Quark TMD in a background field: one-loop renormalization and CSS evolution

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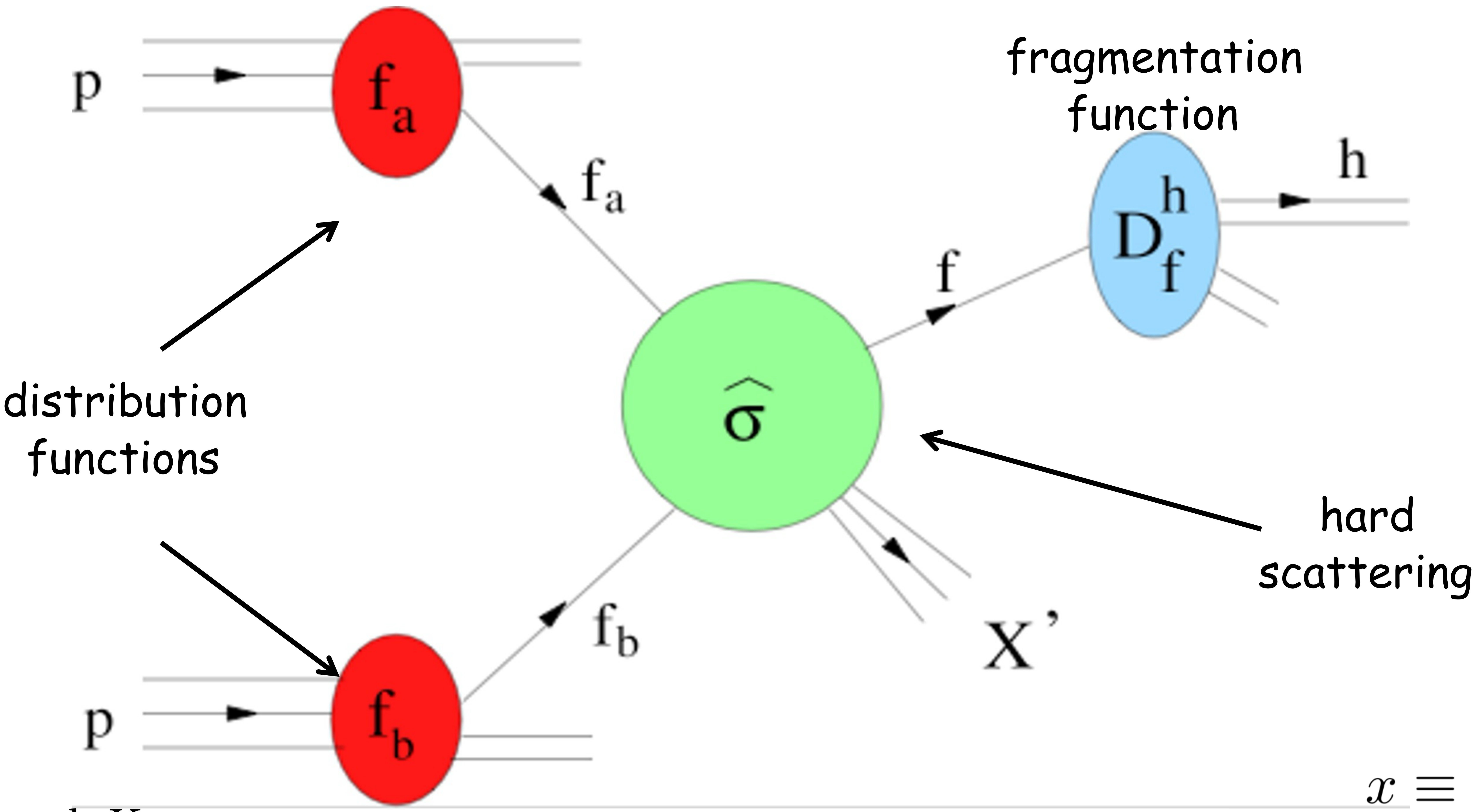
QCD in proton-proton collisions

collinear factorization: separation of soft (long distance) and hard (short distance)



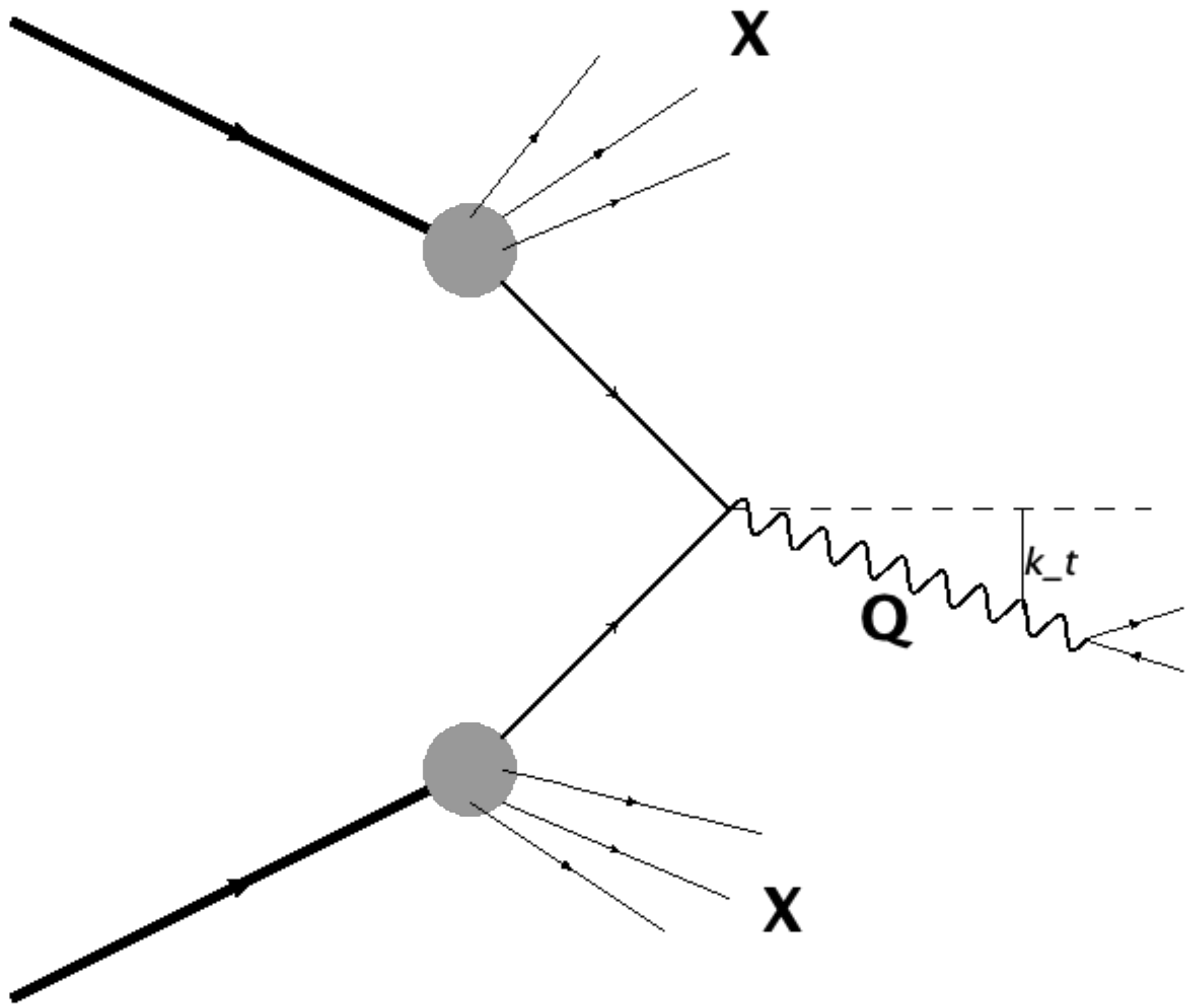
$$x \equiv \frac{p^+}{P^+}$$

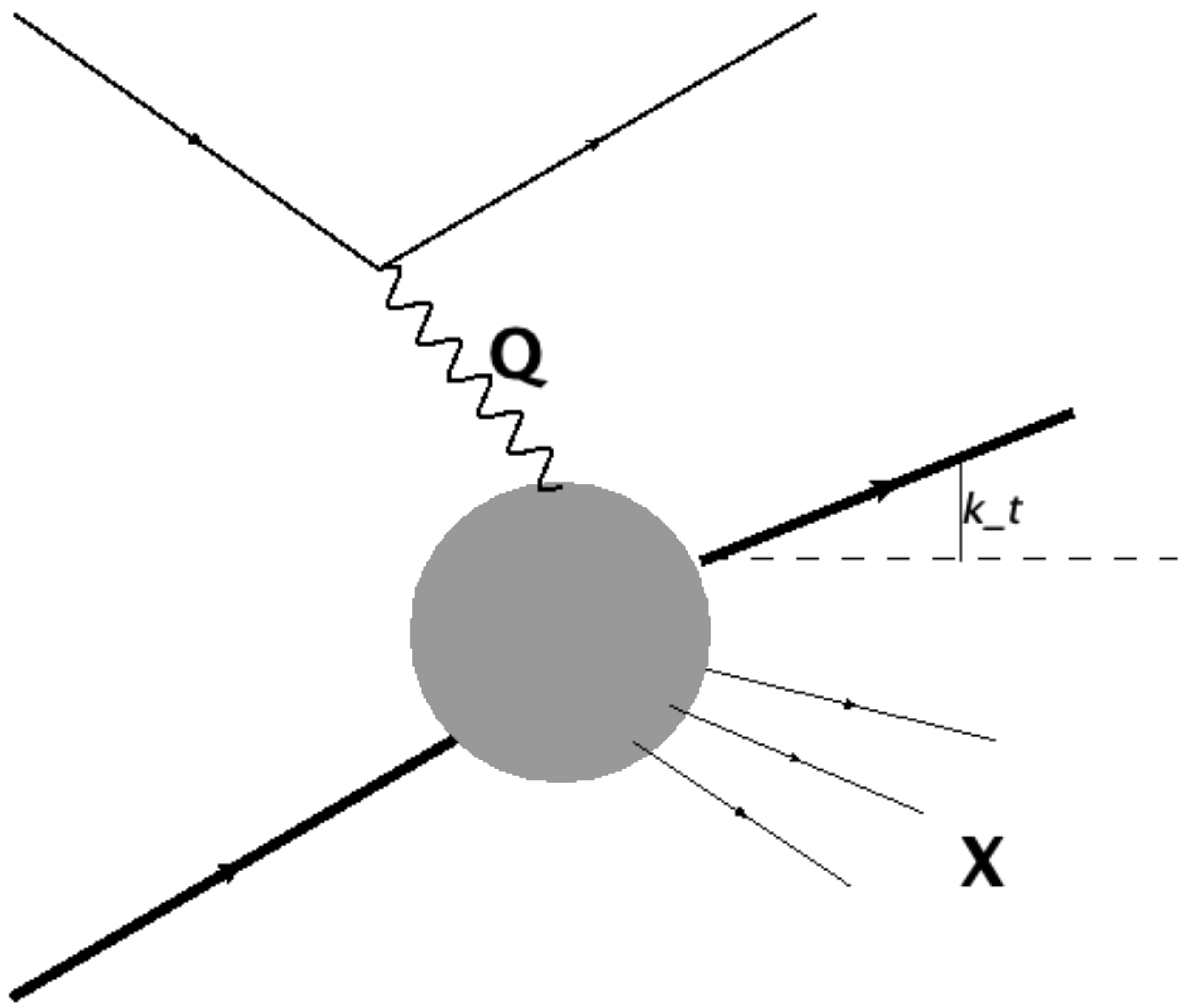
$$\frac{d\sigma^{pp\to h\,X}}{d^2p_t\,dy} \sim f_a(x_1) \otimes f_b(x_2) \otimes \hat{\sigma} \otimes D_f^h(z) + \cdots\cdots$$

