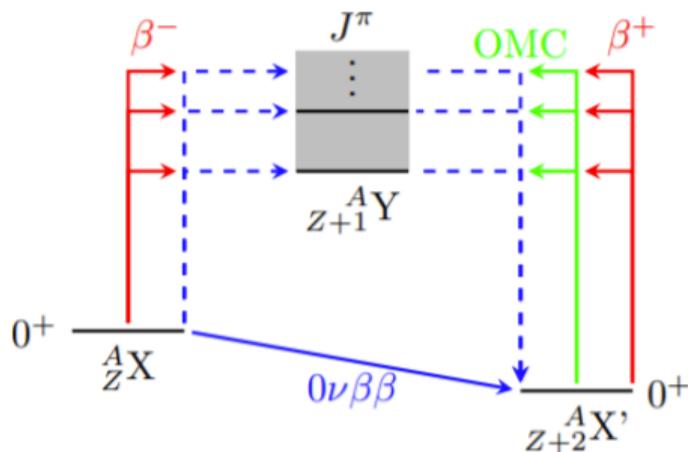
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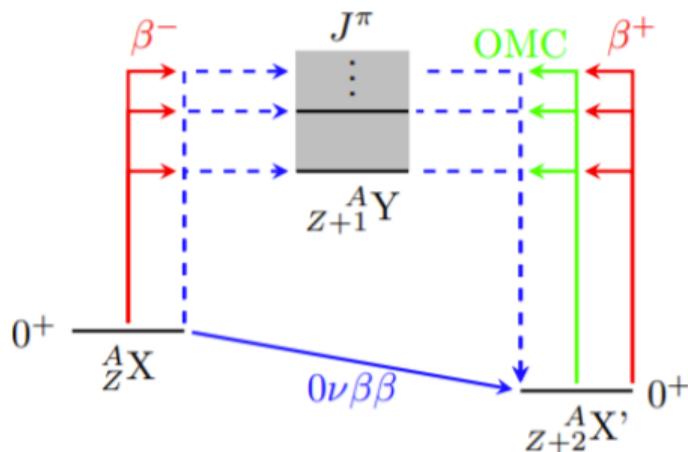
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- ▶ **Violates lepton-number conservation**
- ▶ Requires that **neutrinos are Majorana particles**
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- ▶ Momentum transfer $q \sim 100 \text{ MeV}$



Nuclear matrix elements of neutrinoless double-beta decay

$$M^{0\nu} = \frac{R}{g_A^2} \int \frac{d\mathbf{k}}{2\pi^2} \frac{e^{i\mathbf{k}(\mathbf{x}-\mathbf{y})}}{E_\nu} \sum_n \frac{\langle f | J_\mu(\mathbf{x}) | n \rangle \langle n | J^\mu(\mathbf{y}) | i \rangle}{E_\nu + E_n - \frac{1}{2}(E_i - E_f) - \frac{1}{2}(E_1 - E_2)}$$

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- ▶ Typically used **with most nuclear methods**

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New leading-order short-range nuclear matrix element

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$$M_S^{0\nu} = \frac{2R}{\pi g_A^2} \langle 0_f^+ | | \sum_{m,n} \tau_m^- \tau_n^- \int j_0(qr) h_S(q^2) q^2 dq | | 0_i^+ \rangle,$$

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Unknown coupling in the contact term

- ▶ Axial-vector coupling g_A known from $n \rightarrow p + e^- + \bar{\nu}_e$:

$$g_A = 1.2754(11)$$

D. Dubbers, B. Märkisch, Annu. Rev. Nucl. Part. Sci. 71, 139 (2021)

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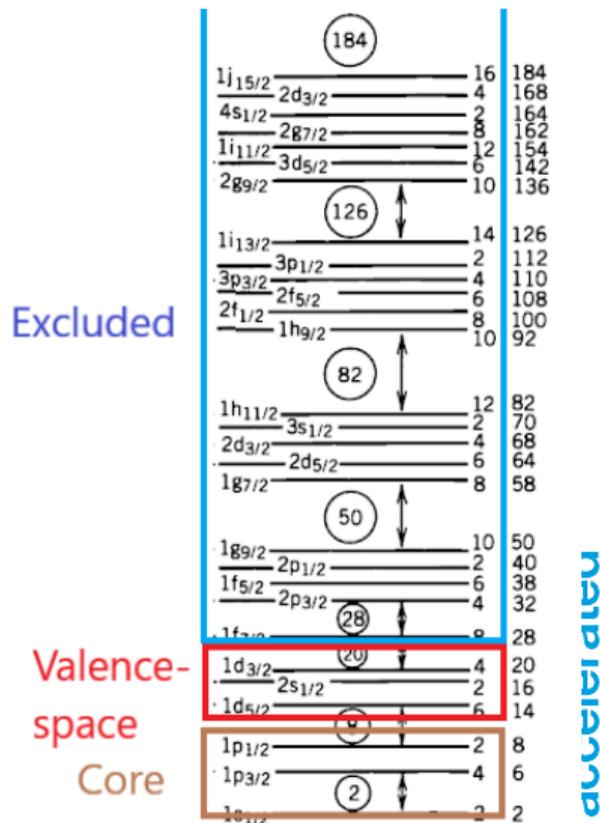
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- ▶ Use charge-independence breaking

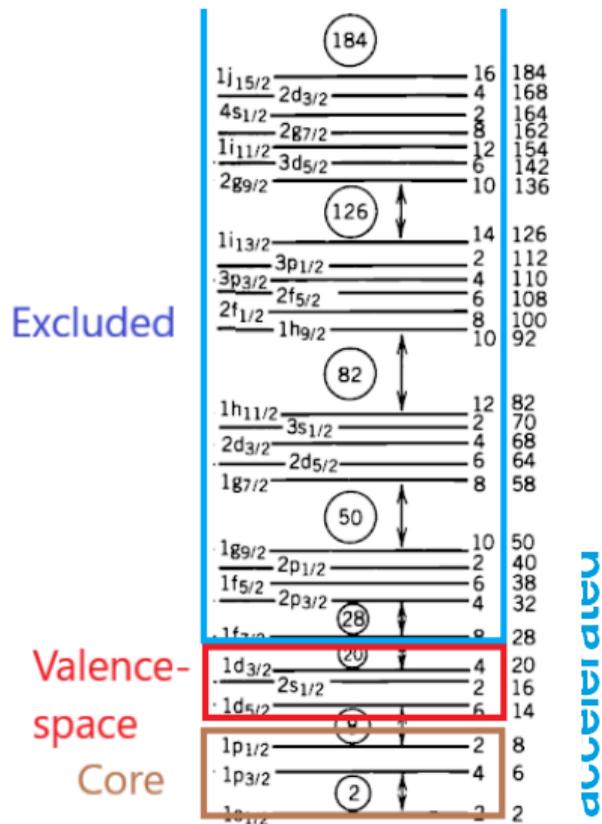
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► Nuclear Shell Model (NSM)



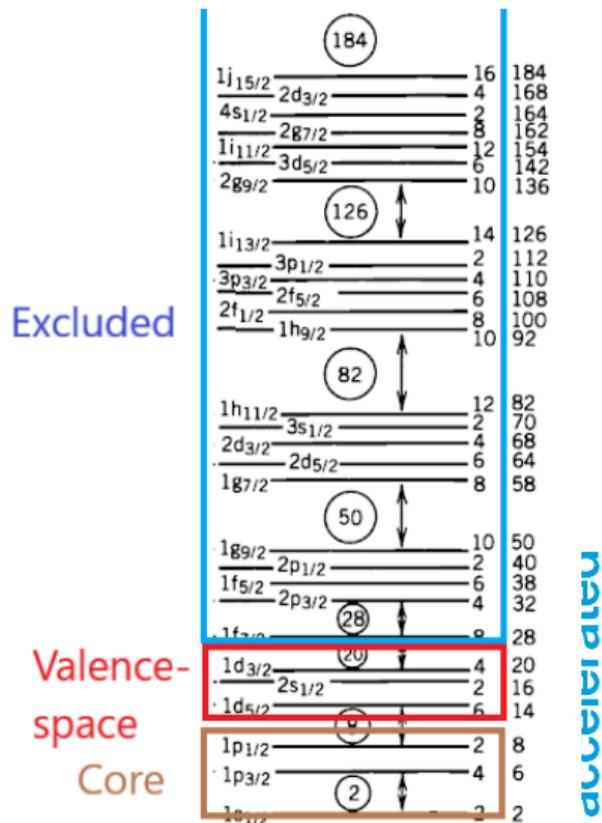
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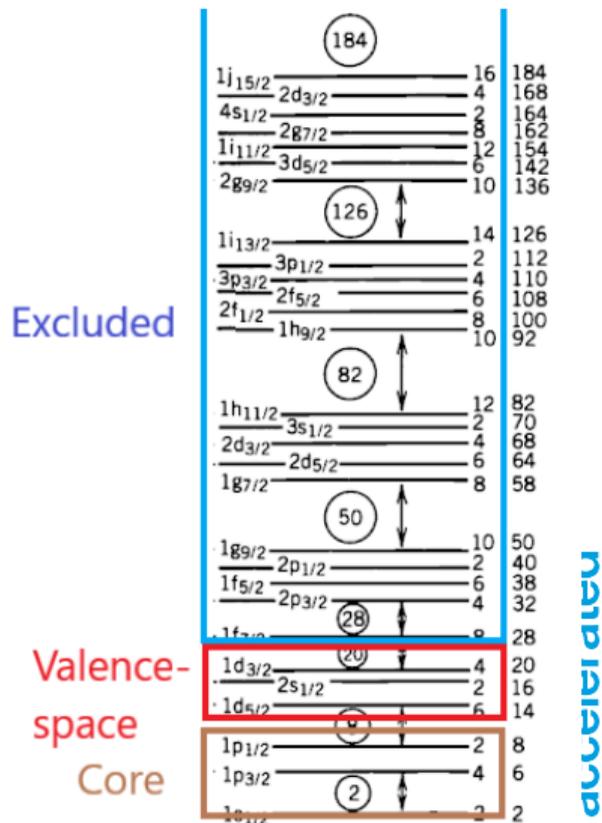
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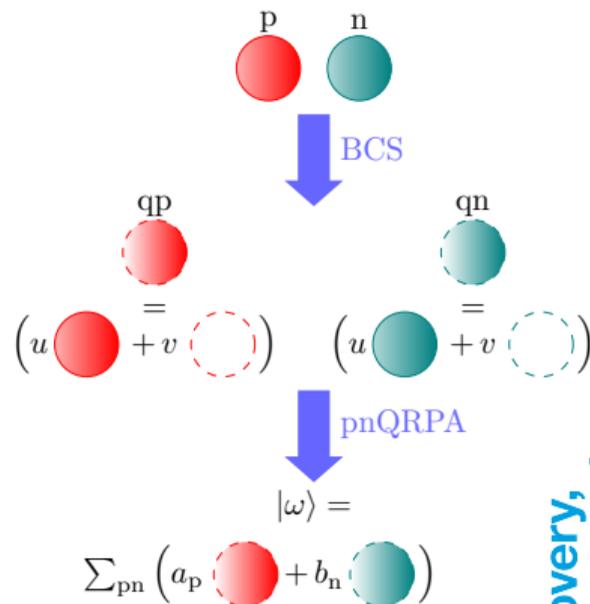
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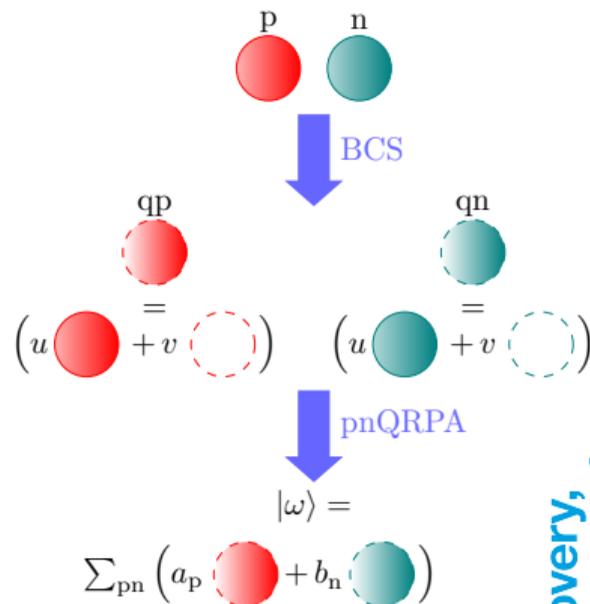


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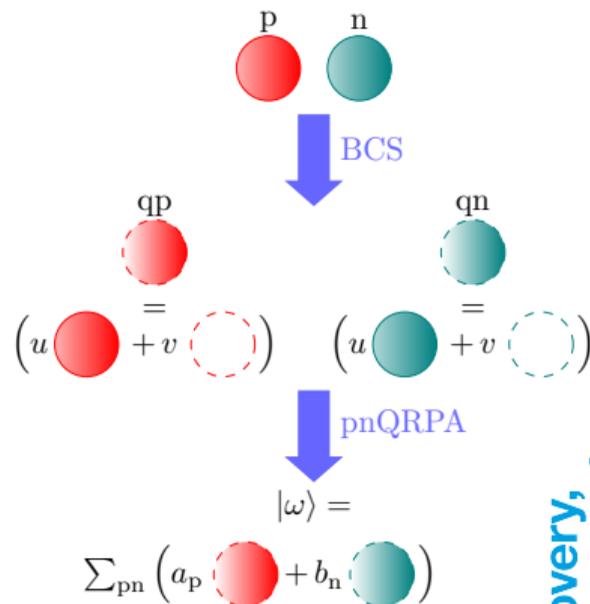


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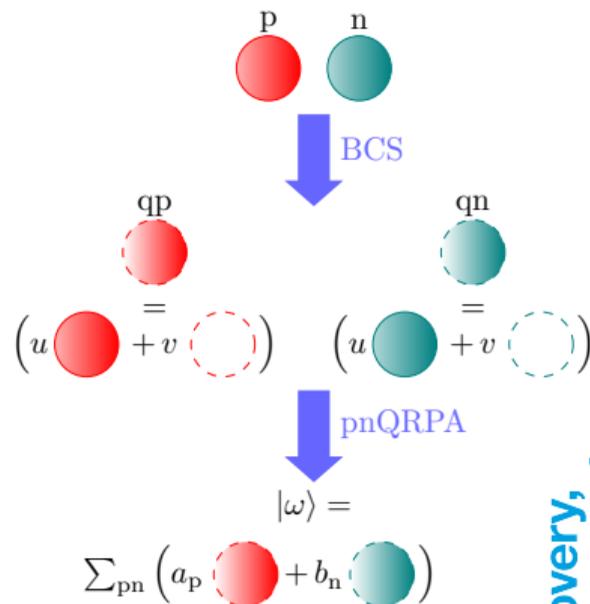


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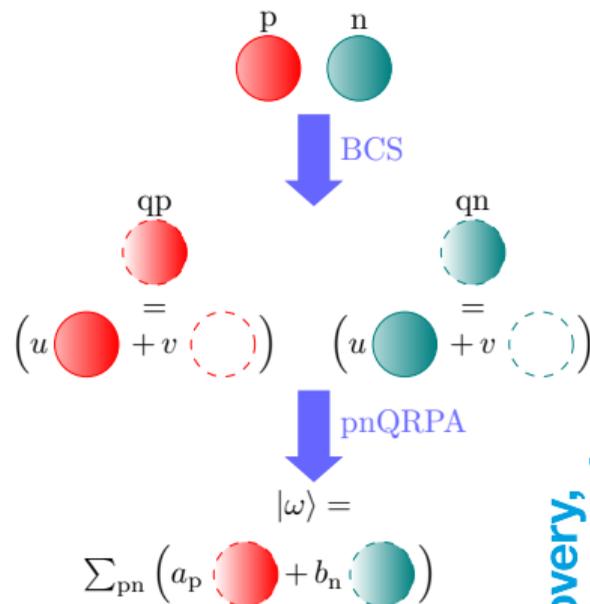
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▶ ...



Contact Term in pnQRPA and NSM

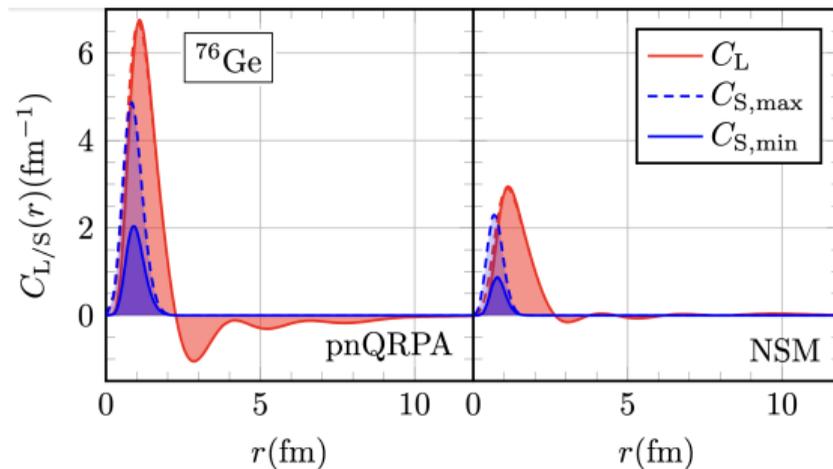
$$\int C_{L/S}(r)dr = M_{L/S}^{0\nu}$$

In pnQRPA:

$$M_S/M_L \approx 30\% - 80\%$$

In NSM:

$$M_S/M_L \approx 15\% - 50\%$$



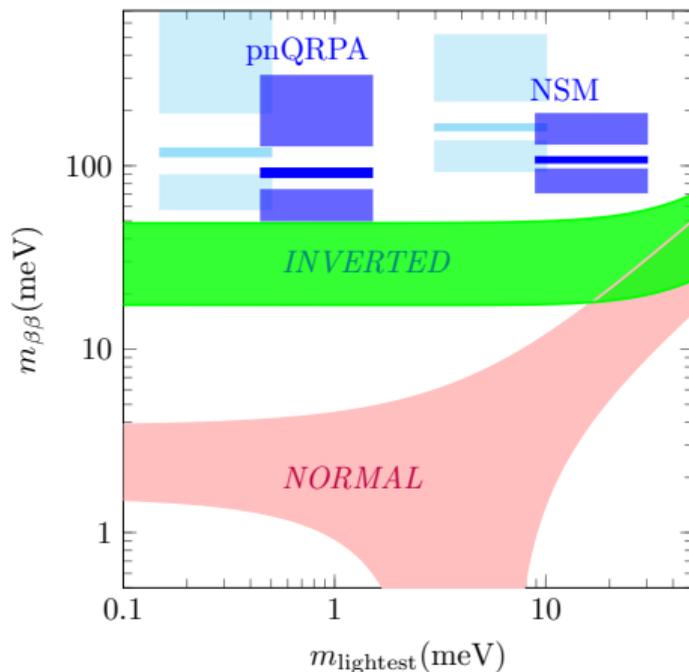
LJ, P. Soriano and J. Menéndez, Phys. Lett. B 823, 136720 (2021)

Effective Neutrino Masses

- ▶ Effective neutrino masses combining the likelihood functions of GERDA (^{76}Ge), CUORE (^{130}Te), EXO-200 (^{136}Xe) and KamLAND-Zen (^{136}Xe)

S. D. Biller, Phys. Rev. D **104**, 012002 (2021)

- ▶ Middle bands: $M_L^{(0\nu)}$
- Lower bands: $M_L^{(0\nu)} + M_S^{(0\nu)}$
- Upper bands: $M_L^{(0\nu)} - M_S^{(0\nu)}$



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Contribution of ultrasoft neutrinos

- Contribution of ultrasoft neutrinos ($|\mathbf{k}| \ll k_F$) to $0\nu\beta\beta$ decay:

V. Cirigliano et al., Phys. Rev. C 97, 065501 (2018)

$$M_{\text{usoft}}^{0\nu} = -\frac{\pi R}{g_A^2} \sum_n \frac{d^{d-1}k}{(2\pi)^{d-1}} \frac{1}{|\mathbf{k}|} \left[\frac{\langle f | J_\mu | n \rangle \langle n | J^\mu | i \rangle}{|\mathbf{k}| + E_2 + E_n - E_i - i\eta} + \frac{\langle f | J_\mu | n \rangle \langle n | J^\mu | i \rangle}{|\mathbf{k}| + E_1 + E_n - E_i - i\eta} \right]$$

