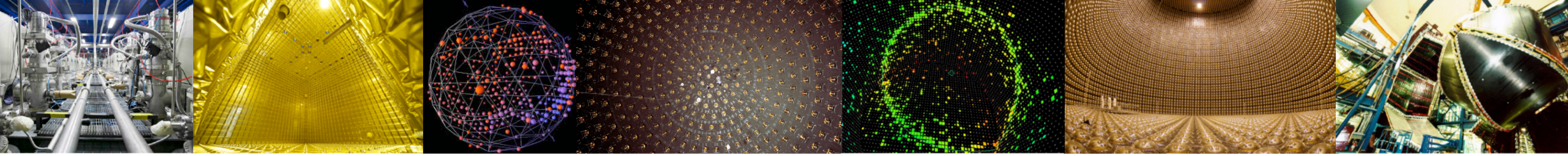


Achilles

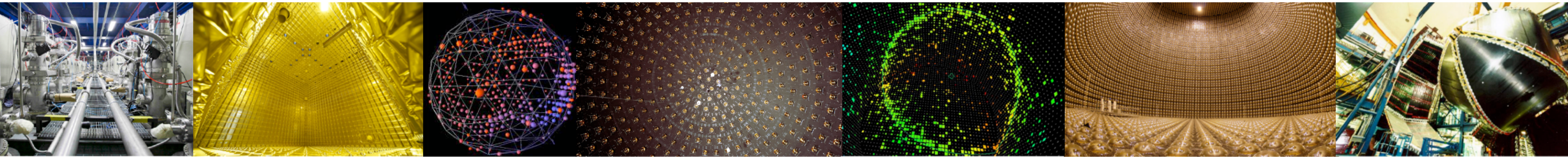
A CHIcagoLand Lepton Event Simulator

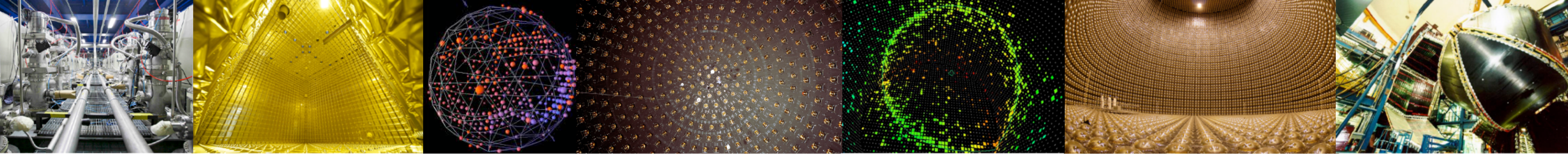
Current Capabilities & Future Plans for Lepton Scattering Modeling and Associated Uncertainties

William Jay — MIT

with Joshua Isaacson, Alessandro Lovato, Pedro Machado, Noemi Rocco, and Luke Pickering

INT Workshop 23-86W
Theory Uncertainties to Empower Neutrino Experiments
30/ October- 3 November 2023



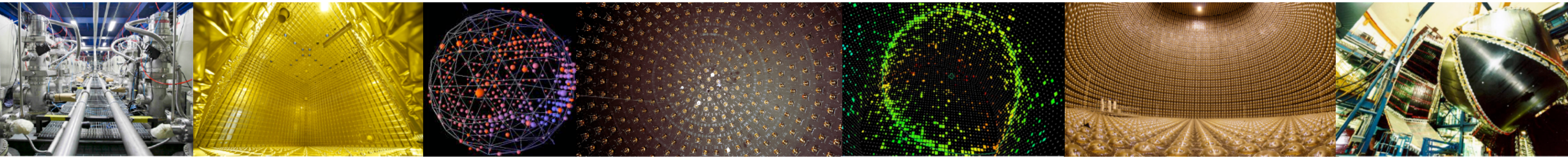


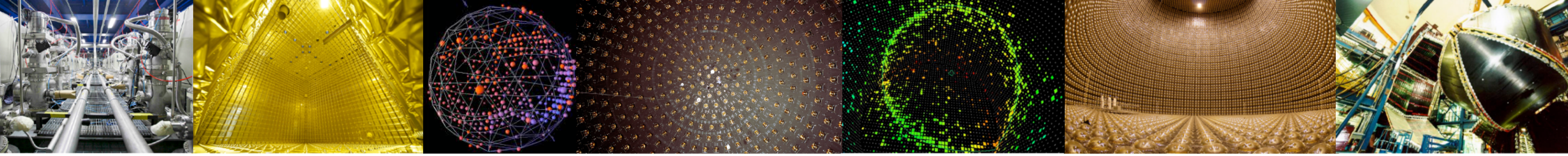
Based largely on:

Isaacson [WJ] et al.
"New approach to intranuclear cascades with quantum Monte Carlo configurations"
***Phys.Rev.C* 103 (2021) 1, 015502 [arXiv:2007.15570]**

Isaacson [WJ] et al.
***"Introducing a novel event generator for electron-nucleus and neutrino-nucleus scattering*"**
***Phys.Rev.D* 107 (2023) 3, 033007 [arXiv:2205.06378]**

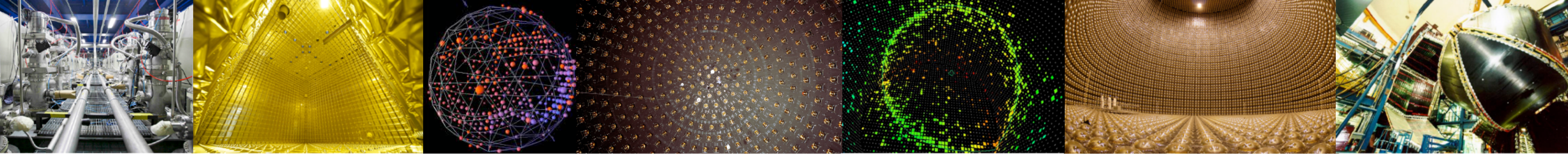
+ unpublished work in progress



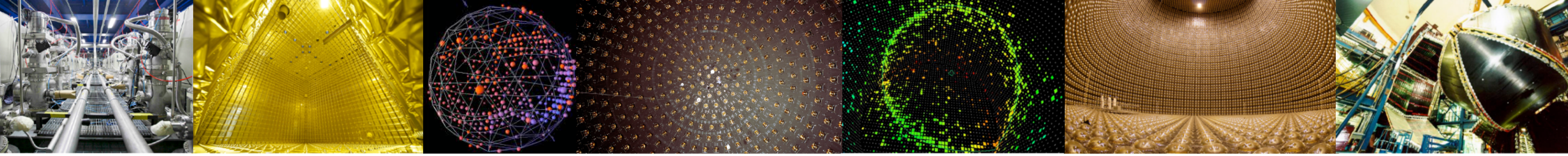


Outline

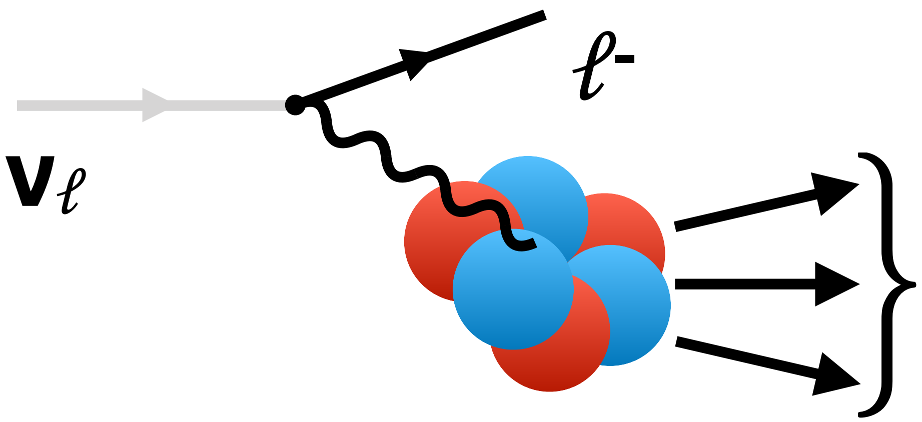
- Motivation
- Achilles overview
  - Theoretical setup
  - Work in progress
- Recent results



Motivation

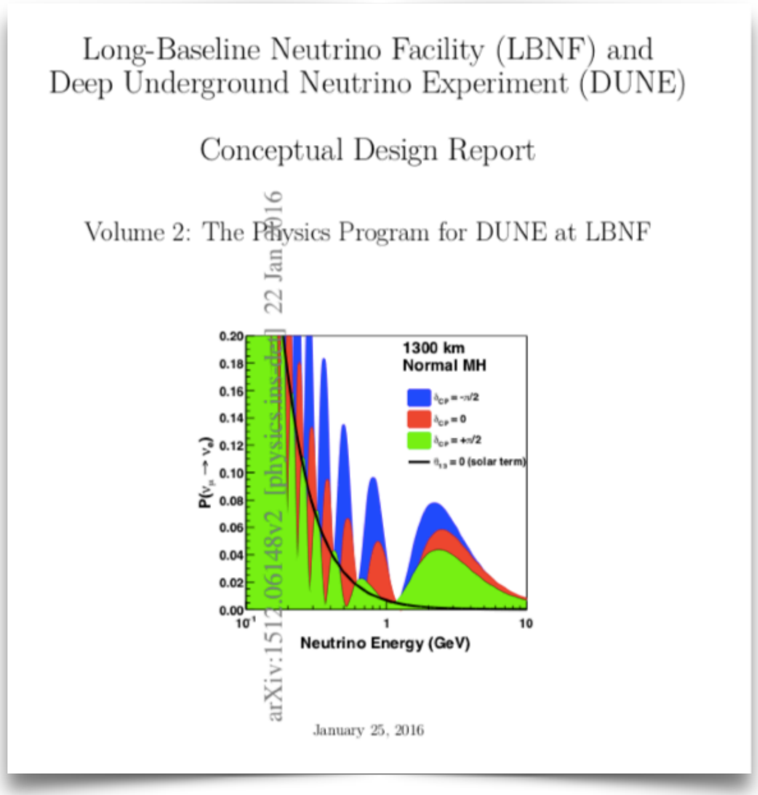


The Challenge
Lepton Event Simulation



- Neutrino experiments like DUNE and T2HK herald the start of the "precision era" for neutrino experiments
- Percent-level theoretical control of neutrino-nucleus scattering cross sections is needed

"Uncertainties exceeding 1% for signal and 5% for backgrounds may result in substantial degradation of the sensitivity to CP violation and the mass hierarchy."

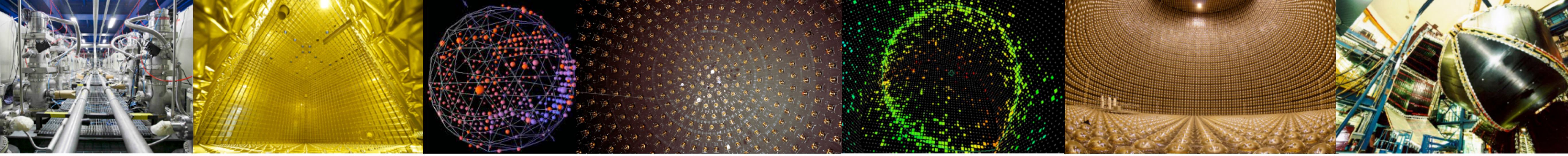

Long-Baseline Neutrino Facility (LBNF) and
Deep Underground Neutrino Experiment (DUNE)

Conceptual Design Report

Volume 2: The Physics Program for DUNE at LBNF

arXiv:1512.06148v2 [physics.ins-det] 22 Jan 2016

January 25, 2016

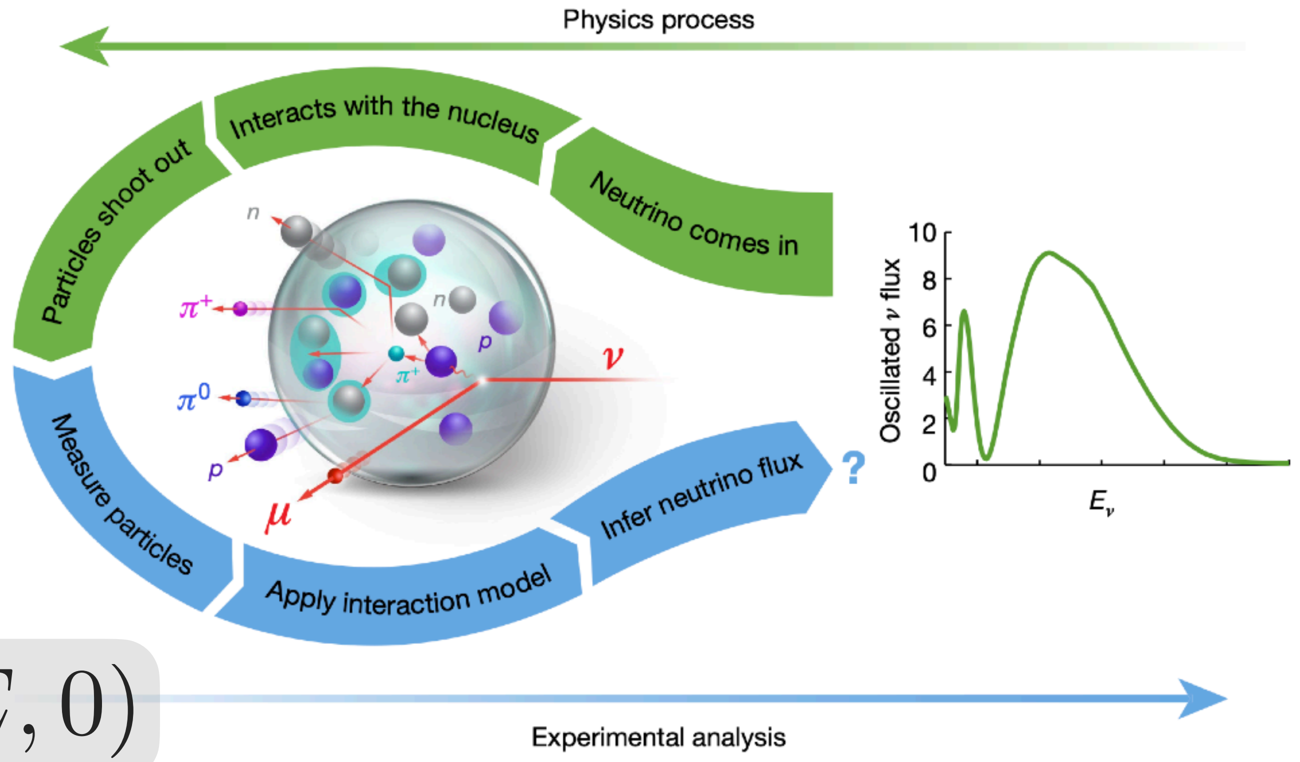


The Challenge

Lepton Event Simulation

Want: Mixing parameters, e.g, angle θ

$$P_{\nu_\mu \to \nu_e} \approx \sin^2(2\theta)\sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$



$$\Phi_e(E,L) \propto P_{\nu_\mu \to \nu_e}(E,L)\Phi_\mu(E,0)$$

Neutrino fluxes. "Measurable."

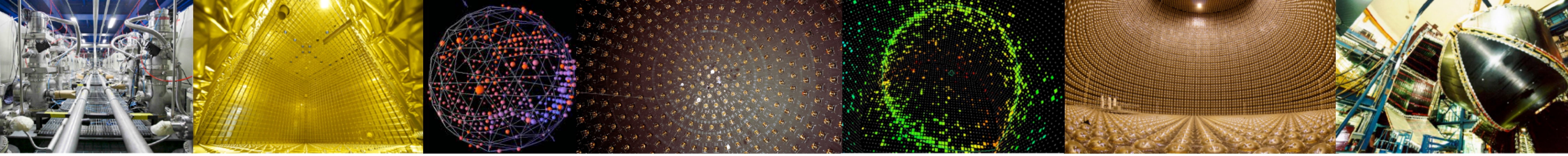
"Smearing matrix"
(Experimental + theoretical)

$$N_\alpha(E_{\rm rec},L) \propto \int dE\, \Phi_\alpha(E,L)\sigma(E) f_{\sigma_i}(E,E_{\rm rec})$$