power corrections

TMD factorization: two hard scales

DY **SIDIS** $Q^2 \gg k_\perp^2 \gg \Lambda_{QCD}^2$





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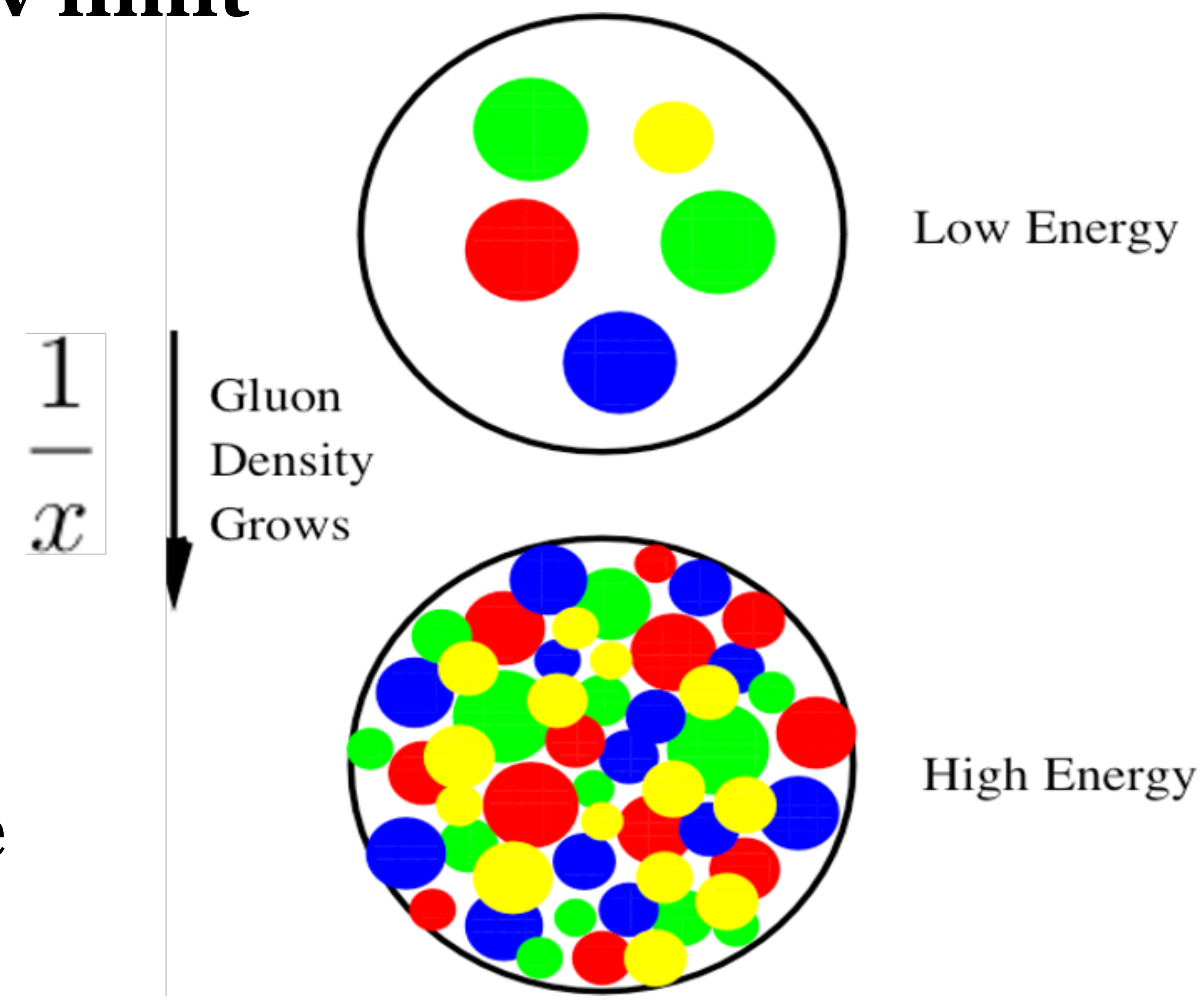
Transverse Momentum Distribution $q(x, \mathbf{x}_\perp, \mu^2, \zeta)$ satisfies the CSS evolution equations

Gribov-Levin-Ryskin

Resolving the nucleus/hadron: Regge-Gribov limit

Q^2 fixed and $\sqrt{S} \to \infty \quad (x \equiv \frac{Q^2}{S} \to 0)$



gluons are radiated into fixed resolved area

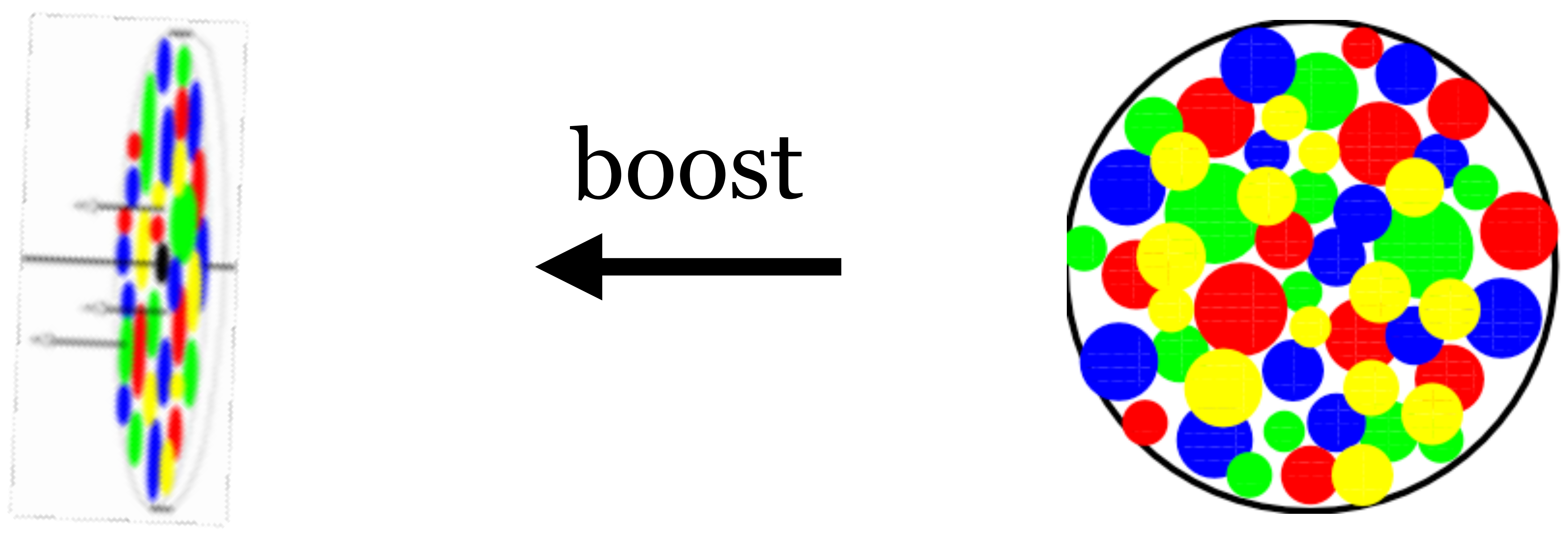
number of gluons increases due to increased longitudinal phase space

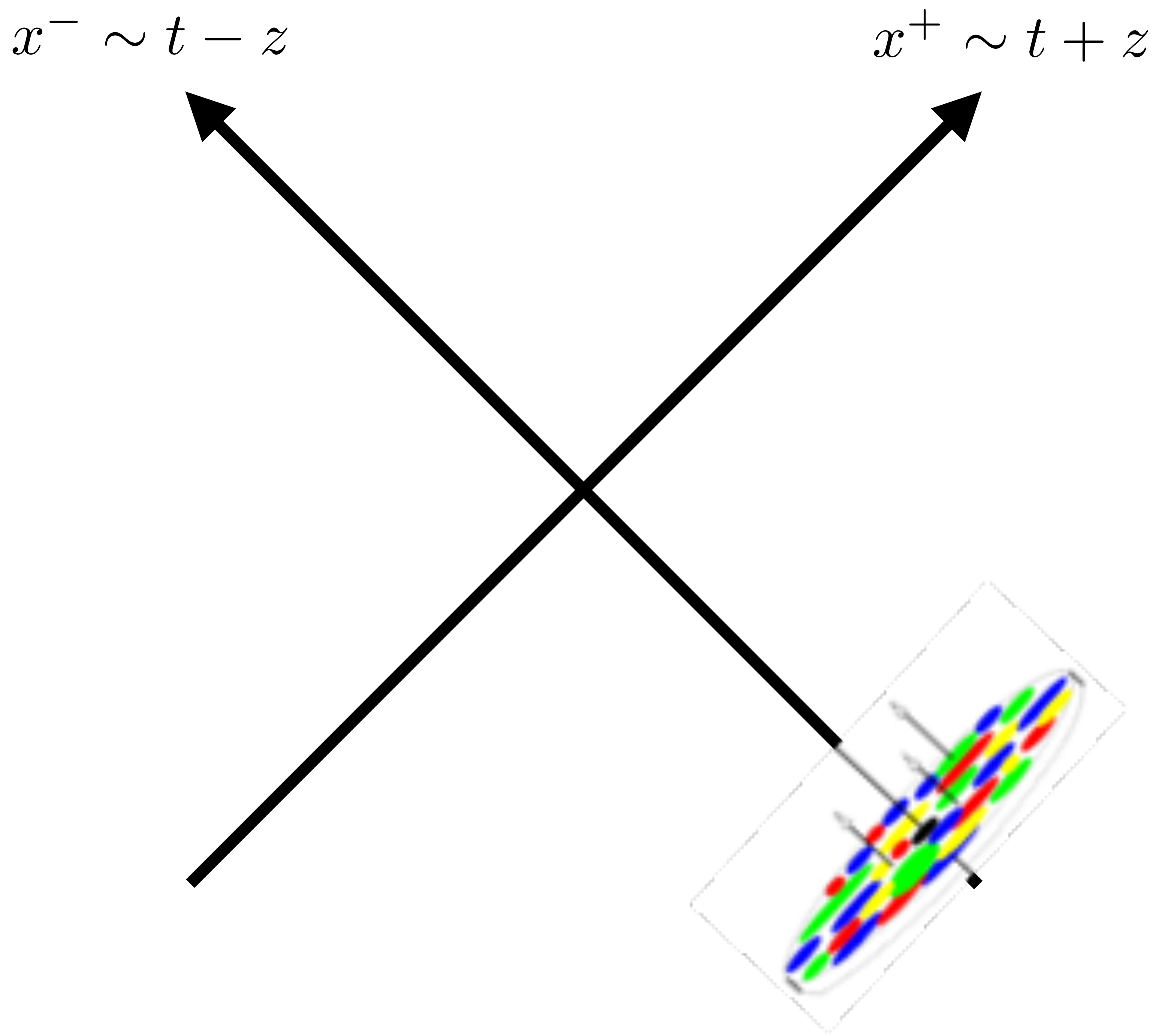
hadron/nucleus becomes a dense state of gluons (CGC) $\quad \frac{\alpha_s}{Q^2}\frac{xG(x,Q^2)}{S_\perp} \sim 1 \quad Q_s^2(x,A,b_\perp) \sim A^{1/3}\left(\frac{1}{x}\right)^{0.3}$

strong color fields

possible universal properties of QCD observables ?

MV: a high energy hadron/nucleus as a classical color field





$$R \longrightarrow \frac{R}{\gamma}$$

$\gamma \sim 100$ RHIC

$\gamma \sim 8000$ LHC

$$J_a^\mu(x) = \delta^{\mu -}\delta(x^+)\rho_a(\mathbf{x}_\perp)$$

color charge

$$D_\mu F^{\mu\nu} = gJ^\nu$$

solution: classical field $A^\mu(x)$

this is the starting point for CGC

Toward precision CGC: inclusive DIS

NLO BK/JIMWLK evolution equations

Kovner, Lublinsky, Mulian (2013)

Balitsky, Chirilli (2007)

NLO corrections to structure functions

Beuf, Lappi, Paatelainen (2022), Beuf (2017)

NLO corrections to SIDIS (+)

Altinoluk, JJM, Marquet (2024)

Bergabo, JJM (2023, 2024)

Caucal, Ferrand, Salazar (2024)

NLO corrections to dihadron/dijets (+)

Bergabo, JJM (2022, 2023)

Iancu, Mulian (2023)

Caucal, Salazar, Schenke, Stebel, Venugopalan (2023), Caucal, Salazar, Schenke, Venugopalan (2022)

Taels, Altinoluk, Beuf, Marquet (2022), Taels (2023)

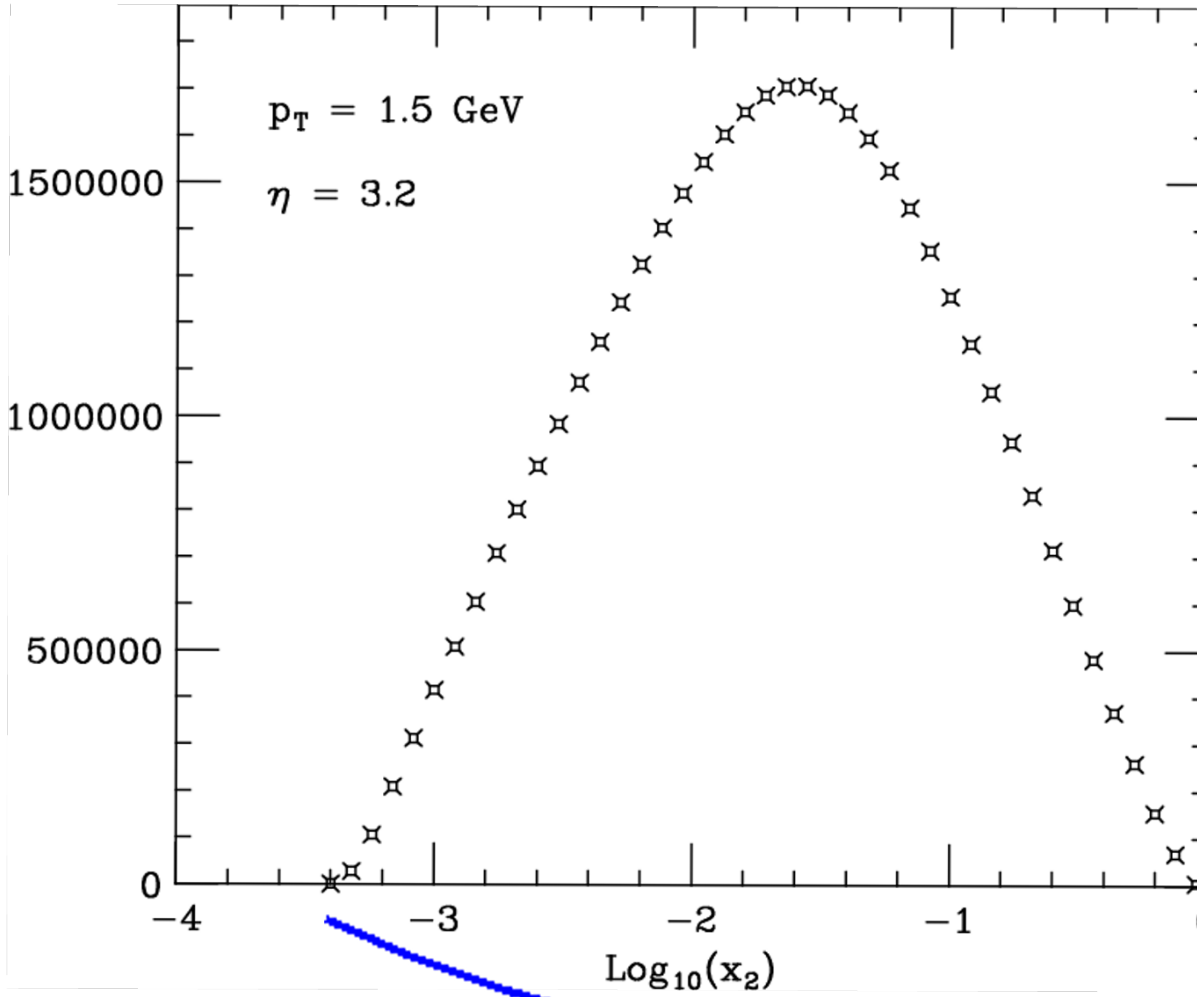
Caucal, Salazar, Venugopalan (2021)