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- Keeping only $\mathbf{k} = 0$ term in the current and assuming $E_1 = E_2 = Q_{\beta\beta}/2 + m_e$:

$$M_{\text{usoft}}^{0\nu}(\mu_{\text{us}}) = \frac{R}{2\pi} \sum_n \langle f | \sum_a \sigma_a \tau_a^+ | n \rangle \langle n | \sum_b \sigma_b \tau_b^+ | i \rangle \\ \times 2 \left(\frac{Q_{\beta\beta}}{2} + m_e + E_n - E_i \right) \left(\ln \frac{\mu_{\text{us}}}{2 \left(\frac{Q_{\beta\beta}}{2} + m_e + E_n - E_i \right)} + 1 \right)$$

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- We take $\mu_{\text{us}} = m_\pi \sim k_F \sim 100 \text{ MeV}$

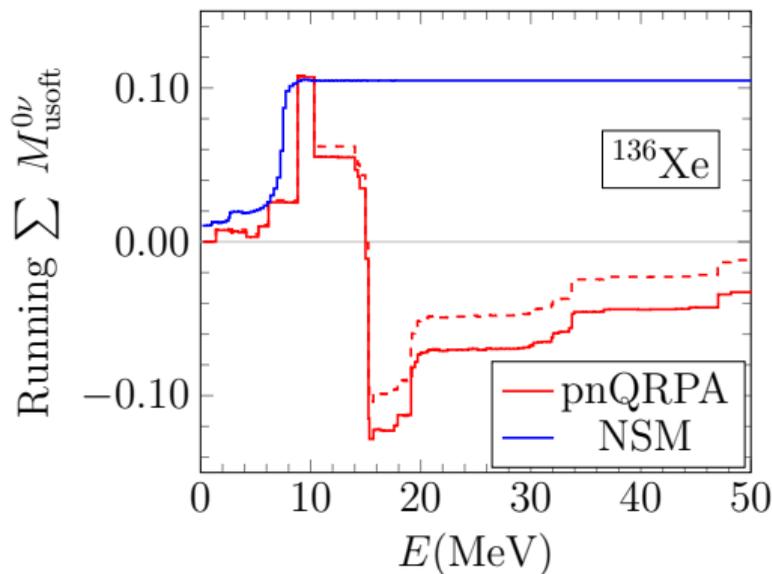
Ultrasoft neutrinos in pnQRPA and nuclear shell model

In pnQRPA:

$$|M_{\text{usoft}}^{0\nu} / M_{\text{L}}^{0\nu}| \approx 1\% - 15\%$$

In NSM:

$$|M_{\text{usoft}}^{0\nu} / M_{\text{L}}^{0\nu}| \approx 1\% - 5\%$$



LJ, P. Soriano, J Menéndez, work in progress

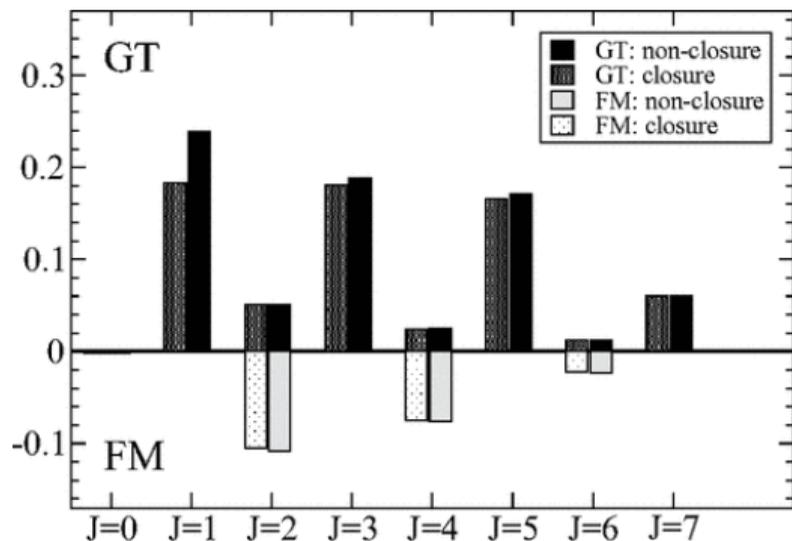
Ultrasoft neutrinos as correction of the closure approximation

- ▶ In nuclear shell model, using **closure approximation** typically decreases

$M_L^{0\nu}$ by $\sim 10\%$

R. A. Sen'kov, M. Horoi, Phys. Rev. C 88, 064312 (2013), Phys. Rev. C

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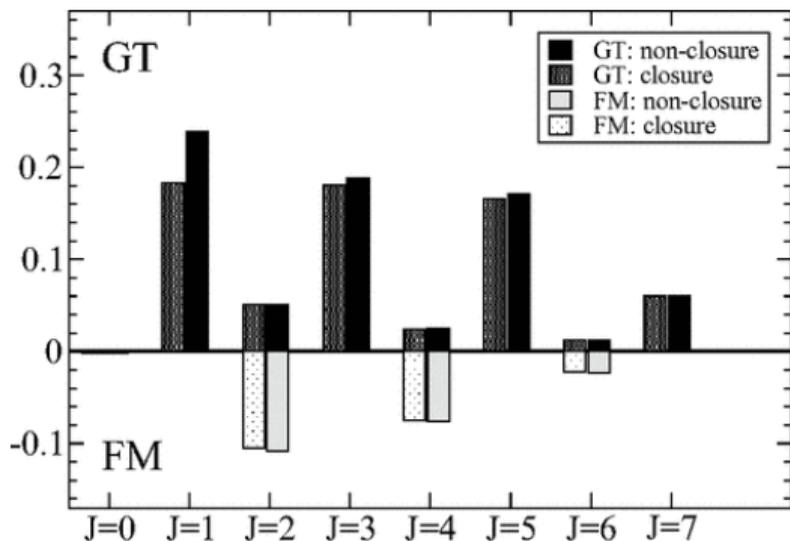
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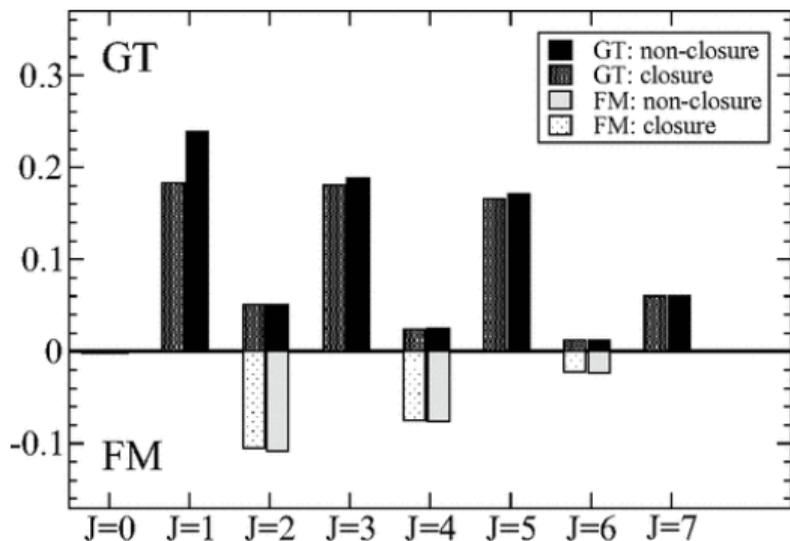
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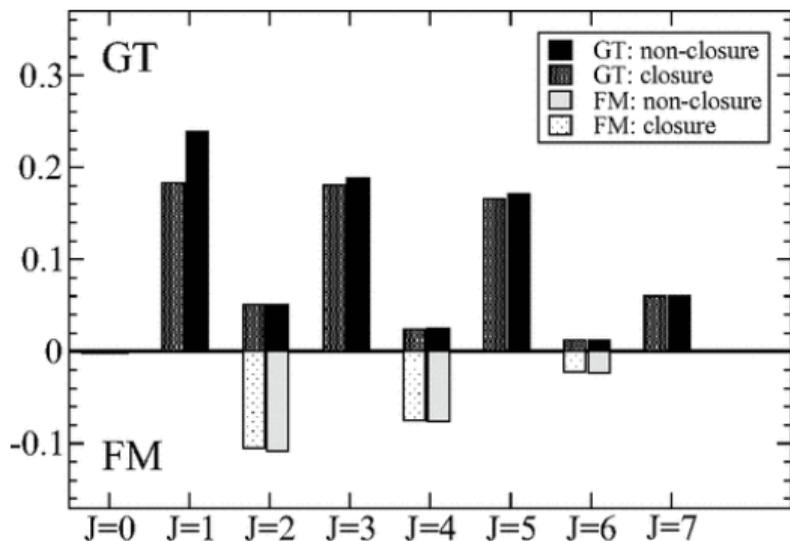
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→ **TODO:** compare $M_L^{0\nu} - M_{L,\text{cl}}^{0\nu}$ with $M_{\text{usoft}}^{0\nu}$ in pnQRPA



R. A. Sen'kov, M. Horoi, Phys. Rev. C 88, 064312 (2013)

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Corrections to $0\nu\beta\beta$ -decay nuclear matrix elements

The contact term

Contribution of ultrasoft neutrinos

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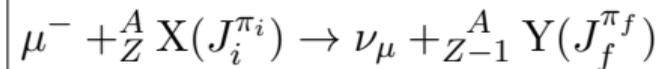
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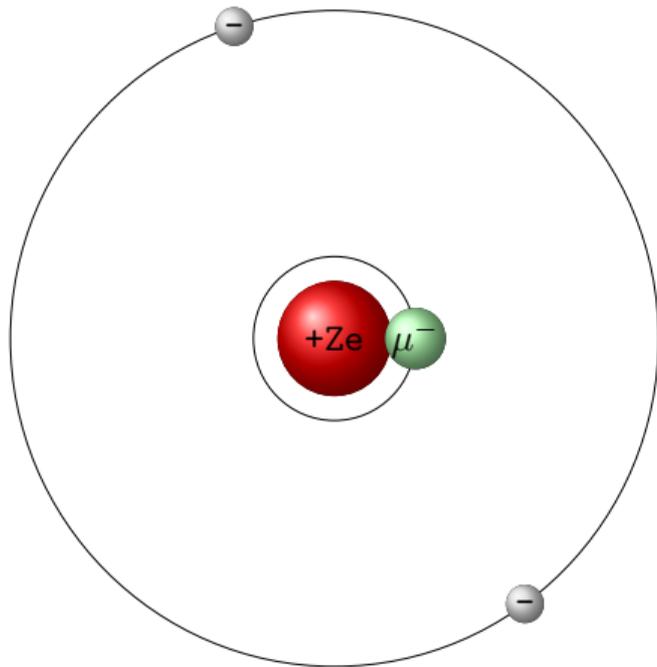
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Summary and Outlook

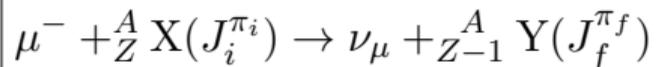
Ordinary Muon Capture (OMC)



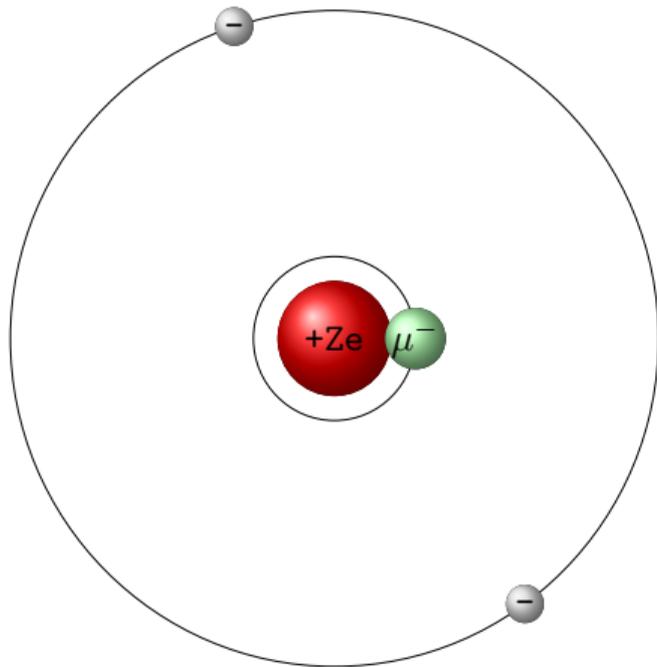
- ▶ A muon can replace an electron in an atom, forming a *muonic atom*



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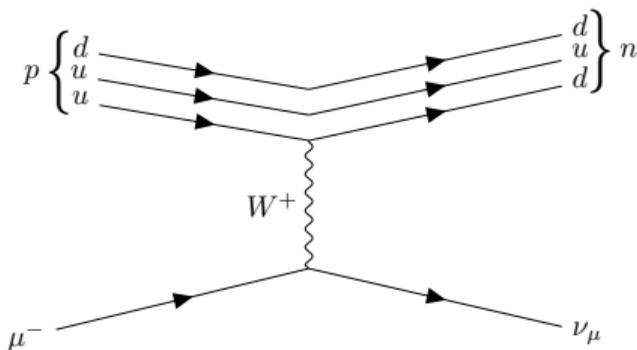
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 - ▶ Eventually bound on **the $1s_{1/2}$ orbit**



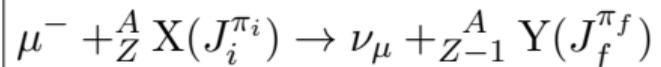
Ordinary Muon Capture (OMC)

$$\mu^- + {}^A_Z X(J_i^{\pi_i}) \rightarrow \nu_\mu + {}^A_{Z-1} Y(J_f^{\pi_f})$$

- ▶ A muon can replace an electron in an atom, forming a *muonic atom*
 - ▶ Eventually bound on **the $1s_{1/2}$ orbit**
- ▶ The muon can then be captured by the positively charged nucleus

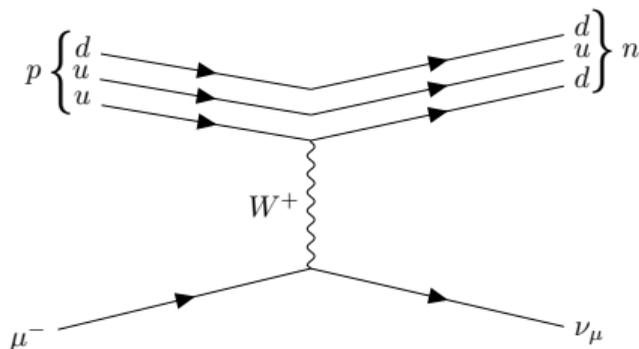
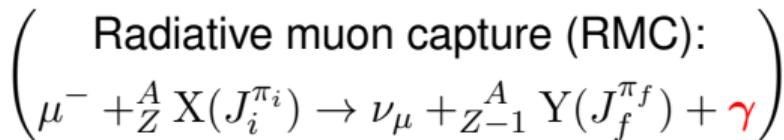


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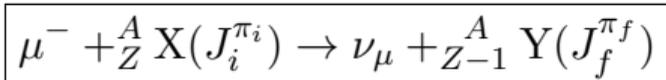
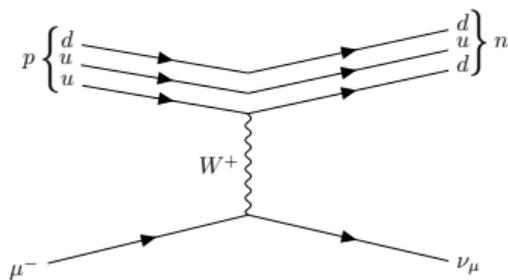


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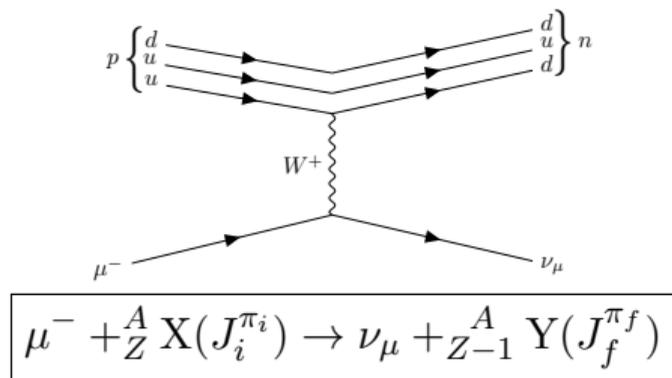
Ordinary = non-radiative



Ordinary Muon Capture (OMC) vs. $0\nu\beta\beta$

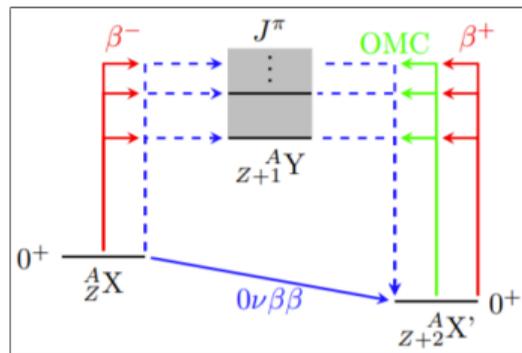
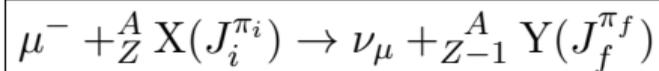
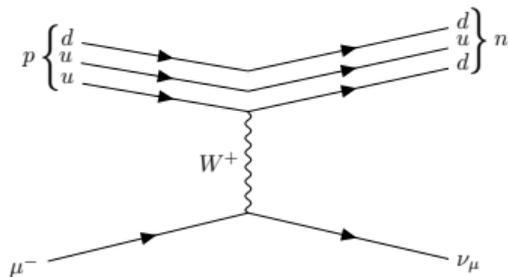


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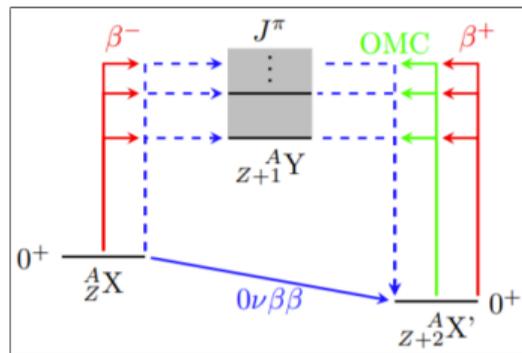
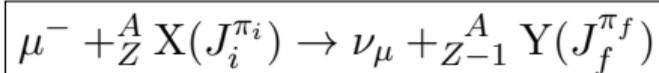
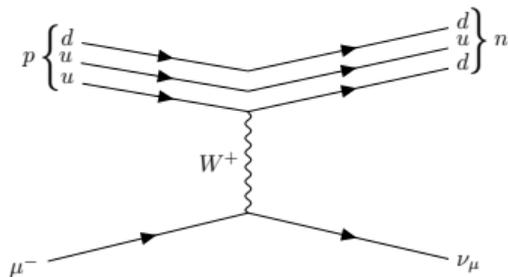
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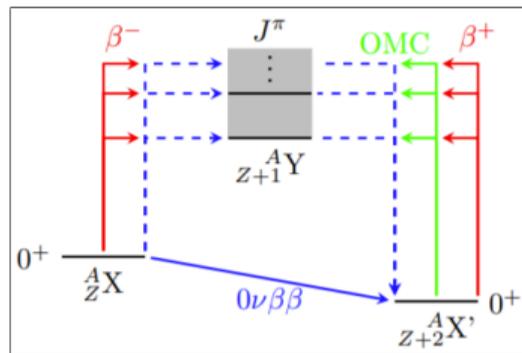
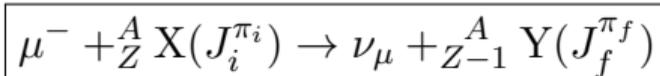
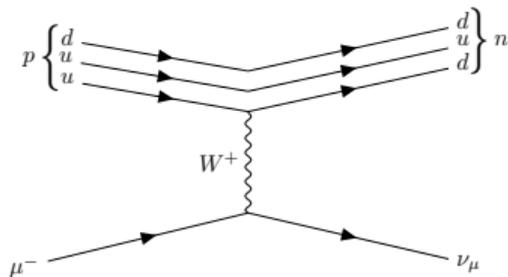
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- ▶ **Both the axial vector coupling g_A and the pseudoscalar coupling g_P involved**

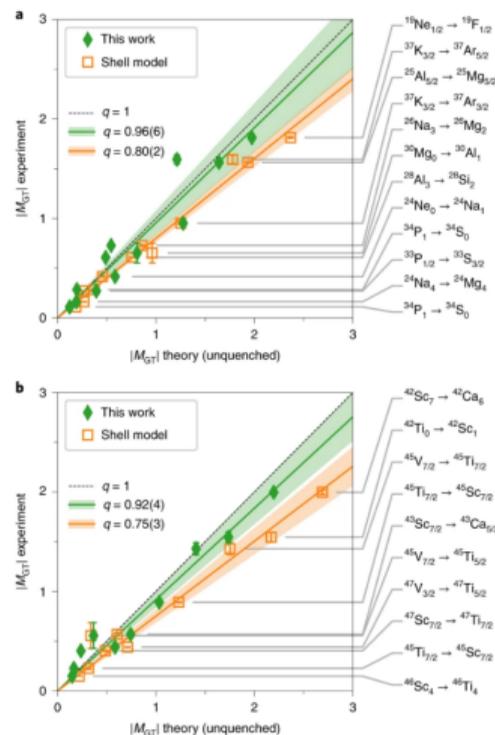
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- ▶ **Both the axial vector coupling g_A and the pseudoscalar coupling g_P involved**
 - Similar to $0\nu\beta\beta$ decay!

