Towards Lattice Calculations of Double Parton Distributions

Max Jaarsma, Rudi Rahn, Wouter Waalewijn

University of Amsterdam

m.jaarsma@uva.nl

INT Workshop 2022
september 15

Progress in extracting PDFs and TMDs from lattice calculations

- We propose a matching relation that allows for lattice calculations of double parton distributions (DPDs)

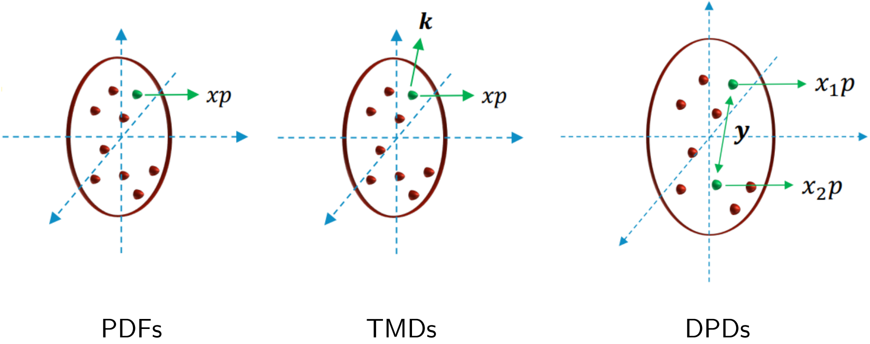
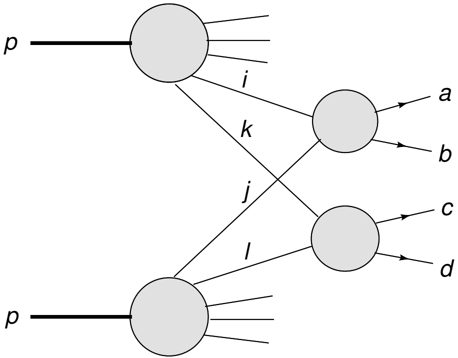


Figure adapted from seminar by J. Gaunt

Motivation - What is double parton scattering?

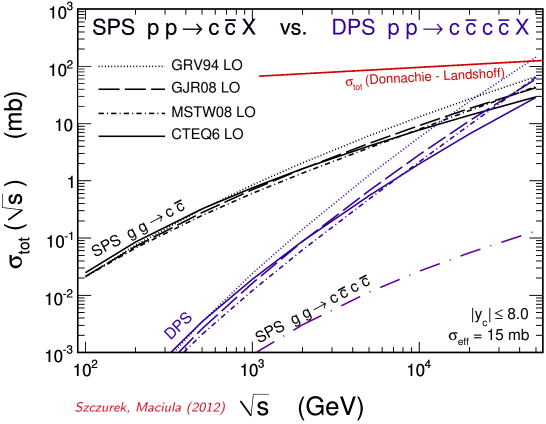
- Two hard scattering partons from each proton



- Higher-twist: DPS is suppressed by $\Lambda^2_{\mathrm{QCD}}/Q^2$ compared to SPS
  - Then why worry about DPS?

(1) Precision: Sizeable contribution in some kinematic regions

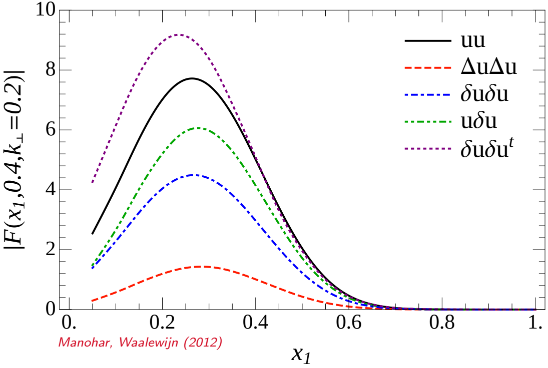
- DPS competes with SPS in production of $c\bar{c}$ pairs as CM energy increases



Szczurek, Maciula (2012)

(2) Curiosity: Understanding the proton

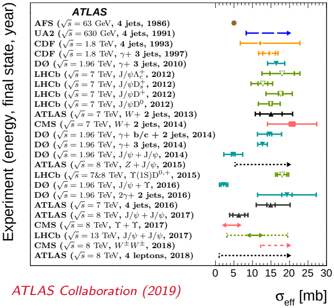
- DPDs probe correlations between partons



Manohar, Waalewijn (2012)

What do we know about the distributions?

