Analytic continuation of the relativistic three-particle scattering amplitudes

Part I

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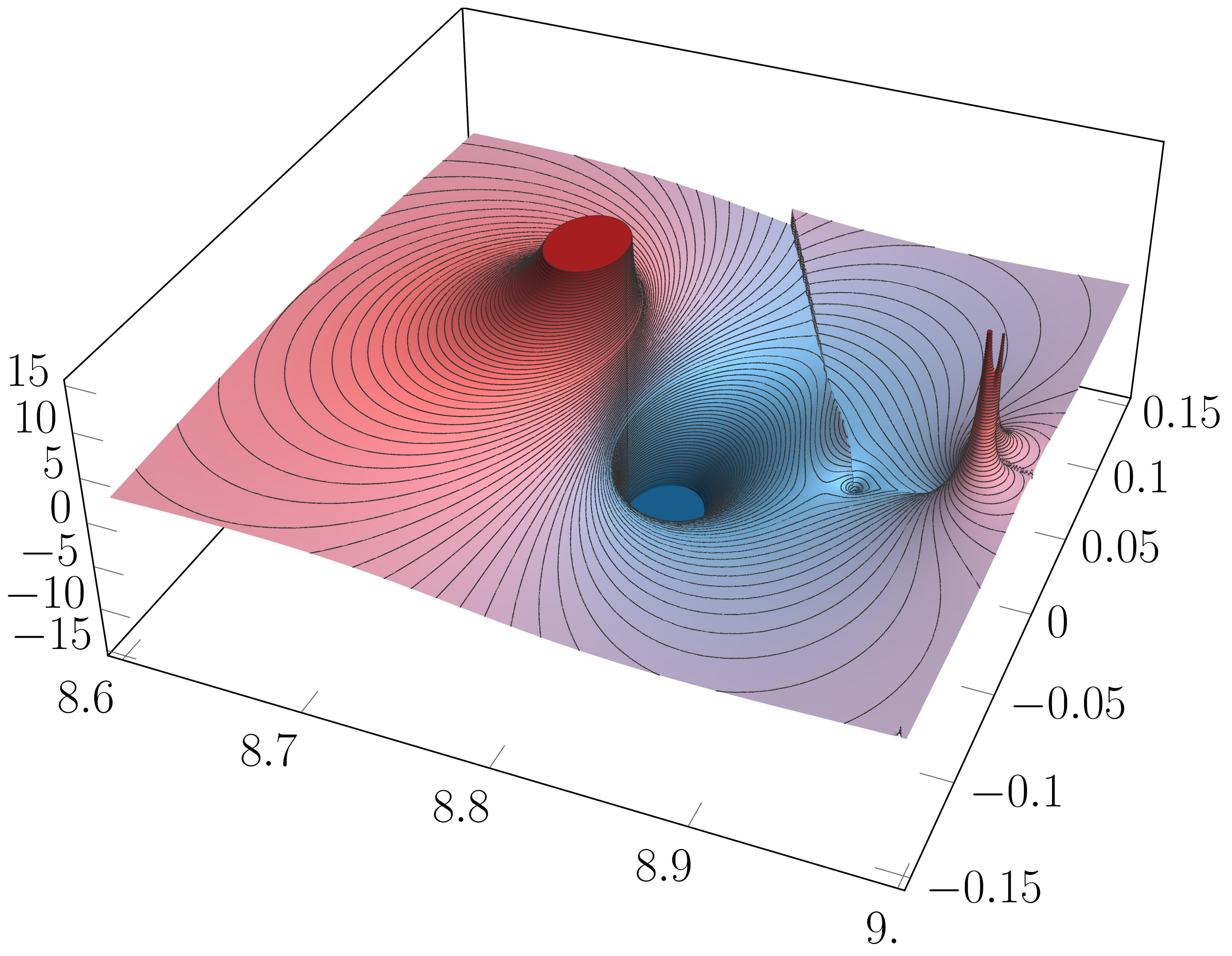
With Sebastian M. Dawid (U Washington) and Raúl Briceño (UC Berkeley)

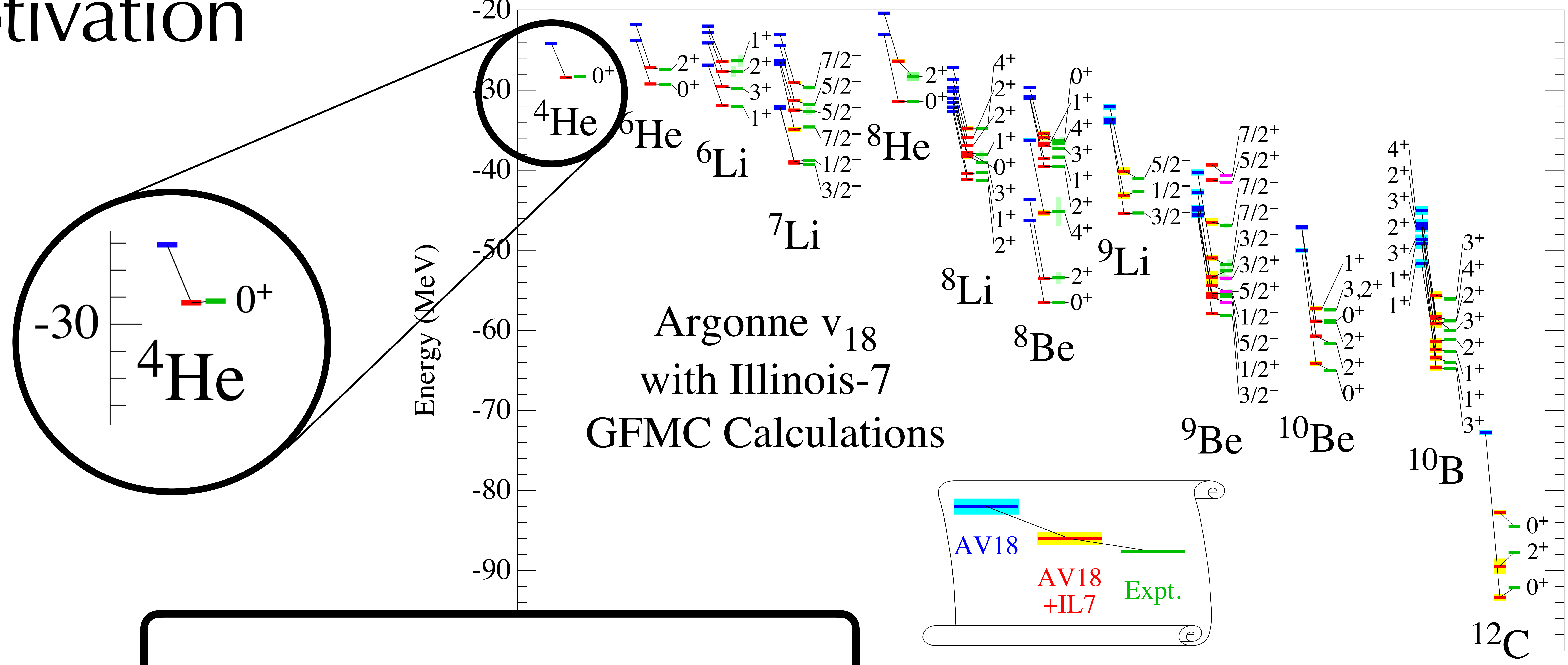


OLD DOMINION UNIVERSITY

Jefferson Lab

Motivation

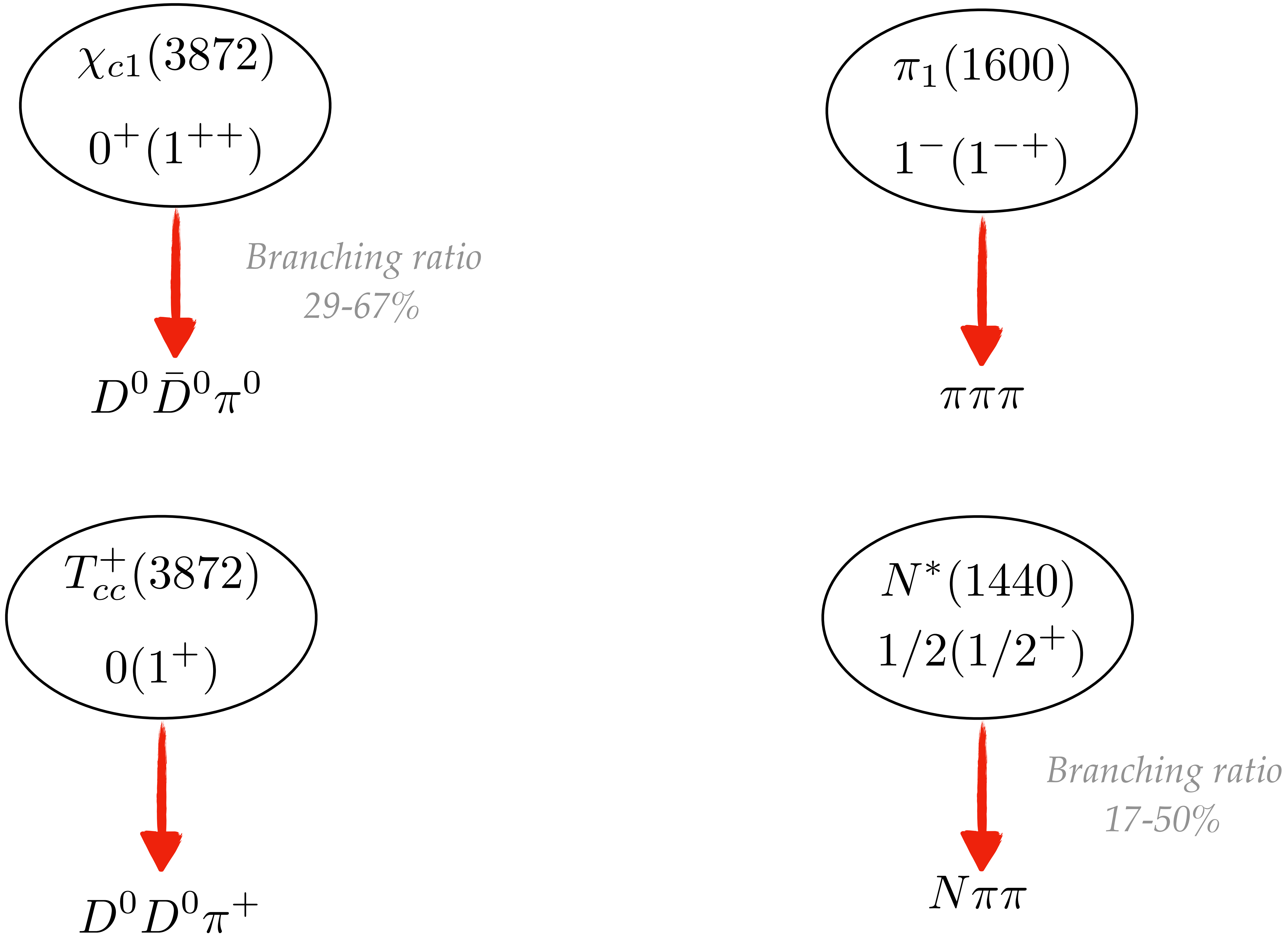
Carlson, Gandolfi, Pederiva, Pieper, Schiavilla, Schmidt, Wiringa, 2015



This model shows contributions of the three body forces in formation of stable nucleus is about 15-20%.