- Recently, **first *ab initio* solution to g_A quenching puzzle** was proposed for β -decay

P. Gysbers *et al.*, *Nature Phys.* **15**, 428 (2019)

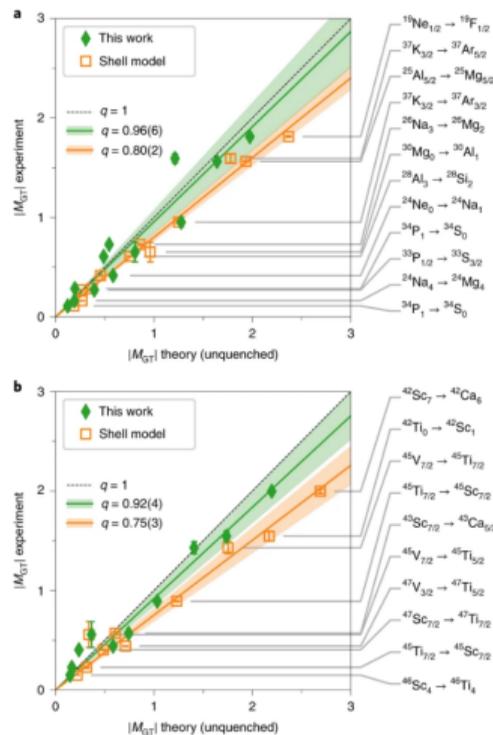


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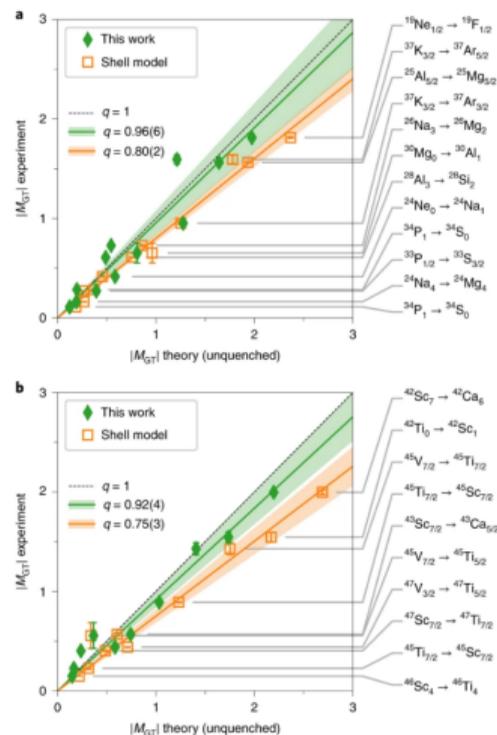


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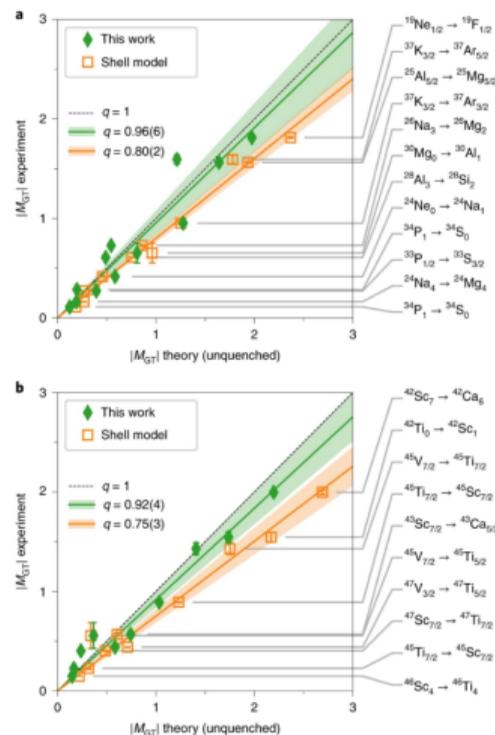


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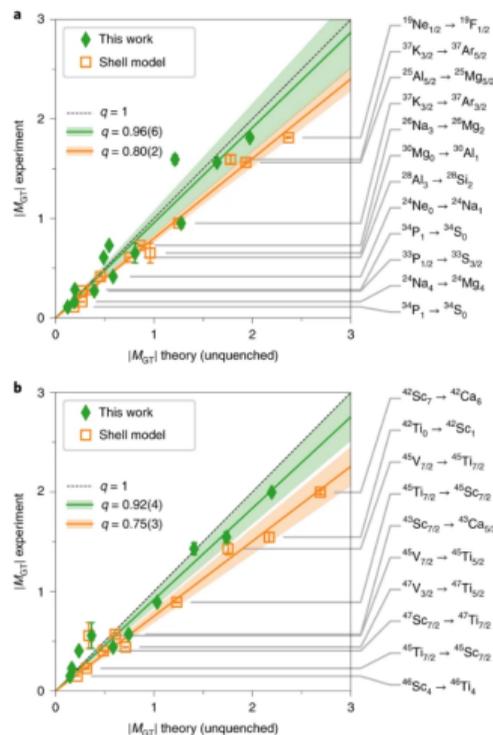


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- ▶ Solution: adding **two-body currents** and **missing correlations**
- ▶ How about **g_A quenching** at high momentum transfer ≈ 100 MeV?
 - ▶ **OMC could provide a hint!**
- ▶ In principle, one could also access the pseudoscalar coupling g_P



Gysbers et al., Nature Phys. 15, 428 (2019)

- Interaction Hamiltonian → capture rate:

$$W(J_i \rightarrow J_f) = \frac{2J_f + 1}{2J_i + 1} \left(1 - \frac{q}{m_\mu + AM} \right) q^2 \sum_{\kappa u} |g_V M_V(\kappa, u) + g_M M_M(\dots) + g_A M_A(\dots) + g_P M_P(\dots)|^2$$

PHYSICAL REVIEW

VOLUME 118, NUMBER 2

APRIL 15, 1960

Theory of Allowed and Forbidden Transitions in Muon Capture Reactions*

MASATO MORITA

Columbia University, New York, New York

AND

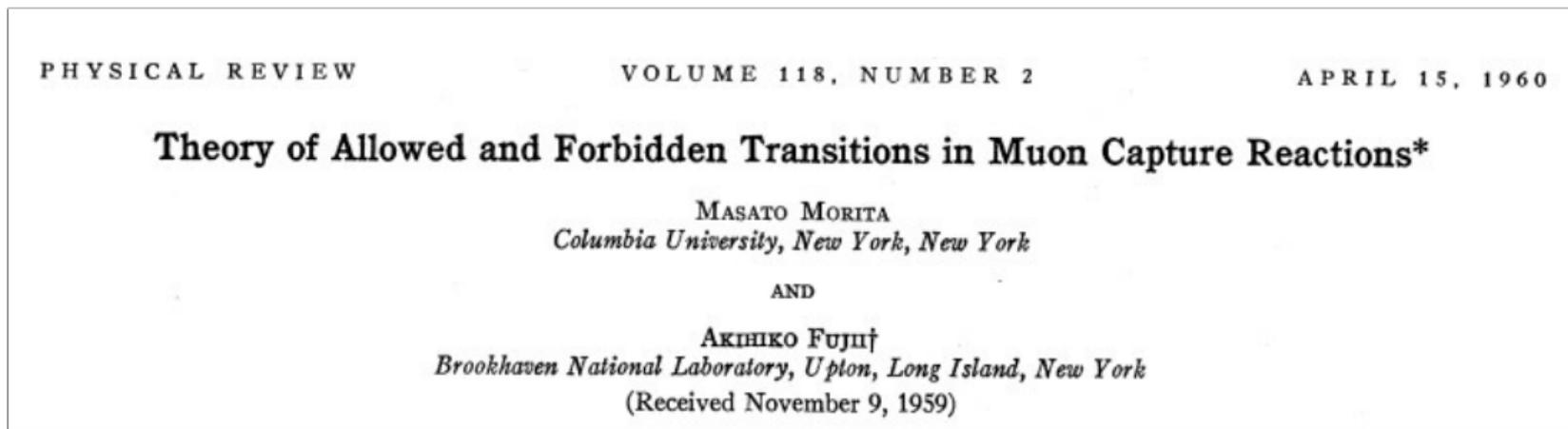
AKIHIKO FUJII†

Brookhaven National Laboratory, Upton, Long Island, New York

(Received November 9, 1959)

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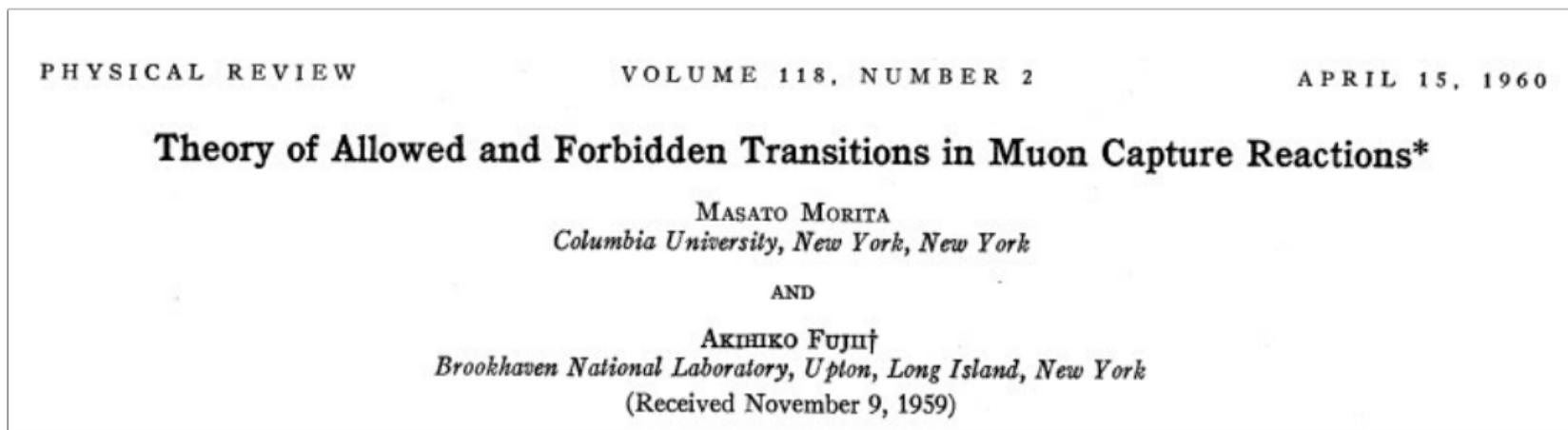
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- ▶ Use realistic bound-muon wave functions

- ▶ Interaction Hamiltonian → capture rate:

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- ▶ Use **realistic bound-muon wave functions**
- ▶ Add the effect of **two-body currents**

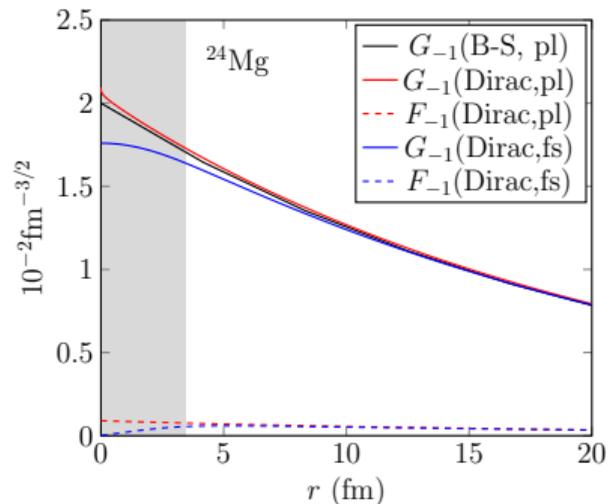
Bound-Muon Wave Functions

- Expand the muon wave function in terms of spherical spinors

$$\psi_{\mu}(\kappa, \mu; \mathbf{r}) = \psi_{\kappa\mu}^{(\mu)} = \begin{bmatrix} -iF_{\kappa}(r)\chi_{-\kappa\mu} \\ G_{\kappa}(r)\chi_{\kappa\mu} \end{bmatrix},$$

where $\kappa = -j(j+1) + l(l+1) - \frac{1}{4}$
 ($\kappa = -1$ for the $1s_{1/2}$ orbit)

B-S = Bethe-Salpeter: $G_{-1} = 2(\alpha Z m'_{\mu})^{\frac{3}{2}} e^{-\alpha Z m'_{\mu} r}$
pl = pointlike
fs = finite size nucleus



LJ, Miyagi, Stroberg, Holt, Kotila, Suhonen,
 Phys. Rev. C **107**, 014327 (2023)

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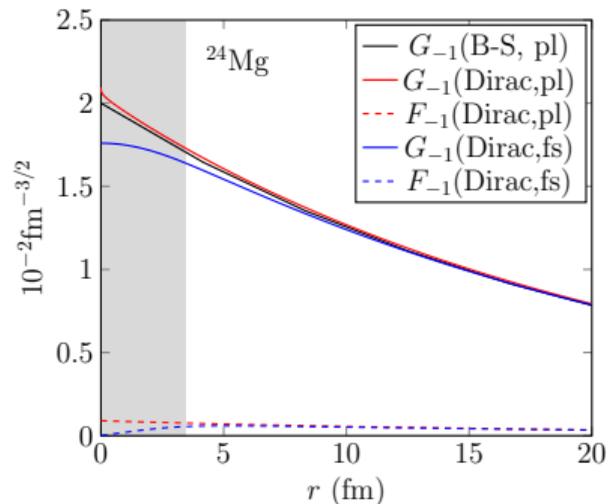
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- Solve the Dirac equations in the Coulomb potential $V(r)$:

$$\begin{cases} \frac{d}{dr}G_{-1} + \frac{1}{r}G_{-1} = \frac{1}{\hbar c}(mc^2 - E + V(r))F_{-1} \\ \frac{d}{dr}F_{-1} - \frac{1}{r}F_{-1} = \frac{1}{\hbar c}(mc^2 + E - V(r))G_{-1} \end{cases}$$

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LJ, Miyagi, Stroberg, Holt, Kotila, Suhonen,
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- One-body currents

$$\mathbf{J}_{i,1b}^3 = \frac{\tau_i^3}{2} \left(g_A \boldsymbol{\sigma}_i - \frac{g_P}{2m_N} \mathbf{q} \cdot \boldsymbol{\sigma}_i \right)$$

+ two-body currents

$$\mathbf{J}_{i,2b}^{\text{eff}} = g_A \frac{\tau_i^3}{2} \left[\delta a(\mathbf{q}^2) \boldsymbol{\sigma}_i + \frac{\delta a^P(\mathbf{q}^2)}{q^2} (\mathbf{q} \cdot \boldsymbol{\sigma}_i) \mathbf{q} \right]$$

Hoferichter, Klos, Schwenk Phys. Lett. B 746, 410 (2015)

Axial-Vector Two-Body Currents (2BCs)

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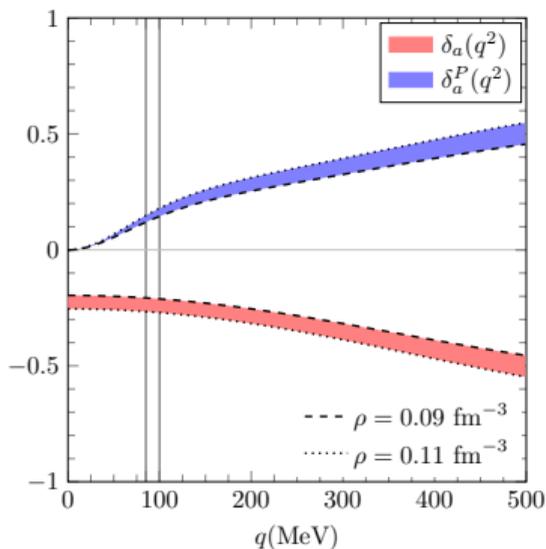
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- Two-body currents approximated by

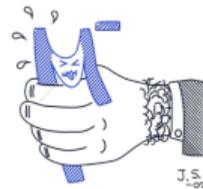
$$\left\{ \begin{array}{l} g_A \rightarrow (1 + \delta a(\mathbf{q}^2)) g_A, \\ g_P \rightarrow (1 - \frac{q^2 + m_\pi^2}{q^2} \delta a^P(\mathbf{q}^2)) g_P \end{array} \right.$$



LJ, Miyagi, Stroberg, Holt, Kotila, Suhonen, Phys. Rev. C 107, 014327 (2023)

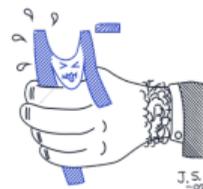
Muon-Capture Studies at PSI, Switzerland

- ▶ Most muon-capture experiments date back to \sim 1960s – 1990s



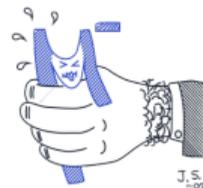
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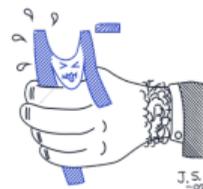
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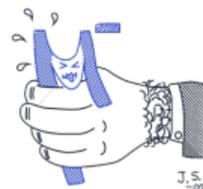


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 - ▶ Potentially partial capture rates for ^{12}C , ^{13}C



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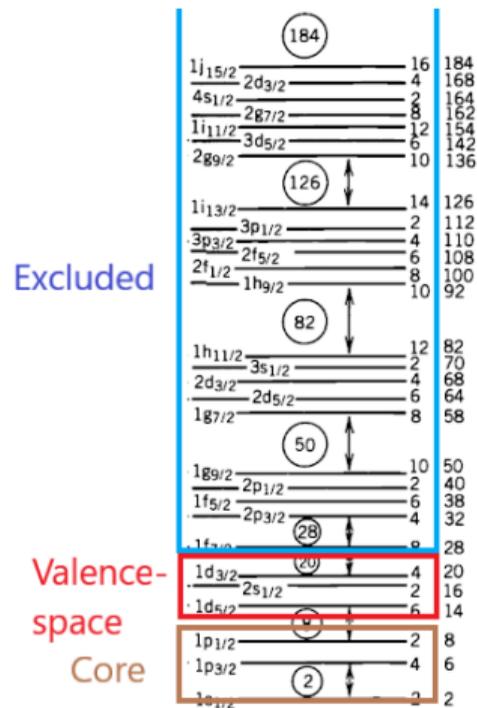
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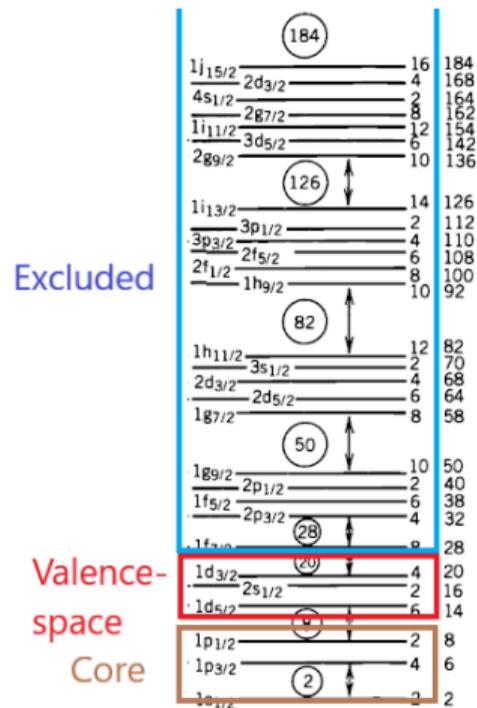
Valence-Space In-Medium Similarity Renormalization Group (VS-IMSRG)

