Thermodynamics of oscillating neutrinos

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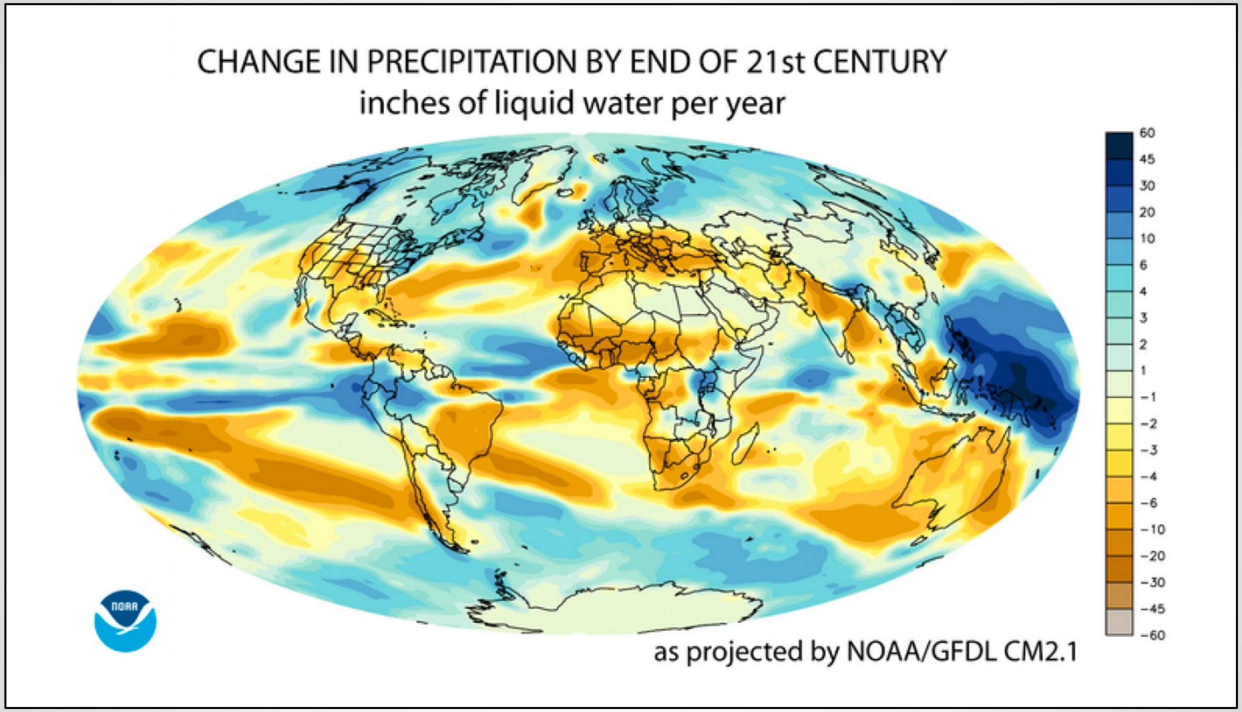
Quantum kinetic equation for density matrix $\rho(t, \boldsymbol{r}, \boldsymbol{p})$:

$$i\left(\partial_t + \hat{\mathbf{p}} \cdot \partial_{\mathbf{r}}\right)\rho = [H, \rho] + iC$$