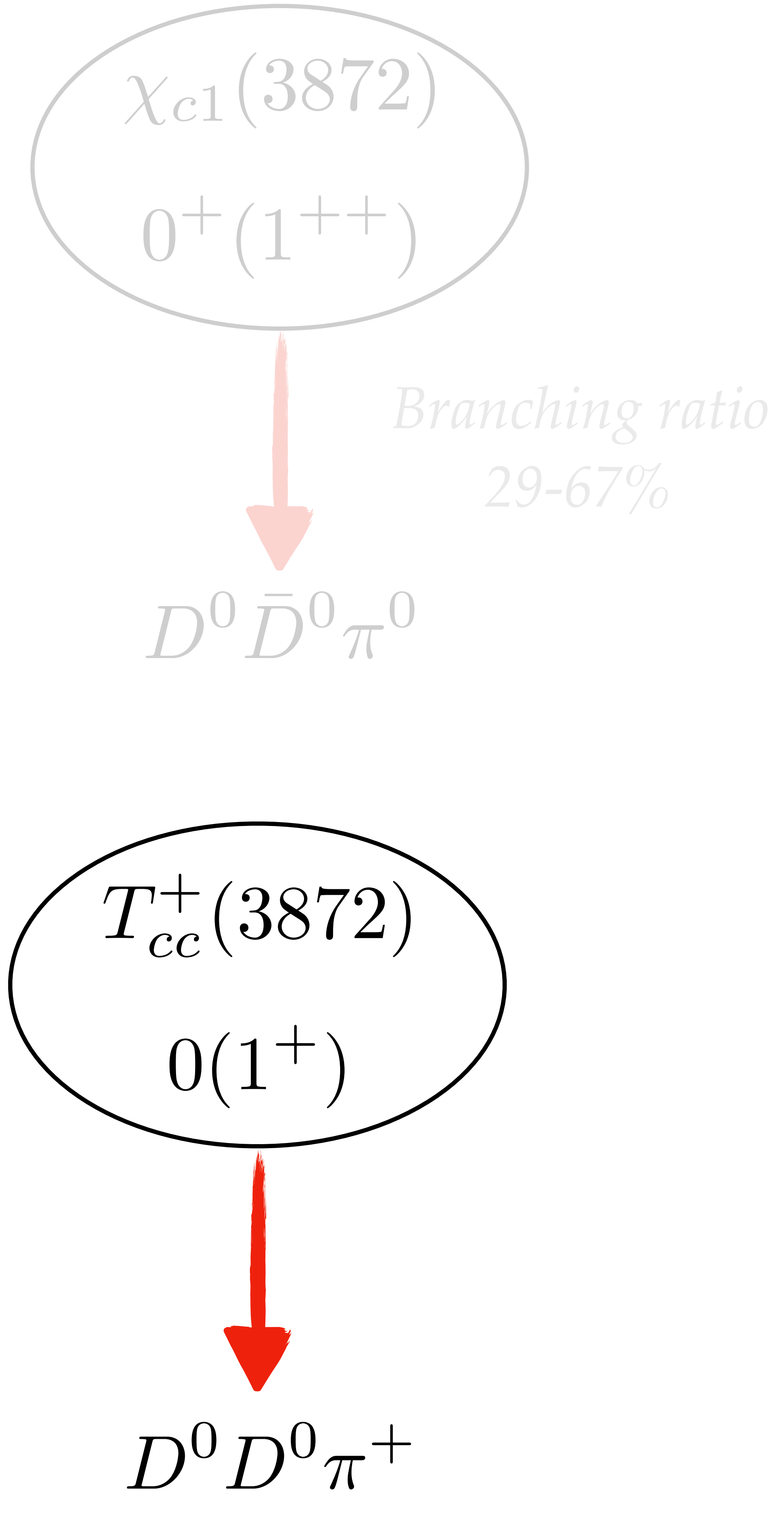
Motivation

Interesting resonances decay to three particle final states

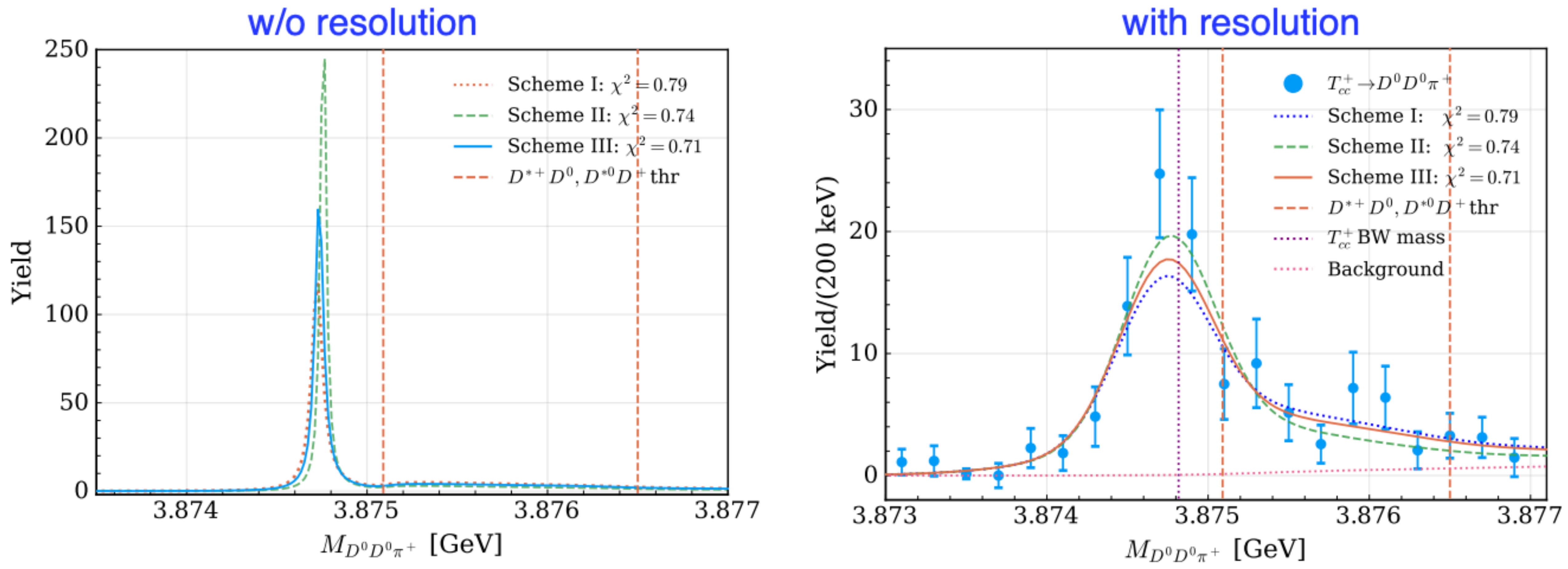


Motivation

Interesting resonances decay to three particle final states



RESULTS AND DISCUSSION



Scheme	I	II	III
Description	2-body unitarity: No OPE, static D^* width	Incomplete 3-body unitarity: No OPE, dynamical D^* width	full 3-body unitarity: OPE + dynamical D^* width
Pole [keV]	$-368^{+43}_{-42} - i(37\pm 0)$	$-333^{+41}_{-36} - i(18\pm 1)$	$-356^{+39}_{-38} - i(28\pm 1)$
χ^2	0.79	0.74	0.71

- Precision needs 3 body dynamics (problem: experimental resolution)
- $r = -2.4 \pm 0.01 \pm 0.85$ fm, but $r_0 = +1.38 \pm 0.01 \pm 0.85$ fm

$\Longrightarrow$ T^+_{cc} qualifies as isoscalar DD^* molecule

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JÜLICH Forschungszentrum

The Recipe

Hansen and Sharpe, 2014, 2015
Mai and Döring, 2017
Briceño, Hansen and Sharpe, 2018
Hansen, Romero-Lopéz and Sharpe, 2020
Blanton and Sharpe, 2020
Jackura, 2022

Finite volume spectroscopy

Infinite-volume three-body K-matrix

Infinite-volume scattering amplitude

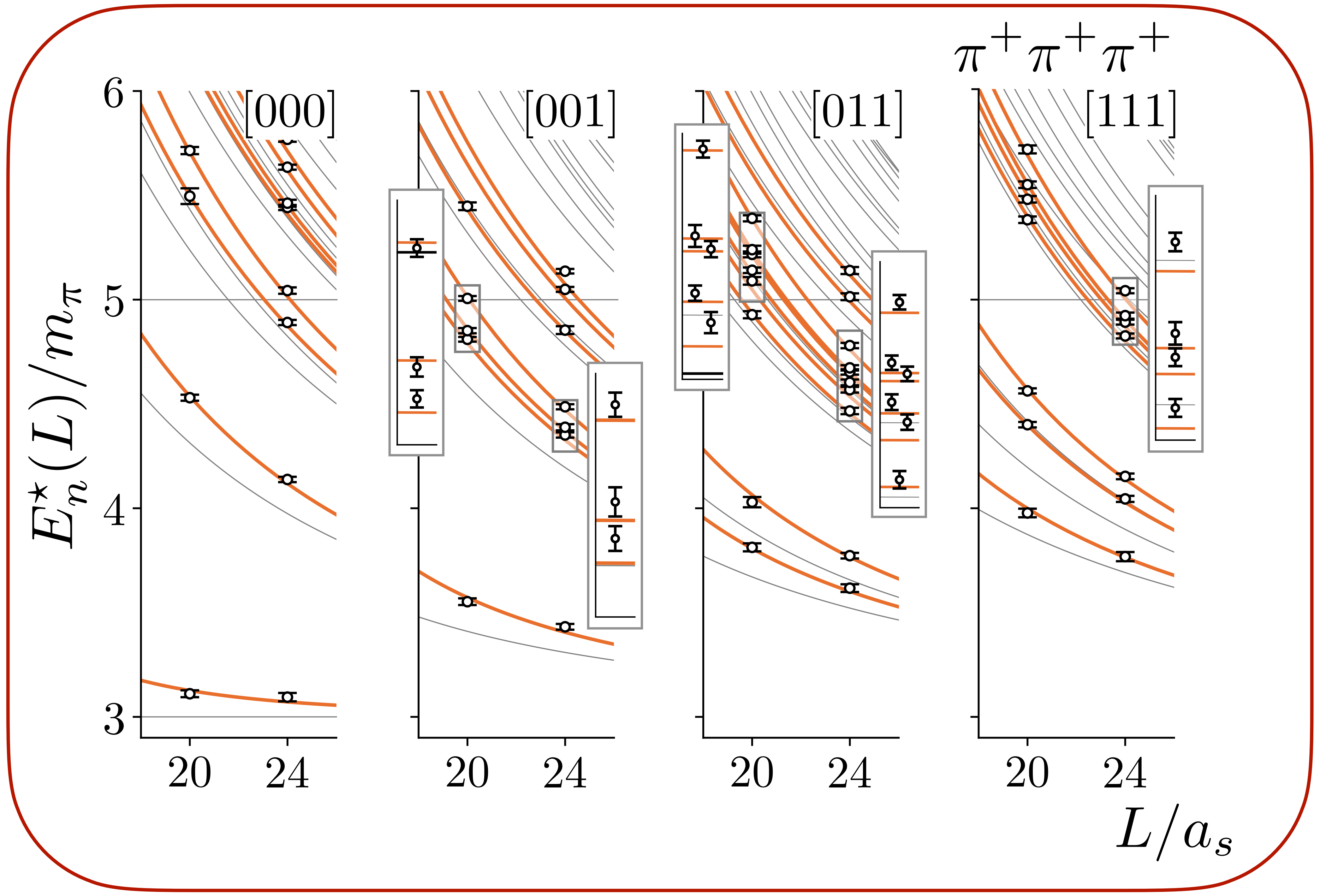
Bound-states and resonances

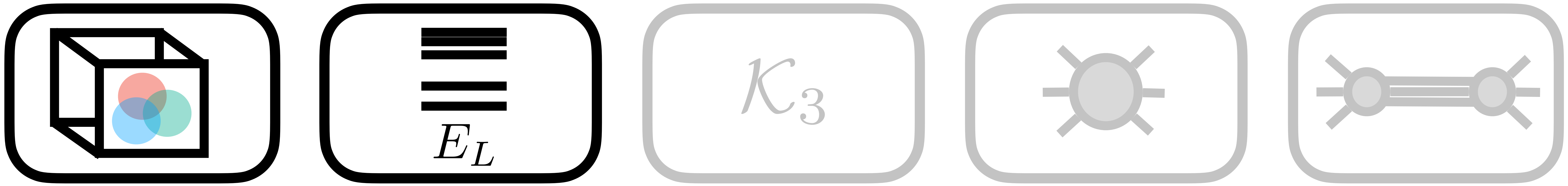
The Recipe

Hansen and Sharpe, 2014, 2015
Mai and Döring, 2017
Briceño, Hansen and Sharpe, 2018
Hansen, Romero-Lopéz and Sharpe, 2020
Blanton and Sharpe, 2020
Jackura, 2022

Hansen, Briceño, Edwards, Thomas and Wilson, 2020

Application of “The Recipe” *For mesonic states*





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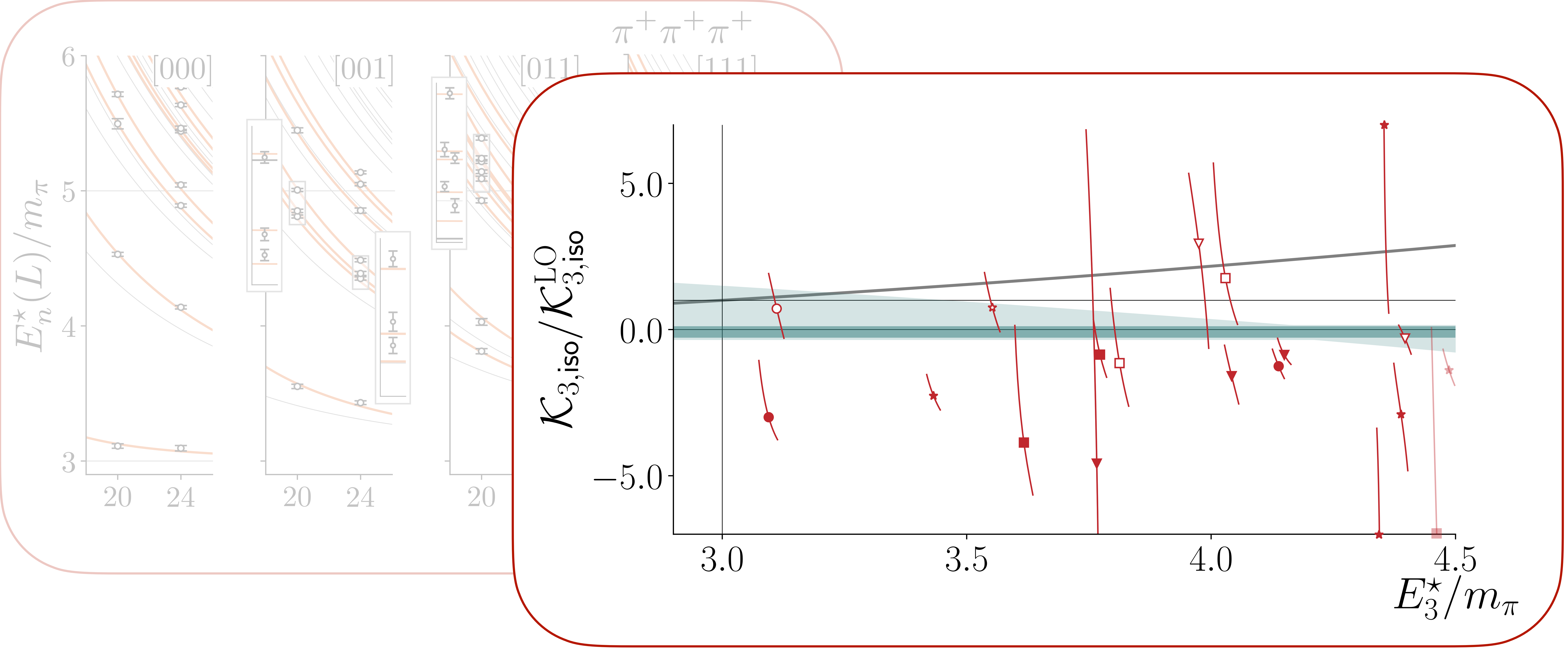
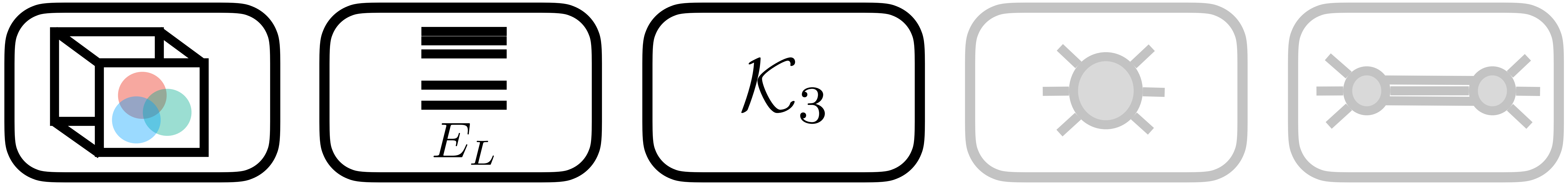
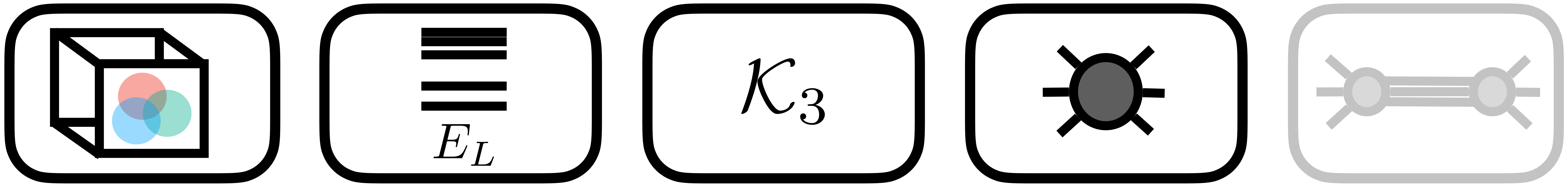
Hansen, Briceño, Edwards, Thomas and Wilson, 2020