Ayala, Hentschinski, JJM, Tejeda-Yeomans (2016,2017),............

Single inclusive pion production in pp at RHIC

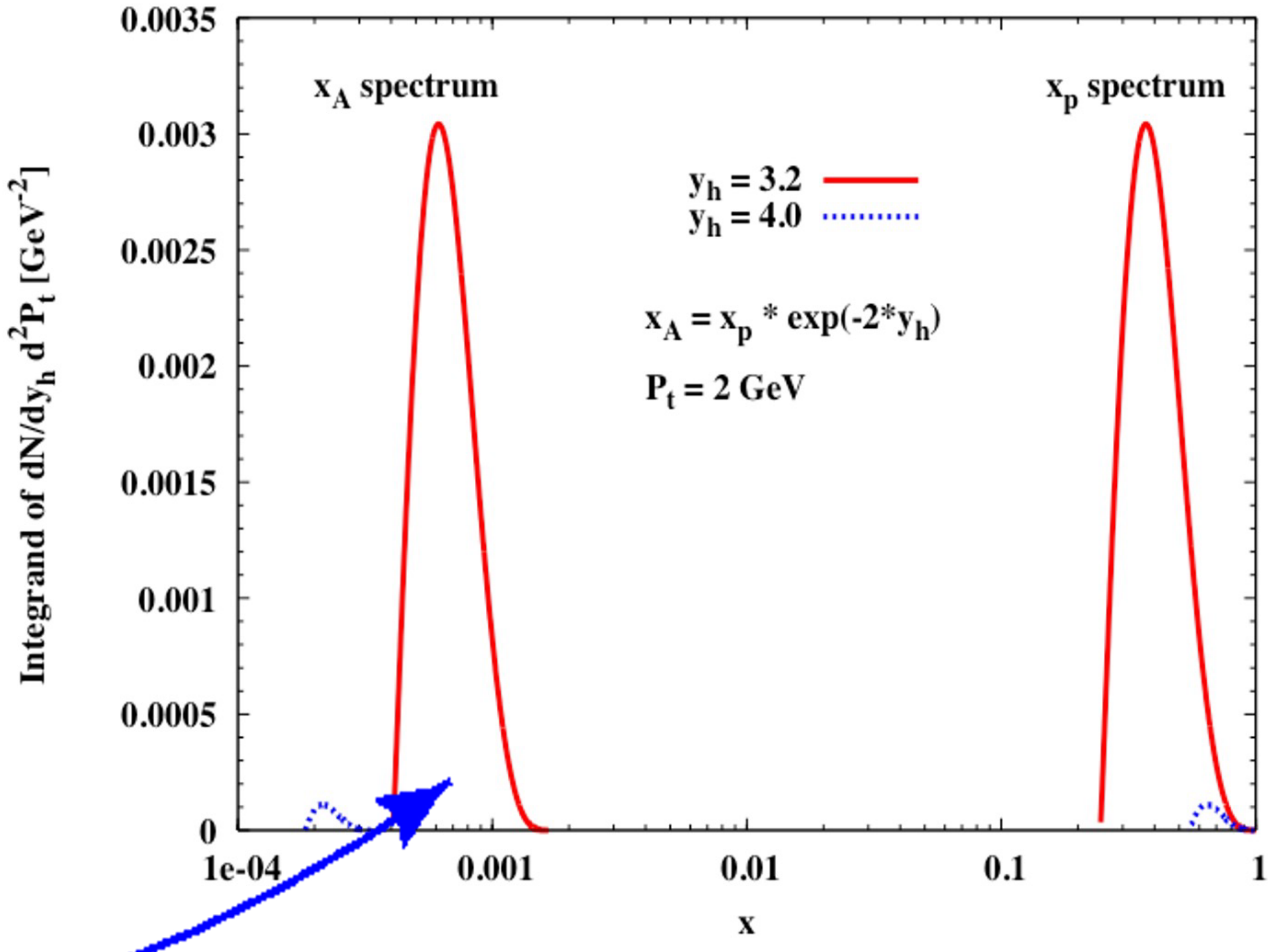
collinear factorization

GSV, PLB603 (2004) 173-183



CGC

DHJ, NPA765 (2006) 57-70



$$\int_{x_{min}}^{1} dx\, xG(x, Q^2) \cdots\cdots \longrightarrow x_{min} G(x_{min}, Q^2) \cdots$$

which kinematics are we in?

A unified formalism for both large and small x ?

Collinear/TMD/... factorization at large x

CGC at small x

this would be tremendously useful for RHIC, LHC, EIC,.. observables

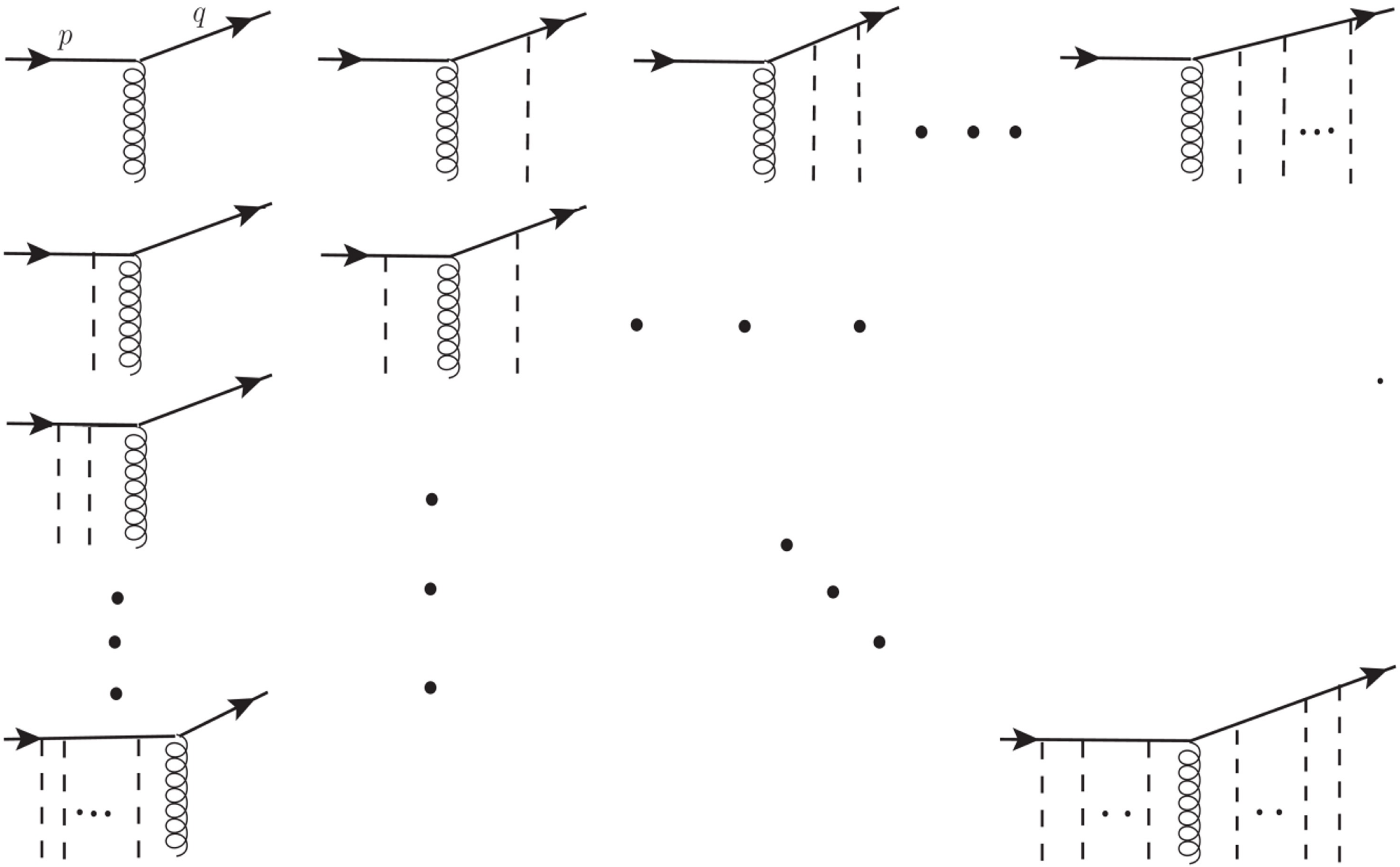
could a background field be the bridge to a unified approach?

already at small x:

connections to TMDs,...

sub-eikonal corrections: Altinoluk et al., Kovchegov et al.

some earlier attempts: JJM, PRD96 (2017), PRD99 (2019), PRD102 (2020)