Event rate

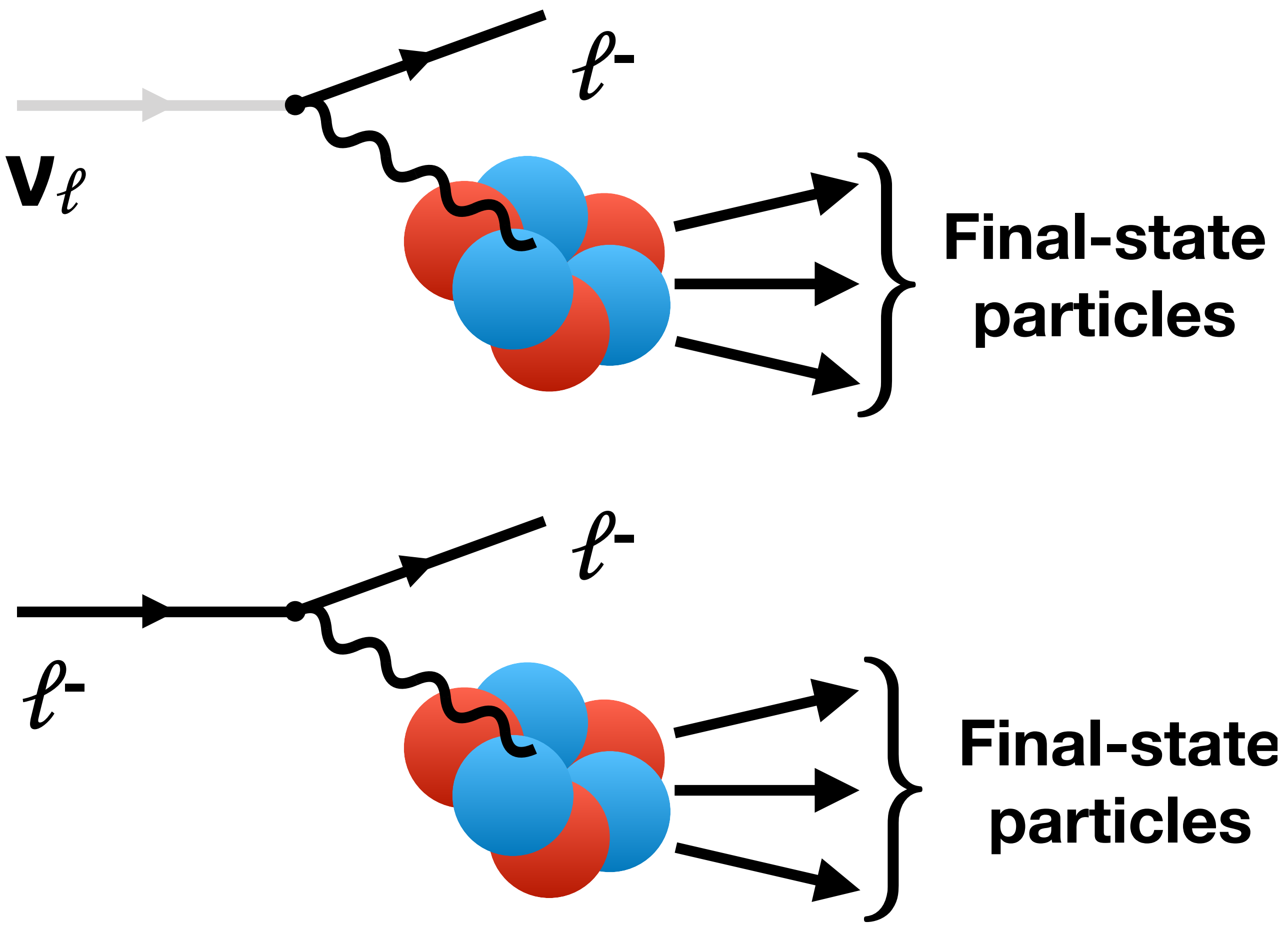
Interaction cross section

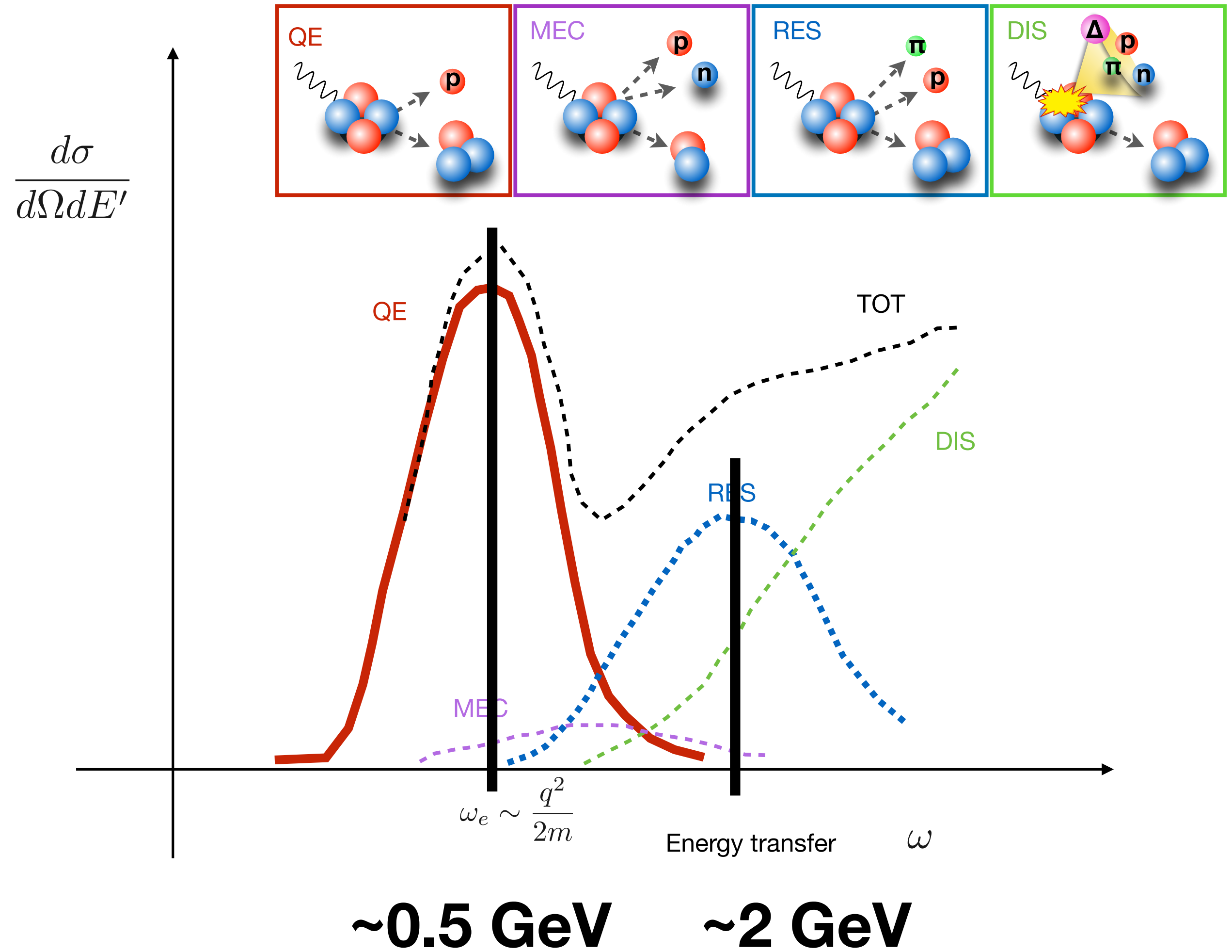
Figure credit:
M. Khachatryan et al. (e4v)
Nature 599, 565 (2021)



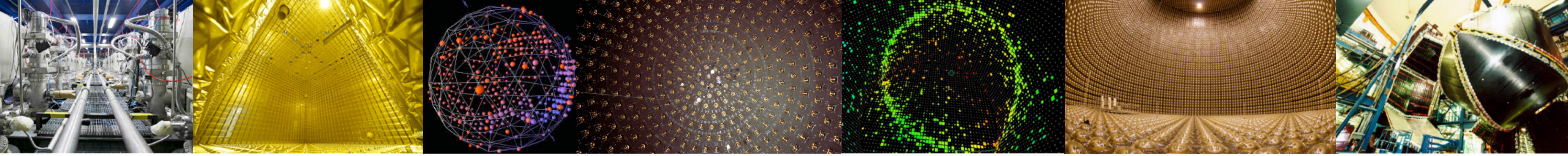
The Challenge

Lepton Event Simulation





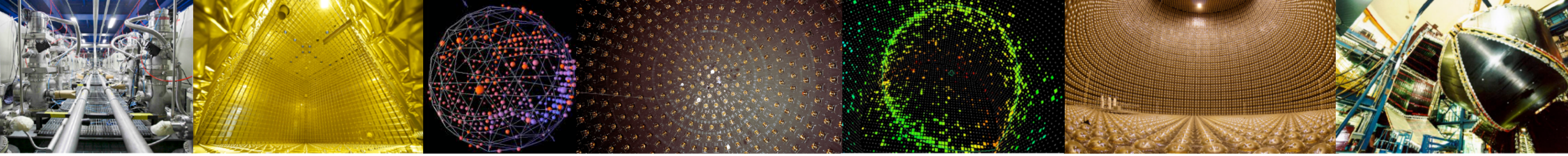
Achilles is a **theory-driven event generator** aiming to be responsive to **current and upcoming experimental needs**



Achilles overview

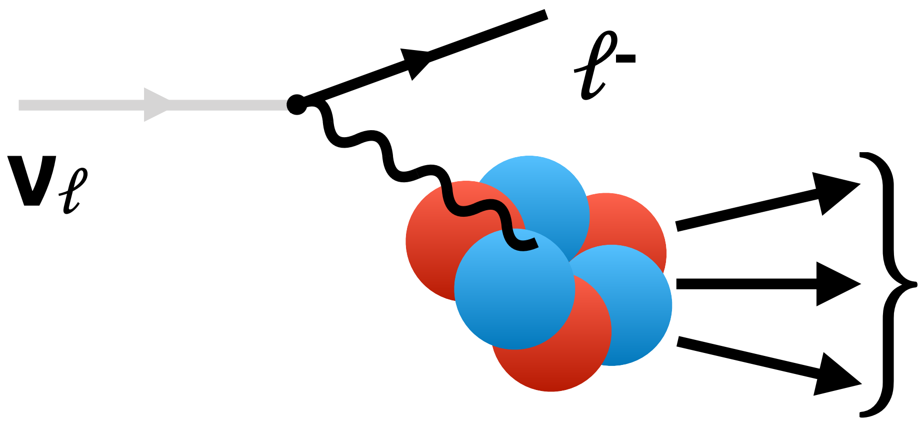
For contrast / context, see other talks

- Steven Gardiner (W 10:40) [MARLEY]
- Yoshinari Hayato (Th 15:00) [NEUT]
- Ulrich Mosel (Th 16:20) [GiBUU]
- Marco Roda (F 09:00) [GENIE]
- Kajetan Niewczas (F 10:10) [NuWro]



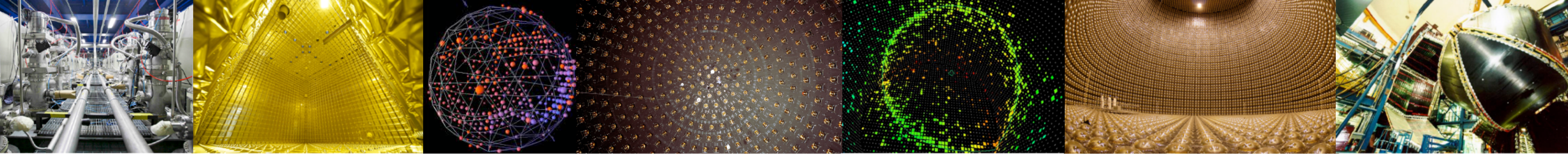
Achilles

Theory-driven: break the problem into well-defined theoretical pieces



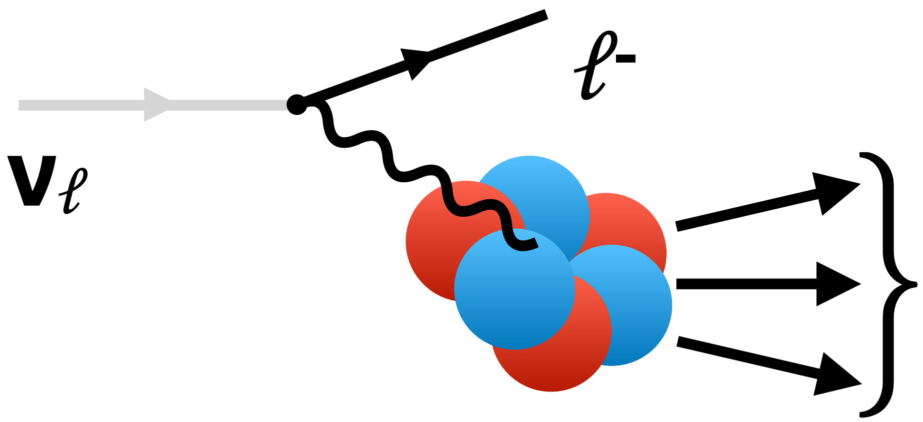
$$d\sigma = \left(\frac{1}{|v_A - v_\ell|} \frac{1}{4E_A^{\text{in}} E_\ell^{\text{in}}} \right) \times \left| \mathcal{M} \right|^2 \times \prod_f \frac{d^3 p_f}{(2\pi)^3} (2\pi)^4 \delta^4 \left(k_A + k_\ell - \sum_f p_f \right)$$

$$d\sigma = (\text{flux}) \times (\text{matrix element}) \times (\text{phase space})$$



The Matrix Element

Approx. 1: Factorization of leptonic & hadronic tensors



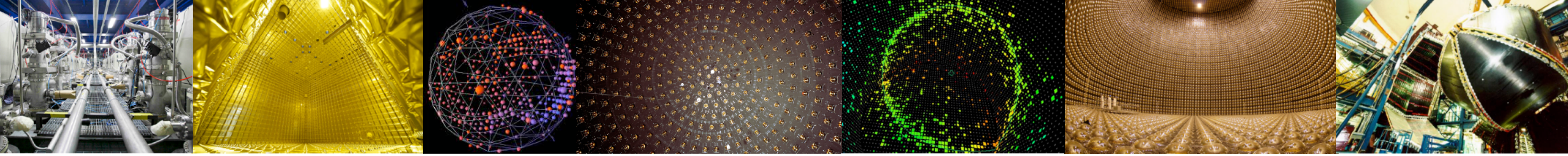
$$|\mathscr{M}|^2 = L_{\mu\nu}\,\frac{1}{P^2}\,W^{\mu\nu} \;\rightarrow\; \langle \Psi_0 | J^\dagger_\mu(q) | \Psi_f \rangle \langle \Psi_f | J_\nu(q) | \Psi_0 \rangle$$

Leptonic tensor:
Known analytically in SM or BSM scenario

Hadronic tensor:
Complicated multi-scale objecting encoding all the hadronic/nuclear physics

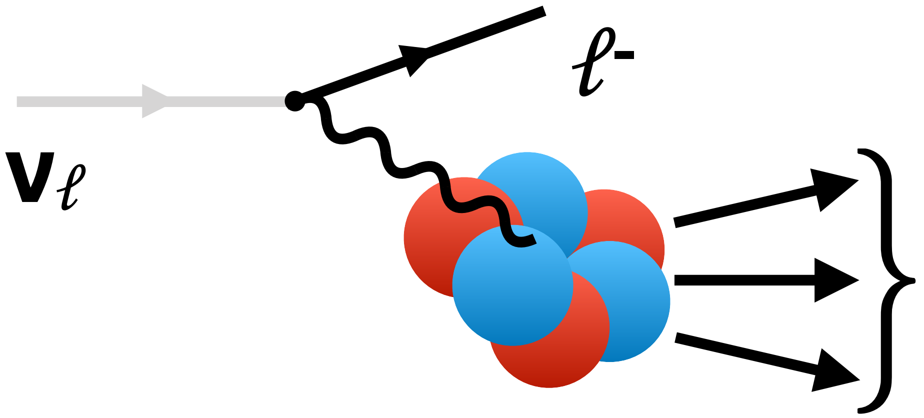
$|\Psi_0\rangle$: Initial state (say, ^{40}Ar or H_2O)

$|\Psi_f\rangle$: Final state (nuclear remnant + outgoing pions, kaons, etc…)



The Matrix Element

Approx. 2: Factorization of primary vertex



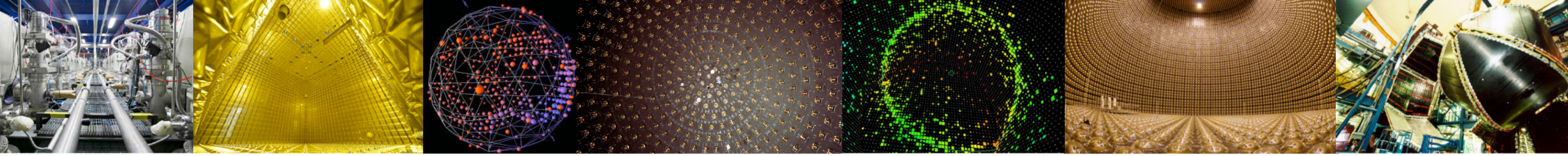
$\mathscr{V}$: Primary-interaction vertex

$\mathscr{P}$: Time evolution to produce observed final states

"Sum coherently over all possible intermediate states p'."
-Quantum mechanics

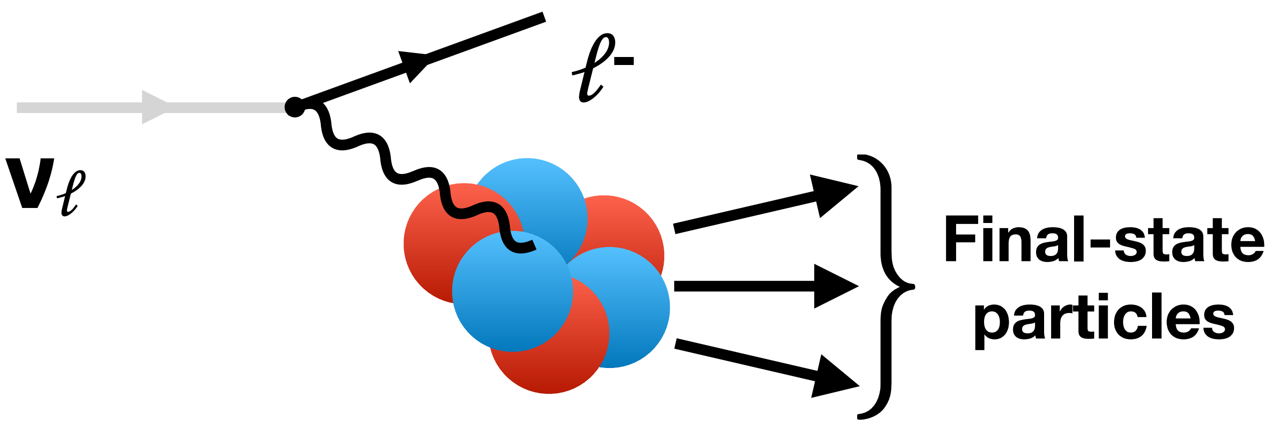
$$|\mathcal{M}(\{k\} \to \{p\})|^2 = \Big| \sum\!\!\!\!\!\!\int_{p'} \mathcal{V}(\{k\} \to \{p'\}) \times \mathcal{P}(\{p'\} \to \{p\}) \Big|^2$$

This is exact, but exponentially hard.
Factorize the problem again.



The Matrix Element

Approx. 2: Factorization of primary vertex



$\mathscr{V}$: Primary-interaction vertex

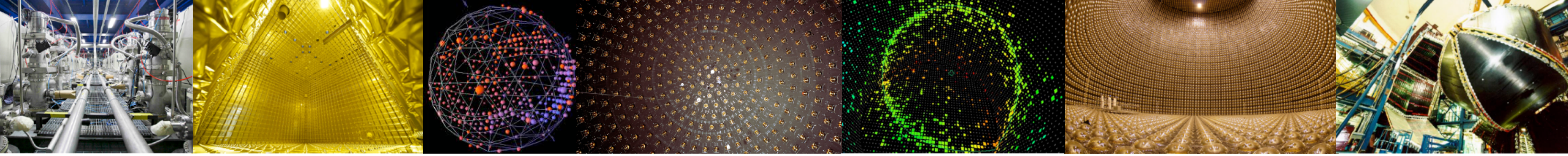
$\mathscr{P}$: Time evolution to produce observed final states

$$|\mathcal{M}(\{k\} \to \{p\})|^2 \simeq \sum\!\!\!\!\!\!\!\int_{p'} |\mathcal{V}(\{k\} \to \{p'\})|^2 \times |\mathcal{P}(\{p'\} \to \{p\})|^2$$

Treat the sum incoherently.
Handle constituents with theoretical care.