Hansen, Briceño, Edwards, Thomas and Wilson, 2020

Application of "The Recipe" *For mesonic states*



Possible Application of "The Recipe" *For baryonic states*

Draper et al. 2023
Hörz et al. 2021

Finite Volume Formalism

Three relativistic neutrons in a finite volume

Zachary T. Draper[a], **Maxwell T. Hansen**[b], **Fernando Romero-López**[c], **and Stephen R. Sharpe**[a]

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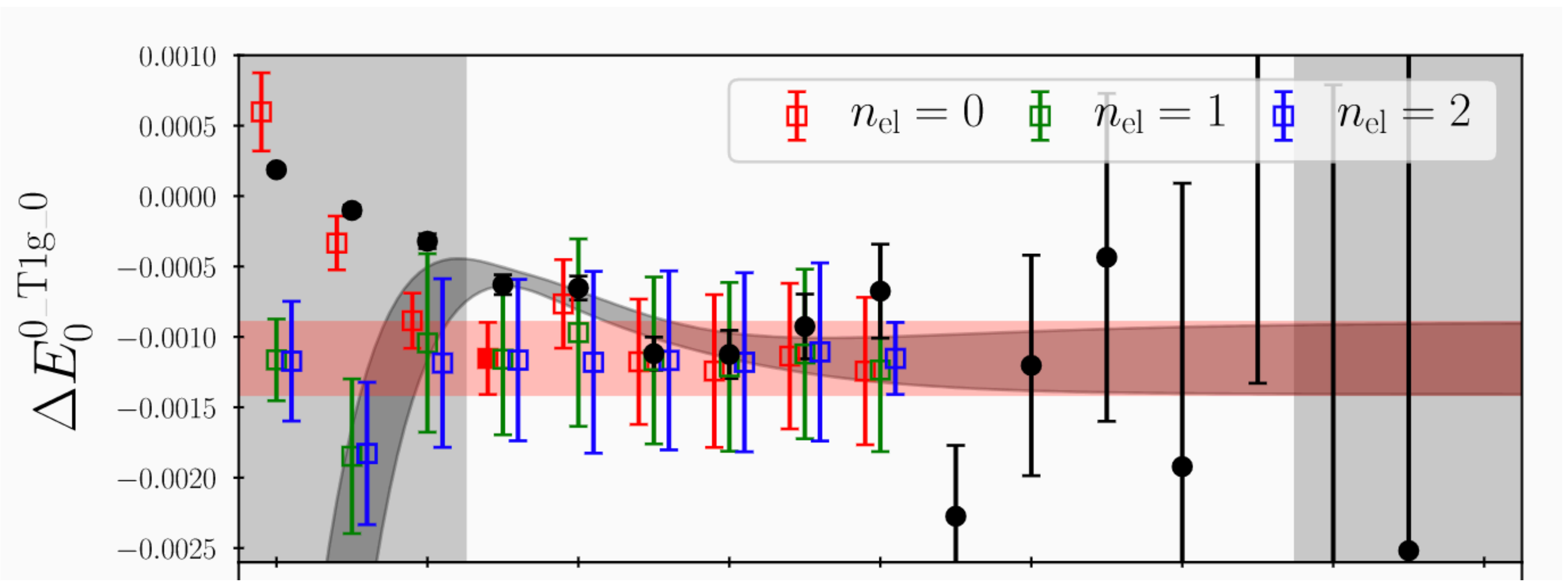
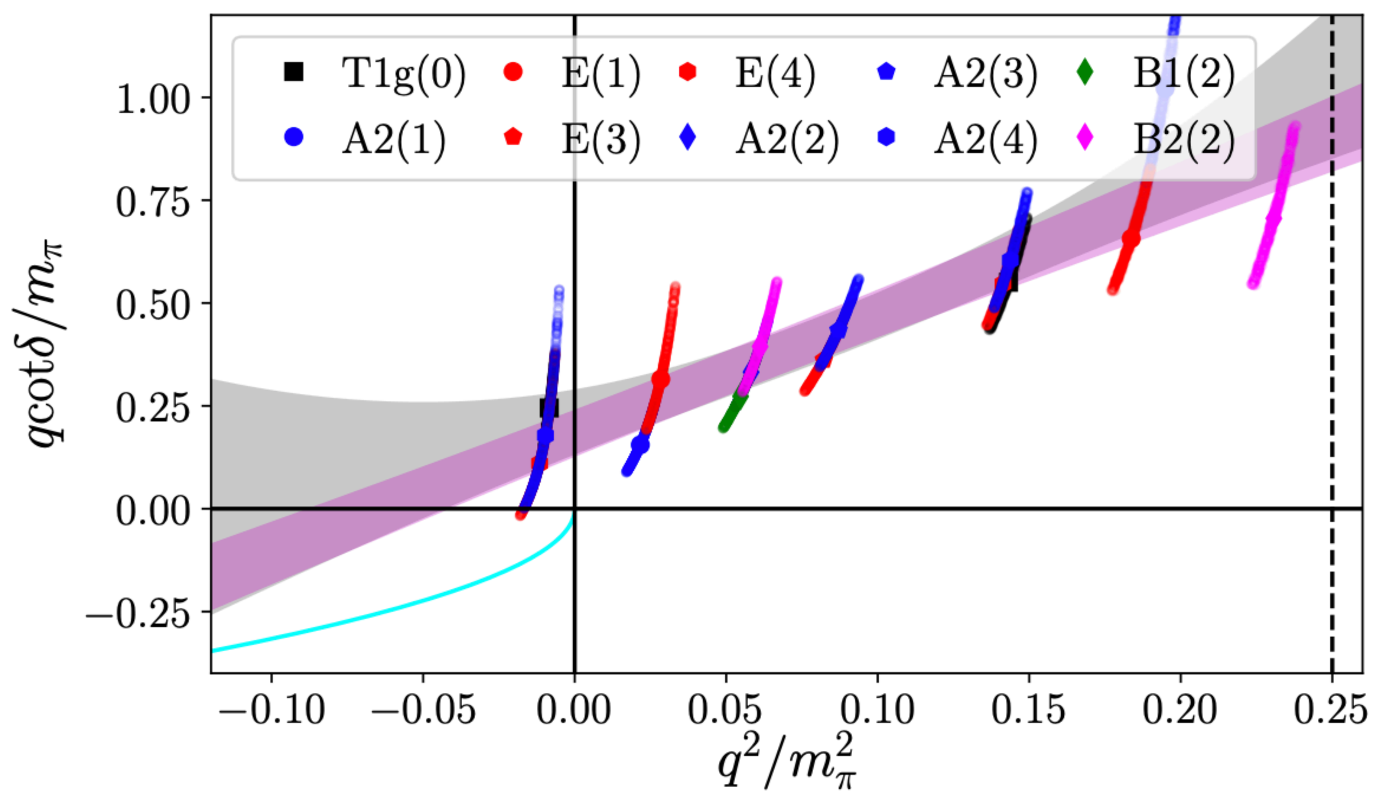
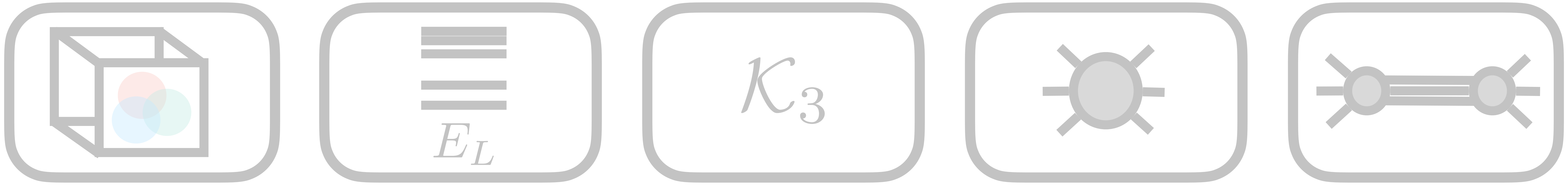
[b] *School of Physics and Astronomy, University of Edinburgh, Edinburgh EH9 3JZ, UK*
[c] *CTP, Massachusetts Institute of Technology, Cambridge, MA 02139, USA*

E-mail: ztd@uw.edu, maxwell.hansen@ed.ac.uk, fernando@mit.edu, srsharpe@uw.edu

Lattice Calculation

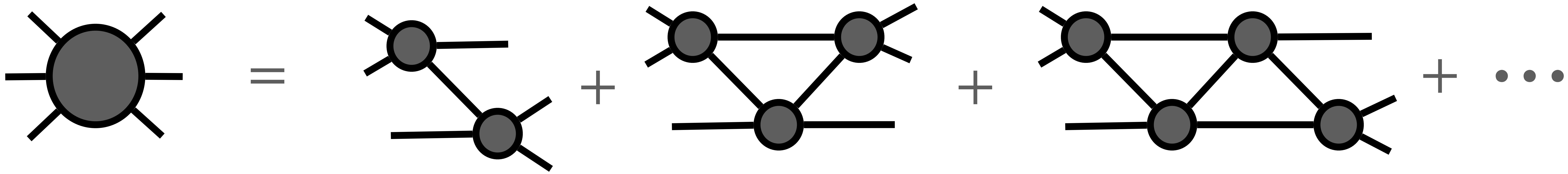






Three Body Scattering Amplitude

Assuming $\boxed{\mathcal{K}_3} = 0$ *, and all identical scalar particles*

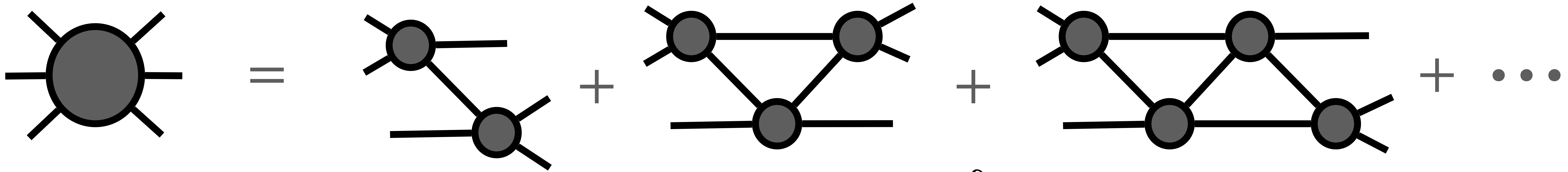


$$\mathcal{D} = -\mathcal{M}_2 G \mathcal{M}_2 - \mathcal{M}_2 \int_q G \mathcal{D}$$

Integral Equation for
Infinite volume three particle scattering amplitude

Three Body Scattering Amplitude

Assuming $\boxed{\mathcal{K}_3} = 0$ *, and all identical scalar particles*



$$\mathcal{D} = -\mathcal{M}_2 G \mathcal{M}_2 - \mathcal{M}_2 \int_q G\mathcal{D}$$