- We choose a Hamiltonian based on the chiral EFT with EM 1.8/2.0 interaction



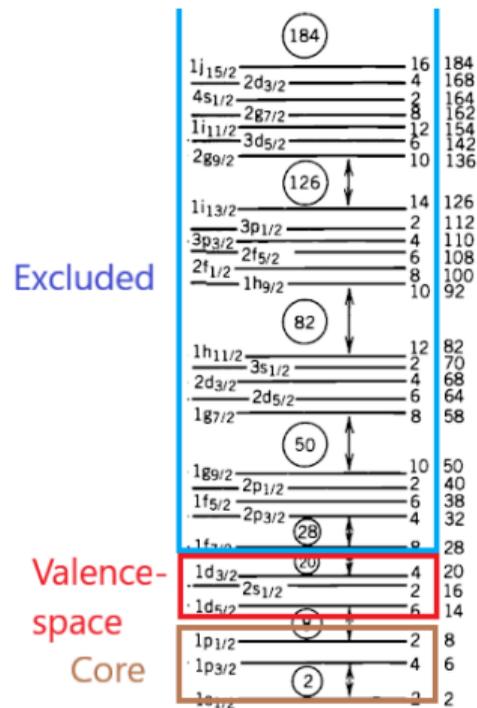
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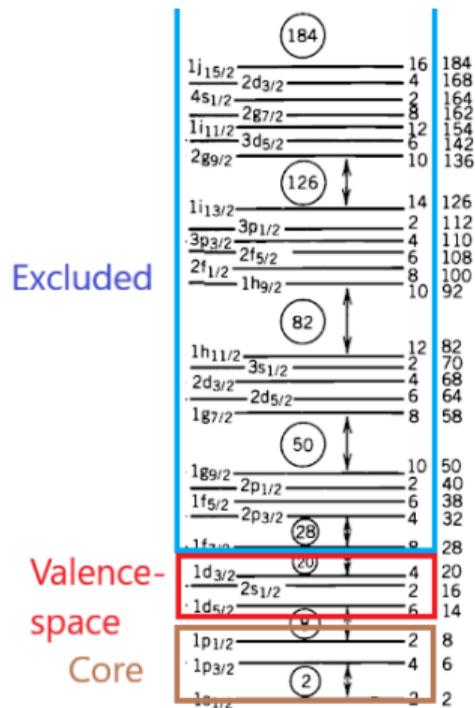
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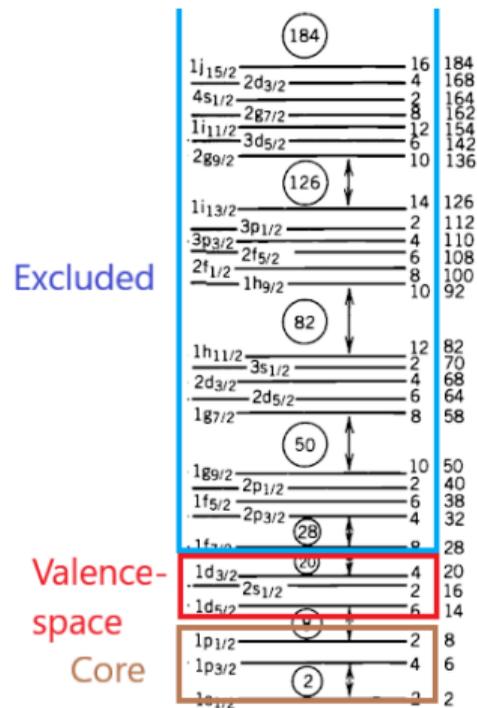
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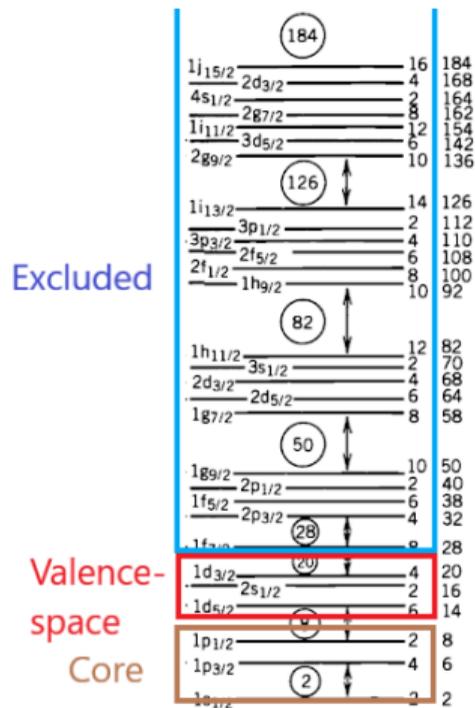
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 - ▶ **Can be applied to medium-heavy to heavy nuclei**



Valence-Space In-Medium Similarity Renormalization Group (VS-IMSRG)

- ▶ We choose a Hamiltonian based on the chiral EFT with EM 1.8/2.0 interaction
 - ▶ **Valence-space Hamiltonian and OMC operators decoupled** with a unitary transformation
 - ▶ **Operators can be made consistent with the Hamiltonian!**
 - ▶ The nuclear many-body problem can then be solved with a shell-model code
 - ▶ **Can be applied to medium-heavy to heavy nuclei**
- **First case: OMC on ^{24}Mg**



Capture Rates to Low-Lying States in ^{24}Na

J_i^π	E_{exp} (MeV)	Rate (10^3 1/s)				
		Exp. ¹	NSM		VS-IMSRG	
			1bc	1bc+2bc	1bc	1bc+2bc
1_1^+	0.472	(21.0 ± 6.6)	4.0	3.0	22.3	15.2
1_2^+	1.347	17.5 ± 2.3	32.7	21.3	7.7	4.9
Sum(1^+)		38.5 ± 8.9	36.7	24.5	30.0	20.0
2_1^+	0.563	17.5 ± 2.1	1.0	0.7	0.5	0.3
2_2^+	1.341	3.4 ± 0.5	3.1	2.5	1.0	0.9
Sum(2^+)		20.9 ± 2.6	4.1	3.2	1.5	1.2

*LJ, Miyagi, Stroberg, Holt, Kotila, Suhonen, Phys. Rev. C **107**, 014327 (2023)*

¹Gorringe *et al.*, *Phys. Rev. C* **60**, 055501 (1999)

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 - ▶ The effect of two-body currents may be overestimated

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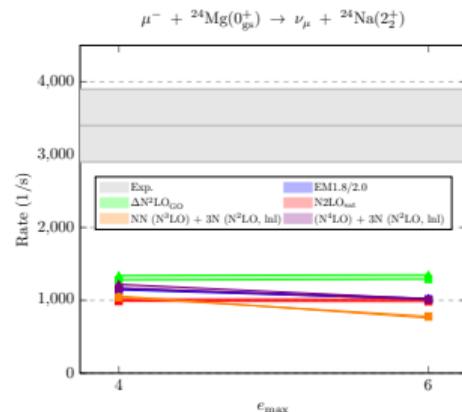
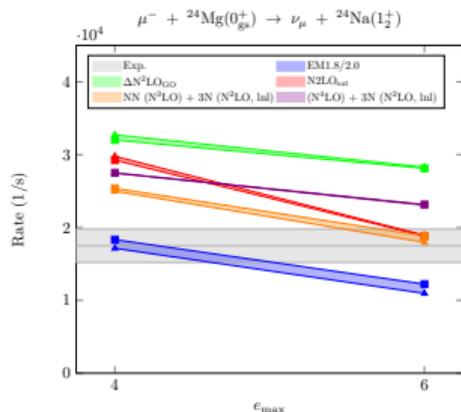
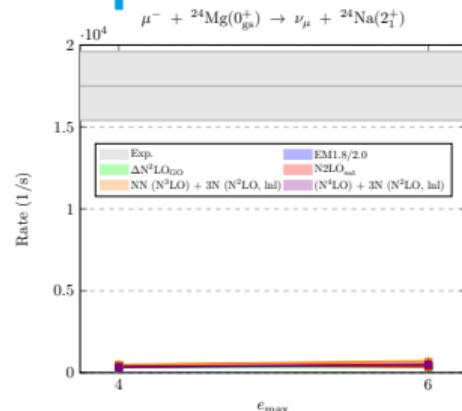
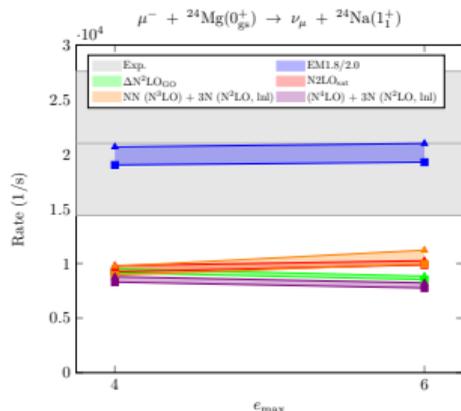
*LJ, Miyagi, Stroberg, Holt, Kotila, Suhonen, Phys. Rev. C **107**, 014327 (2023)*

- ▶ **Rate to the lowest two 1^+ states agrees with experiment**
 - ▶ The effect of two-body currents may be overestimated
- ▶ **1^+ states mixed**
- ▶ **Both NSM and VS-IMSRG notably underestimate the rates to 2^+ states**

¹Gorringe *et al.*, *Phys. Rev. C* **60**, 055501 (1999)

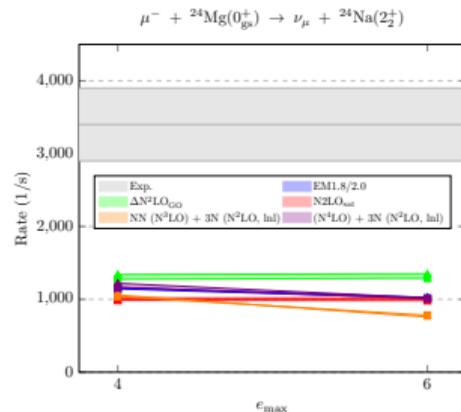
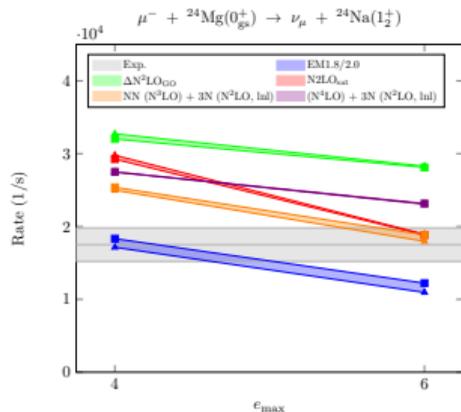
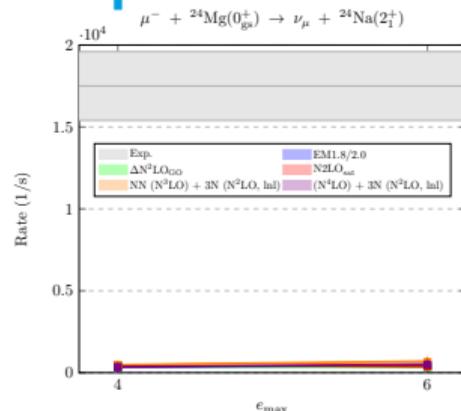
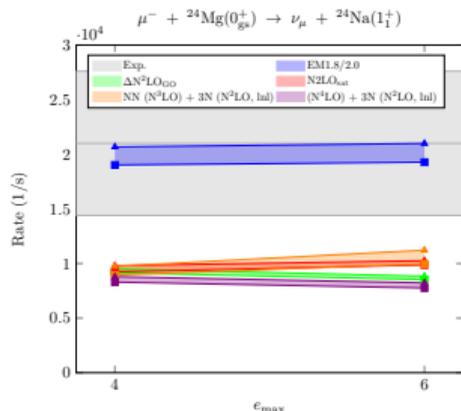
Interaction Dependence

- Rates are sensitive to the interaction



Interaction Dependence

- ▶ Rates are sensitive to the interaction
- ▶ It does not explain the poor agreement with the measured rates to the 2^+ states (on the right)



Introduction

Corrections to $0\nu\beta\beta$ -decay nuclear matrix elements

The contact term

Contribution of ultrasoft neutrinos

Muon capture as a probe of $0\nu\beta\beta$ decay

VS-IMSRG Study on Muon Capture on ^{24}Mg

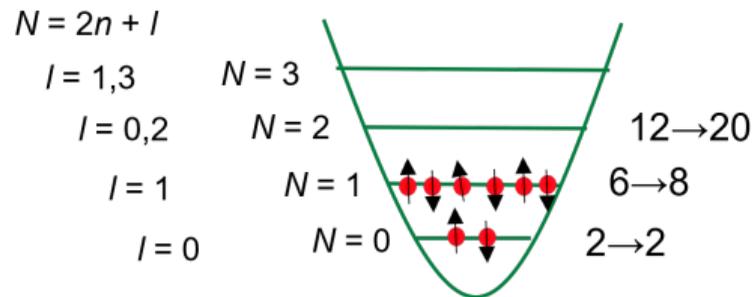
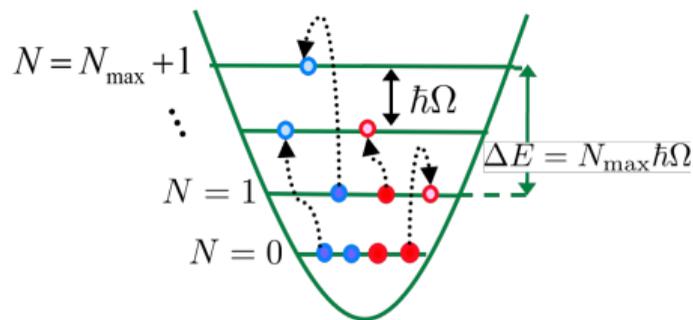
No-Core Shell-Model Studies on Muon Capture on Light Nuclei

Phenomenological study on muon capture on ^{136}Ba

Summary and Outlook

No-Core Shell Model (NCSM)

- OMC operators and one-body transition densities computed in large harmonic-oscillator (HO) basis



$$E = (2n + l + \frac{3}{2})\hbar\Omega$$

Figure courtesy of P. Navrátil

No-Core Shell Model (NCSM)

- ▶ OMC operators and one-body transition densities computed in large harmonic-oscillator (HO) basis
 - ▶ **HO basis truncated with N_{\max}**

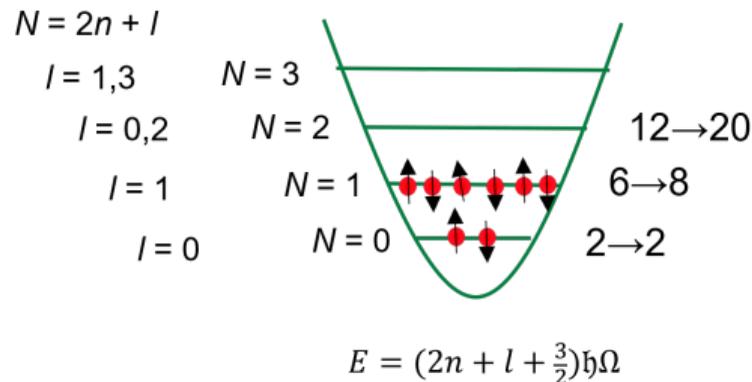
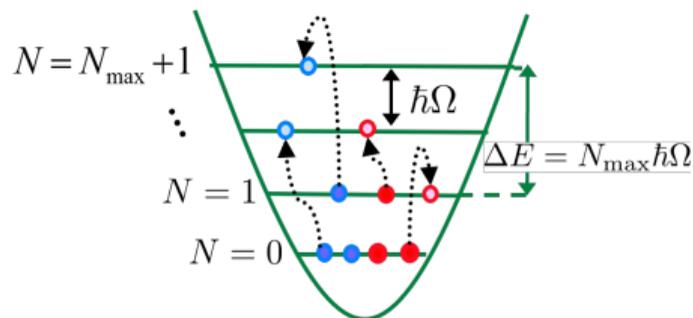


Figure courtesy of P. Navrátil

No-Core Shell Model (NCSM)

- ▶ OMC operators and one-body transition densities computed in large harmonic-oscillator (HO) basis
 - ▶ **HO basis truncated with N_{\max}**
- ▶ Hamiltonian based on the chiral EFT with different interactions:

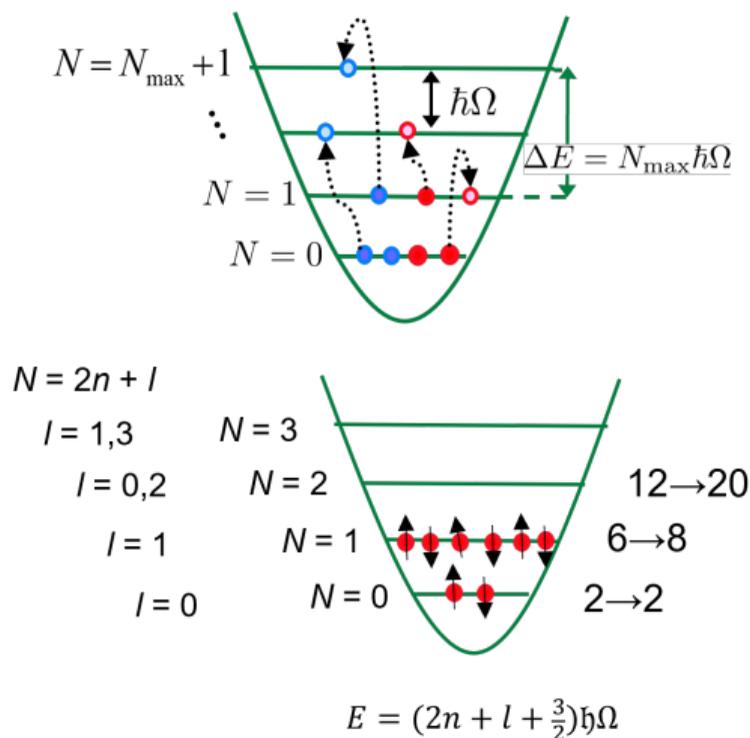


Figure courtesy of P. Navrátil

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- ▶ $NN(N^4LO)+3N(N^2LO,InI)$

Entem, Machleidt, Nosyk, Phys. Rev. C **96**, 024004 (2017) (NN)

Gysbers et al., Nature Phys. **15**, 428 (2019) (3N)

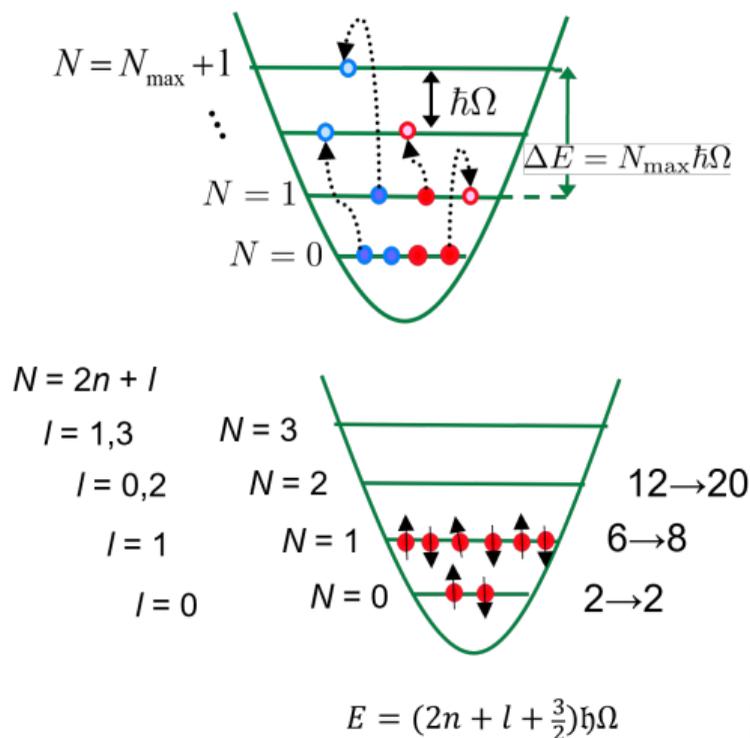


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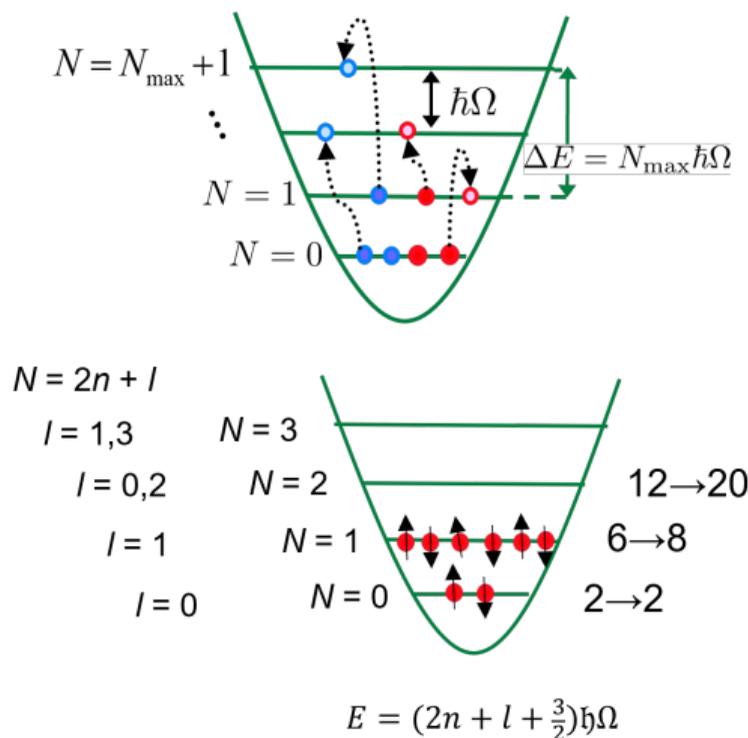


