HIC matter in modified-gravity NSs

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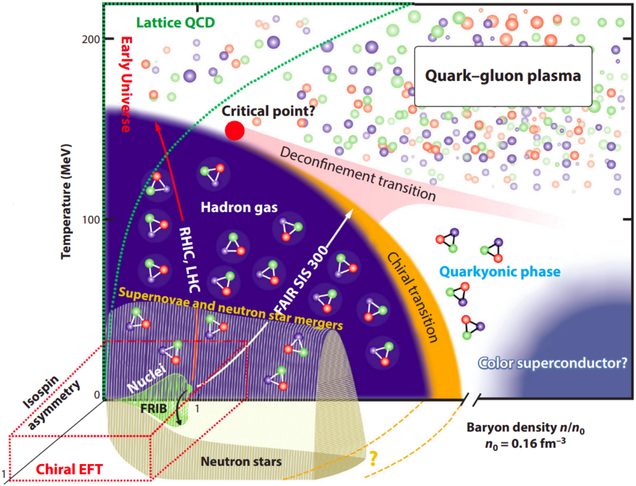
Dense Nuclear Matter Equation of State from Heavy-Ion Collisions
(INT-22-84W, Seattle, Washington, US 2022)

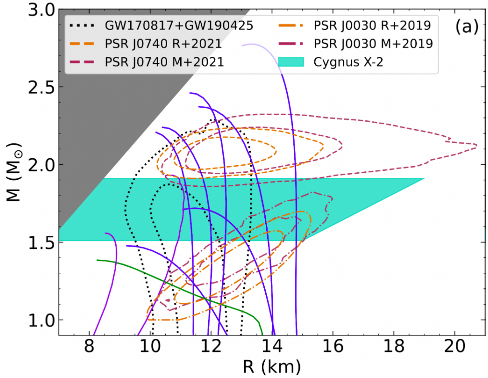
Motivation

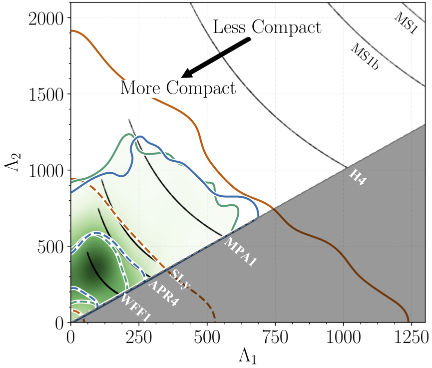


Cartoon of the QCD phase diagram

[Drischler *et al.*, 2021]

Motivation





Mass-Radius diagram from NICER (left) and tidal deformabilities from GW170817 (right)

Motivation

nature physics

LETTERS

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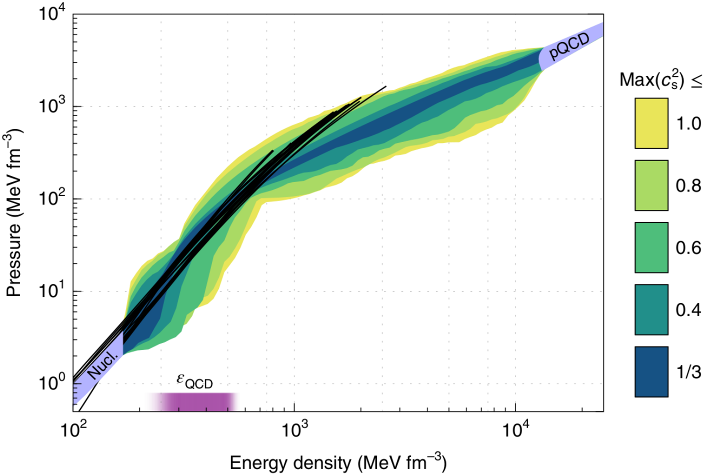
Evidence for quark-matter cores in massive neutron stars

Eemeli Annala[1], Tyler Gorda[2], Aleksi Kurkela[3,4], Joonas Nättilä[5,6,7] and Aleksi Vuorinen[1]

[Nature Phys. 16 (2020) 9, 907-910]

Motivation



Equations of state for QCD matter

[Annala *et al.*, Nature Phys. 2020]

Should we always believe in General Relativity (GR)?