Two-body scattering amplitude

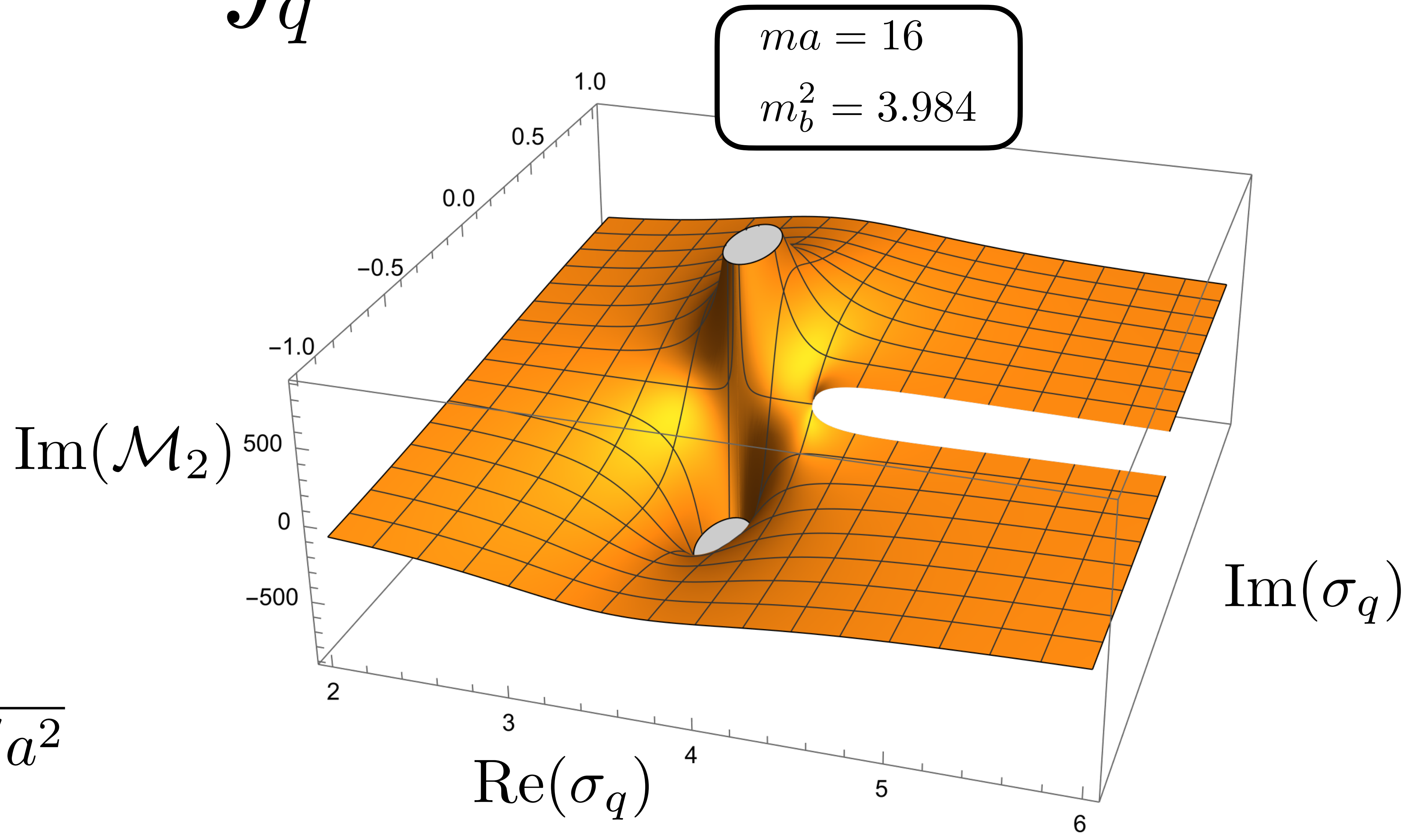
$$\mathcal{M}_2 = \frac{1}{\mathcal{K}_2^{-1} - i\rho}$$

$$= \frac{1}{\rho \cot \delta_0 - i\rho}$$

Leading order effective range expansion

$$= \frac{16\pi\sqrt{\sigma_q}}{-\frac{1}{a} - i\sqrt{\frac{\sigma_q}{4} - m^2}}$$

If a>0, we have a bound-state in the two body system with mass, $m_b = 2\sqrt{m^2 - 1/a^2}$



Three Body Scattering Amplitude

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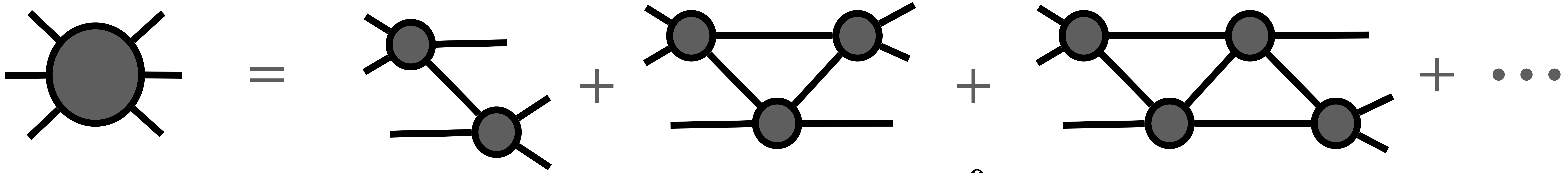
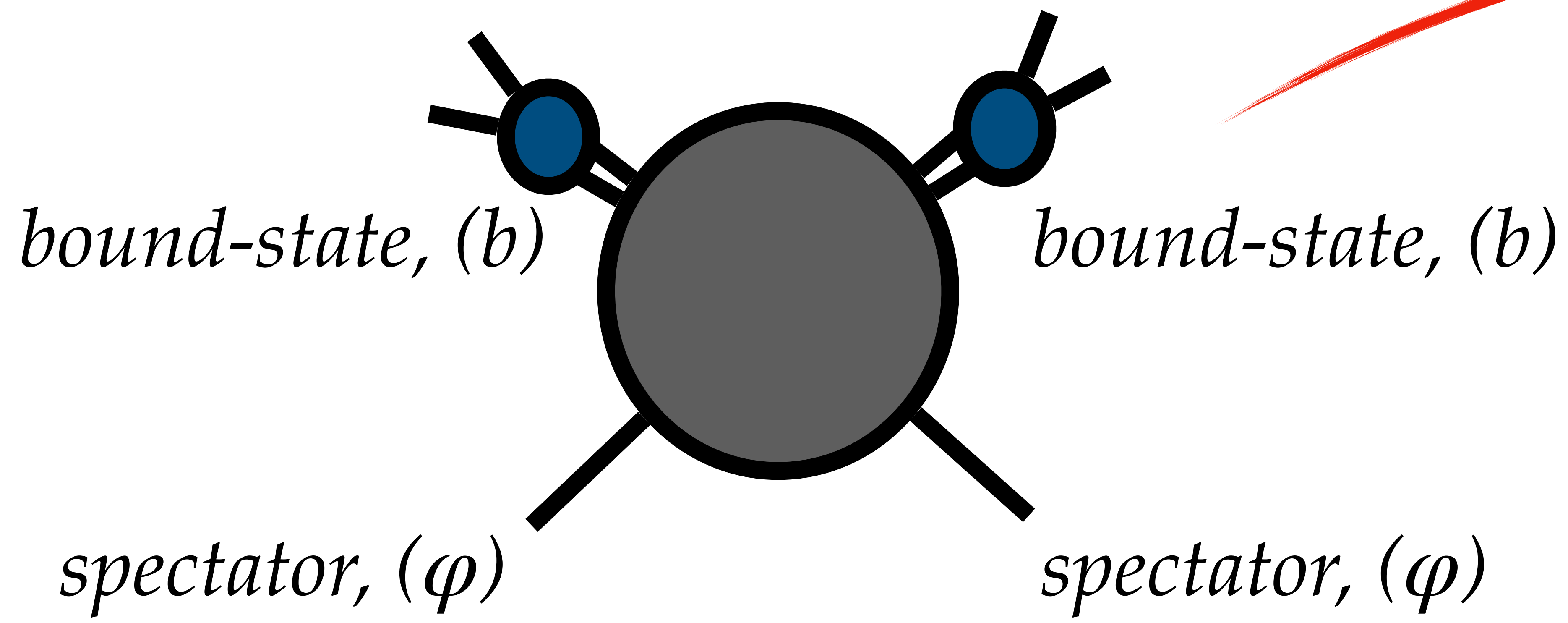


$$\mathcal{D} = -\mathcal{M}_2 G \mathcal{M}_2 - \mathcal{M}_2 \int_q G \mathcal{D}$$

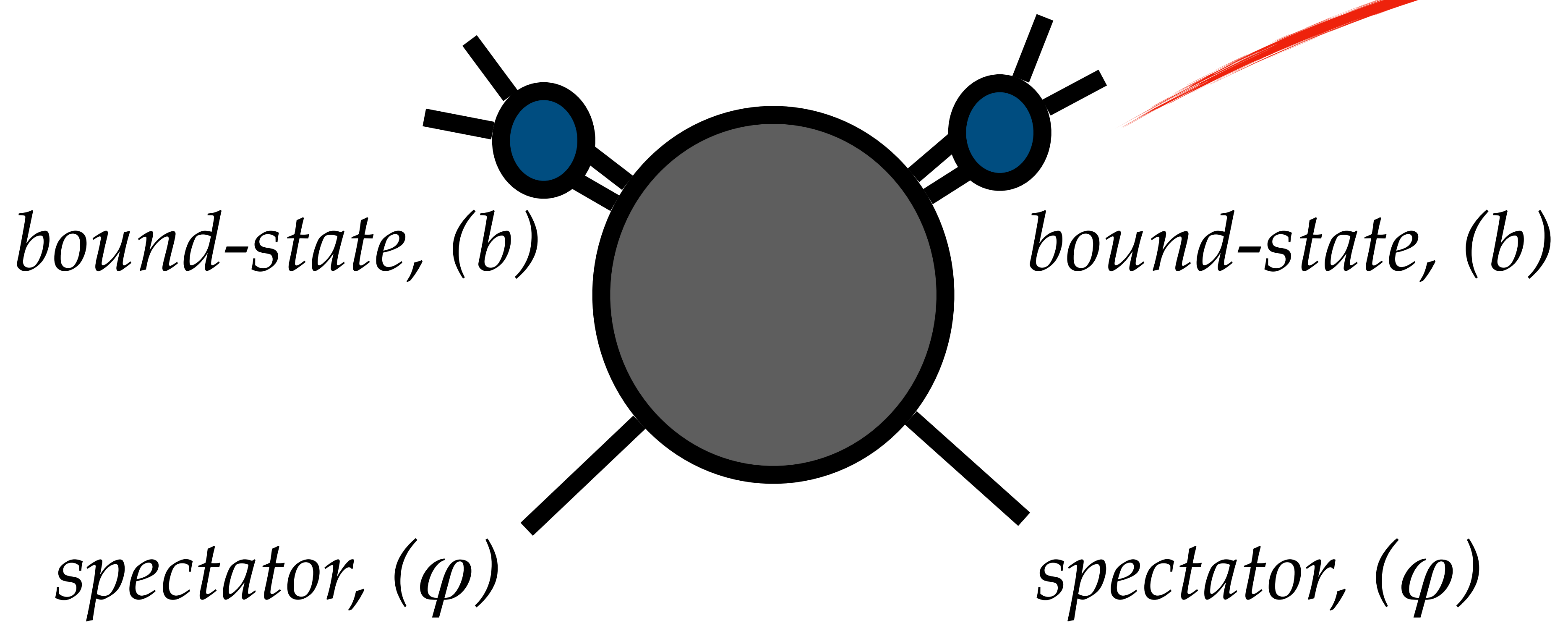


$$\mathcal{D} = \frac{-g}{\sigma_p - m_b^2} \mathcal{M}_{\varphi b} \frac{-g}{\sigma_k - m_b^2}$$

Two-body bound-state residue g^2

Two-body bound-state mass m_b

Consistency checks using Toy Model:



$$\mathcal{D} = \frac{-g}{\sigma_p - m_b^2} \mathcal{M}_{\varphi b} \frac{-g}{\sigma_k - m_b^2}$$

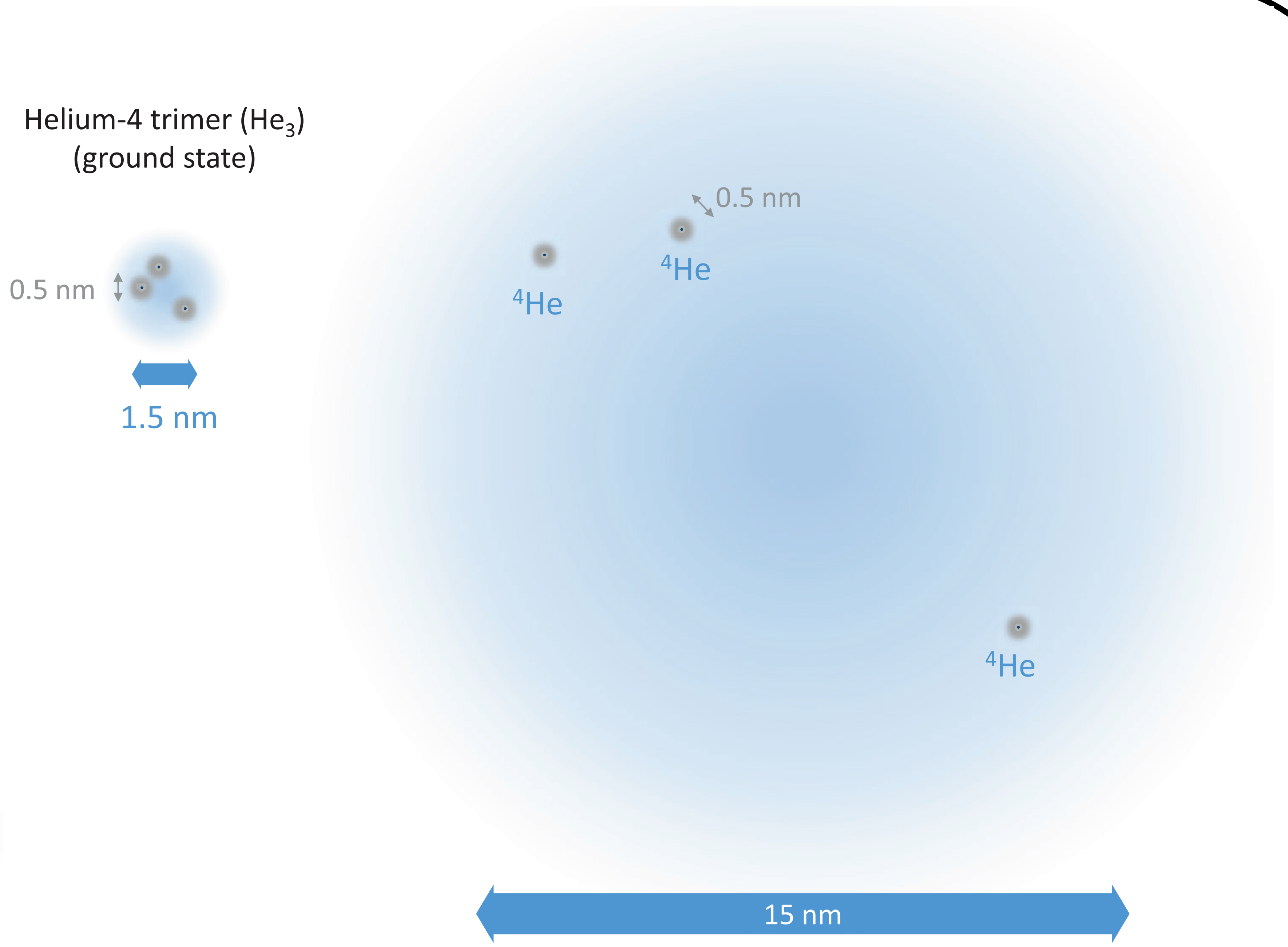
1. System dynamics fixed by two-body scattering length only.
2. Bound state pole in the integration interval, numerically most demanding scenario.
3. Finite volume results are available for comparison *(Romero-López, Sharpe, Blanton, Briceño and Hansen, 2019)*
4. Hints at **Efimov Physics**:

Unitarity Limit: $q \cot \delta_0 = -1/a = 0$

Two body scattering amplitude has a pole at the threshold: $$\mathcal{M}_2 = \frac{16\pi\sqrt{\sigma_q}}{-i\sqrt{\frac{\sigma_q}{4} - m^2}}$$

Geometrically separated, infinite tower of three-body bound-states: $E_{N+1} = E_N/\lambda^2$

$\lambda = 22.69438$



Vitaly Efimov, 1970
Naidon and Endo, 2016

Solving Integral Equations

1. Introduce $\mathcal{D} = \mathcal{M}_2 d \mathcal{M}_2$

2. Project onto partial wave, $\ell = 0$ sector

$$d_S(p,s,k) = -G_S(p,s,k) - \int_0^{q_{max}} dq \frac{q^2}{(2\pi)^2 \omega_q} G_S(p,s,q) \mathcal{M}_2(q,s) d_S(q,s,k)$$

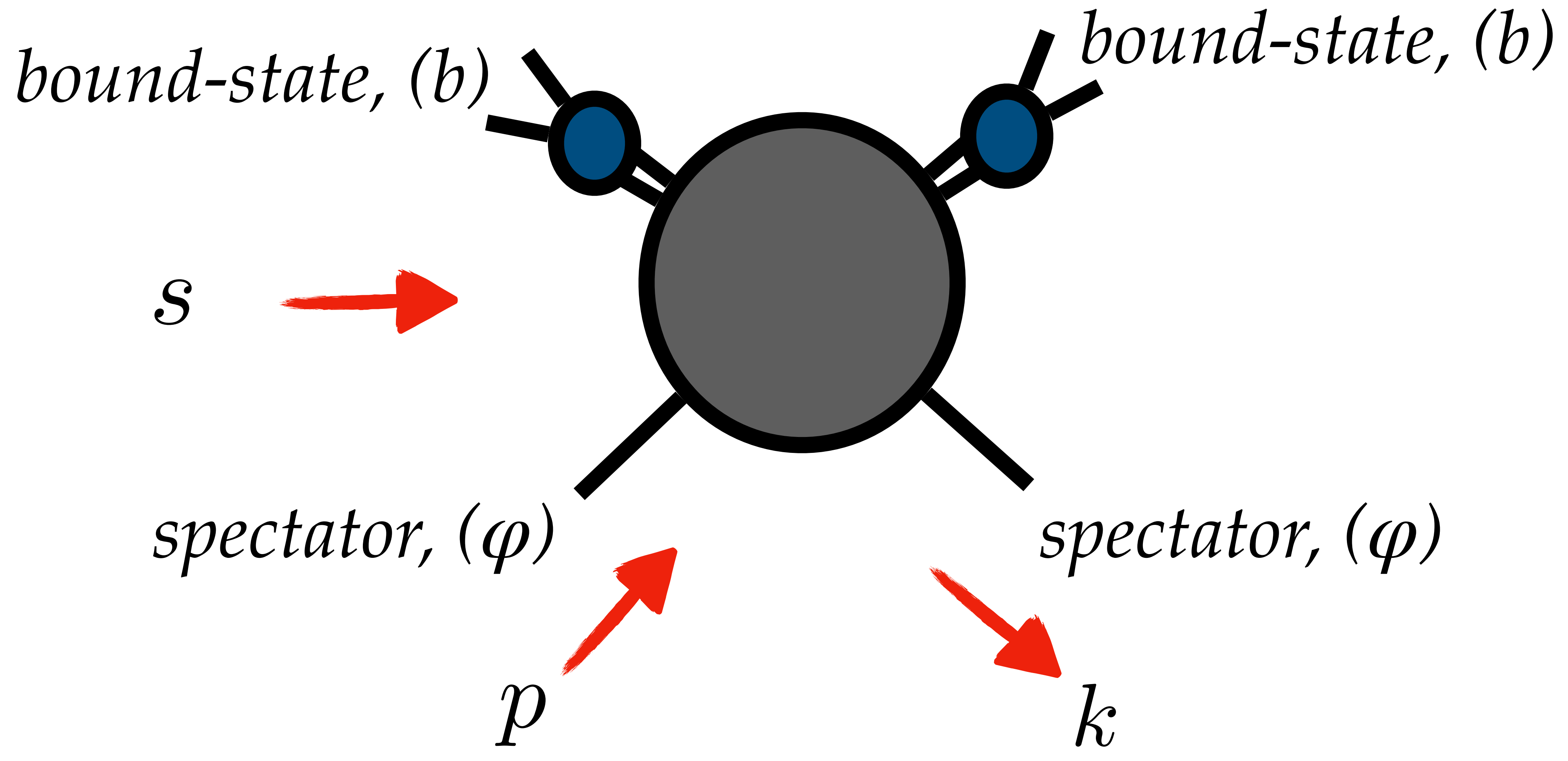
3. Soften singularities using non-zero epsilon, or isolate them

4. Write as matrix equation:

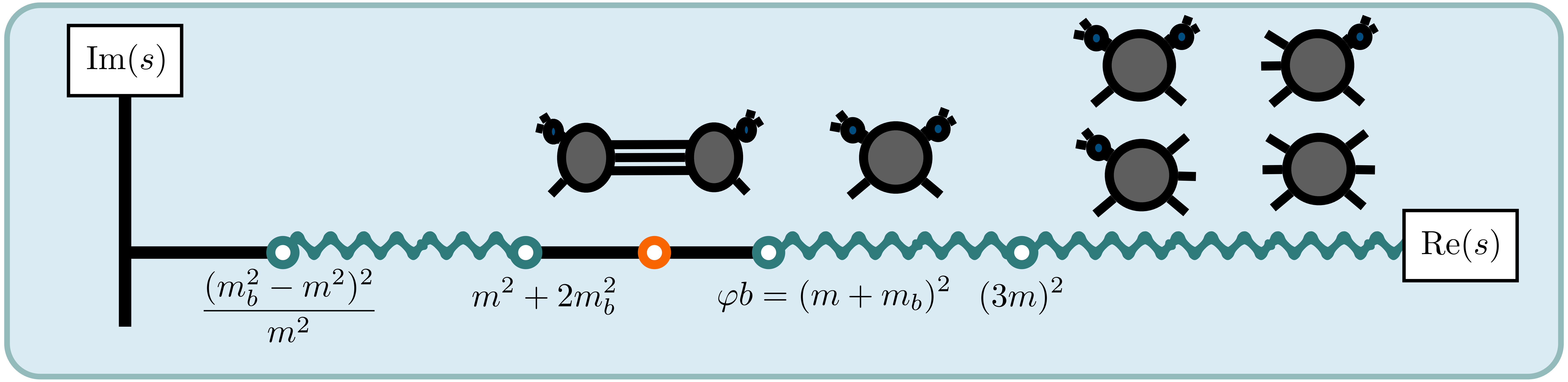
$$d(N,\epsilon) = -G(\epsilon) - P \cdot G(\epsilon) \cdot \mathcal{M}_2(\epsilon) \cdot d(N,\epsilon)$$

5. Recover the exact result by taking $N \to \infty$, $\epsilon \to 0$ limit

6. $\mathcal{M}_{\varphi b} = g^2 \lim_{\sigma_p, \sigma_k \to m_b^2} d_S$

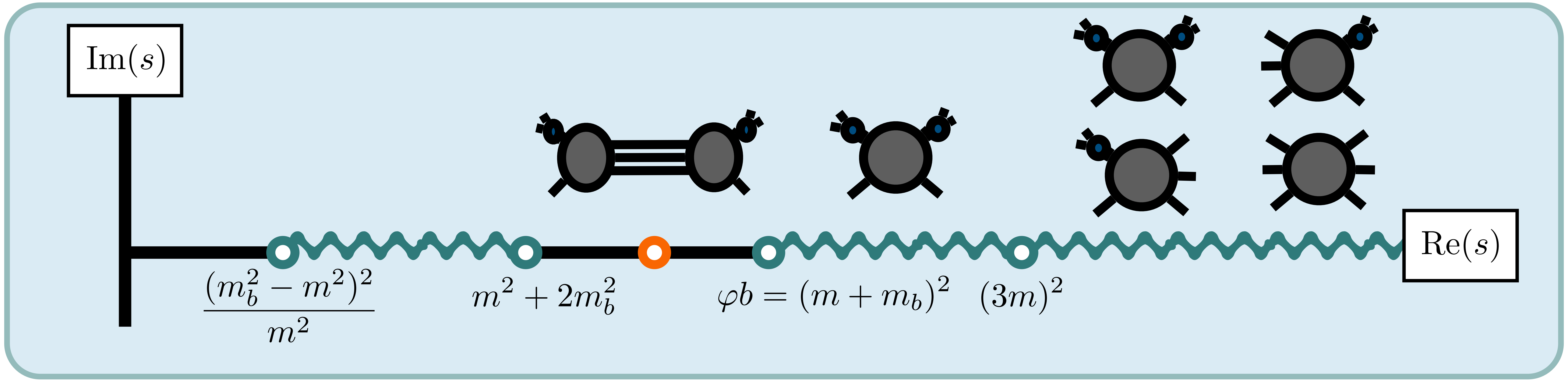


Kinematic Region of Interest



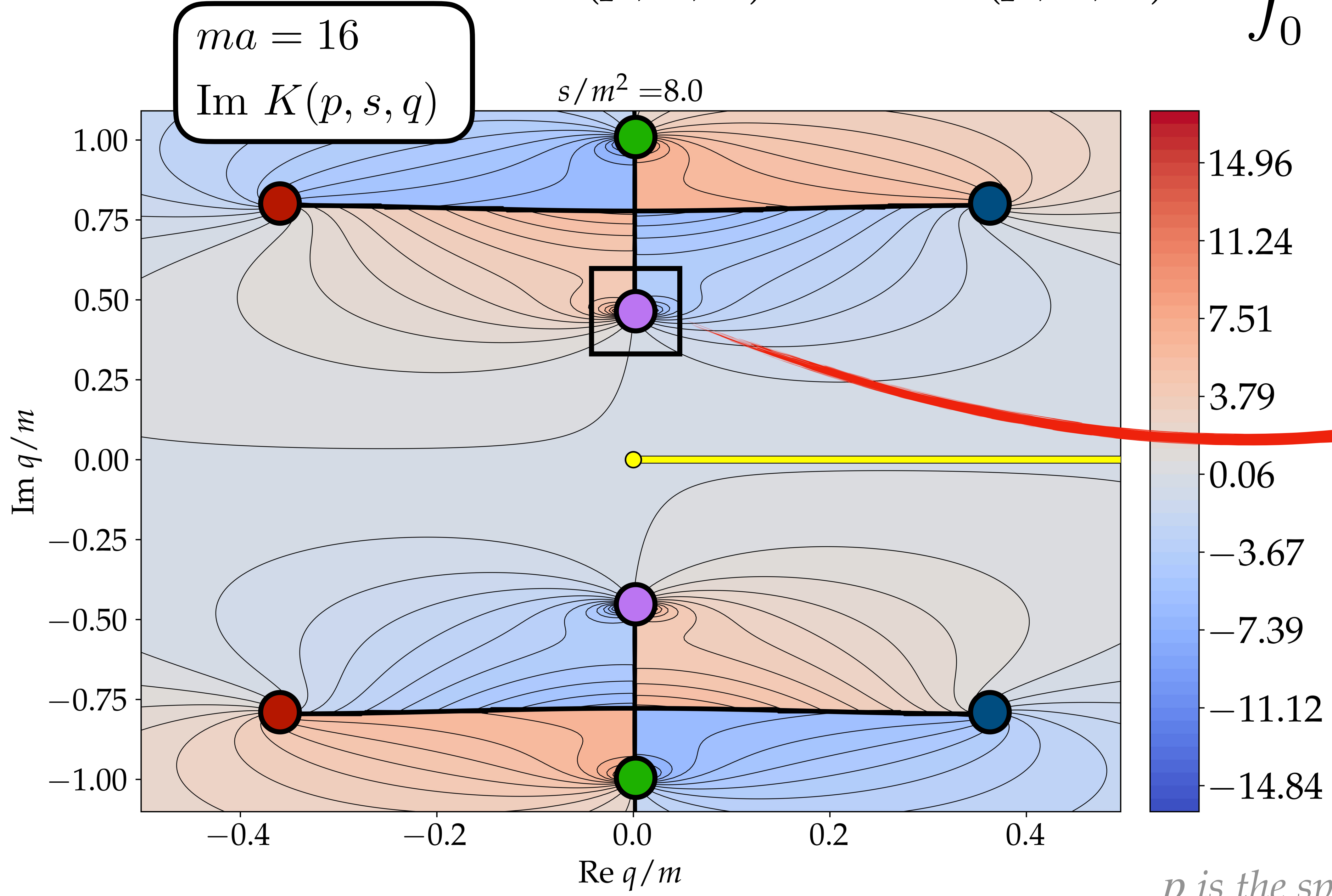
$$d_S(p,s,k) = -G_S(p,s,k) - \int_0^{q_{max}} dq \frac{q^2}{(2\pi)^2 \omega_q} G_S(p,s,q) \mathcal{M}_2(q,s) d_S(q,s,k)$$

