

$0\nu\beta\beta$ in effective field theory

Wouter Dekens

with

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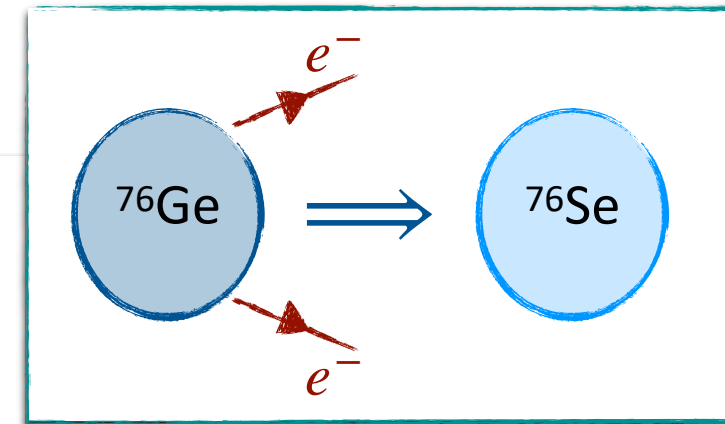


Neutrinoless Double beta decay

2

What is $0\nu\beta\beta$?

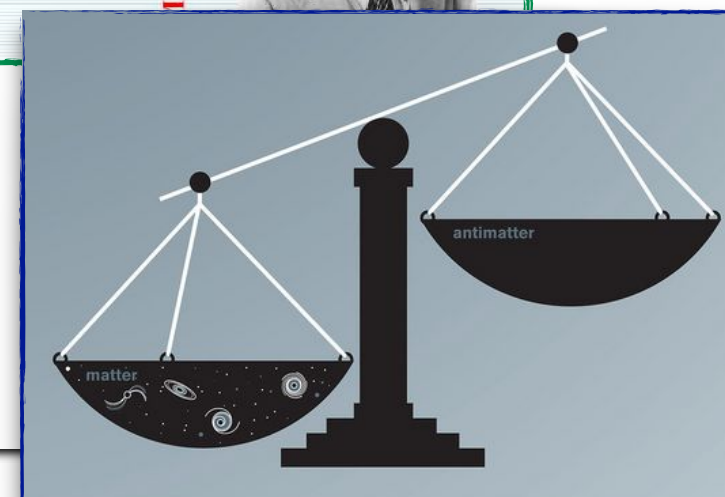
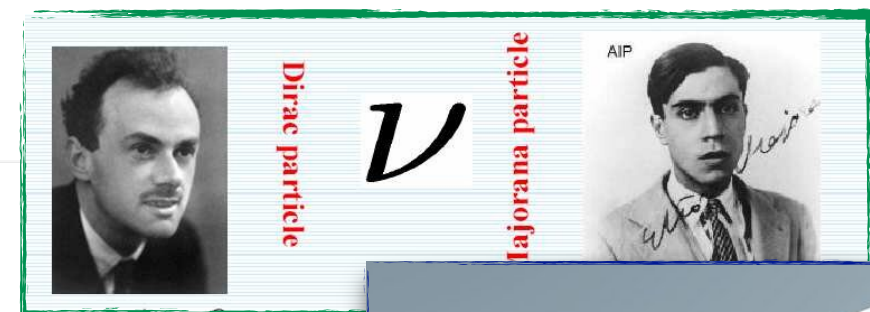
- $A(Z, N) \rightarrow A(Z + 2, N - 2) + 2e^- + 0\bar{\nu}$
 - Lepton Number Violating (LNV)
- Very strong limits on the half-life
 $\sim 10^{16} \times [\text{age of the universe}]$
- Future reach, 1-2 orders of improvement



Gerda (Germanium)	Cuore (Tellurium)	KamLAND-zen (Xenon)
$> 1.8 \cdot 10^{26} \text{ yr}$	$> 3.2 \cdot 10^{25} \text{ yr}$	$> 3.8 \cdot 10^{26} \text{ yr}$

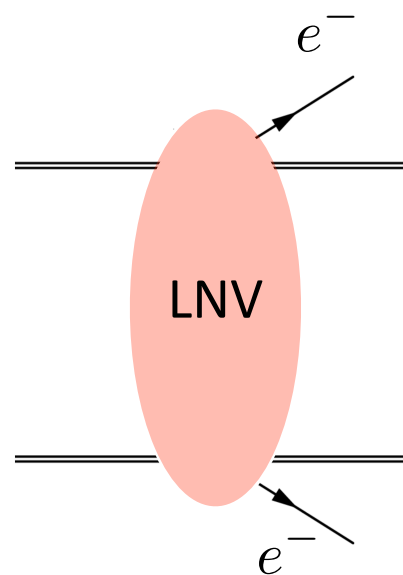
Why look for $0\nu\beta\beta$?

- Detection sheds light on:
 - **Neutrino masses**
 - Observation would show that ν 's are Majorana particles
 - **Matter-antimatter asymmetry**

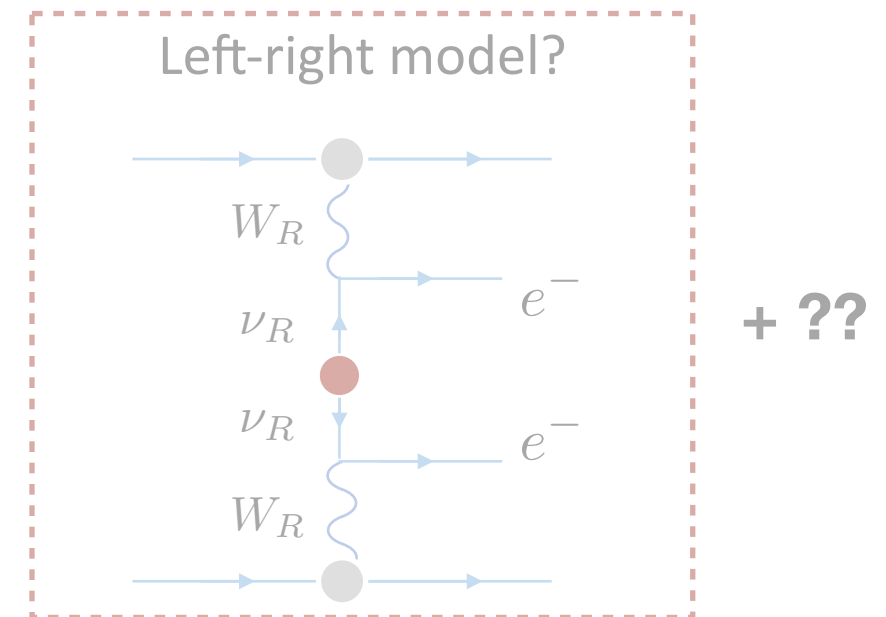
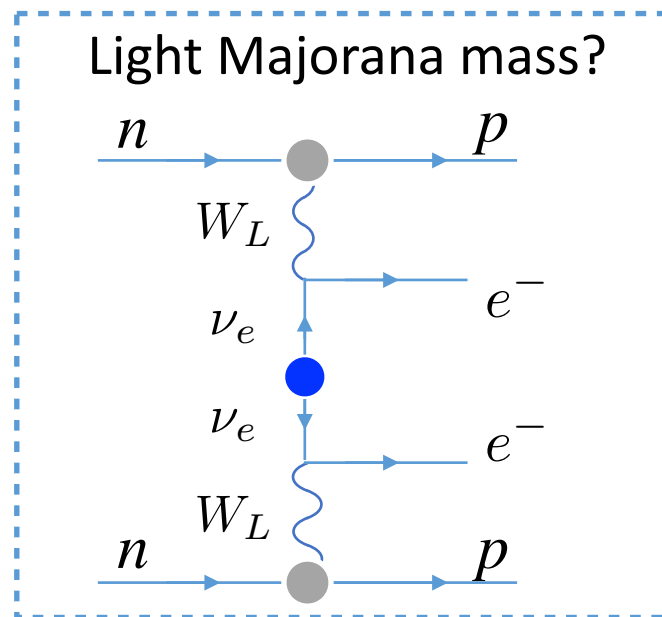


Neutrinoless Double beta decay

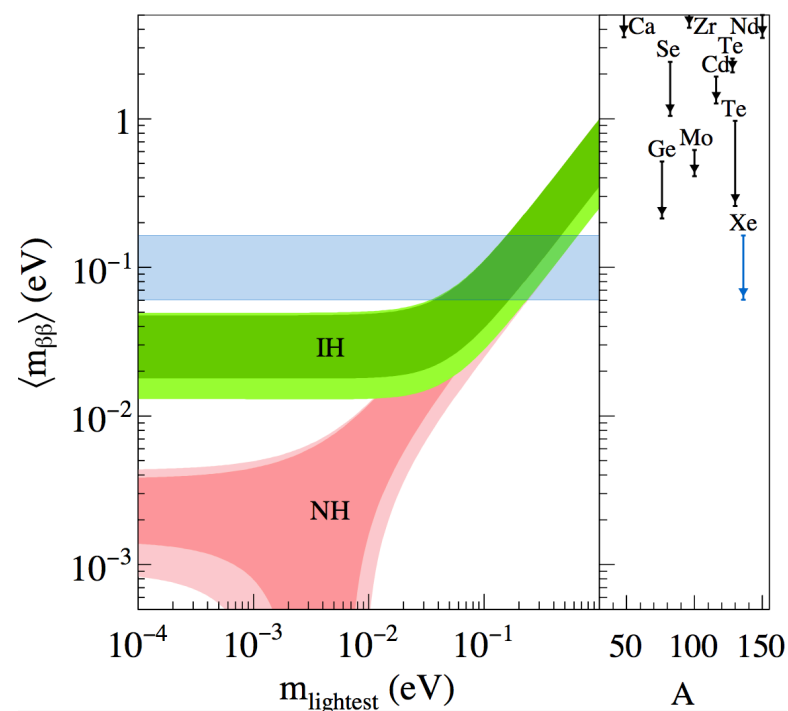
What could be responsible?



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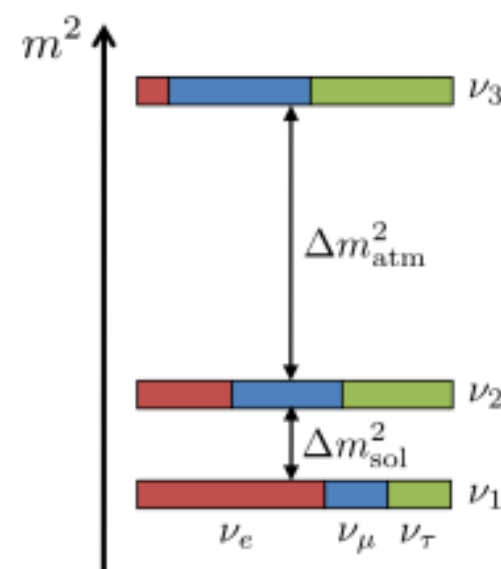
Majorana mass mechanism



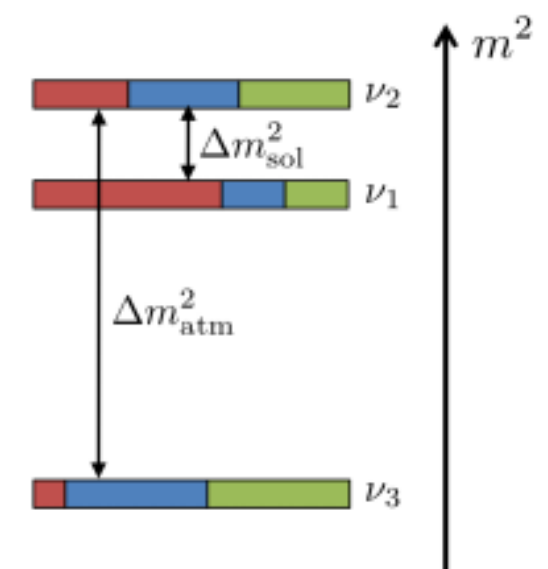
- Probes very high BSM scales, $\Lambda \sim 10^{15}$ GeV

Depends on hierarchy of neutrino masses

normal hierarchy (NH)

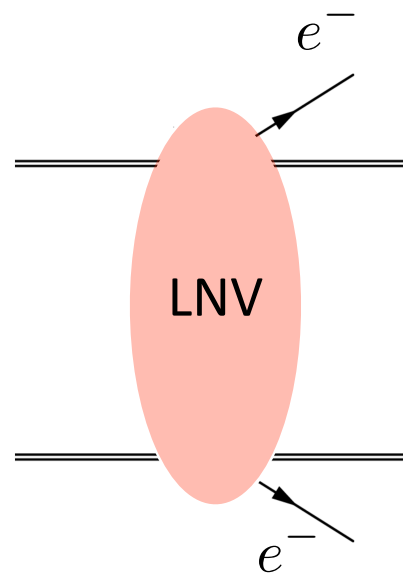


inverted hierarchy (IH)

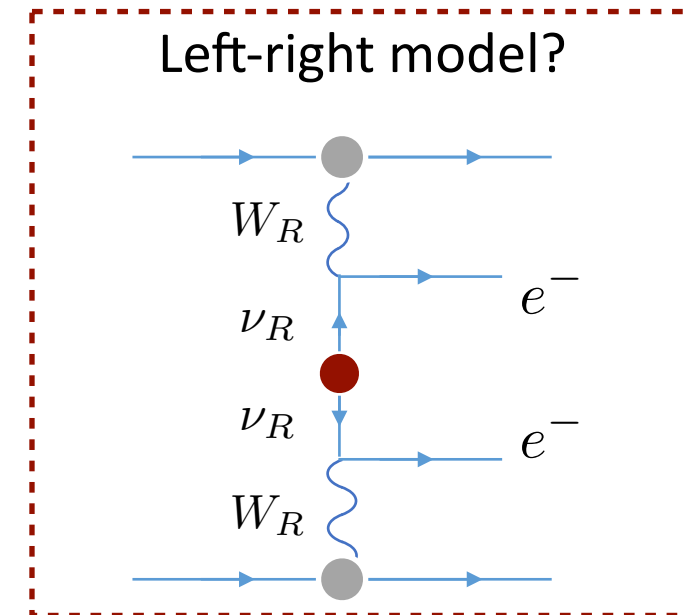
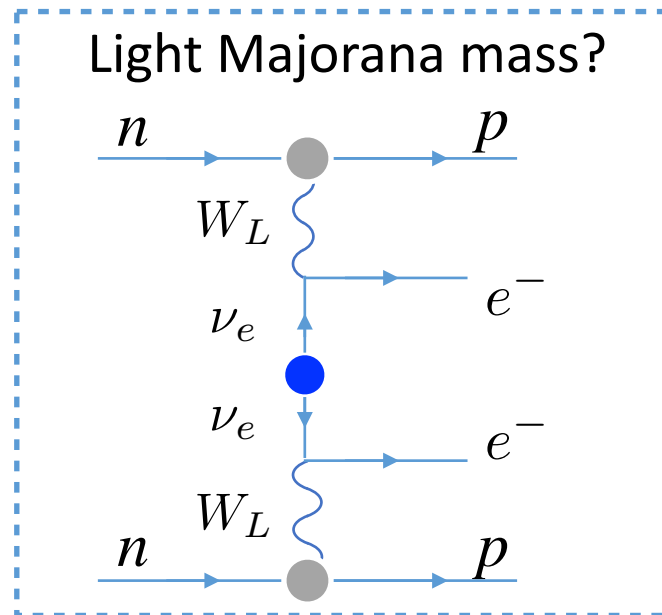


Neutrinoless Double beta decay

What could be responsible?

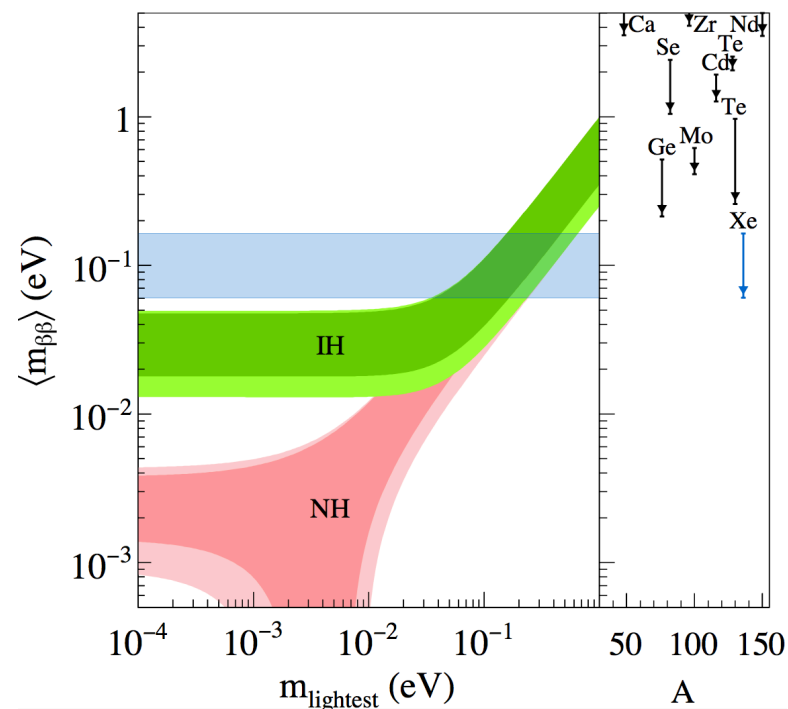


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+ ??

Majorana mass mechanism



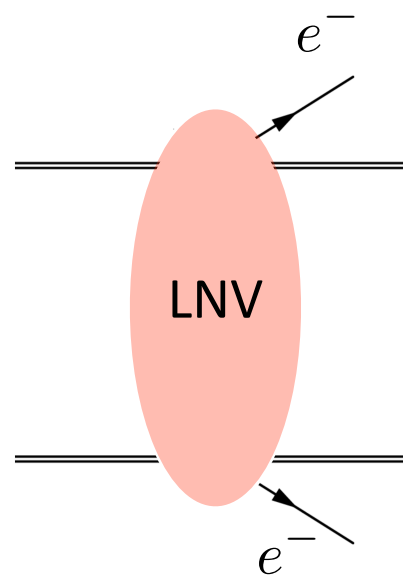
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Heavy BSM mechanisms

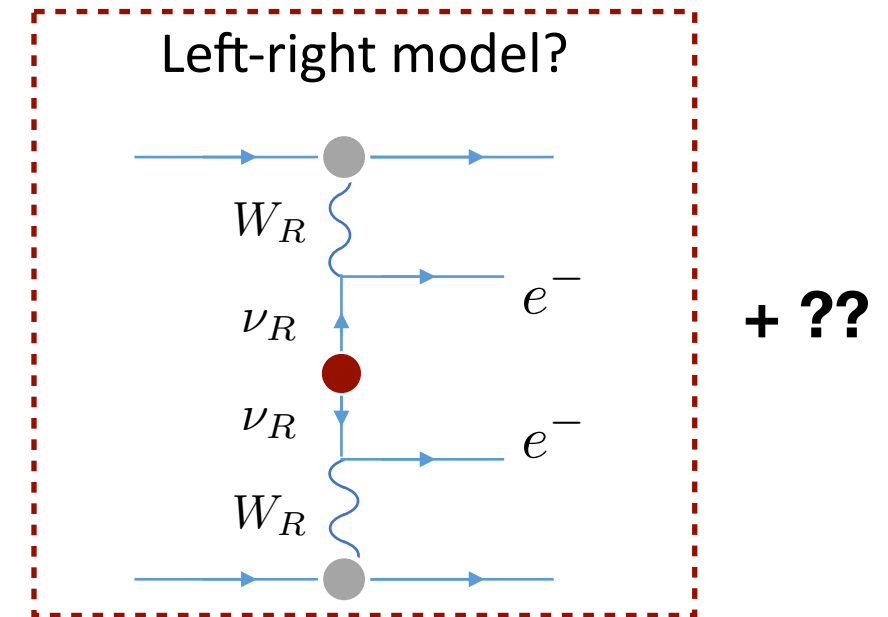
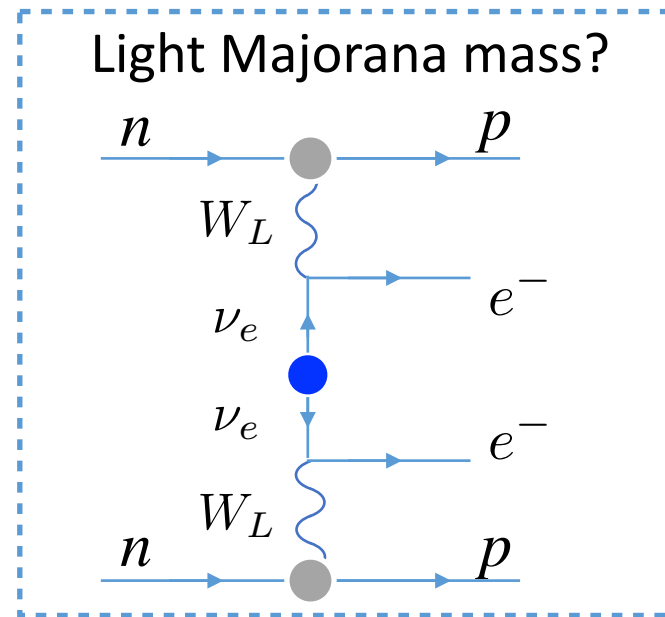
- Many possible scenarios
 - Left-right model,
 - R-parity violating SUSY
 - Leptoquarks...

Neutrinoless Double beta decay

What could be responsible?

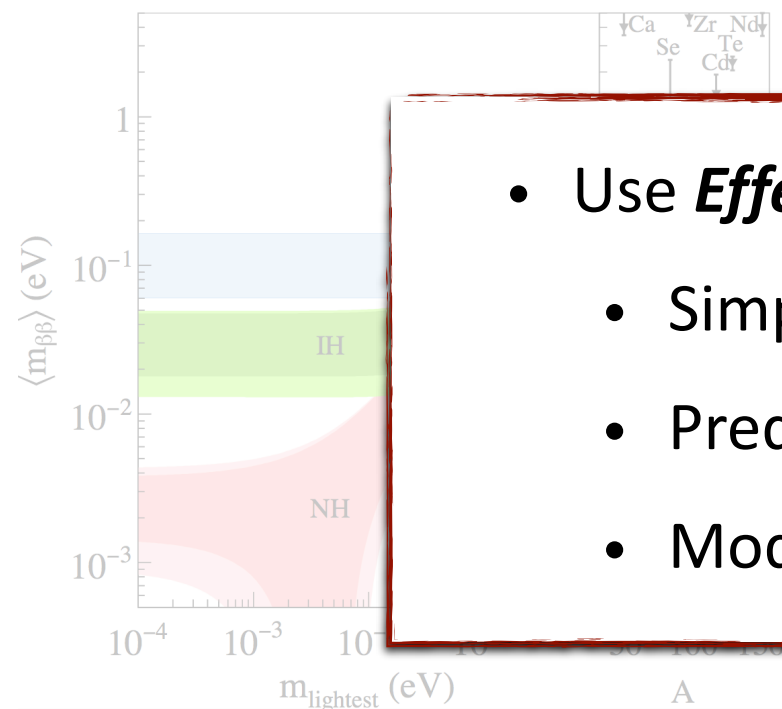


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Majorana mass mechanism

Heavy BSM mechanisms



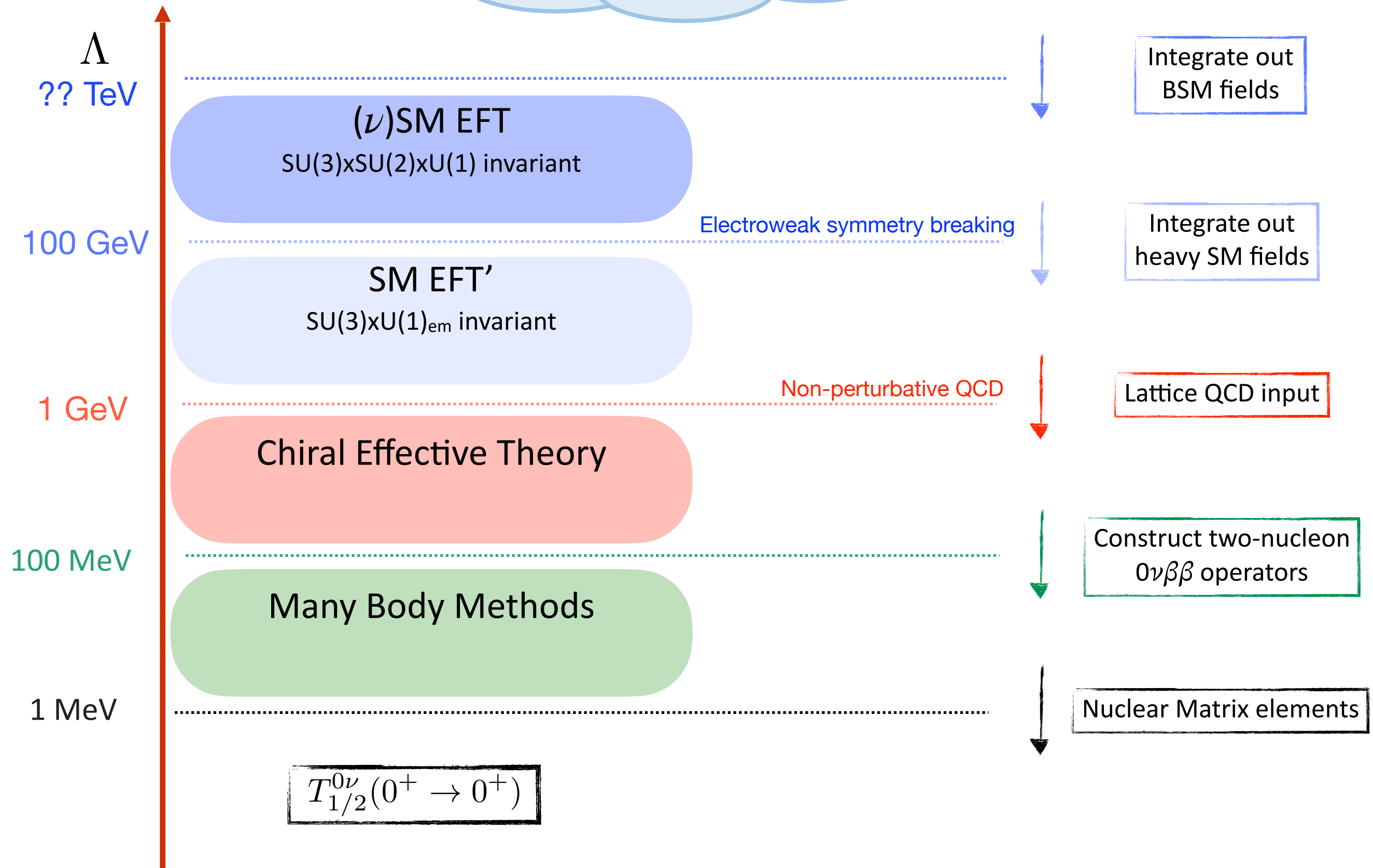
• Many possible scenarios

- Use **Effective Theories**

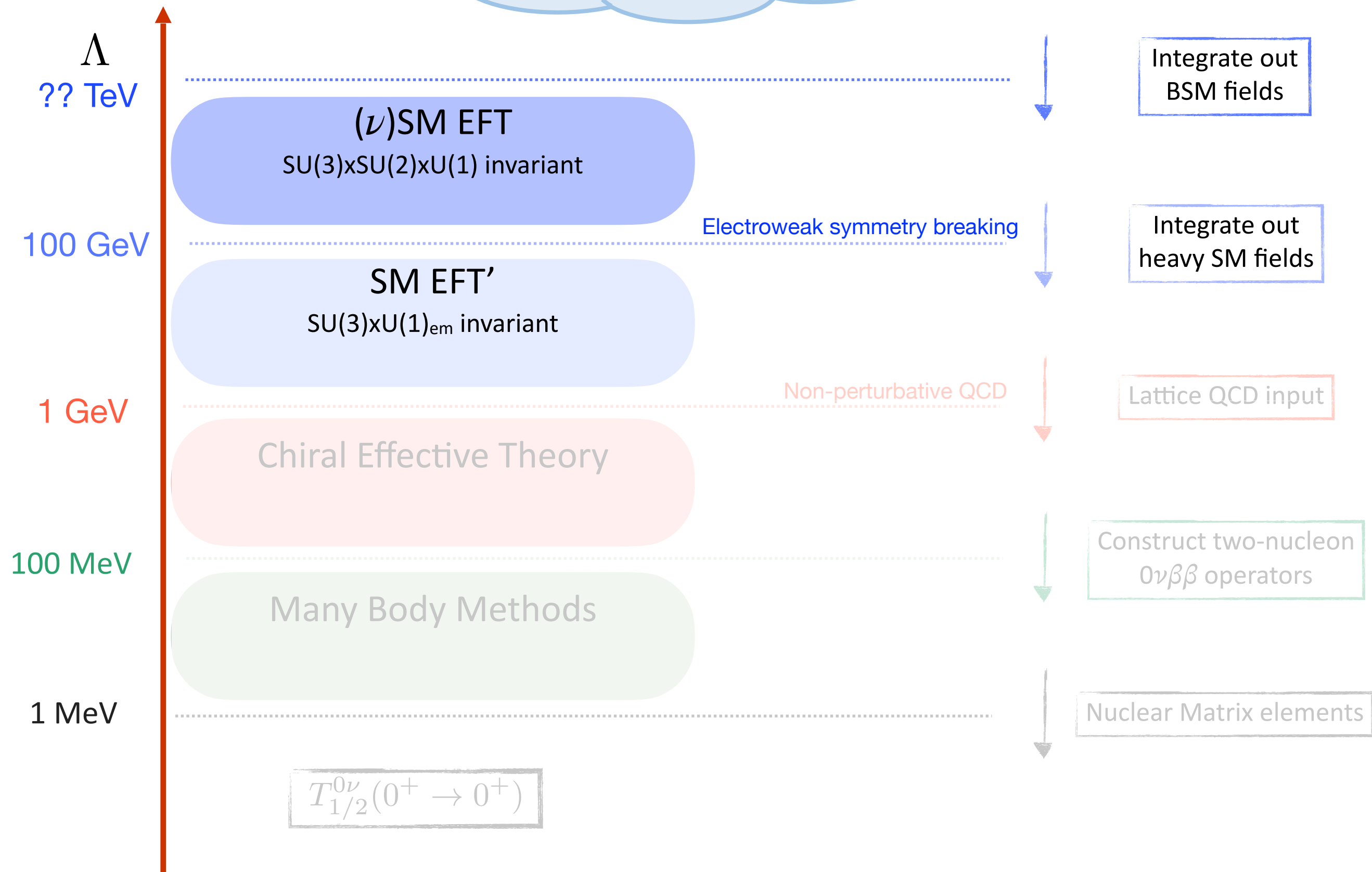
- Simplifies calculations
- Predictions with controlled uncertainties
- Model independent, describes any BSM scenario