- If applied to cosmological problems, e.g. early-time inflation and late-time accelerated expansion of the Universe, GR is apparently insufficient even adding a Λ constant or dark-energy component.

- Simple extensions like '$\mathcal{L}_{\mathrm{H-E}} \sim R \to f(R) = R + \alpha R^2$' (known as the Starobinsky model) solve these problems without a Λ constant or inflaton field.

- In fact, this Starobinsky model shows very good agreement with the *Planck 2018* data for the inflationary epoch via the analysis of the CMB anisotropies.

Modified theories of gravity $f(R)$

- The corresponding action in the Jordan frame is given by

$$I = \frac{1}{16\pi}\int d^4x\sqrt{-g}f(R) + I_m,$$

which produces the modified Einstein's field equations given by

$$f_R R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}f + [g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu]f_R = 8\pi T_{\mu\nu},$$

where $T_{\mu\nu}$ is the matter energy-momentum tensor and $\Box \equiv \nabla_\mu\nabla^\mu$ is the d'Alembertian operator.

- Notice that now the Ricci scalar satisfies the equation

$$3\Box f_R(R) + Rf_R(R) - 2f(R) = 8\pi T$$

which in the outside vacuum case, $T = 0$, has non-trivial solutions in contrast to $R = 0$ in GR.

Starobinsky TOV equations

- As usual, for compact-star interiors we consider the static and spherically-symmetric spacetime element as

$$ds^2 = -e^{2\psi(r)}dt^2 + e^{2\lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2).$$

- The modified TOV equations within $f(R) = R + \alpha R^2$ gravity are

$$\frac{d\psi}{dr} = \frac{1}{4r(1+2\alpha R + \alpha r R')}\left[r^2 e^{2\lambda}(16\pi p - \alpha R^2) + \ldots\right],$$

$$\frac{d\lambda}{dr} = \frac{1}{4r(1+2\alpha R + \alpha r R')}\left\{2(1+2\alpha R)\left(1-e^{2\lambda}\right) + \ldots\right\},$$

$$\frac{d^2R}{dr^2} = \frac{e^{2\lambda}}{6\alpha}\left[R + 8\pi(3p-\rho)\right] + \left(\lambda' - \psi' - \frac{2}{r}\right)R',$$

$$\frac{dp}{dr} = -(\rho + p)\psi'.$$

As in GR, the stellar surface is found when $p(r = r_{\rm sur}) = 0$.

Boundary conditions for R^2-gravity stars

- In order to ensure the regularity of the geometry at the stellar center, we establish the following boundary conditions

$$\rho(0) = \rho_c, \qquad \psi(0) = \psi_c, \qquad \lambda(0) = 0,$$
$$R(0) = R_c, \qquad R'(0) = 0,$$

where "ρ_c" and "R_c" are the values of the central energy density and central scalar curvature, respectively.

- Besides, it is also useful to settle the junction conditions as

$$\psi_{in}(r_{\rm sur}) = \psi_{out}(r_{\rm sur}), \qquad \lambda_{in}(r_{\rm sur}) = \lambda_{out}(r_{\rm sur}),$$
$$R_{in}(r_{\rm sur}) = R_{out}(r_{\rm sur}), \qquad R'_{in}(r_{\rm sur}) = R'_{out}(r_{\rm sur}).$$

Boundary conditions for R^2−gravity stars

- We can define a mass parameter in $R^2 - -$gravity just as in Einstein's theory

$$m = 4\pi \int r^2 \rho dr + \alpha \int \left\{ \frac{R^2}{4} - \frac{R}{r^2} \frac{d}{dr} \left[r(1 - e^{-2\lambda}) \right] + \frac{1}{e^{2\lambda}} [...] \right\} r^2 dr.$$

- If $\alpha \neq 0$, even in the outer region of a compact star this '$m(r)$' generates an extra mass contribution due to the Ricci scalar and its derivatives sometimes called "gravitational sphere".
- Besides, asymptotic flatness requires

$$\lim_{r\to\infty} R(r) = 0, \qquad \lim_{r\to\infty} m(r) = \text{constant}.$$

- So, R_c must be chosen appropriately at infinity. Thus, the total gravitational mass of the star M is determined from the asymptotic behavior

$$M \equiv \lim_{r\to\infty} \frac{r}{2} \left(1 - \frac{1}{e^{2\lambda}} \right).$$