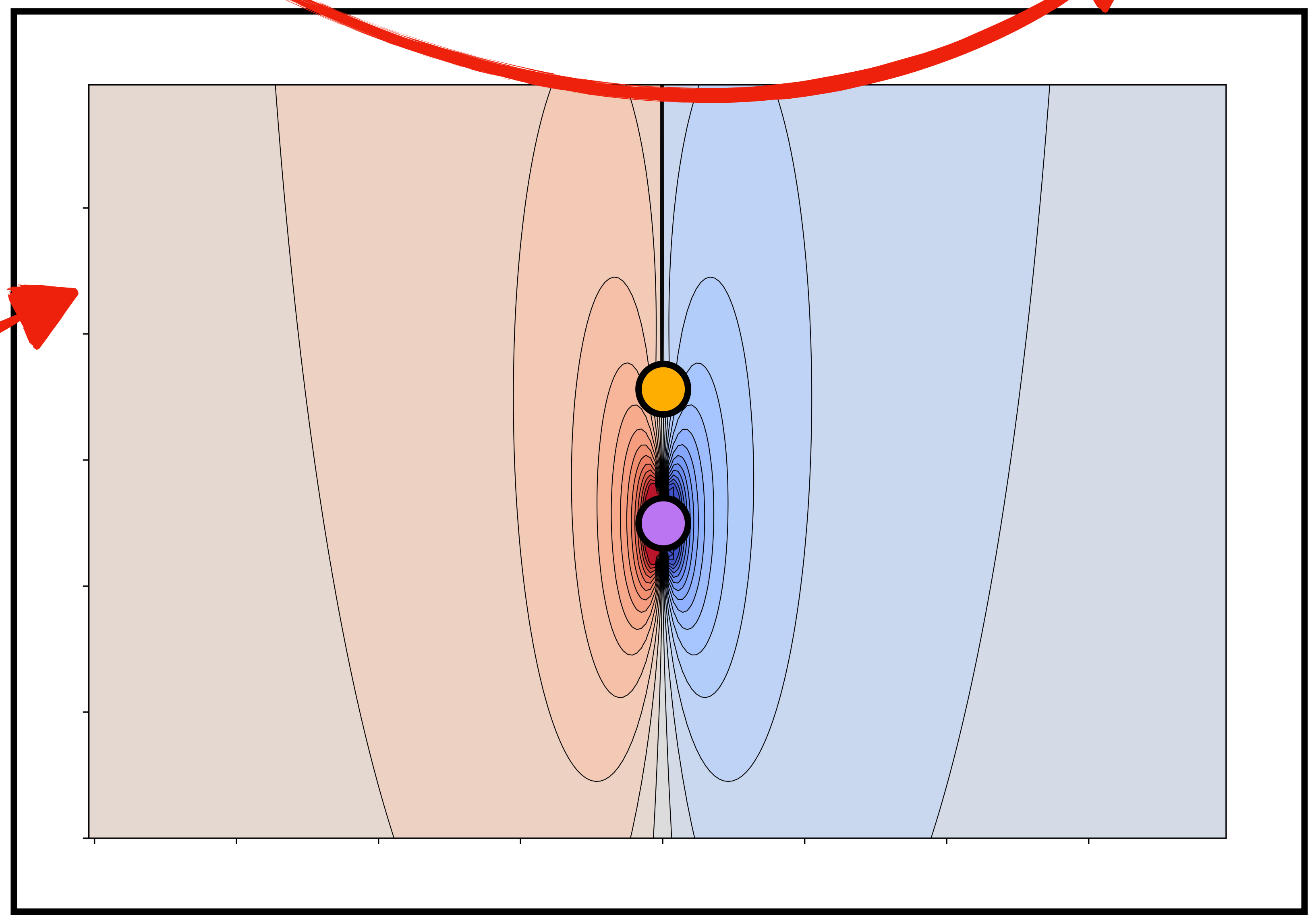
Singularities in Region of Interest



$$d_S(p,s,k) = -G_S(p,s,k) - \int_0^{q_{max}} dq \frac{q^2}{(2\pi)^2 \omega_q} G_S(p,s,q) \mathcal{M}_2(q,s) d_S(q,s,k)$$

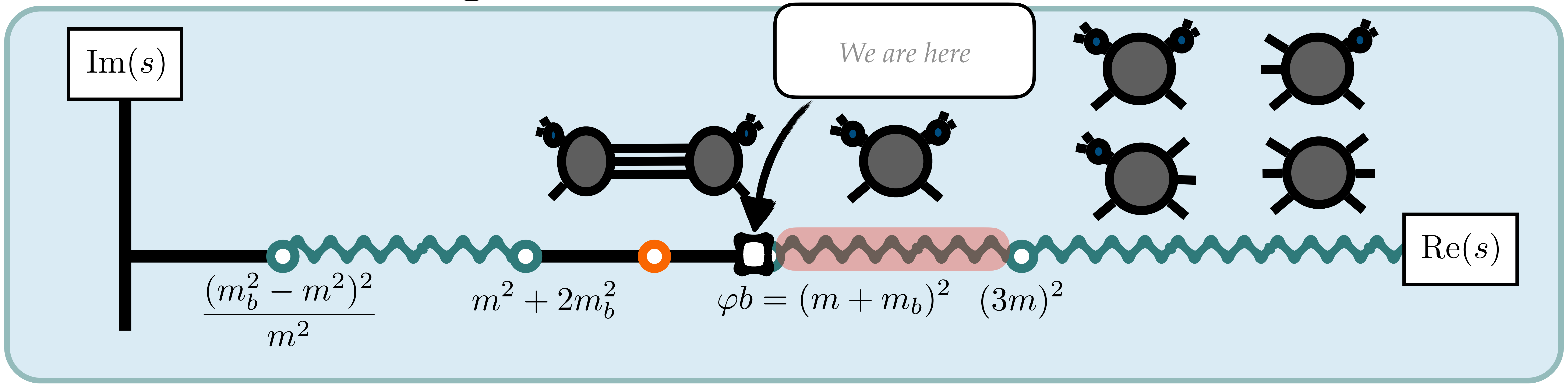
$K(p,s,q)$



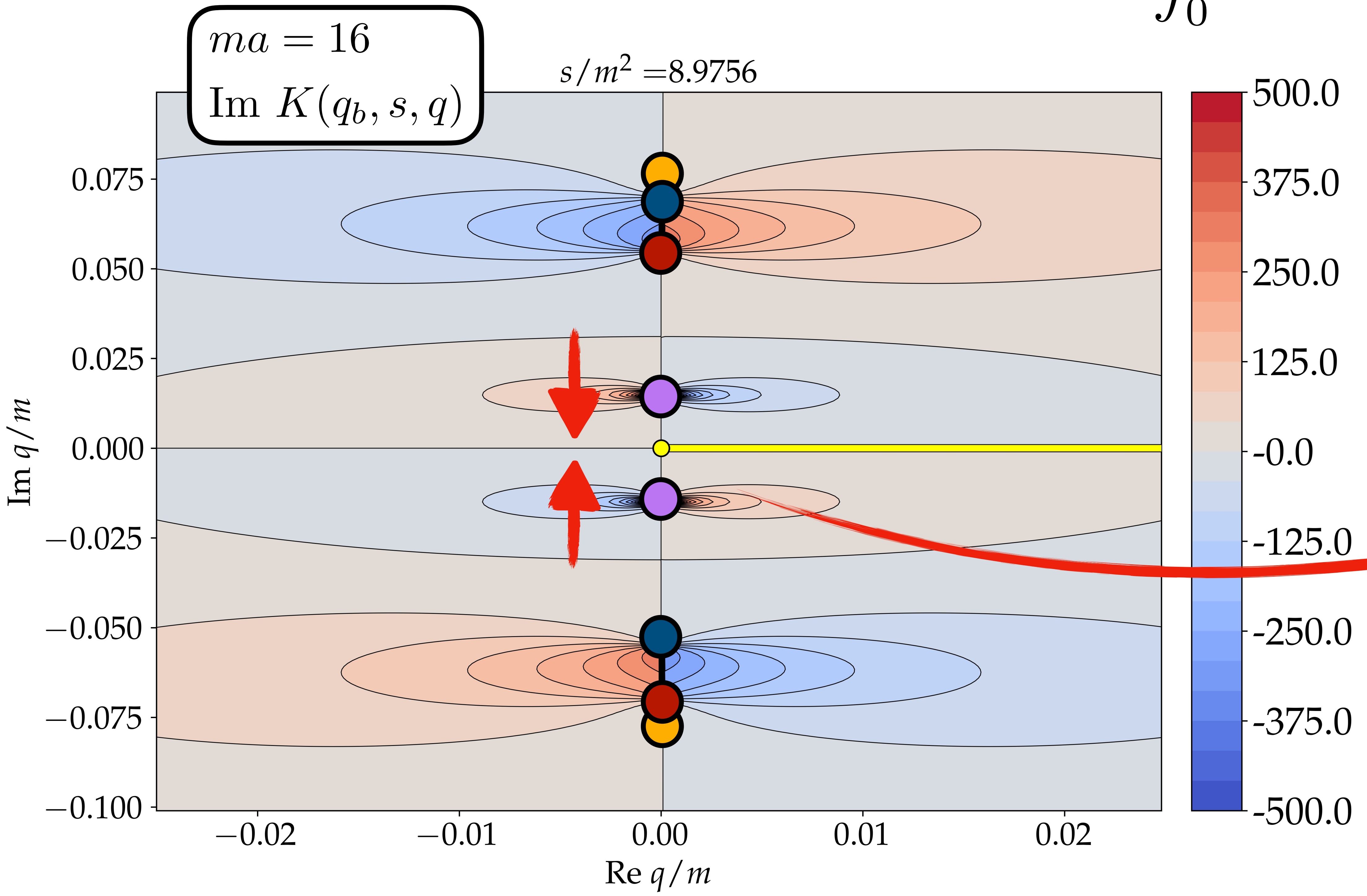


p is the spectator momenta corresponding to two-body subchannel energy, $\sigma_p = 2m^2$

Singularities in Region of Interest



$$d_S(p, s, k) = -G_S(p, s, k) - \int_0^{q_{max}} dq \frac{q^2}{(2\pi)^2 \omega_q} G_S(p, s, q) \mathcal{M}_2(q, s) d_S(q, s, k)$$