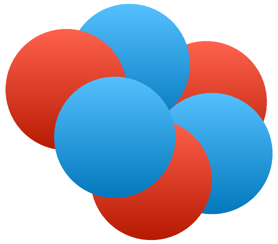
Similar to dressing hard-scattering cross sections with parton showers in LHC context

See talk by Stephen Mrenna (M 15:40) *Uncertainties in LHC Physics Modelling*



The Primary-interaction vertex
Approx. 3: Choose DOF, Factorization Scheme

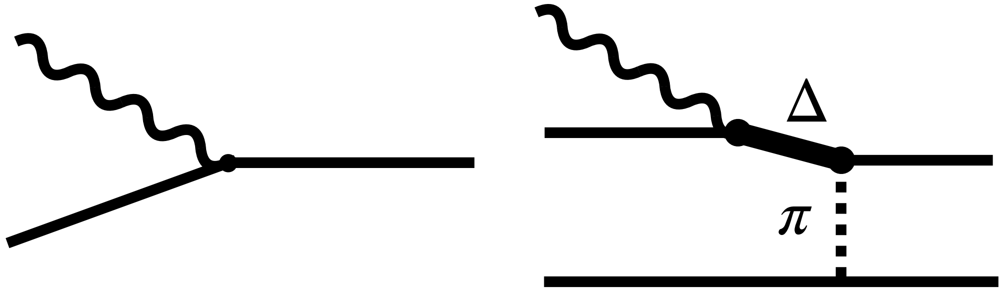
- Take nucleons as initial-state DOF



Spatial distribution from nuclear many-body theory: QMC. Quasi-exact.

- Take electroweak currents from nuclear EFT:

$$J^\mu(q) = \sum_i j_i^\mu(q) + \sum_{i<j} j_{ij}^\mu(q) + \cdots$$



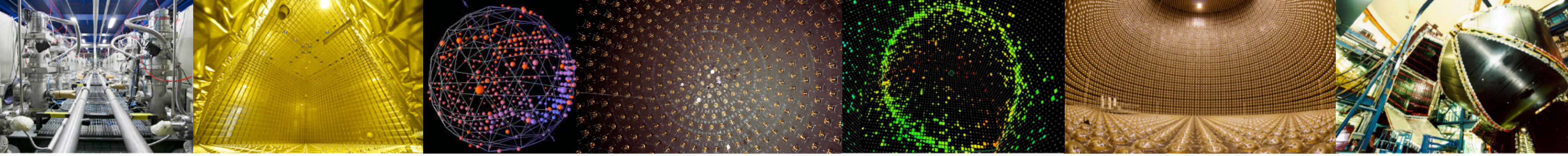
(among others)

- Choose a factorization scheme: the *impulse approximation*:

$$|\Psi_f\rangle = |\mathbf{p}\rangle \otimes |\Psi_f^{A-1}\rangle$$

"For momentum transfer $|\mathbf{q}| \gtrsim 400\ \mathrm{MeV}$, external probes resolve individual nucleons."

Compare with talks by
Lovato (T 09:00), Adreoli (T 9:40), Steinberg (T 11:10), Sobczyk (T 10:40), González Jiménez (T 14:20),



The Primary-interaction vertex

Approx. 3: Choose DOF, Factorization Scheme

$$W_N^{\mu\nu} = \langle \Psi_0 | J^{\mu\dagger}(q) | \Psi_f \rangle \langle \Psi_f | J^{\nu}(q) | \Psi_0 \rangle$$

With the *impulse approximation* $|\Psi_f\rangle = |\mathbf{p}\rangle \otimes |\Psi_f^{A-1}\rangle$,

$$W_{\mathrm{N}}^{\mu\nu} \propto S_n \otimes w_n^{\mu\nu}$$

Nuclear hadronic tensor

Nuclear spectral function (exactly calculable in nuclear many-body theory up to ~O16)

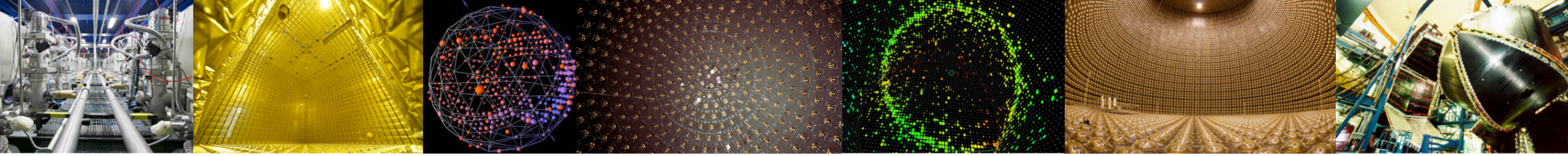
= "probability to find a nucleon with a given momentum inside the nucleus"

Nucleon hadronic tensor (experimentally measurable or calculable with lattice QCD)

For QE scattering:

$$w_n^{\mu\nu} \sim |\text{form factor}|^2$$

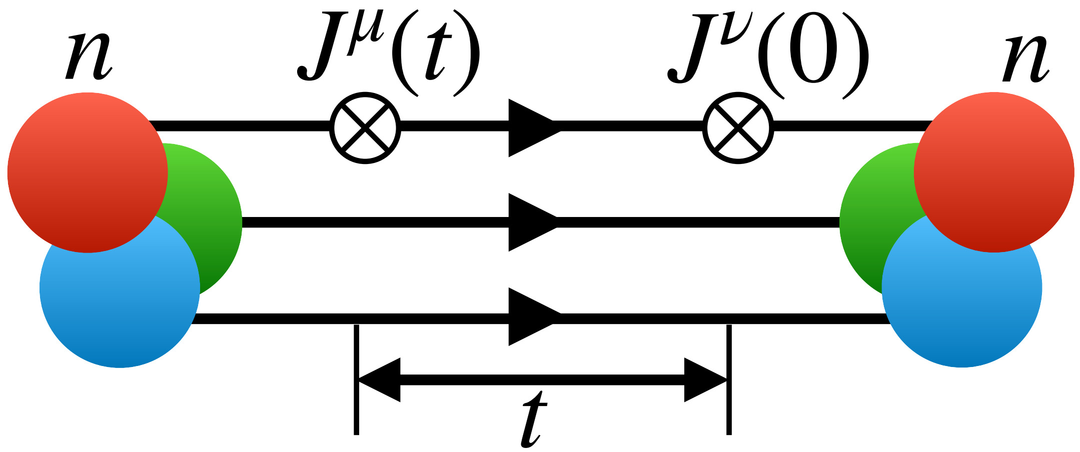
See talks by
R. Gupta (M 14:10)
A. Meyer (M 14:40)



The Hadronic Tensor and Lattice QCD

A brief detour

$$w_n^{\mu\nu}(t, \mathbf{q}) \propto$$



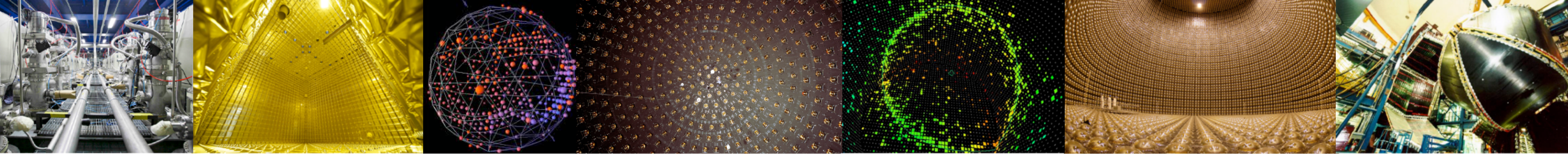
- Fully inclusive: all possible states propagate between the current insertions: $N\pi$, $N\pi\pi$, $N\pi\pi\pi$, $N\pi\pi\pi\pi$, …

- Would provide a vital bounding constraint, grounded in QCD for shallow inelastic kinematics, on exclusive models used in generators

- Complementary to QCD on $\langle N | J_\mu | N\pi \rangle$ form factors for Δ-resonance physics
    - See also: A. Grebe's talk (T 15:00) *Towards lattice QCD calculations of pion production*

- Key technical point: scattering happens in real time

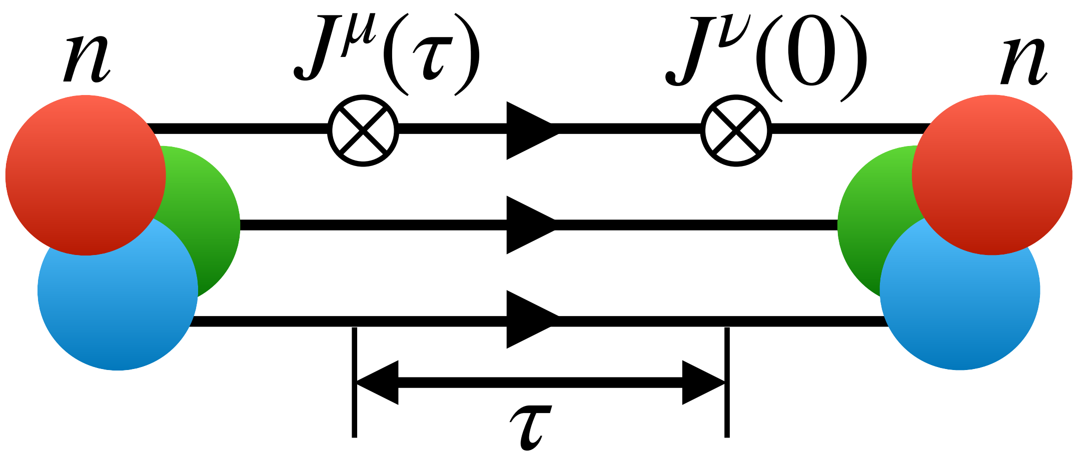
$\Longrightarrow$ Cross sections / inclusive structure functions need analytic continuation $t \to \omega + i\epsilon$



The Hadronic Tensor and Lattice QCD

A brief detour

$$w_n^{\mu\nu}(\tau, \mathbf{q}) \propto$$

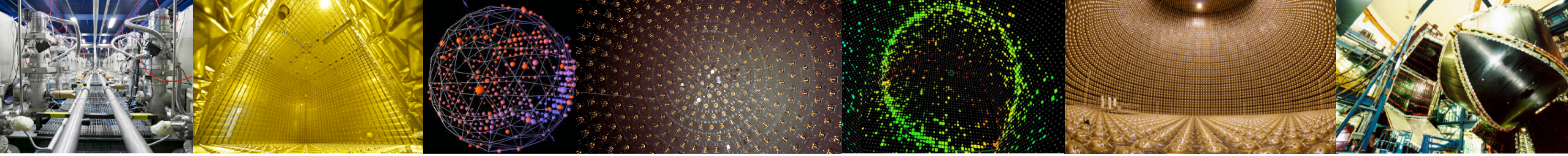


- Key technical point: scattering happens in real time

  $\Longrightarrow$ Cross sections / inclusive structure functions need analytic continuation $t \to \omega + i\epsilon$

- Inverse Laplace transform $w_n^{\mu\nu}(\tau, \mathbf{q}) = \int \frac{d\omega}{2\pi} w_n^{\mu\nu}(\omega, \mathbf{q}) e^{-\omega\tau}$

- Recent work on rigorous bounding of uncertainties (from complex analysis, with a theorem!)
  - See T. Bergamaschi, WJ, P.R. Oare PRD 108 (2023) 7, 074516 [arXiv:2305.16190] + refs therein
  - Close connection to Euclidean response functions in nuclear theory community



The Hadronic Tensor and Lattice QCD

A brief detour

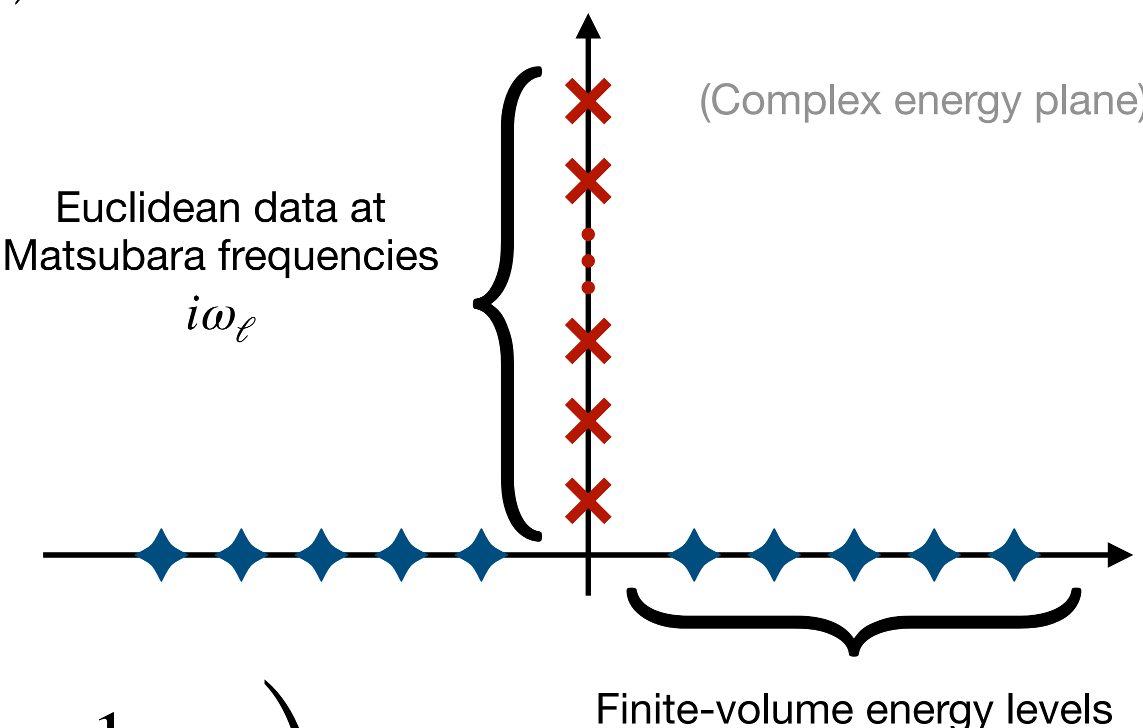
Lattice QCD calculations occur in Euclidean time:

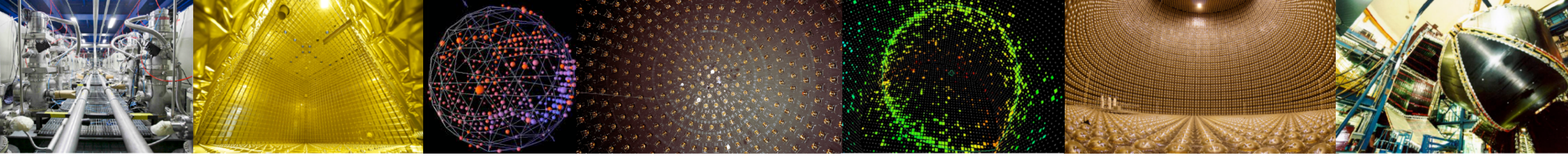
$$G(\tau) = \sum_n \left|\langle 0|\mathcal{O}|n\rangle\right|^2 \left(e^{-E_n\tau} + e^{-E_n(\beta-\tau)}\right)$$

- In frequency space:

$$G(i\omega_\ell) = \int d\tau\, e^{i\omega_\ell \tau} G(\tau)$$

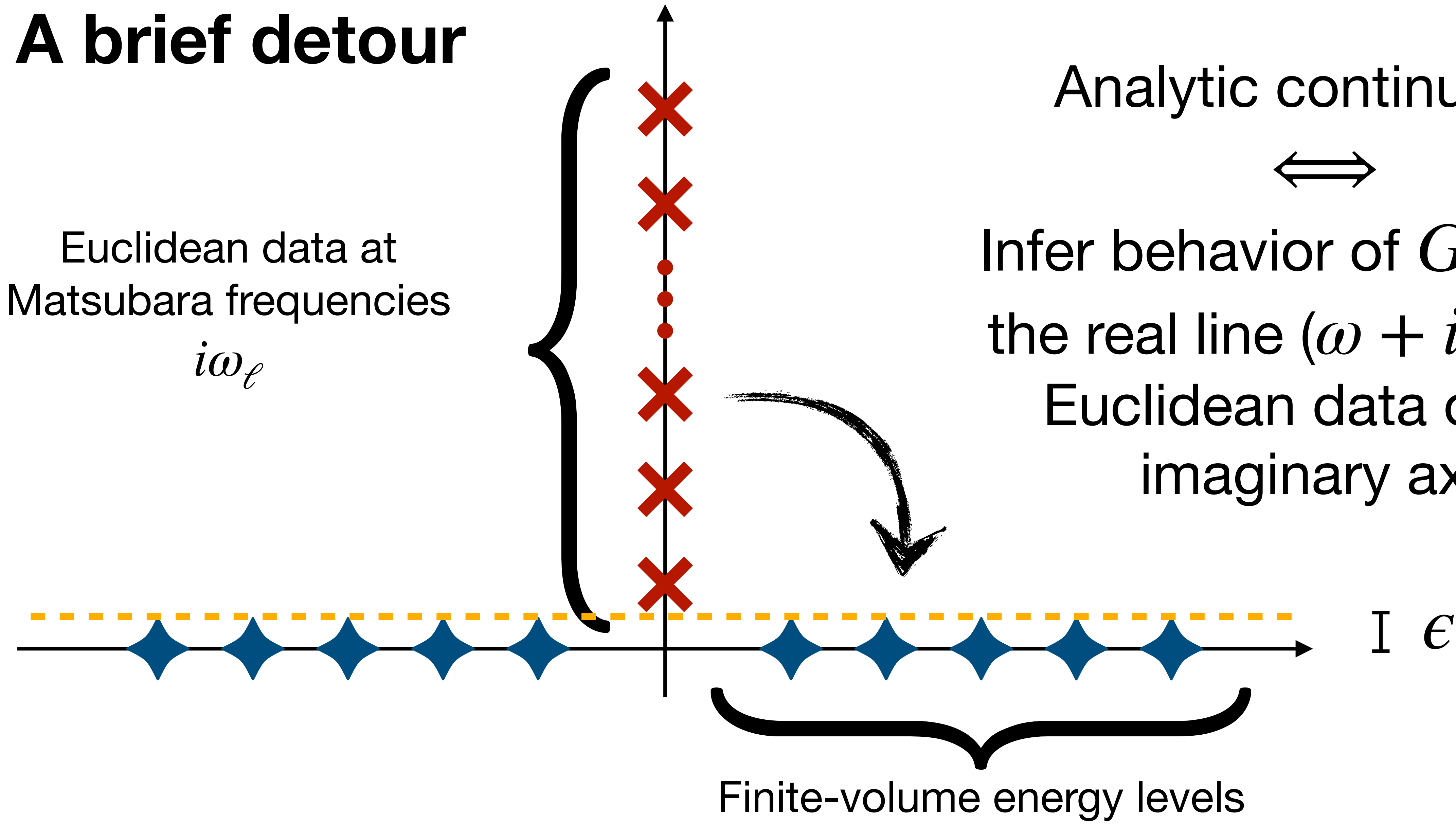
$$= \sum_n \left|\langle 0|\mathcal{O}|n\rangle\right|^2 \left(\frac{1}{E_n + i\omega_\ell} + \frac{1}{E_n - i\omega_\ell}\right)$$