Thermal and dense QCD:
Simple prescription but a challenging calculation

- The total pressure of a QCD can be obtained from

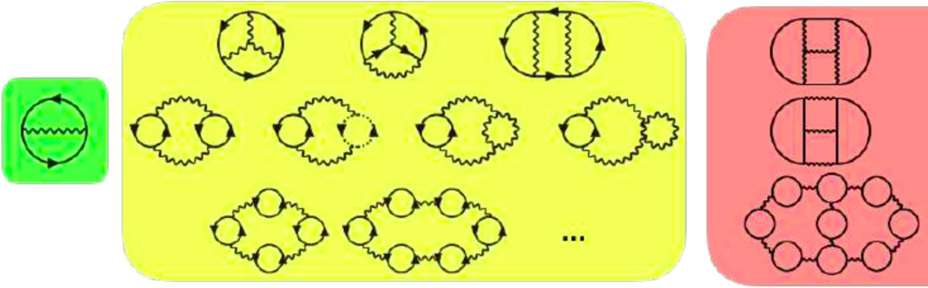
$$P(T, \{\mu_i\}) = T \log \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}A_\mu e^{-\int d^3x \int_0^{1/T} d\tau \mathcal{L}_{\mathrm{QCD}}}.$$

- For $T \neq 0$ and $\mu \lesssim T$: Lattice-gauge-field theory methods **apply**.

- For $\mu \gtrsim T$: **Unfeasible** due to the fermionic sign problem.

- Perturbative control at low densities (chiral effective field theory) and at ultra-high densities (perturbative QCD), both in the **cold limit**.

Cold and dense perturbative QCD (pQCD)



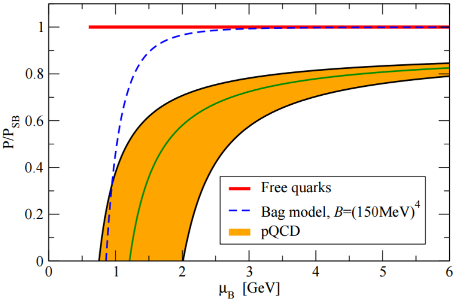
$$P(\mu_B)/P_{\text{free}} \sim 1 + \underbrace{c_1 g^2}_{NLO} + \underbrace{c_2 g^4 + c_2' g^4 \log g}_{NNLO} + \underbrace{c_3' g^6 \log^2 g + c_3'' g^6 \log g + \ldots}_{N^3LO}$$

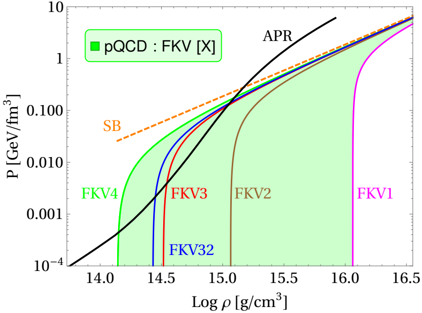
pQCD quark stars within R^2−gravity

The 3-loop result of Kurkela *et al.* (2010) can be cast in a simple pocket formula for the pressure given by [Fraga et al., 2014]

$$p = p_{\mathrm{SB}}(\mu_B)\left(c_1 - \frac{a(X)}{(\mu_B/\mathrm{GeV}) - b(X)}\right) ,$$

where $p_{\mathrm{SB}}(\mu_B) = (3/4\pi^2)(\mu_B/3)^4$ is the Stefan-Boltzmann pressure.