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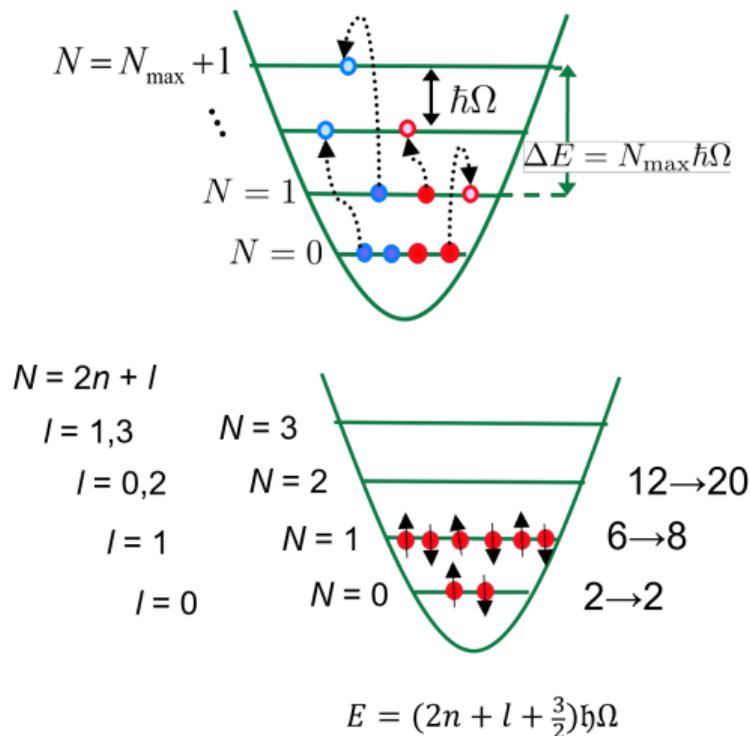


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→ **OMC on ${}^6\text{Li}$, ${}^{12}\text{C}$ and ${}^{16}\text{O}$**

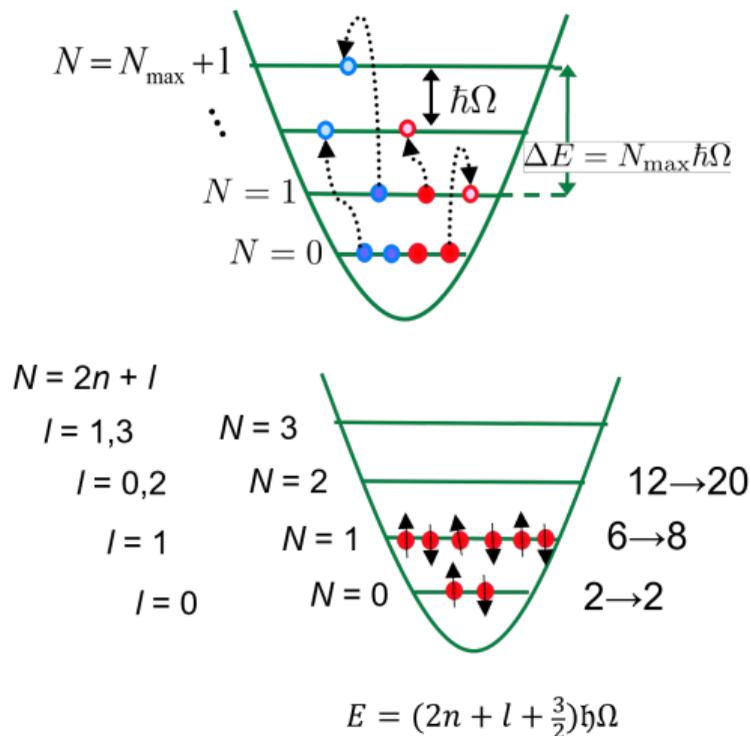
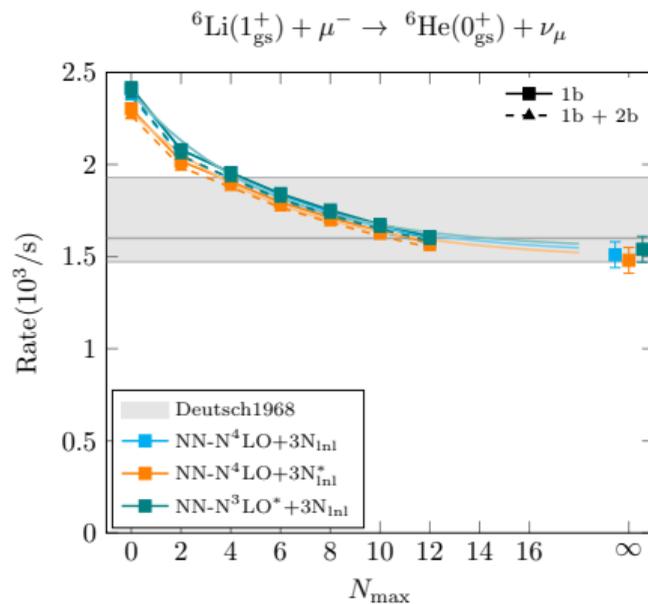


Figure courtesy of P. Navrátil

Capture Rates to the Ground State of ${}^6\text{He}$

- NCSM in keeping with experiment



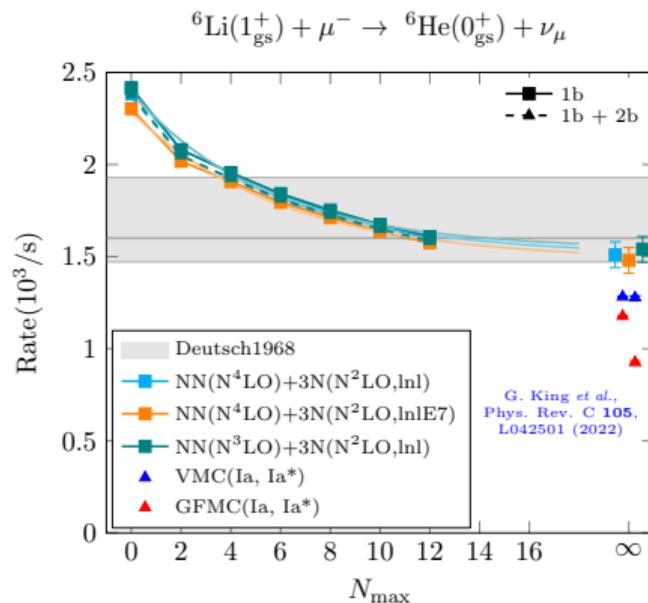
LJ, Navrátil, Kotila, Kravvaris,

work in progress

Capture Rates to the Ground State of ${}^6\text{He}$

- ▶ NCSM in keeping with experiment
- ▶ The rates can be compared with the variational (VMC) and Green's function Monte-Carlo (GFMC) calculations

King *et al.*, Phys. Rev. C **105**, L042501 (2022)

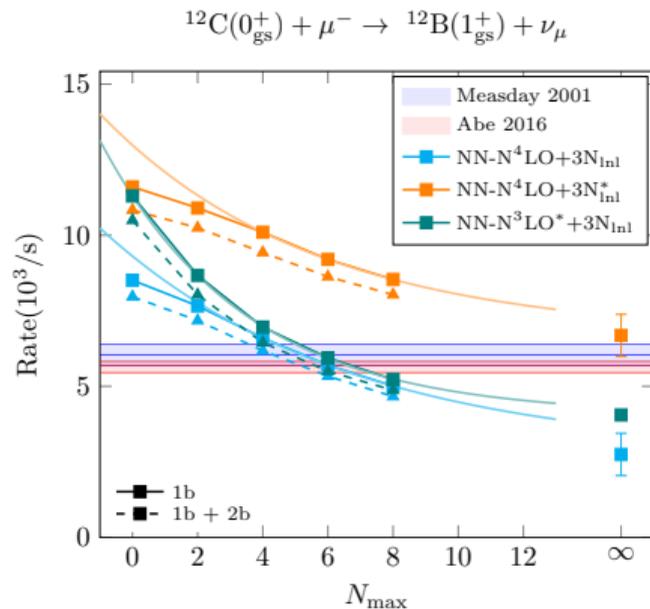


LJ, Navrátil, Kotila, Kravvaris,

work in progress

Capture Rates to the Ground State of ^{12}B

► Interaction dependence

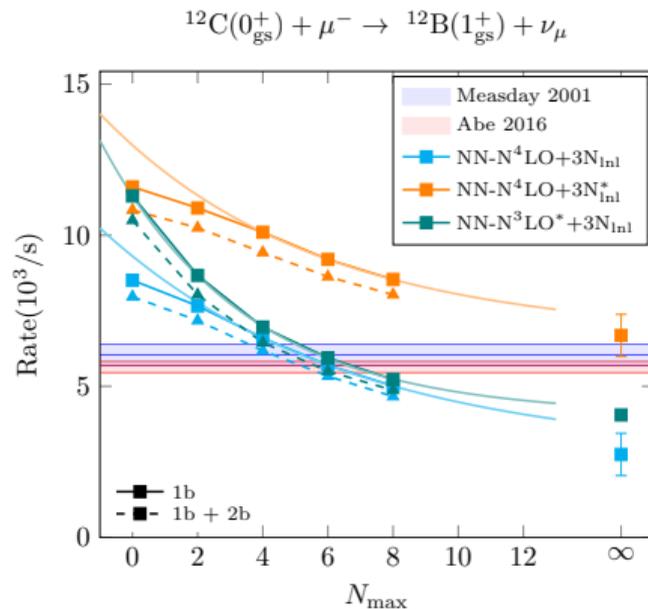


LJ, Navrátil, Kotila, Kravvaris,

work in progress

Capture Rates to the Ground State of ^{12}B

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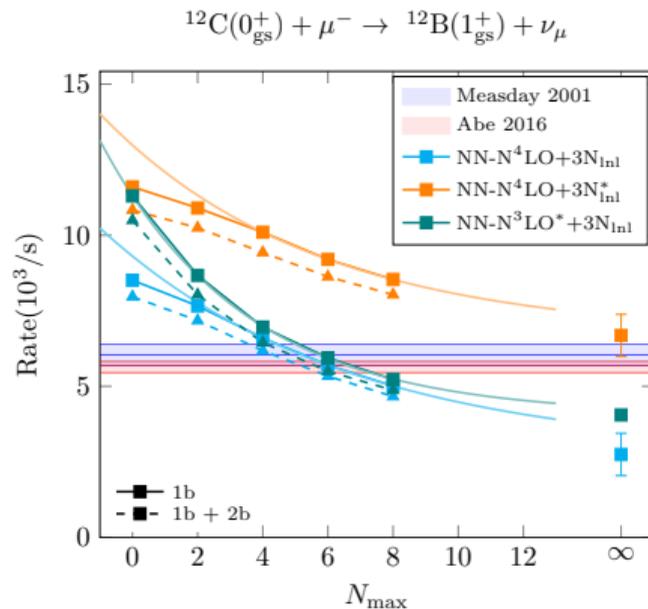


LJ, Navrátil, Kotila, Kravvaris,

work in progress

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- ▶ Converge slow (clustering effects?)



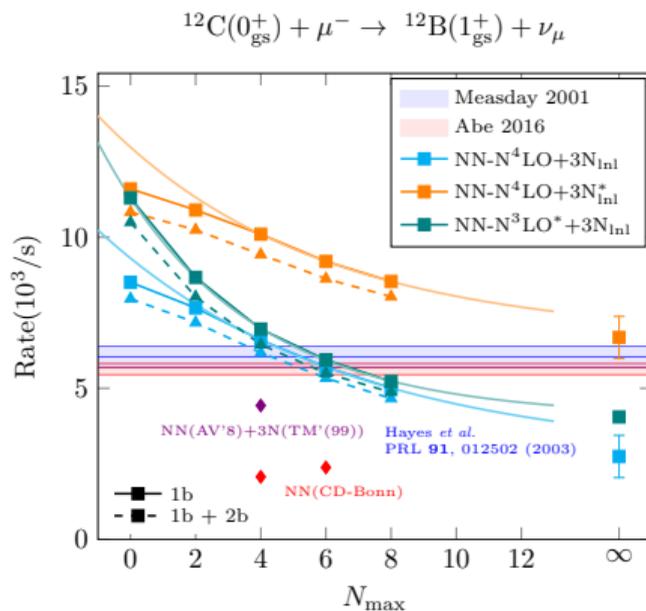
LJ, Navrátil, Kotila, Kravvaris,

work in progress

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Hayes *et al.*, *Phys. Rev. Lett.* **91**, 012502 (2003)

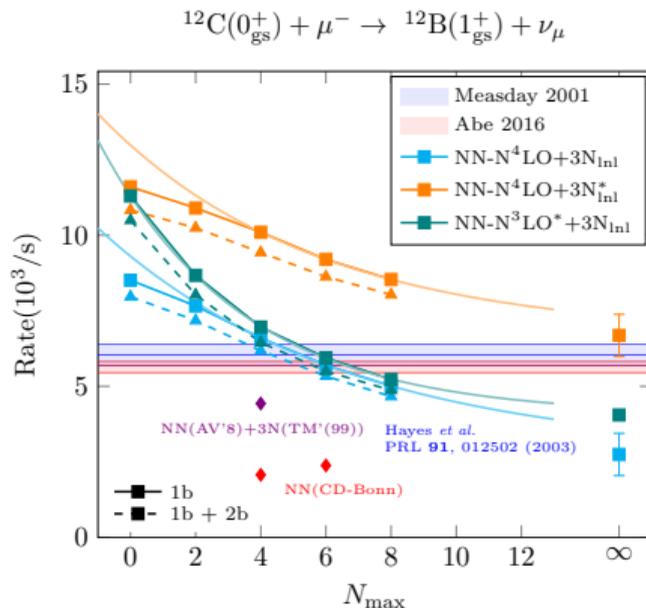


LJ, Navrátil, Kotila, Kravvaris,

work in progress

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- Hayes et al., Phys. Rev. Lett. 91, 012502 (2003)*
- ▶ 3-body forces essential to reproduce the measured rate

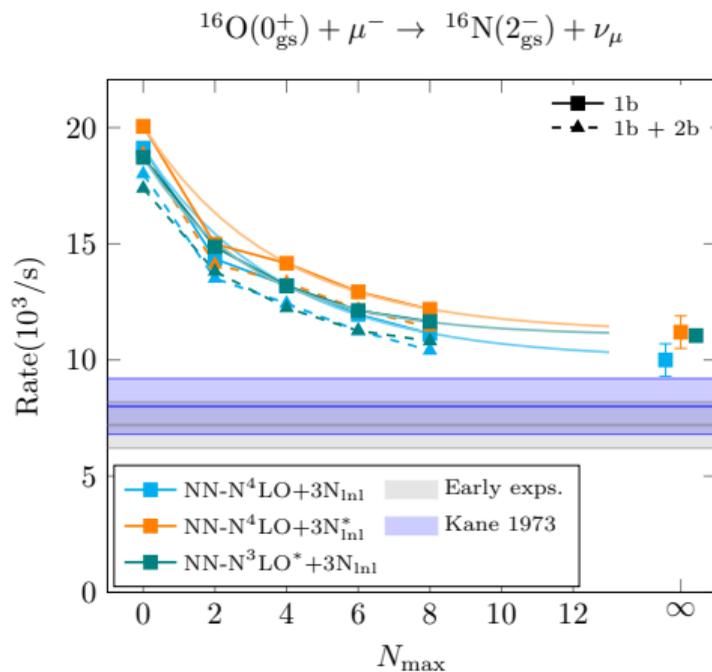


LJ, Navrátil, Kotila, Kravvaris,

work in progress

Capture Rates to the ground state of ^{16}N

- NCSM describes well the complex systems ^{16}O and ^{16}N

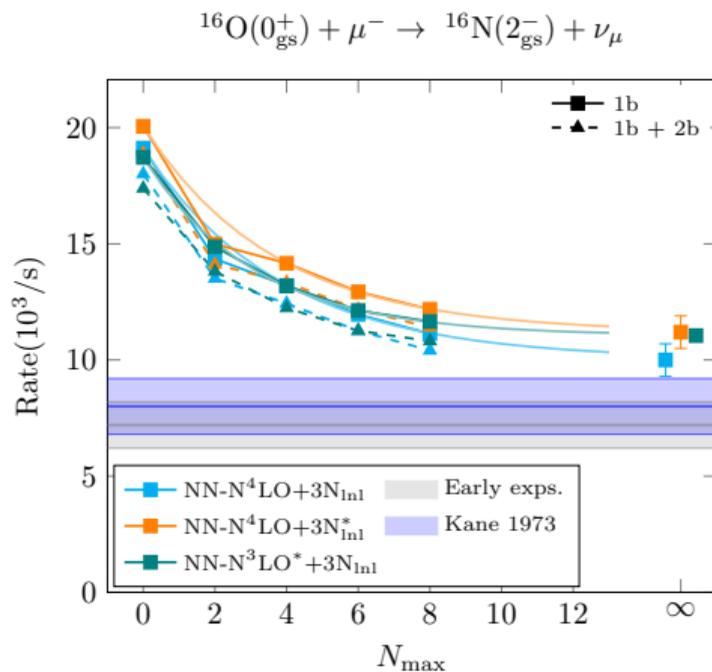


LJ, Navrátil, Kotila, Kravvaris,

work in progress

Capture Rates to the ground state of ^{16}N

- ▶ NCSM describes well the complex systems ^{16}O and ^{16}N
- ▶ Less sensitive to the interaction than $^{12}\text{C}(\mu^-, \nu_\mu)^{12}\text{B}$

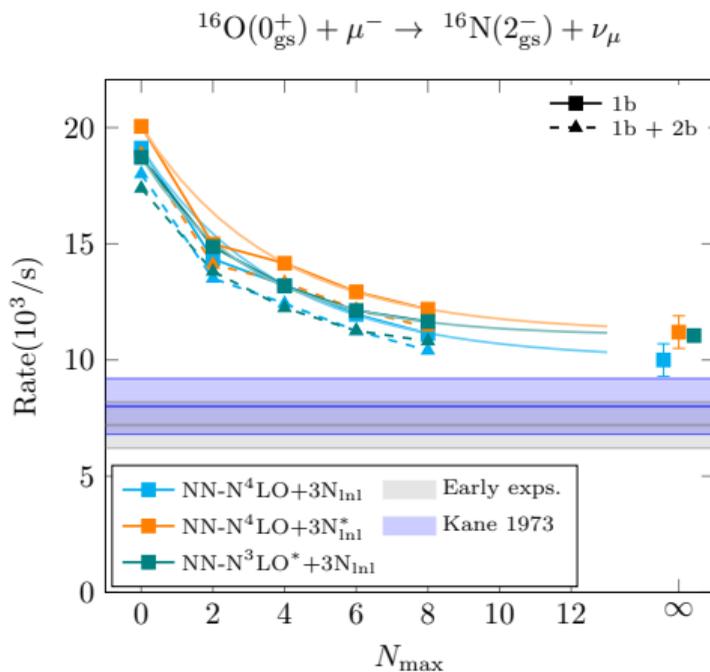


LJ, Navrátil, Kotila, Kravvaris,

work in progress

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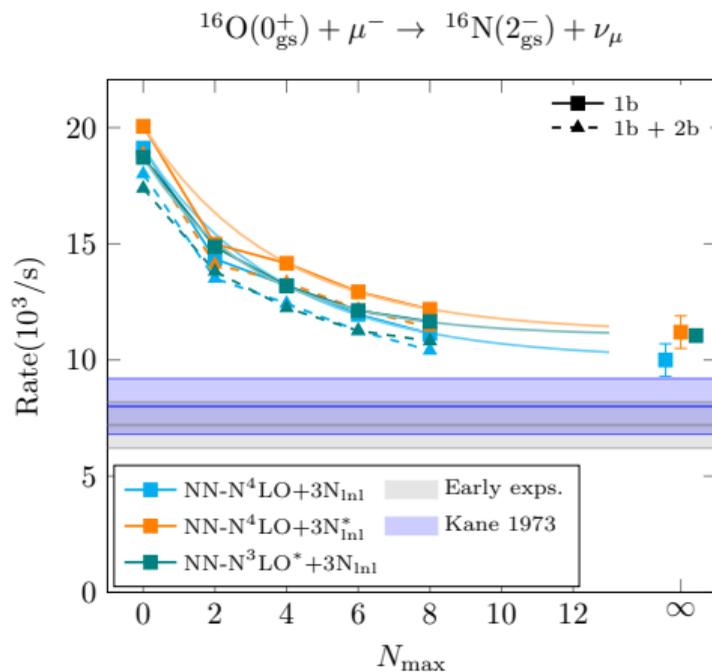