standard pQCD evolution equations

in a

background field

work done with T. Altinoluk and G. Beuf

DGLAP: arXive:2305.11079

CSS (quark TMD): arXiv:2505.20467

see also: Mukherjee, Skokov, Tarasov, Tiwari

arXiv:2311.16402

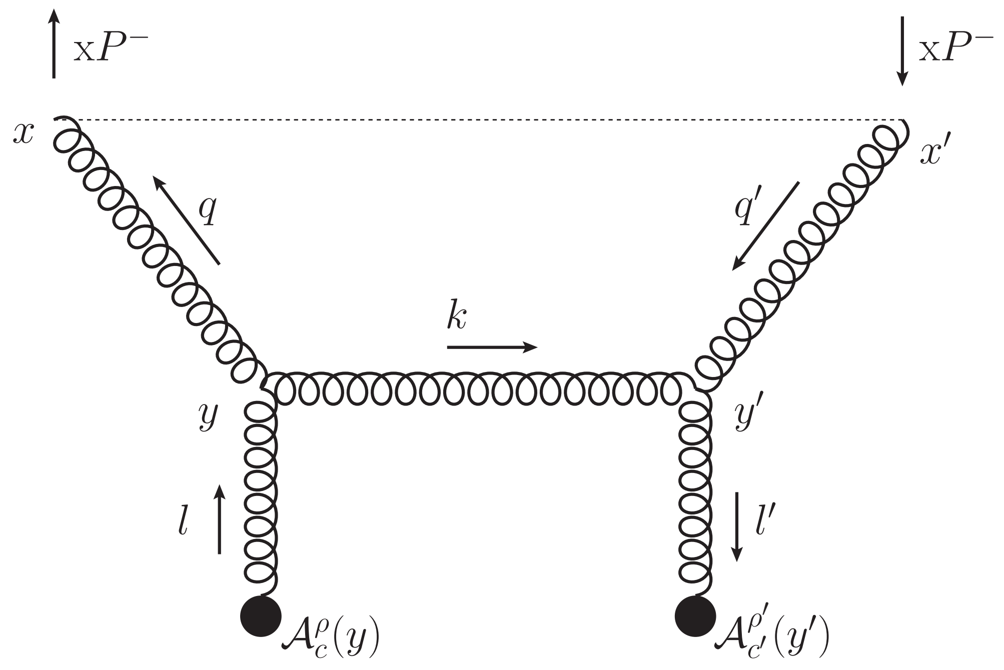
arXiv:2502.15889

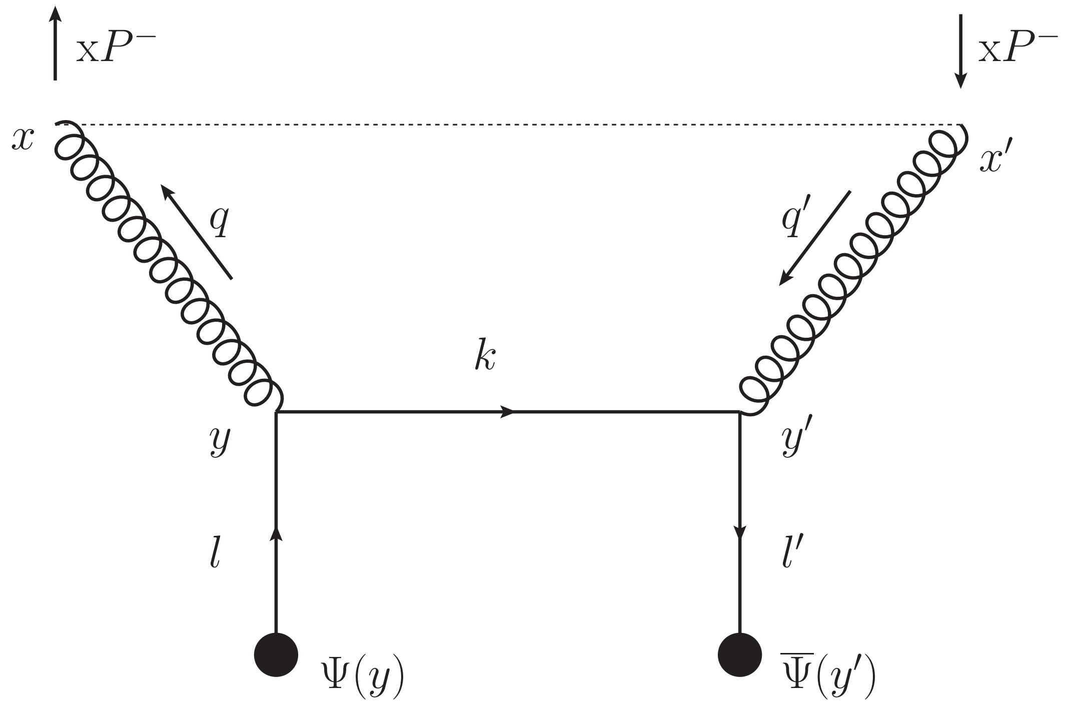
Collinear factorization: DGLAP evolution of parton PDFs

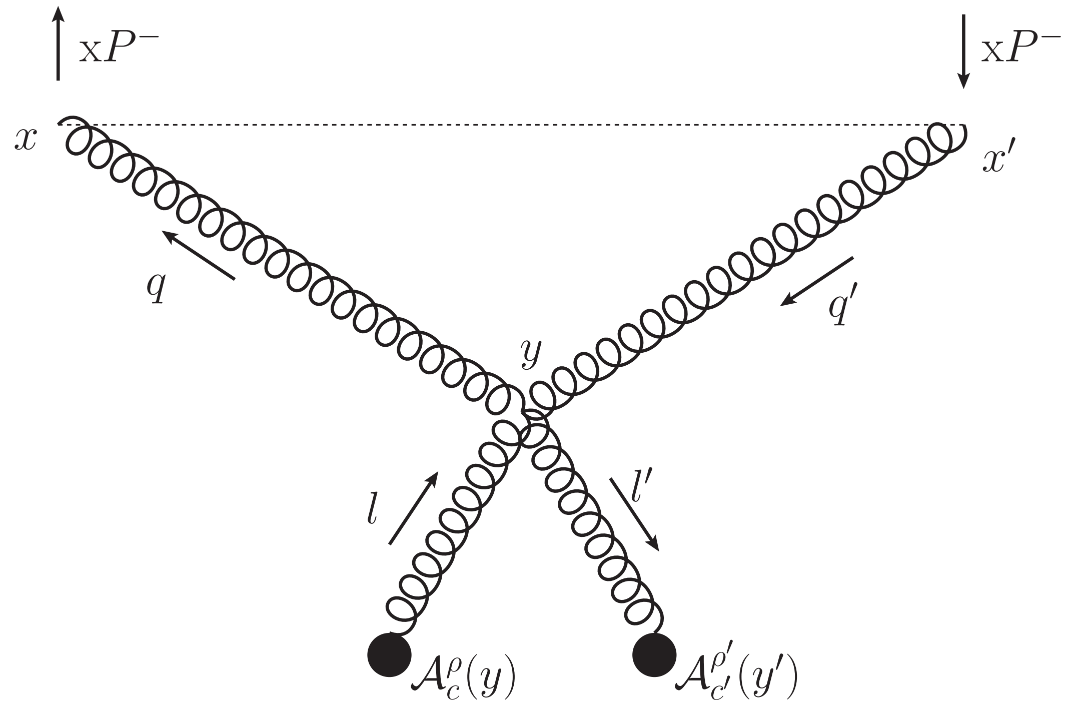
one-loop renormalization of gluon distribution function in <u>light cone gauge using background fields</u>

Bassetto, Heinrich, Kunszt, Vogelsang: 9805283

Altinoluk-Beuf-JJM: 2305.11079







renormalization of YM in light cone gauge

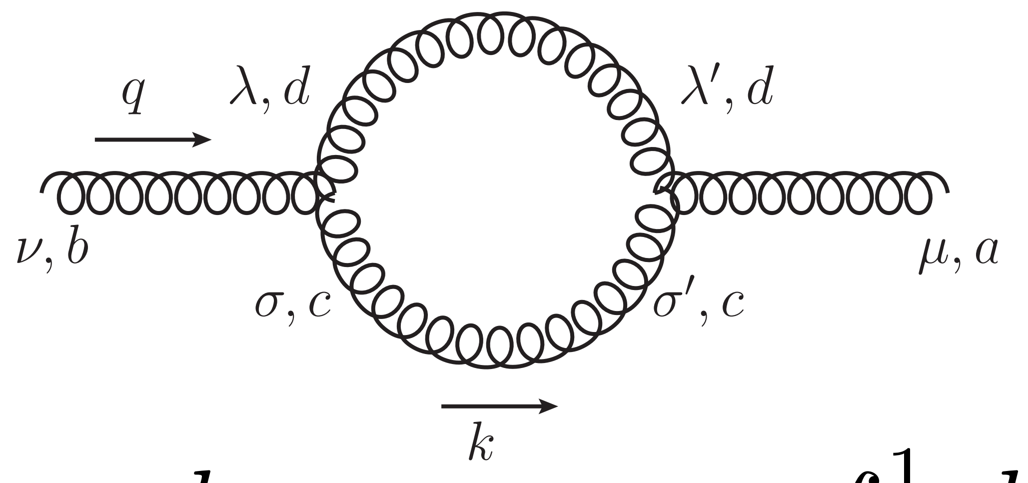
Bassetto, Dalbosco, Soldati: PRD33, 617 (1986)

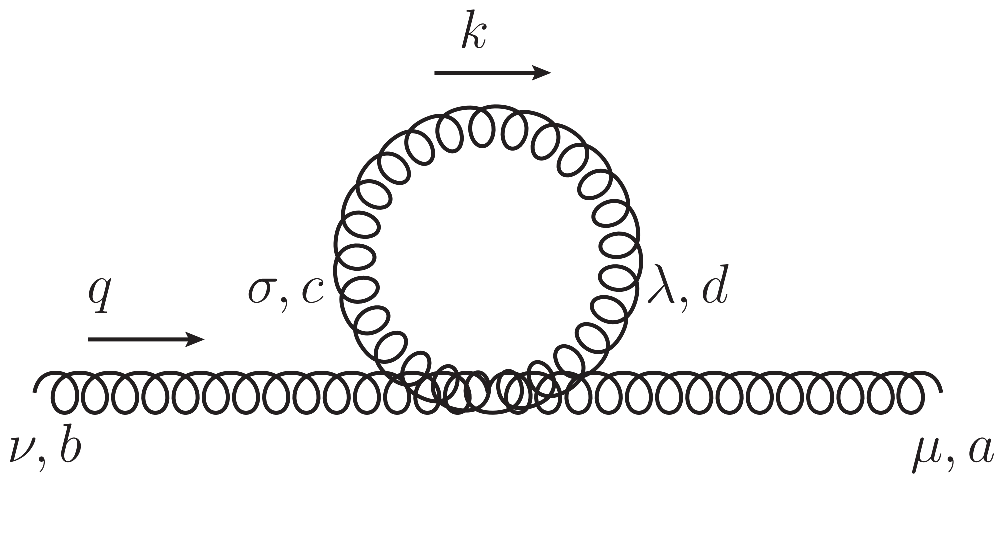
$$A^{(0)a}_{\mu} = Z_3^{\frac{1}{2}} \left[A^a_{\mu} - \left(1 - \tilde{Z}_3^{-1}\right) n_{\mu}\, \Omega^a \right]$$

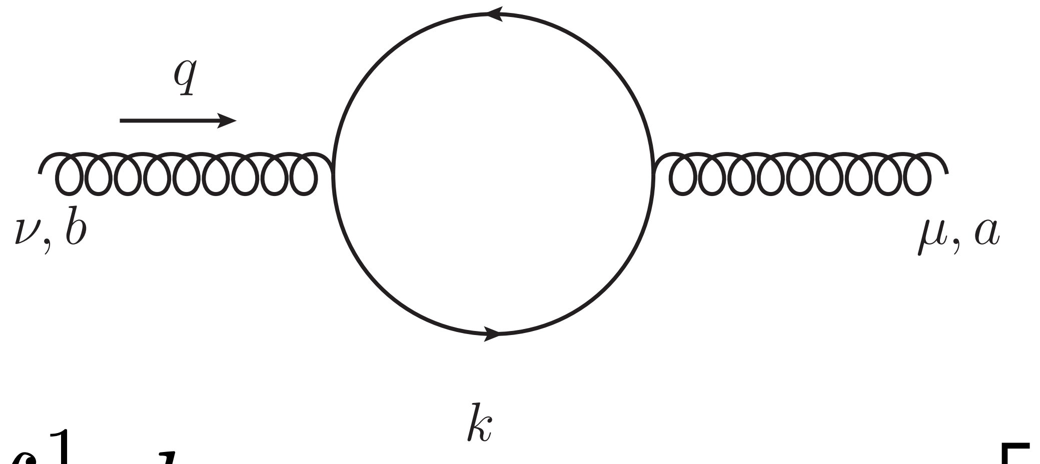
$$\Lambda^{(0)a} = Z_3^{-\frac{1}{2}}\, \Lambda^a$$

$$g_0 = \mu^{\epsilon}\, Z_3^{-\frac{1}{2}}\, g(\mu^2)$$

$$\Omega \equiv \frac{n^{\mu}\bar{n}^{\nu}}{(\bar{n}\cdot n)}\, (n^{\rho} D_{\rho})^{-1}\, F_{\mu\nu}$$







$$\mu^2 \frac{d}{d\mu^2} g(\mathrm{x}, \mu^2) = \int_{\mathrm{x}}^{1} \frac{d\mathrm{z}}{\mathrm{z}}\; P_{gg}(\mathrm{z}, g, 0)\; g\left(\frac{\mathrm{x}}{\mathrm{z}}, \mu^2\right) + \int_{\mathrm{x}}^{1} \frac{d\mathrm{z}}{\mathrm{z}}\; P_{gq}(\mathrm{z}, g, 0)\; \sum_f \left[q_f\left(\frac{\mathrm{x}}{\mathrm{z}}, \mu^2\right) + \bar{q}_f\left(\frac{\mathrm{x}}{\mathrm{z}}, \mu^2\right) \right]$$

TMD factorization: Collins-Soper-Sterman (CSS) evolution of quark TMD

$$q(\mathrm{x}, \mathbf{b}; \mu^2, \zeta) = \lim_{Y^+\to+\infty} \int \frac{\mathrm{d}b^+}{2\pi}\, e^{-i\mathrm{x}P^- b^+} \left\langle P \middle| \overline{\Psi}(b^+, \mathbf{b}, 0^-)\, \frac{\gamma^-}{2}\, U^\dagger(Y^+, \mathbf{b}, 0^-; b^+, \mathbf{b}, 0^-) \right.$$
$$\left. \times\; U(Y^+, \mathbf{b}, 0^-; Y^+, 0_\perp, 0^-)\, U(Y^+, 0_\perp, 0^-; 0)\Psi(0) \middle| P \right\rangle \qquad \text{with } \mathrm{x} > 0$$

$$U(Y^+, \mathbf{b}, 0^-; b^+, \mathbf{b}, 0^-) = \mathcal{P} \exp\left\{ -i\mu^\epsilon g \int_{b^+}^{Y^+} \mathrm{d}x^+ t^a A_a^-(x^+, \mathbf{b}, 0^-) \right\}$$

$$U(Y^+, \mathbf{x}+\mathbf{b}, 0^-; Y^+, \mathbf{x}, 0^-) = \mathcal{P} \exp\left\{ -i\mu^\epsilon g \int_0^1 d\tau\, \mathbf{b}^i\, t^a\, A_i^a(Y^+, \mathbf{x}+\tau\mathbf{b}, 0^-) \right\}$$