Particle advection — **Flavor mixing** — **Collisions**

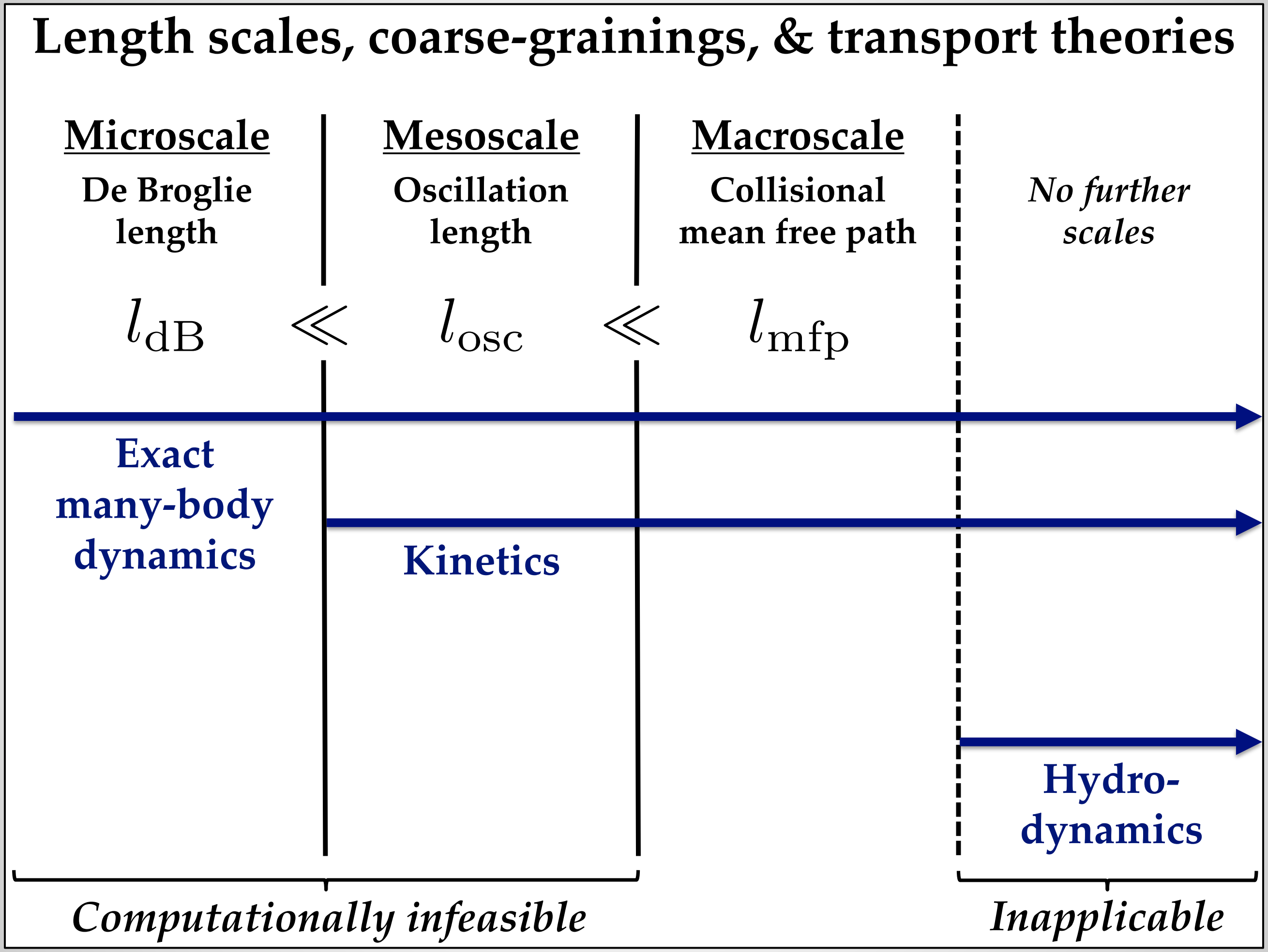
Dolgov, SJNP (1981); Stodolsky, PRD (1987); Nötzold & Raffelt, NPB (1988); Pantaleone, PLB (1992); Sigl & Raffelt, NPB (1993); Raffelt, Sigl, & Stodolsky, PRL (1993); Raffelt & Sigl, AP (1993); Loreti & Balantekin, PRD (1994); Yamada, PRD (2000); Friedland & Lunardini, PRD (2003); Strack & Burrows, PRD (2005); Cardall, PRD (2008); Volpe, Väänänen, & Espinoza, PRD (2013); Vlasenko, Fuller, & Cirigliano, PRD (2014); Kartavtsev, Raffelt, & Vogel, PRD (2015); Stirner, Sigl, & Raffelt, JCAP (2018); Nagakura, PRD (2022); **Johns**, 2305.04916; plus many others