- Constraints from experiment
  - Only measurements on σ_{eff} with large disagreements $\rightarrow$ parton correlations
- Model calculations
- First moment on the lattice *Bali, Diehl, Gläßle, Schäfer, Zimmermann (2021)*
  - Only the lowest few moments are accessible on the lattice



ATLAS Collaboration (2019)

Motivation for extending LaMET to DPDs

Although DPDs play a significant role at the LHC, not much is known about them

Outline

- Lightspeed introduction to double parton distributions
  - Factorization and definitions
- Lightcone correlators on the lattice
  - Quick summary of the quasi-PDF approach
  - Extension to TMDs
- Extending LaMET to double parton distributions
  - What do we need?
  - Conjectured matching relation + 1-loop result
- Outlook: how do we proceed from here?

Conclusion

LaMET can (with some effort) be applied to double parton distributions

Lightspeed introducion to double parton scattering

Let's get formal - Factorization

- Cross section of DPS process factorizes as Hard $\otimes$ DPDs $\otimes$ Soft.

$$
\begin{aligned}
\mathrm{d}\sigma^{\mathsf{DPS}} = \left(\frac{4\pi\alpha^2 Q_q^2}{3N_c s}\right)\frac{1}{q_1^2 q_2^2}\int \mathrm{d}^2\mathbf{b}_\perp \\
\times \Big\{ \left[{}^1F_{qq}\,{}^1F_{\bar{q}\bar{q}} + {}^1F_{\Delta q\Delta q}\,{}^1F_{\Delta\bar{q}\Delta\bar{q}} + {}^1F_{q\bar{q}}\,{}^1F_{\bar{q}q} + {}^1F_{\Delta q\Delta\bar{q}}\,{}^1F_{\Delta\bar{q}\Delta q}\right]{}^{11}S \\
+ \tfrac{2N_c}{C_F}\left[{}^8F_{qq}\,{}^8F_{\bar{q}\bar{q}} + {}^8F_{\Delta q\Delta q}\,{}^8F_{\Delta\bar{q}\Delta\bar{q}} + {}^8F_{q\bar{q}}\,{}^8F_{\bar{q}q} + {}^8F_{\Delta q\Delta\bar{q}}\,{}^8F_{\Delta\bar{q}\Delta q}\right]{}^{88}S \\
+ \text{interference terms}\Big\}
\end{aligned}
$$

- Many different color and spin structures

Manohar, Waalewijn (2012)
Gaunt (2014)
Diehl, Gaunt, Ostermeier, Ploessl, Schäfer (2015)

Let's get formal - Definitions

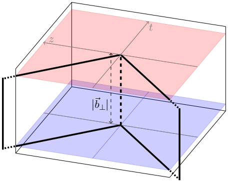
- DPDs can be expressed as hadronic lightcone correlators. For F_{qq}:

$$
\begin{aligned}
{}^{R}F_{a_1 a_2} = -\pi P^+ \int \frac{db_1^-}{2\pi}\frac{db_2^-}{2\pi}\frac{db_3^-}{2\pi}\; & e^{-ix_1 P^+ b_1^-} e^{-ix_2 P^+ b_2^-} e^{ix_1 P^+ b_3^-} \\
& \times \langle P|\, T^\dagger \Big[\bar\psi_n(0^+, b_1^-, \mathbf{b}_\perp)\Gamma_{a_1} R_1\Big]_i \Big[\bar\psi_n(b_2^-)\Gamma_{a_2} R_2\Big]_j \\
& \times T\Big[\psi_n(0^+, b_3^-, \mathbf{b}_\perp)\Big]_i \Big[\psi_n(0)\Big]_j \,|P\rangle
\end{aligned}
$$

  - different spin and color structures allowed
- Soft factors can be written as vacuum matrix elements of Wilson loops

$$
{}^{11}S = 1\,, \qquad {}^{88}S = \frac{1}{2N_c C_F}\,\langle 0|\, \mathrm{tr}\big[\mathcal{S}\big]\mathrm{tr}\big[\mathcal{S}^\dagger\big]\,|0\rangle - \frac{1}{2N_c C_F}
$$

$\mathcal{S}$:



Diehl, Ostermeier, Schäfer (2011)
Manohar, Waalewijn (2012)
Diehl, Nagar (2019)