UV divergences: use dim reg $D = 4 - 2\epsilon$

rapidity divergences: use “pure rapidity regulator” η: $\left[\frac{k^-}{k^+}\,\frac{\nu^+}{\nu^-}\right]^{\eta/2}$ Ebert et al., arXiv:1812.08189

with $\zeta \equiv \dfrac{2(\mathrm{x}P^-)^2\nu^+}{\nu^-}$

$$\Psi(x) = \psi(x) + \delta\Psi(x)$$
$$A^\mu_a(x) = \mathcal{A}^\mu_a(x) + \delta A^\mu_a(x)$$

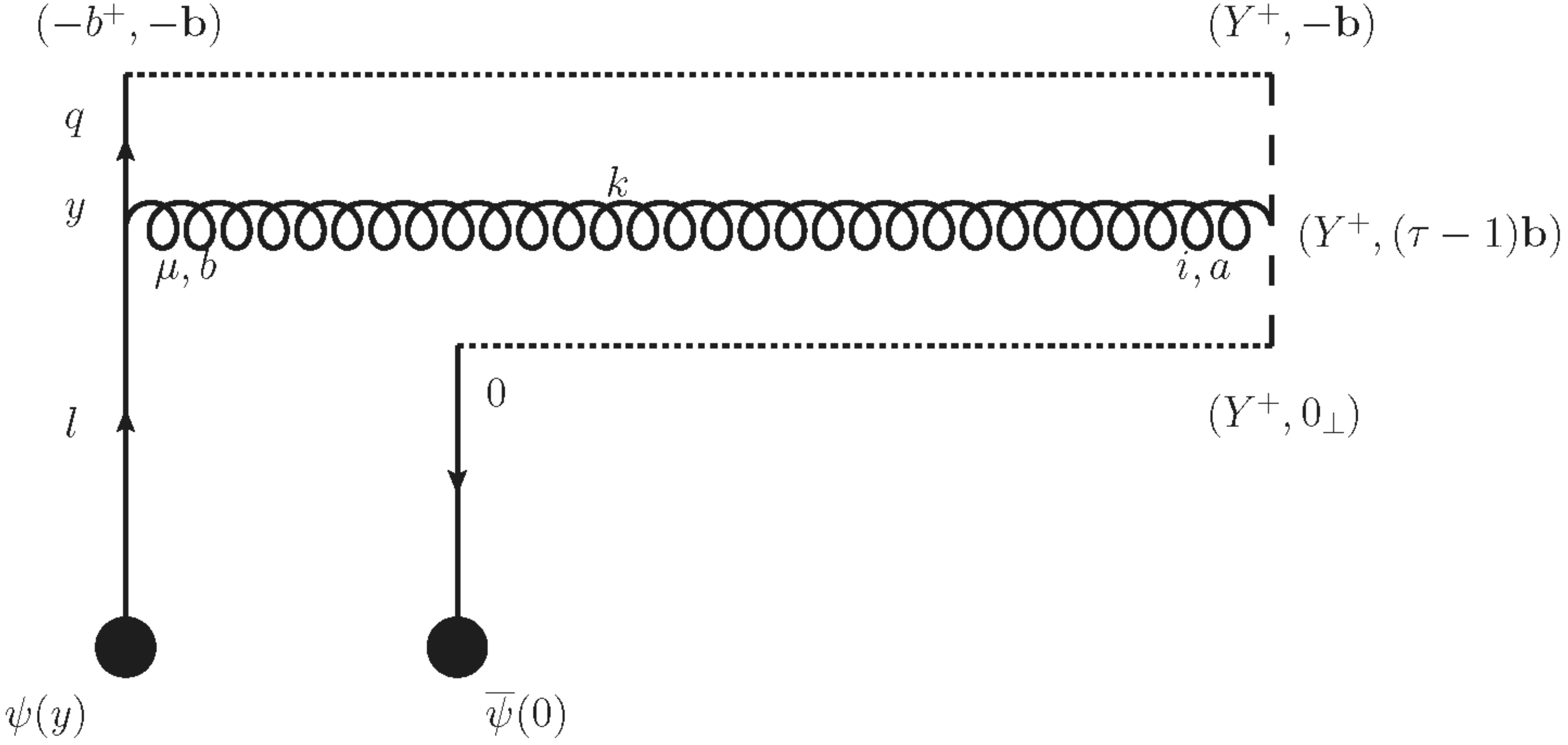
$$\langle P|\mathcal{T}\Big[\overline{\Psi}(b^+,\mathbf{b},0^-)\,\frac{\gamma^-}{2}\,U(Y^+,\mathbf{b},0^-;Y^+,0_\perp,0^-)\,\Psi(0)\Big]|P\rangle_c = \langle P|\mathcal{T}\Big[\overline{\psi}(b^+,\mathbf{b},0^-)\,\frac{\gamma^-}{2}\,\psi(0)\Big]|P\rangle_c$$
$$+\langle P|\mathcal{T}\Big[\overline{\delta\Psi}(b^+,\mathbf{b},0^-)\,\frac{\gamma^-}{2}\,\delta\Psi(0)\Big]|P\rangle_c$$
$$+\langle P|\mathcal{T}\Big[\overline{\psi}(b^+,\mathbf{b},0^-)\,\frac{\gamma^-}{2}\,\Big\{-i\mu^\epsilon g\int_0^1 d\tau\,\mathbf{b}^i\,t^a\,\delta A^a_i(Y^+,\tau\mathbf{b},0^-)\Big\}\,\delta\Psi(0)\Big]|P\rangle_c$$
$$+\langle P|\mathcal{T}\Big[\overline{\delta\Psi}(b^+,\mathbf{b},0^-)\,\frac{\gamma^-}{2}\,\Big\{-i\mu^\epsilon g\int_0^1 d\tau\,\mathbf{b}^i\,t^a\,\delta A^a_i(Y^+,\tau\mathbf{b},0^-)\Big\}\,\psi(0)\Big]|P\rangle_c$$
$$+\langle P|\mathcal{T}\Big[\overline{\psi}(b^+,\mathbf{b},0^-)\,\frac{\gamma^-}{2}\,\frac{1}{2}\mathcal{P}\Big\{-i\mu^\epsilon g\int_0^1 d\tau\,\mathbf{b}^i\,t^a\,\delta A^a_i(Y^+,\tau\mathbf{b},0^-)\Big\}^2\,\psi(0)\Big]|P\rangle_c$$
$$+\cdots$$

$$q^{\mathrm{LO}}(\mathrm{x},\mathbf{b};\mu^2) = \int\frac{\mathrm{d}b^+}{2\pi}\,e^{-i\mathrm{x}P^-b^+}\langle P|\mathcal{T}\Big[\overline{\psi}(b^+,\mathbf{b},0^-)\,\frac{\gamma^-}{2}\,\psi(0)\Big]|P\rangle_c$$

use spacetime translation invariance to shift fields when convenient

$$q(\mathrm{x}, \mathbf{b}; \mu^2, \zeta)\Big|_{\mathrm{rad.}\infty} = \lim_{Y^+ \to +\infty} \int \frac{\mathrm{d}b^+}{(2\pi)}\, e^{-i\mathrm{x}P^- b^+} \langle P \big| \mathcal{T}\Big[\overline{\psi}(0)\, \frac{\gamma^-}{2}\, (-i) g \mu^\epsilon \int_0^1 d\tau\, (-1)\mathbf{b}^i\, t^a\, \delta A_a^i(Y^+, (\tau - 1)\mathbf{b}, 0^-)$$

$$\times\, \delta\Psi(-b^+, -\mathbf{b}, 0^-)\Big]\big| P \rangle_c \Big|_{\overline{\psi}\psi}$$



identical contribution from radiation by the other leg

take $Y^+ \to +\infty$

then must take $\eta \longrightarrow 0$ at finite ϵ

$$q(\mathrm{x}, \mathbf{b}; \mu^2, \zeta)\Big|_{\mathrm{rad.}\infty} = \lim_{Y^+ \to +\infty} (-ig\mu^\epsilon)^2 \int \frac{\mathrm{d}b^+}{(2\pi)}\, e^{-i\mathrm{x}P^- b^+} \int d^{4-2\epsilon}y \int_0^1 d\tau\, (-1)\mathbf{b}^i\, \delta_{ab} G_{0,F}^{i\mu}(Y^+, (\tau - 1)\mathbf{b}, 0^-; y)$$

$$\times \langle P \big| \mathcal{T}\, \overline{\psi}(0)\, \frac{\gamma^-}{2} t^a\, S_{0,F}(-b, -\mathbf{b}, 0^-; y) \gamma_\mu t^b\, \psi(y) \big| P \rangle_c$$

gluon propagator in the light cone gauge

$$n_\mu A^\mu = A^- = 0 \quad \text{with} \quad n^2 = 0$$

$$\tilde{G}^{\mu\nu}_{0,F}(k) = \frac{i}{(k^2 + i0)} \left\{ -g^{\mu\nu} + \frac{(k^\mu n^\nu + n^\mu k^\nu)}{[n\cdot k]} \right\}$$

Mandelstam-Leibbrandt (ML) prescription

$$\frac{1}{[n\cdot k]} \equiv \frac{(\bar{n}\cdot k)}{(n\cdot k)(\bar{n}\cdot k) + i0} = \frac{\theta(\bar{n}\cdot k)}{[n\cdot k + i0]} + \frac{\theta(-\bar{n}\cdot k)}{[n\cdot k - i0]}$$

with $\bar{n} \cdot A = A^+$

$$\tilde{G}^{\mu\nu}_{0,F}(k) = \frac{i}{(k^2 + i0^+)} \left\{ -g^{\mu\nu} + \frac{2(\bar{n}\cdot k)}{\mathbf{k}^2}(k^\mu n^\nu + n^\mu k^\nu) \right\} - i \left[\frac{\theta(\bar{n}\cdot k)}{(n\cdot k + i0)} + \frac{\theta(-\bar{n}\cdot k)}{(n\cdot k - i0)} \right] \frac{(k^\mu n^\nu + n^\mu k^\nu)}{\mathbf{k}^2}$$

$$q(\mathrm{x},\mathbf{b};\mu^2,\zeta)\Big|_{\mathrm{rad.}\infty} = \lim_{Y^+\to+\infty} \mu^{2\epsilon}\alpha_s C_F \int d^{4-2\epsilon}y \, \langle P|\mathcal{T}\,\overline{\psi}_\alpha(0)\,\psi_\beta(y)|P\rangle_c e^{i\mathrm{x}P^-y^+} \int \frac{\mathrm{d}^{2-2\epsilon}\mathbf{l}}{(2\pi)^{2-2\epsilon}} \int \frac{\mathrm{d}l^+}{(2\pi)} e^{il^+y^-}$$
$$\times\; e^{-i\mathbf{l}\cdot(\mathbf{y}+\mathbf{b})} \int \frac{\mathrm{d}^{2-2\epsilon}\mathbf{k}}{(2\pi)^{2-2\epsilon}} \frac{\mathbf{b}^i}{[\mathbf{k}\cdot\mathbf{b}+i0]} \left[e^{i\mathbf{k}\cdot\mathbf{b}}-1\right] \int_0^\infty \frac{\mathrm{d}k^+}{(2\pi)} \frac{1}{[2\mathrm{x}P^-(l^+-k^+)-(\mathbf{l}-\mathbf{k})^2+i0]}$$
$$\times \Bigg\{ \Big[-\mathrm{x}P^-\gamma^-\gamma^+\gamma^i + (\mathbf{l}^j-\mathbf{k}^j)\gamma^-\gamma^j\gamma^i \Big]_{\alpha\beta} \frac{e^{-i\frac{(\mathbf{k}^2-i0)}{2k^+}(Y^+-y^+)}}{2k^+}$$
$$+ \frac{2\mathrm{x}P^-\,\mathbf{k}^i\,\gamma^-_{\alpha\beta}}{\mathbf{k}^2}\left[e^{-i\frac{(\mathbf{k}^2-i0)}{2k^+}(Y^+-y^+)}-1\right]\Bigg\}$$

light cone pole contribution

this is finite at both $k^+\longrightarrow 0$ and $k^+\longrightarrow\infty$ but if we take $Y^+\longrightarrow\infty$ it will be divergent as $k^+\longrightarrow\infty$

need the "pure rapidity regulator": Ebert et al., arXiv:1812.08189

$$q(\mathrm{x},\mathbf{b};\mu^2,\zeta)\Big|_{\mathrm{rad.}\infty} = -\frac{\mu^{2\epsilon}\alpha_s C_F}{2\pi} \int d^{4-2\epsilon}y\,\langle P|\mathcal{T}\,\overline{\psi}(0)\,\gamma^-\psi(y)|P\rangle_c\, e^{i\mathrm{x}P^-y^+} \int \frac{\mathrm{d}^{2-2\epsilon}\mathbf{l}}{(2\pi)^{2-2\epsilon}} \int \frac{\mathrm{d}l^+}{(2\pi)} e^{il^+y^-}\; e^{-i\mathbf{l}\cdot(\mathbf{y}+\mathbf{b})}$$
$$\times \int \frac{\mathrm{d}^{2-2\epsilon}\mathbf{k}}{(2\pi)^{2-2\epsilon}} \frac{\left[e^{i\mathbf{k}\cdot\mathbf{b}}-1\right]}{\mathbf{k}^2} \int_0^\infty \mathrm{d}k^+ \frac{2\mathrm{x}P^-}{[2\mathrm{x}P^-(l^+-k^+)-(\mathbf{l}-\mathbf{k})^2+i0]} \left[\frac{\mathbf{k}^2}{2(k^+)^2}\,\frac{\nu^+}{\nu^-}\right]^{\eta/2}$$