• Probes very high BSM scales, $\Lambda \sim 10^{15}$ GeV

Outline

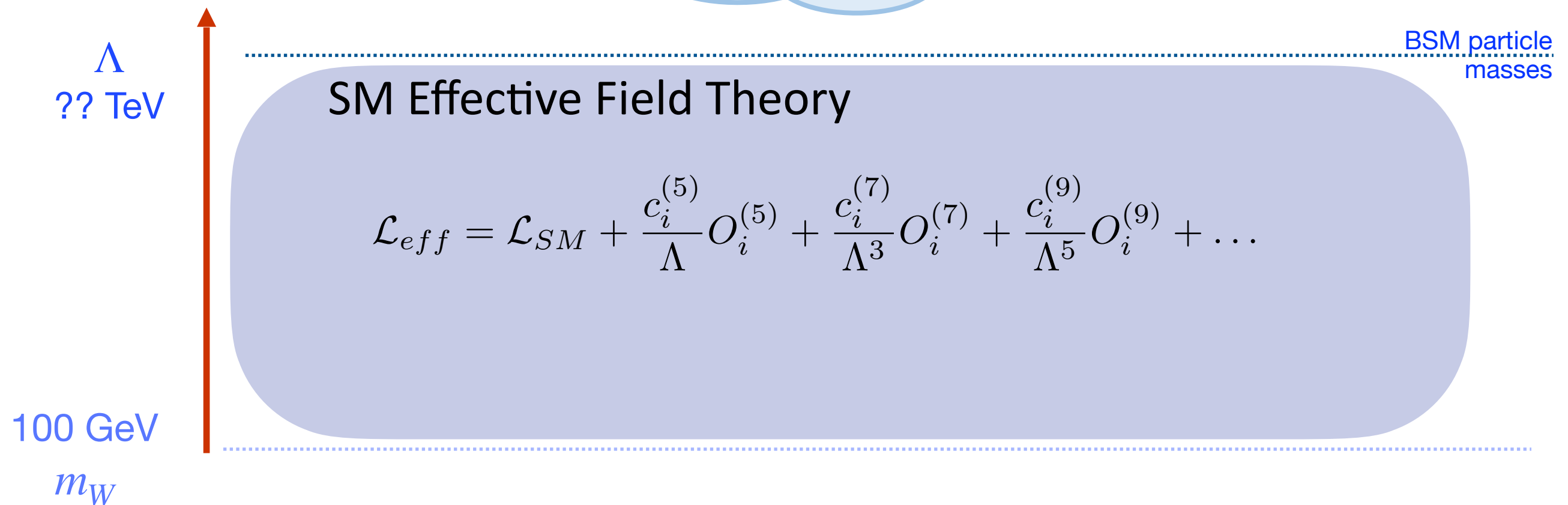


Outline



LNV from heavy BSM physics

Lepton-number violating
BSM physics

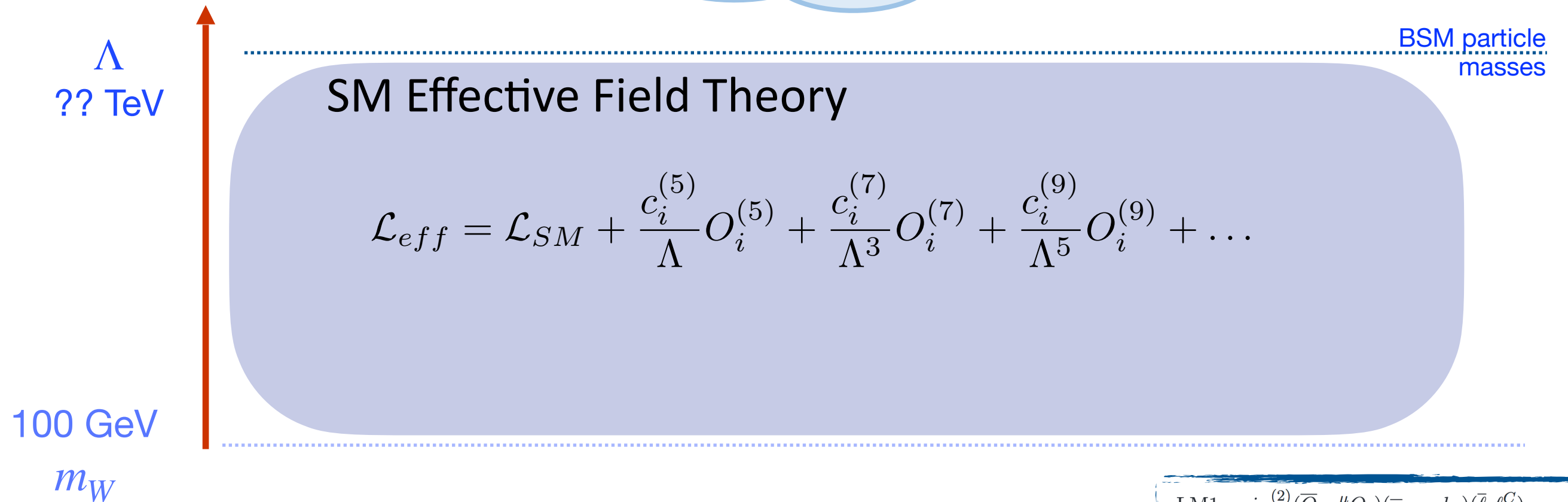


- Consists of the SM + effective interactions
 - Expansion in E/Λ , with E the energy scale of the process
 - Higher dimensional interactions suppressed by $(E/\Lambda)^n$

SMEFT

LNV from heavy BSM physics

Lepton-number violating
BSM physics



- Consists of the SM + effective interactions
 - Expansion in E/Λ , with E the energy scale of the process
 - Higher dimensional interactions suppressed by $(E/\Lambda)^n$

- LNV interactions appear at odd dimensions
- Complete basis known to dimension 9

LM1	$= i\sigma_{ab}^{(2)} (\bar{Q}_a \gamma^\mu Q_c) (\bar{u}_R \gamma_\mu d_R) (\bar{\ell}_b \ell_c^C)$	
LM2	$= i\sigma_{ab}^{(2)} (\bar{Q}_a \gamma^\mu \lambda^A Q_c) (\bar{u}_R \gamma_\mu \lambda^A d_R) (\bar{\ell}_b \ell_c^C)$	
LM3	$= (\bar{u}_R Q_a) (\bar{u}_R Q_b) (\bar{\ell}_a \ell_b^C)$	
LM4	$= (\bar{u}_R \lambda^A Q_a) (\bar{u}_R \lambda^A Q_b) (\bar{\ell}_a \ell_b^C)$	
LM5	$= i\sigma_{ab}^{(2)} i\sigma_{cd}^{(2)} (\bar{Q}_a d_R) (\bar{Q}_c d_R) (\bar{\ell}_b \ell_d^C)$	
LM6	$= i\sigma_{ab}^{(2)} i\sigma_{cd}^{(2)} (\bar{Q}_a \lambda^A d_R) (\bar{Q}_c \lambda^A d_R) (\bar{\ell}_b \ell_d^C)$	
LM7	$= (\bar{u}_R \gamma^\mu d_R) (\bar{u}_R \gamma_\mu d_R) (\bar{e}_R e_R^C)$	
LM8	5 : $\psi^4 D + \text{h.c.}$	
LM9	$\mathcal{O}_{LL\bar{d}uD}^{(1)}$	$\epsilon_{ij} (\bar{d} \gamma_\mu u) (L^i C D^\mu L^j)$
LM10	$\mathcal{O}_{LL\bar{d}uD}^{(2)}$	$\epsilon_{ij} (\bar{d} \gamma_\mu u) (L^i C \sigma^{\mu\nu} D_\nu L^j)$
LM11	$\mathcal{O}_{\bar{L}QddD}^{(1)}$	$(Q C \gamma_\mu d) (\bar{L} D^\mu d)$
	$\mathcal{O}_{\bar{L}QddD}^{(2)}$	$(\bar{L} \gamma_\mu Q) (d C D^\mu d)$
	$\mathcal{O}_{dd\bar{d}eD}$	$(\bar{e} \gamma_\mu d) (d C D^\mu d)$

SMEFT

LNV from heavy BSM physics

Lepton-number violating
BSM physics

BSM particle
masses

Λ
?? TeV

SM Effective Field Theory

$$\mathcal{L}_{eff} = \mathcal{L}_{SM} + \frac{c_i^{(5)}}{\Lambda} O_i^{(5)} + \frac{c_i^{(7)}}{\Lambda^3} O_i^{(7)} + \frac{c_i^{(9)}}{\Lambda^5} O_i^{(9)} + \dots$$

**Majorana mass
Mechanism**

**Other mechanisms
Due to heavy BSM**

100 GeV

m_W

- Consists of the SM + effective interactions
 - Expansion in E/Λ , with E the energy scale of the process
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- LNV interactions appear at odd dimensions
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$$\begin{aligned} \text{LM1} &= i\sigma_{ab}^{(2)} (\bar{Q}_a \gamma^\mu Q_c) (\bar{u}_R \gamma_\mu d_R) (\bar{\ell}_b \ell_c^C) \\ \text{LM2} &= i\sigma_{ab}^{(2)} (\bar{Q}_a \gamma^\mu \lambda^A Q_c) (\bar{u}_R \gamma_\mu \lambda^A d_R) (\bar{\ell}_b \ell_c^C) \\ \text{LM3} &= (\bar{u}_R Q_a) (\bar{u}_R Q_b) (\bar{\ell}_a \ell_b^C) \\ \text{LM4} &= (\bar{u}_R \lambda^A Q_a) (\bar{u}_R \lambda^A Q_b) (\bar{\ell}_a \ell_b^C) \\ \text{LM5} &= i\sigma_{ab}^{(2)} i\sigma_{cd}^{(2)} (\bar{Q}_a d_R) (\bar{Q}_c d_R) (\bar{\ell}_b \ell_d^C) \\ \text{LM6} &= i\sigma_{ab}^{(2)} i\sigma_{cd}^{(2)} (\bar{Q}_a \lambda^A d_R) (\bar{Q}_c \lambda^A d_R) (\bar{\ell}_b \ell_d^C) \\ \text{LM7} &= (\bar{u}_R \gamma^\mu d_R) (\bar{u}_R \gamma_\mu d_R) (\bar{e}_R e_R^C) \end{aligned}$$

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SMEFT

LNV from heavy BSM physics

Why dimension >5?

- Naive scaling:

$$\mathcal{A}_{0\nu\beta\beta} \sim \frac{c_5}{\Lambda} \left[1 + \left(\frac{v}{\Lambda} \right)^2 \frac{c_7}{c_5} + \left(\frac{v}{\Lambda} \right)^4 \frac{c_9}{c_5} \right] \stackrel{?}{=} \frac{c_5}{\Lambda} \left[1 + \mathcal{O}(v^2/\Lambda^2) \right]$$

SMEFT

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Related to dim-5 induced ν masses: $m_\nu \sim c_5 v^2 / \Lambda$

- For natural couplings, $c_5 = \mathcal{O}(1) \implies \Lambda = \mathcal{O}(10^{15} \text{ GeV})$
 - **Dimension-7, -9 irrelevant in this case**
- However, lower BSM scales, $\Lambda \sim (1 - 100) \text{ TeV}$ would imply $c_5 \ll 1$
 - Relative enhancement of higher-dimensional terms due to $c_{7,9}/c_5 \gg 1$

SMEFT

LNV from heavy BSM physics

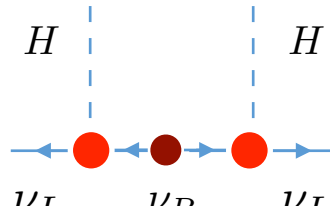
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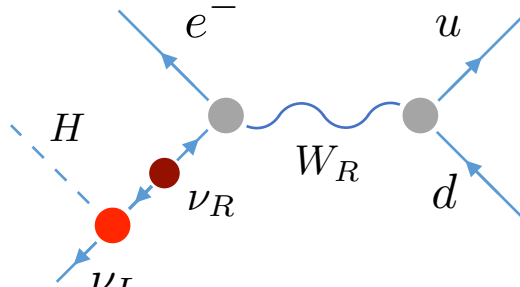
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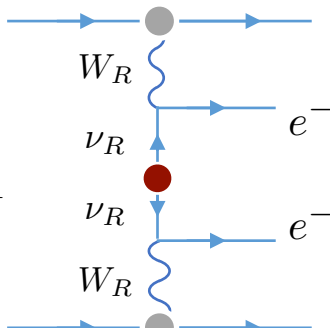
Left-right model example:

$$c_5 \sim y_e^2$$


A Feynman diagram showing a neutrino mass insertion. A horizontal line represents a neutrino, with a red dot in the middle. Above the line, two vertical dashed lines labeled 'H' are connected to the red dot. Below the line, the labels ν_L , ν_R , and ν_L are placed from left to right.

$$c_7 \sim y_e$$


A Feynman diagram showing a neutrino mass insertion and a charged current interaction. A horizontal line represents a neutrino, with a red dot in the middle. Above the line, a blue line labeled e^- enters a vertex, and a blue line labeled u exits. Below the line, a blue line labeled ν_R enters a vertex, and a blue line labeled ν_L exits. A wavy line labeled W_R connects the two vertices. A vertical dashed line labeled 'H' is connected to the red dot.

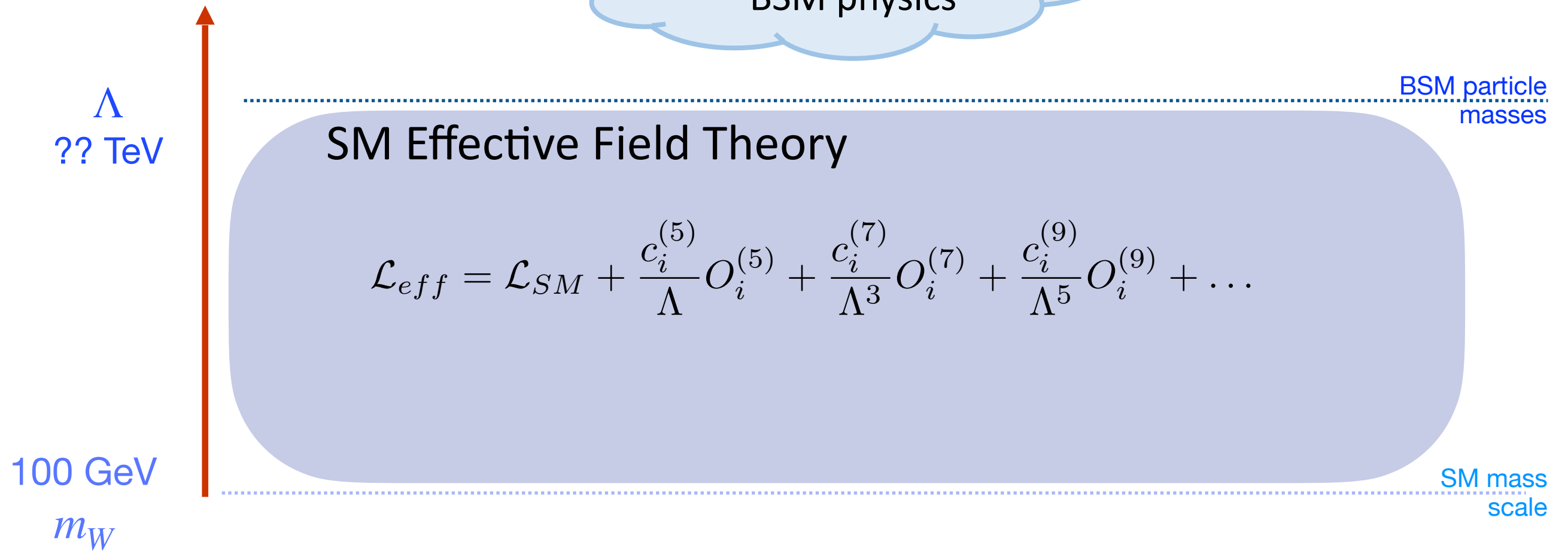
$$c_9 \sim 1$$


A Feynman diagram showing a neutrino mass insertion and a charged current interaction. A horizontal line represents a neutrino, with a red dot in the middle. Above the line, a blue line labeled e^- enters a vertex, and a blue line labeled e^- exits. Below the line, a blue line labeled ν_R enters a vertex, and a blue line labeled ν_R exits. Two wavy lines labeled W_R connect the two vertices.

SMEFT

LNV from heavy BSM physics

Lepton-number violating
BSM physics

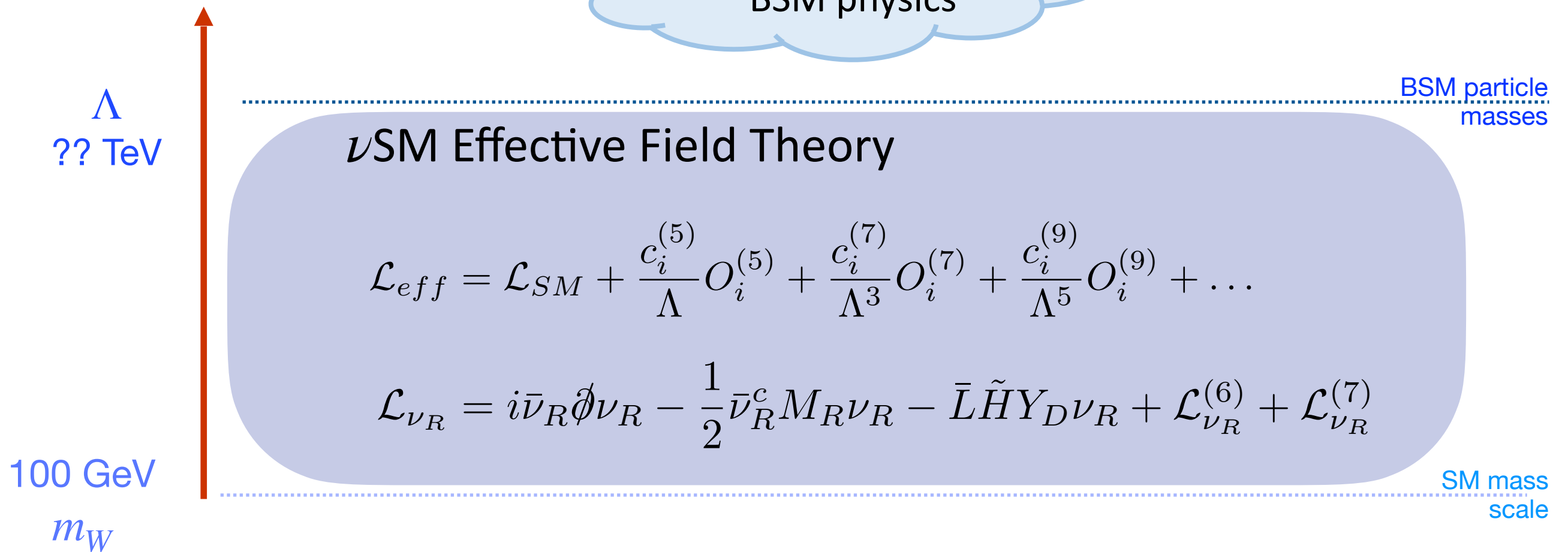


- SMEFT: heavy BSM described by operators with SM fields
 - Complete basis known to dimension 9

SMEFT

LNV from heavy BSM physics

Lepton-number violating
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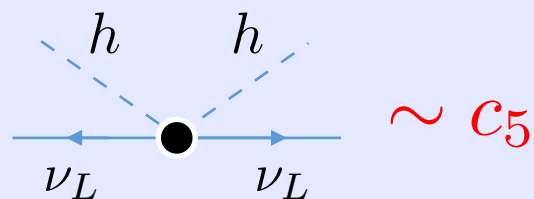


- SMEFT: heavy BSM described by operators with SM fields
 - Complete basis known to dimension 9
- ν SMEFT: extends SMEFT to include sterile neutrinos ν_R
 - Basis known to dimension 7

Low-energy operators

At/below the weak scale*

(ν)SMEFT: SU(3)xSU(2)xU(1) invariant



M_{EW}
 $\sim 100 \text{ GeV}$



Dim-3

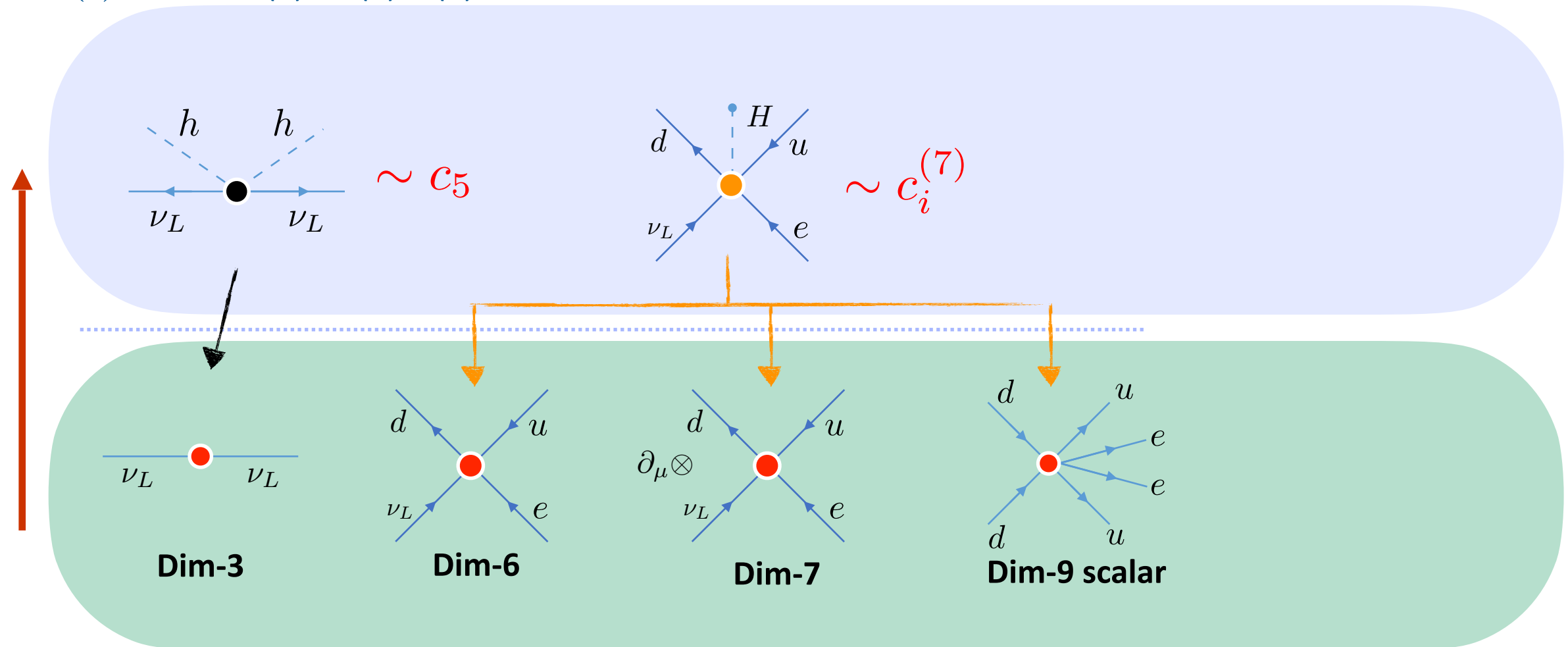
(ν)LEFT: SU(3)xU(1) invariant

* very similar for operators involving ν_R

Low-energy operators

At/below the weak scale*

(ν)SMEFT: SU(3)xSU(2)xU(1) invariant



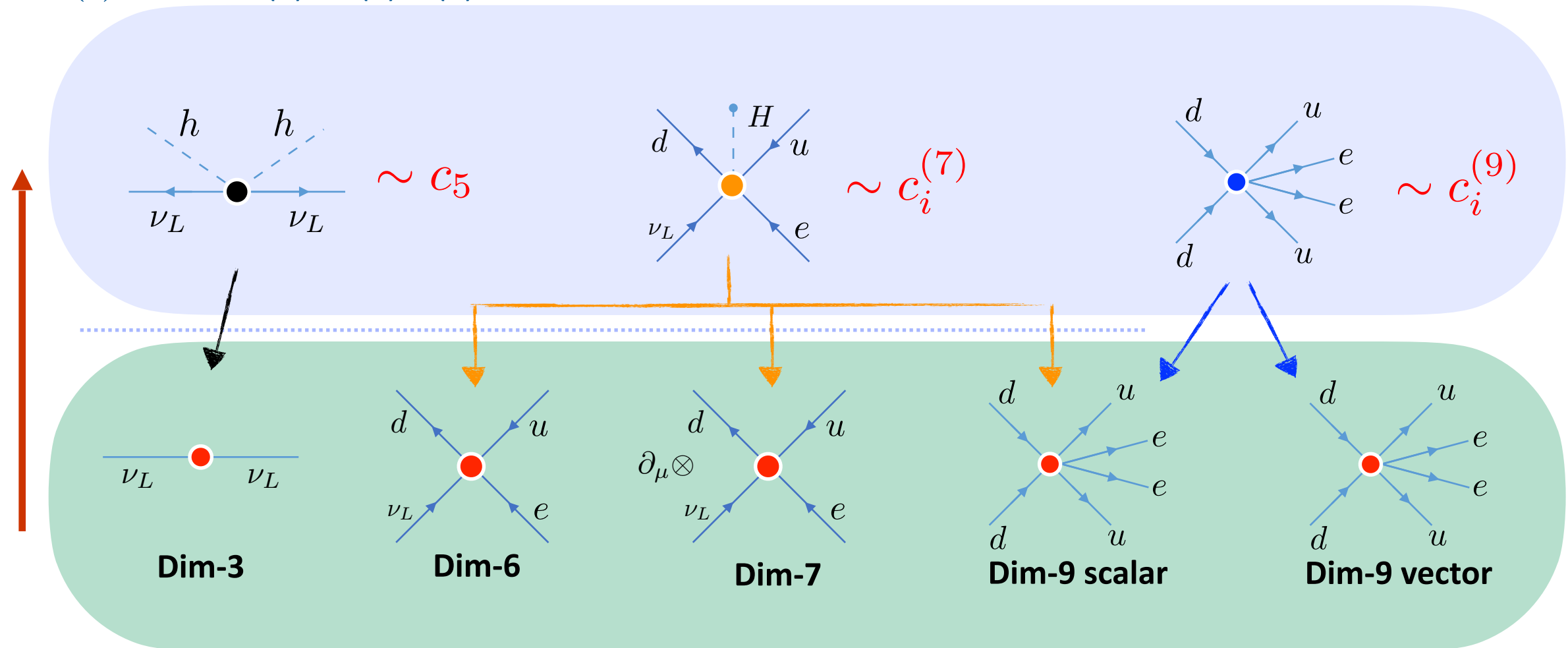
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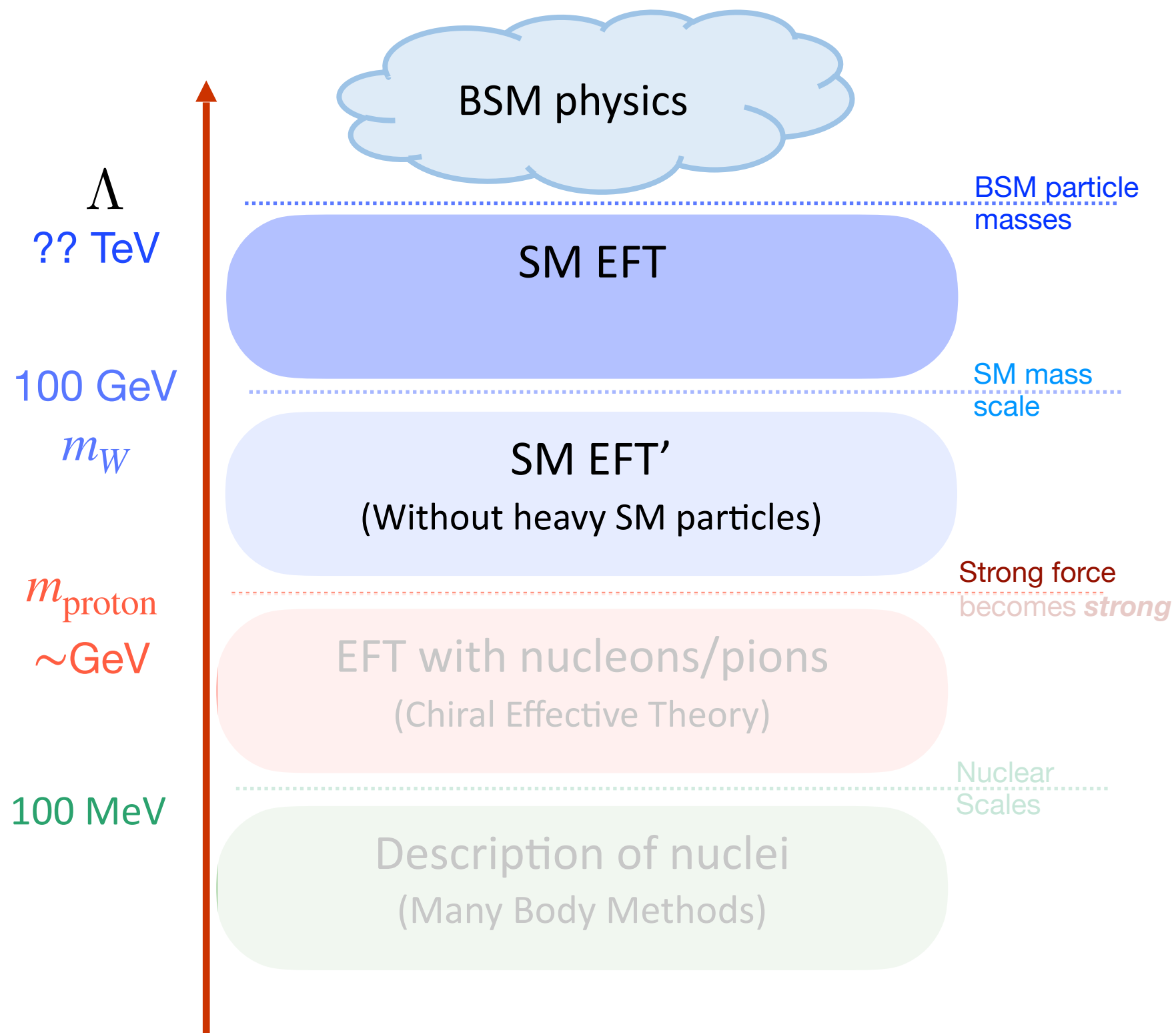
At/below the weak scale*

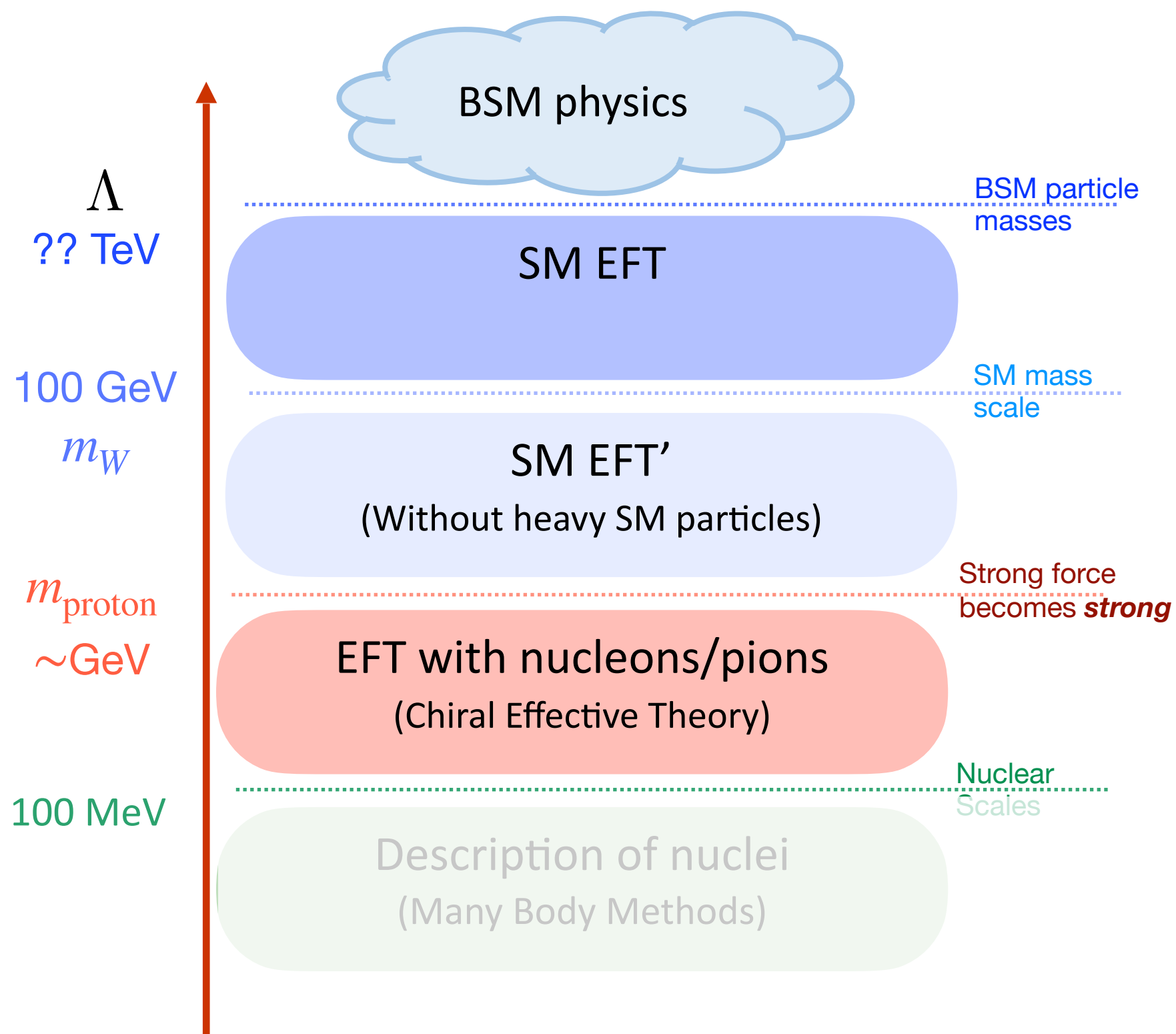
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From quarks to nucleons