LJ, Navrátil, Kotila, Kravvaris,

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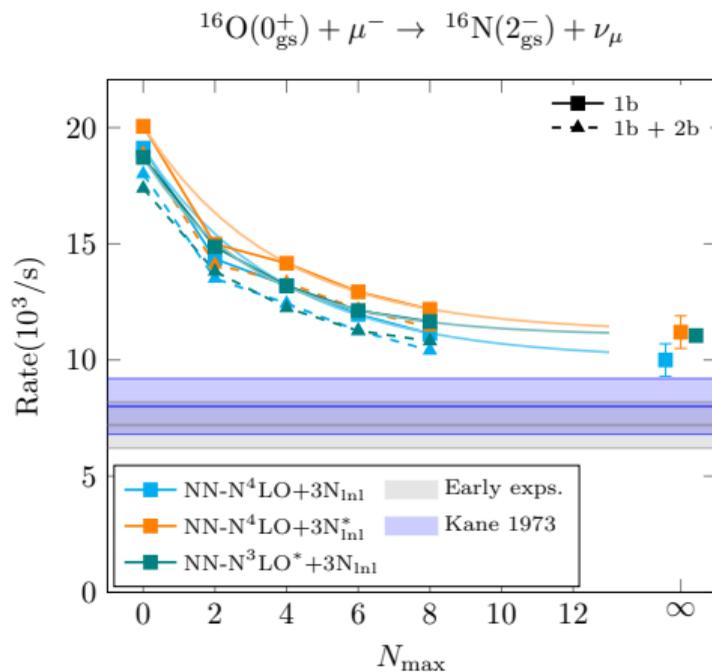


LJ, Navrátil, Kotila, Kravvaris,

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 - Theory estimates based on NCSM

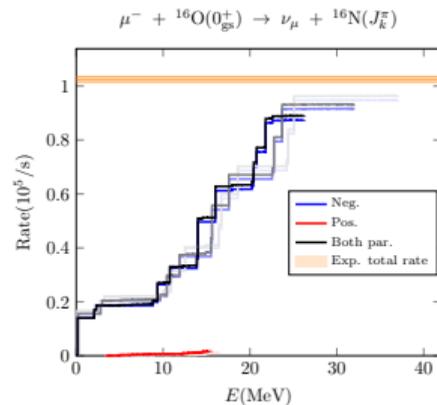
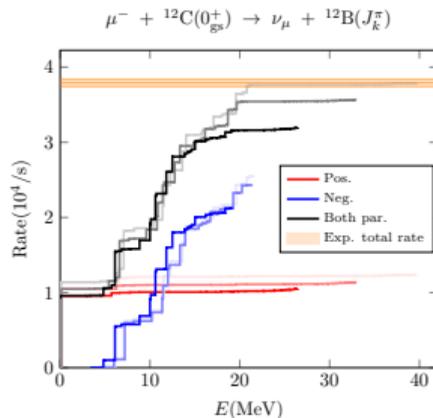


LJ, Navrátil, Kotila, Kravvaris,

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Total Muon-Capture Rates in ^{12}B and ^{16}N

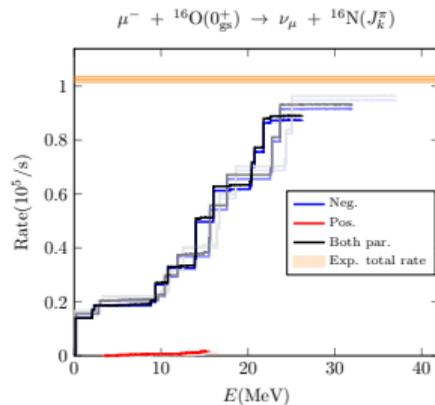
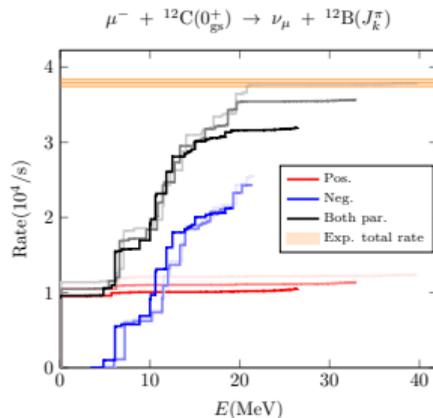
- Color gradient: increasing N_{max}
 (3,5,7 for ^{12}C and
 2,4,6 for ^{16}O)



LJ, Navrátil, Kotila, Kravvaris, work in progress

Total Muon-Capture Rates in ^{12}B and ^{16}N

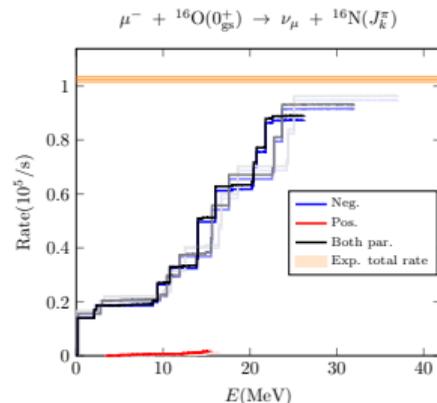
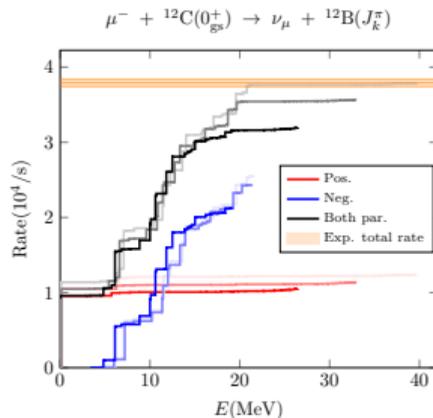
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- ▶ Rates obtained summing over ~ 50
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LJ, Navrátil, Kotila, Kravvaris, work in progress

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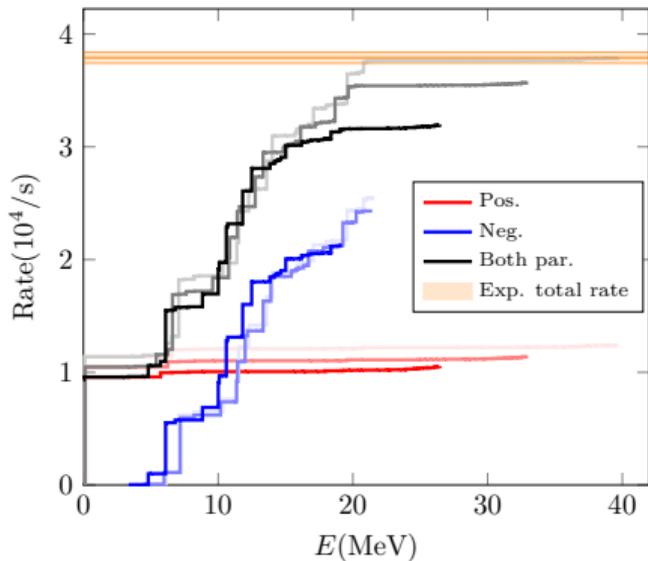
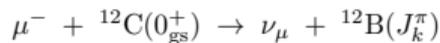
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(**3,5,7** for ^{12}C and
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- ▶ Rates obtained summing over ~ 50
final states of each parity
- ▶ Summing up **the rates up to ~ 20
MeV**, we capture **$\sim 85\%$ of the
total rate** in both ^{12}B and ^{16}N



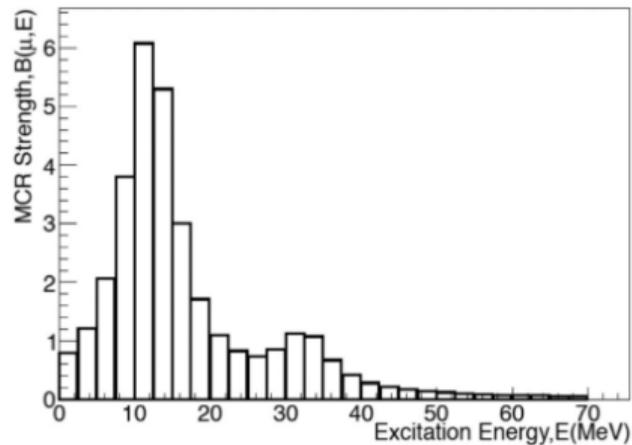
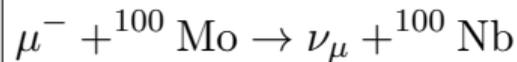
LJ, Navrátil, Kotila, Kravvaris, work in progress

Total Muon-Capture Rates

Calculation:



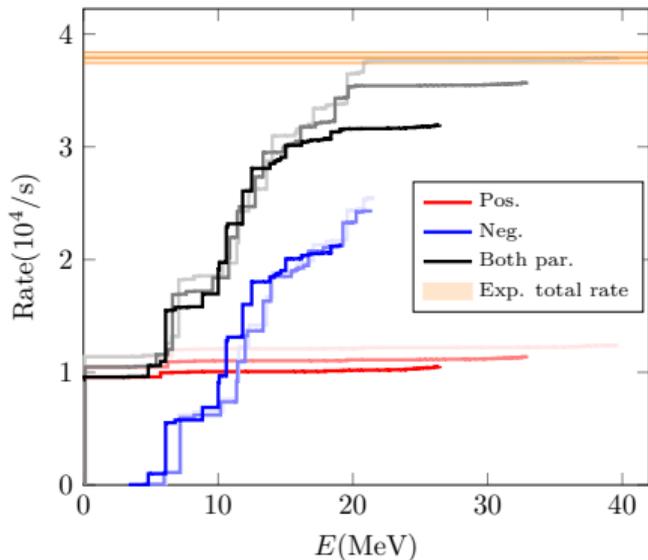
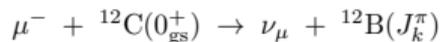
Experiment:



Hashim et al., *Phys. Rev. C* **97**, 014617 (2018)

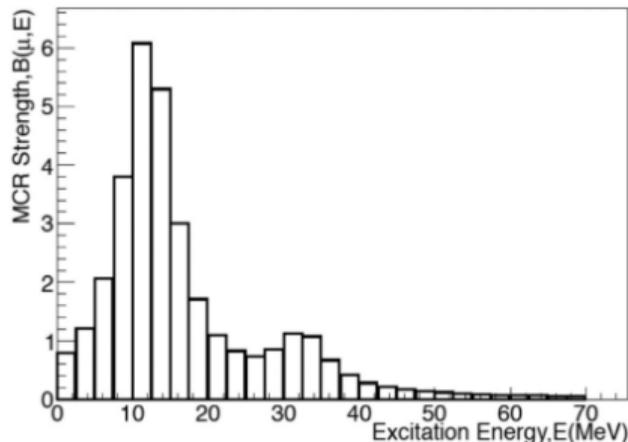
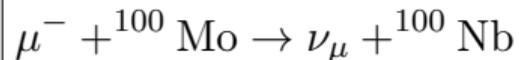
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Missing potentially important contribution from high energies

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Hashim et al., *Phys. Rev. C* **97**, 014617 (2018)

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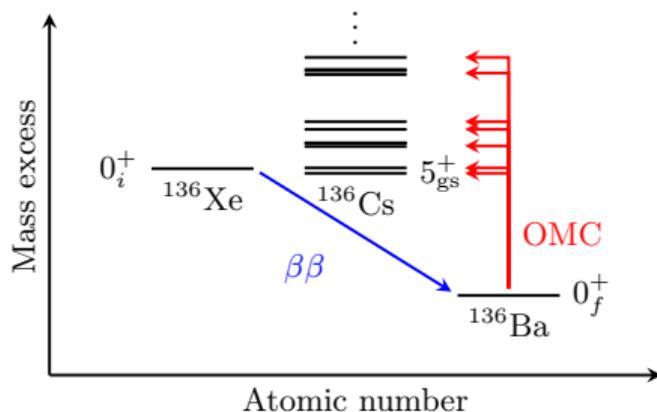
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Phenomenological study on muon capture on ^{136}Ba

Summary and Outlook

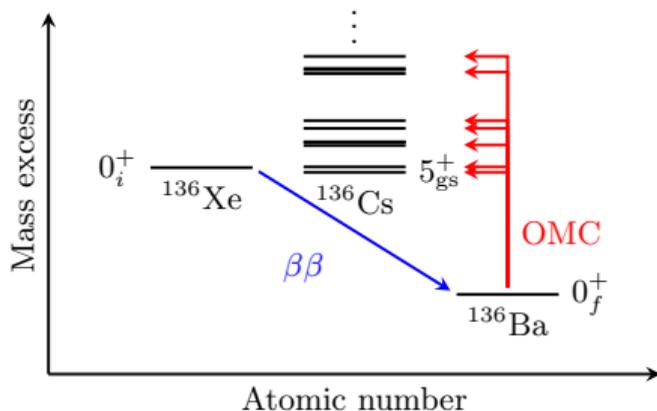
Muon capture on ^{136}Ba

- OMC on ^{136}Ba one of the candidates to be measured by the MONUMENT collaboration



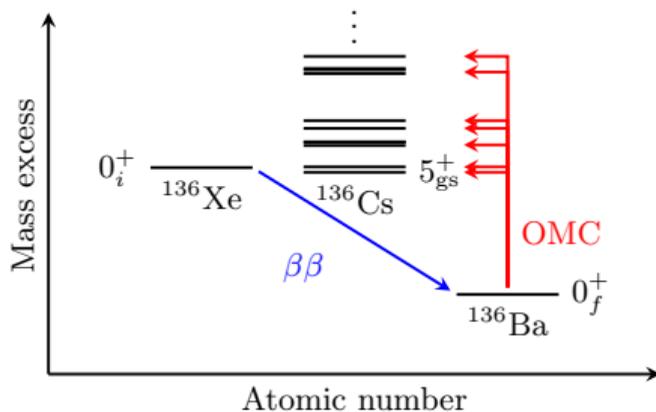
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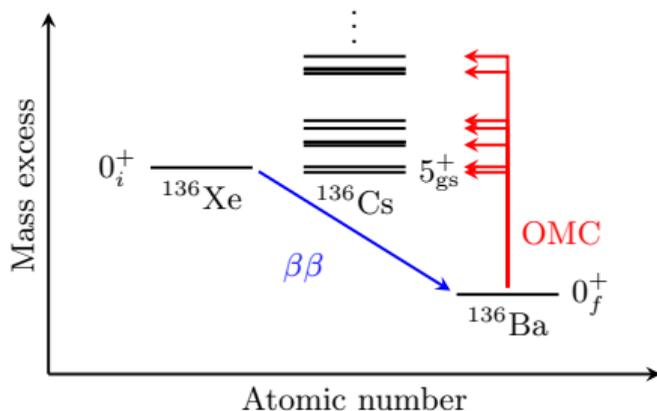
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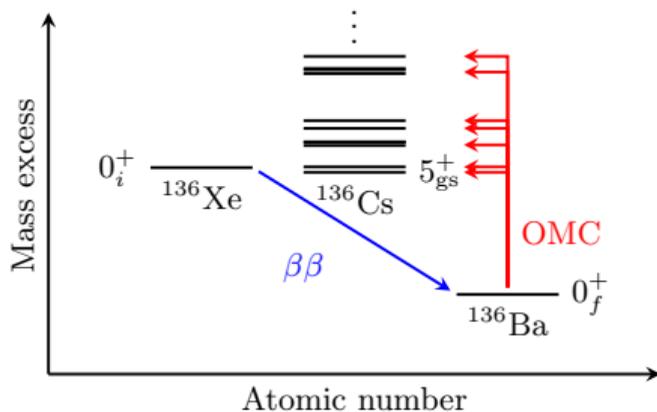
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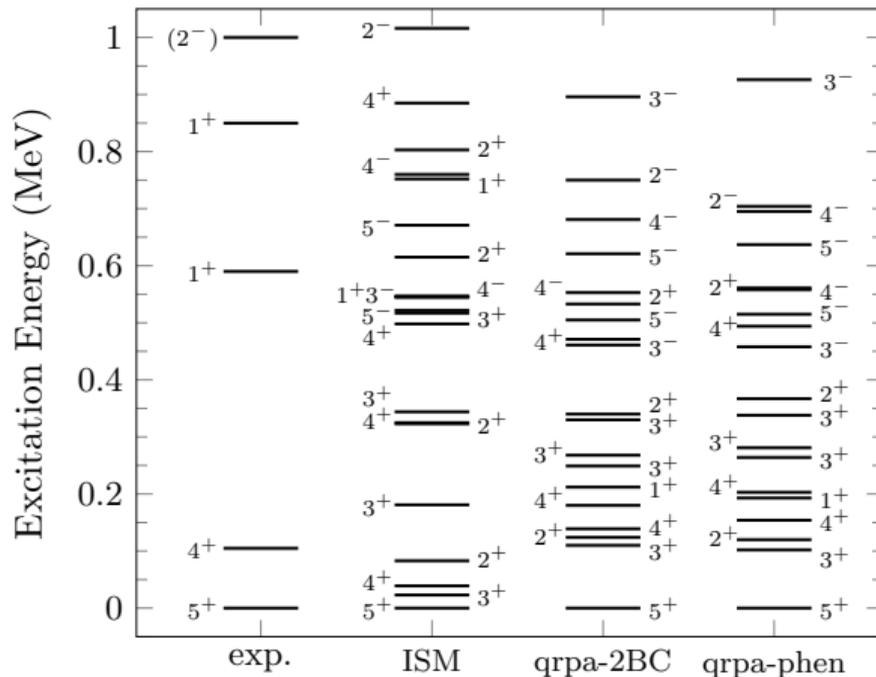
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- ▶ Solution: (phenomenological) **nuclear shell model and proton-neutron QRPA**



Excitation energies in ^{136}Cs ($J \leq 5$)

- The shell-model and pnQRPA energies are surprisingly similar



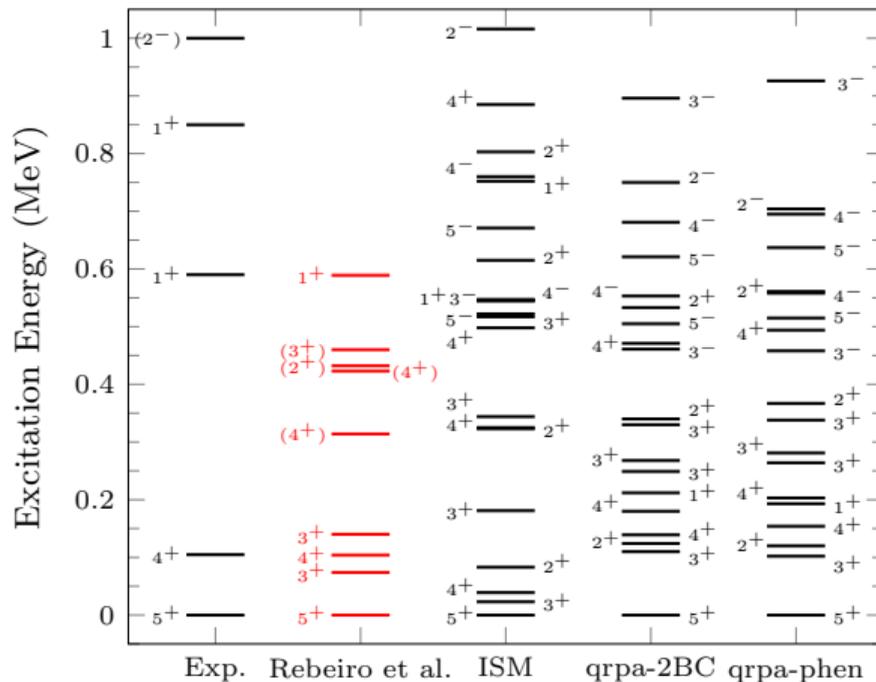
P. Gimeno, LJ, J. Kotila, M. Ramalho, J. Suhonen,