Lightcone correlators on the lattice

Summary of quasi-PDF approach

- Lightcone-PDF is defined as hadronic lightcone correlator
  - Cannot be calculated on the lattice directly
- Quasi-PDF has same definition, but with space-like separated fields
  - Can be calculated on the lattice
- Boosting the quasi-PDF takes you closer to lightcone-PDF
- The two differ by an order of non-commuting limits
  - $P^z \to \infty$ and $\Lambda \to \infty$
- Asymptotic freedom of QCD guarantees that difference in order of limits is perturbative
- Quasi-PDFs can be matched perturbatively onto physical lightcone-PDFs

Case study: TMDs

- Rapidity divergences: regularize and subtract
  - Physical and quasi-TMD defined in terms of beam and soft functions

$$f = \frac{B}{\sqrt{S}}\,, \qquad \tilde{f} = \frac{\tilde{B}}{\sqrt{\tilde{S}}}$$

- Rapidity scale dependence
  - Collins-Soper evolution

$$\frac{\mathrm{d}}{\mathrm{d}\log\zeta} f(x, b_\perp, \mu, \zeta) = \gamma_\zeta(b_\perp, \mu) f(x, b_\perp, \mu, \zeta)$$

- Rapidity scale dependence enters matching relation

$$\tilde{f}(x, b_\perp, \mu, \tilde{\zeta}, x\tilde{P}^z) = C\big(x\tilde{P}^z, \mu\big)\ \exp\left[\frac{1}{2}\gamma_\zeta(\mu, b_\perp)\log\left(\frac{\tilde{\zeta}}{\zeta}\right)\right] f(x, b_\perp, \mu, \zeta)$$

Ji, Liu, Liu (2020)
Ebert, Schindler, Stewart, Zhao (2022)

Applying LaMET to double parton distributions

What do we need?

- (1) Lattice calculable ingredients
  - Replace lightcone correlators with equal-time correlators
- (2) Factorization formula relating quasi- and lightcone-DPDs
  - With TMD case as starting point
- (3) Perturbative matching kernel
  - Consistency check: does IR behaviour match up?

(1) Lattice calculable ingredients

- Define quasi-DPD

$$
{}^{R}\tilde{F}_{a_1 a_2} = -\pi P^+ \int \frac{db_1^z}{2\pi}\frac{db_2^z}{2\pi}\frac{db_3^z}{2\pi}\; e^{ix_1 P^z b_1^z} e^{ix_2 P^z b_2^z} e^{-ix_1 P^z b_3^z}
\times \langle P|\, T^\dagger \Big[\bar{\psi}_z(0,\mathbf{b}_\perp, b_1^z)\tilde{\Gamma}_{a_1} R_1\Big]_i \Big[\bar{\psi}_z(b_2^z)\tilde{\Gamma}_{a_2} R_2\Big]_j
\times T\Big[\psi_n(0,\mathbf{b}_\perp, b_3^z)\Big]_i \Big[\psi_n(0)\Big]_j \,|P\rangle
$$

- Define quasi DPS soft function

$$
{}^{88}\tilde{S} = \frac{1}{2N_c C_F}\,\langle 0|\, \mathrm{tr}\big[\tilde{\mathcal{S}}\big]\mathrm{tr}\big[\tilde{\mathcal{S}}^\dagger\big]\,|0\rangle - \frac{1}{2N_c C_F}
$$

  - where $\tilde{\mathcal{S}}$ is the same Wilson loop as for the quasi-TMD
  - Two sets of oppositely directed staples $\rightarrow$ not straightforward for lattice

(2) Matching relation - where to start?

Ultraviolet and large rapidity behaviour are key

Matching relation encodes the difference in the UV and large rapidity behaviour

- How do DPDs behave in the ultraviolet regime?
  - Convolution in momentum fractions
  - Mixing between color and spin structures
  - Mixing between parton flavors
  - Mixing with single PDFs ($F_{q\bar{q}}$ mixes with f_g)

Manohar, Waalewijn (2012)
Diehl, Nagar (2014)
Diehl, Gaunt (2016)
Diehl, Gaunt, Ploessl, Schäfer (2019)
Diehl, Gaunt, Ploessl (2021)

- What is the large rapidity behaviour of DPDs?
  - Regulate rapidity divergences using your favourite regulator and subtract $\sqrt{S}$
  - Rapidity scale dependence described by Collins-Soper kernel

$$\frac{d}{d\log\zeta}\,{}^{R}F^{\text{sub}}_{a_1 a_2} = \tfrac{1}{2}\,{}^{R}\gamma_\zeta(b_\perp,\mu)\,{}^{R}F^{\text{sub}}_{a_1 a_2} \qquad\qquad \zeta = 4x_1x_2(P^+)^2$$