Compare to climate modeling…

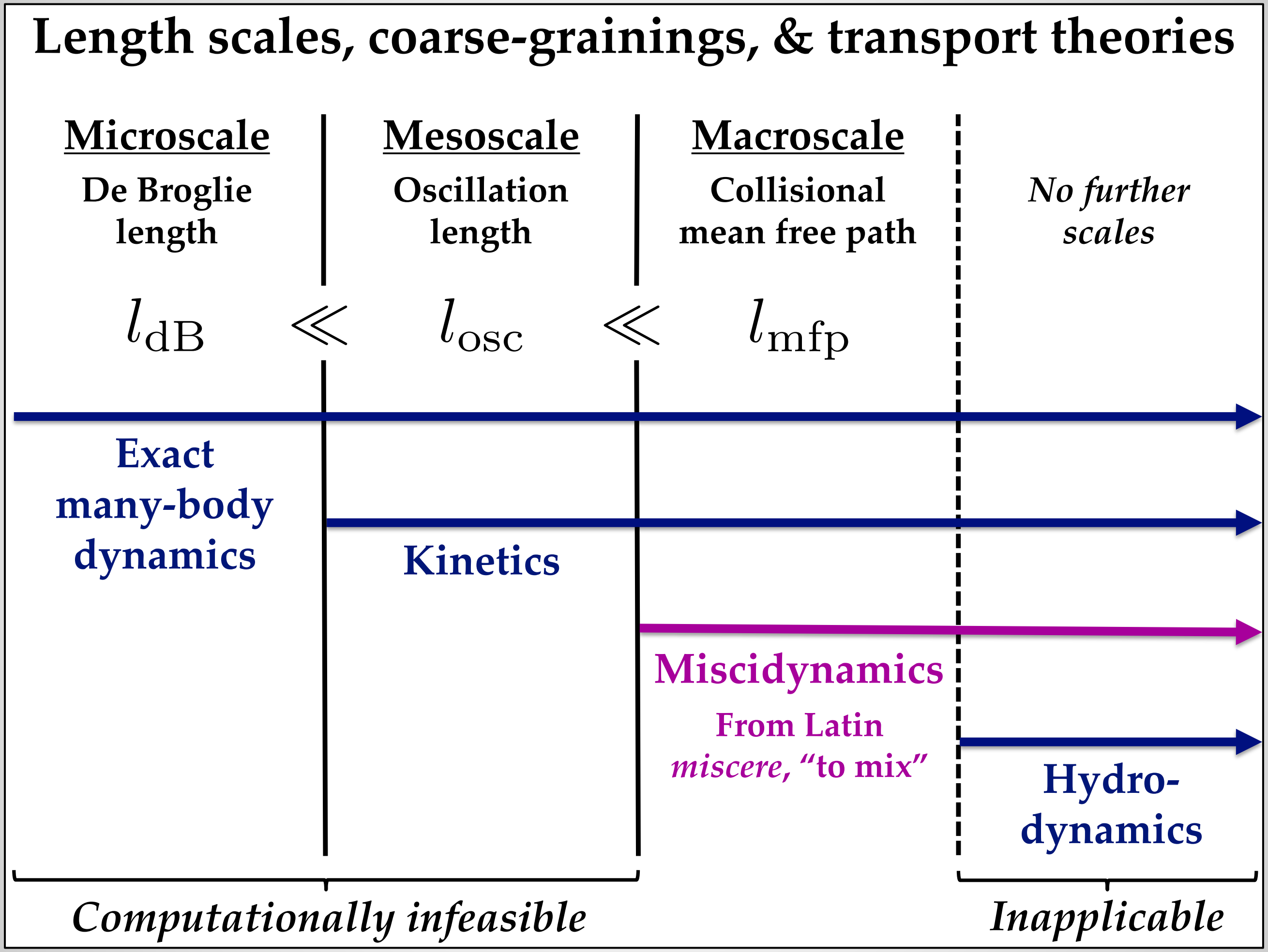


NOAA Geophysical Fluid Dynamics Laboratory

Without hydrodynamics, this field would not have gotten far.