The Hadronic Tensor and Lattice QCD

A brief detour

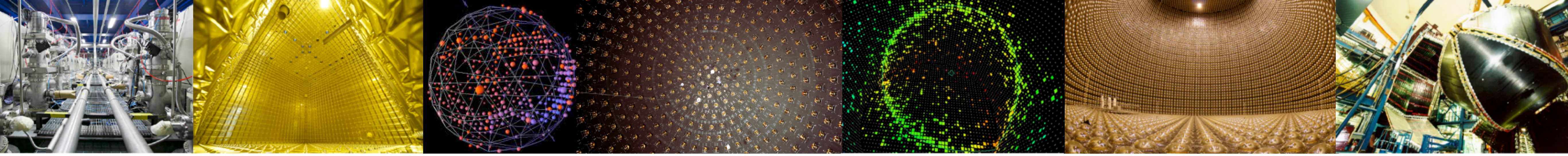


Analytic continuation

$\Longleftrightarrow$

Infer behavior of $G(z)$ near the real line $(\omega + i\epsilon)$ given Euclidean data on the imaginary axis

$\rho^\epsilon(\omega) \equiv \frac{1}{\pi} \mathrm{Im}\, G(\omega + i\epsilon)$ can be viewed as smeared spectral function

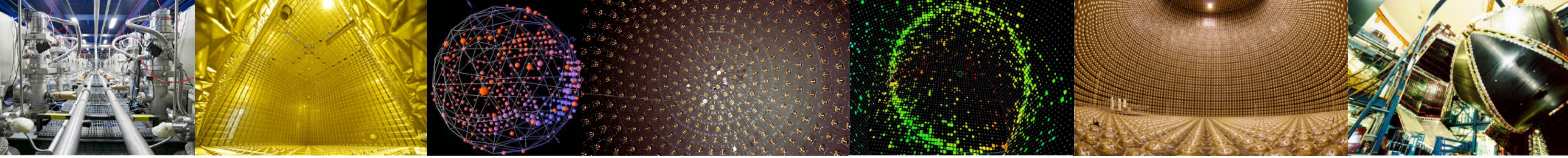


The Hadronic Tensor and Lattice QCD

A brief detour

R. Nevanlinna
Ann. Acad. Sci. Fenn. Ser. A 13 (1919)
Ann. Acad. Sci. Fenn. Ser. A 32 (1929)

- The problem of analytic continuation is amenable to techniques from *Nevanlinna-Pick interpolation*
- **Theorem (Nevanlinna, 1919/1929):** Computes the <u>*space of functions*</u> in the upper half-plane which
    1. Interpolate the given set of Euclidean data and
    2. Are analytic in the upper-half plane.
- Applicability to field-theory problems first recognized by Fey, Yeh, and Gull [arXiv:2010.04572]
- Existence of rigorous error bounds first recognized in our [arXiv:2305.16190]



The Hadronic Tensor and Lattice QCD

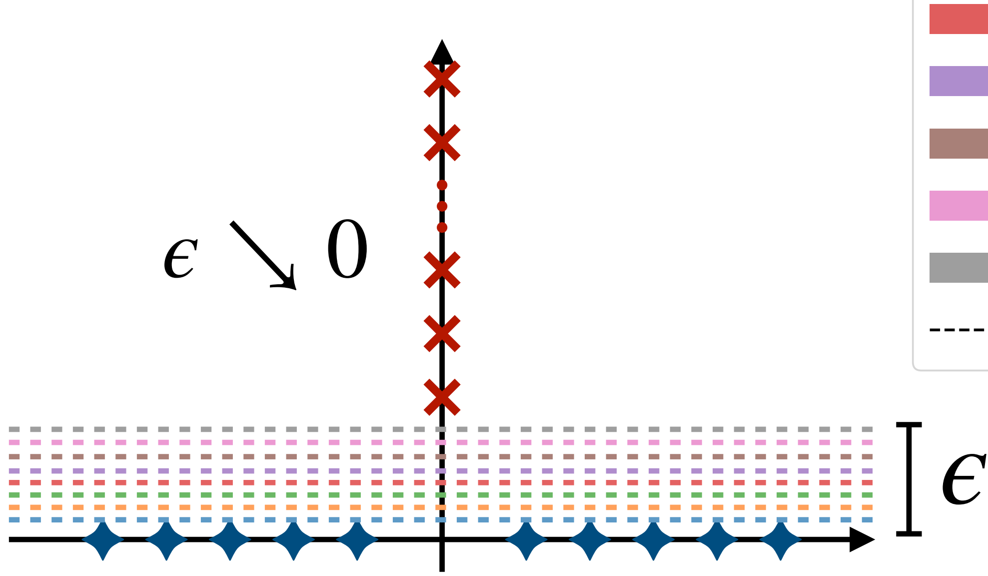
A brief detour

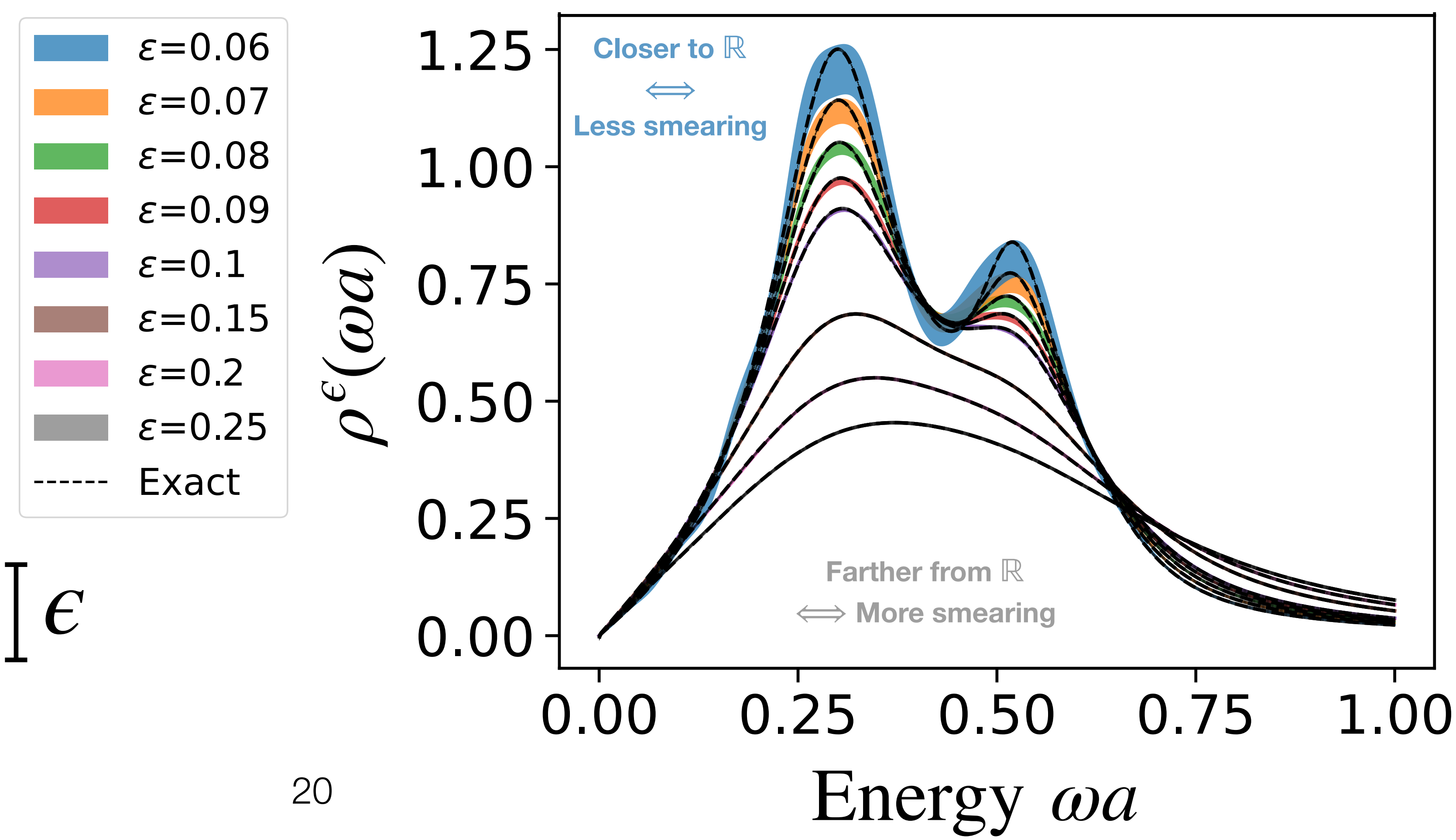
Example: Reconstructing a parameterization of the R-ratio $\sigma(e^+e^- \to \mathrm{hadrons})/(4\pi\alpha^2/3s)$

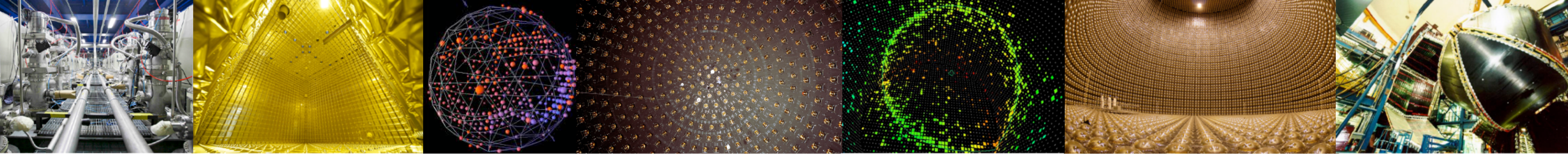
- Energies rescaled to line in unit interval $\implies$ lattice units with $a \approx 0.07\ \mathrm{fm}$, so $am_\rho \approx 0.25$

✓ Spectral peaks from $\rho(770)/\omega(782)$ and $\phi(1020)$ cleary visible in reconstructions

✓ Exact answer is contained within the rigorous bounding envelope of the "Wertevorrat"



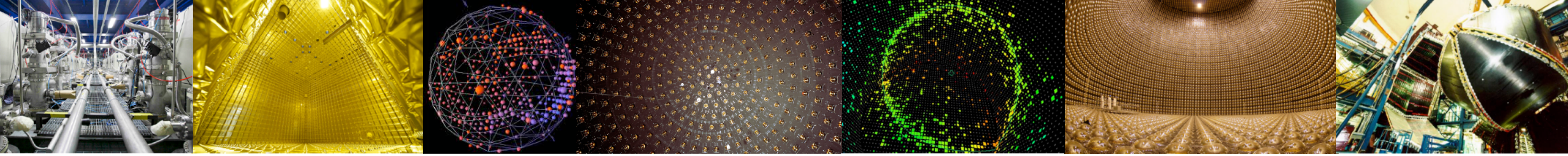




Subsequent time evolution

Approx. 4: Intranuclear cascade

$$|\mathcal{M}(\{k\} \to \{p\})|^2 \simeq \sum\!\!\!\!\!\!\!\int_{p'} |\mathcal{V}(\{k\} \to \{p'\})|^2 \times |\mathcal{P}(\{p'\} \to \{p\})|^2$$



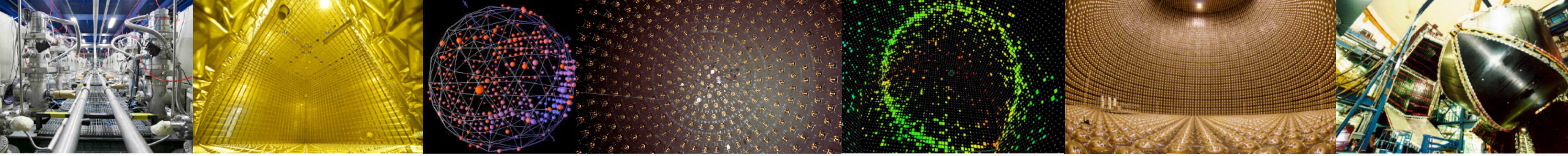
Subsequent time evolution

Approx. 4: Intranuclear cascade

$$|\mathcal{M}(\{k\} \to \{p\})|^2 \simeq \sum\!\!\!\!\!\!\!\int_{p'} |\mathcal{V}(\{k\} \to \{p'\})|^2 \times |\mathcal{P}(\{p'\} \to \{p\})|^2$$

- *Intranuclear Cascade (INC)*
  - Scatter nucleons quantum mechanically
  - Propagate nucleons classically, with in-medium corrections

(Neglect interference between successive scattering events in propagation)

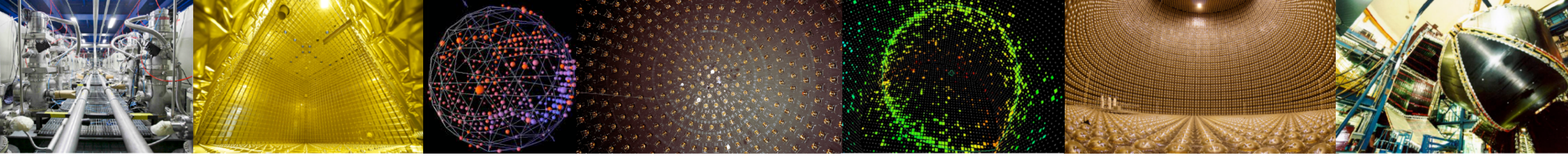


Subsequent time evolution

Approx. 4: Intranuclear cascade

$$|\mathcal{M}(\{k\} \to \{p\})|^2 \simeq \sum\!\!\!\!\!\!\!\int_{p'} |\mathcal{V}(\{k\} \to \{p'\})|^2 \times |\mathcal{P}(\{p'\} \to \{p\})|^2$$

- The initial configuration of nucleons is taken from:
  - Spatial distribution: quantum Monte Carlo, retaining correlations
  - Momenta: local Fermi gas model



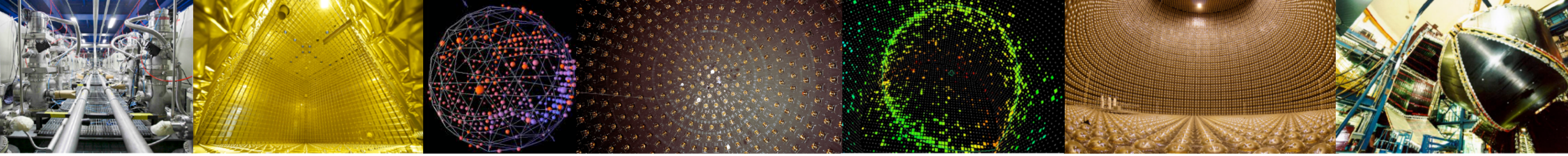
Subsequent time evolution

Approx. 4: Intranuclear cascade

$$|\mathcal{M}(\{k\} \to \{p\})|^2 \simeq \sum\!\!\!\!\!\!\!\int_{p'} |\mathcal{V}(\{k\} \to \{p'\})|^2 \times |\mathcal{P}(\{p'\} \to \{p\})|^2$$

The quantum mechanical scattering model:

- Utilizes measured NN cross sections, e.g., from from SAID database with GEANT4 or NASA parameterization
- Scatters probabilistically according to the impact parameter: $P(b) = \exp(-\pi b^2/\sigma)$
    - ☑ $\lambda^{-1} = \rho\sigma$ for the mean free path λ
    - ☑ Total probability integrates to the cross section σ
- Incorporates Pauli blocking and formation zone to constrain possible scatterings



Subsequent time evolution

Approx. 4: Intranuclear cascade

$$|\mathcal{M}(\{k\} \to \{p\})|^2 \simeq \sum\!\!\!\!\!\!\int_{p'} |\mathcal{V}(\{k\} \to \{p'\})|^2 \times |\mathcal{P}(\{p'\} \to \{p\})|^2$$

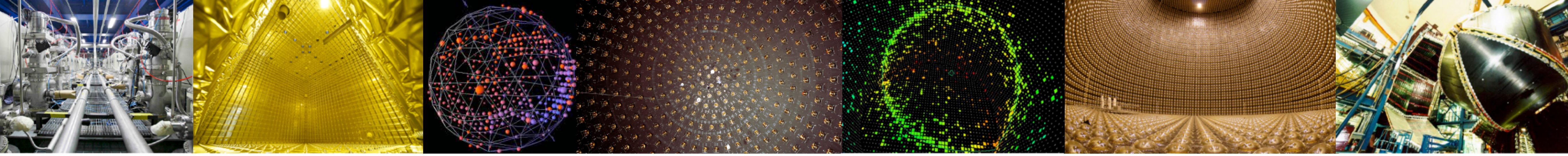
Classical propagation in the background nucleus creates an effective optical potential which induces two effects:

1. Short-distance: $$\frac{d\sigma}{d\Omega} \longrightarrow \left(\frac{d\sigma}{d\Omega}\right)_{\text{in medium}}$$

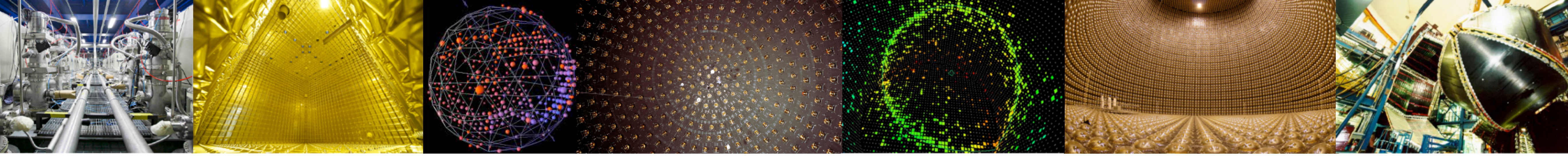
(In-medium corrections to NN interactions)

2. Long-distance: $$\dot{\mathbf{p}} = -\partial_{\mathbf{q}} H \quad \dot{\mathbf{q}} = +\partial_{\mathbf{p}} H$$

(Classical evolution in background potential)



Achilles overview
(A few recent developments)