η plays the role of ϵ in dim reg of transverse momentum integrals change of variables $k^+ \longrightarrow z \equiv \frac{2\mathrm{x}P^- k^+}{2\mathrm{x}P^- k^+ + \mathbf{k}^2}$ and expand around $\eta = 0$

$$q(\mathrm{x}, \mathbf{b}; \mu^2, \zeta)\Big|_{\mathrm{rad}.\infty} = -\frac{\mu^{2\epsilon}\alpha_s C_F}{2\pi} \int d^{4-2\epsilon}y \, \langle P \big| \mathcal{T}\, \overline{\psi}(0)\, \gamma^- \psi(y) \big| P \rangle_c \, e^{i\mathrm{x}P^- y^+} \int \frac{\mathrm{d}^{2-2\epsilon}\mathbf{l}}{(2\pi)^{2-2\epsilon}} \int \frac{\mathrm{d}l^+}{(2\pi)} \, e^{il^+ y^-} \, e^{-i\mathbf{l}\cdot(\mathbf{y}+\mathbf{b})}$$
$$\times \int \frac{\mathrm{d}^{2-2\epsilon}\mathbf{k}}{(2\pi)^{2-2\epsilon}} \left[e^{i\mathbf{k}\cdot\mathbf{b}} - 1\right] \left\{ -\frac{1}{\eta} \frac{\zeta^{\eta/2}}{(\mathbf{k}^2)^{1+\frac{\eta}{2}}} + \frac{1}{\mathbf{k}^2} \log\left(\frac{(\mathbf{l}-\mathbf{k})^2 - 2\mathrm{x}P^- l^+ - i0}{\mathbf{k}^2} \right) + O(\eta) \right\}$$

the second term is finite even at $\epsilon = 0$ and does not depend on ζ so it does not contribute to CSS evolution

first term is independent of $l^+, \mathbf{l}$

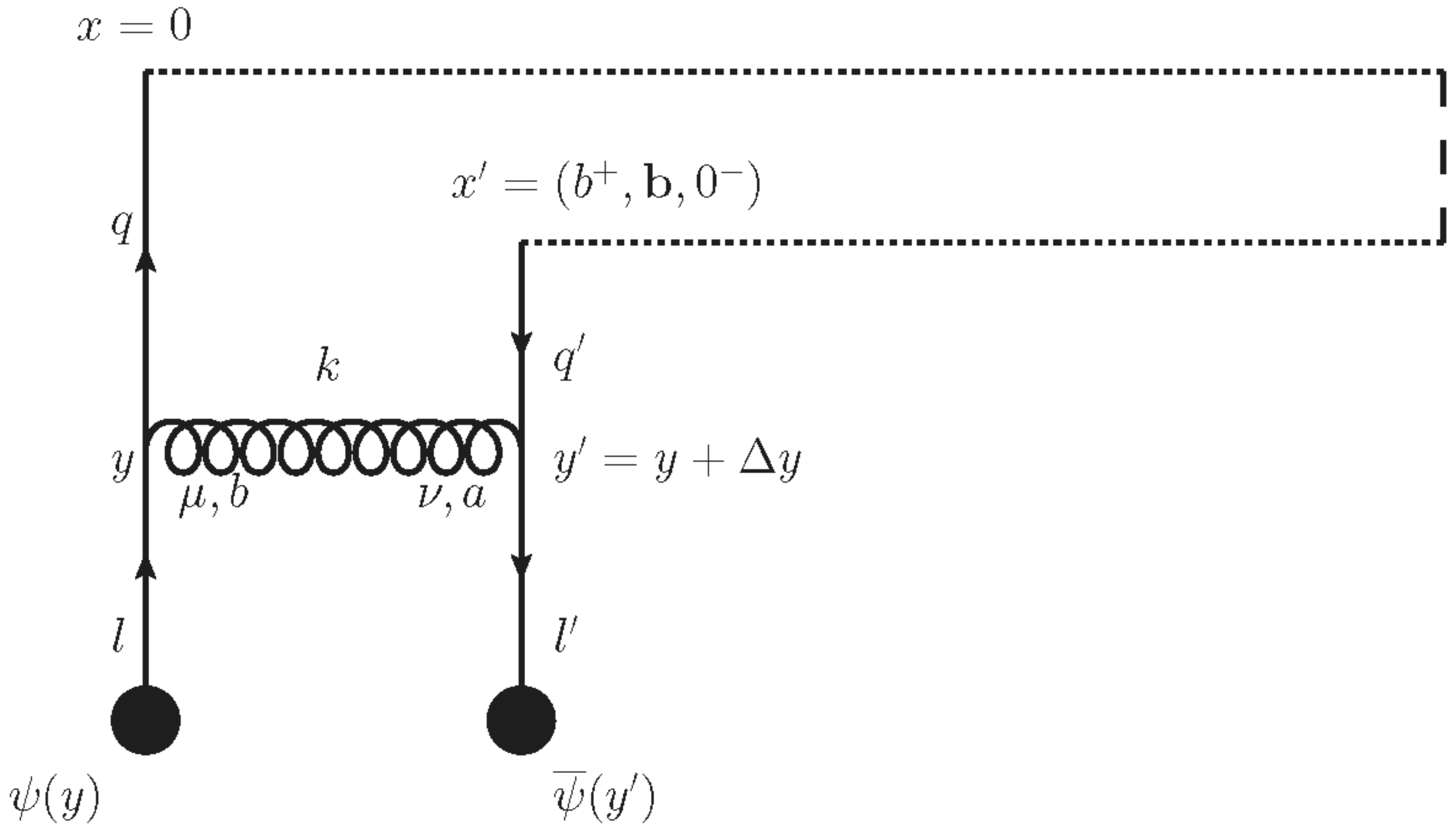
$\mathbf{k}$ integral can be done

$$q(\mathrm{x}, \mathbf{b}; \mu^2, \zeta)\Big|_{\mathrm{rad}.\infty} = \frac{\alpha_s C_F}{4\pi} \Gamma(-\epsilon) \left[\pi \mu^2 \mathbf{b}^2\right]^\epsilon \left[\frac{2}{\eta} + \log\left(\frac{\zeta\, \mathbf{b}^2}{c_0^2} \right) - \Psi(-\epsilon) + \Psi(1) \right] q^{\mathrm{LO}}(\mathrm{x}, \mathbf{b}; \mu^2)$$
$$+ \text{finite NLO} + O(\eta)$$

with $c_0 \equiv 2e^{\Psi(1)} = 2e^{-\gamma_E}$

there is an identical contribution from radiation by the other leg

quark-quark ladder diagram



$$q(\mathrm{x}, \mathbf{b}; \mu^2, \zeta)\big|_{\text{q2q-ladder}} = -g^2\, C_F\, \mu^{2\epsilon} \int \mathrm{d}^{4-2\epsilon}\Delta y \left\langle P \middle| \mathcal{T}\big[\bar{\psi}_\beta(\Delta y)\, \psi_\alpha(0)\big] \middle| P \right\rangle_c$$

$$G_{0,F}^{\nu\mu}(\Delta y, 0) \int \frac{\mathrm{d}b^+}{(2\pi)} e^{-i\mathrm{x}P^- b^+} \int \mathrm{d}^{4-2\epsilon} y \left[\gamma_\nu\, S_F(\Delta y + y; b^+, \mathbf{b}, 0^-)\, \frac{\gamma^-}{2}\, S_F(0, y)\, \gamma_\mu\right]_{\beta\alpha}$$

$$q(\mathrm{x}, \mathbf{b}; \mu^2, \zeta)\big|_{\text{q2q-ladder}} = \frac{\alpha_s C_F}{2\pi}\, \mu^{2\epsilon} \int \mathrm{d}^{4-2\epsilon}\Delta y \left\langle P \middle| \mathcal{T}\big[\bar{\psi}_\beta(\Delta y)\, \psi_\alpha(0)\big] \middle| P \right\rangle_c e^{-i\mathrm{x}P^- \Delta y^+} \int \frac{\mathrm{d}l^+}{2\pi}\, e^{-il^+ \Delta y^-}$$

$$\int \frac{\mathrm{d}^{2-2\epsilon}\mathbf{l}}{(2\pi)^{2-2\epsilon}} e^{i\mathbf{l}\cdot(\Delta\mathbf{y} - \mathbf{b})} \int_0^1 \mathrm{dz} \int \frac{\mathrm{d}^{2-2\epsilon}\mathbf{K}}{(2\pi)^{2-2\epsilon}}\, \frac{e^{i\mathbf{b}\cdot\left[\mathbf{K} + (1-\mathrm{z})\mathbf{l}\right]}}{\left[\mathbf{K}^2 + (1-\mathrm{z})(\mathrm{z}\mathbf{l}^2 - 2\mathrm{x}P^-\mathbf{l}^+) - i0\right]^2}$$

$$\Bigg\{ e^{-i\Delta y^+ \frac{(1-\mathrm{z})}{\mathrm{z}} \mathrm{x}P^-}\, \frac{(1-\mathrm{z})}{\mathrm{z}} \left[2(1-\epsilon_s)\left((\mathrm{x}P^-)^2\gamma^+ + \frac{\gamma^-}{2}\big(\mathbf{K} - \mathrm{z}\mathbf{l}\big)^2\right) - 2\epsilon_s\, \mathrm{x}P^-(\mathbf{K}^i - \mathrm{z}\mathbf{l}^i)\gamma^i\right]$$

$$+ \left[e^{-i\Delta y^+ \frac{(1-\mathrm{z})}{\mathrm{z}} \mathrm{x}P^-} - 1\right] \left[\left[\frac{2}{(1-\mathrm{z})}\Big(\mathbf{K} + (1-\mathrm{z})\mathbf{l}\Big)^2 - 2\mathbf{l}\cdot\Big(\mathbf{K} + (1-\mathrm{z})\mathbf{l}\Big)\right]\gamma^-\right.$$

$$\left. - 2\mathrm{x}P^-\Big(\mathbf{K}^i + (1-\mathrm{z})\mathbf{l}^i\Big)\gamma^i\right]\Bigg\}_{\beta\alpha}$$

light cone pole contribution

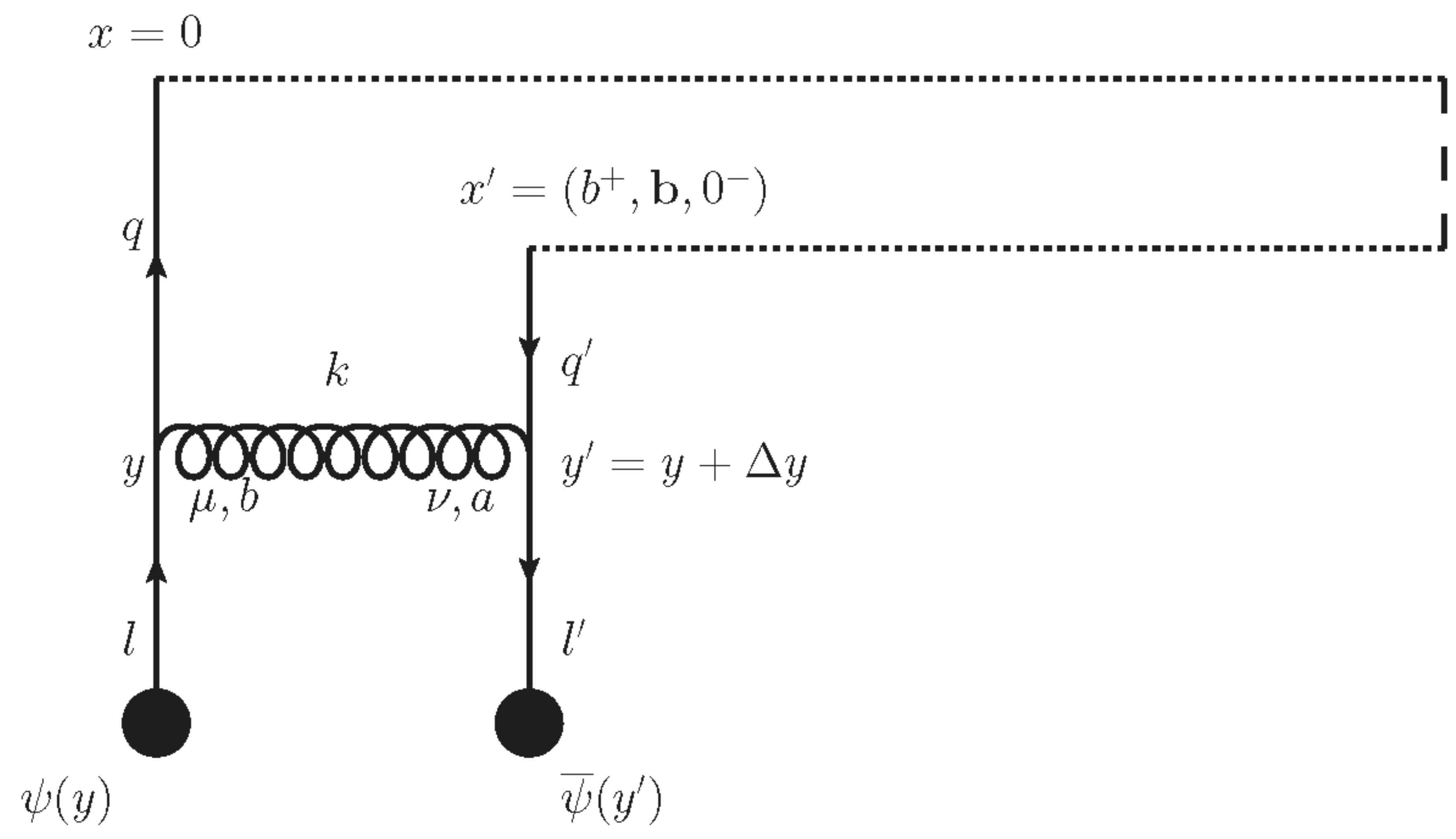
is regular as $z \longrightarrow 1$

quark-quark ladder diagram

the only potential problem is when $\mathbf{K} \longrightarrow 0$ and $z \longrightarrow 1$