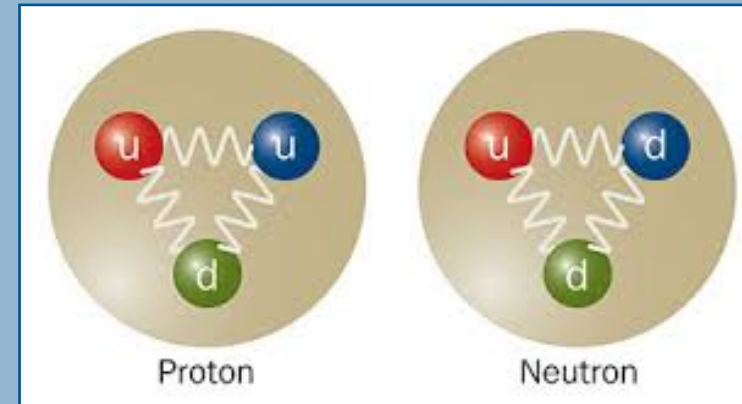
Majorana mass

M_{QCD}
1 GeV



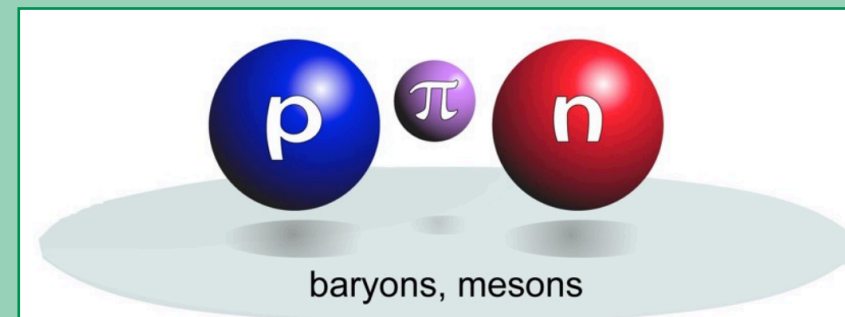
SM Effective Theory

In terms of quarks, gluons, leptons



Chiral Effective Theory

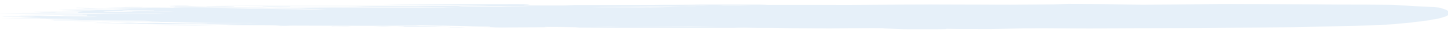
In terms of nucleons, pions, leptons



- Possible hadronic interactions determined by (chiral) symmetries
- Interaction strengths determined by (unknown) coupling constants
- Expansion in $Q/\Lambda_\chi \sim m_\pi/\Lambda_\chi$, where $\Lambda_\chi \sim 1 \text{ GeV}$
 - Need “power counting” to estimate couplings

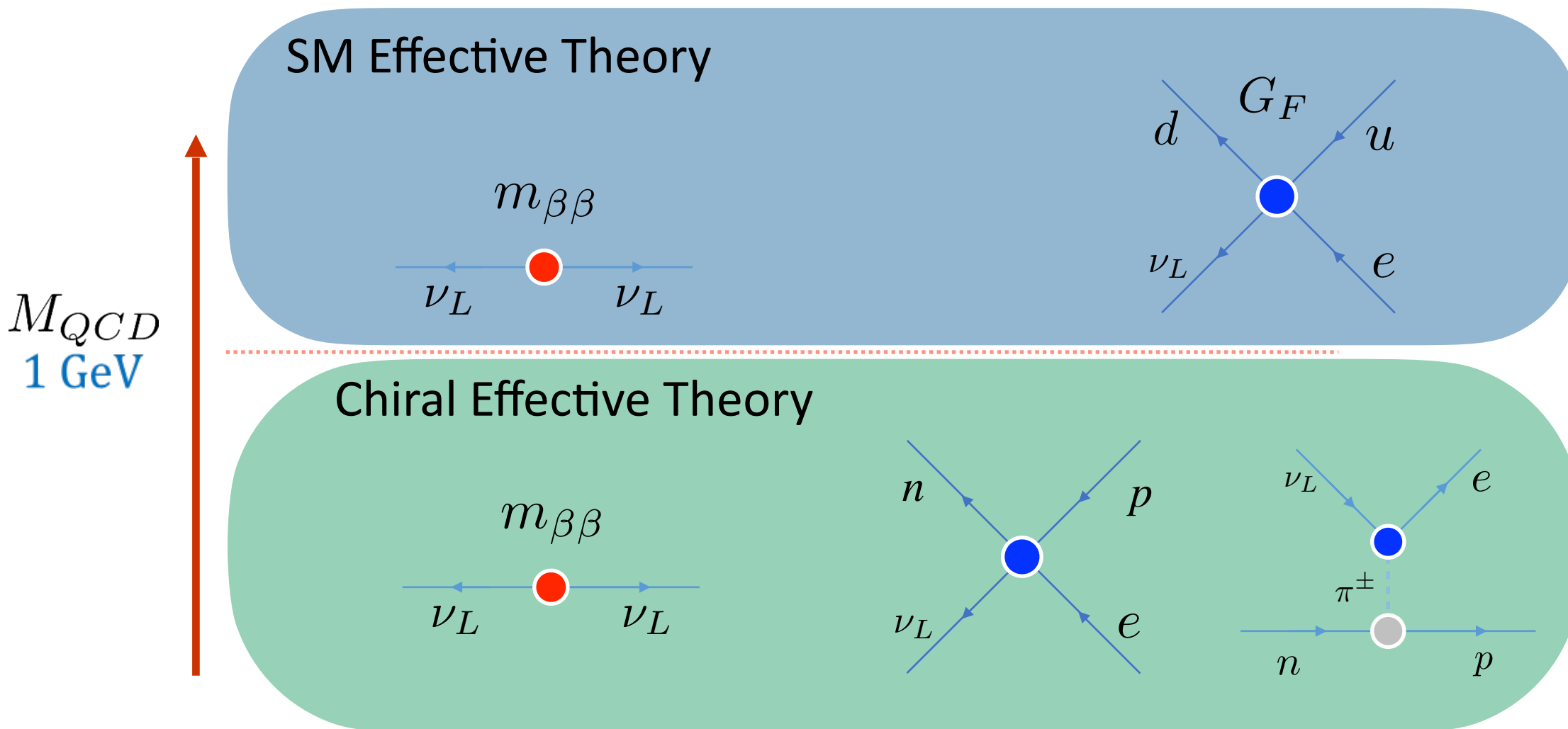
Majorana mass mechanism

Dimension-5 operator



From quarks to nucleons

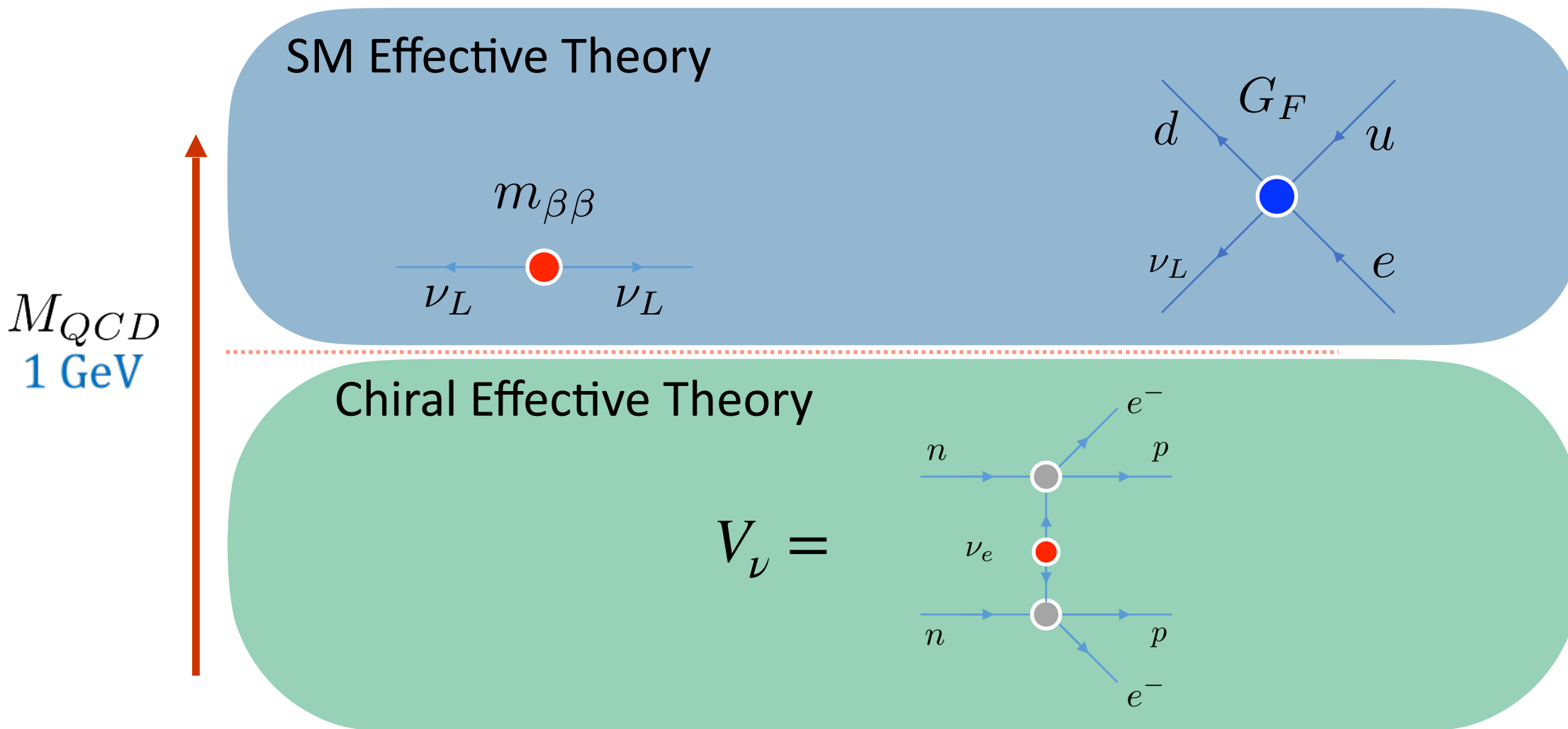
Majorana mass



- At LO in Weinberg counting, only need one-nucleon currents
- Needed low-energy constants are known

From quarks to nucleons

Majorana mass



Decay rate

$$\Gamma_{0\nu\beta\beta} \propto \boxed{M_\nu^2} \boxed{m_{\beta\beta}^2}$$

Fundamental Physics
• Effective neutrino mass

$$m_{\beta\beta} = \sum_i m_i U_{ei}^2$$

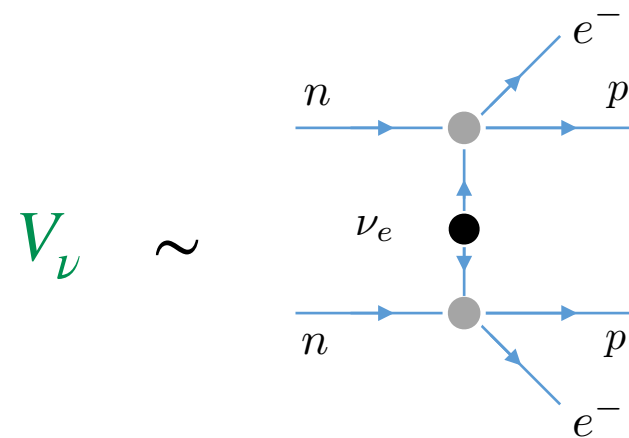
Nuclear Matrix Element $M_\nu \sim \langle \psi_{Se} | V_\nu | \psi_{Ge} \rangle$

Majorana-mass mechanism

Dimension five operator

Theory input: Nuclear Matrix Element

$$M_\nu \sim \langle \psi_{\text{Se}} V_\nu \psi_{\text{Ge}} \rangle$$



Weinberg counting:
No unknown hadronic couplings

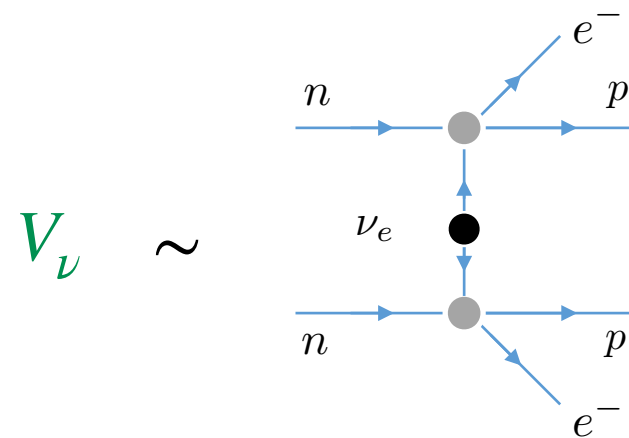
Majorana-mass mechanism

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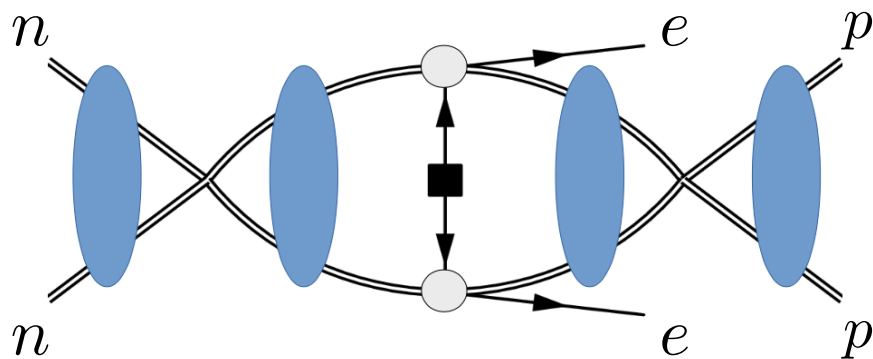
See Bira's talk last Friday

$$M_\nu \sim \langle \psi_{\text{Se}} V_\nu \psi_{\text{Ge}} \rangle$$



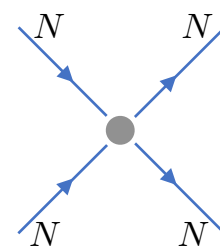
Weinberg counting:
No unknown hadronic couplings

Combined with strong interactions

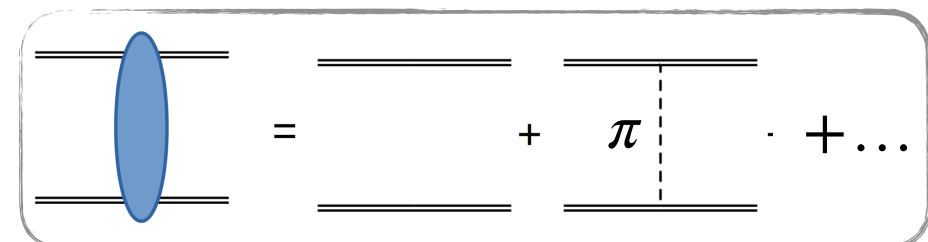


Divergent diagrams $\Rightarrow$ inconsistent theory!

Strong interactions



Short-distance
interaction



Pion exchange

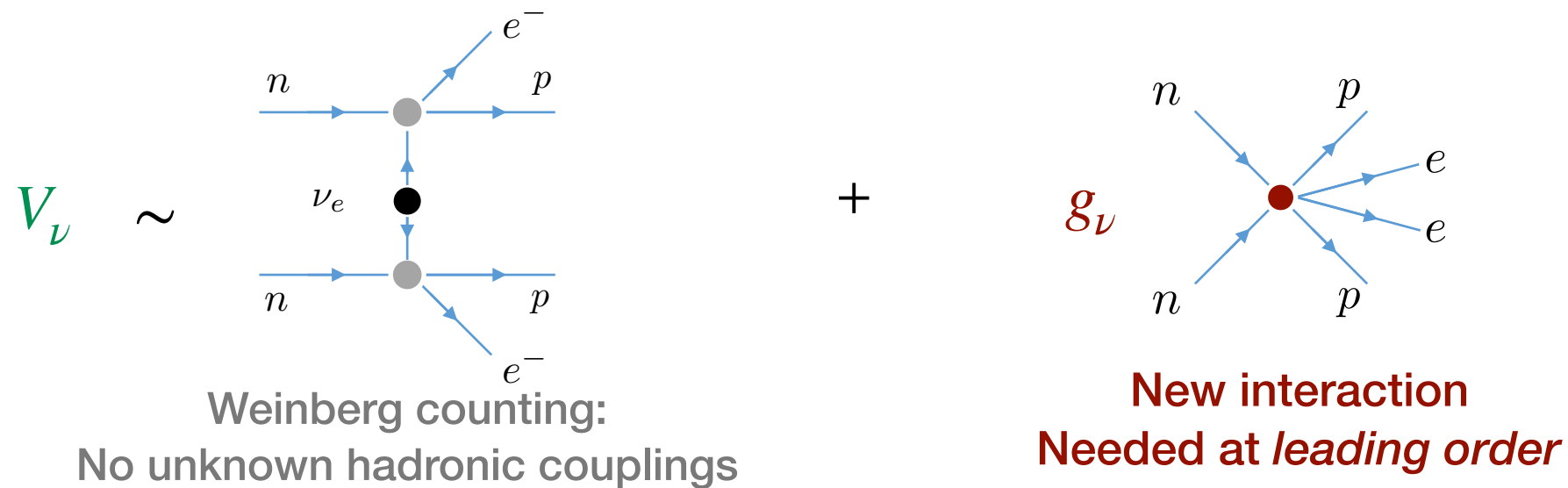
Majorana-mass mechanism

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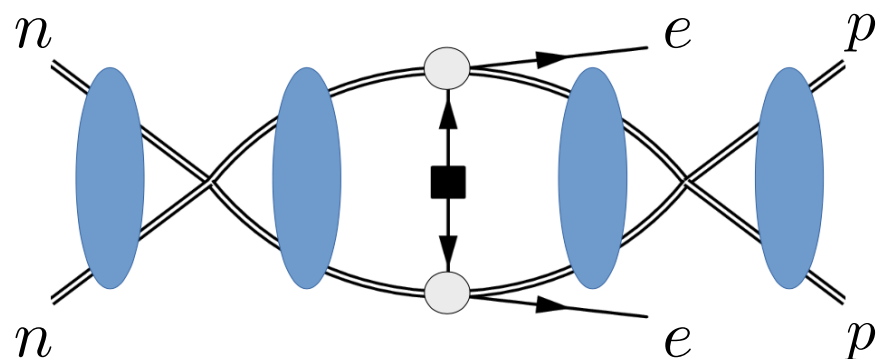
Theory input: Nuclear Matrix Element

See Bira's talk last Friday

$$M_\nu \sim \langle \psi_{Se} \ V_\nu \ \psi_{Ge} \rangle$$

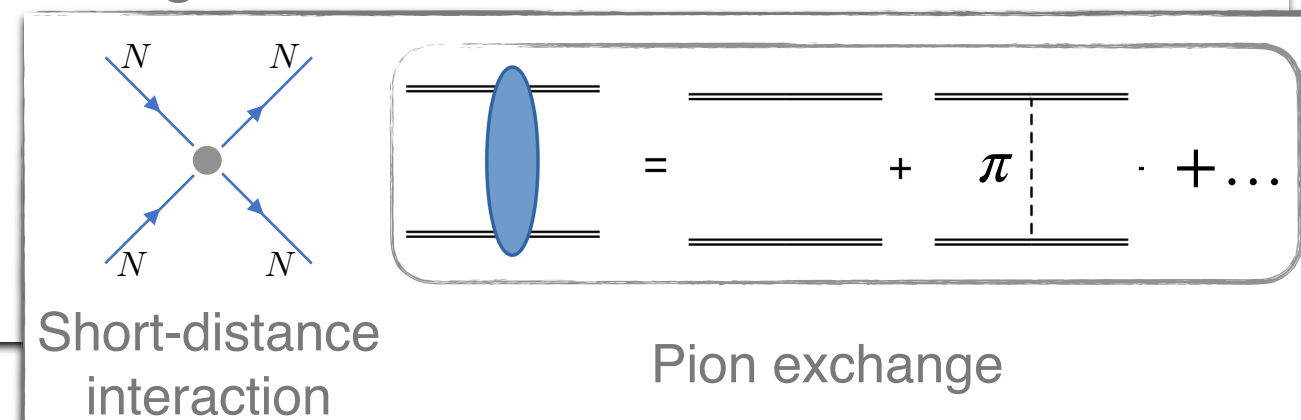


Combined with strong interactions



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Strong interactions



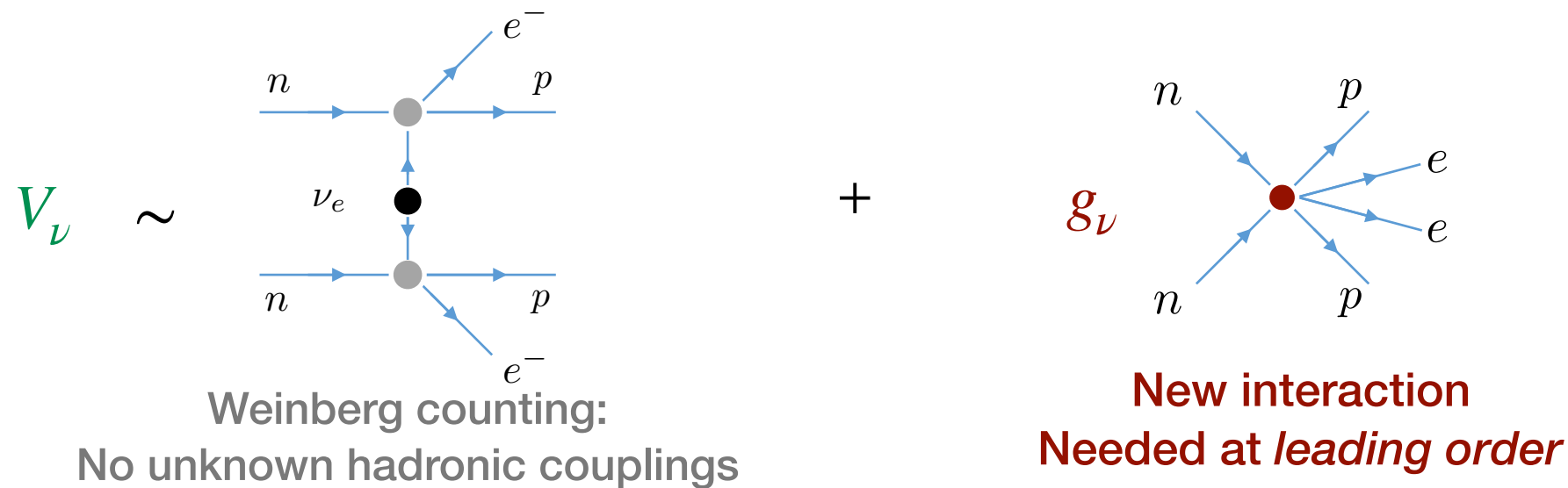
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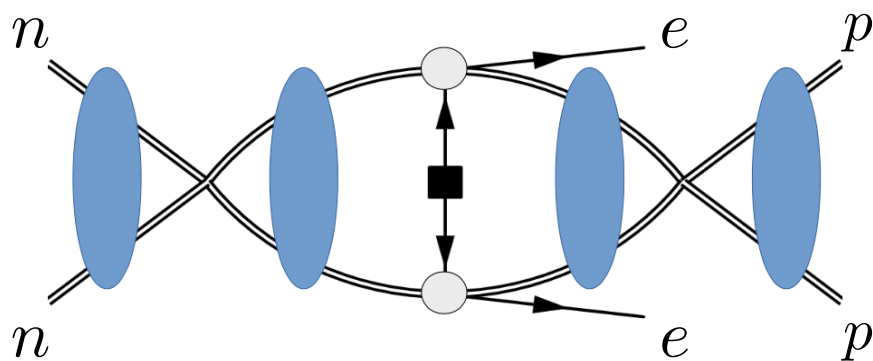
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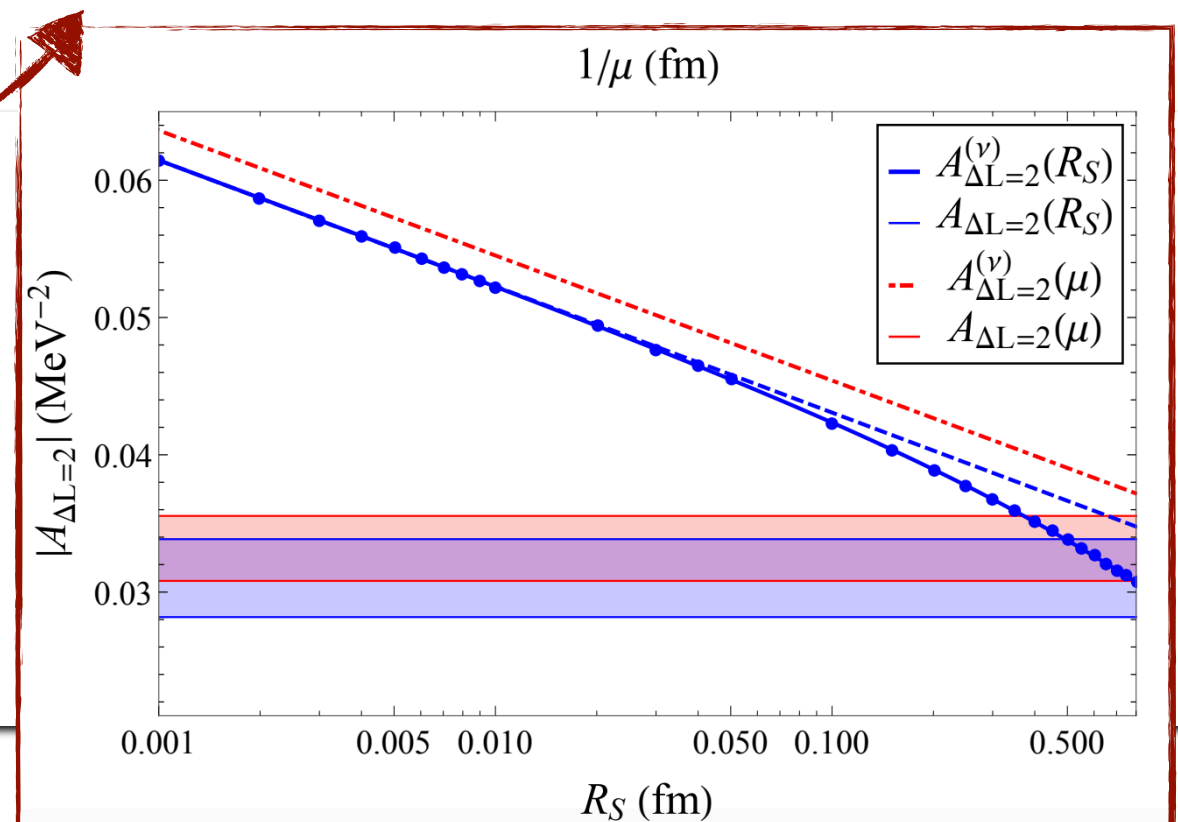
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Combined with strong interactions



Divergent diagrams $\Rightarrow$ inconsistent theory!