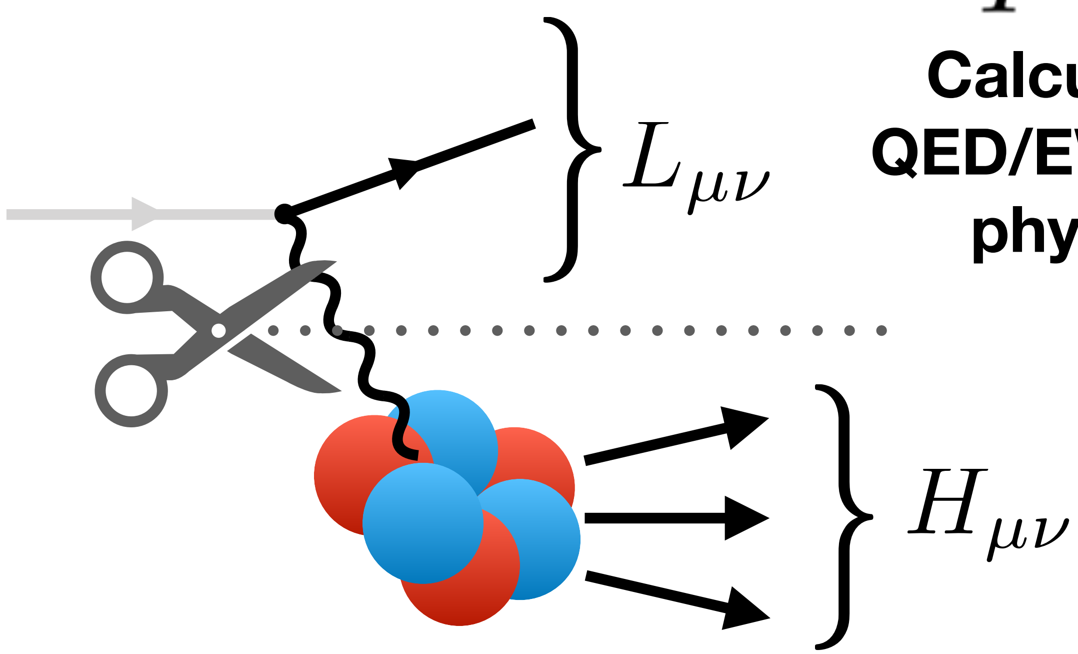
Achilles — Recent updates

Isaacson et al.
PRD 105 (2022) 9, 096006
[arXiv:2110.15319]

Factorization of leptonic and hadronic tensors

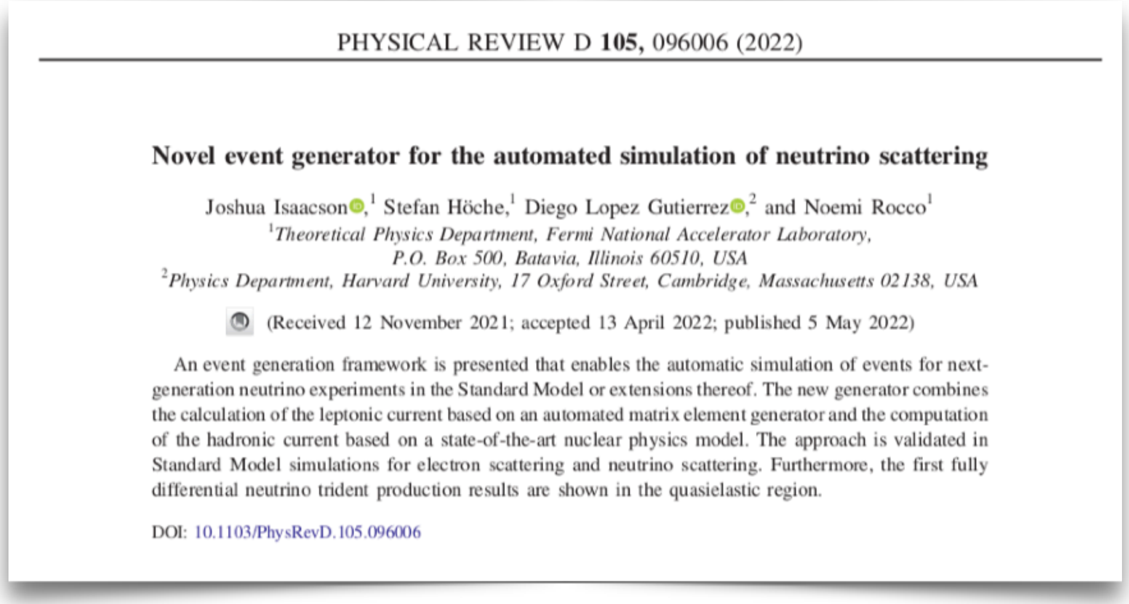
- Automated specification of leptonic tensor (including BSM possibilities)
- Key involvement: Diego Lopez Gutierrez [Undergrad @ Macalester → PhD @ Wash. U. St Louis]
- Uses tools developed by LHC event generation community: Sherpa, Comix, FeynRules, UFO files

$$|\mathcal{M}|^2 = L_{\mu\nu} W^{\mu\nu} \frac{1}{P^2}$$



Calculable QED/EW/BSM physics

Nuclear/hadronic physics of initial interaction and subsequent evolution


PHYSICAL REVIEW D **105**, 096006 (2022)

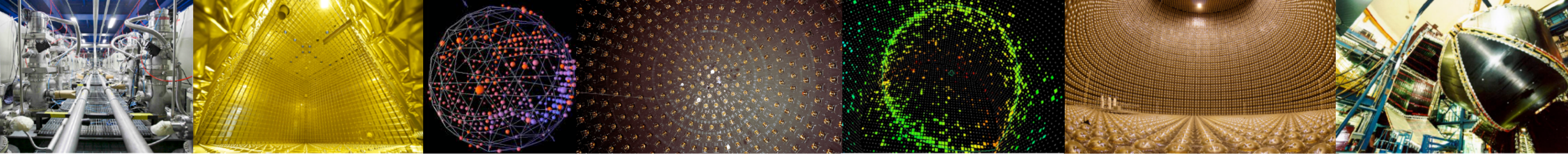
Novel event generator for the automated simulation of neutrino scattering

Joshua Isaacson,[1] Stefan Höche,[1] Diego Lopez Gutierrez,[2] and Noemi Rocco[1]
[1]Theoretical Physics Department, Fermi National Accelerator Laboratory, P.O. Box 500, Batavia, Illinois 60510, USA
[2]Physics Department, Harvard University, 17 Oxford Street, Cambridge, Massachusetts 02138, USA

(Received 12 November 2021; accepted 13 April 2022; published 5 May 2022)

An event generation framework is presented that enables the automatic simulation of events for next-generation neutrino experiments in the Standard Model or extensions thereof. The new generator combines the calculation of the leptonic current based on an automated matrix element generator and the computation of the hadronic current based on a state-of-the-art nuclear physics model. The approach is validated in Standard Model simulations for electron scattering and neutrino scattering. Furthermore, the first fully differential neutrino trident production results are shown in the quasielastic region.

DOI: 10.1103/PhysRevD.105.096006



Achilles — Work in progress

New API for nuclear models

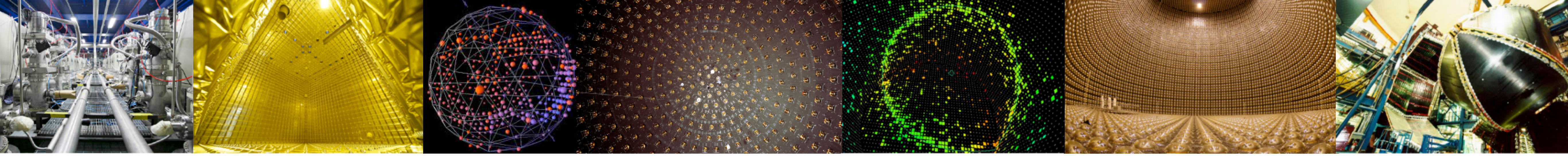
- We have new API/extendible interface for nuclear models
- The API supports models implemented in Fortran or CPP. Extension to models in python is straightforward if there is community interest
- Allows, e.g., for different nuclear spectral functions

$$W_{\mathrm{N}}^{\mu\nu} \propto S_n \otimes w_n^{\mu\nu}$$

Nuclear hadronic tensor

Nuclear spectral function

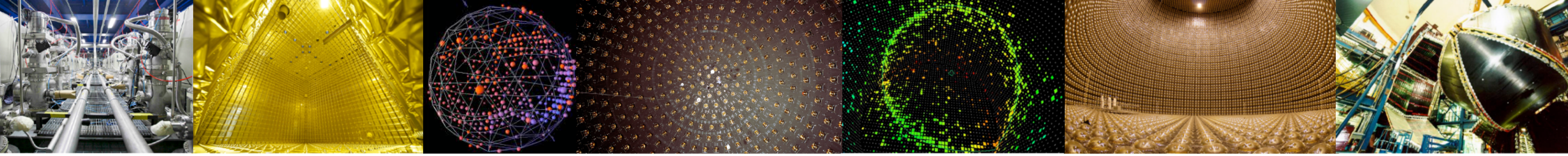
Nucleon hadronic function



Achilles — Work in progress

Resonant production

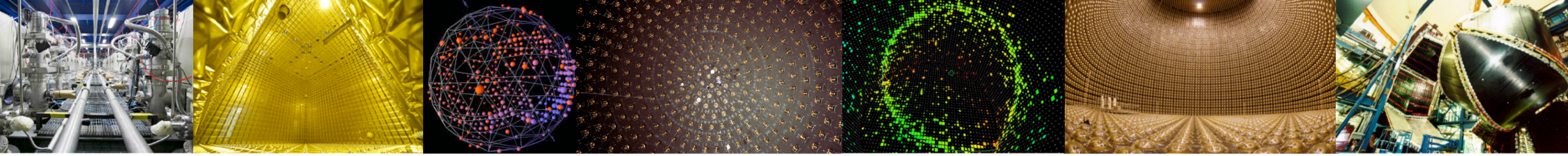
- First Achilles paper focused on QE scattering: $2 \to 2$ scattering
- Resonant scattering (e.g., $\ell N \to \ell N\pi$) is $2 \to 3$ scattering
- Preliminary implementation [Noemi Rocco] of the dynamical coupled channel (DCC) model of resonant scattering.
  - See Rocco et al., *PRC* 100 (2019) 4, 045503 [arXiv:1907.01093]
  - Fundamental input to Achilles: DCC $\to \langle N | J_\mu | N\pi \rangle$ matrix elements with fully exclusive kinematics
- Working on cascade model including pion production



Achilles — Work in progress

New “process grouping” for multiple processes

- Accommodates charged-currents and neutral-current scattering in the same run with correct event fractions
- Handles different beam particles (e.g., different neutrino flavors and/or charged leptons from detector environment)
- Allows for different scattering mechanisms (e.g, QE and resonance) in the same run with correct event fractions

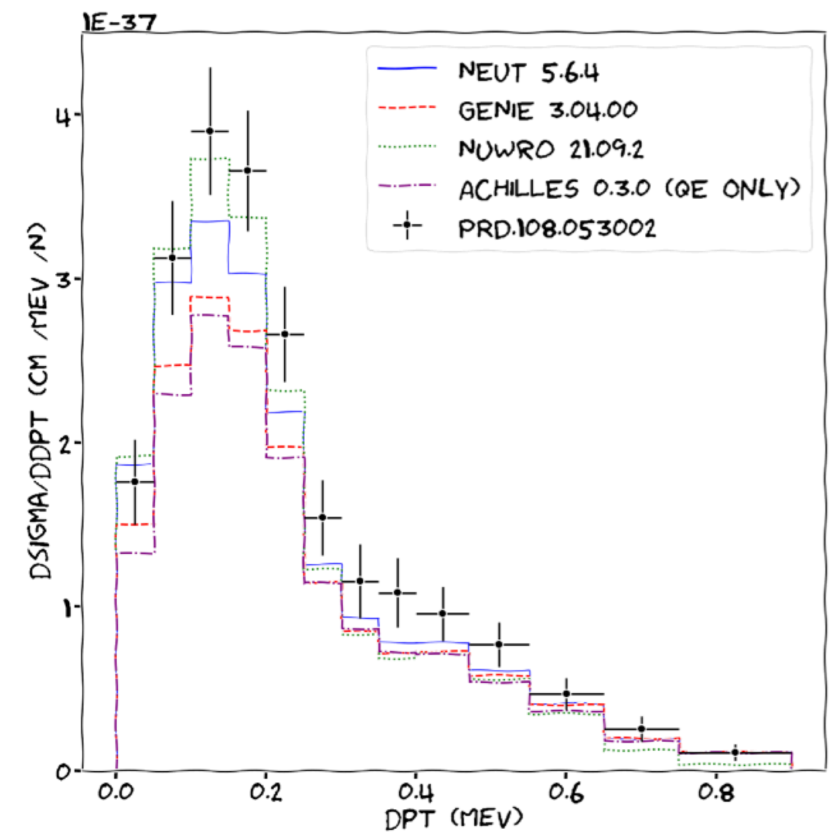


NuHepMC

S. Gardener, J. Isaacson, L. Pickering
[arXiv:2310.13211]

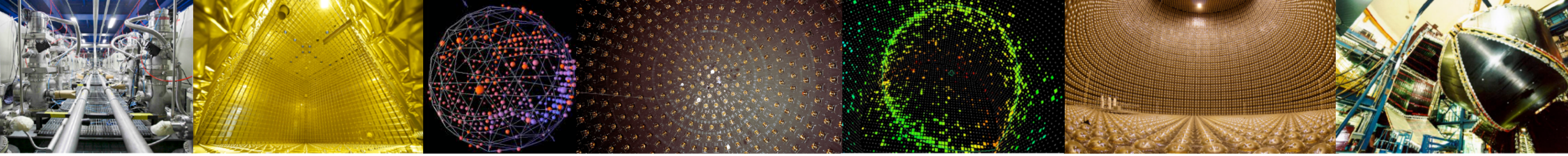
Standardized event record format for neutrino event generators

- NuHepMC is the default output format for Achilles
- NuHepMC gives a framework for uncertainty quantifications via the “Generator Run metadata”
    - Example: G.R.7 Event Weights - Can specify a *vector* of event weights. These can be used with “*in situ* parameter variation” to constrain model uncertainties.
    - See talk by Stephen Mrenna (M 15:40) for how this used already by LHC event generators like Pythia
- NuHepMC streamlines the pipeline for data/theory comparisons using the NUISANCE framework

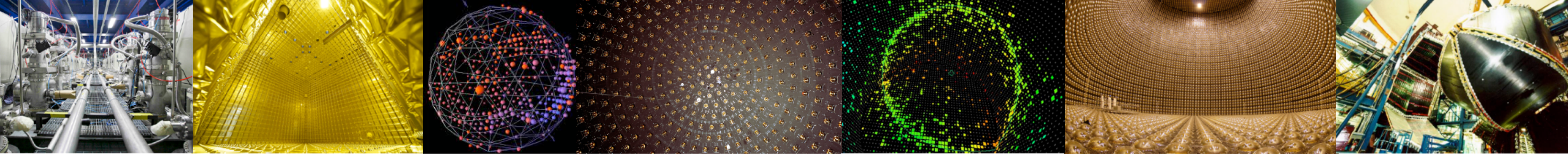


“This comparison was made with the NUISANCE framework, which before this implementation of NuHepMC would have to have been built against GENIE, NEUT, and NuWro binaries of compatible versions to be able to generate the predictions shown in the figure.”

See talk by Steven Gardiner (W 10:40)



Recent results



Achilles: Comparison to experiment

PRD 107 (2023) 3, 033007 [arXiv:2205.06378]

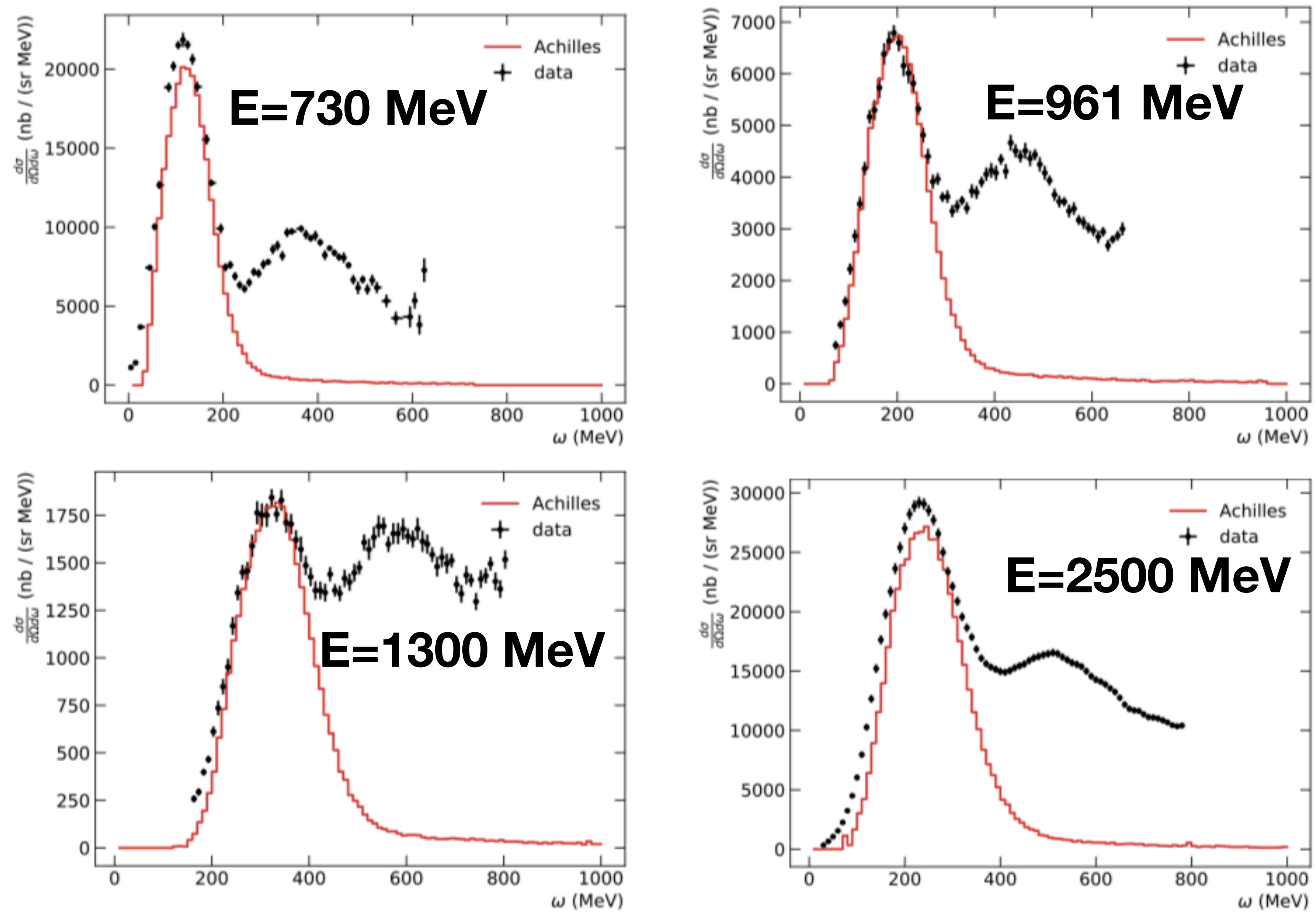
J. S. O'Connell *et al.*, Phys. Rev. C **35**, 1063 (1987).
R. M. Sealock *et al.*, Phys. Rev. Lett. **62**, 1350 (1989).
D. Zeller, Investigation of the structure of the C-12 nucleus by high-energy electron scattering, Other thesis, Karlsruhe University, 1973.