Johns, 2306.14982



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Varieties of equilibration

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- ## Small scales

  Classical: Collisionless relaxation in galaxies & plasmas. Turbulence.
  Quantum: **Mixing equilibration**.

To formulate the thermodynamics of oscillating neutrinos, we need to define an appropriate entropy:

$$S = -\int d\mathbf{p}\ \mathrm{Tr}\left[\overline{\rho_{\mathbf{p}}}\log\overline{\rho_{\mathbf{p}}} + (1-\overline{\rho_{\mathbf{p}}})\log(1-\overline{\rho_{\mathbf{p}}})\right]$$

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Spatially coarse-grained density matrix

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Spatially coarse-grained density matrix

Second law of thermodynamics

S is maximal at equilibrium, subject to conservation laws.

We then appeal to two physical principles: **scale separation** & **ergodicity**.

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Equilibrium distribution of collisionless neutrino matter:

$$\rho_{\mathbf{p}}^{\mathrm{eq}} = \frac{1}{e^{\beta\left(H_{\mathbf{p}}^{\mathrm{eq}} - \mu_{\mathbf{p}}\right)} + 1}$$

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<u>**Third law of thermodynamics**</u>

The unique ($S = 0$) ground state:

$$\left(\rho_{\mathbf{p}}^{\mathrm{eq}}\right)_{IJ} \xrightarrow{T \to 0} \begin{cases} \delta_{IJ} & \left(H_{\mathbf{p}}^{\mathrm{eq}}\right)_{IJ} \leq \mu_{\mathbf{p}} \\ 0 & \left(H_{\mathbf{p}}^{\mathrm{eq}}\right)_{IJ} > \mu_{\mathbf{p}} \end{cases}$$

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First law of thermodynamics

$$\Delta U = W + Q$$

with W and Q appropriately defined.

$$\Delta U = \overbrace{\frac{1}{N_f} H_0 \Delta P_0 + \frac{1}{2} \vec{H} \cdot \Delta |\vec{P}| \hat{P}}^{\equiv Q^{\mathrm{env}}} + \overbrace{\frac{1}{2} |\vec{H}| |\vec{P}| \Delta \left(\hat{H} \cdot \hat{P} \right)}^{\equiv Q^{\mathrm{kin}}}$$

$$+ \underbrace{\frac{1}{N_f} \Delta H_0 P_0 + \frac{1}{2} \Delta |\vec{H}| |\vec{P}| \hat{H} \cdot \hat{P}}_{\equiv W}$$

$$\rho = \frac{1}{N_f} P_0 + \frac{1}{2} \vec{P} \cdot \vec{\Lambda}$$

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From here it's easy to show that quantum-adiabatic effects (MSW, spectral swaps, MNR) are **adiabatic processes in the thermodynamic sense**.