Majorana-mass mechanism

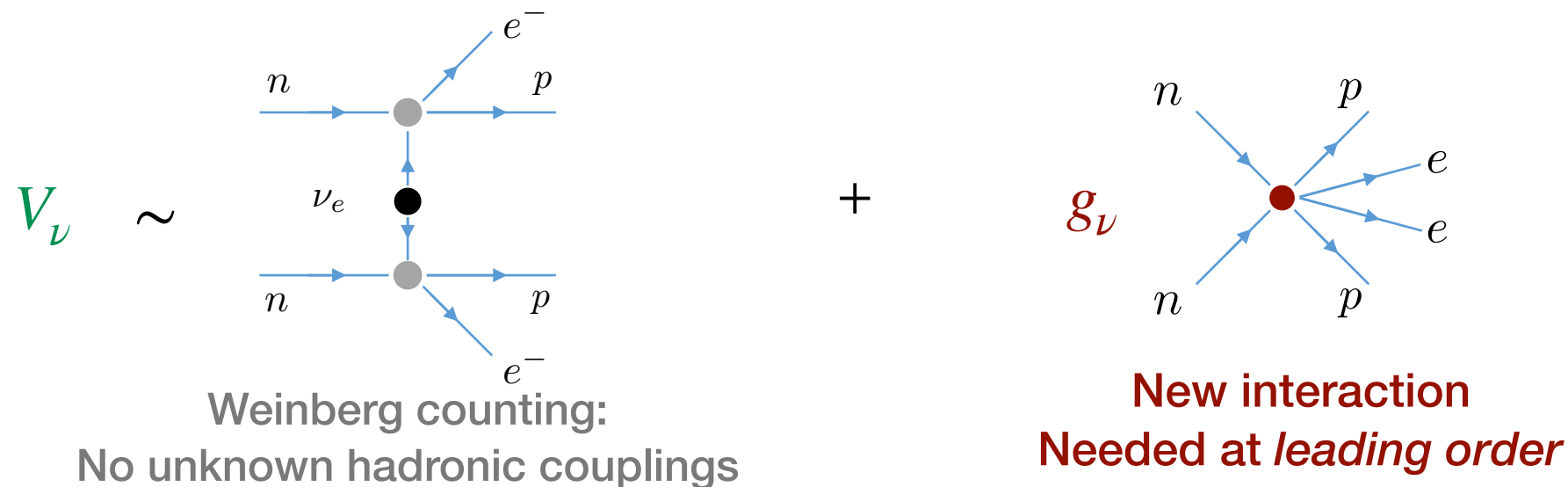
29

Dimension five operator

Theory input: Nuclear Matrix Element

See Bira's talk last Friday

$$M_\nu \sim \langle \psi_{\text{Se}} V_\nu \psi_{\text{Ge}} \rangle$$



Hadronic matrix element g_ν^{NN} is poorly known

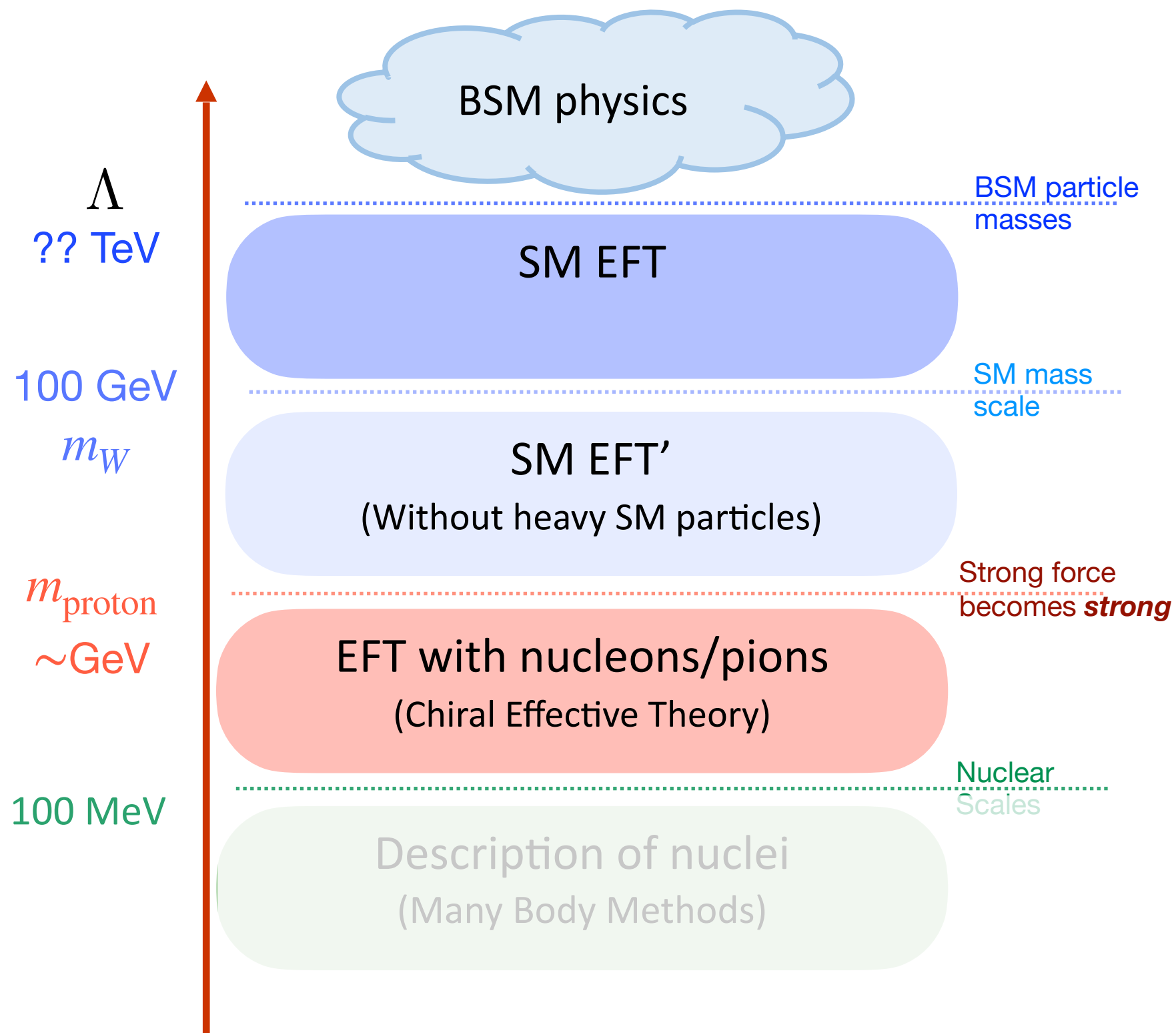
See Anthony's talk

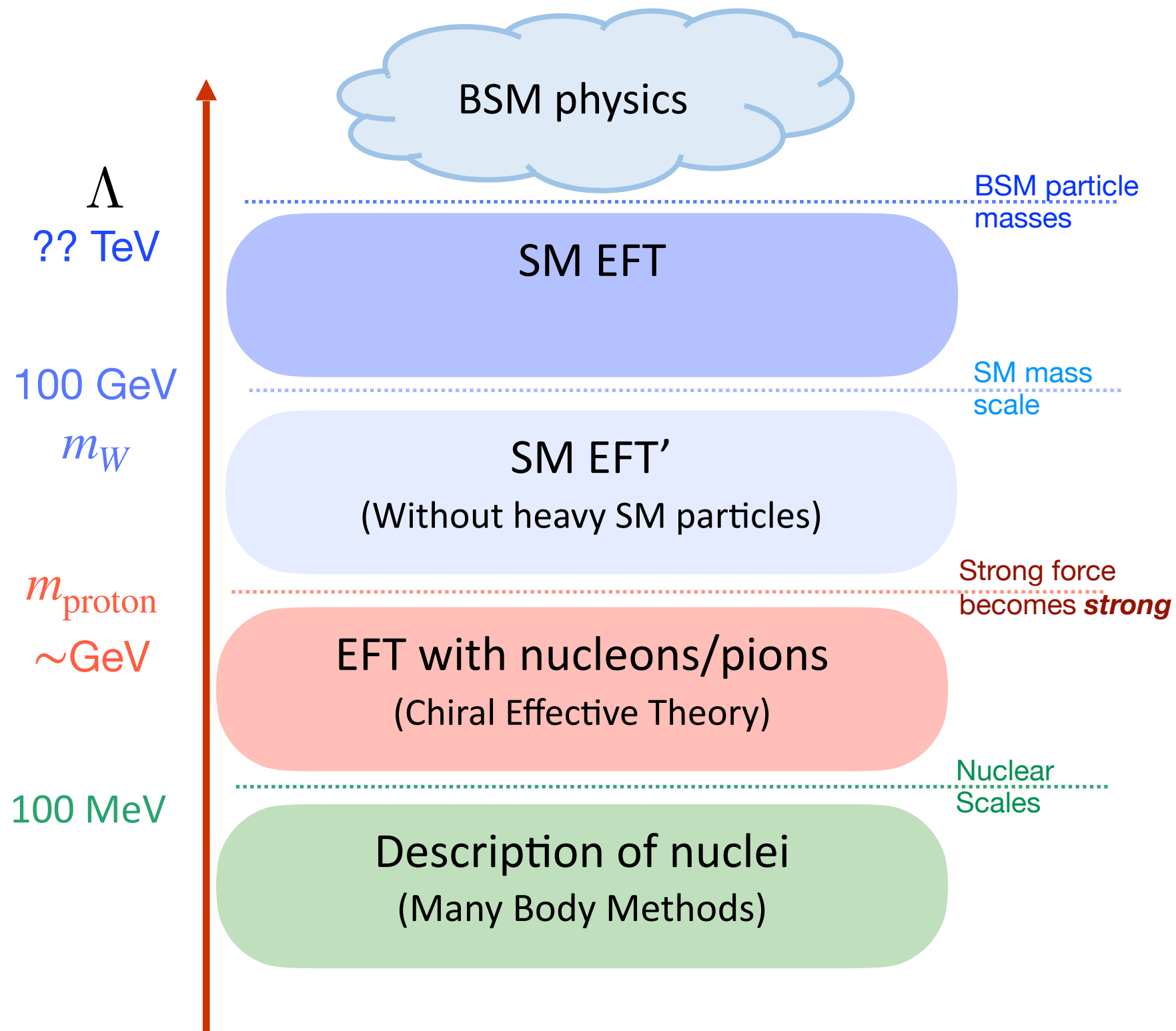
- Could be determined from Lattice QCD
- Active area of research

Davoudi & Kadam, '20, '21
Feng et al, '19; Detmold & Murphy, '20

- Currently only (model) estimates available:
 - Large- N_c , dispersive-like “Cottingham” method, connection to CIB
 - Estimates give $\tilde{g}_\nu^{NN} \sim \mathcal{O}(1)$ with $\sim 30\%$ uncertainty

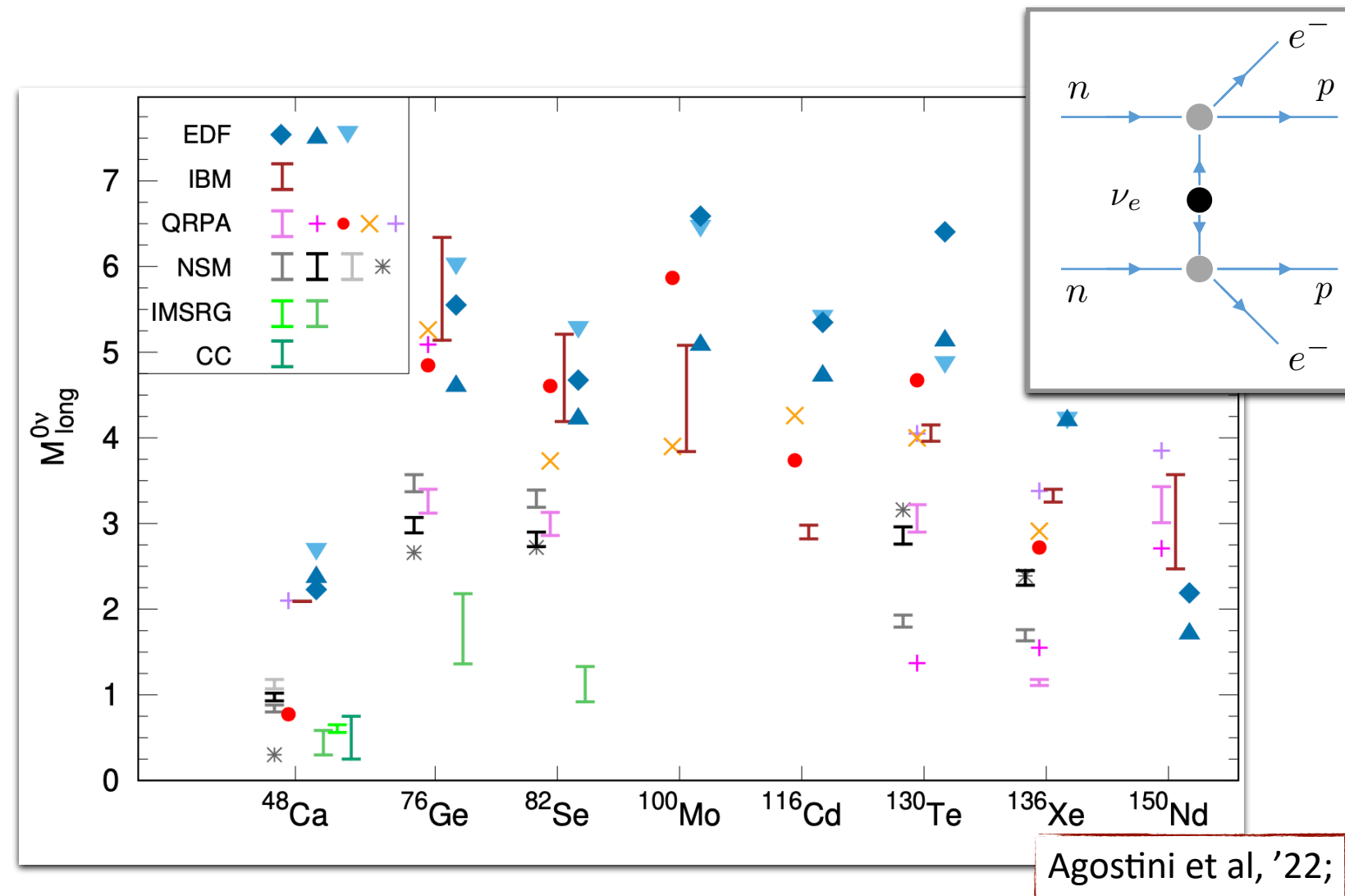
Cirigliano, et al, '19, '20, '21; Richardson et al, '21





Nuclear Matrix Elements

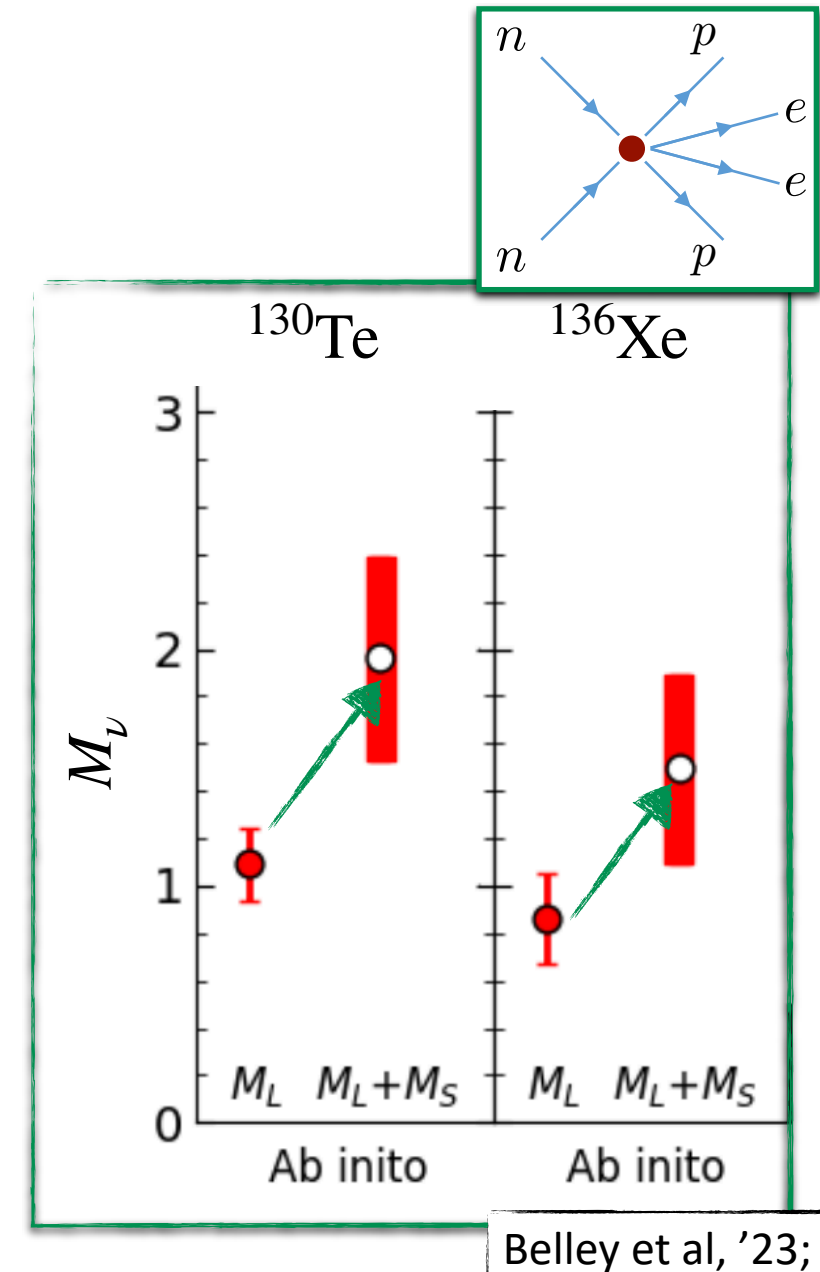
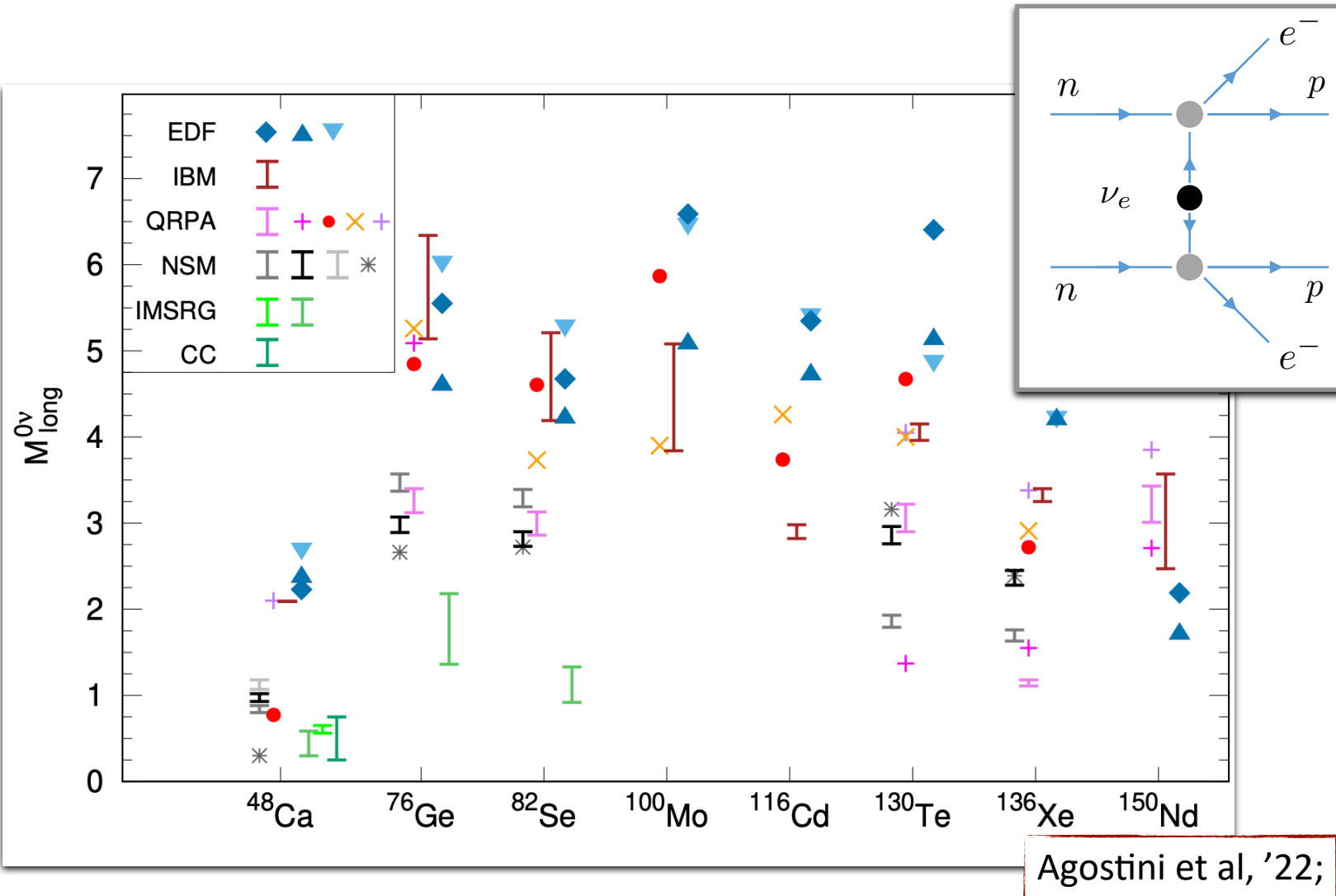
Majorana mass



- Nuclear Matrix Elements differ by a factor 2-3 between methods

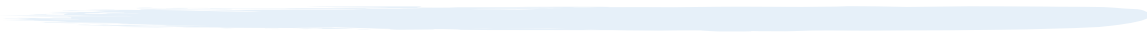
Nuclear Matrix Elements

Majorana mass



- Nuclear Matrix Elements differ by a factor 2-3 between methods
- More recently, *ab initio* calculations appearing
 - Find additive contribution of short-range interaction $\sim g_\nu$

Majorana-mass mechanism Beyond LO



Majorana-mass mechanism

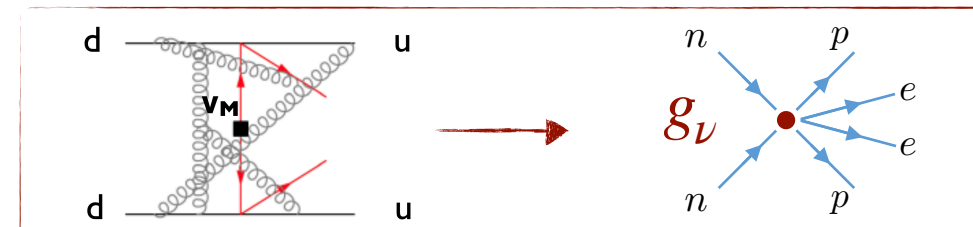
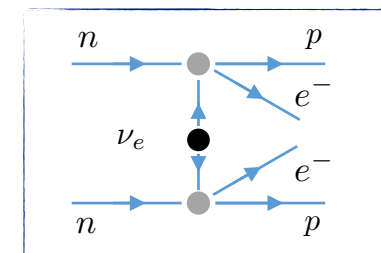
Dimension five operator

- Contributions to A_ν can be divided into ν momentum regions:

$$A_\nu \sim \sum_n \int \frac{d^4 q}{(2\pi)^4} \frac{1}{q^2} \left[\frac{\langle f J_W n \rangle \langle n J_W i \rangle}{q_0 + E_i - E_n + i\epsilon} + \frac{\langle f J_W n \rangle \langle n J_W i \rangle}{-q_0 + E_i - E_n + i\epsilon} \right]$$

Leading order:

- Potential ν s:** $q_0 \ll \vec{q} \sim m_\pi$
 - Leading-order potential
- Hard ν s:** $q_0 \sim \vec{q} \sim \Lambda_\chi \sim \text{GeV}$
 - Contact interaction



Majorana-mass mechanism

Dimension five operator

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NLO:

- No corrections to the $0\nu\beta\beta$ operator

Majorana-mass mechanism

Dimension five operator

- Contributions to A_ν can be divided into ν momentum regions:

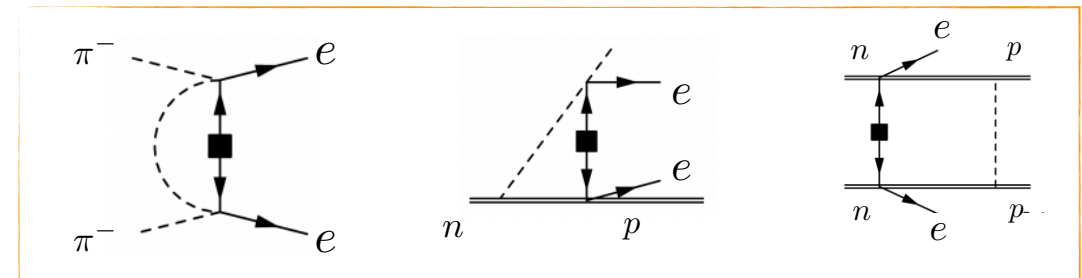
$$A_\nu \sim \sum_n \int \frac{d^4 q}{(2\pi)^4} \frac{1}{q^2} \left[\frac{\langle f J_W n \rangle \langle n J_W i \rangle}{q_0 + E_i - E_n + i\epsilon} + \frac{\langle f J_W n \rangle \langle n J_W i \rangle}{-q_0 + E_i - E_n + i\epsilon} \right]$$

NLO:

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N2LO:

- Soft ν s:** $q_0 \sim \vec{q} \sim m_\pi$
 - Loops & (partially known) counterterms



Majorana-mass mechanism

Dimension five operator

- Contributions to A_ν can be divided into ν momentum regions:

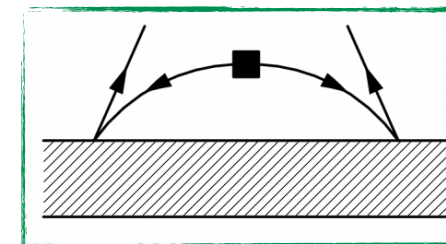
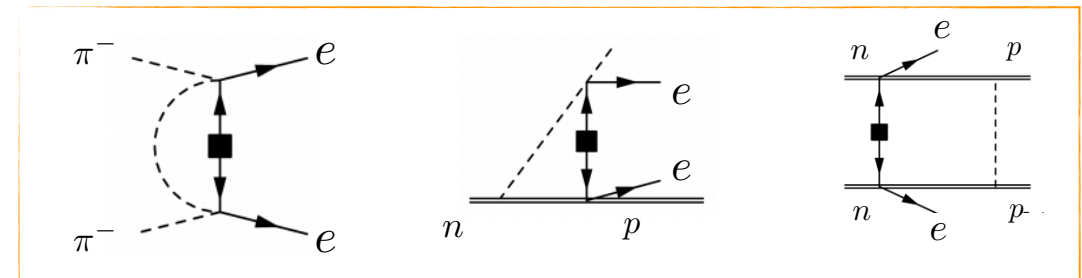
$$A_\nu \sim \sum_n \int \frac{d^4 q}{(2\pi)^4} \frac{1}{q^2} \left[\frac{\langle f J_W n \rangle \langle n J_W i \rangle}{q_0 + E_i - E_n + i\epsilon} + \frac{\langle f J_W n \rangle \langle n J_W i \rangle}{-q_0 + E_i - E_n + i\epsilon} \right]$$

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- Ultrasoft ν s:** $q_0 \sim \vec{q} \sim E_i - E_n$
 - Intermediate state dependence
 - Similar to $2\nu\beta\beta$



Majorana-mass mechanism

Dimension five operator

- Contributions to A_ν can be divided into ν momentum regions:

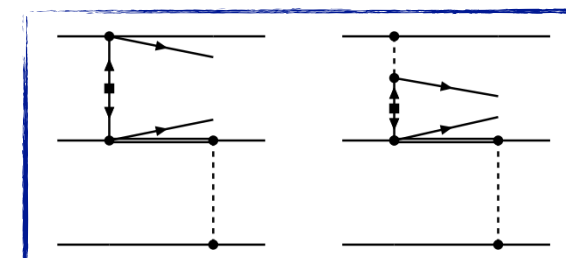
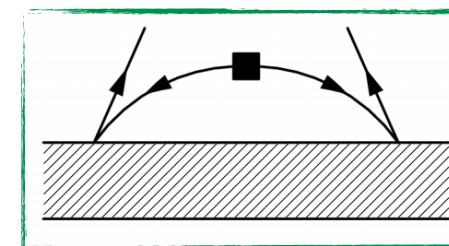
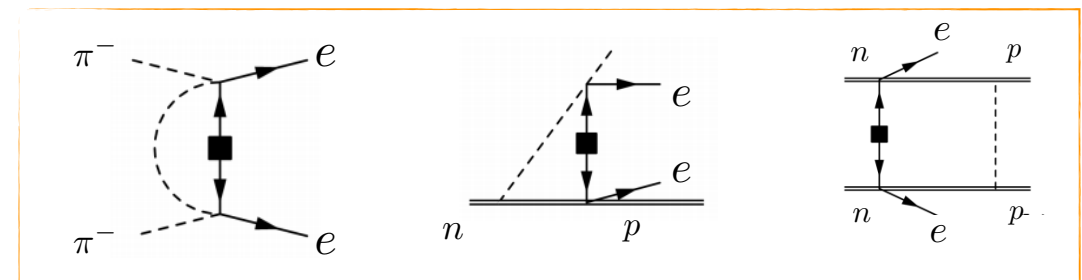
$$A_\nu \sim \sum_n \int \frac{d^4 q}{(2\pi)^4} \frac{1}{q^2} \left[\frac{\langle f J_W n \rangle \langle n J_W i \rangle}{q_0 + E_i - E_n + i\epsilon} + \frac{\langle f J_W n \rangle \langle n J_W i \rangle}{-q_0 + E_i - E_n + i\epsilon} \right]$$

NLO:

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N2LO:

- Soft ν s:** $q_0 \sim \vec{q} \sim m_\pi$
 - Loops & (partially known) counterterms
- Ultrasoft ν s:** $q_0 \sim \vec{q} \sim E_i - E_n$
 - Intermediate state dependence
 - Similar to $2\nu\beta\beta$
- Potential ν s:** $q_0 \ll \vec{q} \sim m_\pi$
 - Induce three-body terms at N2LO (and N3LO)



Majorana-mass mechanism

Dimension five operator

- Contributions to A_ν can be divided into ν momentum regions:

$$A_\nu \sim \sum_n \int \frac{d^4 q}{(2\pi)^4} \frac{1}{q^2} \left[\frac{\langle f | J_W | n \rangle \langle n | J_W | i \rangle}{q_0 + E_i - E_n + i\epsilon} + \frac{\langle f | J_W | n \rangle \langle n | J_W | i \rangle}{-q_0 + E_i - E_n + i\epsilon} \right]$$

NLO:

- No corrections to the $0\nu\beta\beta$ operator

N2LO:

- These N2LO effects have been evaluated in
 - Light nuclei

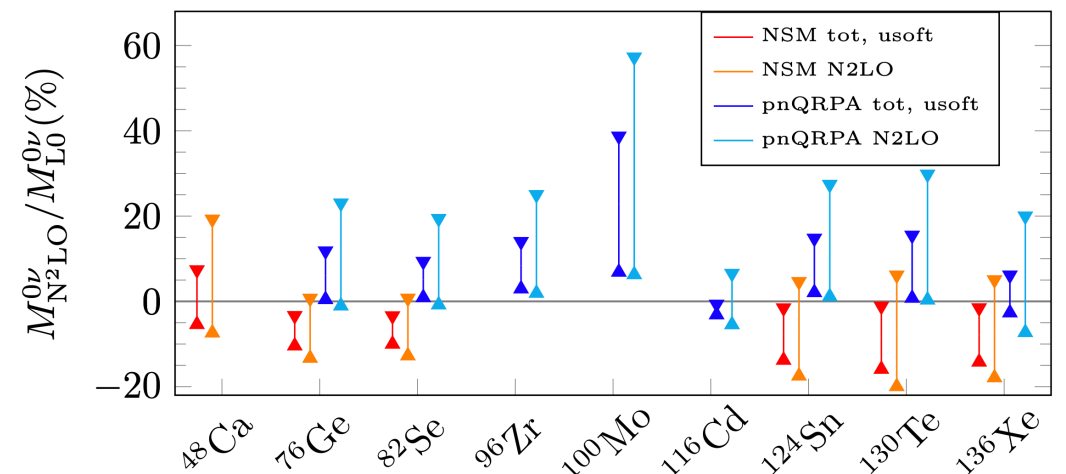
Chambers-Wall et al, '25; Pastore et al '18

- Heavier systems

Belley et al '24; Castillo et al '25

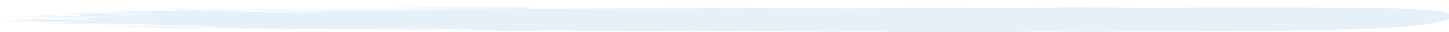
- Found to be up to $\mathcal{O}(30\%)$ of LO NME