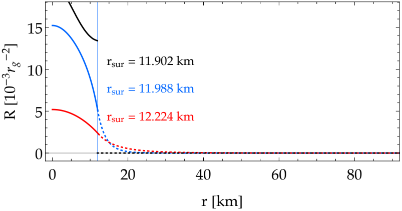


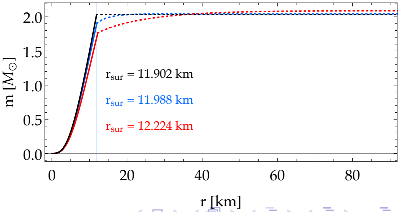
Pressures for the MIT bag model (B) and perturbative QCD (FKV)

[arXiv:1311.5154,2112.09950]

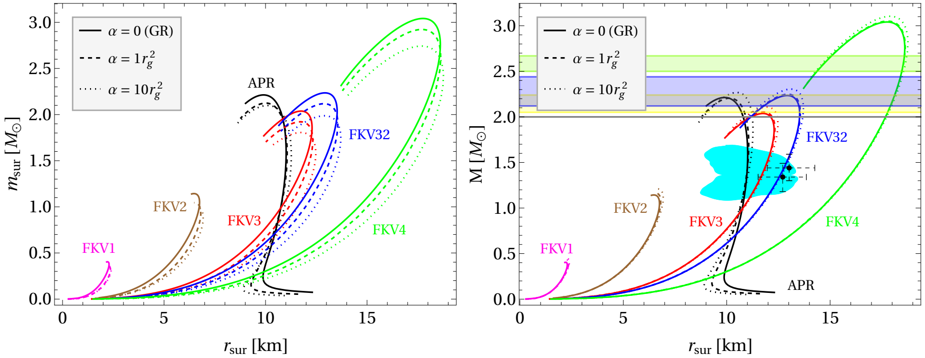
Maximal parameters of R^2−quark stars

EoS	α $[r_g^2]$	R_c $[10^{-3} r_g^{-2}]$	ρ_c $[10^{15}\mathrm{g/cm^3}]$	r_{sur} [km]	m_{sur} $[M_\odot]$
FKV3	0	21.160	1.693	11.752	2.037
	1	15.735	1.775	11.784	1.921
	10	5.284	1.912	11.939	1.800
FVK32	0	17.687	1.400	12.927	2.235
	1	13.682	1.459	12.956	2.117
	10	4.981	1.587	13.100	1.982
FVK4	0	9.727	0.744	17.755	3.041
	1	8.278	0.763	17.775	2.922
	10	3.925	0.829	17.900	2.738



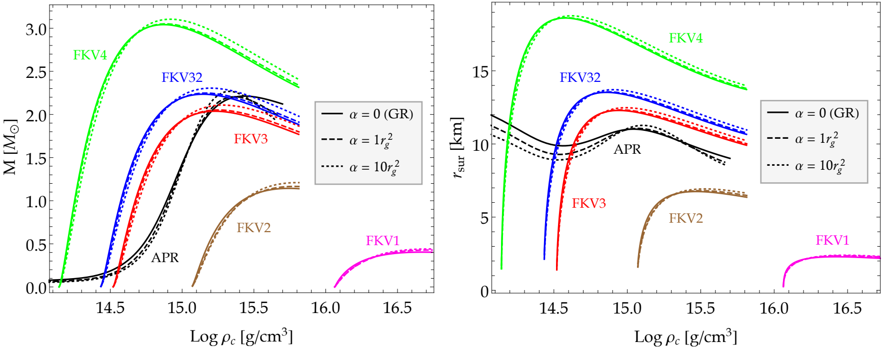


Structure of R^2−quark stars



Solutions of the Starobinsky TOV equations [JCJ *et al.*, 2022]

Mechanical stability of R^2−quark stars



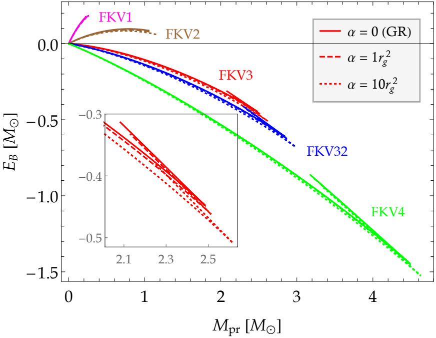
Effects on their masses and radii [JCJ *et al.*, 2022]