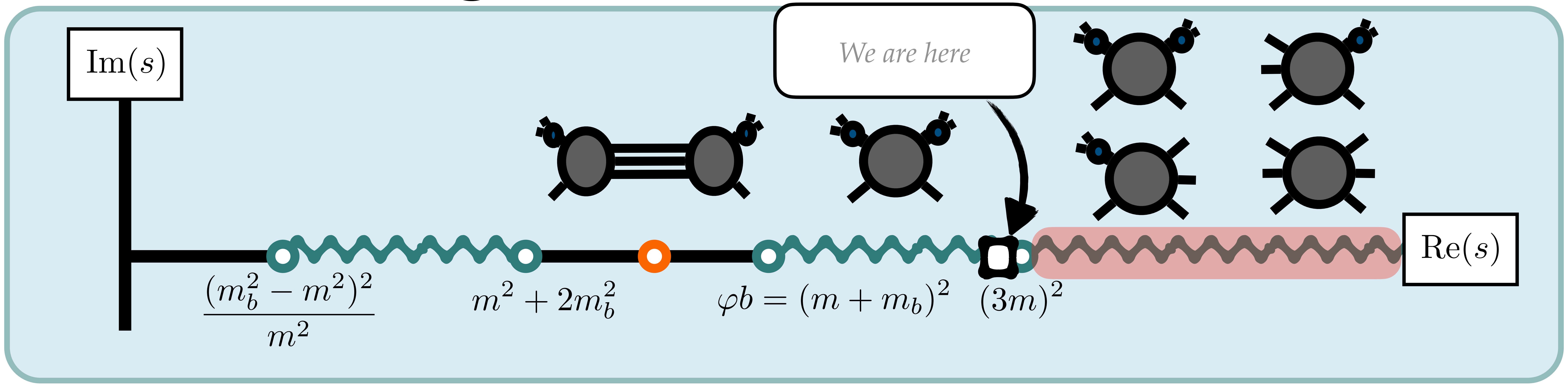


Two-body poles enclosing the end-point of integration

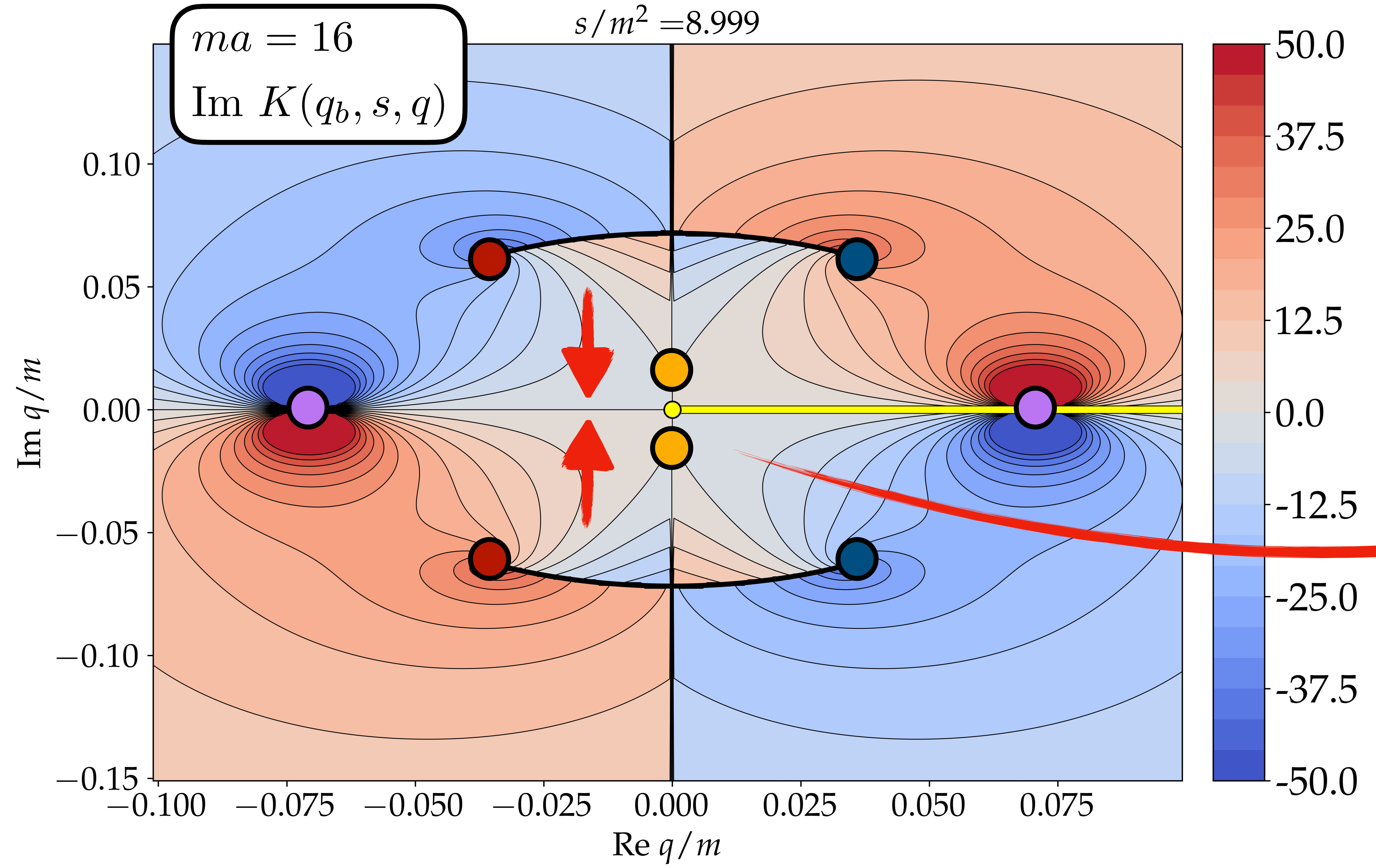
$$K(p, s, q) = \frac{q^2}{(2\pi)^2 \omega_q} G_S(p, s, q) \mathcal{M}_2(q, s)$$

q_b is the spectator momenta corresponding to two-body subchannel energy, $\sigma_q = m_b^2$

Singularities in Region of Interest



$$d_S(p,s,k) = -G_S(p,s,k) - \int_0^{q_{max}} dq \frac{q^2}{(2\pi)^2\omega_q} G_S(p,s,q)\mathcal{M}_2(q,s)d_S(q,s,k)$$

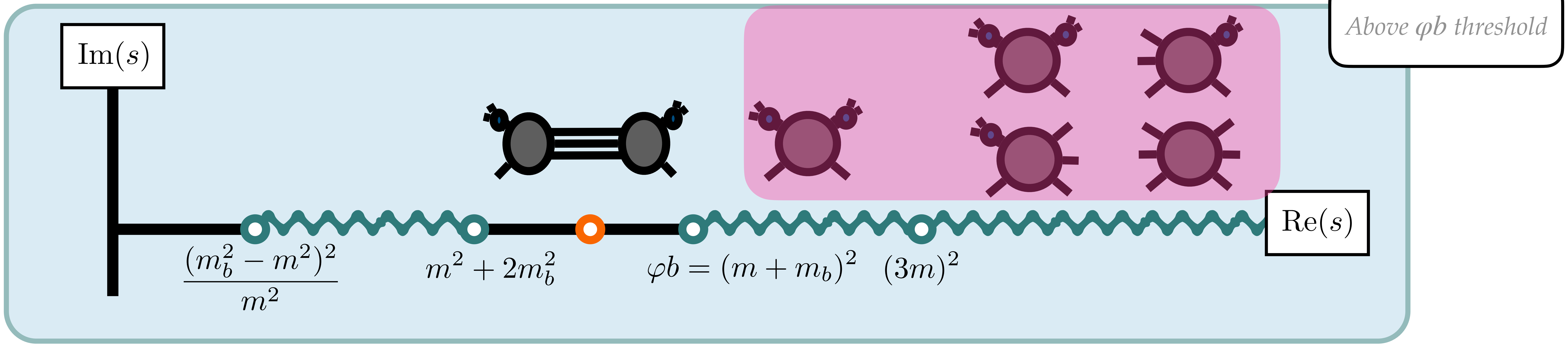


Two-body branch points enclosing one end-point of the integration interval

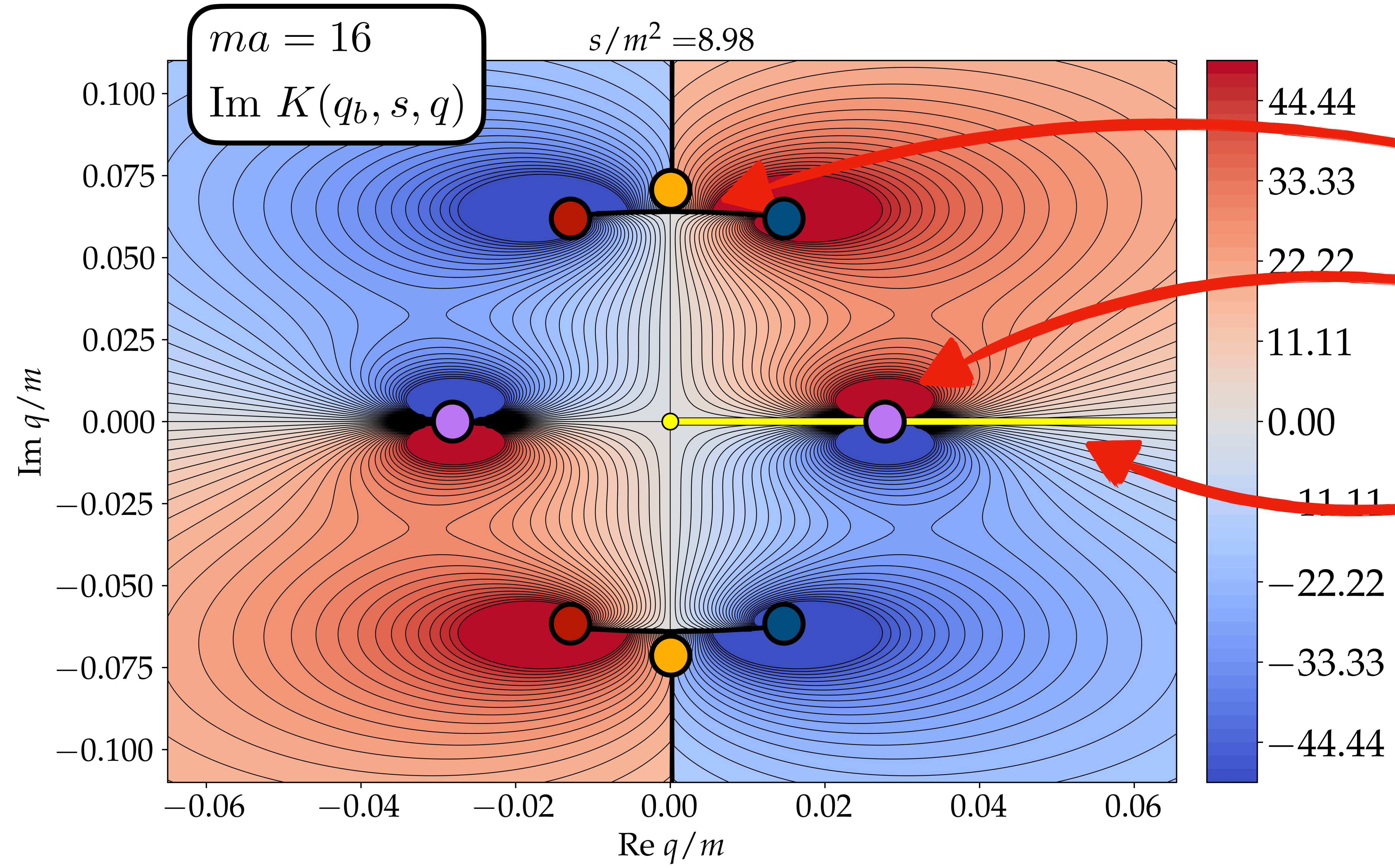
$$K(p,s,q) = \frac{q^2}{(2\pi)^2\omega_q} G_S(p,s,q)\mathcal{M}_2(q,s)$$

q_b is the spectator momenta corresponding to two-body bound-state energy, $\sigma_q = m_b^2$

Complications in obtaining amplitude



$$d_S(p,s,k) = -G_S(p,s,k) - \int_0^{q_{max}} dq \frac{q^2}{(2\pi)^2\omega_q} G_S(p,s,q)\mathcal{M}_2(q,s) d_S(q,s,k)$$



Set $p, k = q_b$ spectator momenta corresponding to two body bound state

Singularity cut from the one particle exchange function

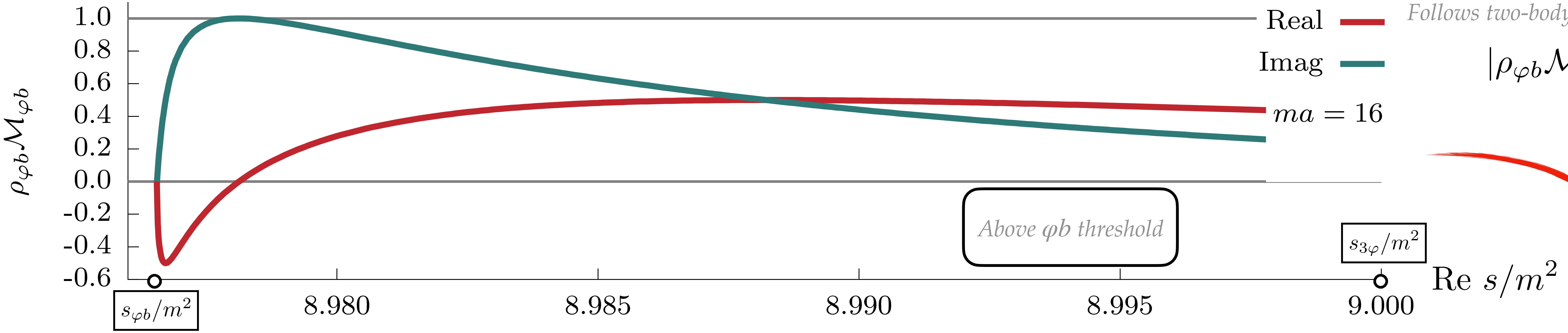
Two body amplitude has a pole in the integration region