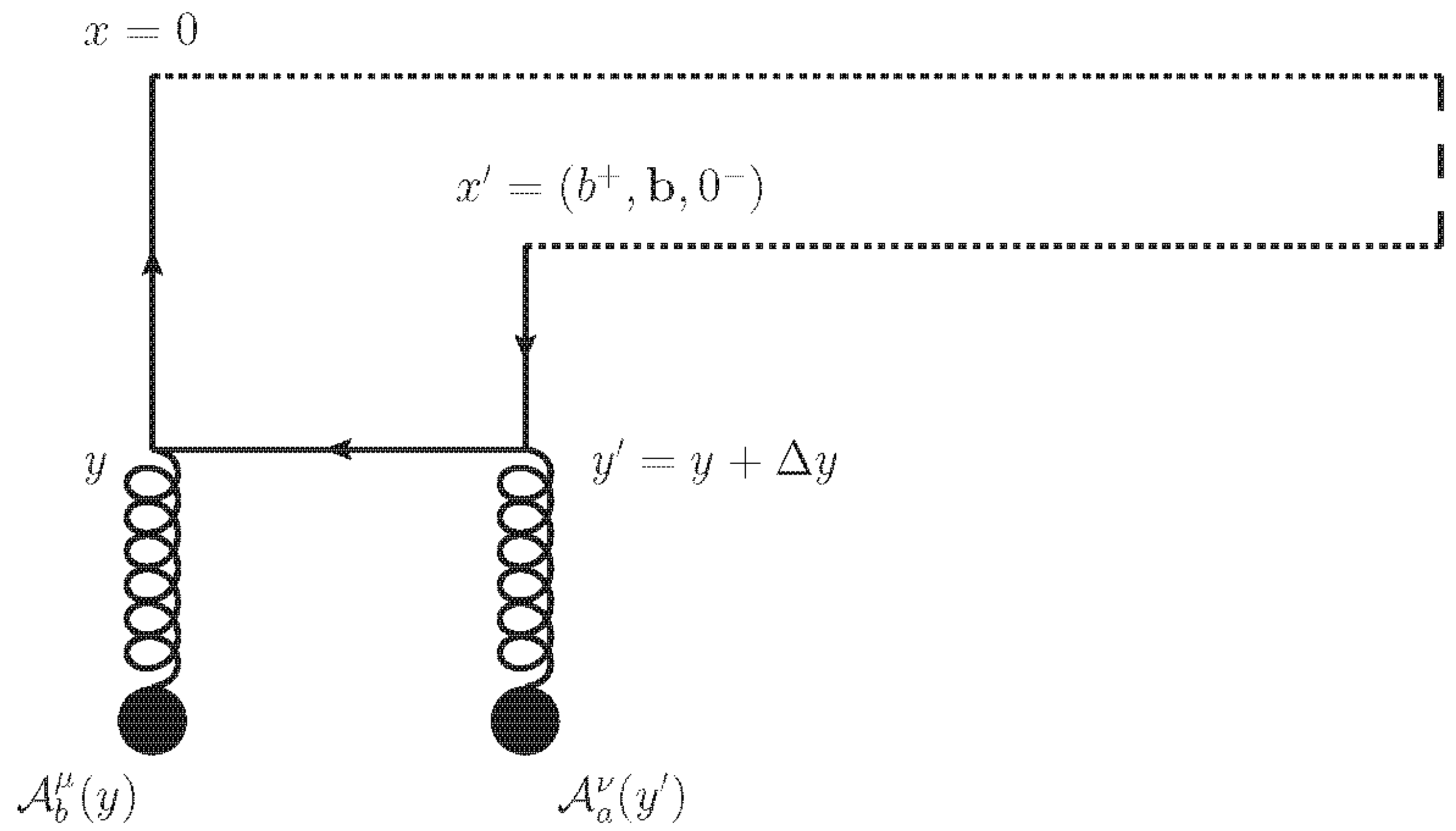
can be shown to be finite

for $\mathbf{b} = 0$ there is a UV divergence (DGLAP)



quark-gluon ladder diagram

similarly can be shown to be finite



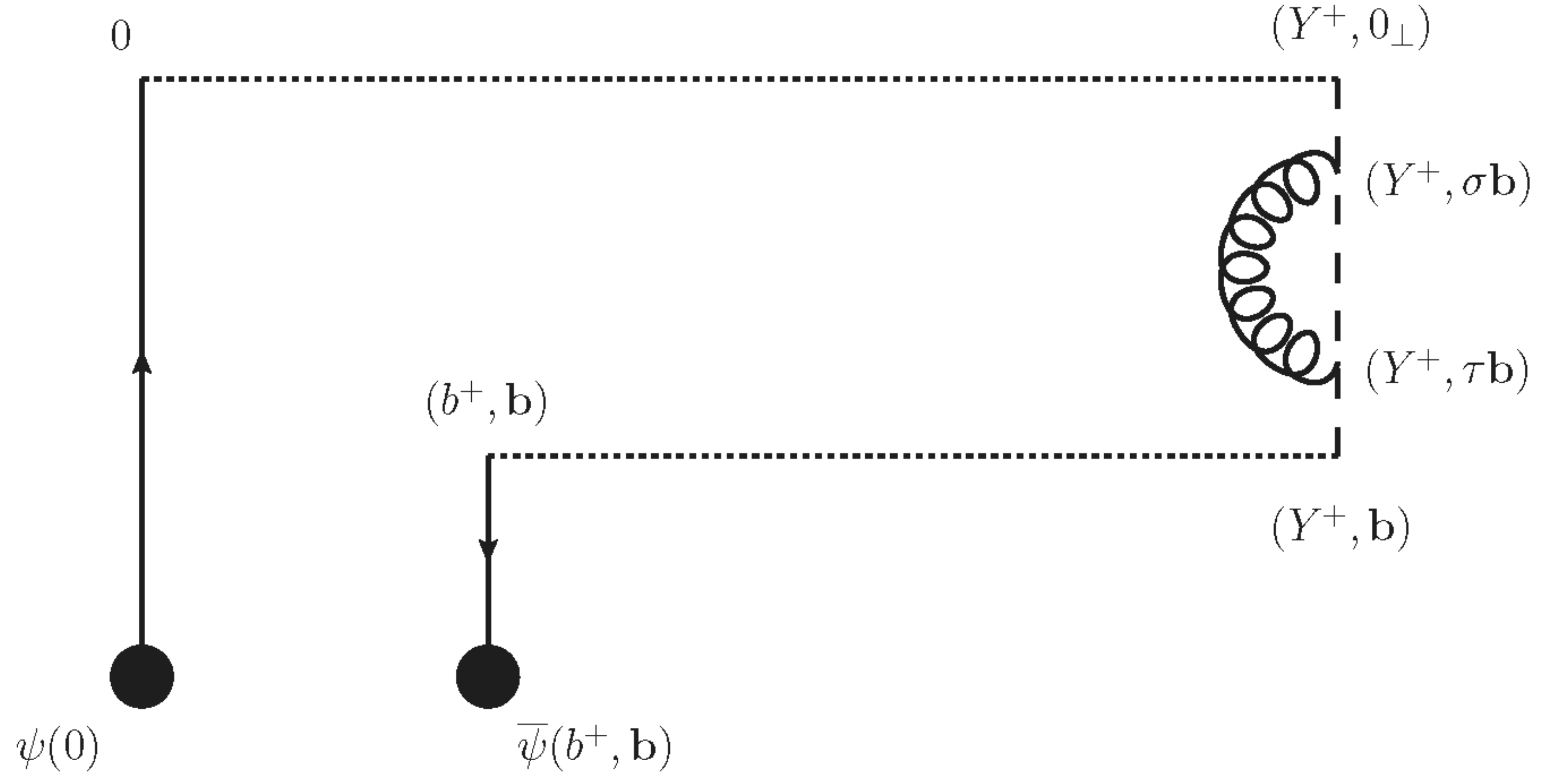
transverse Wilson line self-energy at infinity

$$q(\mathrm{x},\mathbf{b};\mu^2,\zeta) = \int \frac{\mathrm{d}b^+}{2\pi}\, e^{-i\mathrm{x}P^- b^+} \langle P\big|\mathcal{T}\Big[\bar{\psi}(b^+,\mathbf{b},0^-)\frac{\gamma^-}{2}\,\frac{1}{2}\mathcal{P}\left\{-i\mu^\epsilon g\int_0^1 d\tau\,\mathbf{b}^i\,t^a\,\delta A_i^a(Y^+,\tau\mathbf{b},0^-)\right\}^2\,\psi(0)\Big]\big|P\rangle_c$$

$$= -g^2\mu^{2\epsilon}\int_0^1 \mathrm{d}\tau\int_0^\tau \mathrm{d}\sigma\;\mathbf{b}^i\mathbf{b}^j\;t^a t^b\;\delta_{ab}\,G^{ij}_{0,F}(Y^+,\tau\mathbf{b},0^-;Y^+,\sigma\mathbf{b},0^-)$$

involves only the transverse components of the gluon propagator

light cone gauge singularity is not present



contribution of this diagram is then

$$q(\mathrm{x},\mathbf{b};\mu^2,\zeta) = \frac{\alpha_s C_F}{2\pi}\frac{\Gamma(1-\epsilon)}{\epsilon(1-2\epsilon)}\left(\pi\mu^2\mathbf{b}^2\right)^\epsilon q^{\mathrm{LO}}(\mathrm{x},\mathbf{b};\mu^2)$$

contribution of this diagram is cancelled by a soft factor which must be included in the proper definition of the quark TMD

then the full result is

$$q_{\mathrm{unsub.}}(\mathrm{x},\mathbf{b};\mu^2,\zeta) = q^{\mathrm{LO}}(\mathrm{x},\mathbf{b};\mu^2)\left\{1+\frac{\alpha_s C_F}{2\pi}\,\Gamma(-\epsilon)\left[\pi\mu^2\mathbf{b}^2\right]^\epsilon\left[\frac{2}{\eta}+\log\left(\frac{\zeta\mathbf{b}^2}{c_0^2}\right)-\Psi(-\epsilon)+\Psi(1)\right]\right\}$$

$$+\,\text{finite NLO} + O(\alpha_s^2)$$

Renormalization in light cone gauge

bare and renormalized fields/coupling in Lagrangian are related by

$$\Psi^{(0)} = \sqrt{Z_2\,\tilde{Z}_2}\,\left[1-\left(1-\tilde{Z}_2^{-1}\right)\frac{\bar{n}\!\!\!/\,n\!\!\!/}{2\bar{n}\cdot n}\right]\Psi$$

$$A_\mu^{(0)a} = Z_3{}^{\frac{1}{2}}\,\left[A_\mu^a-\left(1-\tilde{Z}_3^{-1}\right)n_\mu\,\Omega^a\right]$$

$$\Lambda^{(0)a} = Z_3{}^{-\frac{1}{2}}\,\Lambda^a$$

$$g_0 = \mu^\epsilon\,Z_3{}^{-\frac{1}{2}}\,g(\mu^2)$$

non-multiplicative renormalization

one-loop QCD: Bassetto, Dalbosco, Soldati, PRD33, 617 (1987)

$$Z_2 = 1+\frac{g^2C_F}{16\pi^2}\,\frac{S_\epsilon}{\epsilon}+O(g^4) = 1+\frac{\alpha_sC_F}{4\pi}\,\frac{S_\epsilon}{\epsilon}+O(\alpha_s^2)$$

$$\tilde{Z}_2 = 1-\frac{g^2C_F}{8\pi^2}\,\frac{S_\epsilon}{\epsilon}+O(g^4) = 1-\frac{\alpha_sC_F}{2\pi}\,\frac{S_\epsilon}{\epsilon}+O(\alpha_s^2)$$

$$\frac{Z_2}{\tilde{Z}_2} = 1+\frac{3\alpha_sC_F}{4\pi}\,\frac{S_\epsilon}{\epsilon}+O(\alpha_s^2) \qquad S_\epsilon\equiv(4\pi e^{-\gamma_E})^\epsilon$$