10.20944/preprints202304.0899.v1 (submitted to Universe)

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B. M. Rebeiro *et al.*, arXiv:2301.11371 (2023)



P. Gimeno, L.J, J. Kotila, M. Ramalho, J. Suhonen,

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Muon capture rates to low-lying states in ^{136}Cs

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P. Gimeno, L.J. Kotila, M. Ramalho, J. Suhonen, 10.20944/preprints202304.0899.v1 (submitted to Universe)

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- ▶ Similar study ongoing for OMC on $^{128,130}\text{Xe}$

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- ▶ Phenomenological methods still needed for heavy/difficult systems

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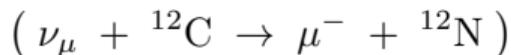
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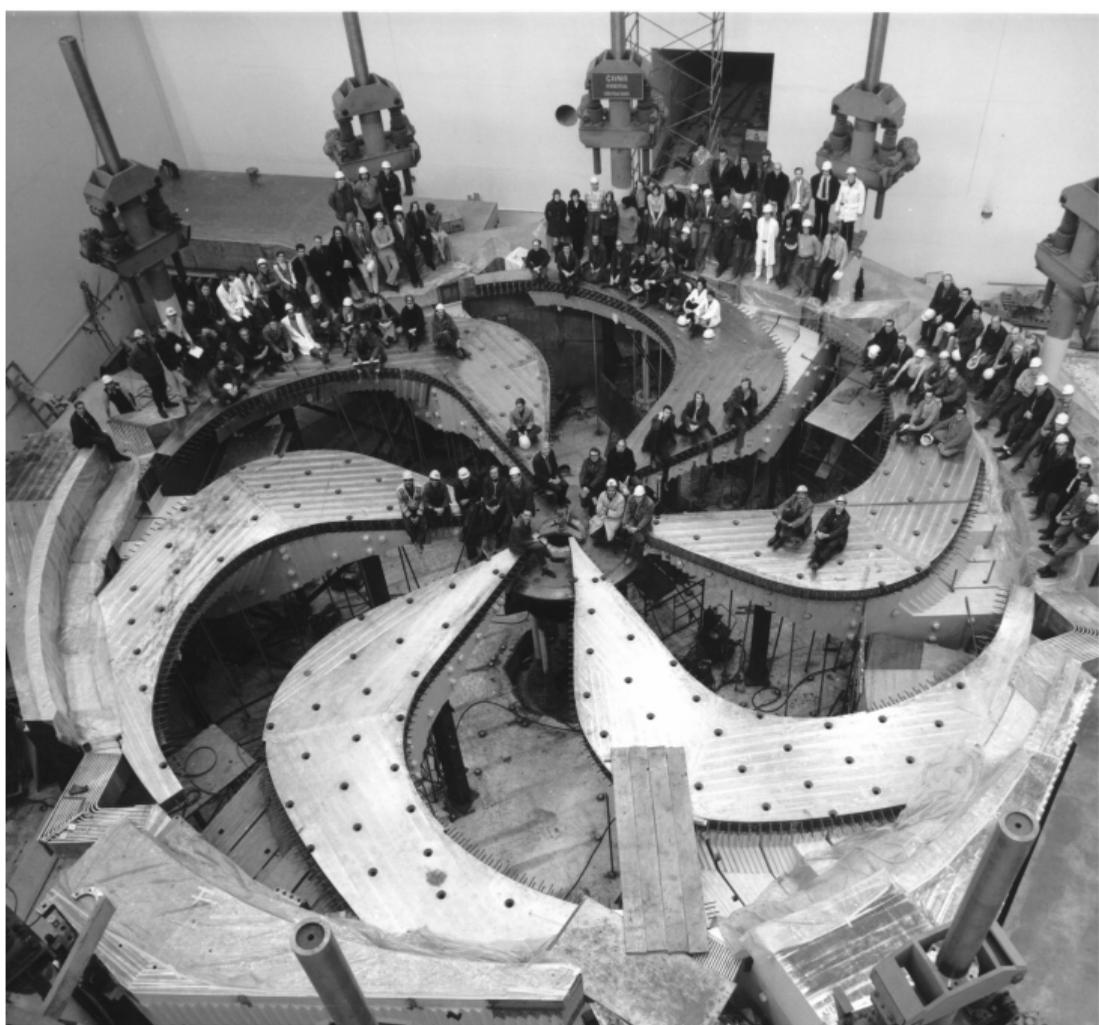
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 - ▶ ^{12}C and ^{16}O are both of interest in **neutrino-scattering experiments**



Thank you
Merci



- Rates written in terms of reduced one-body matrix elements:

$$(\Psi_f || \sum_{s=1}^A \hat{O}_{kwux}(\mathbf{r}_s, \mathbf{p}_s) || \Psi_i) = \frac{1}{\sqrt{2u+1}} \sum_{pn} (n || \hat{O}_{kwux}(\mathbf{r}_s, \mathbf{p}_s) || p) (\Psi_f || [a_n^\dagger \tilde{a}_p]_u || \Psi_i)$$

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Axial-Vector Two-Body Currents (2BCs)

- ▶ One-body (1b) axial-vector currents given by

$$\mathbf{J}_{i,1b}^3 = \frac{\tau_i^3}{2} \left(g_A \boldsymbol{\sigma}_i - \frac{g_P}{2m_N} \mathbf{q} \cdot \boldsymbol{\sigma}_i \right),$$

where $g_P = (2m_N q / (q^2 + m_\pi^2)) g_A$

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- Additional **pion-exchange, pion-pole, and contact** two-body (2b) currents

Hoferichter, Klos, Schwenk *Phys. Lett. B* **746**, 410 (2015)

$$\begin{aligned} \mathbf{J}_{12}^3 = & -\frac{g_A}{2F_\pi^2} [\tau_1 \times \tau_2]^3 \left[c_4 \left(1 - \frac{\mathbf{q}}{q^2 + M_\pi} \mathbf{q} \cdot \right) (\boldsymbol{\sigma}_1 \times \mathbf{k}_2) + \frac{c_6}{4} (\boldsymbol{\sigma}_1 \times \mathbf{q}) + i \frac{\mathbf{p}_1 + \mathbf{p}'_1}{4m_N} \right] \frac{\boldsymbol{\sigma}_2 \cdot \mathbf{k}_2}{M_\pi^2 + k_2^2} \\ & - \frac{g_A}{F_\pi^2} \tau_2^3 \left[c_3 \left(1 - \frac{\mathbf{q}}{q^2 + M_\pi} \mathbf{q} \cdot \right) \mathbf{k}_2 + 2c_1 M_\pi^2 \frac{\mathbf{q}}{q^2 + M_\pi^2} \right] \frac{\boldsymbol{\sigma}_2 \cdot \mathbf{k}_2}{M_\pi^2 + k_2^2} \\ & - d_1 \tau_1^3 \left(1 - \frac{\mathbf{q}}{q^2 + M_\pi^2} \mathbf{q} \cdot \right) \boldsymbol{\sigma}_1 + (1 \leftrightarrow 2) - d_2 (\tau_1 \times \tau_2)^3 (\boldsymbol{\sigma}_1 \times \boldsymbol{\sigma}_2) \left(1 - \frac{\mathbf{q}}{q^2 + M_\pi^2} \mathbf{q} \cdot \right) \end{aligned}$$

where $\mathbf{k}_i = \mathbf{p}'_i - \mathbf{p}_i$ and $\mathbf{q} = -\mathbf{k}_1 - \mathbf{k}_2$

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Hoferichter, Menéndez, Schwenk, *Phys. Rev. D* **102**,074018 (2020)

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$$\rightarrow \mathbf{J}_{i,2b}^{\text{eff}} = g_A \frac{\tau_i^3}{2} \left[\delta a(\mathbf{q}^2) \boldsymbol{\sigma}_i + \frac{\delta a^P(\mathbf{q}^2)}{\mathbf{q}^2} (\mathbf{q} \cdot \boldsymbol{\sigma}_i) \mathbf{q} \right],$$

where

$$\delta a(\mathbf{q}^2) = -\frac{\rho}{F_\pi^2} \left[\frac{c_4}{3} [3I_2^\sigma(\rho, \mathbf{q}) - I_1^\sigma(\rho, |\mathbf{q}|)] - \frac{1}{3} \left(c_3 - \frac{1}{4m_N} \right) I_1^\sigma(\rho, |\mathbf{q}|) - \frac{c_6}{12} I_{c6}(\rho, |\mathbf{q}|) - \frac{c_D}{4g_A \Lambda_\chi} \right],$$

$$\begin{aligned} \delta a^P(\mathbf{q}^2) = & \frac{\rho}{F_\pi^2} \left[-2(c_3 - 2c_1) \frac{m_\pi^2 \mathbf{q}^2}{(m_\pi^2 + \mathbf{q}^2)^2} + \frac{1}{3} \left(c_3 + c_4 - \frac{1}{4m_N} \right) I^P(\rho, |\mathbf{q}|) - \left(\frac{c_6}{12} - \frac{2}{3} \frac{c_1 m_\pi^2}{m_\pi^2 + \mathbf{q}^2} \right) I_{c6}(\rho, |\mathbf{q}|) \right. \\ & \left. - \frac{\mathbf{q}^2}{m_\pi^2 + \mathbf{q}^2} \left(\frac{c_3}{3} [I_1^\sigma(\rho, |\mathbf{q}|) + I^P(\rho, |\mathbf{q}|)] + \frac{c_4}{3} [I_1^\sigma(\rho, |\mathbf{q}|) + I^P(\rho, |\mathbf{q}|) - 3I_2^\sigma(\rho, |\mathbf{q}|)] \right) - \frac{c_D}{4g_A \Lambda_\chi} \frac{\mathbf{q}^2}{m_\pi^2 + \mathbf{q}^2} \right] \end{aligned}$$

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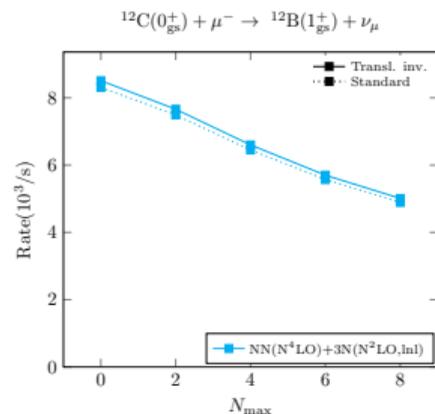
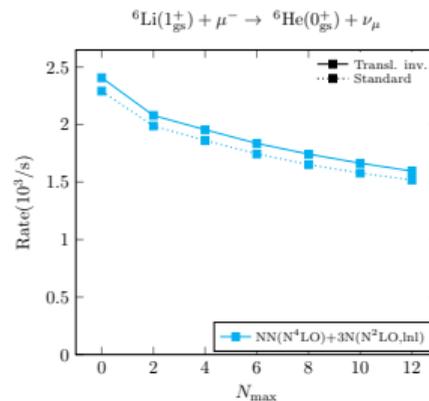
- ▶ Working with A single-particle coordinates and separating the center-of-mass motion:



$$\Psi_{SD}^A = \sum_{N=0}^{N_{\max}} \sum_i c_{Nj}^{\text{SD}} \Phi_{SD}^{\text{HO}}{}_{Nj}(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_A) = \Psi^A \Psi_{CM}(\mathbf{R}_{CM})$$

Removing Spurious Center-of-Mass Motion

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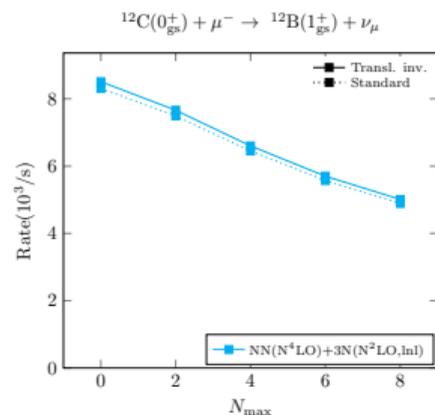
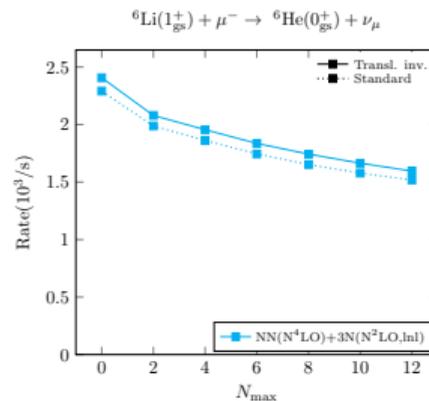
Navrátil, *Phys. Rev. C* **104**, 064322 (2021)

$$\begin{aligned}
 & (\Psi_f || \sum_{s=1}^A \hat{O}_s(\mathbf{r}_s - \mathbf{R}_{\text{CM}}, \mathbf{p}_s - \mathbf{P}) || \Psi_i) \\
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where

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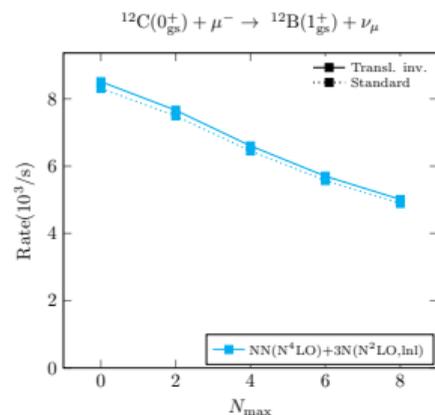
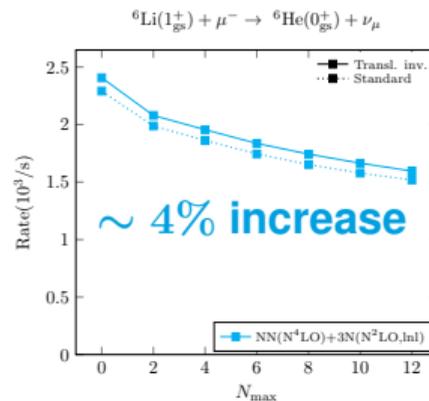
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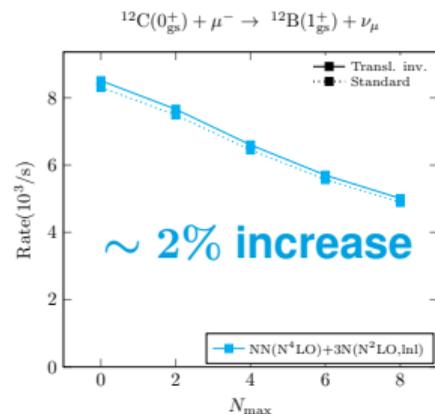
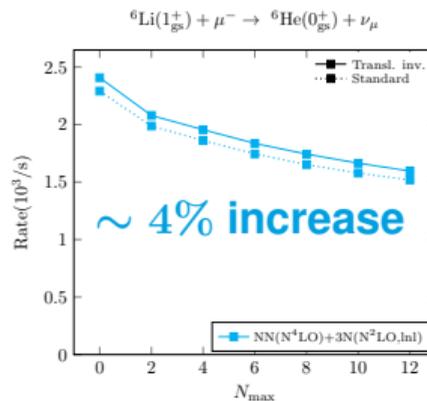
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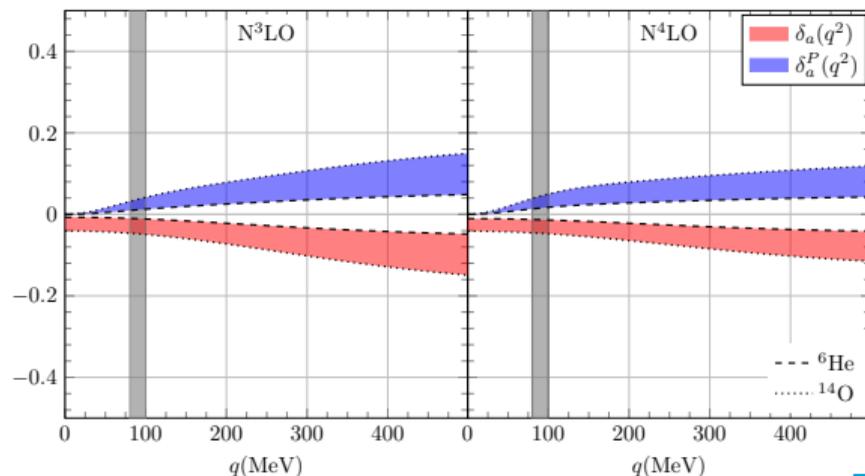
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Two-Body Currents

- Fermi-gas density ρ adjusted so that $\delta_a(0)$ reproduces the effect of exact two-body currents in

P. Gysbers et al., Nature Phys. 15, 428 (2019)



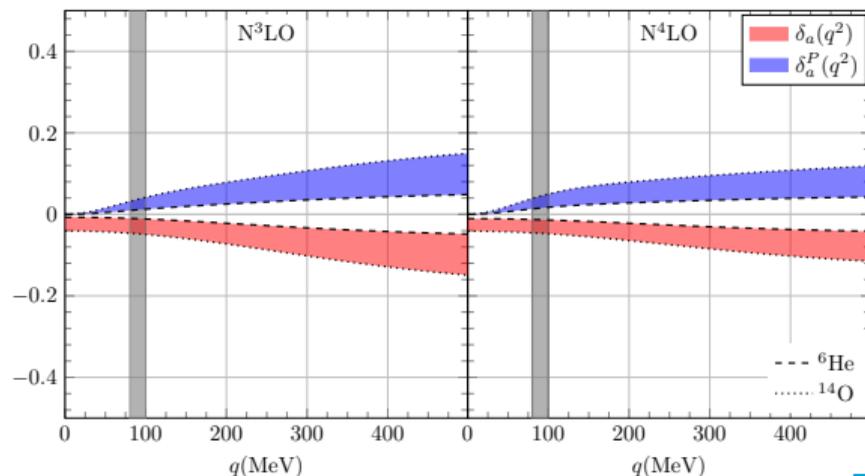
*LJ, Navrátil, Kotila and Kravvaris,
work in progress*

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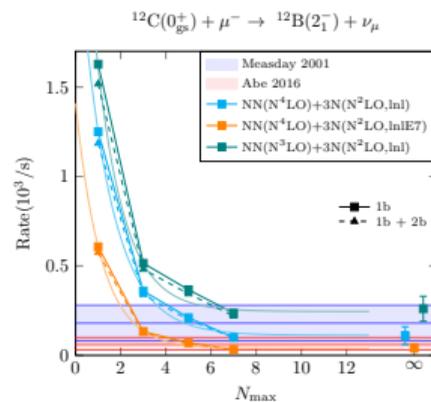
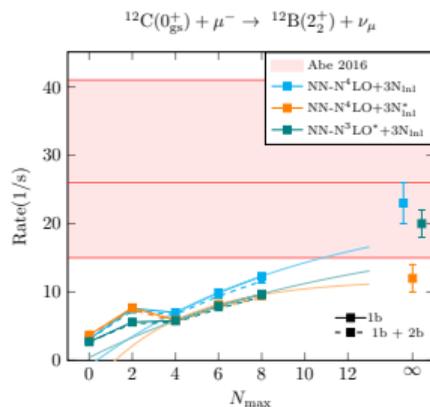
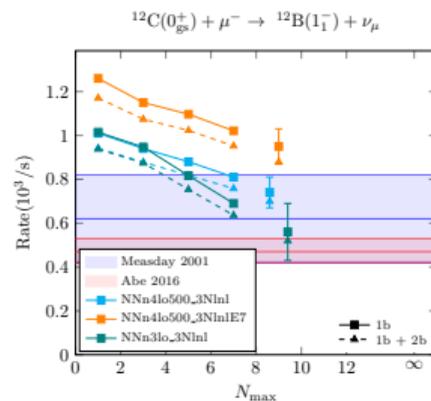
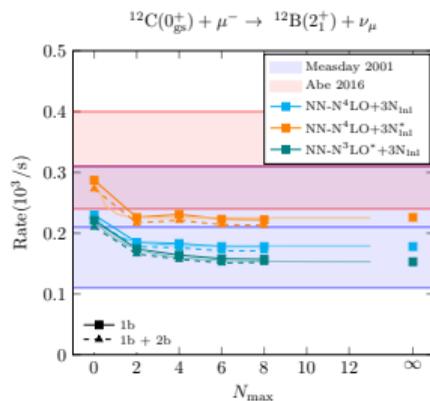
- ▶ Two-body currents typically **reduce** the OMC rates by $\sim 1 - 2\%$ in ${}^6\text{Li}$ and by $\lesssim 10\%$ in ${}^{12}\text{C}$ and ${}^{16}\text{O}$