Baryon stability of R^2-quark stars

- The binding energy is defined as $E_B = M - M_{\rm pr}$, where $M_{\rm pr}$ is the baryonc mass obtainable as follows

$$M_{\rm pr} = m_B N_B = 4\pi m_{\rm B} \int_0^{r_{\rm sur}} e^{\lambda(r)} r^2 n_B(r) dr,$$

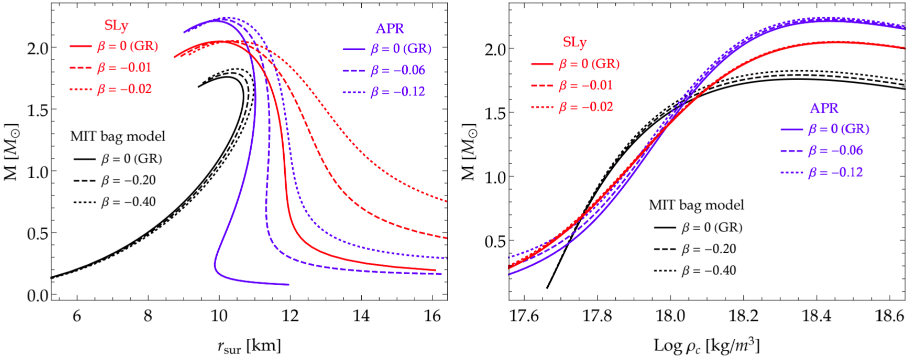
being '$n_B(r)$' is the baryon number density profile and '$m_{\rm B}$' the neutron mass.



Effects on their binding energies [JCJ *et al.*, 2022]

OTHER EXAMPLES: $f(R,T) = R + 2\beta T$

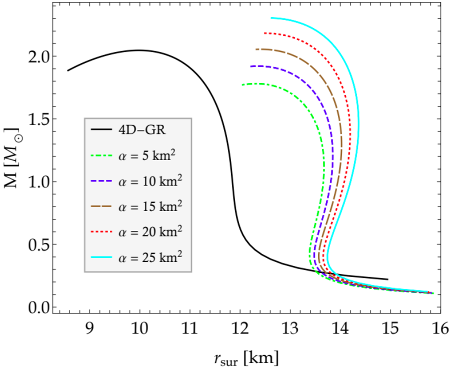
In this case 'T' is the trace of the energy-momentum tensor and 'β' a free parameter.

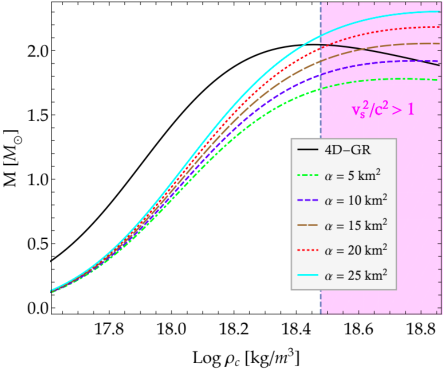


Modified mass-radius relations [arXiv:2012.03342]

OTHER EXAMPLES: $f(R,\mathcal{G}) = R - 2\Lambda + \alpha\mathcal{G}$ (Einstein-Gauss-Bonnet)

The Gauss-Bonnet invariant is defined as
$\mathcal{G} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\sigma\rho}R^{\mu\nu\sigma\rho}$ and 'α' another free parameter.





Results using the SLy nuclear EoS [arXiv:2107.03859]

Summary and lessons

- Most (if not all) M, R and $\bar{\Lambda}$ indirect measurements assume that GR is valid in static NSs and NS mergers when interprete their findings. Besides, few (if not any) studies considering 1st-order transitions or pQCD results in the EoS are assumed a priori in their analysis.

- We have shown that even in simple extensions of GR, the $M's$ and $R's$ of quark stars display a difference of ~ 15 percent with GR results.

- **Are the nuclear matter EoSs from astrophysics consistent with heavy-ion collision observables in the range $\rho < 4\rho_0$?**
  Modified gravity shows that there is a bias introduced at around 'ρ_0' when using astrophysics to constrain the nuclear-matter EoS. Systematically this might introduce higher error bands for $P = P(n_B)$ and even worse when applying the same reasoning to NS mergers.

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