

Glueballs from functional methods

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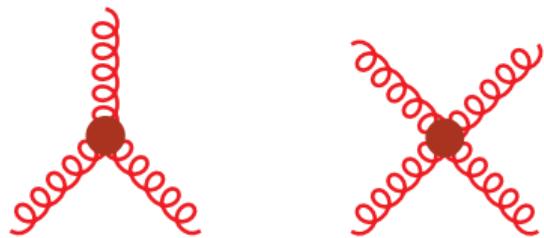
In collaboration with:
Christian S. Fischer
Hèlios Sanchis-Alepuz

[Eur.Phys.J.C 80, arXiv:2004.00415 → J=0](#)
[Eur.Phys.J.C 80, arXiv:2110.09180 → J=0,2,3,4](#)
[vConf21, arXiv:2111.10197 → +higher terms](#)
[HADRON2021, arXiv:2201.05163 → +higher terms](#)



Glueballs

Non-Abelian nature of QCD → self-interaction of force fields.



Mass dynamically created from **massless** (due to gauge invariance) gluons.

Theory:

Glueballs from gauge inv. operators, e.g., $F_{\mu\nu}F^{\mu\nu}$.

→ Mixing of operators with equal quantum numbers.

Experiment:

Production in glue-rich environments, e.g., $p\bar{p}$ annihilation (PANDA), pomeron exchange in pp (central exclusive production), radiative J/ψ decays

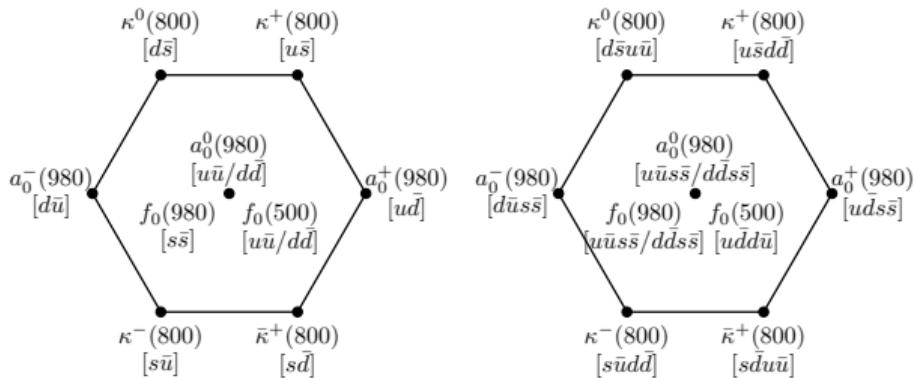
Reviews on glueballs: [Klempt, Zaitsev, Phys.Rept.454 (2007); Mathieu, Kochelev, Vento, Int.J.Mod.Phys.18 (2009); Crede, Meyer, Prog.Part.Nucl.Phys.63 (2009); Ochs, J.Phys.G40 (2013); Llanes-Estrada, EPJST 230 (2021)]

Scalar sector

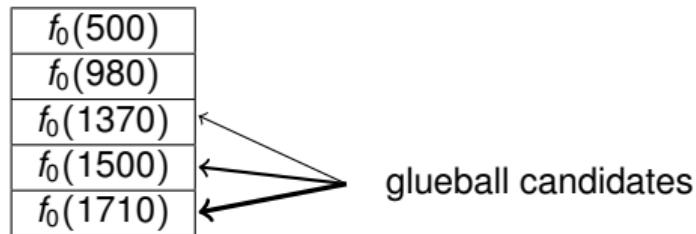
Classification not always easy, e.g., **scalar sector $J^{PC} = 0^{++}$** :

- $q\bar{q}$ mesons, tetraquarks: (inverted) mass hierarchy?

[Jaffe, Phys. Rev. D 15 (1977)]



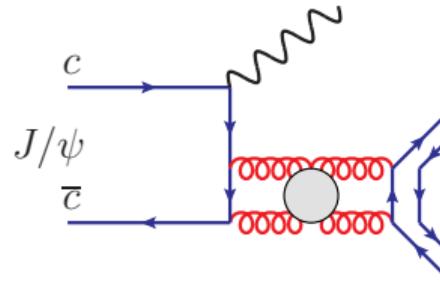
- Glueballs?