Inclusive e-C hadronic cross section

$$\frac{d\sigma}{d\Omega d\omega}$$

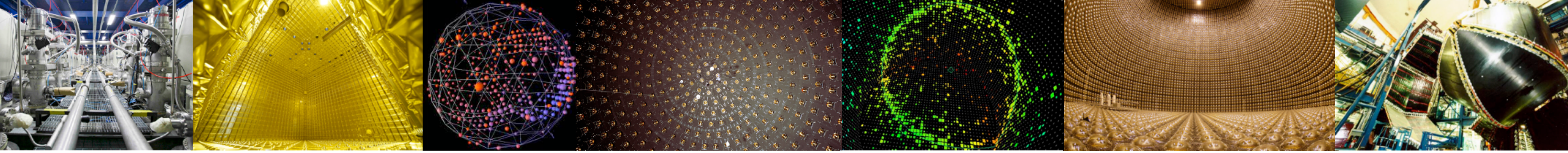
Fixed outgoing electron angle θ=37° to match experimental settings

Differential in outgoing electron energy ω



Beyond firsts peak: Neglected MEC and resonance contributions

Good agreement = Validation of initial model for QE interaction



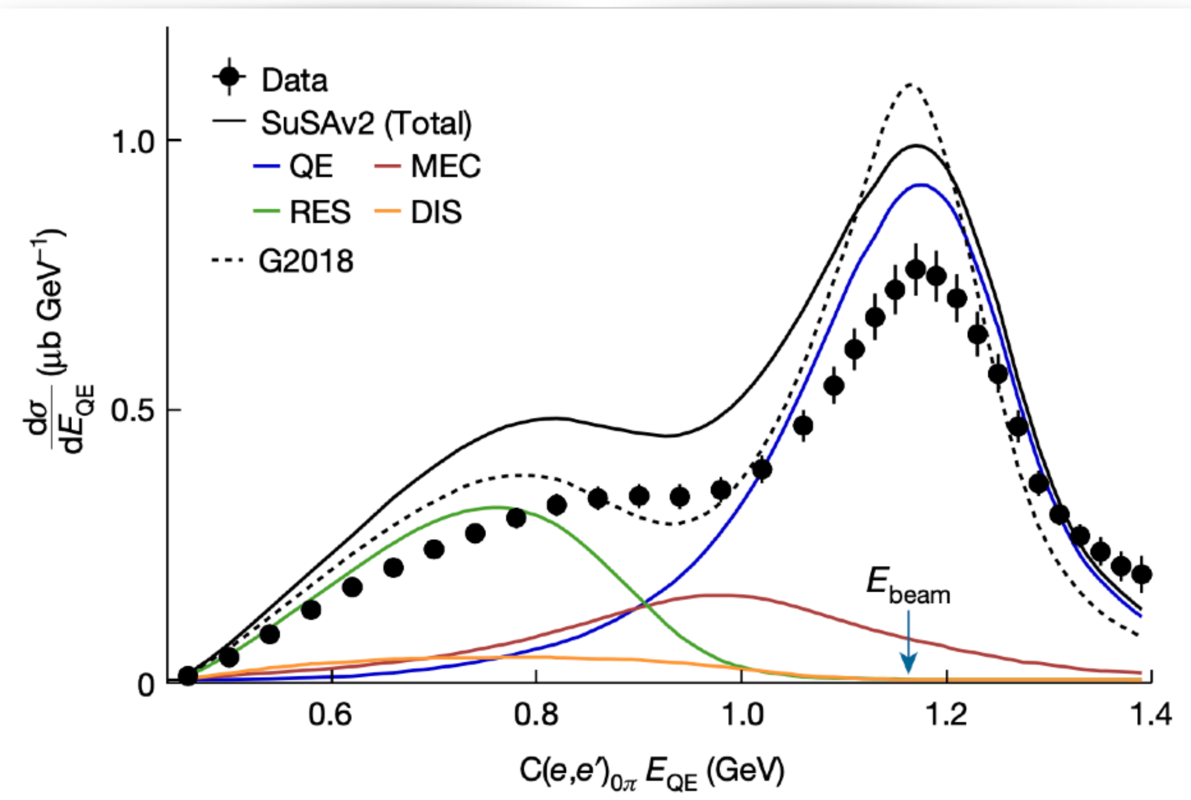
Achilles: Comparison to experiment

PRD 107 (2023) 3, 033007 [arXiv:2205.06378]

CLAS and e4v collaborations
Nature 599 (2021) 7886, 565-570

- **Inclusive e-C hadronic cross section**
- **Analysis by e4v to mimic kinematic setup for QE vA scattering**

$$E_{\mathrm{QE}} = \frac{2m_N\epsilon + 2m_N E_\ell - m_\ell^2}{2\left(m_N - E_\ell + p_\ell \cos\theta_\ell\right)}$$



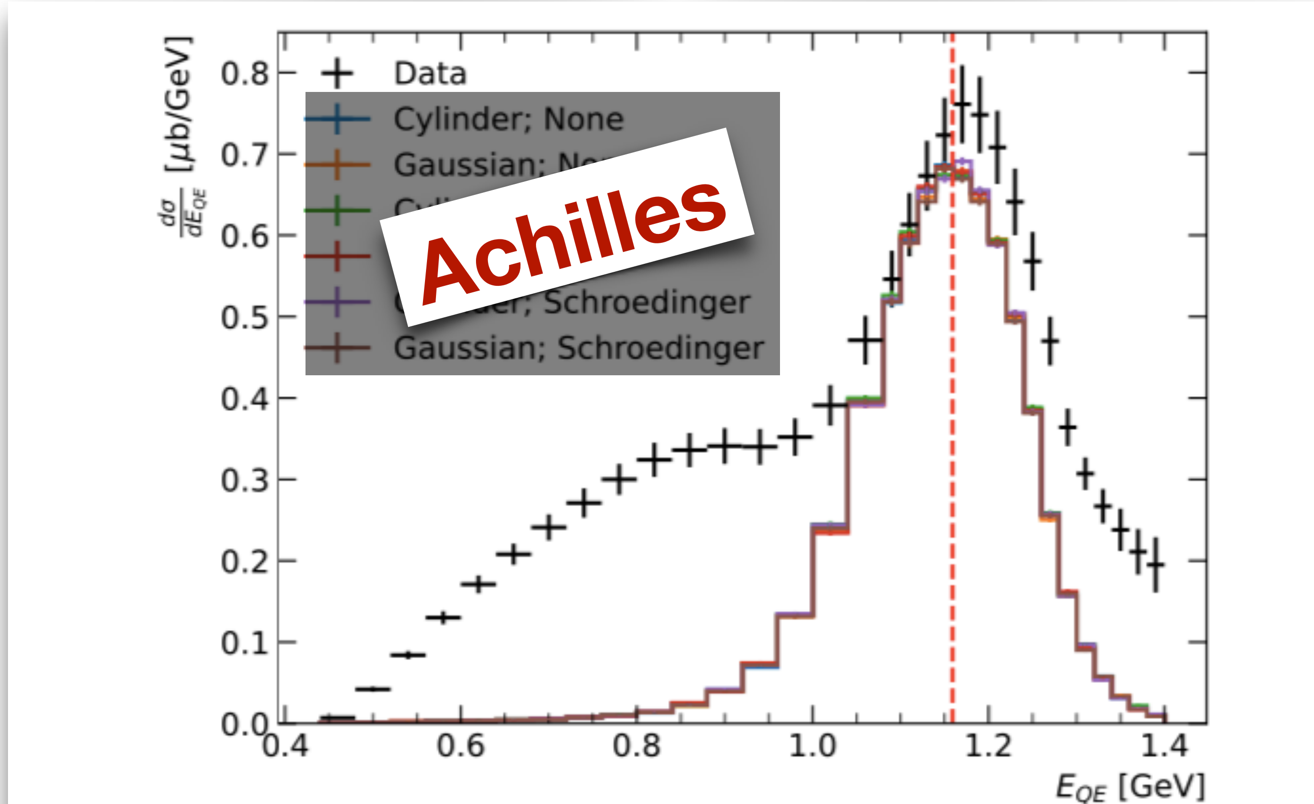
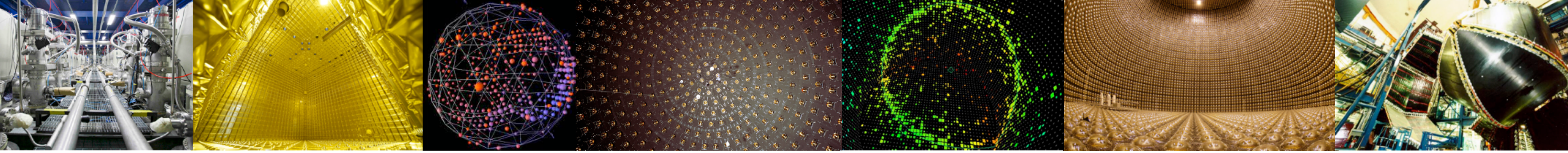


FIG. 4: Comparison of the quasielastic energy reconstructed for an electron beam of 1159 MeV. Data is taken from Ref. [69]. The definition of E_{QE} can be found in Eq. 31. The red dashed vertical line marks the true beam energy.

- **Low E_{QE}: MEC and resonance contributions**
- **High E_{QE}: interference effects (neglected)**



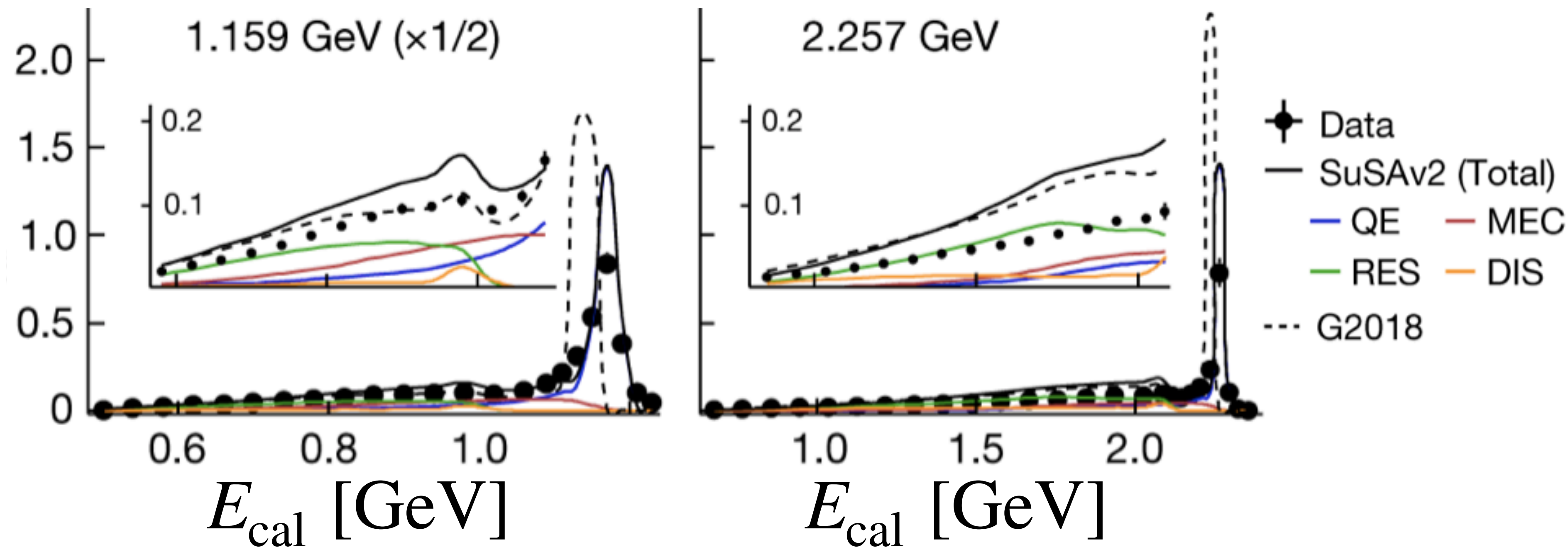
Achilles: Comparison to experiment

PRD 107 (2023) 3, 033007 [arXiv:2205.06378]

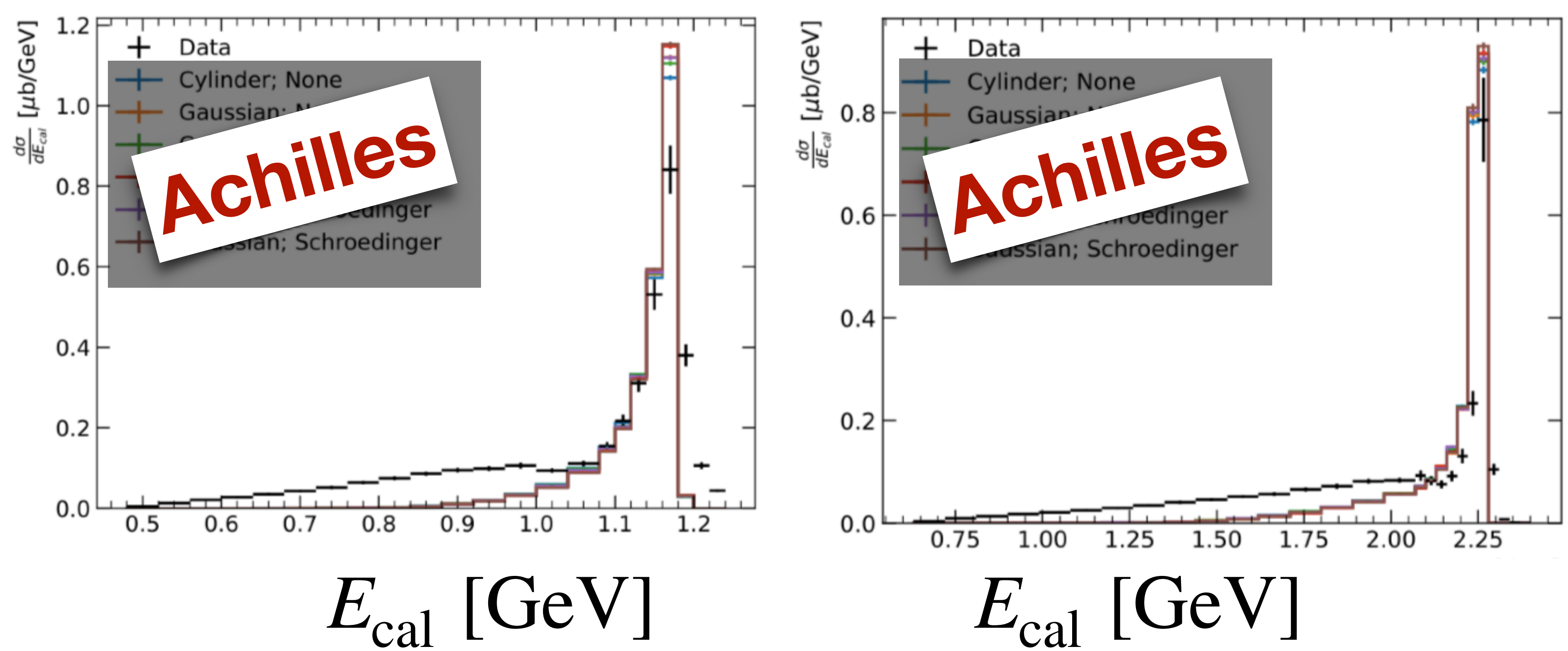
CLAS and e4v collaborations
Nature 599 (2021) 7886, 565-570

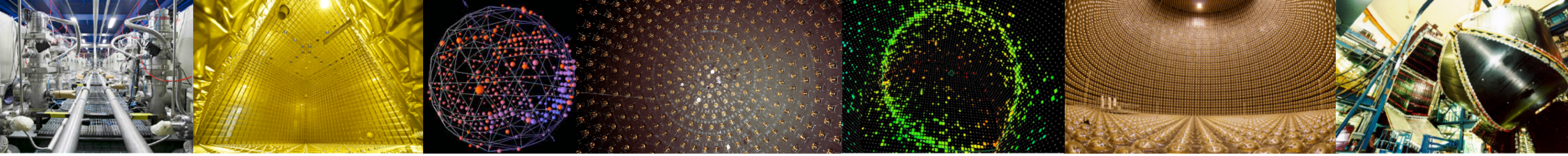
E_{cal} = "Calorimetric energy" = "sum of final-state energies"

Data + simulation from e4v paper $\frac{d\sigma}{dE_{cal}}$



Same e4v data vs Achilles $\frac{d\sigma}{dE_{cal}}$





Achilles: Recent Results

Application: Correlated decays in neutrino experiments

J. Isaacson et al.
[arXiv:2303.08104]

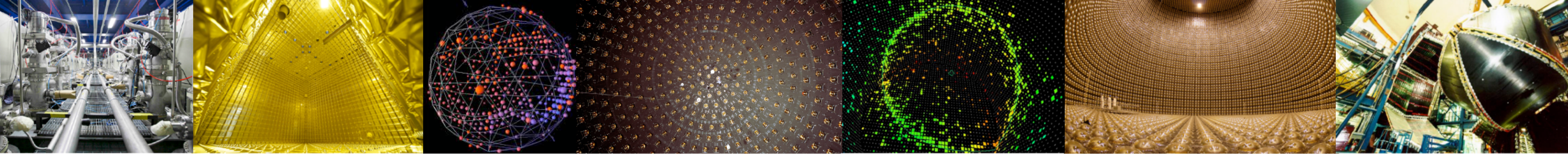
Key involvement: Sherry Wang
[Undergrad @ Northwestern]

Motivation: ν_τ is perhaps the least understood elementary particle

- DUNE: O(few hundred) ν_τ events per year → Accurate theoretical predictions critical
- Outgoing/decaying τ is polarized → Induces correlations in final-state particles
- Standard Model predicts:
  - τ polarization perpendicular to the lepton-scattering plane *vanishes*
  - τ polarization components within the lepton-scattering plane do <u>not</u> vanish
- Other generators have often treated ν_τ interactions as for ν_e, ν_μ → “outgoing τ as LH only”

Results

- First fully differential predictions for ν_τ scattering at DUNE energies, including all spin correlations and all τ decay channels
- Calculated using generic interface between Achilles and Sherpa
- Correlations between production and decay are *automatically* maintained