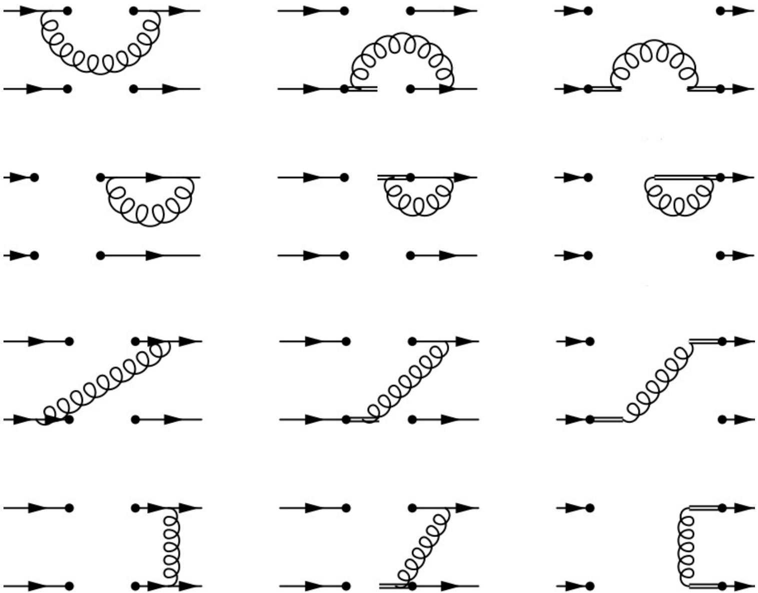
(2) Matching relation - educated guess

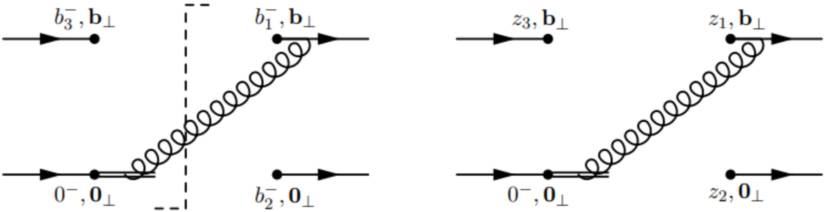
Conjecture: $\tilde{F}$ = perturbative kernel $\otimes$ rapidity evolution $\otimes$ F

$$
\begin{aligned}
&{}^{R}\tilde{F}^{\text{sub}}_{a_1 a_2}(x_1, x_2, b_\perp, \mu, \tilde{\zeta}), P^z \\
&= \int \frac{\mathrm{d}y_1}{y_1}\frac{\mathrm{d}y_2}{y_2}\, {}^{RR'}C_{a_1 a_2, a_1' a_2'}\left(\frac{x_1}{y_1}, \frac{x_2}{y_2}, \frac{(x_1 P^z)^2}{\mu^2}, \frac{(x_2 P^z)^2}{\mu^2}, \frac{\tilde{\zeta}}{\mu^2}\right) \\
&\qquad \times \exp\left[\frac{1}{2}\, {}^{R'}\gamma_\zeta(b_\perp, \mu) \log\left(\frac{\tilde{\zeta}}{\zeta}\right)\right] {}^{R'}F^{\text{sub}}_{a_1' a_2'}(x_1, x_2, b_\perp, \mu, \zeta) \\
&\quad + \text{mixing between flavors} \\
&\quad + \text{mixing with single PDFs} \\
&\quad + \mathcal{O}\left(\frac{1}{x_{1,2} b_\perp P^z}, \frac{\Lambda^2_{\text{QCD}}}{(x_{1,2}P^z)^2}\right)
\end{aligned}
$$

Consistency check: IR agreement

Matching kernel must be free of infrared logarithms ($\log(b_\perp \mu)$)





- Take the difference between lightcone- and quasi- diagrams

$$\Delta^{\text{example}} = -4\pi\delta(1-x_1)\delta(1-x_2)\left[\frac{1}{\epsilon_{\text{ir}}} + \log\left(\frac{\mu^2\mathbf{b}_\perp^2}{b_0^2}\right)\right]\left[2 + \log\left(\frac{\delta^2}{p_1^+ p_2^+}\right)\right]$$
$$+ \text{ other terms}$$

- Infrared poles and logs spotted!
  - These drop out when soft factor subtraction is performed and all diagrams are combined

What did we find?