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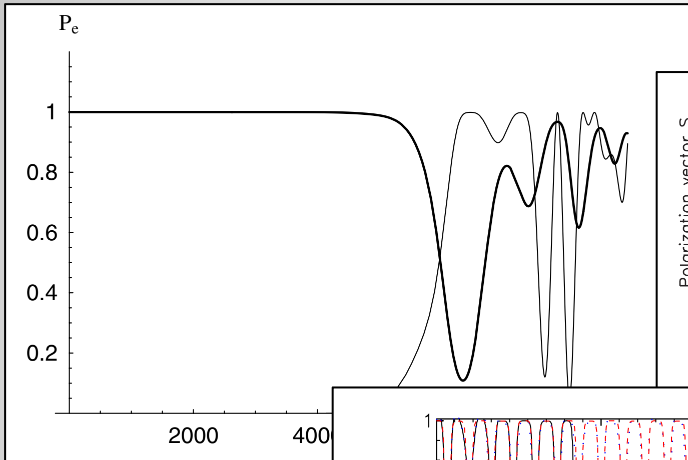
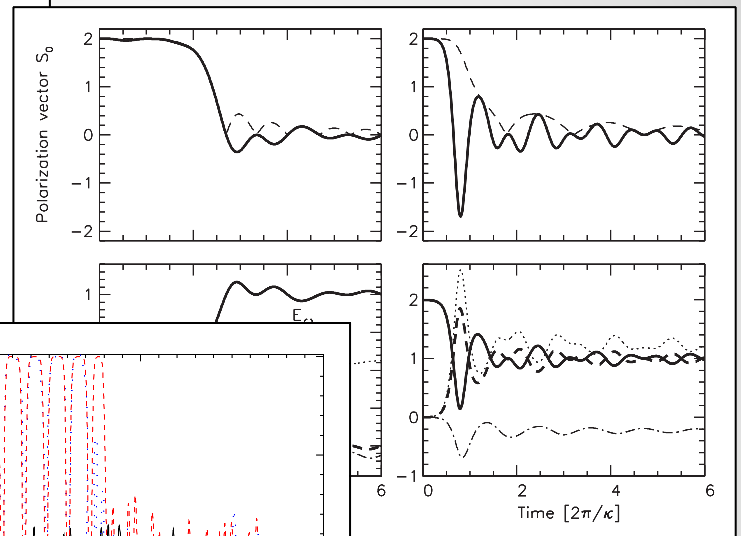
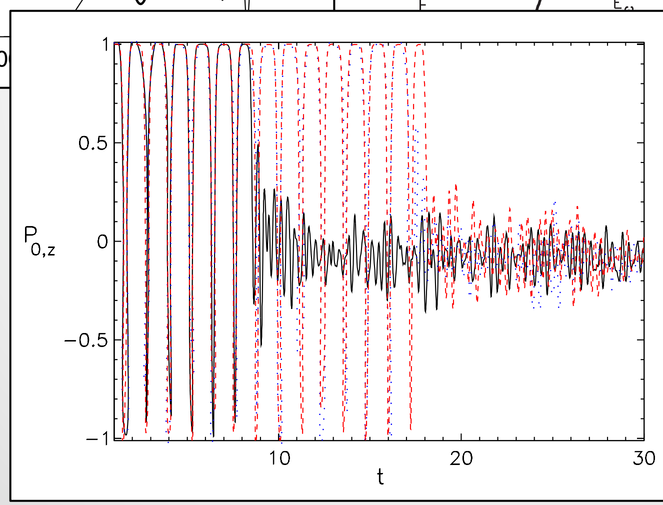
Quasi-steady states are the numerically observed outcomes of collisionless instabilities:

Sawyer, PRD (2005)



Raffelt & Sigl, PRD (2007)





Mangano, Mirizzi, & Saviano, PRD (2014)

\+ many more papers
on fast instabilities

Bhattacharyya & Dasgupta, PRD (2020)
Richers, Willcox, & Ford, PRD (2021)
Nagakura & Zaizen, PRL (2022) & others

Thermodynamics predicts the mean asymptotic distributions.