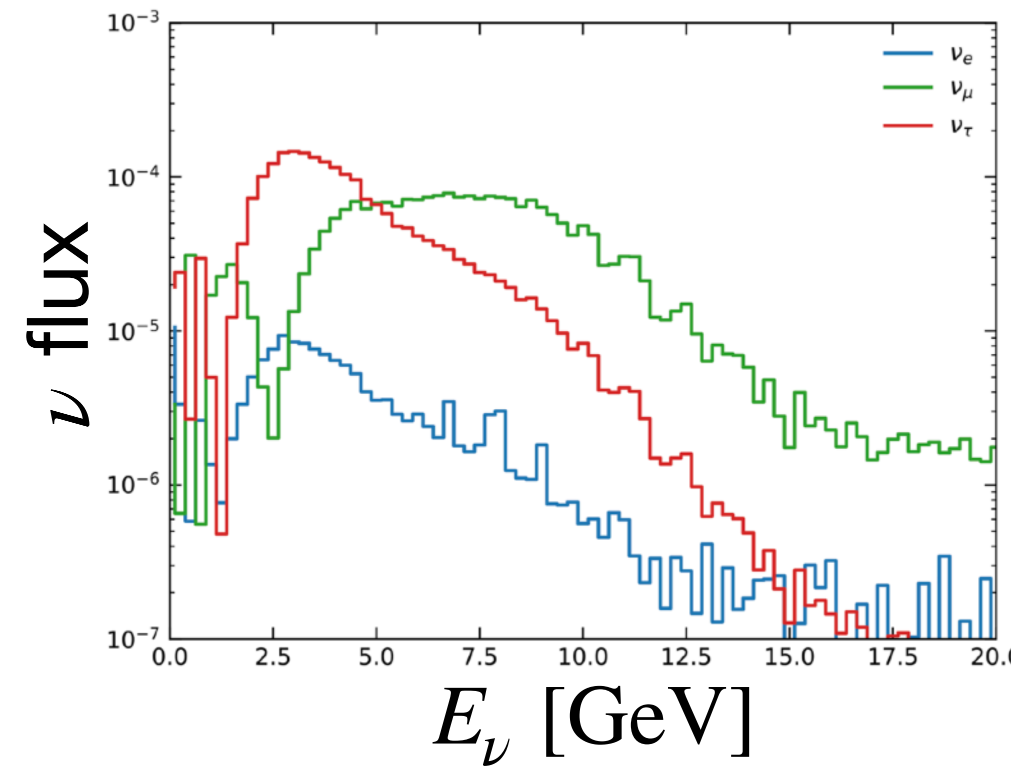
Achilles: Recent Results

J. Isaacson et al.
[arXiv:2303.08104]

Application: Correlated decays in neutrino experiments

Momentum Fraction Distributions

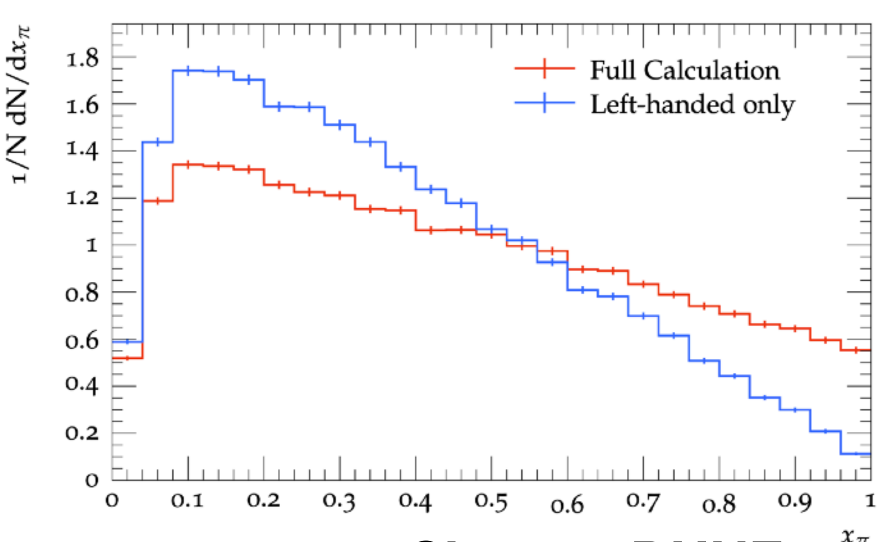
- Benchmarking done against analytic results in collinear ($p_\tau \to \infty$) limit, monochromatic beams
- Final results calculated using realistic DUNE fluxes



$$\frac{1}{N}\frac{dN}{dx_i}$$

Momentum fraction

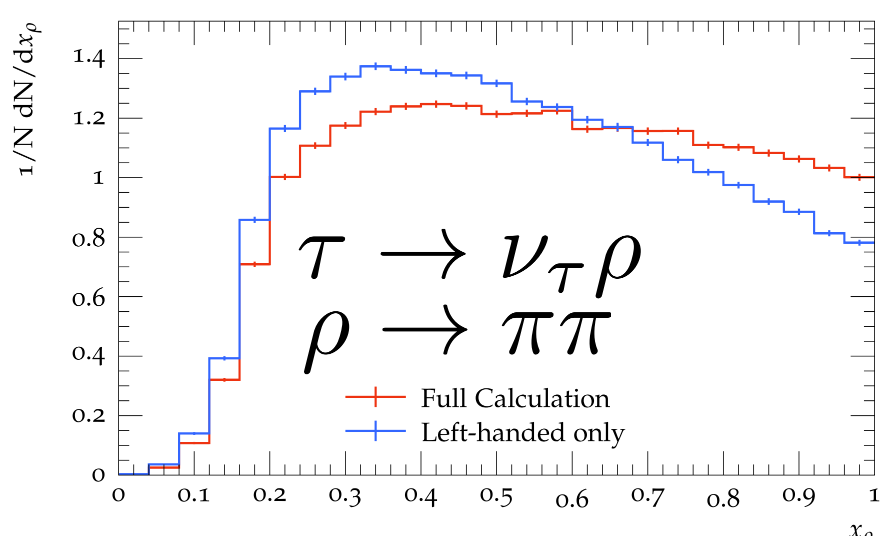
$x_i = E_i / E_\tau$



x_π

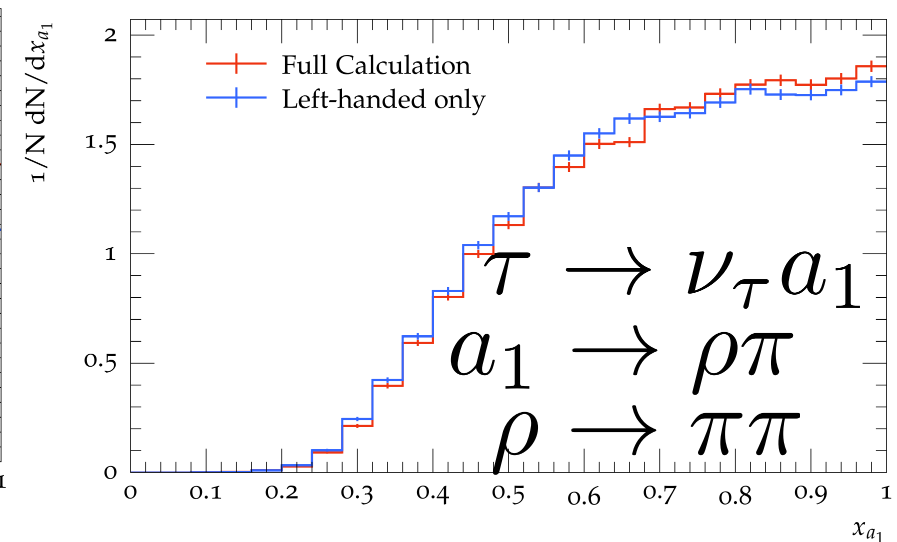
Clean at DUNE

$\mathscr{B}(1\pi) \sim 10\,\%$

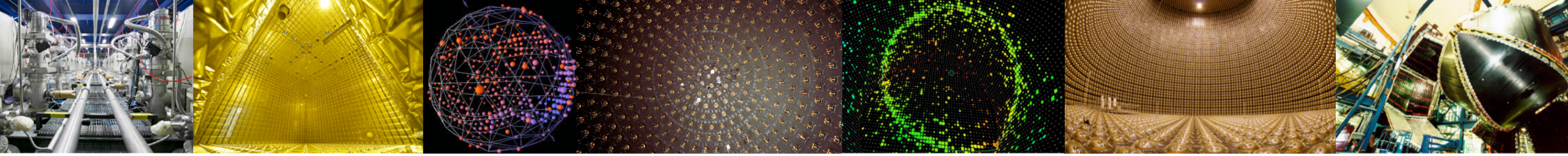


x_ρ

$\mathscr{B}(2\pi) \sim 25\,\%$



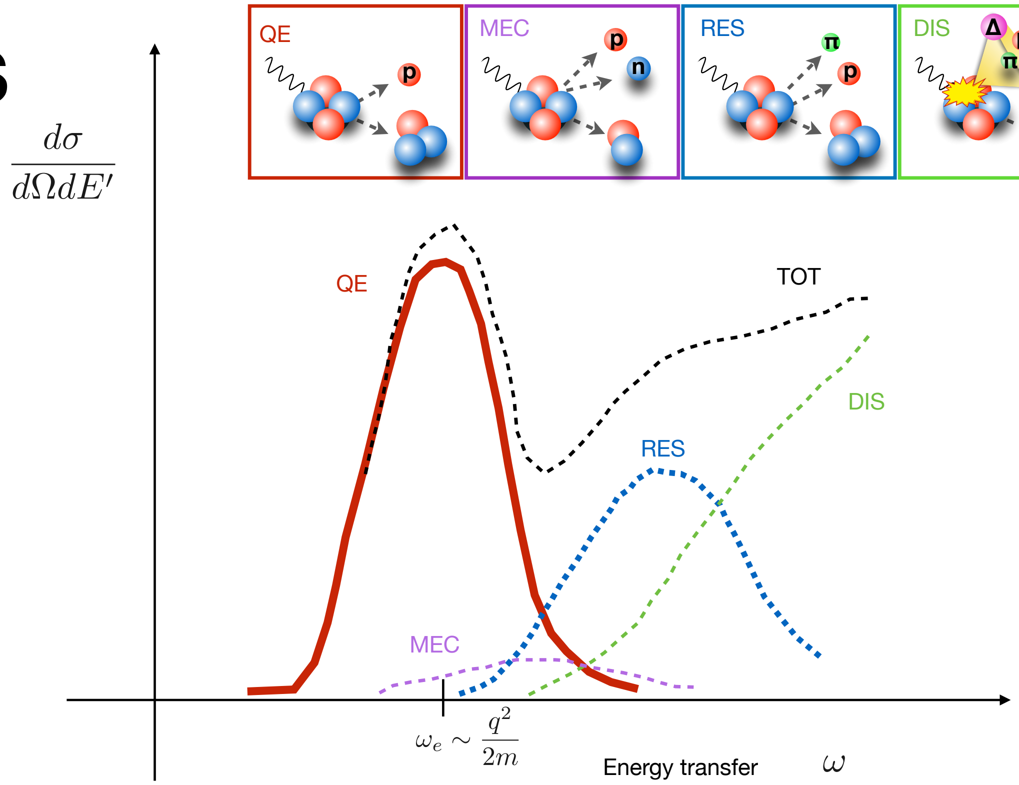
x_{a_1}



Achilles — What's next?

More production processes

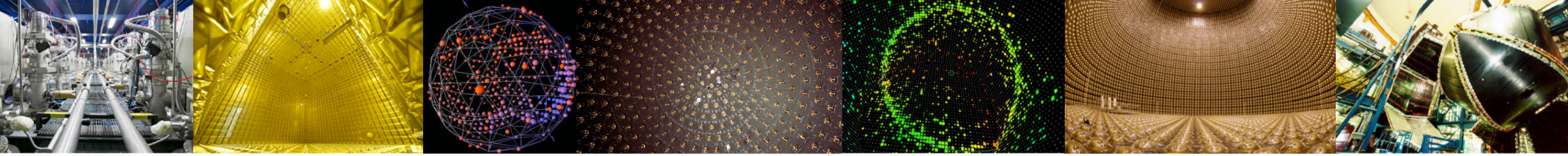
- Published generator: QE scattering only
- Near-term goals: particle production (+decay) at the initial interaction and cascade
- Initial "hard interaction"
  - **Meson-exchange currents** in the spectral function formalism
  - **Resonant scattering** in the dynamical coupled channel formalism (coming very soon!)
  - Longer term: consistent treatment of DIS



- Cascade
  - **Pion production**
  - **Propagation/decay of** Δ

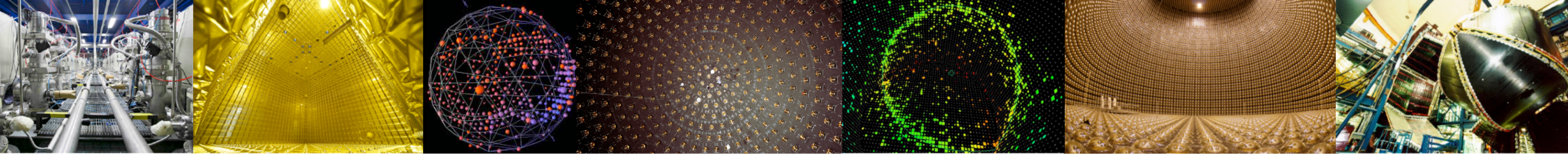
$$NN \rightarrow N\Delta \rightarrow NN\pi$$

(Can take from data. Lattice calculations will always help.)

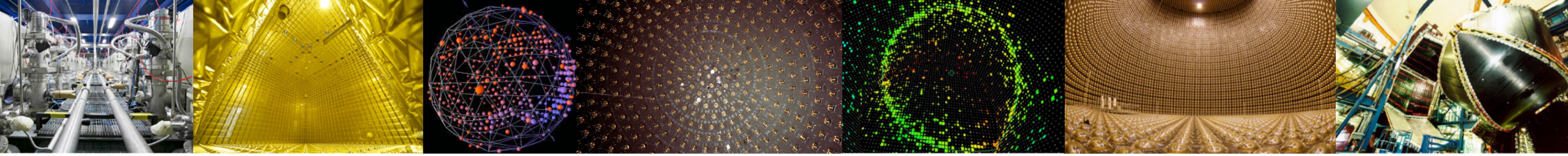


Achilles — Summary

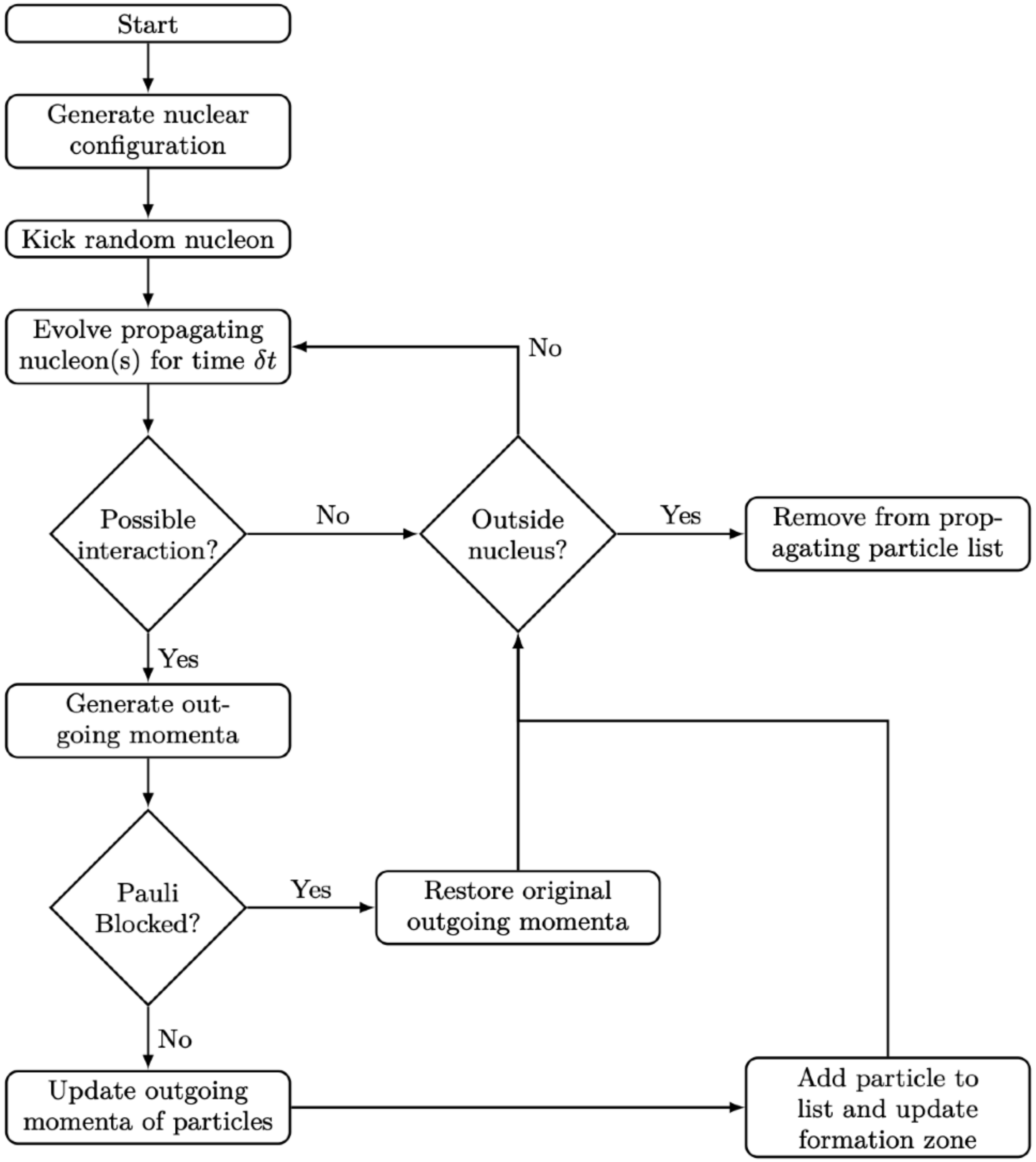
- **Achilles aspires to be a theory-driven event generator, with consistent treatment of known theoretical uncertainties**
- **Observations**:
  - Robustly quantifying systematic errors is generally a tough problem
  - Once chosen, correctly propagating systematics errors is comparatively easy
  - For uncertainties in the “hard interaction” the theoretical uncertainty amounts to an uncertainty in the overall event weight, which is straightforward to propagate
- **Achilles employs a modular design to factorize physically different processes:**
  - Leptonic vs hadronic tensors,
  - Nuclear vs hadronic physics
  - Primary interaction vertex vs intranuclear cascade
- Achilles currently supports quasi-elastic scattering (e.g., spectral function formalism)
- Support for more processes is coming soon!