- No mixing of color and spin structures at one-loop
  - Generalisation to higher orders expected, but not proven

- Matching kernel of color-summed DPD greatly simplifies

$$ {}^{1}C_{a_1 a_2}\left(\frac{x_1}{y_1}, \frac{x_2}{y_2}, \frac{\mu}{|y_1|P^z}, \frac{\mu}{|y_2|P^z}\right) \overset{?}{=} \mathcal{C}_{a_1}\left(\frac{x_1}{y_1}, \frac{\mu}{|y_1|P^z}\right)\mathcal{C}_{a_2}\left(\frac{x_2}{y_2}, \frac{\mu}{|y_2|P^z}\right) $$

  - True to higher orders? Maybe OPE can tell

- Color-correlated kernel also simplifies

$$ \begin{aligned} {}^{8}C^{(1)}_{a_1 a_2} &= \left(1 - \frac{N}{2C_F}\right){}^{1}C^{(1)}_{a_1 a_2} + \delta\big(1 - \tfrac{x_1}{y_1}\big)\delta\big(1 - \tfrac{x_2}{y_2}\big) \\ &\times N_c\left[2\log\left(\frac{\tilde{\zeta}}{\mu^2}\right) - \tfrac{1}{2}\log^2\left(\frac{(2y_1P^z)^2}{\mu^2}\right) - \tfrac{1}{2}\log^2\left(\frac{(2y_2P^z)^2}{\mu^2}\right) - \frac{5}{2} + \frac{\pi^2}{6}\right] \end{aligned} $$

  - Generalisation to higher orders?

But most importantly . . .

No infrared logs at one-loop

Perturbative nature of matching kernel consistent with one-loop result

How do we proceed from here?

- Proof of factorization
  - Proof on operator level is desired, but formalism not available
- Lattice calculable soft function
  - Difficulties in constructing lattice calculable soft function (two opposite light-like staples)
  - Relate DPS reduced soft function to meson form factor?
- Putting it on the lattice
  - Mixing and renormalization
- Include mixing in the matching
  - Mixing with flavors and mixing with single PDFs

Conclusions

Conclusion

Achievement unlocked: formulating DPDs on the lattice

Successfully applied LaMET to DPDs, paving the way for lattice calculations of double parton distributions.

- Defined a lattice calculable quasi-DPD
- Conjectured a matching relation
- One-loop consistency check: no infrared logs
- Some interesting findings:
  - No mixing between color- and spin structures
  - Relation to single-PDF matching kernel
  - Relation between color-summed/correlated kernels

Thank you for your attention!

Backup slides

- Color-summed DPD ${}^{1}F_{qq}$:

$$
\begin{aligned}
{}^{1}F_{qq} = -\pi P^{+} \int \frac{db_1^-}{2\pi}\frac{db_2^-}{2\pi}\frac{db_3^-}{2\pi}\; e^{-ix_1P^+b_1^-}e^{-ix_2P^+b_2^-}e^{ix_1P^+b_3^-} \\
\times \langle P|\, T^{\dagger}\Big[\bar{\psi}(0^+, b_1^-, \mathbf{b}_\perp)\gamma^+ W[b_1 \leftarrow b_3]\Big]_i \Big[\bar{\psi}(b_2^-)\gamma^+ W[b_2 \leftarrow 0]\Big]_j \\
\times T\Big[\psi(0^+, b_3^-, \mathbf{b}_\perp)\Big]_i \Big[\psi(0)\Big]_j \,|P\rangle
\end{aligned}
$$

- Color-correlated DPD ${}^{8}F_{qq}$:

$$
\begin{aligned}
{}^{8}\tilde{F}_{qq} = -\frac{T_F}{N}\,{}^{1}\tilde{F}_{a_1a_2} - \pi P^{+} \int \frac{b_1^-}{2\pi}\frac{b_2^-}{2\pi}\frac{b_3^-}{2\pi}\; e^{-ix_1P^+b_1^-}e^{-ix_2P^+b_2^-}e^{ix_1P^+b_3^-} \\
\times \langle P|\, \Big[\bar{\psi}(b_1)\mathcal{W}_{\sqsupset}\big[b_1 \leftarrow 0\big]\gamma^+\Big]_{i\alpha} \Big[\bar{\psi}(b_2)\mathcal{W}_{\sqsupset}\big[b_2 \leftarrow b_3\big]\gamma^+\Big]_{j\beta} \\
\times \psi_{j\alpha}(b_3)\psi_{i\beta}(0)\,|P\rangle\,,
\end{aligned}
$$

Mixing with single-PDFs