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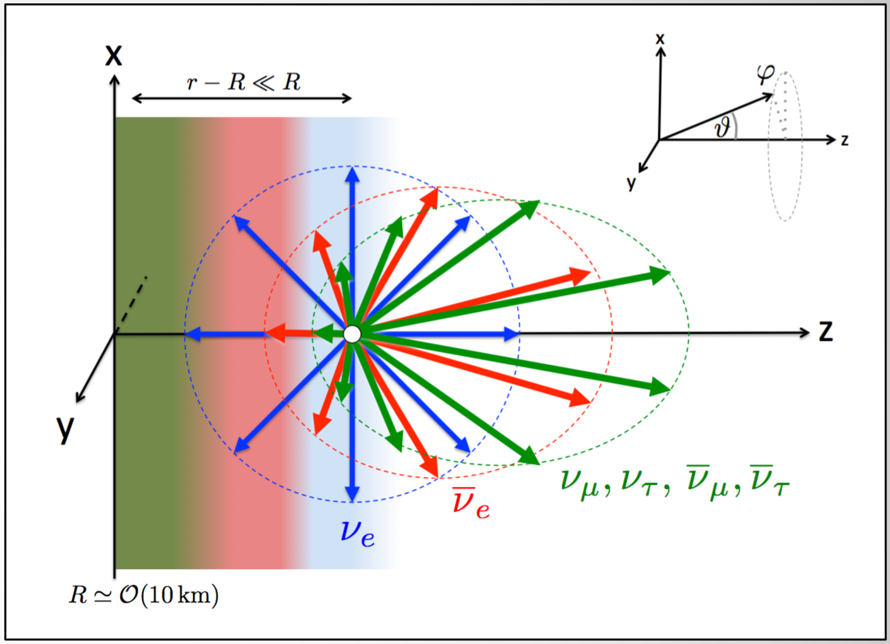
Thermodynamics also explains the association between fast instabilities and **angular crossings**.



Sawyer, PRL (2016); Chakraborty, Hansen, Izaguirre, & Raffelt, JCAP (2016); Dasgupta, Mirizzi, & Sen, JCAP (2017) [*figure*]; Izaguirre, Raffelt, & Tamborra, PRL (2017); Capozzi, Dasgupta, Lisi, Marrone, & Mirizzi, PRD (2017); Abbar & Duan, PRD (2018); Capozzi, Dasgupta, Mirizzi, Sen, & Sigl, PRL (2019); Martin, Yi, & Duan, PLB (2020); **Johns**, Nagakura, Fuller, & Burrows, PRD (2020a); **Johns** & Nagakura, PRD (2021); Nagakura, Burrows, **Johns**, & Fuller, PRD (2021); Morinaga, PRD (2022); Dasgupta, PRL (2022); & many others

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Non-collective neutrino oscillations

Work **The MSW effect**

Wolfenstein, PRD (1978)
Mikheyev & Smirnov, SJNP (1985)
Bethe, PRL (1986)

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Heat (internal) — **Kinematic decoherence**

Nussinov, PLB (1976)
Kayser, PRD (1981)
Kiers, Nussinov, & Weiss, PRD (1996)

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Heat (external)

Collisional decoherence

Harris & Stodolsky, PLB (1978)
Thomson, PRA (1992)
Raffelt, Sigl, & Stodolsky, PRL (1993)

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Collective neutrino oscillations

Work **MSW-like effects** **Spectral swaps. Matter–neutrino resonances.**

Duan, Fuller, Carlson, & Qian, PRD (2006)
Raffelt & Smirnov, PRD (2007)
Malkus, Friedland, & McLaughlin, 1403.5797

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Heat (internal)

Collisionless instabilities
Slow instabilities. Fast instabilities.

Kostelecký & Samuel, PLB (1993)
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Johns, PRL (2023)
Xiong, **Johns**, Wu, & Duan, 2212.03750
Liu, Zaizen, & Yamada, PRD (2023)

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Local mixing equilibrium

$$\rho \longrightarrow \rho^{\text{eq}}$$

The miscidynamic equation

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What changes need to be made to current simulations?

(1) Distribution functions ⟶ Density matrices.

(2) Add off-diagonals to collision terms.

(3) Re-equilibrate ρ after each step.

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Main idea #2. We've outlined the thermodynamic theory of oscillating neutrinos. The primary equilibration mechanism is collisionless phase-space transfer.

Main idea #3. A transport theory—miscidynamics—follows from the assumption of local equilibrium. It might enable the accurate incorporation of neutrino mixing into simulations.