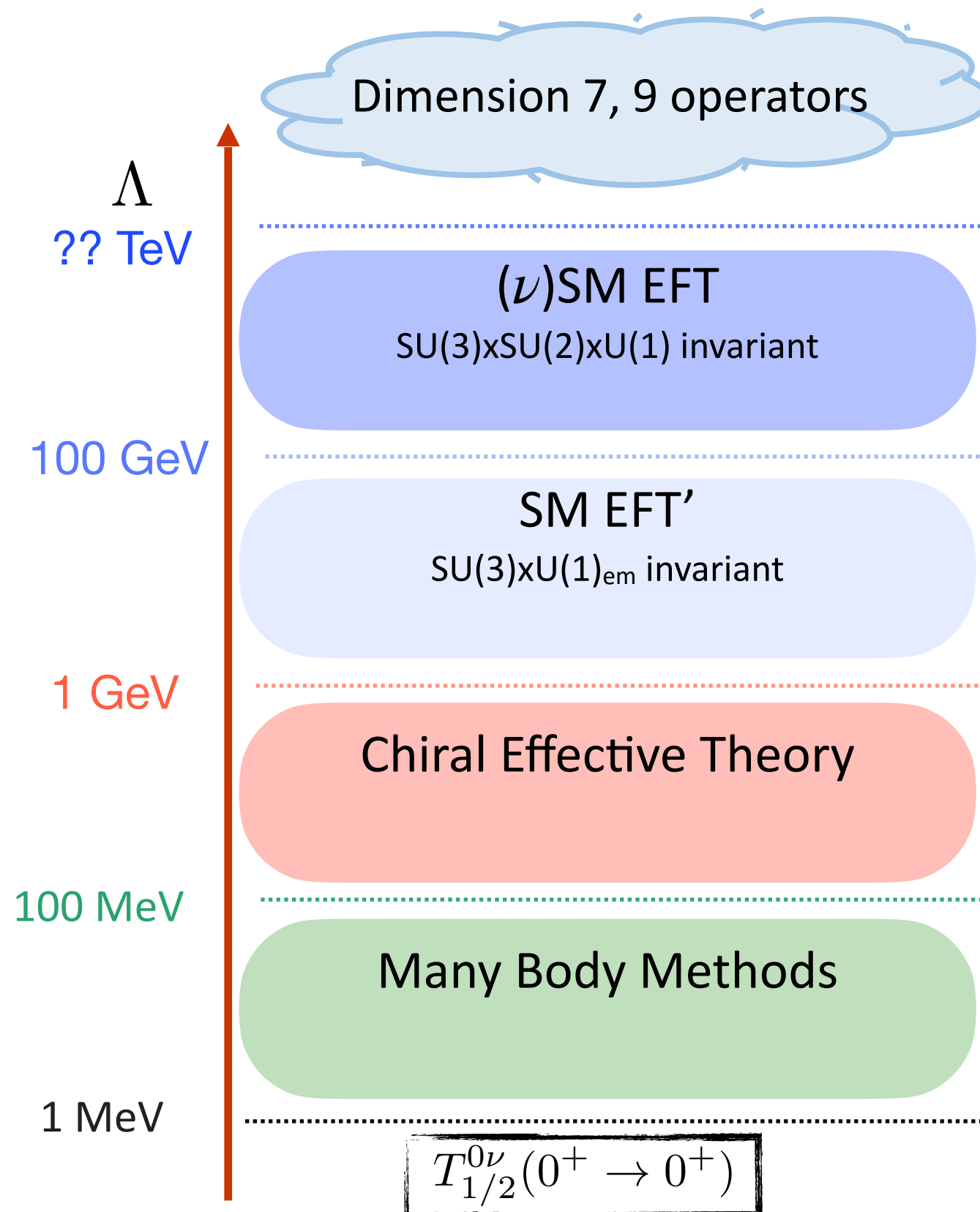
See Daniel's & Amelia's talks



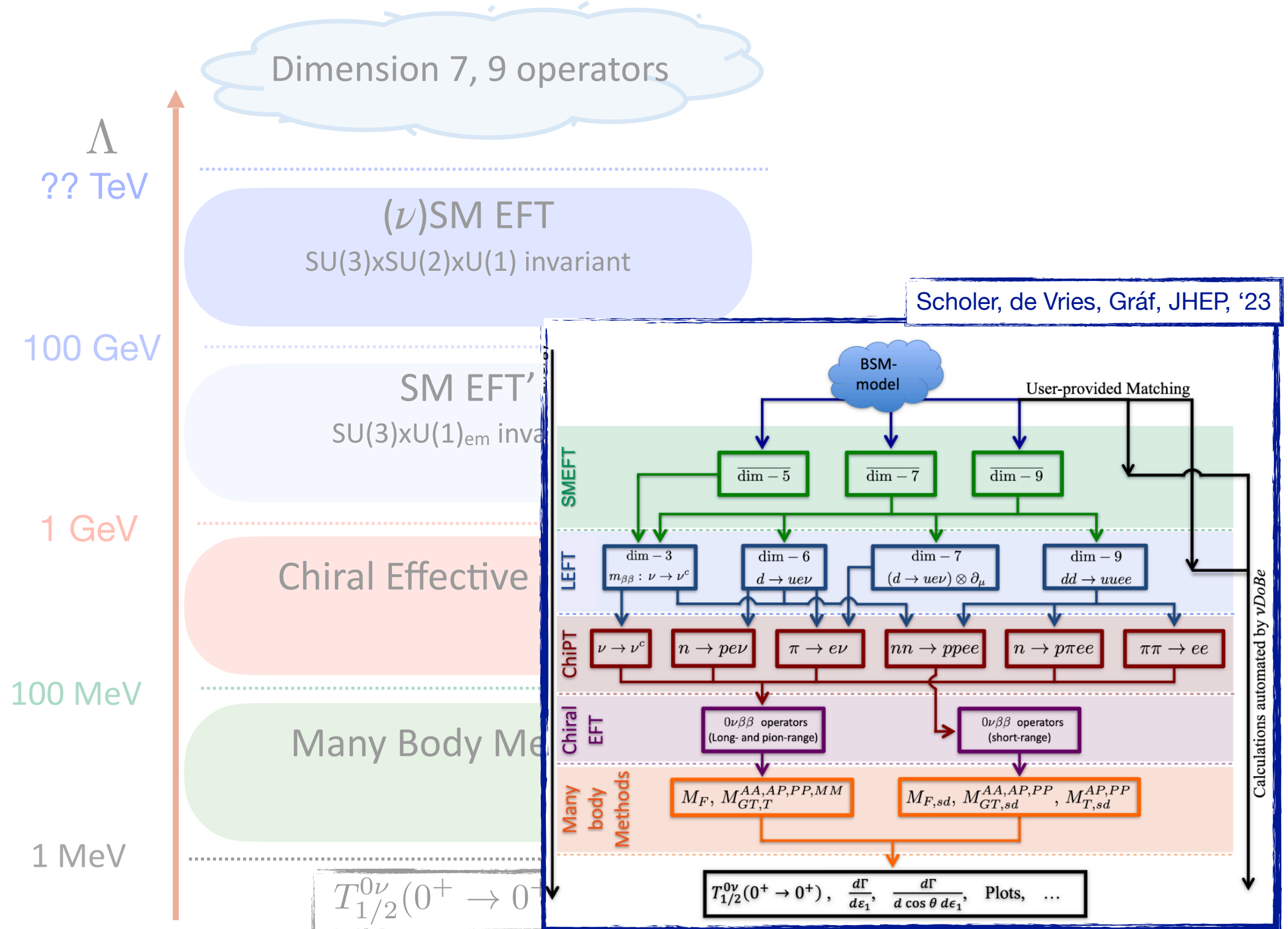
Other mechanisms

Higher-dimensional operators $\sim (m_W/\Lambda)^3, (m_W/\Lambda)^5$





- Can go through the same steps for the higher-dimensional interactions
 - Involves additional hadronic & nuclear matrix elements



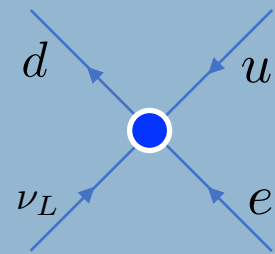
- Can go through the same steps for the higher-dimensional interactions
 - Involves additional hadronic & nuclear matrix elements
- These steps have been automatized: **Python package νDoBe**

Chiral EFT

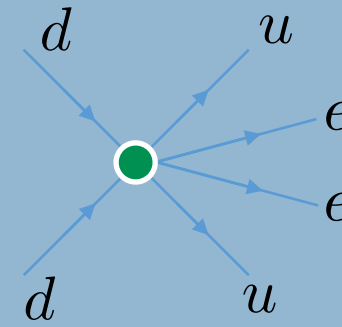
Higher-dimensional interactions

M_{QCD}
1 GeV

SM Effective Theory



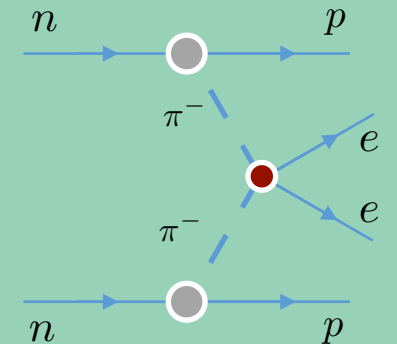
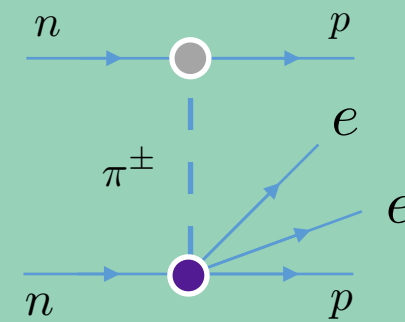
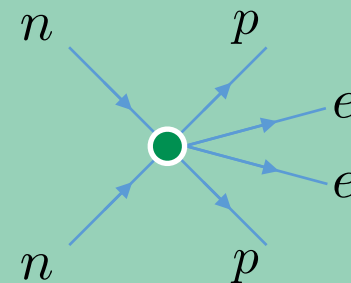
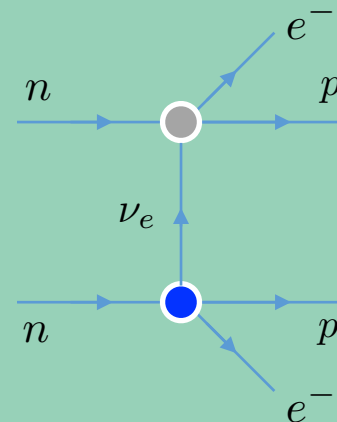
Dim 6 & 7



Dim 9

Chiral Effective Theory

$V_{d=7,9} =$



- Higher-dimensional interactions involve
 - Additional hadronic interactions
 - Only the $\pi\pi$ couplings known

Detmold et al, '22; Nicholson et al.'18
Ramsey-Musolf '03; Cirigliano, (WD) et al. '17

- Can go through same steps for sterile neutrinos

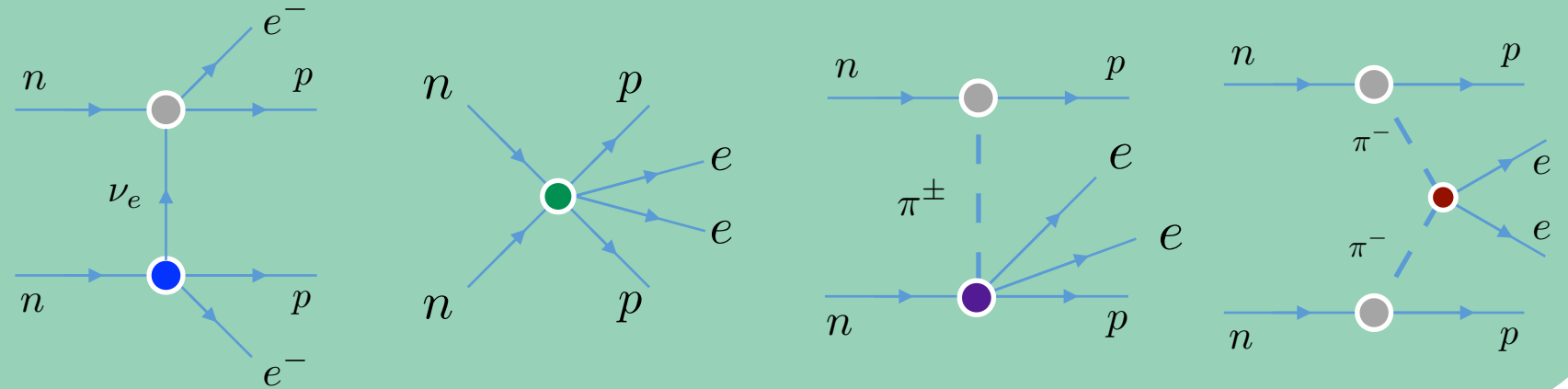
Chiral EFT

Higher-dimensional interactions

M_{QCD}
1 GeV

Chiral Effective Theory

$V_{d=7,9} =$



From nucleons to nuclei

- Requires Nuclear Matrix Elements $M_\nu \sim \langle \psi_{Se} V_{d=7,9} \psi_{Ge} \rangle$
 - In total 15 of them
 - Have been computed in the literature
 - Similar uncertainties as the Majorana-mass mechanism

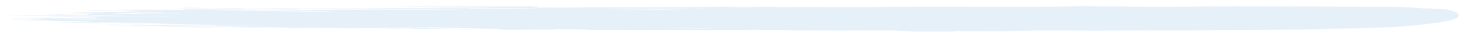
NMEs	⁷⁶ Ge			
	[74]	[31]	[81]	[82, 83]
M_F	-1.74	-0.67	-0.59	-0.68
M_{GT}^{AA}	5.48	3.50	3.15	5.06
M_{GT}^{AP}	-2.02	-0.25	-0.94	-0.92
M_{GT}^{PP}	0.66	0.33	0.30	0.24

NMEs	⁷⁶ Ge			
	[74]	[31]	[81]	[82, 83]
$M_{F, sd}$	-3.46	-1.55	-1.46	-1.1
$M_{GT, sd}^{AA}$	11.1	4.03	4.87	3.62
$M_{GT, sd}^{AP}$	-5.35	-2.37	-2.26	-1.37
$M_{GT, sd}^{PP}$	1.99	0.85	0.82	0.42
$M_{T, sd}^{AP}$	-0.85	0.01	-0.05	-0.97
$M_{T, sd}^{PP}$	0.32	0.00	0.02	0.38

See Alex Todd's talk

Phenomenology

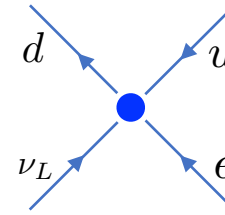
Heavy LNV physics



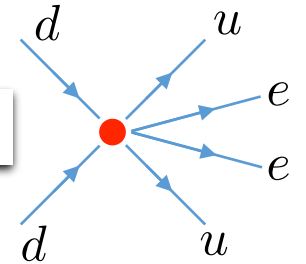
Phenomenology

From heavy new physics

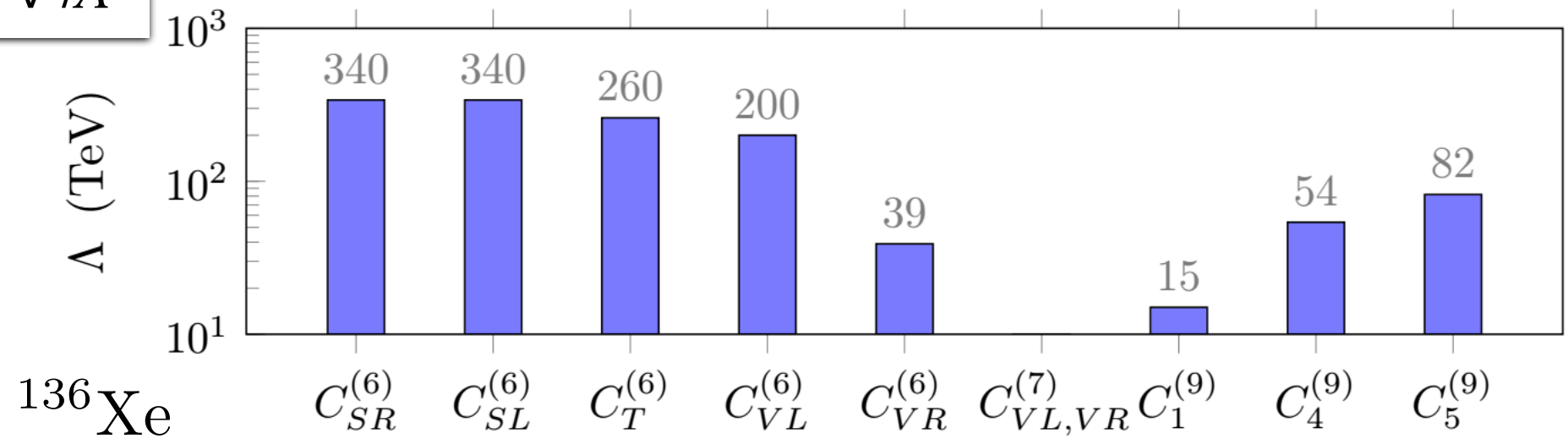
Dim 6 & 7



Dim 9



- Couplings with $C_i \sim v^3/\Lambda^3$

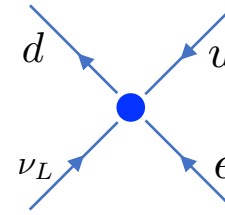


- O(1) uncertainties:
 - Unknown LECs
 - Nuclear Matrix elements

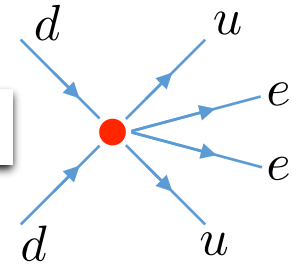
Phenomenology

From heavy new physics

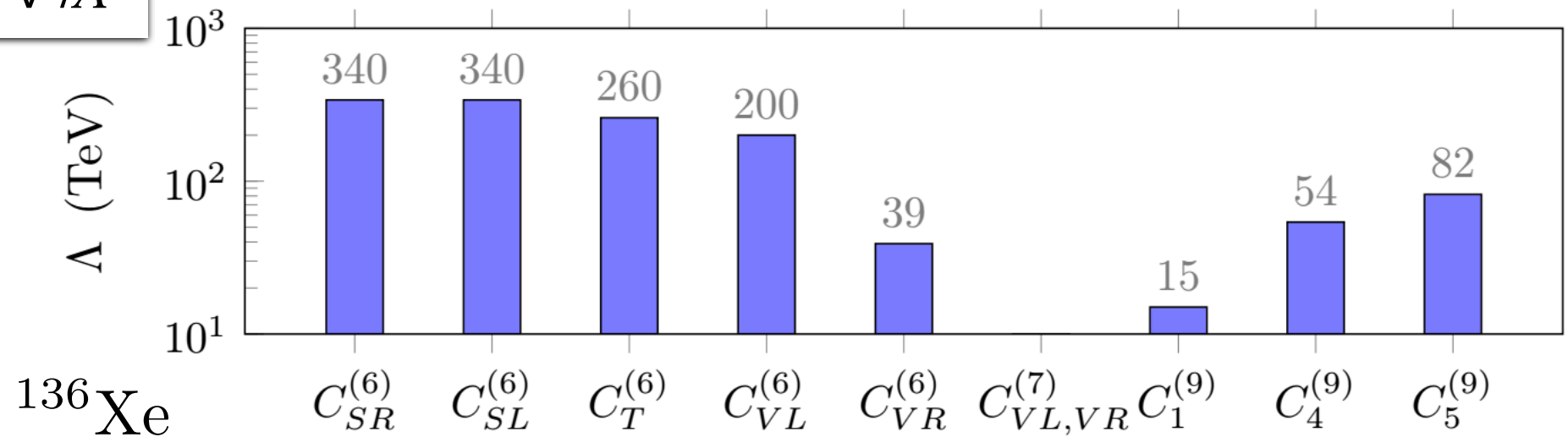
Dim 6 & 7



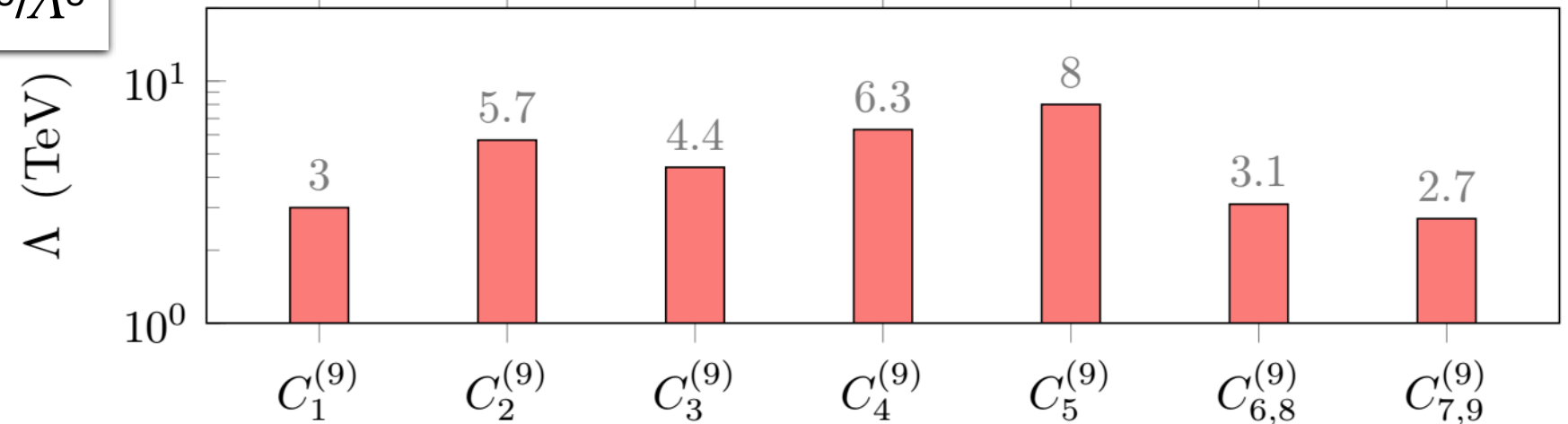
Dim 9



- Couplings with $C_i \sim v^3/\Lambda^3$

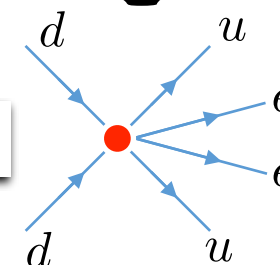


- Couplings with $C_i \sim v^5/\Lambda^5$



- O(1) uncertainties:
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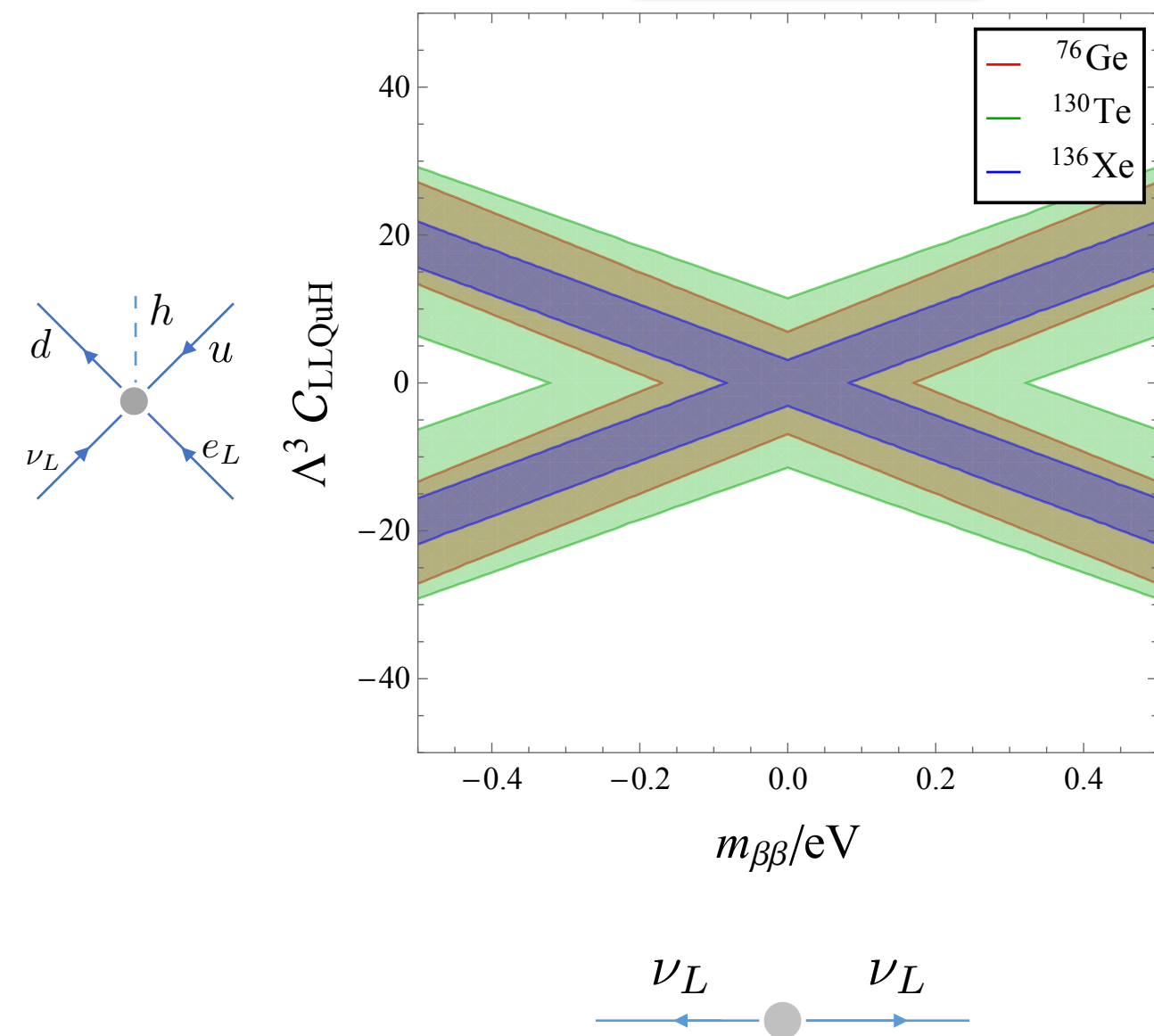
Dim 9



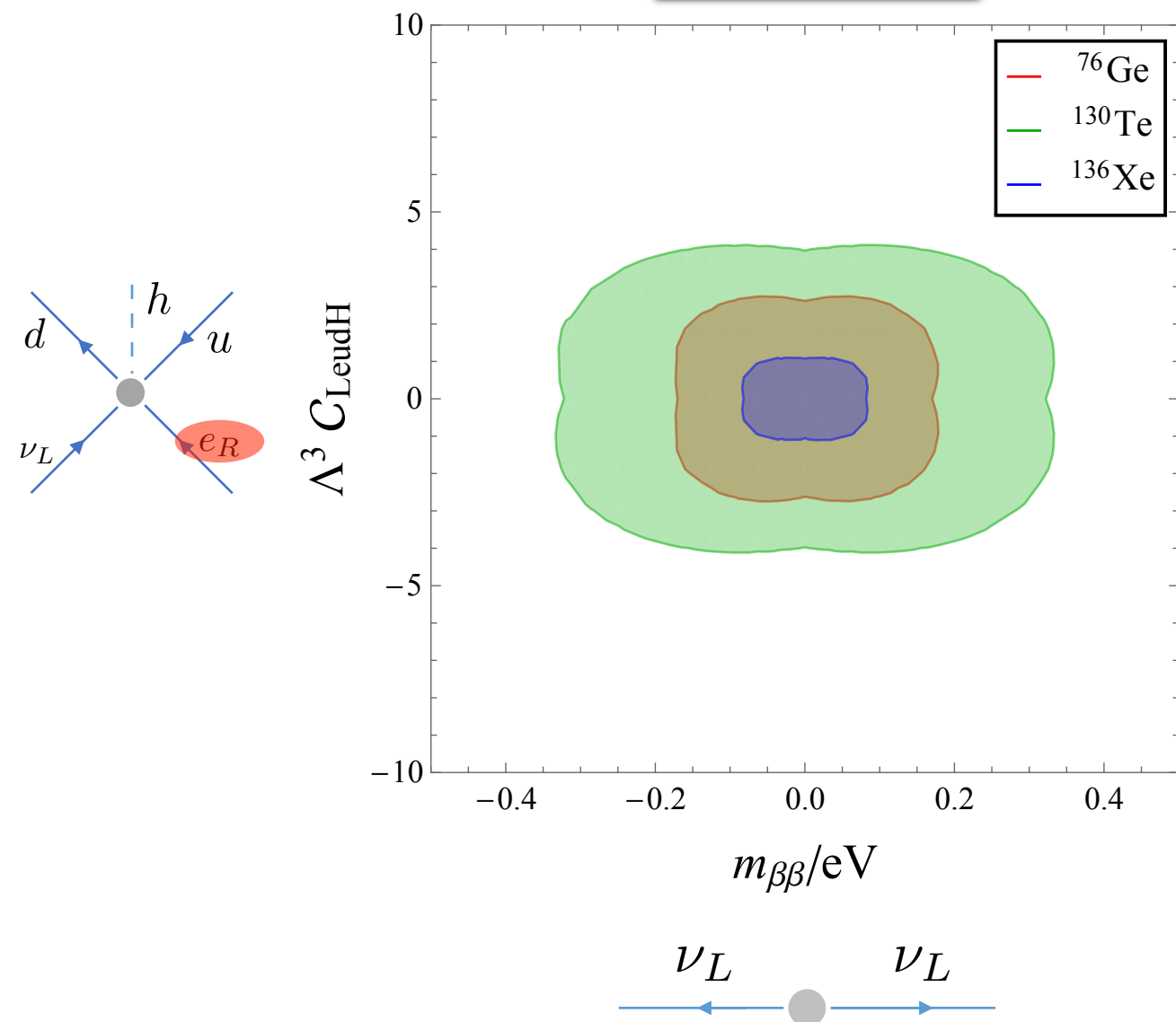
Phenomenology

From heavy new physics

$\Lambda=600$ TeV



$\Lambda=40$ TeV



Light LNV physics

Sterile neutrinos

Based on: 2402.07993, 2303.04168

with

**S. Urrutia Quiroga, D. Castillo, V. Plakkot,
G. Zhou, P. Soriano, J. Menéndez,
V. Cirigliano, J. de Vries, E. Mereghetti**

Effective Field Theory

For heavy $\Delta L = 2$ physics

$$\mathcal{L}_{eff} = \mathcal{L}_{SM} + \frac{c_i^{(5)}}{\Lambda} O_i^{(5)} + \frac{c_i^{(7)}}{\Lambda^3} O_i^{(7)} + \frac{c_i^{(9)}}{\Lambda^5} O_i^{(9)} + \dots$$

For light sterile neutrinos ν_R

- ν_R 's could solve open questions:
 - Neutrino masses
 - Leptogenesis
 - Dark matter candidate

Canetti et al. '13

Boyarski et al. '19

Effective Field Theory

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For light sterile neutrinos ν_R

- ν_R 's could solve open questions:
 - Neutrino masses
 - Leptogenesis
 - Dark matter candidate
- Many BSM scenarios: Left-Right/Leptoquarks/GUTs

Canetti et al. '13

Boyarski et al. '19

Effective Field Theory

For heavy $\Delta L = 2$ physics

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For light sterile neutrinos ν_R

- Can be described in the same framework (ν SMEFT):

$$\mathcal{L}_{\nu_R} = i\bar{\nu}_R \not{\partial} \nu_R - \frac{1}{2} \bar{\nu}_R^c M_R \nu_R - \bar{L} \tilde{H} Y_D \nu_R + \mathcal{L}_{\nu_R}^{(6)} + \mathcal{L}_{\nu_R}^{(7)}$$

Effective Field Theory

For heavy $\Delta L = 2$ physics

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- Majorana mass
(L violating)

Effective Field Theory

For heavy $\Delta L = 2$ physics

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- Majorana mass
(L violating)

- Dirac mass
(L conserving)

Effective Field Theory

For heavy $\Delta L = 2$ physics

$$\mathcal{L}_{eff} = \mathcal{L}_{SM} + \frac{c_i^{(5)}}{\Lambda} O_i^{(5)} + \frac{c_i^{(7)}}{\Lambda^3} O_i^{(7)} + \frac{c_i^{(9)}}{\Lambda^5} O_i^{(9)} + \dots$$

For light sterile neutrinos ν_R

- Can be described in the same framework (ν SMEFT):

Liao & Ma, '17

$$\mathcal{L}_{\nu_R} = i\bar{\nu}_R \not{\partial} \nu_R - \frac{1}{2} \bar{\nu}_R^c M_R \nu_R - \bar{L} \tilde{H} Y_D \nu_R + \mathcal{L}_{\nu_R}^{(6)} + \mathcal{L}_{\nu_R}^{(7)}$$

- Majorana mass
(L violating)

- Dirac mass
(L conserving)

- Dimension-6 (L-conserving)
- Dimension-7 operators (L-violating)
- Induced by heavy BSM physics

Minimal ν_R scenario

- Add n singlets, ν_R , to the SM:

$$\mathcal{L}_{\nu_R} = i\bar{\nu}_R \not{\partial} \nu_R - \frac{1}{2} \bar{\nu}_R^c M_R \nu_R - \bar{L} \tilde{H} Y_D \nu_R + \cancel{\mathcal{L}_{\nu_R}^{(6)}} + \cancel{\mathcal{L}_{\nu_R}^{(7)}}$$