Momentum integration interval

$$K(p,s,q) = \frac{q^2}{(2\pi)^2\omega_q} G_S(p,s,q)\mathcal{M}_2(q,s)$$

$\mathcal{M}_{\phi b}$ on the real s axis

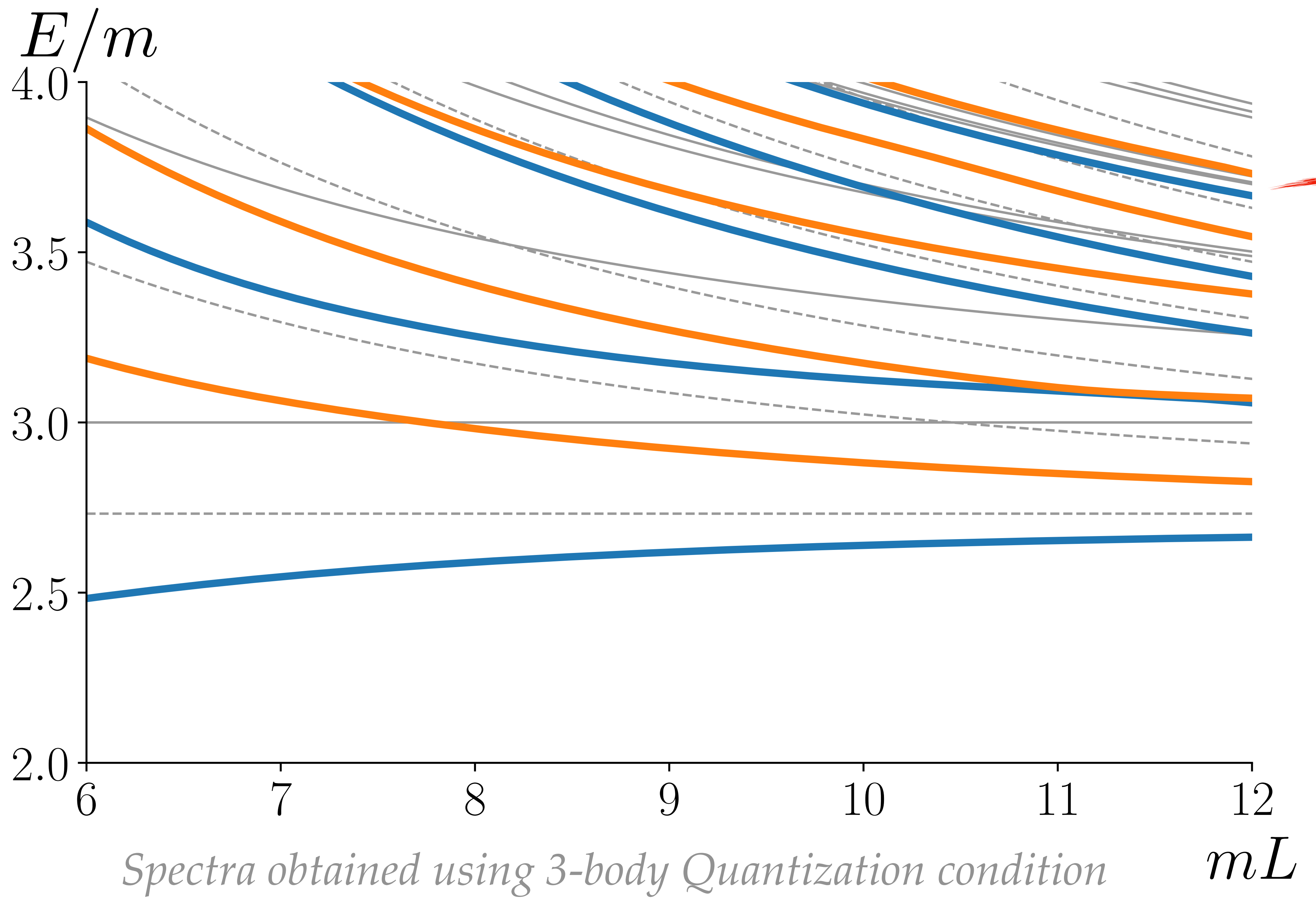
Romero-López, Sharpe, Blanton, Briceño and Hansen, 2019
Jackura, Briceño, Dawid, Islam, and McCarty, 2020



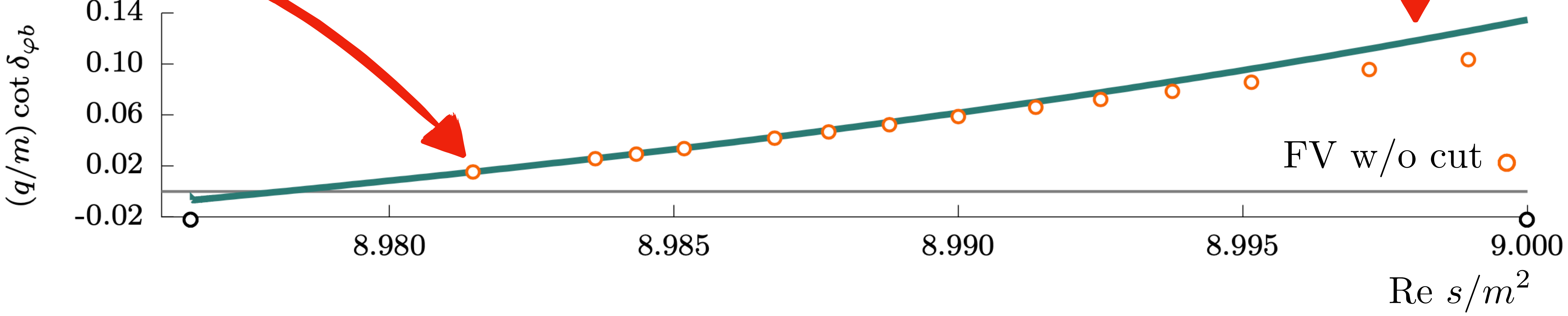
Follows two-body unitarity condition:

$$|\rho_{\varphi b}\mathcal{M}_{\varphi b}| \leq 1$$

$$q\cot\delta_{\varphi b} = 8\pi\sqrt{s}\ \mathrm{Re}\mathcal{M}_{\varphi b}^{-1}$$



Spectra obtained using 3-body Quantization condition
Phase shift calculated using 2-body Lüscher formalism



φb Scattering Length, b_0

Romero-López, Sharpe, Blanton, Briceño and Hansen, 2019
Jackura, Briceño, Dawid, Islam, and McCarty, 2020
Bedaque, Hammer, van Kolck, 1999

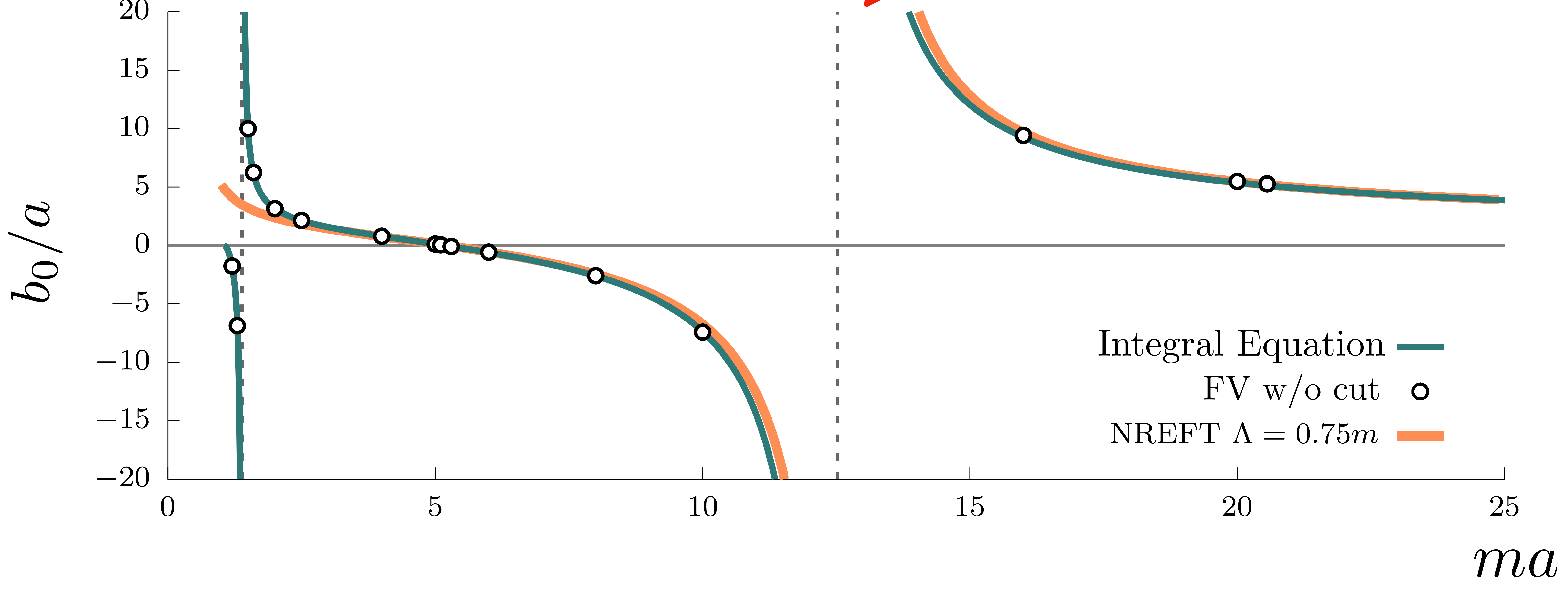
Fit to leading order effective range expansion to get φb scattering length, b_0

$$\lim_{q\to 0} q \cot\delta_{\varphi b} = -\frac{1}{b_0}$$

$$\mathcal{M}_{\varphi b} = \frac{8\pi\sqrt{s}}{q\cot\delta_{\varphi b} - iq}$$

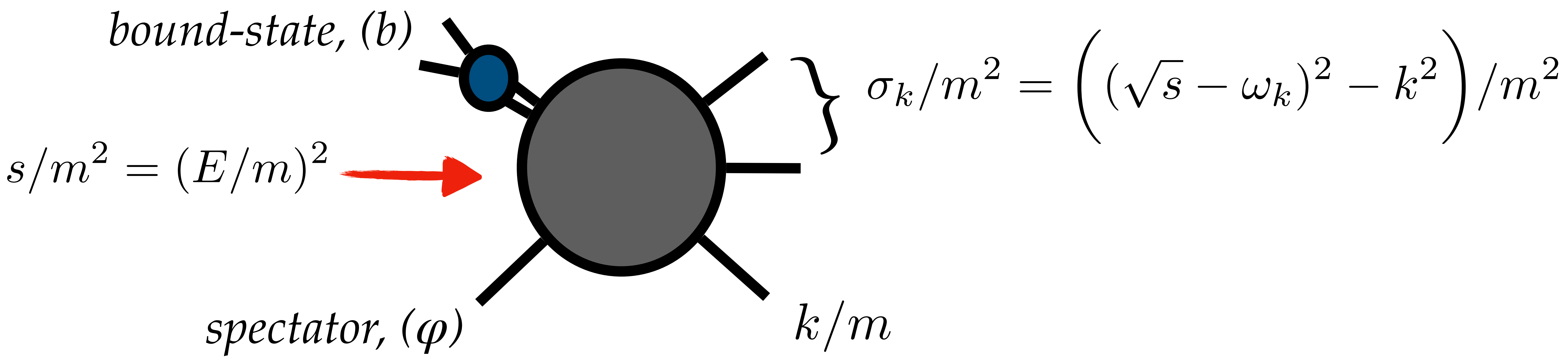
0

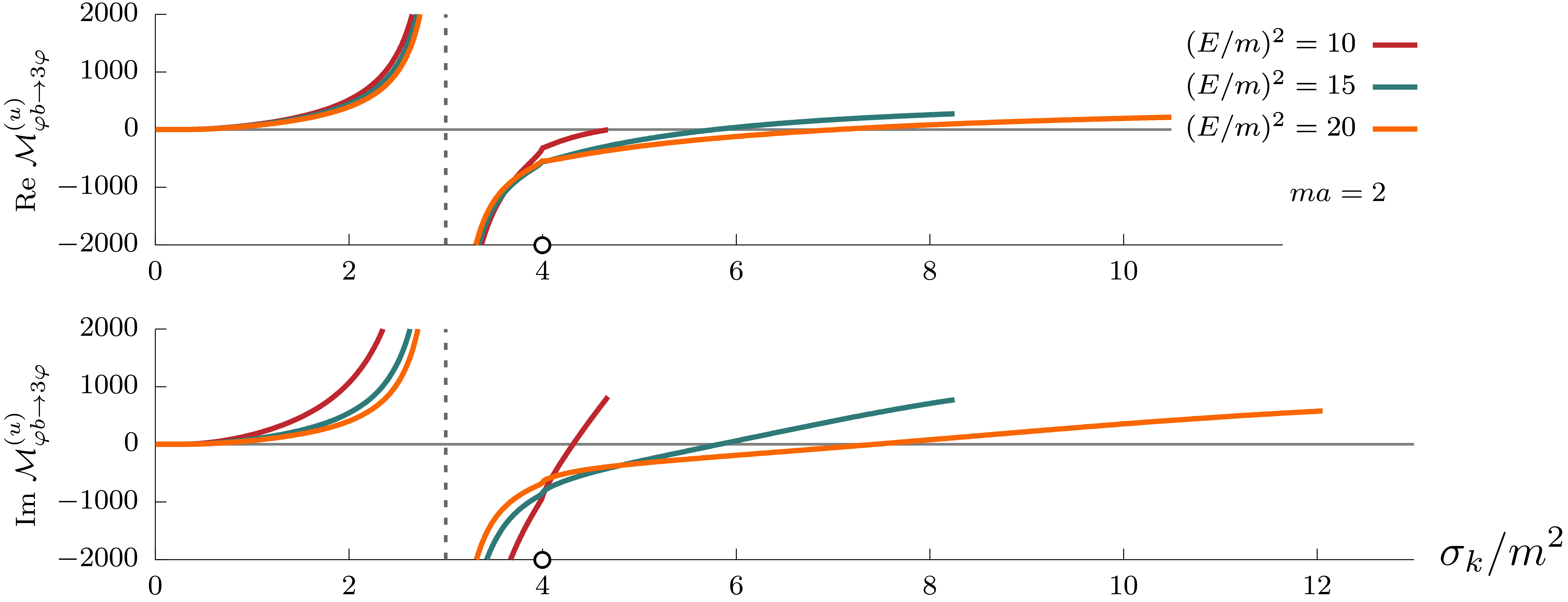
Hints at possible trimers



Three body breakups

Romero-López, Sharpe, Blanton, Briceño and Hansen, 2019
Jackura, Briceño, Dawid, Islam, and McCarty, 2020





Summary

- We have addressed the complications in studying bound-state-spectator amplitude above threshold and shown possible solutions to them
- We have performed consistency checks with toy model that provides confidence for both finite-volume and infinite-volume formalisms
- Solutions hint at possible trimers below φb threshold
- Moving singularities can create complication in the complex plane - will be discussed in Part II.

Thank you