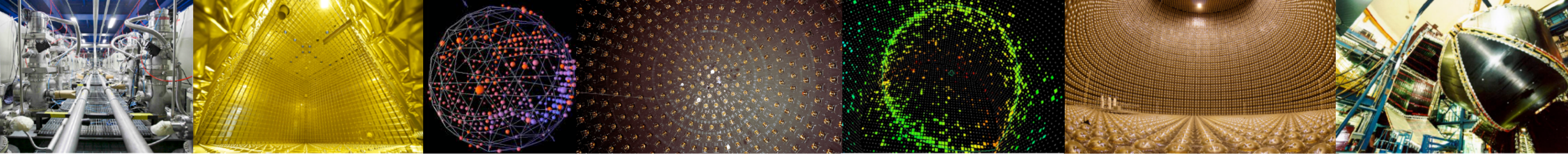


Backup



Cascade — Algorithm





Effective Background Potentials

- Three-parameter non-relativistic potential
- Parameters obtained by a fit to single-particle energy of nuclear matter (Urbana v_{14} + TNI Hamiltonian)
- Consistent with variational ground-state calculations of Wiringa, Fiks, and Fabrocini

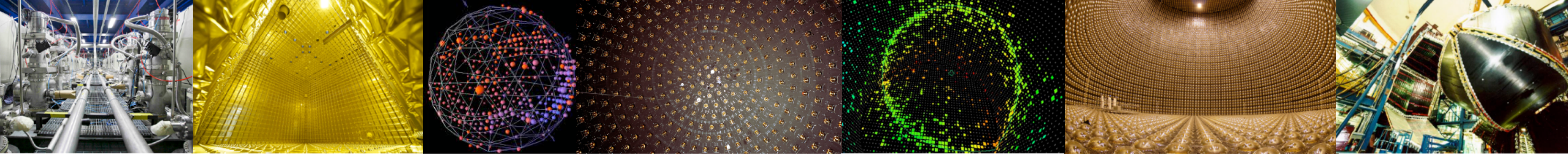
$$U(p', r) = \alpha[\rho(r)] + \frac{\beta[\rho(r)]}{1 + (p'/\Lambda[\rho(r)])^2}$$

$$\begin{aligned}
\alpha(\rho) &= 15.52(\rho/\rho_0) + 24.93(\rho/\rho_0)^2 \text{ MeV}, \\
\beta(\rho) &= -116(\rho/\rho_0) \text{ MeV}, \\
\Lambda(\rho) &= 3.29 - 0.373(\rho/\rho_0) \text{ fm}^{-1}, \\
\rho_0 &= 0.16 \text{ fm}^{-3}
\end{aligned}$$

[102] A. Nikolakopoulos, R. González-Jiménez, N. Jachowicz, K. Niewczas, F. Sánchez, and J. M. Udías, (2022), arXiv:2202.01689 [nucl-th].

[103] R. B. Wiringa, Phys. Rev. C **38**, 2967 (1988).

[104] R. B. Wiringa, V. Fiks, and A. Fabrocini, Phys. Rev. C **38**, 1010 (1988).

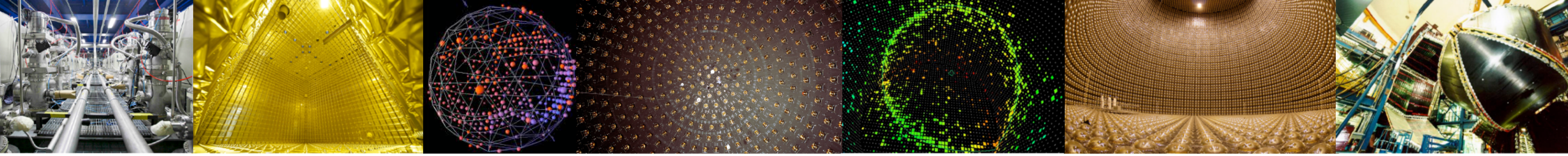


Effective Background Potentials

- Potential fitted from proton-nucleus cross section data to determine global proton-nucleus optical potentials for energies between 20 - 1040 MeV for several nuclear targets

- Taken from work by Cooper, Hama, and Clark

[97] E. D. Cooper, S. Hama, and B. C. Clark, Phys. Rev. C **80**, 034605 (2009).

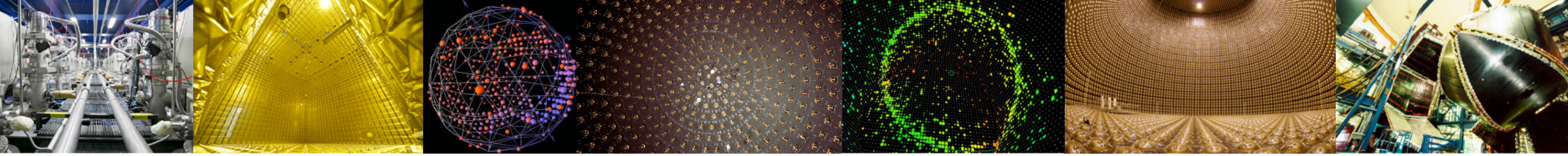


In-medium corrections

[107] V. R. Pandharipande and S. C. Pieper, Phys. Rev. C **45**, 791 (1992).

$$\frac{d\sigma'}{d\Omega} = \frac{|\mathbf{p}'_1 - \mathbf{p}'_2|}{m} \left| \frac{\mathbf{p}'_1}{m^*(p'_1, \rho)} - \frac{\mathbf{p}'_2}{m^*(p'_2, \rho)} \right|^{-1} \frac{m^*\left(\sqrt{(p'^2_3 + p'^2_4)/2}, \rho\right)}{m} \frac{d\sigma}{d\Omega}$$

$$m^*(p', \rho) = p' \left(\frac{p'}{m} + \frac{dU(p', \rho)}{dp'} \right)^{-1}$$

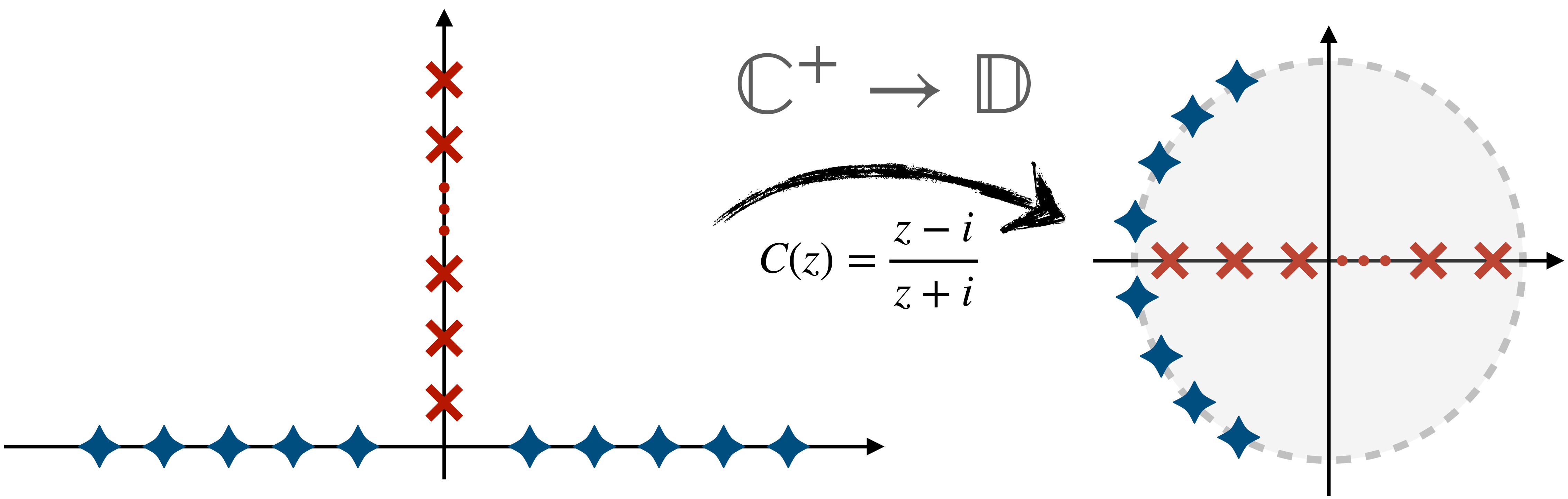


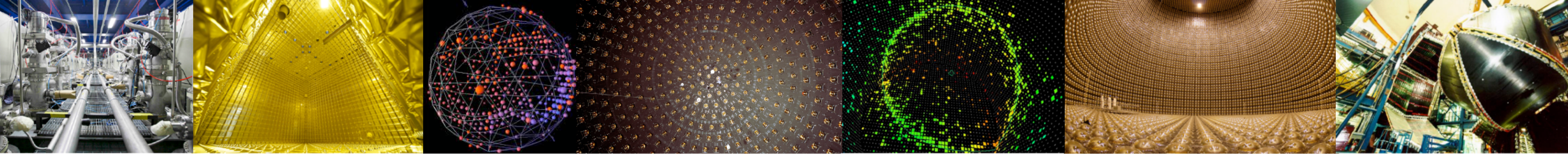
Analytic Continuation

Conformal maps

- Recall: analytic functions are defined by convergent power series in an open set around each nonsingular point
- Radius of convergence is determined by the location of the nearest pole

So change coordinates!





Analytic Continuation

The technical problem

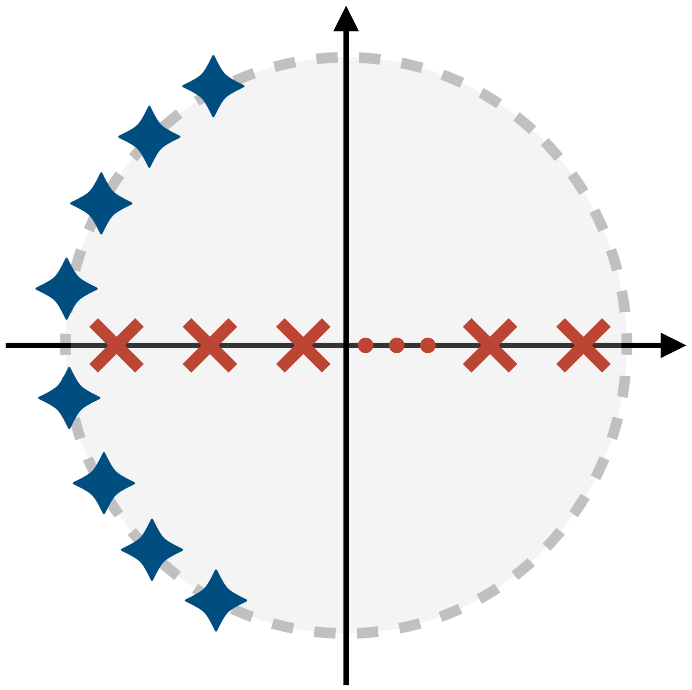
- Recall: analytic functions are defined by convergent power series in an open set around each nonsingular point
- Radius of convergence is determined by the location of the nearest pole
- The Cayley transform maps the problem to the unit disk.
- **Given Euclidean data**

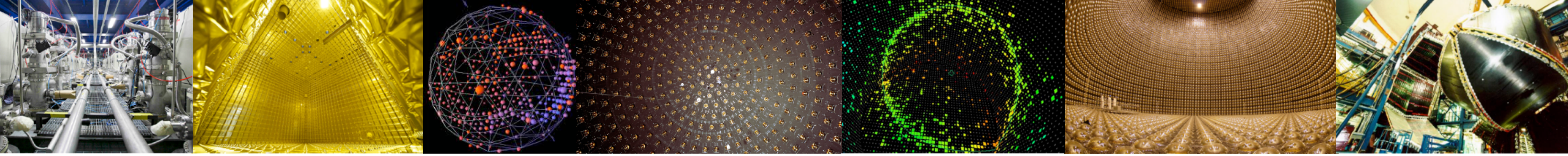
$$\{i\omega_\ell\} \to \zeta_\ell \subset \mathbb{D},$$

$$\{G(i\omega_\ell)\} \mapsto w_\ell \subset \mathbb{D},$$

construct an analytic function $f(\zeta)$

on the disk such that $f(\zeta_\ell) = w_l$**.**





Analytic Continuation

R. Nevanlinna
Ann. Acad. Sci. Fenn. Ser. A 13 (1919)
Ann. Acad. Sci. Fenn. Ser. A 32 (1929)

Nevanlinna's Theorem

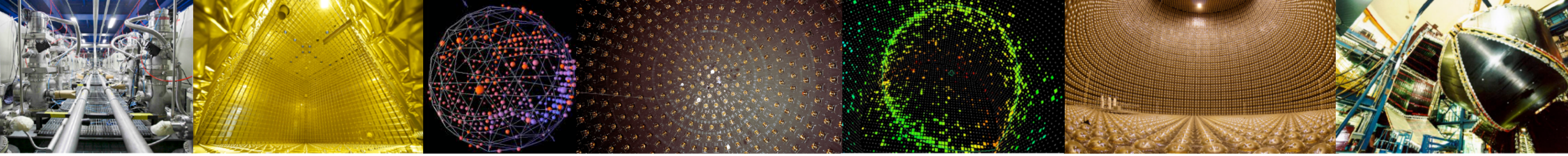
- **Theorem (Nevanlinna, 1919/1929):**
  - Any solution to the interpolation problem with N points can be written in the form

$$f(\zeta) = \frac{P_N(\zeta) f_N(\zeta) + Q_N(\zeta)}{R_N(\zeta) f_N(\zeta) + S_N(\zeta)}$$

where the coefficient functions P_N, Q_N, R_N, S_N are calculable using an inductive formula in terms of the input data $\{\zeta_\ell\}$ and $\{w_\ell\}$and an arbitrary analytic function $f_N(\zeta) : \mathbb{D} \to \mathbb{D}$.

Derivation: See our preprint [arXiv:2305.16190], which follows modern treatment by mathematician Nicolau [https://mat.uab.cat/~artur/data/nevanlinna-pick.pdf]

- $P_N, Q_N, R_N, S_N \Longleftrightarrow$"*Nevanlinna coefficients*"
- Arbitrary function $f_N(\zeta) \Longleftrightarrow$ "Freedom to specify further Euclidean data to constrain the interpolating function"



Analytic Continuation

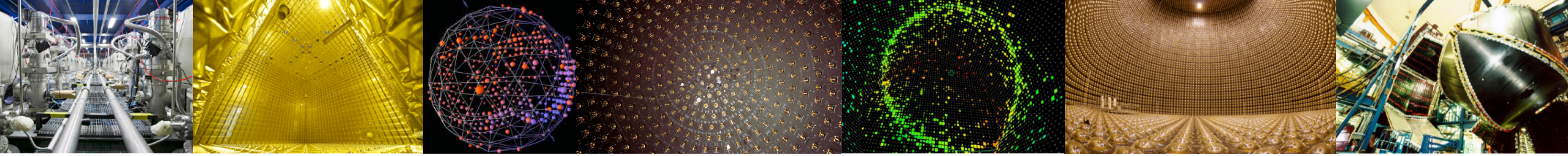
The full space of solutions

- Question: For fixed N and ζ, **what are the possible values that an interpolating function $f(\zeta)$ can take,** by varying possible values of the arbitrary function $f_N(\zeta) \in \mathbb{D}$?

- Answer: The space of possible values is given by the *Wertevorrat* $\Delta_N(\zeta)$, which is the disk of radius $r_N(\zeta)$ and centered at $c_N(\zeta)$.

$$c_N = \frac{P_N \overline{(-R_N/S_N)} + Q_N}{R_N \overline{(-R_N/S_N)} + S_N} \qquad r_N = \frac{|P_N S_N - Q_N R_N|}{|S_N|^2 - |R_N|^2}$$

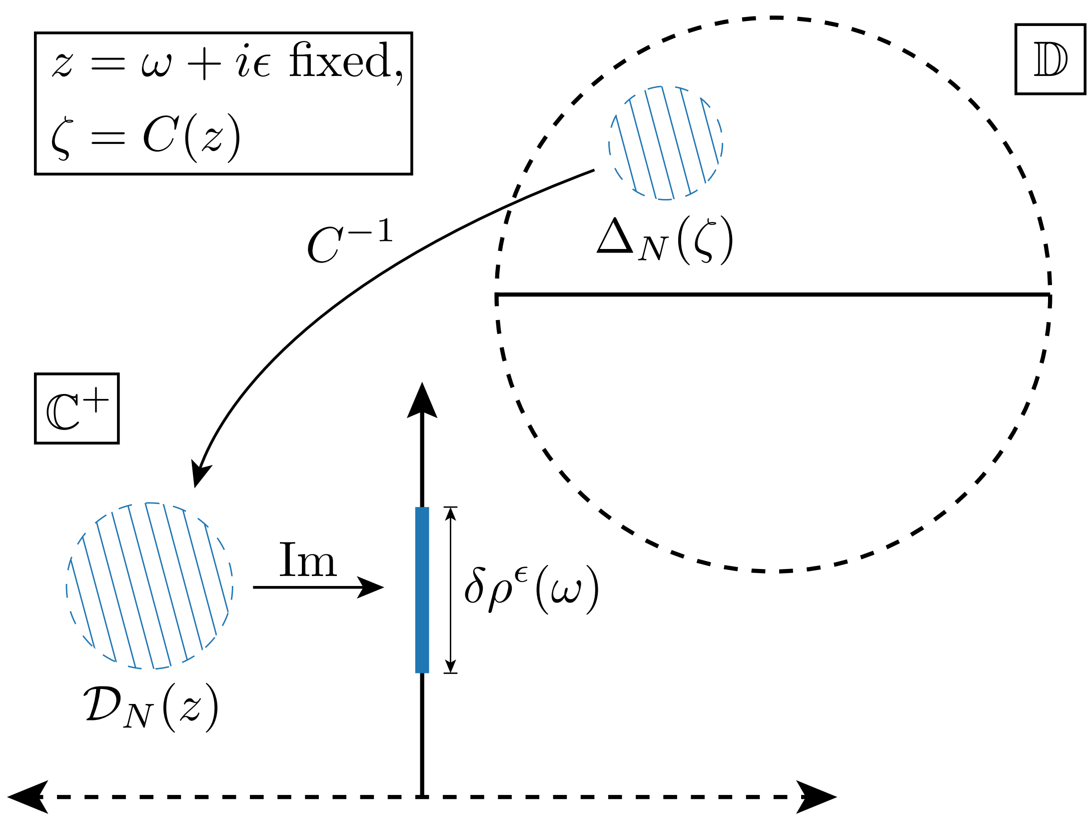
- The Wertevorrat $\Delta_N(\zeta)$ rigorously contains the full infinite family of all possible analytic continuations at each point $\zeta \in \mathbb{D}$.



Analytic Continuation

The Wertevorrat and rigorous bounds on $\rho^{\epsilon}(\omega)$

- Finally we need to map the Wertevorrat back to the upper half plane. Use the inverse Cayley transform $z = C^{-1}(\zeta)$.



$$\rho^{\epsilon}(\omega) = \frac{1}{\pi} \mathrm{Im}\, G(\omega + i\epsilon)$$

$$\delta\rho^{\epsilon}(\omega) = \frac{1}{\pi} \Big[\max \mathrm{Im}\, \partial D_N(\omega + i\epsilon) - \min \mathrm{Im}\, \partial D_N(\omega + i\epsilon) \Big]$$