Minimal ν_R scenario

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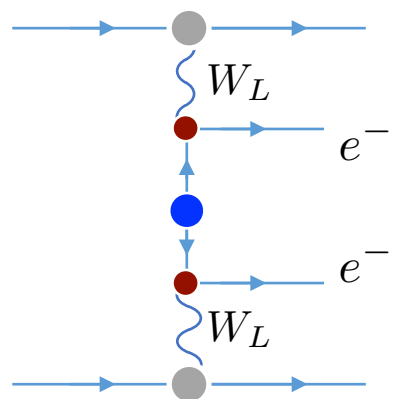
$$\mathcal{L}_{\nu_R} \xrightarrow{\text{EWSB}} -\frac{1}{2}\bar{N}^c M_\nu N \quad N = \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} \quad M_\nu = \begin{pmatrix} 0 & \frac{v}{\sqrt{2}}Y_\nu^T \\ \frac{v}{\sqrt{2}}Y_\nu & M_R^\dagger \end{pmatrix}$$

Minimal ν_R scenario

- Add n singlets, ν_R , to the SM:

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- Usually leading contributions $\sim m_{\beta\beta}$ vanish in this case:



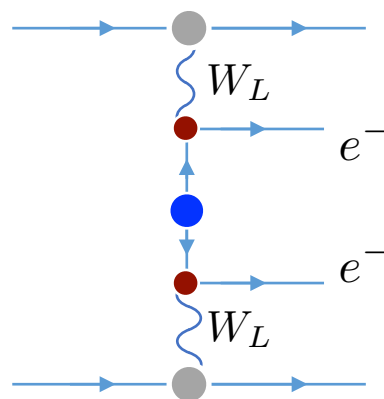
$$\propto \sum_{i=1}^{3+n} \frac{m_i}{q^2 - m_i^2} U_{ei}^2$$

Minimal ν_R scenario

- Add n singlets, ν_R , to the SM:

$$\mathcal{L}_{\nu_R} \xrightarrow{\text{EWSB}} -\frac{1}{2}\bar{N}^c M_\nu N \quad N = \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} \quad M_\nu = \begin{pmatrix} 0 & \frac{v}{\sqrt{2}} Y_\nu^T \\ \frac{v}{\sqrt{2}} Y_\nu & M_R^\dagger \end{pmatrix}$$

- Usually leading contributions $\sim m_{\beta\beta}$ vanish in this case:



$$\propto \sum_{i=1}^{3+n} \frac{m_i}{q^2 - \underbrace{m_i^2}_{\simeq 0}} U_{ei}^2 \simeq \frac{1}{q^2} (M_\nu)_{ee}^* = 0$$

Minimal ν_R scenario

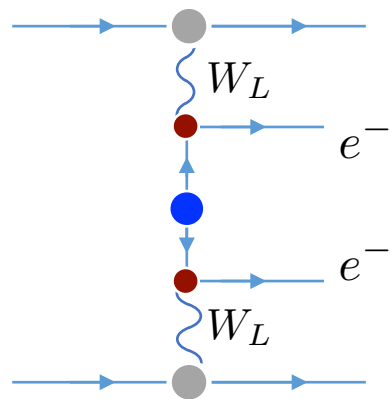
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$$\mathcal{L}_{\nu_R} \xrightarrow{\text{EWSB}} -\frac{1}{2}\bar{N}^c M_\nu N$$

$$N = \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix}$$

$$M_\nu = \begin{pmatrix} 0 & \frac{v}{\sqrt{2}} Y_\nu^T \\ \frac{v}{\sqrt{2}} Y_\nu & M_R^\dagger \end{pmatrix}$$

- Usually leading contributions $\sim m_{\beta\beta}$ vanish in this case:



$$\propto \sum_{i=1}^{3+n} \frac{m_i}{q^2 - m_i^2} U_{ei}^2 \simeq \frac{1}{q^2} (M_\nu)_{ee}^* = 0$$

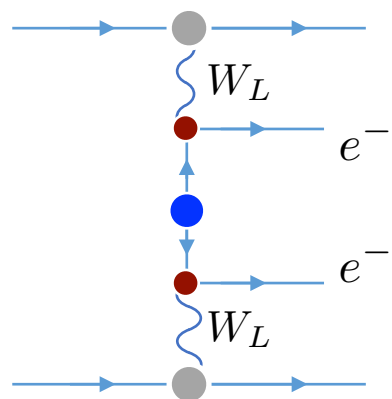
$\xrightarrow{\quad} \simeq 0$

Minimal ν_R scenario

- Add n singlets, ν_R , to the SM:

$$\mathcal{L}_{\nu_R} \xrightarrow{\text{EWSB}} -\frac{1}{2}\bar{N}^c M_\nu N \quad N = \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} \quad M_\nu = \begin{pmatrix} 0 & \frac{v}{\sqrt{2}} Y_\nu^T \\ \frac{v}{\sqrt{2}} Y_\nu & M_R^\dagger \end{pmatrix}$$

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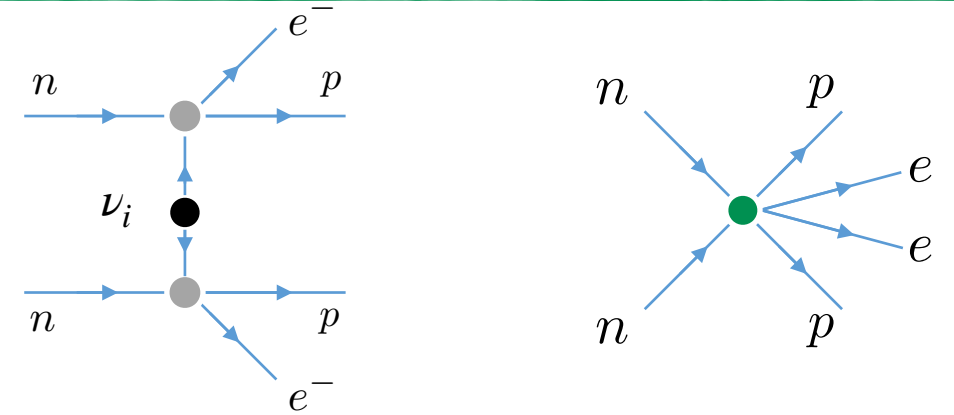
$$\propto \sum_{i=1}^{3+n} \frac{m_i}{q^2 - m_i^2} U_{ei}^2 \simeq \frac{1}{q^2} (M_\nu)_{ee}^* = 0$$

Cannot neglect this m_i
dependence!

ν_i contributions

'Usual' contributions:

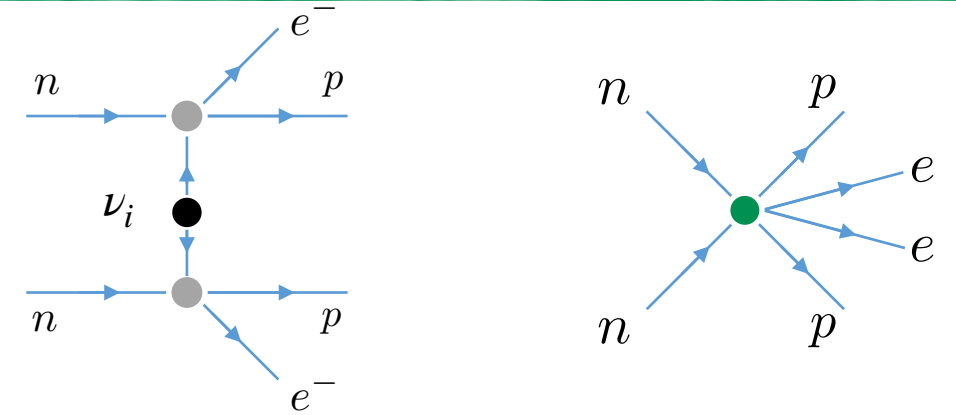
- Similar to $m_{\beta\beta}$ case:
- NMEs and LECs now m_i dependent



ν_i contributions

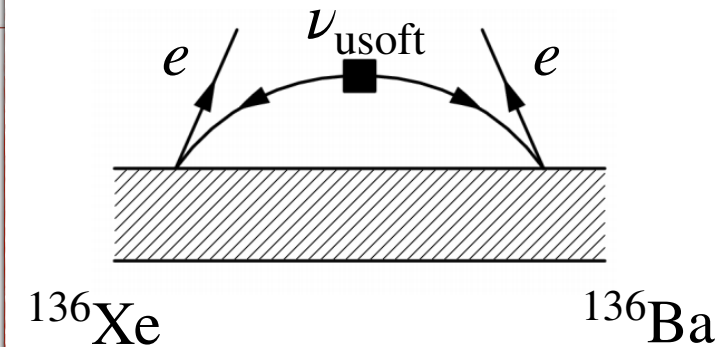
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New: 'Ultrasoft' neutrinos

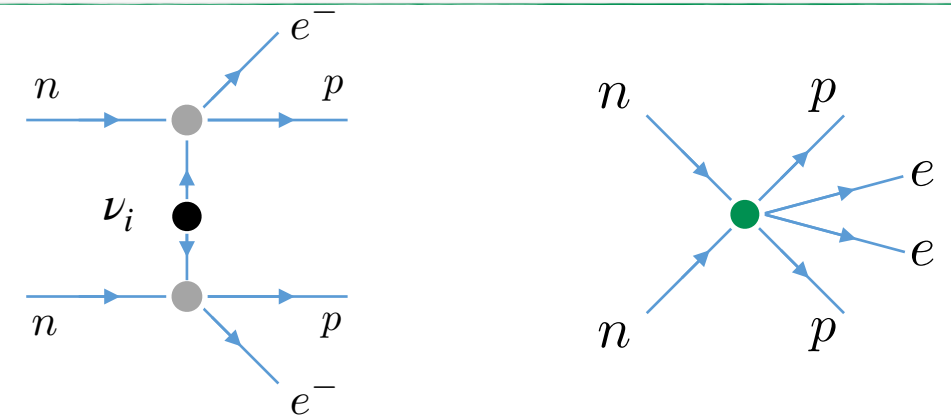
- See the nucleus as a whole, have momenta $q^0 \sim \vec{q} \sim k_F^2/m_N \sim Q$
- Usually N2LO effect, now leading order



ν_i contributions

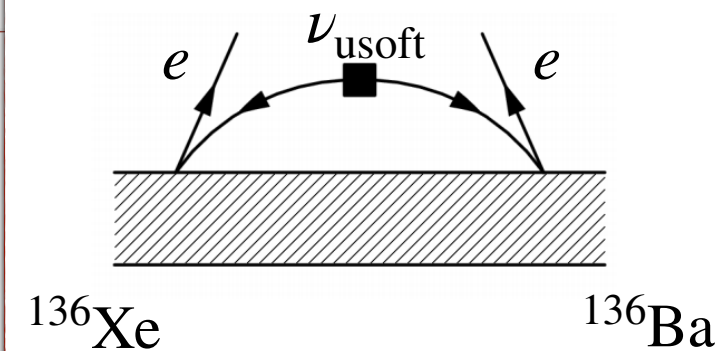
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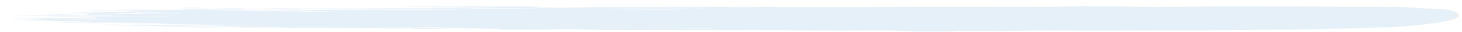


$$A_{\nu}^{\text{usoft}} \sim \sum_n \langle f | J_{\mu} | n \rangle \langle n | J^{\mu} | i \rangle \times \begin{cases} \frac{m_i}{k_F}, & \Delta E \lesssim m_i \lesssim k_F \\ \frac{m_i^2}{4\pi k_F \Delta E} \ln \frac{m_i}{\Delta E}, & m_i \lesssim \Delta E \end{cases}$$

- Depends on intermediate state energies, $\Delta E \equiv E_n + E_e - E_i$
- Overlap integrals

Phenomenology:

A 3+1 toy model

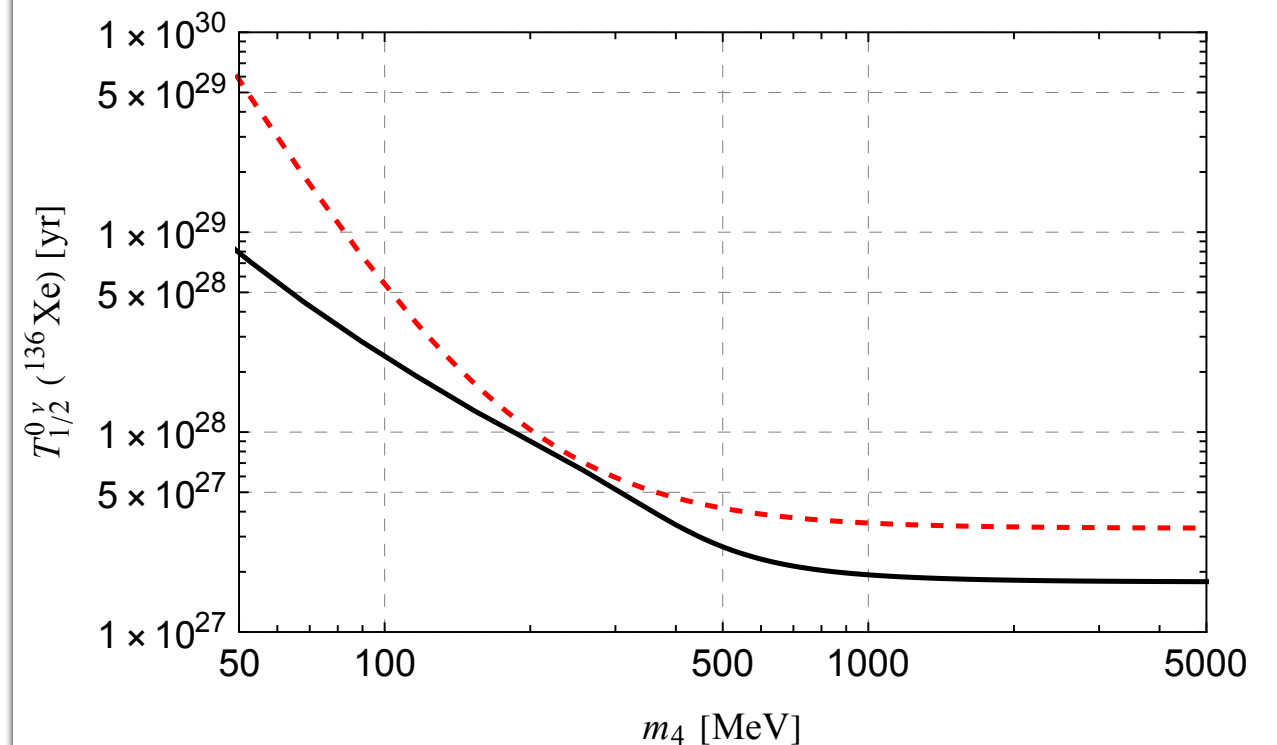
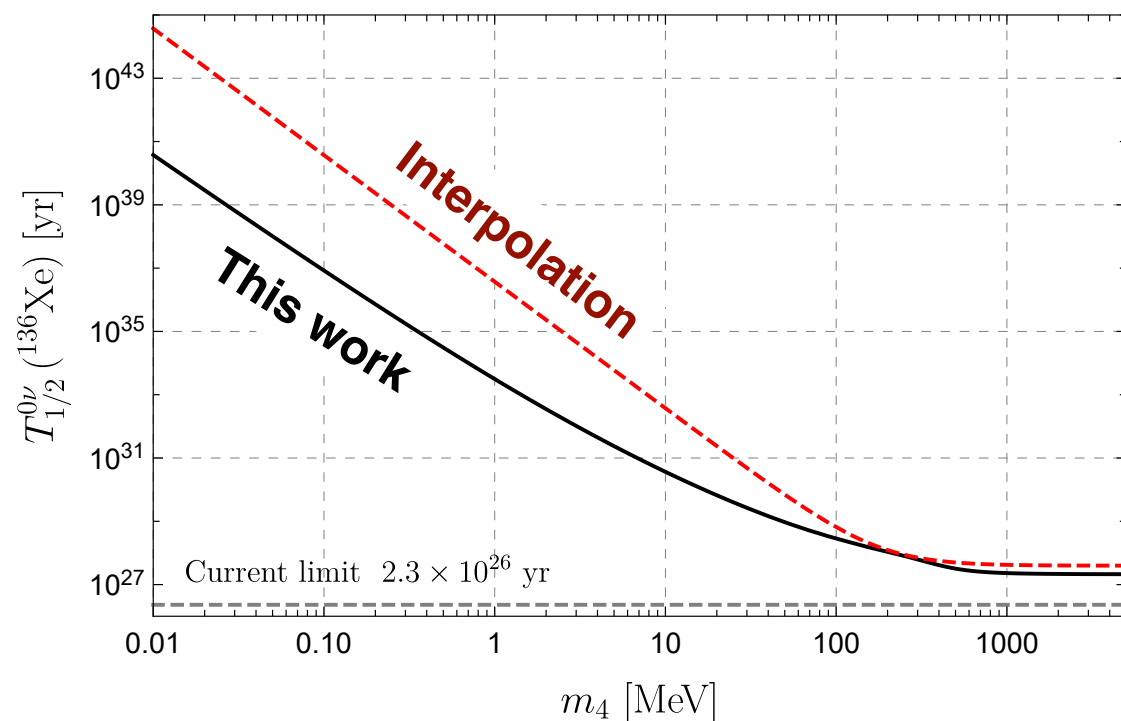


Toy model: 3+1

- Add just one sterile neutrino to the SM
 - Not realistic: does not reproduce all masses/mixings
- Simple scenario to test impact
 - Similar features to more realistic cases

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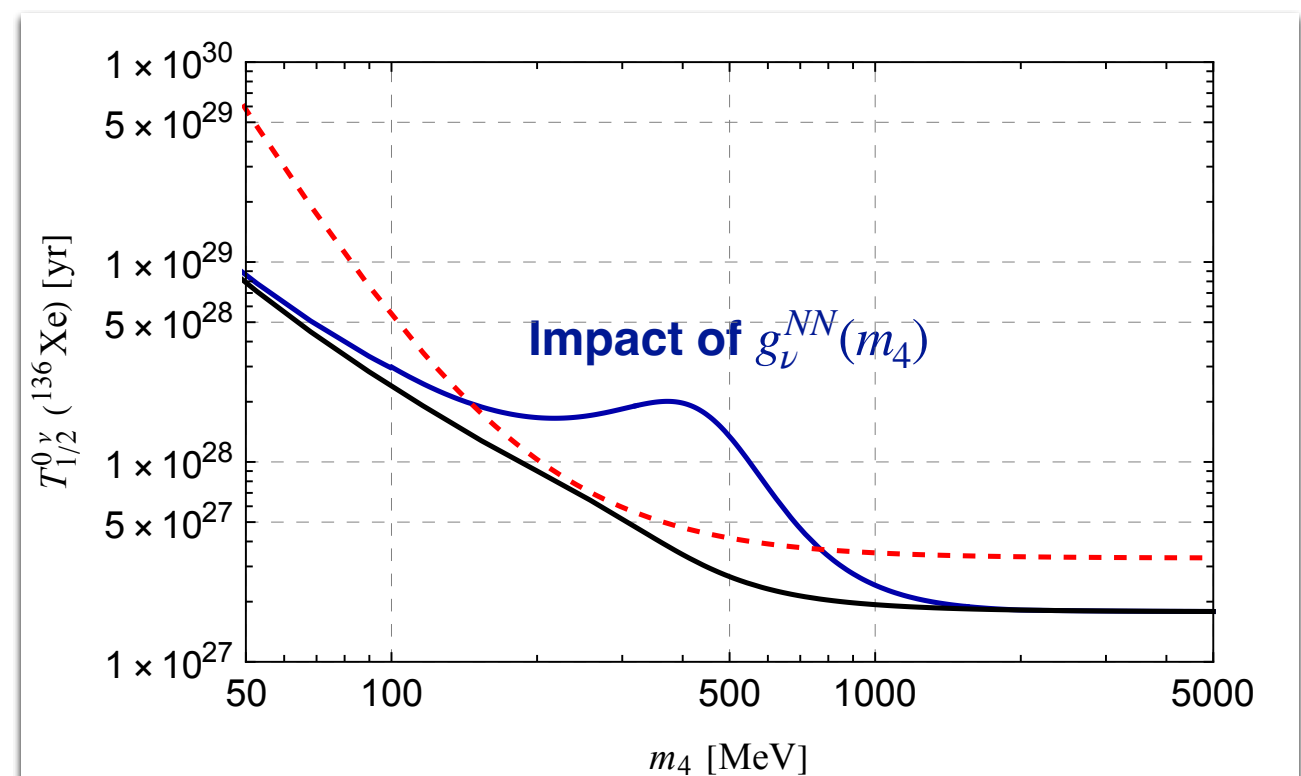
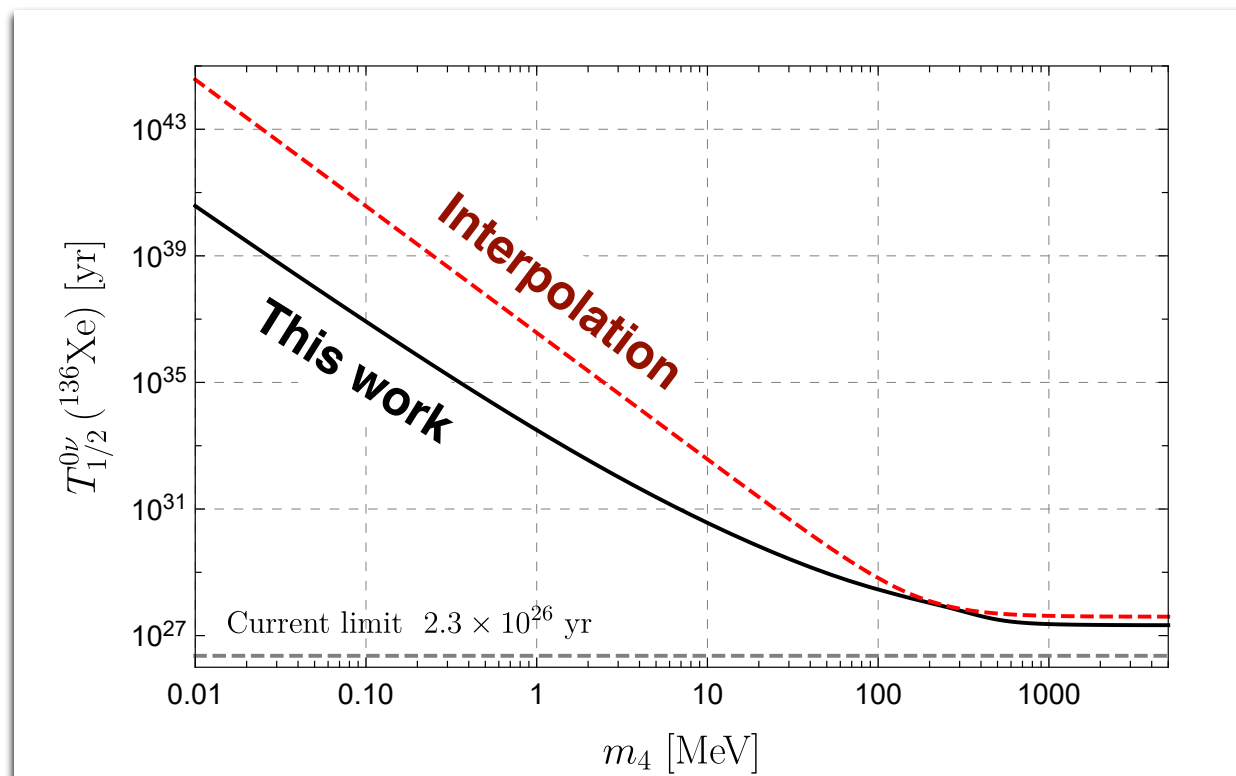
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 - Ultrasoft neutrinos for $m_4 \ll m_\pi$
 - $g_\nu^{NN}(m_4)$ for $m_4 \gtrsim \text{GeV}$

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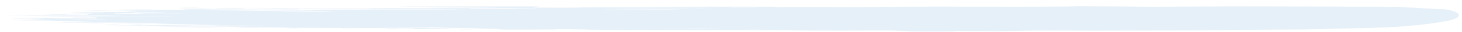


- Largest differences:
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- ‘Cottingham’ estimate of m_ν dependence of g_{ν}^{NN}
 - Significant impact for $m_4 \gtrsim m_\pi$

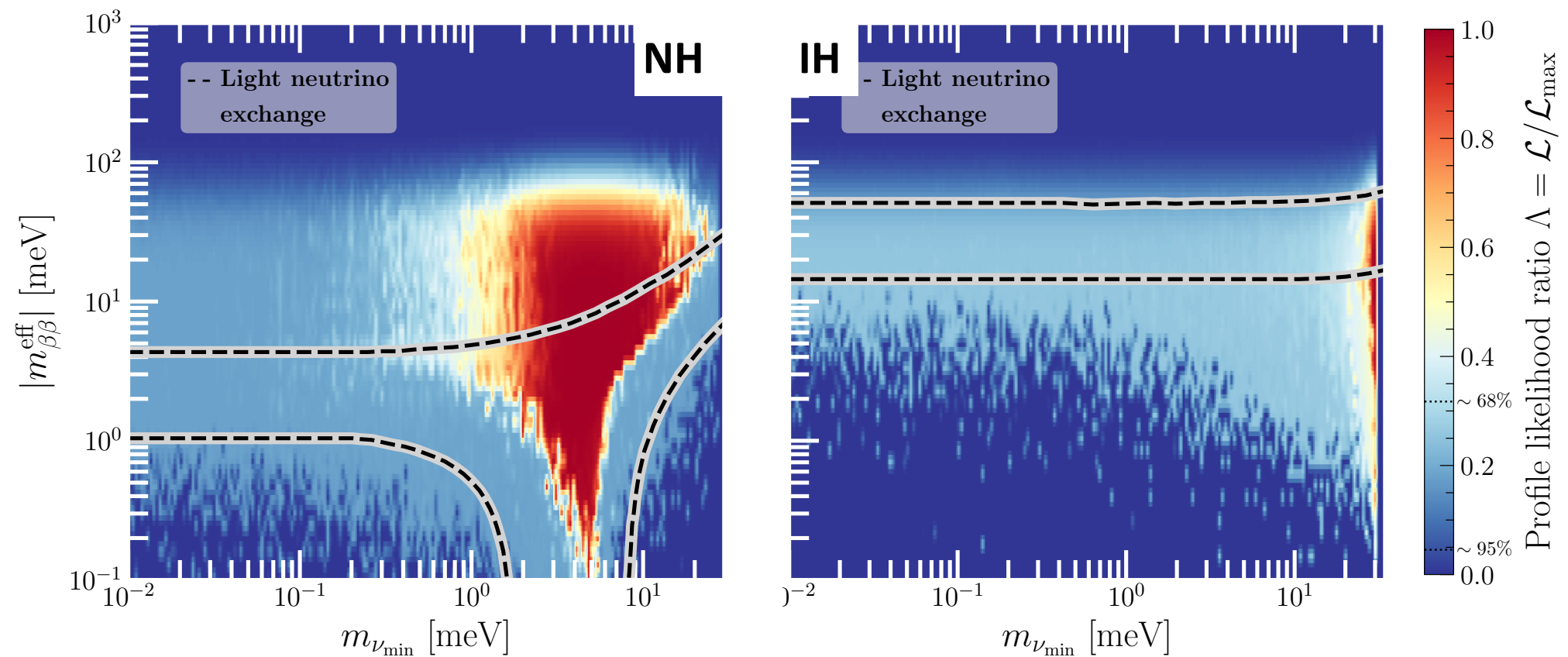
Phenomenology:

3+3 seesaw models



- Add 3 sterile neutrinos to the SM
- What do we expect for $0\nu\beta\beta$ given all other experimental information?
 - Oscillation experiments, direct searches, electroweak precision ...