transverse component of gauge field

$$A_i^{(0)a} = Z_3{}^{\frac{1}{2}}\,A_i^a$$

$$g_0\,A_i^{(0)a} = \mu^\epsilon\,g\,A_i^a$$

<u>good</u>/bad components of Dirac field

$$\frac{\bar{n}\!\!\!/\,n\!\!\!/}{2\bar{n}\cdot n}\Psi^{(0)} = \frac{\sqrt{Z_2}}{\sqrt{\tilde{Z}_2}}\,\frac{\bar{n}\!\!\!/\,n\!\!\!/}{2\bar{n}\cdot n}\Psi$$

$$\frac{n\!\!\!/\,\bar{n}\!\!\!/}{2\bar{n}\cdot n}\Psi^{(0)} = \sqrt{Z_2\,\tilde{Z}_2}\,\frac{n\!\!\!/\,\bar{n}\!\!\!/}{2\bar{n}\cdot n}\Psi$$

toward CSS: putting things together

$$q^{(0)}_{\text{unsub.}}(\mathrm{x}, \mathbf{b}; \zeta) = \frac{Z_2}{\tilde{Z}_2}\, q^{\text{n.r.}}_{\text{unsub.}}(\mathrm{x}, \mathbf{b}; \mu^2, \zeta)$$

bare quark TMD — quark TMD with norm. fields/couplings

rap. div not subtracted — rap. div not subtracted

remove the rapidity divergence

$$q^{\text{n.r.}}_{\text{sub.}}(\mathrm{x}, \mathbf{b}; \mu^2, \zeta) = \lim_{\eta \to 0} Z_{\text{rap.}}\, q^{\text{n.r.}}_{\text{unsub.}}(\mathrm{x}, \mathbf{b}; \mu^2, \zeta)$$

with $Z_{\text{rap.}} = 1 - \dfrac{\alpha_s C_F}{\pi}\, \dfrac{\Gamma(-\epsilon)}{\eta} \left[\pi \mu^2 \mathbf{b}^2\right]^{\epsilon} + O(\alpha_s^2)$

$$q^{\text{n.r.}}_{\text{sub.}}(\mathrm{x}, \mathbf{b}; \mu^2, \zeta) = q^{\text{LO}}(\mathrm{x}, \mathbf{b}; \mu^2) \left\{ 1 + \frac{\alpha_s C_F}{2\pi}\, \Gamma(-\epsilon) \left[\pi \mu^2 \mathbf{b}^2\right]^{\epsilon} \left[\log\left(\frac{\zeta \mathbf{b}^2}{c_0^2}\right) - \Psi(-\epsilon) + \Psi(1) \right] \right\}$$

$$+ \text{finite NLO} + O(\alpha_s^2)$$

this still has UV divergences, remove by Z_{UV} with $Z_{UV} = 1 - \dfrac{\alpha_s C_F}{2\pi} \left[\dfrac{S_\epsilon}{\epsilon^2} + \left(\log\left(\dfrac{\mu^2}{\zeta}\right) + \dfrac{3}{2} \right) \dfrac{S_\epsilon}{\epsilon} \right] + O(\alpha_s^2)$

fully renormalized TMD $q(\mathrm{x}, \mathbf{b}; \mu^2, \zeta) = Z_{UV}\, q^{(0)}_{\text{sub.}}(\mathrm{x}, \mathbf{b}; \zeta) = Z_{UV}\, \dfrac{Z_2}{\tilde{Z}_2}\, q^{\text{n.r.}}_{\text{sub.}}(\mathrm{x}, \mathbf{b}; \mu^2, \zeta)$

bare TMD is independent of μ^2

$$\mu^2 \frac{d}{d\mu^2} \log\Big(q(\mathrm{x}, \mathbf{b}; \mu^2, \zeta) \Big) = \mu^2 \frac{d}{d\mu^2} \log Z_{UV}$$

$\dfrac{Z_2}{\tilde{Z}_2}$ is independent of ζ

$$\zeta \frac{d}{d\zeta} \log q(\mathrm{x}, \mathbf{b}; \mu^2, \zeta) = \zeta \frac{d}{d\zeta} \log Z_{UV} + \zeta \frac{d}{d\zeta} \log q^{\text{n.r.}}_{\text{sub.}}(\mathrm{x}, \mathbf{b}; \mu^2, \zeta)$$

CSS evolution of quark TMD

$$\mu^2 \frac{d}{d\mu^2} q(\mathrm{x}, \mathbf{b}; \mu^2, \zeta) = \left\{ \frac{\alpha_s C_F}{2\pi} \left[\log\left(\frac{\mu^2}{\zeta} \right) + \frac{3}{2} \right] + O(\alpha_s^2) \right\} q(\mathrm{x}, \mathbf{b}; \mu^2, \zeta)$$

$$\zeta \frac{d}{d\zeta} q(\mathrm{x}, \mathbf{b}; \mu^2, \zeta) = \left\{ -\frac{\alpha_s C_F}{2\pi} \log\left(\frac{\mu^2 \mathbf{b}^2}{c_0^2} \right) + O(\alpha_s^2) \right\} q(\mathrm{x}, \mathbf{b}; \mu^2, \zeta)$$

SUMMARY

CSS evolution of quark TMD as renormalization of a non-local composite operator

target light cone gauge using background field to one-loop: CSS evolution

Mandelstam-Leibbrandt prescription is robust

various rapidity regulators used by the practitioners

effect on soft factors?

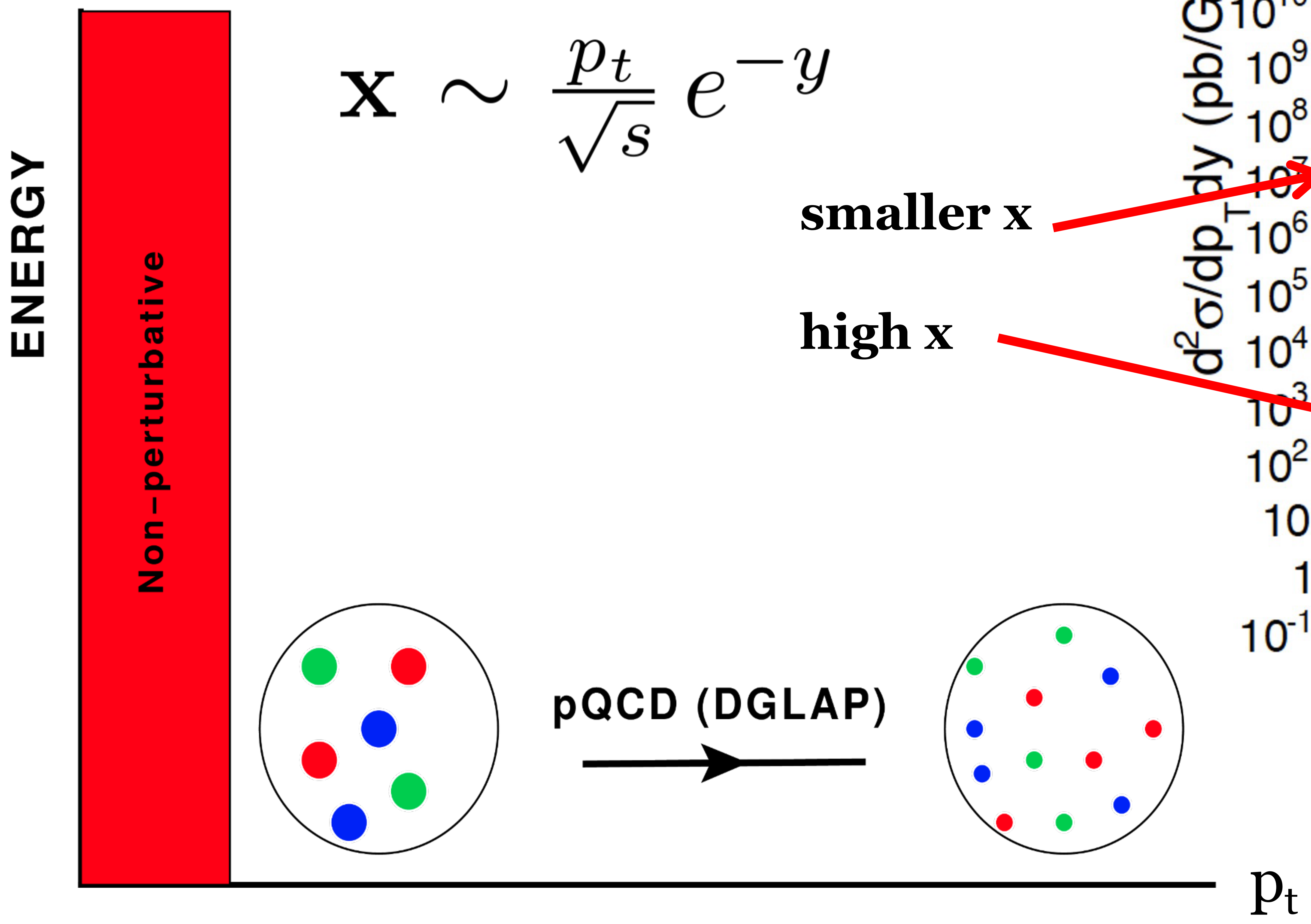
relation to projectile light cone gauge ?

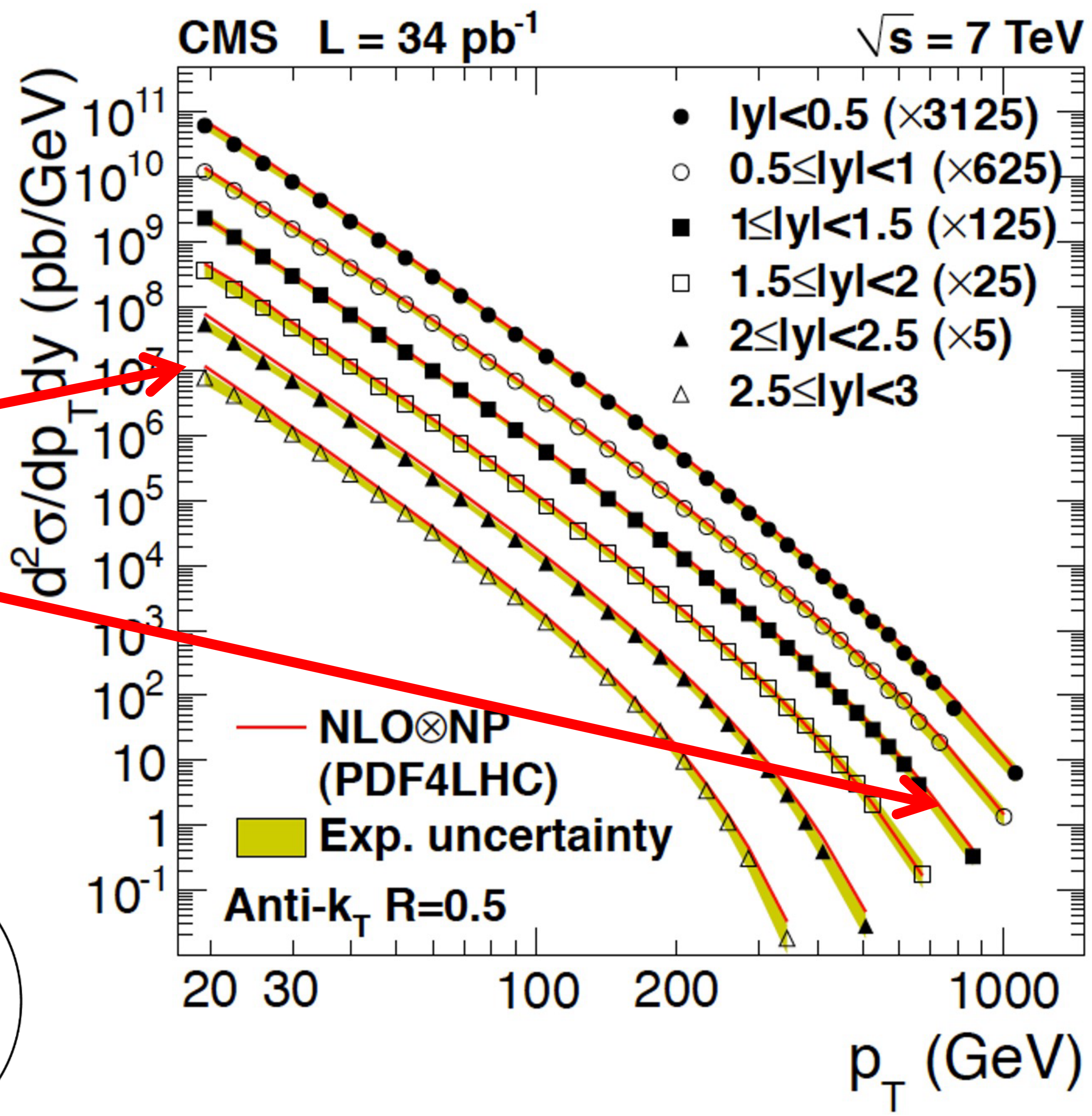
CGC calculations

valuable lessons for constructing a unified formalism of small and large x

pQCD: the standard paradigm

$$\mathbf{x} \sim \frac{p_t}{\sqrt{s}}\, e^{-y}$$





bulk of QCD phenomena happens at low p_t (<u>small x</u>)