*LJ, Navrátil, Kotila and Kravvaris,
work in progress*

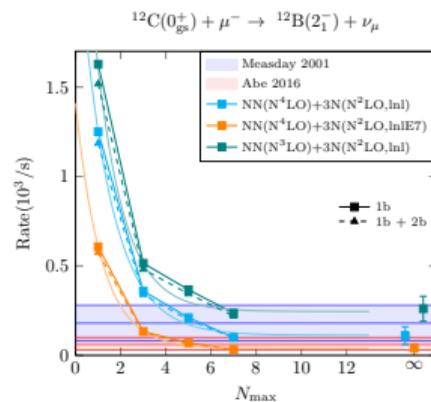
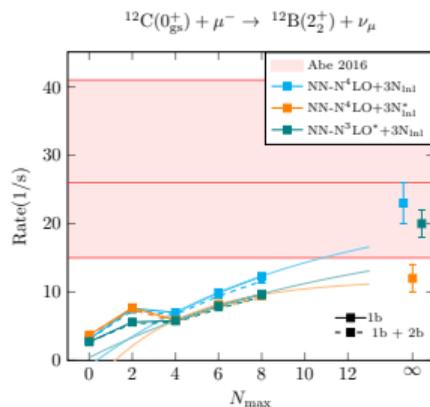
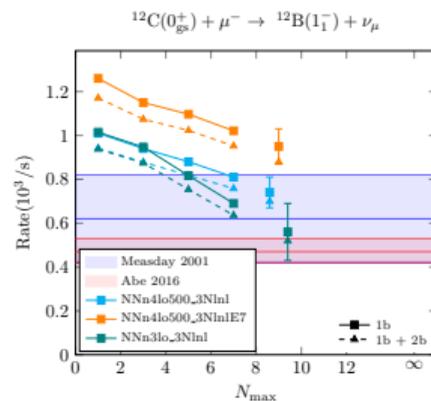
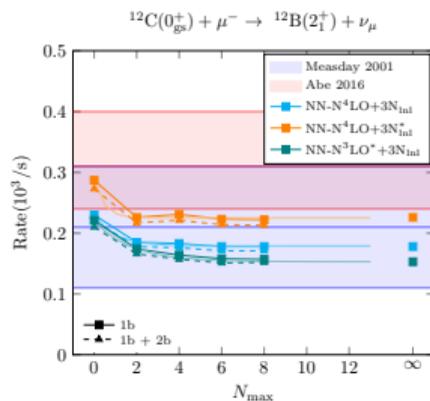
Capture Rates to Low-Lying States in ^{12}B

► Interaction dependence



Capture Rates to Low-Lying States in ^{12}B

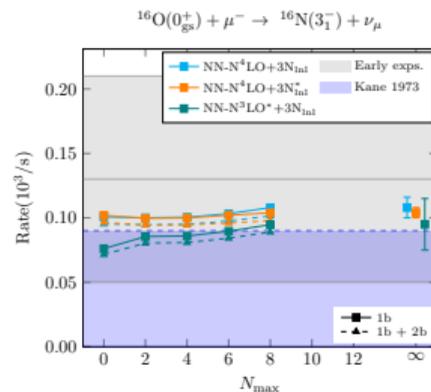
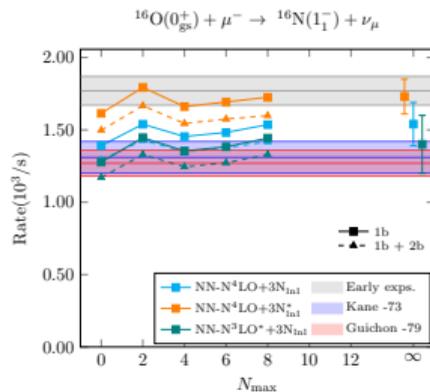
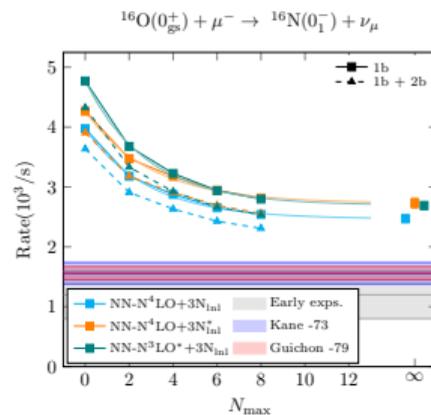
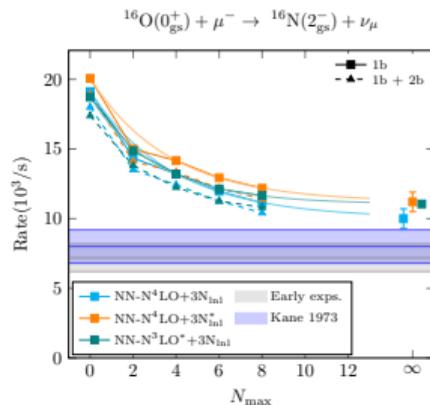
- ▶ Interaction dependence
- ▶ Adding the E_7 spin-orbit term improves agreement with experiment



LJ, Navrátil, Kotila, Kravvaris, work in progress

Capture Rates to Low-Lying States in ^{16}N

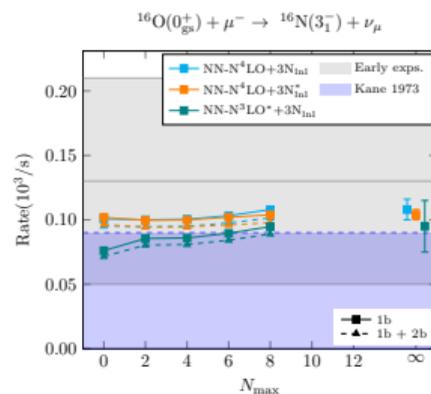
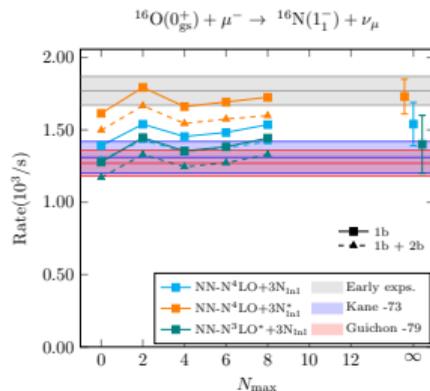
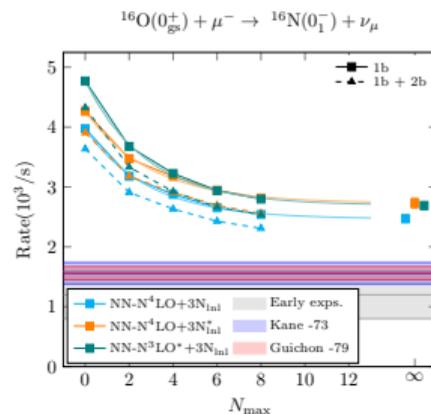
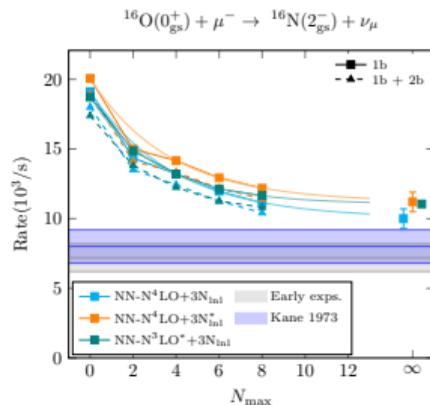
- NCSM describes well the complex systems ^{16}O and ^{16}N



LJ, Navrátil, Kotila, Kravvaris, work in progress

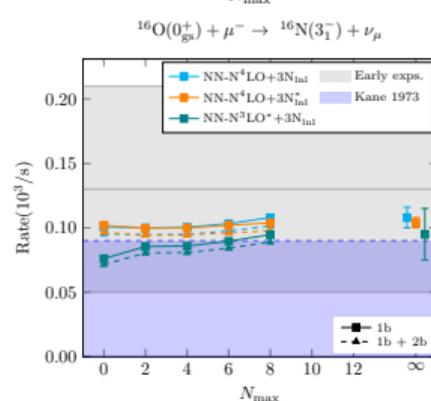
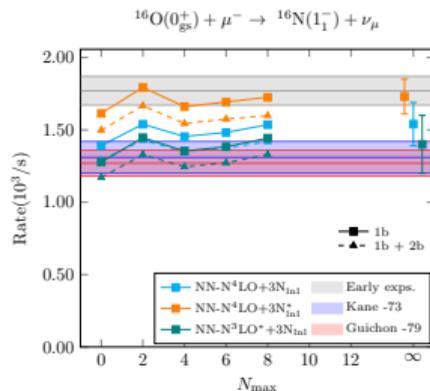
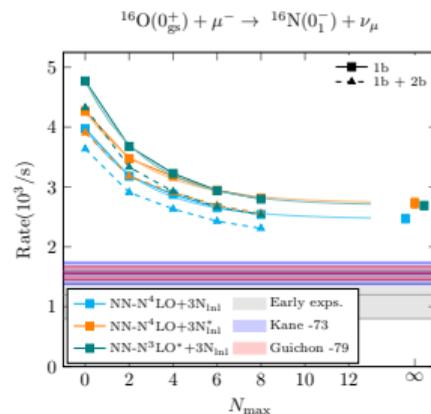
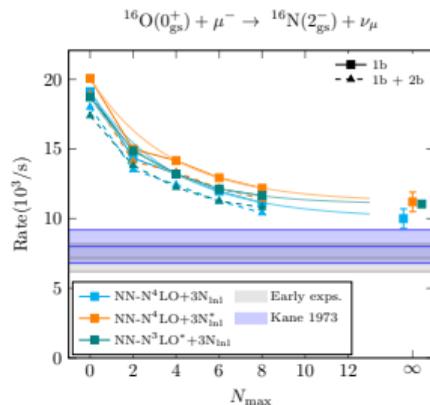
Capture Rates to Low-Lying States in ^{16}N

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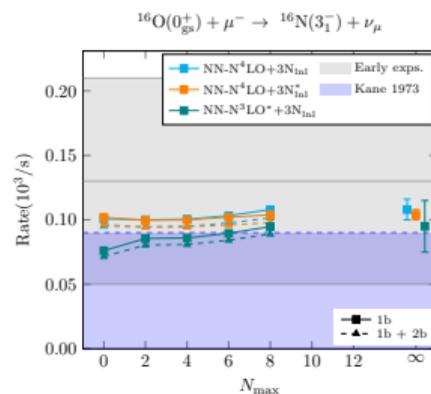
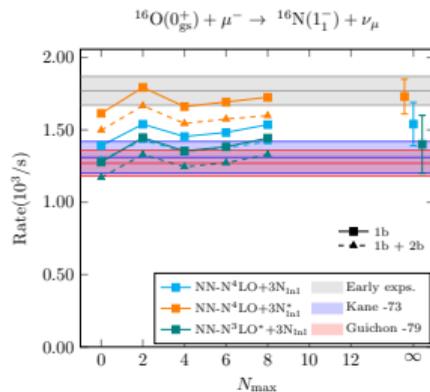
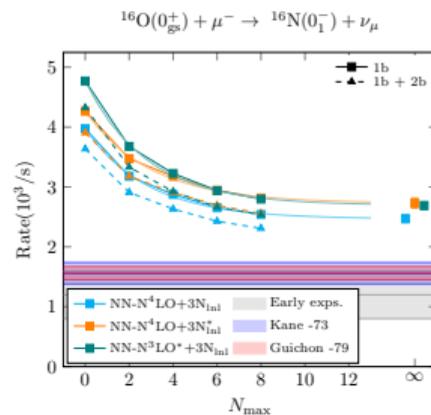
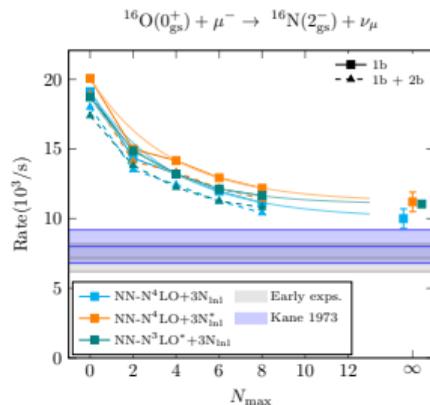
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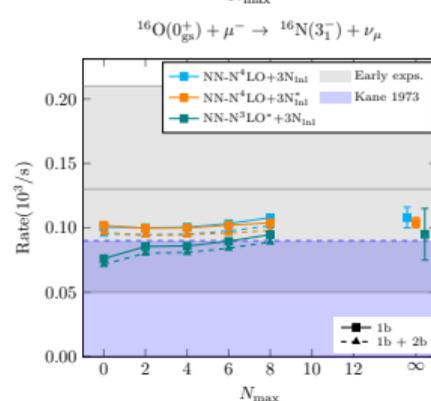
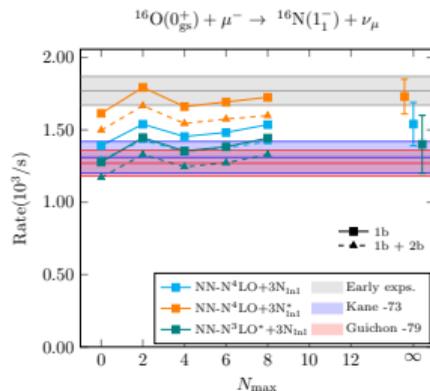
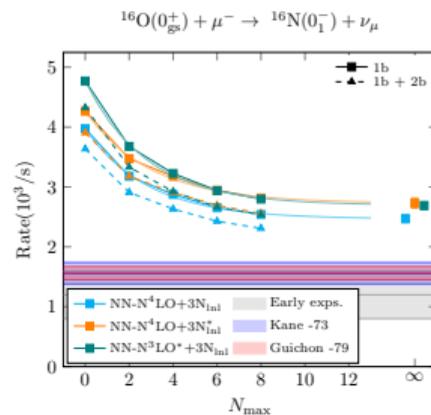
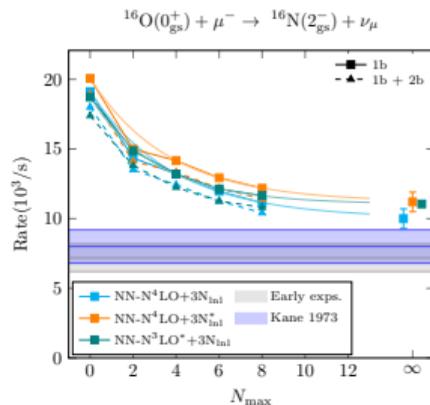
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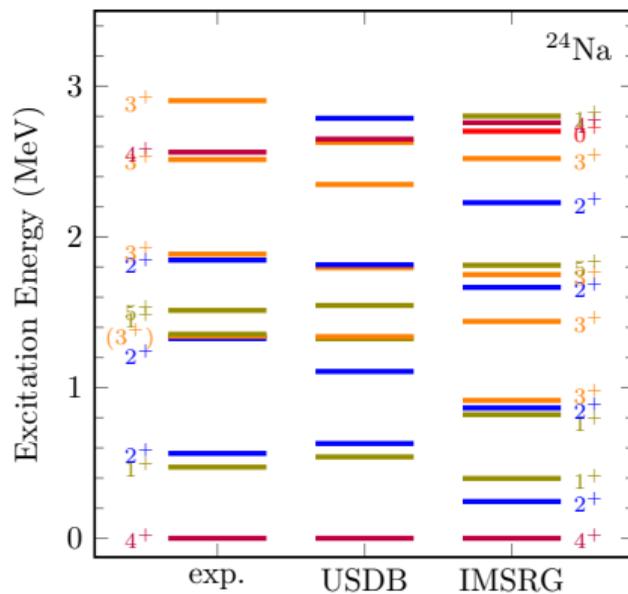
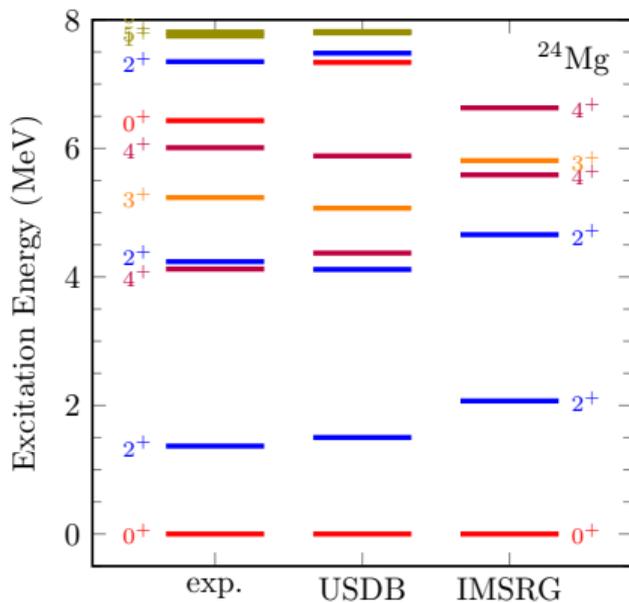
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Excitation Energies in the $A = 24$ Systems



Electromagnetic Moments in the $A = 24$ Systems

Nucleus	J_i^π	$E(\text{MeV})$			$\mu(\mu_N)$			$Q(e^2\text{fm}^2)$		
		exp.	NSM	IMSRG	exp.	NSM	IMSRG	exp.	NSM	IMSRG
^{24}Mg	2^+	1.369	1.502	1.981	1.08(3)	1.008	1.033	-29(3)	-19.346	-12.9
^{24}Mg	4^+	4.123	4.372	5.327	1.7(12)	2.021	2.096	-		
^{24}Mg	2^+	4.238	4.116	4.327	1.3(4)	1.011	1.085	-		
^{24}Mg	4^+	6.010	5.882	6.347	2.1(16)	2.015	2.089	-		
^{24}Na	4^+	0.0	0.0	0.0	1.6903(8)	1.533	1.485	-		
^{24}Na	1^+	0.472	0.540	0.397	-1.931(3)	-1.385	-0.344	-		

β Decays of the $A = 24$ Systems

Nucleus	$J_i \rightarrow J_f$	$\log ft$		
		exp.	NSM	IMSRG
^{24}Na	$1_1^+ \rightarrow 0_1^+$	5.80	5.188–5.223	4.448–4.545
^{24}Na	$4_{\text{gs}}^+ \rightarrow 4_1^+$	6.11	5.416–5.461	5.795–5.866
^{24}Na	$4_{\text{gs}}^+ \rightarrow 3_1^+$	6.60	5.727–5.773	6.342–6.422

Excitation Energies of ^{12}B

J_i^π	Interaction	$E_{\text{exc.}}$ (MeV)			Exp.
		$N_{\text{max}} = 4$	$N_{\text{max}} = 6$	$N_{\text{max}} = 8(\text{IT})$	
1_1^+	NN(N ⁴ LO)-3Nlnl	0.0	0.0	0.0	0.0
	NN(N ⁴ LO)-3NlnIE7	0.135	0.000	0.000	
2_1^+	NN(N ⁴ LO)-3Nlnl	0.251	0.465	0.538	0.953
	NN(N ⁴ LO)-3NlnIE7	0.000	0.027	0.097	
0_1^+	NN(N ⁴ LO)-3Nlnl	2.073	1.831	1.713	2.723
	NN(N ⁴ LO)-3NlnIE7	3.306	2.909	2.761	
2_2^+	NN(N ⁴ LO)-3Nlnl	3.816	3.490	3.344	3.760
	NN(N ⁴ LO)-3NlnIE7	4.919	4.463	4.281	

Excitation Energies of ^{16}N

J_i^π	Interaction	$E_{\text{exc.}}$ (MeV)			Exp.
		$N_{\text{max}} = 4$	$N_{\text{max}} = 6$	$N_{\text{max}} = 8(\text{IT})$	
2_1^-	NN(N^4LO)-3Nlnl	0.154	0.087	0.064	0.0
	NN(N^4LO)-3NlnIE7	0.214	0.146	0.133	
0_1^-	NN(N^4LO)-3Nlnl	2.245	1.487	1.010	0.120
	NN(N^4LO)-3NlnIE7	2.807	2.065	1.606	
3_1^-	NN(N^4LO)-3Nlnl	0.000	0.000	0.000	0.298
	NN(N^4LO)-3NlnIE7	0.000	0.000	0.000	
1_1^-	NN(N^4LO)-3Nlnl	2.561	1.833	1.363	0.397
	NN(N^4LO)-3NlnIE7	2.985	2.310	1.869	