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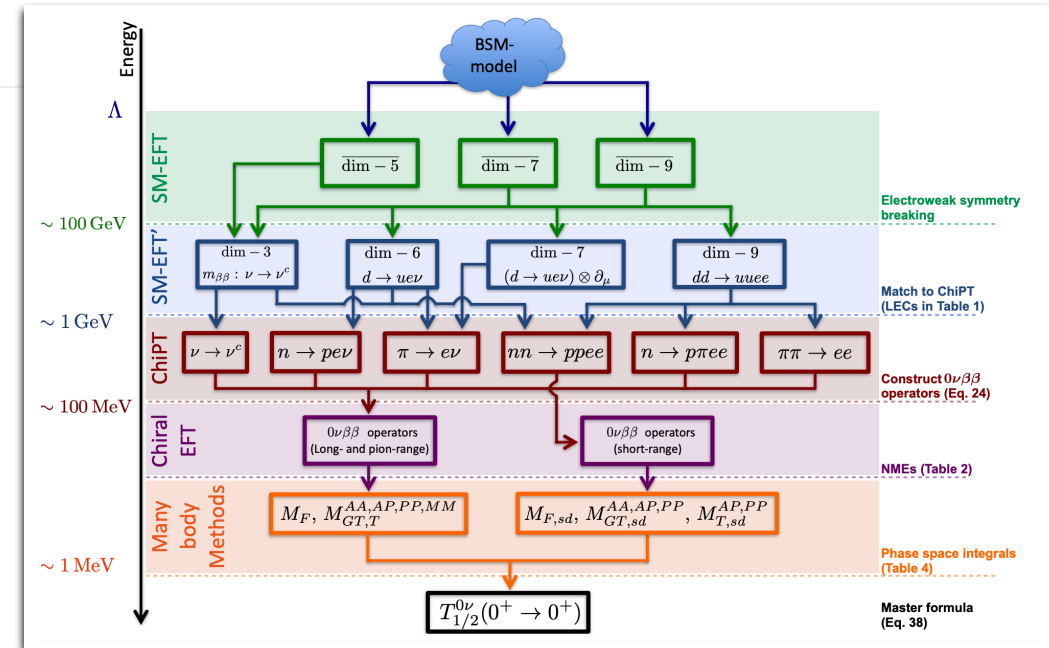


S. Urrutia Quiroga,
V. Cirigliano, WD, '25

- Even in minimal models $T_{1/2}^{0\nu}$ could be significantly larger than for the Majorana mass
 - Similar in 3+2 scenarios

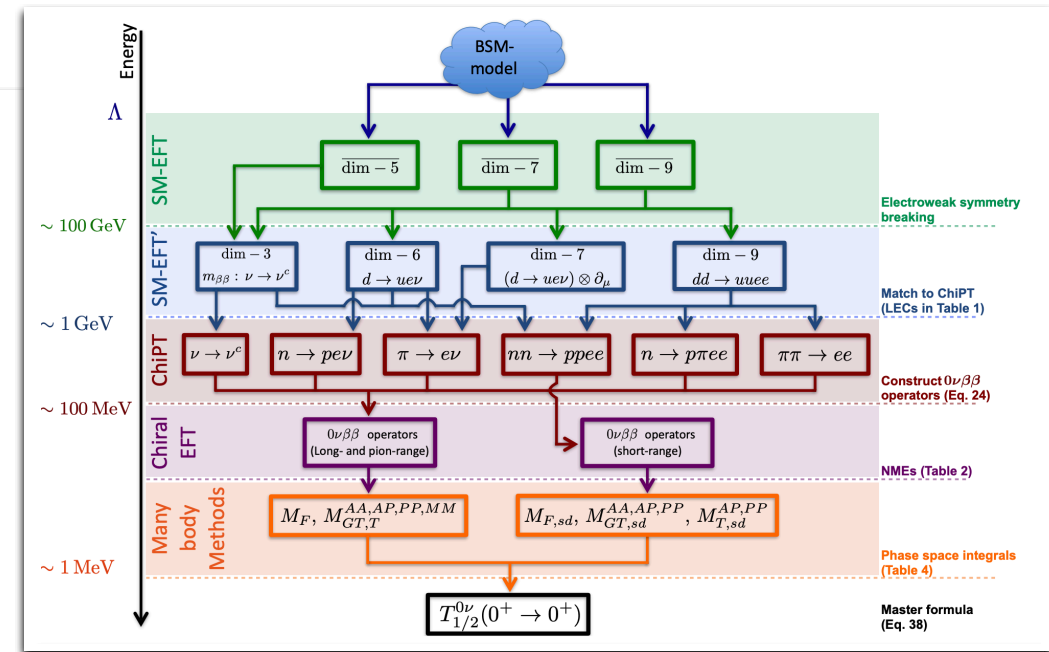
Summary

- EFTs can systematically describe $\Delta L=2$ sources
- Majorana-mass mechanism (dim-5)
- Dimension-7 & -9 sources
- Effects from ν_R

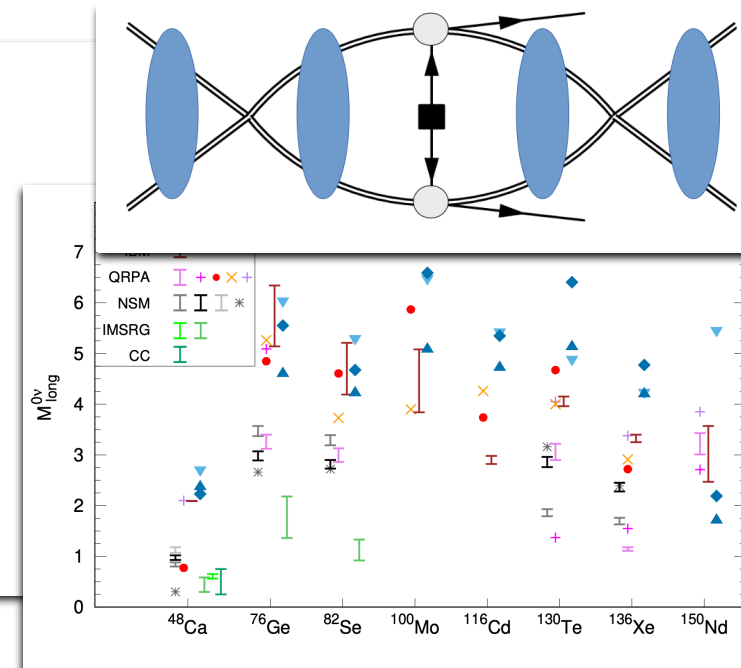


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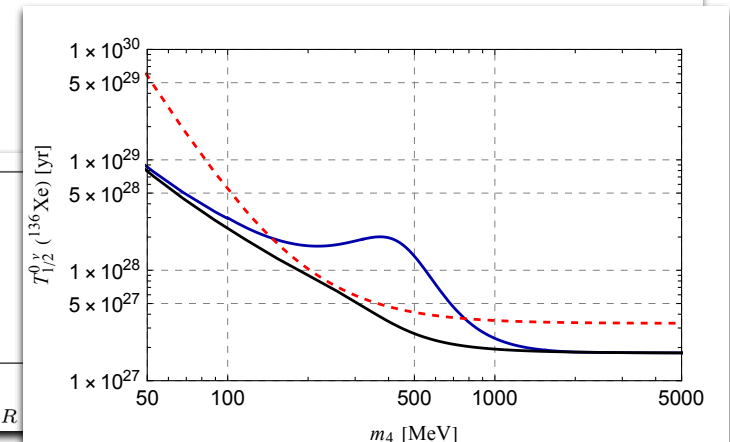
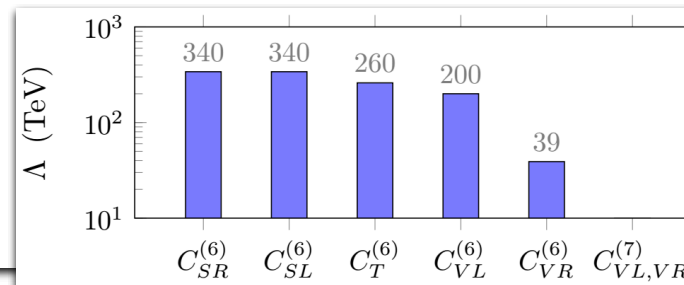
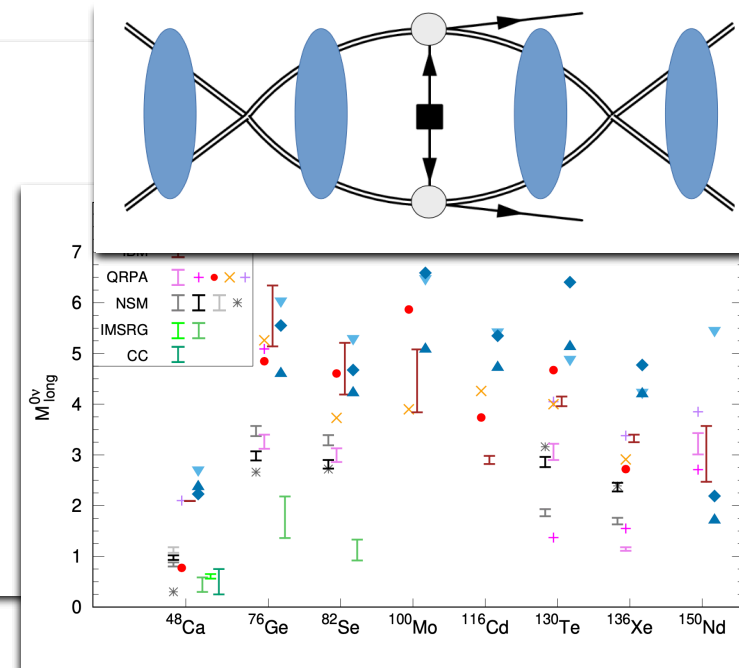
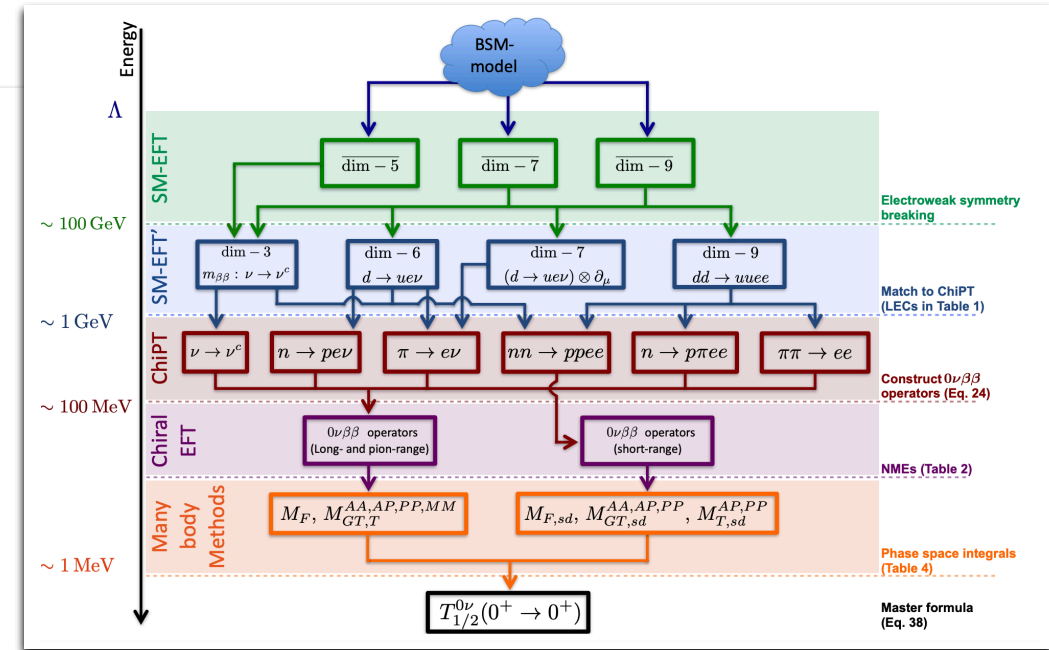
- EFT leads to additional effects:
 - Contact term needed at leading order
 - Large effects of light ultrasoft ν_R
- Identifies needed Nuclear Matrix Elements



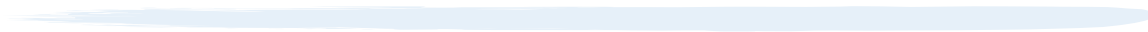
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- EFT leads to additional effects:
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 - Identifies needed LECs & Nuclear Matrix Elements

- $0\nu\beta\beta$ probes
 - Heavy BSM up to $O(100)$ TeV
 - Interactions with ν_R to $O(10)$ TeV



Back up slides



Disentangling operators

What if a $0\nu\beta\beta$ signal is measured?

- Measurement in a single isotope could be due to any operator
- Could measure the rate in several nuclei, however
 - Different isotopes have similar sensitivity to LNV operators
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- Instead look at angular & energy distributions of the leptons

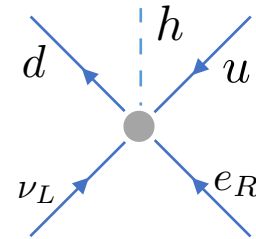
Disentangling operators

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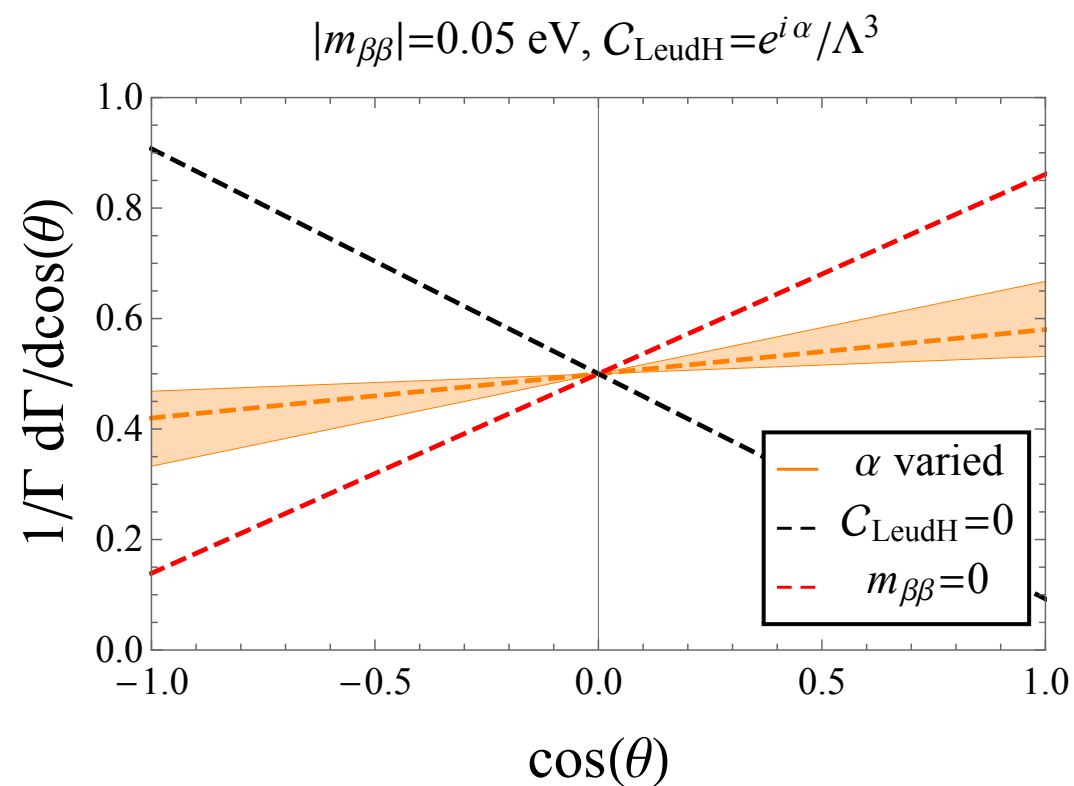
- Picking the allowed values



$$m_{\beta\beta} = 0.05 \text{ eV}$$



$$C_{\text{LeudH}} = e^{i\alpha} / \Lambda^3 \quad \Lambda = 40 \text{ TeV}$$



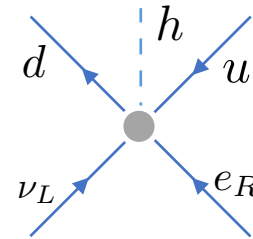
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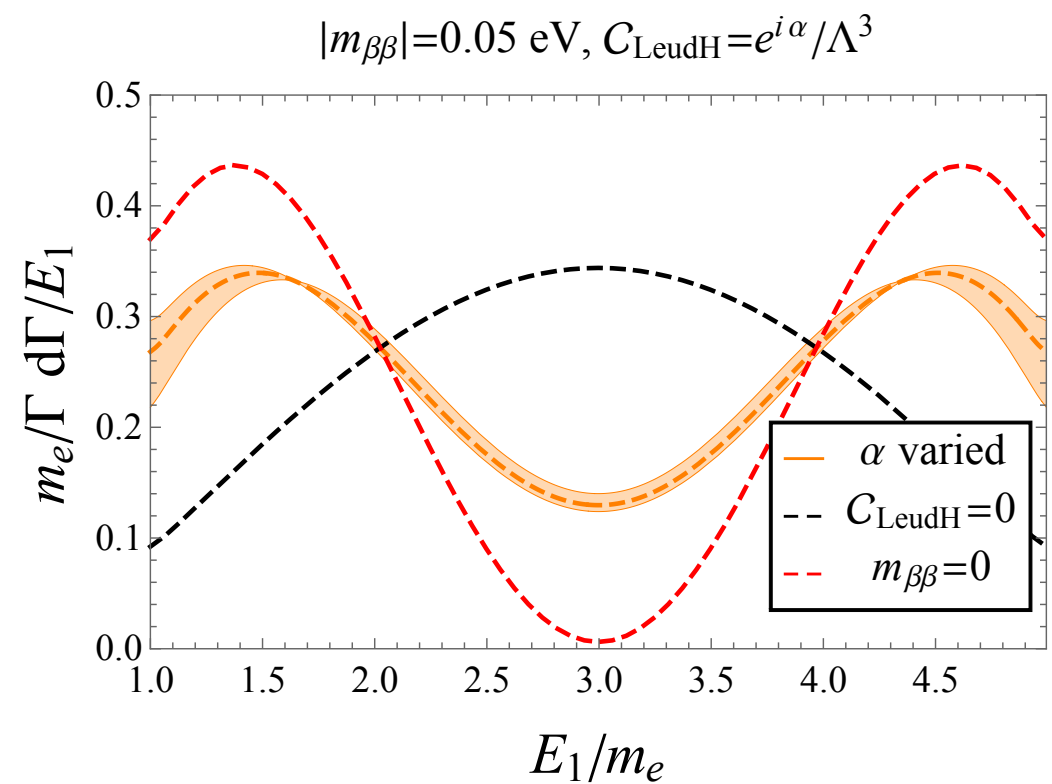
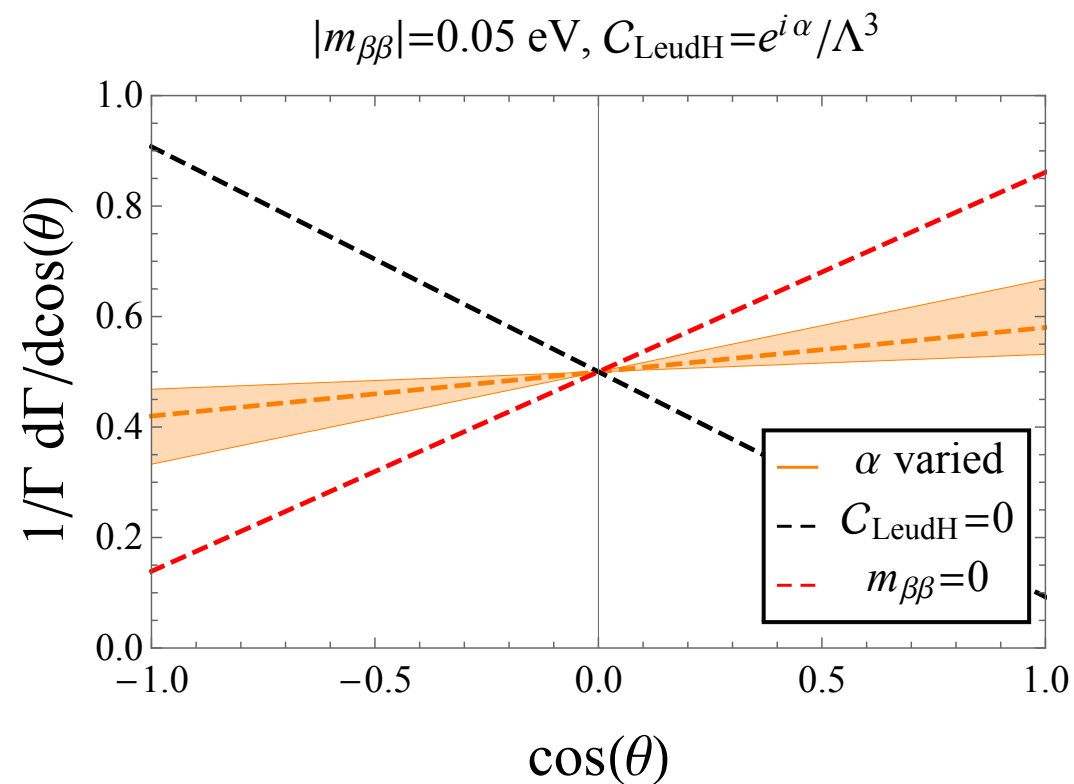
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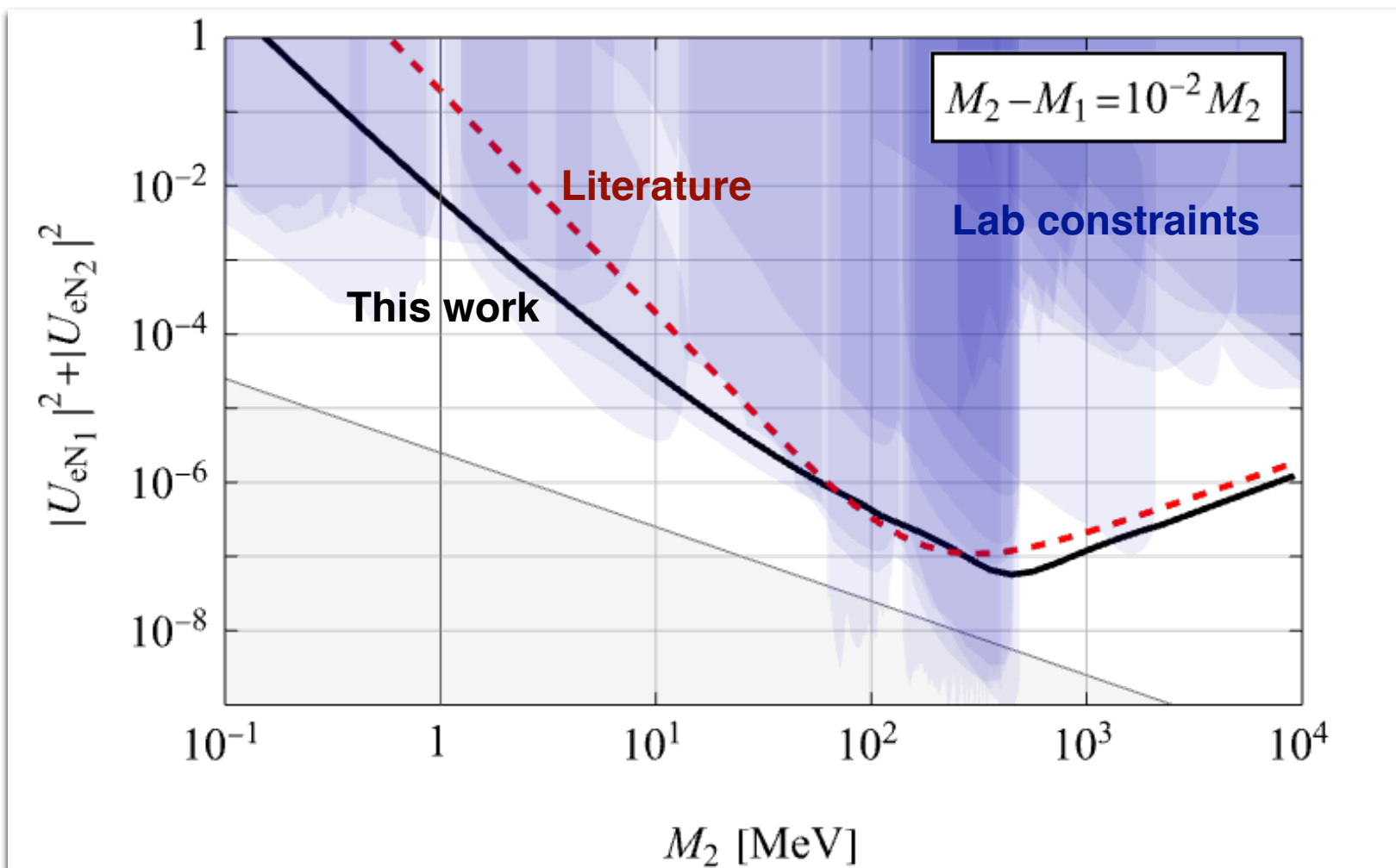
More ν_R scenarios

Toy model: 1+1+1 pseudo-Dirac

- Involves 1 active, two sterile neutrinos
 - Assume steriles much heavier than the active neutrinos; $M_1 \simeq M_2 \gg m_\nu$
 - Two heavier ν 's, form a pseudo-Dirac pair
 - Light neutrino mass proportional to LNV parameter (opposite to seesaw)

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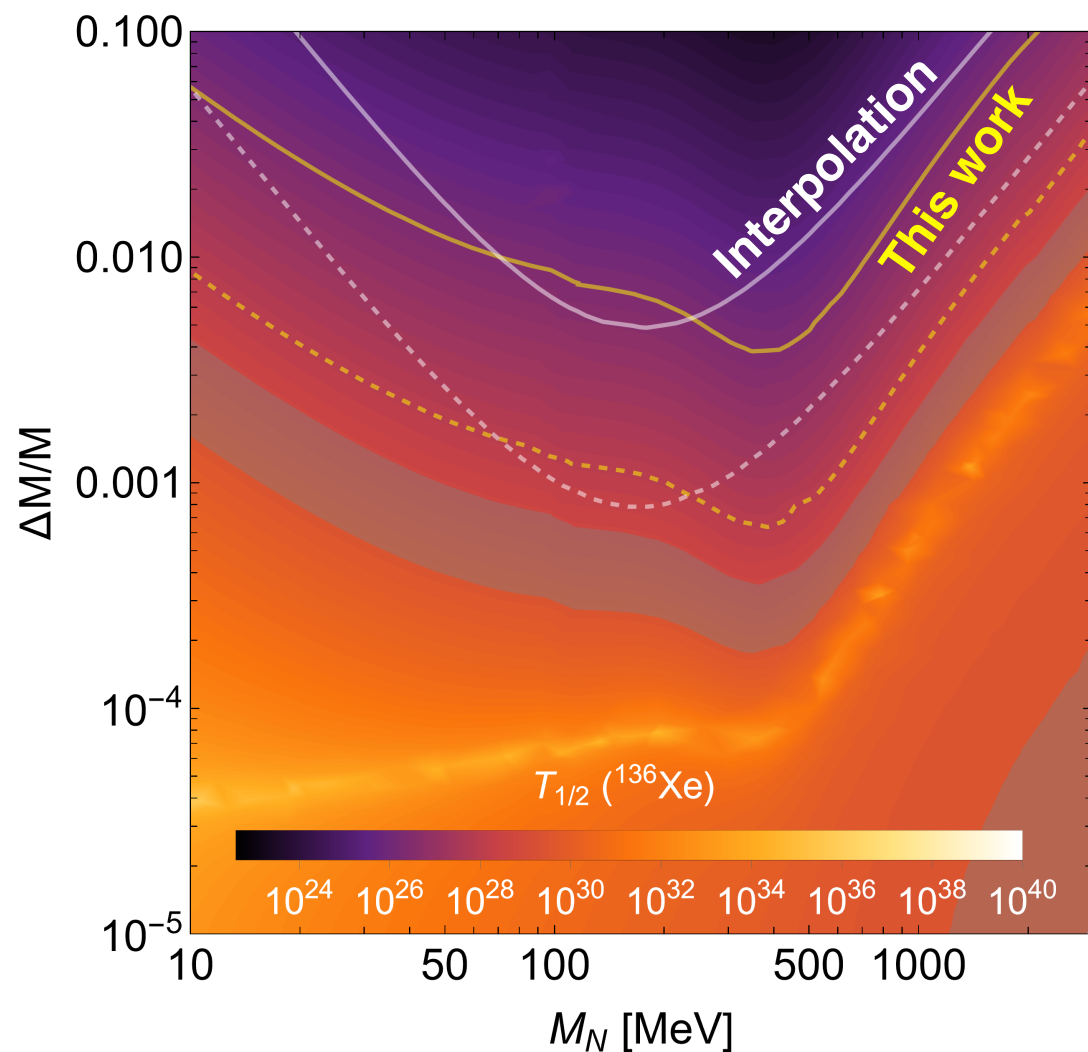
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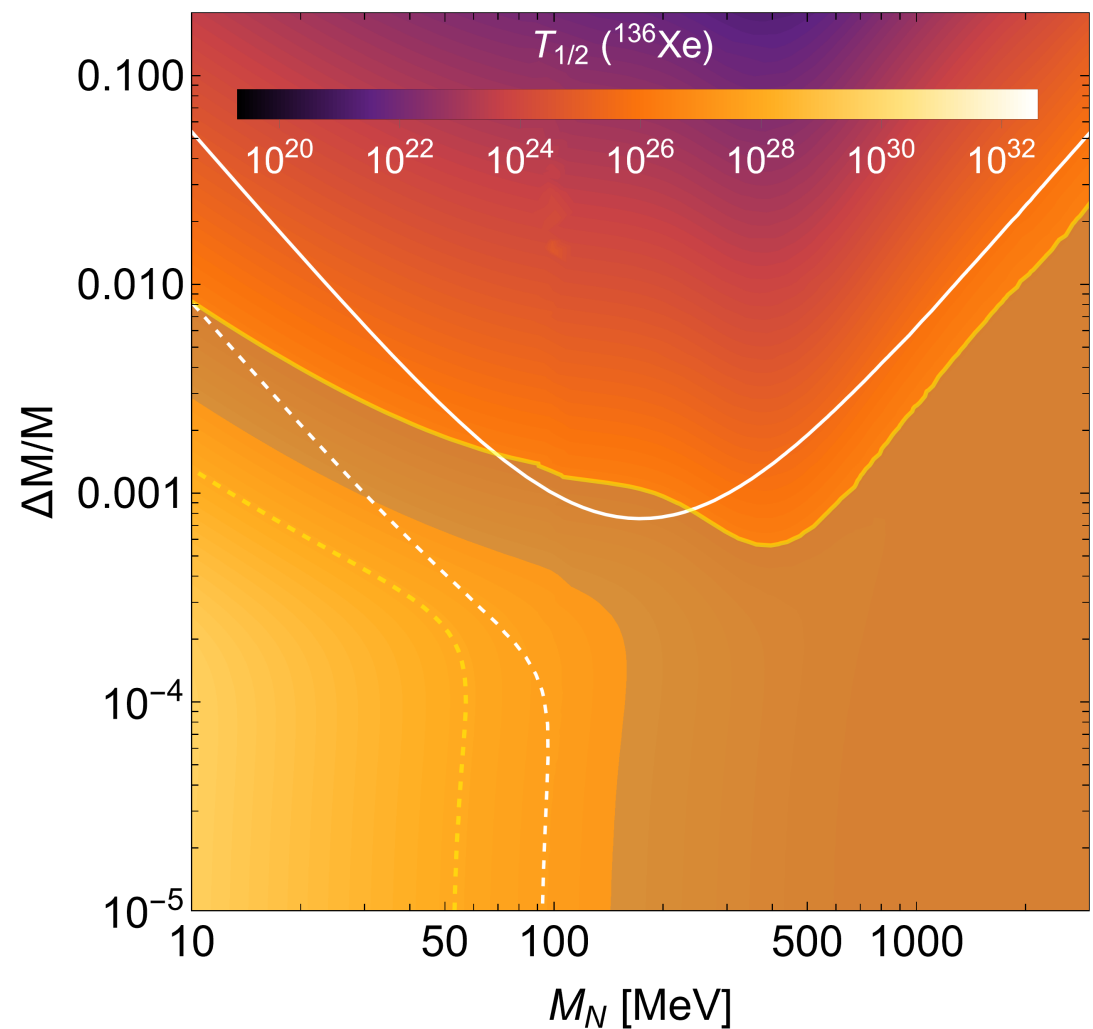
3+2 scenario

- Add 2 sterile neutrinos to the SM
 - Can now fit the oscillation data
 - Involves more parameters, for some choice can derive bounds:

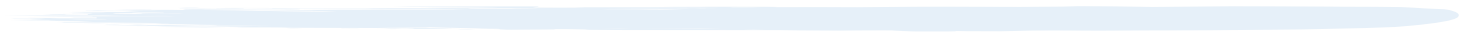
• Normal hierarchy



• Inverted hierarchy



Sterile neutrinos
+ higher-dimensional $\mathcal{O}$'s



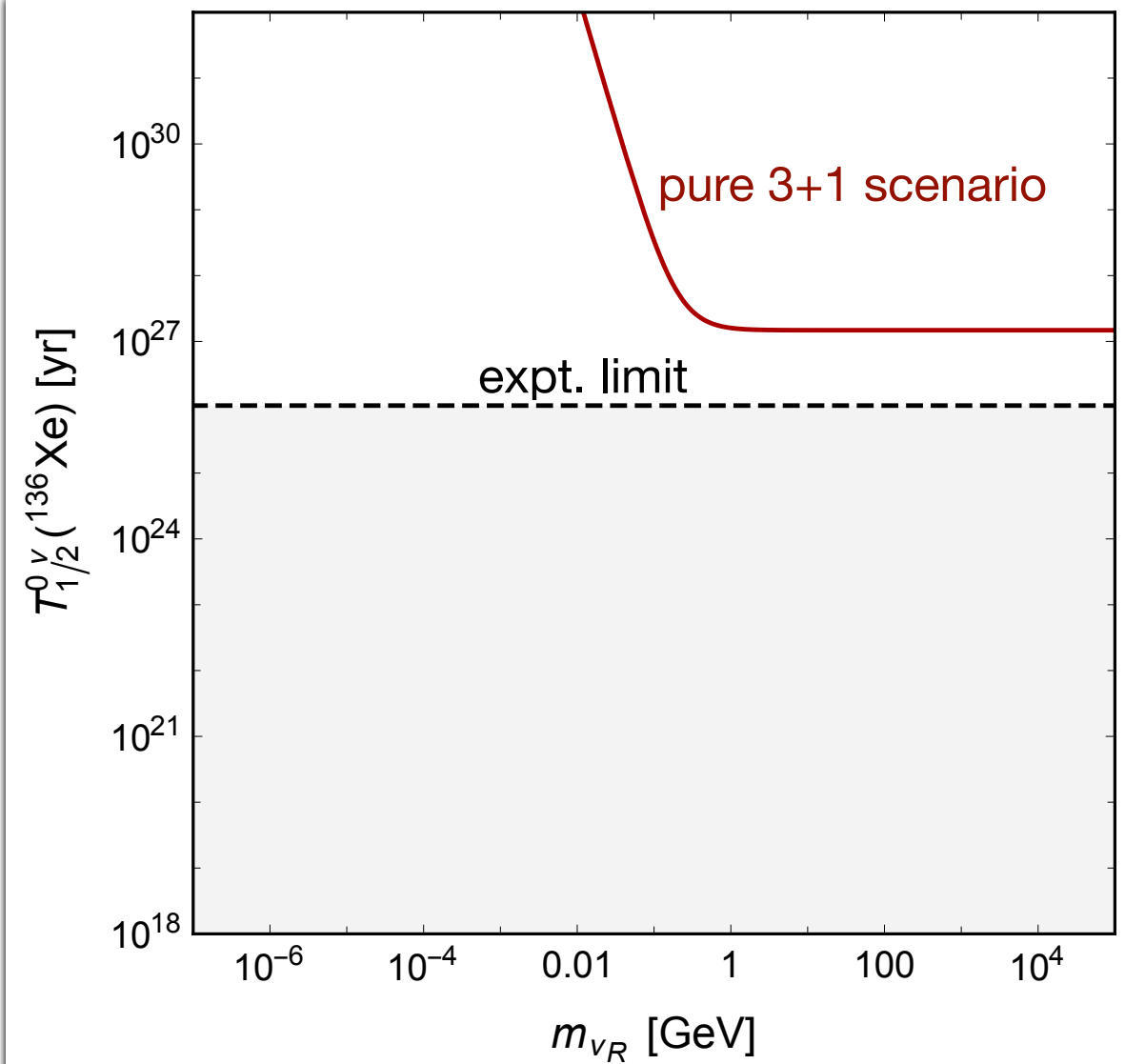
Phenomenology

From heavy new physics + light ν_R

Example with ν_R

- Toy Model
 - SM + 1 light ν_R

O(100%) uncertainties not shown



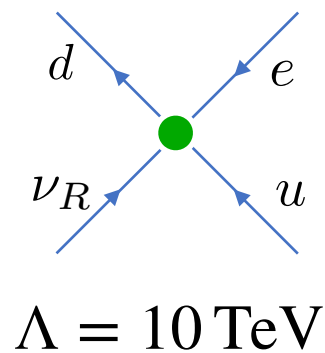
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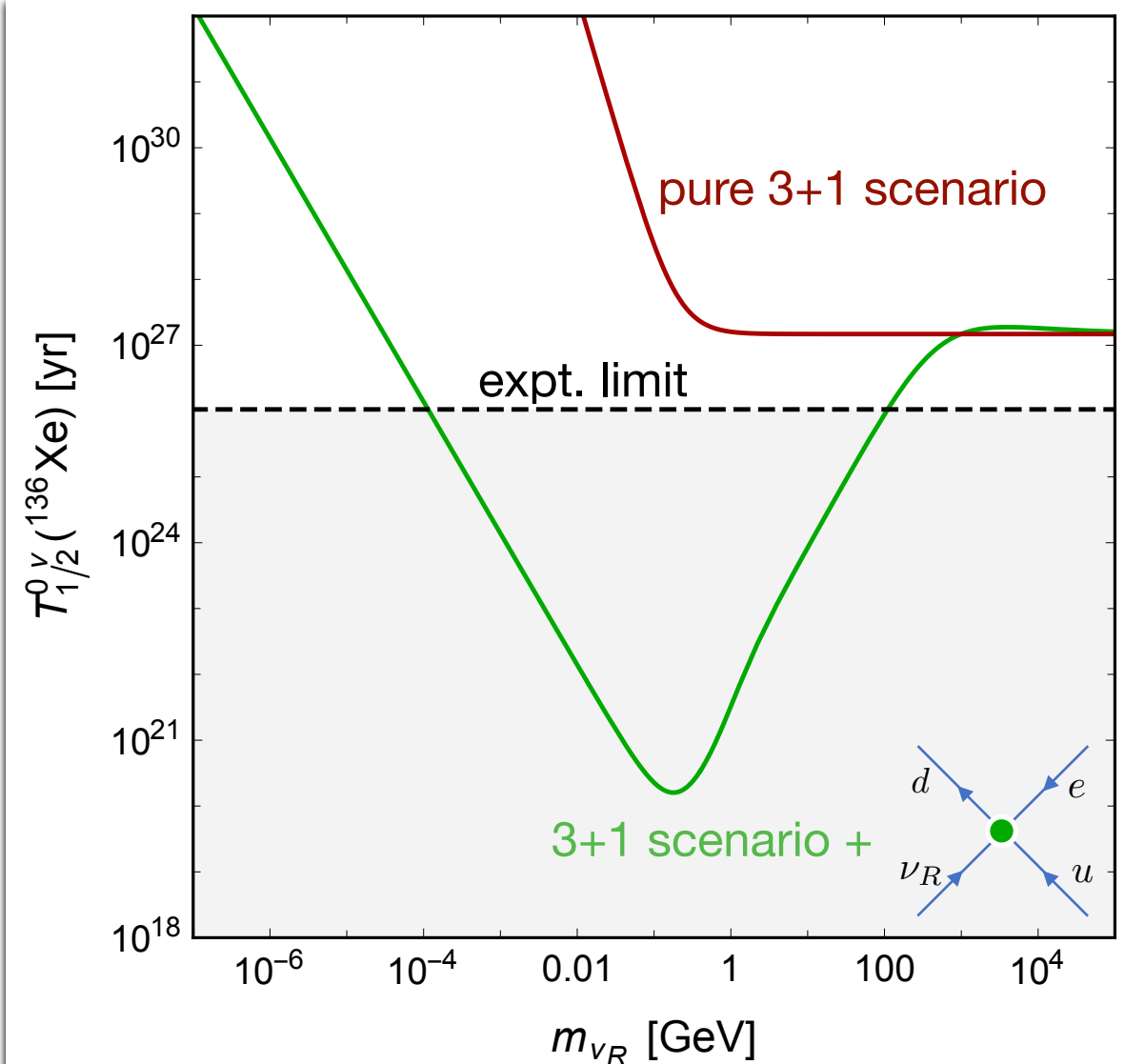
Example with ν_R

- Toy Model
 - SM + 1 light ν_R
- Added dimension-six interaction

- SM + 1 light ν_R +



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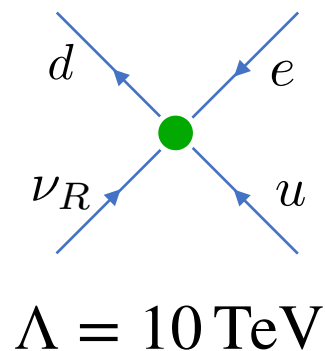
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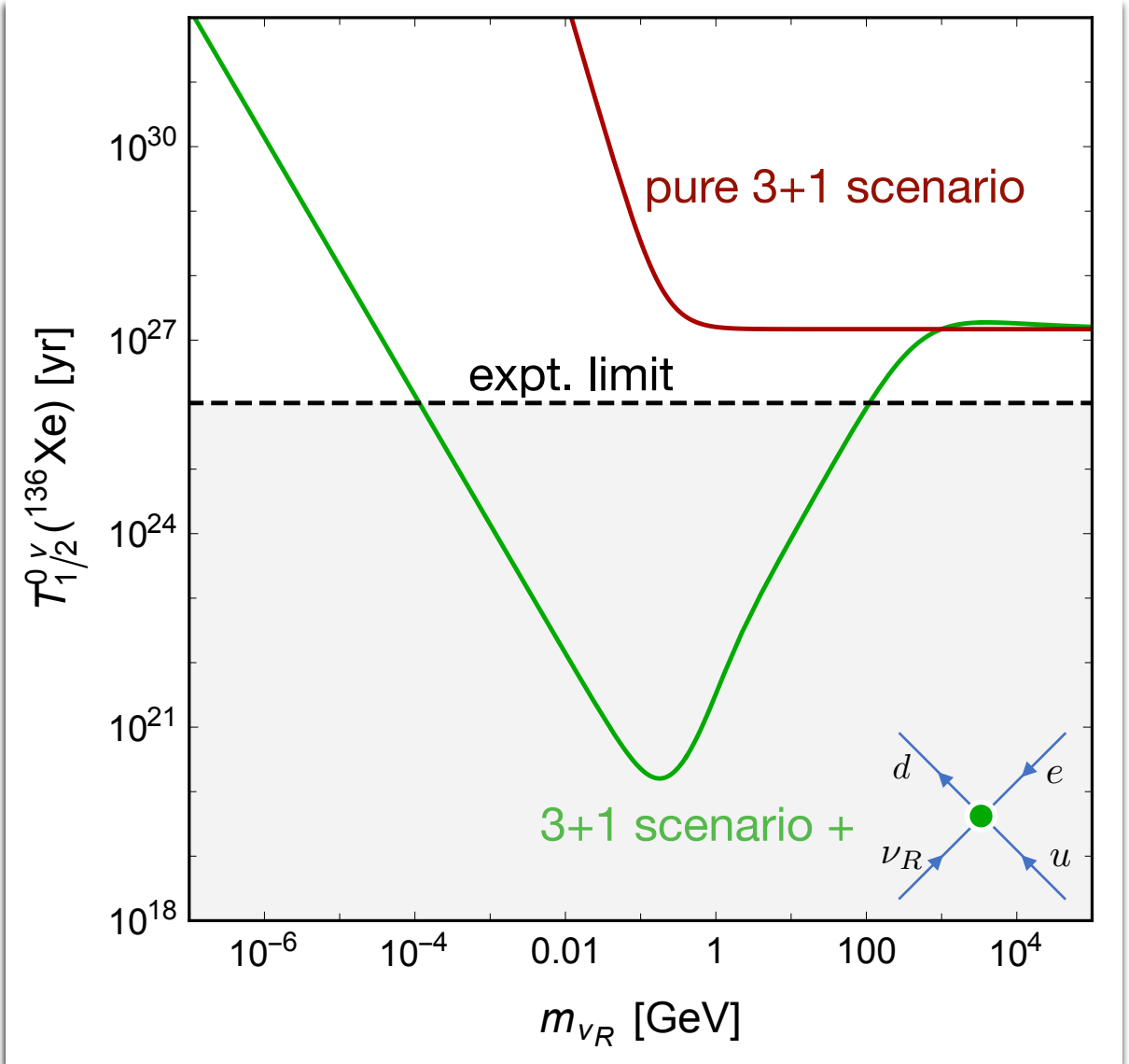
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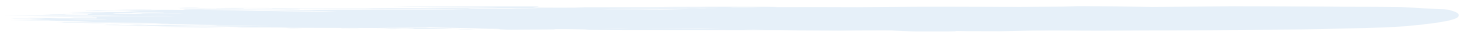
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- Higher dimensional ν_R terms can have a large impact!

$g_{\nu}^{NN} :$

Cottingham estimate

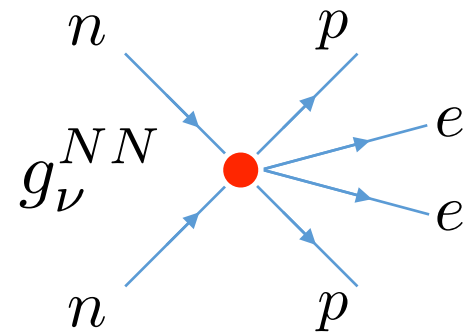
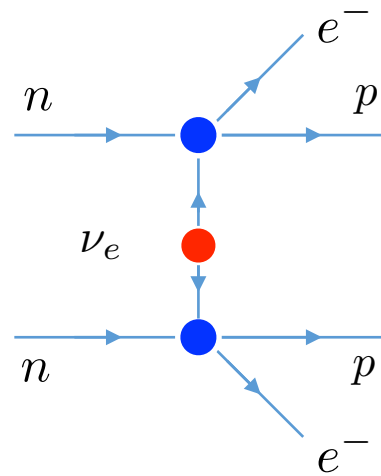


Need for a counter term

New interaction needed at leading order to get physical amplitudes:

$$\mathcal{L}_{CT} = 2G_F^2 V_{ud}^2 m_{\beta\beta} g_\nu^{NN} \bar{p}n \bar{p}n \bar{e}_L C \bar{e}_L^T$$

$$V_{\Delta L=2} = V_\nu + V_{\nu,CT} =$$

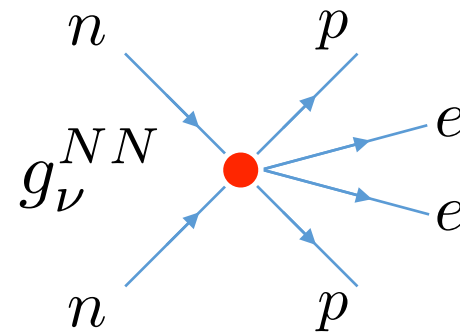
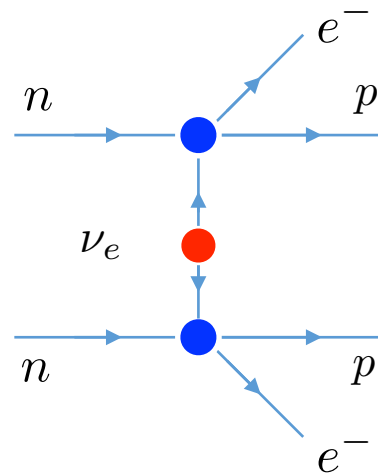


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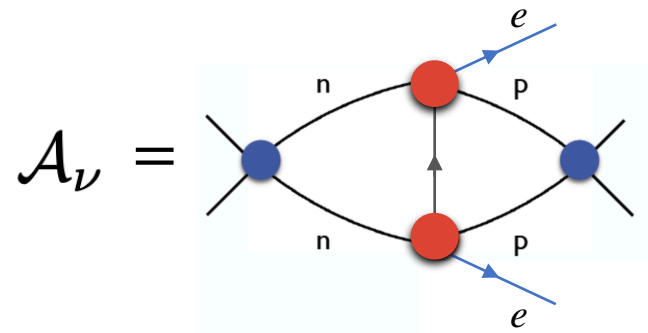
- g_ν^{NN} to be determined from a lattice calculation of $\mathcal{A}(nn \rightarrow ppe^-e^-)$
- Area of active research

Davoudi and Kadam, '20, '21
Feng et al, '20

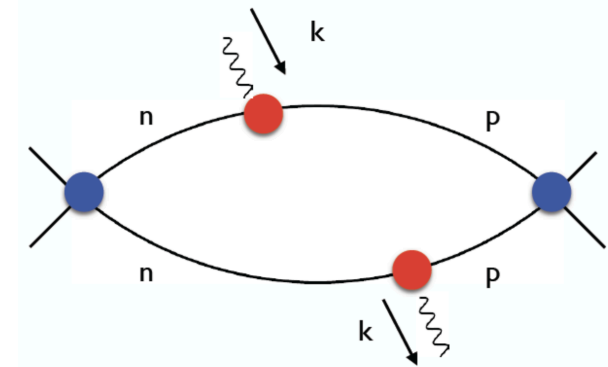
Determination of the counterterm

- Analogy to the Cottingham approach for pion/nucleon mass differences Cirigliano et al, '20, '21

$$\mathcal{A}_\nu \propto \int \frac{d^4 k}{(2\pi)^4} \frac{g_{\mu\nu}}{k^2 + i\epsilon} \int d^4 x e^{ik \cdot x} \langle pp | T \{ j_W^\mu(x) j_W^\nu(0) \} | nn \rangle$$



$$\propto \int dk a(k) = \int \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2 + i\epsilon}$$



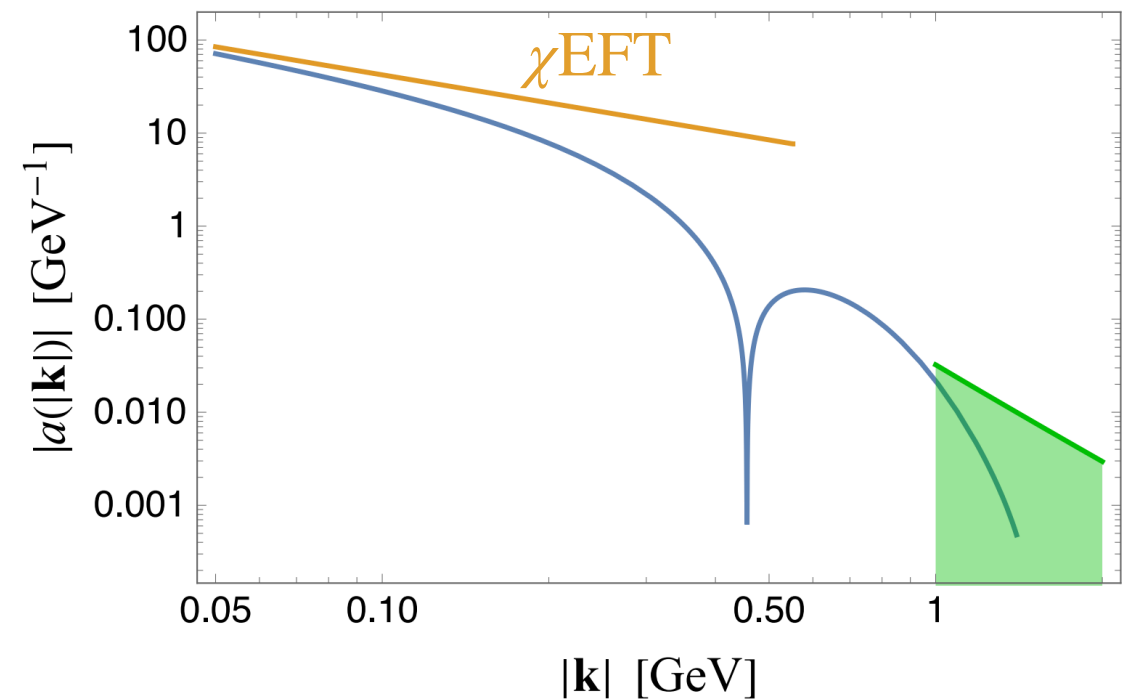
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$$\mathcal{A}_\nu = \text{Diagram 1} \propto \int dk a(k) = \int \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2 + i\epsilon} \text{Diagram 2}$$

- Estimate the $\mathcal{A}_\nu$ by constraining the **integrand**
 - $k \ll \Lambda_\chi$ region determined by χ EFT



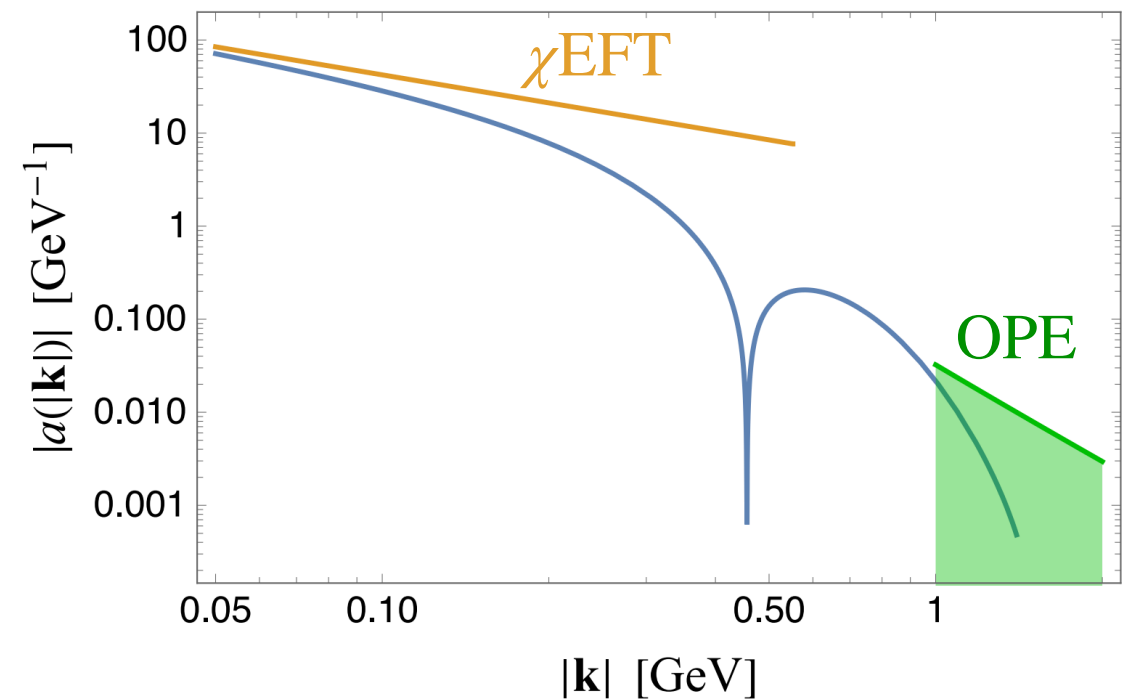
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 - $k \gg \text{GeV}$ region determined by OPE



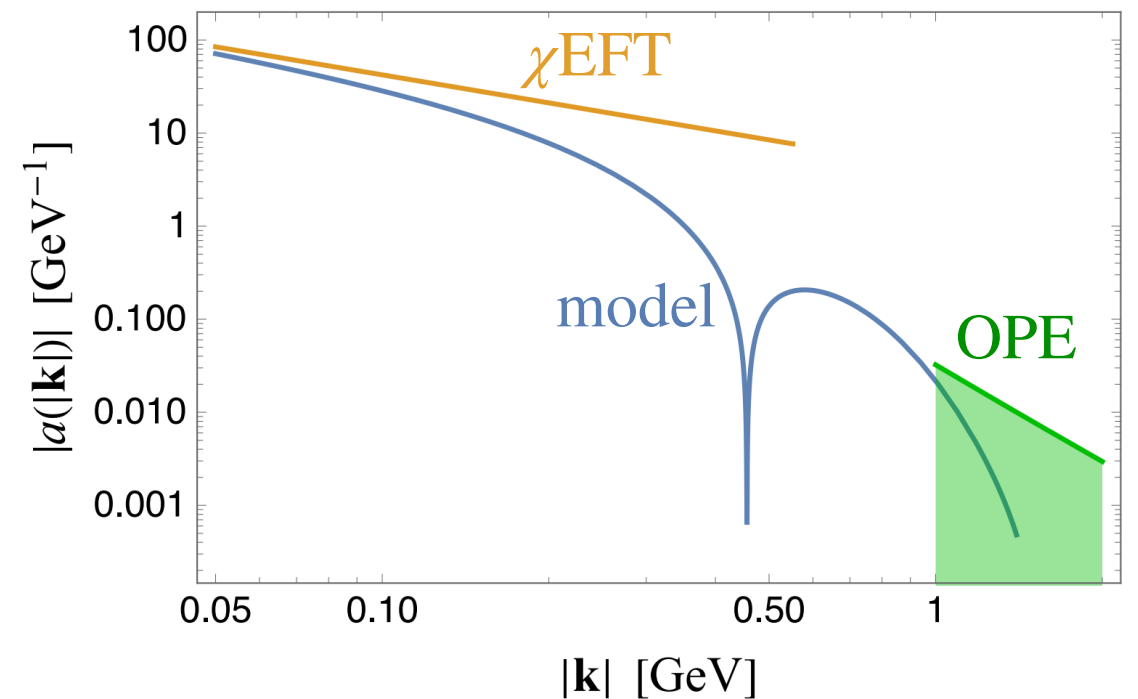
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$$A_\nu = \text{[Feynman diagram: bubble with two vertices, two nucleon lines (n, p), and two electron lines (e)]} \propto \int dk a(k) = \int \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2 + i\epsilon} \text{[Feynman diagram: bubble with two vertices, two nucleon lines (n, p), and two wavy lines (k)]}$$

- Estimate the A_ν by constraining the **integrand**
 - $k \ll \Lambda_\chi$ region determined by χ EFT
 - $k \gg \text{GeV}$ region determined by OPE
- Model** intermediate region using:
 - Form factors
 - Off-shell effects from NN intermediate states



Determination of the counterterm

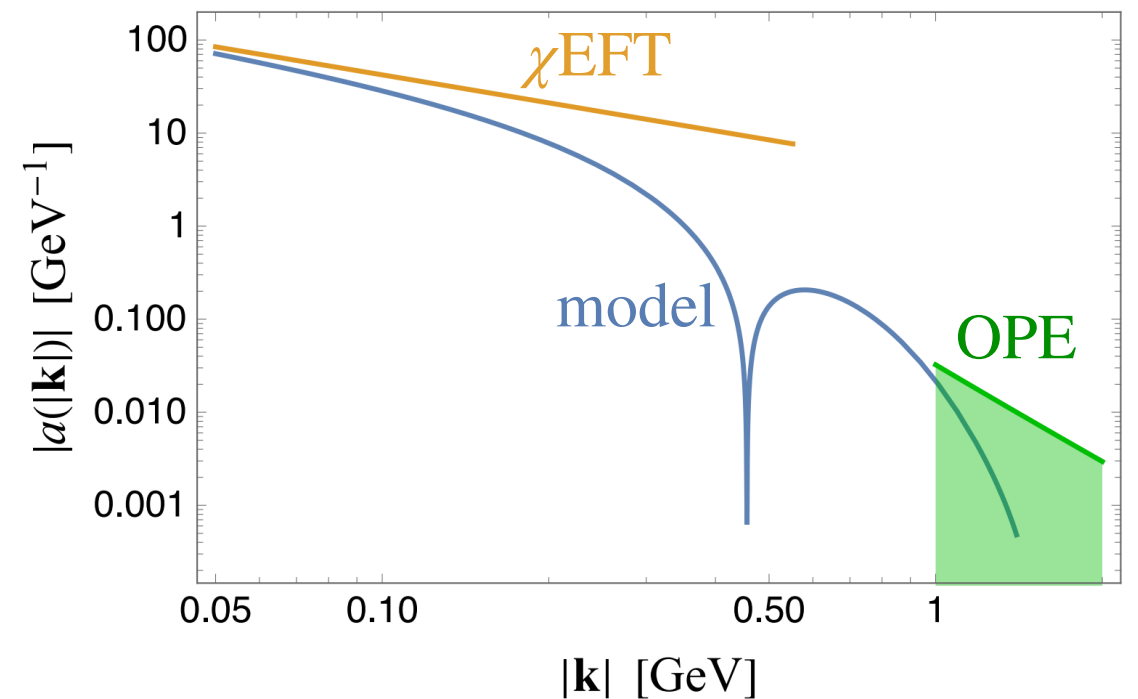
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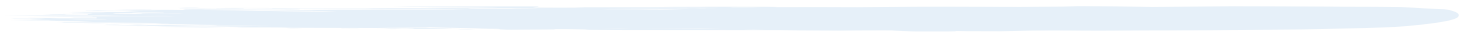
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- Gives $\tilde{g}_\nu^{NN}(\mu = m_\pi) = 1.3(6)$ in $\overline{\text{MS}}$
- Estimated 30% uncertainty
- Validated in isospin-breaking observables
- Consistent with large- N_c estimate

Richardson et al, '21

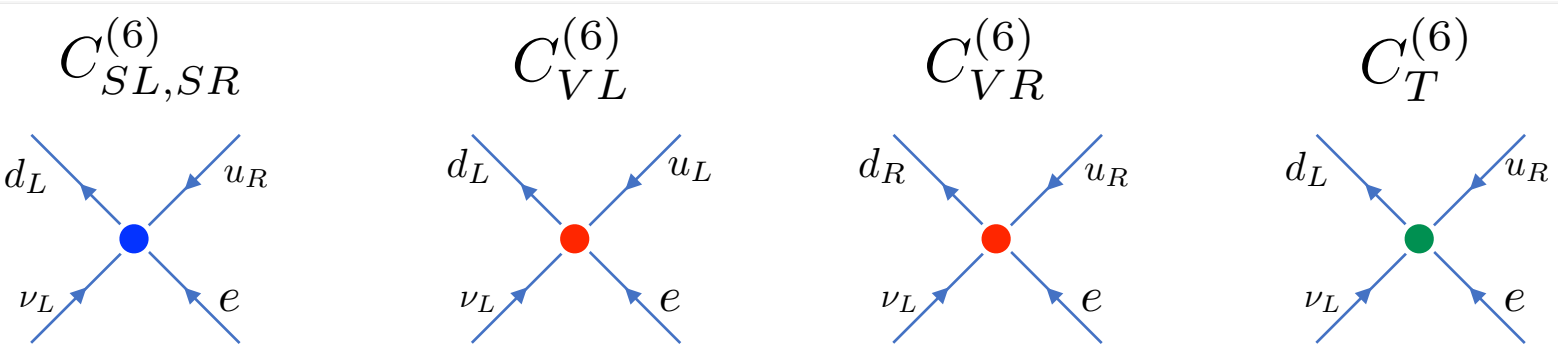


Low energy
constants

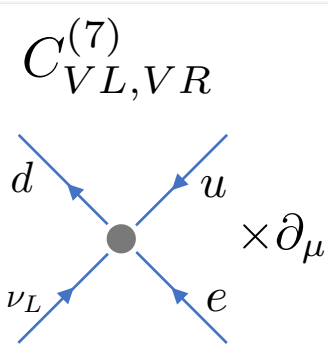


LECs

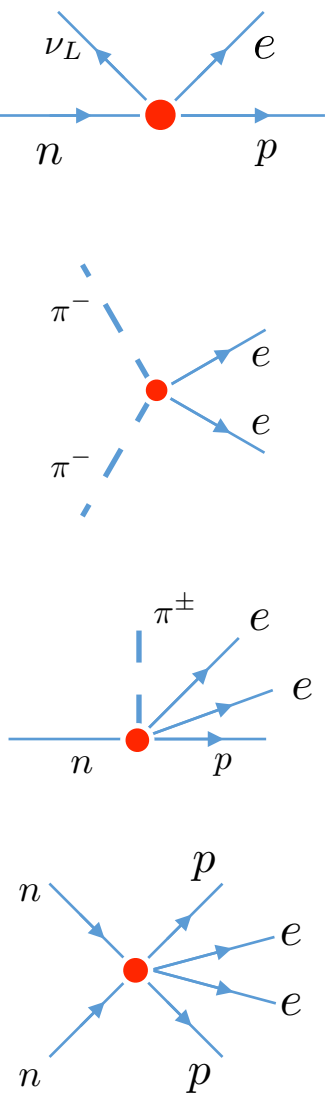
Dimension 6



Dimension 7



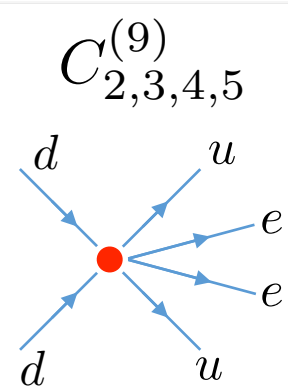
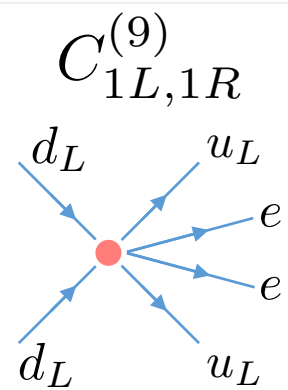
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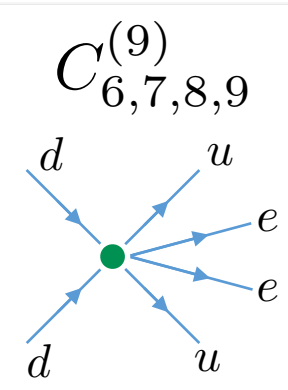
Scalar charge	Vector/axial charge	Vector/axial charge	Tensor charge	Scalar charge
			NLO LEC	
			x1	
	x1		x1	
	x1	Non-NDA	x1	
		x2		

LECs

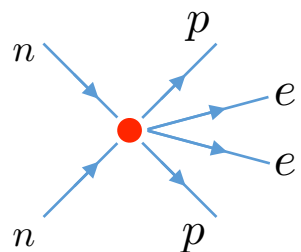
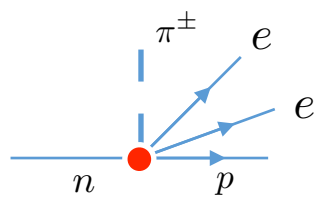
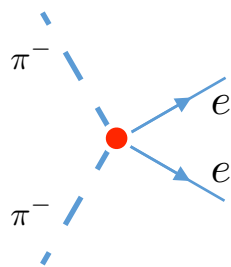
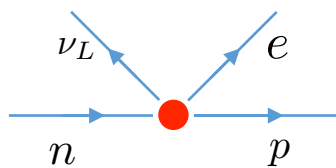
Dimension 9 - scalar



Dimension 9 - vector



Low energy constants



x1	x4	
x1		x2
x1	x4	x2