Functional review:

[Eichmann, Fischer,
Santowsky, Wallbott,
Few-Body Syst. 61 (2020)]

Scalar glueballs from J/ψ decay

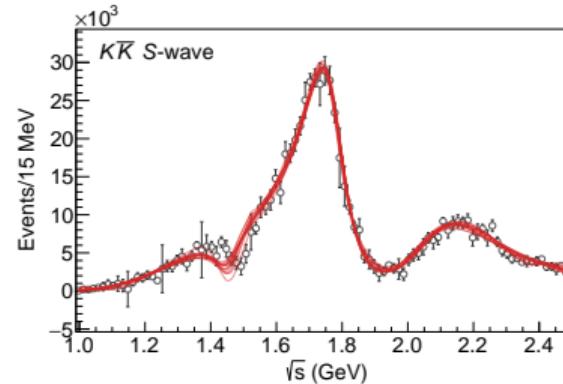
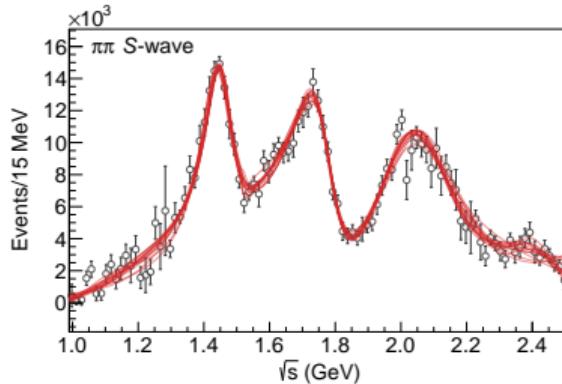


Coupled-channel analyses of exp. data (BESIII):

- +add. data, largest overlap with $f_0(1770)$
- largest overlap with $f_0(1710)$

[Sarantsev, Denisenko, Thoma, Klempt, Phys. Lett. B 816 (2021)]

[JPAC Coll., Rodas et al., Eur.Phys.J.C 82 (2022)]



Glueball studies

- **Reviews on glueballs:** [Klempt, Zaitsev, Phys.Rept.454 (2007); Mathieu, Kochelev, Vento, Int.J.Mod.Phys.18 (2009); Crede, Meyer, Prog.Part.Nucl.Phys.63 (2009); Ochs, J.Phys.G40 (2013); Llanes-Estrada, EPJST 230 (2021)]
- **Lattice:** [Morningstar, Peardon, Phys. Rev. D60 (1999); Athenodorou, Teper, JHEP11 (2020); Gregory et al., JHEP10 (2012); Brett et al., AIP Conf.Proc. 2249 (2020); Chen et al., 2111.11929; ...]
- **Hamiltonian many body methods:** [Szczepaniak, Swanson, Ji, Cotanch, PRL 76 (1996); Szczepaniak, Swanson, Phys. Lett. B 577 (2003); ...]
- **Chiral Lagrangians:** [Janowski, Parganlija, Giacosa, Rischke, Phys. Rev. D 84 (2011); Eshraim, Janowski, Giacosa, Rischke, Phys. Rev. D 87 (2013); ...]
- **Holographic QCD:** [Brower, Mathur, Tan, Nucl. Phys. B 587 (2000); Colangelo, De Fazio, Jugeau, Nicotri, Phys. Lett. B 652 (2007); Brüner, Parganlija, Rebhan, Phys. REv. D 93 (2016); ...]
- **Gribov-Zwanziger framework:** [Dudal, Guimaraes, Sorella, Phys. Lett. B 732 (2014)]
- **Functional studies:** [Meyers, Swanson, Phys.Rev.D87 (2013); Sanchis-Alepuz, Fischer, Kellermann, von Smekal, Phys.Rev.D92, (2015); Souza et al., Eur.Phys.J.A56 (2020); Kaptari, Kämpfer, Few Body Syst.61 (2020); MQH, Phys.Rev.D 101 (2020); MQH, Fischer, Sanchis-Alepuz, Eur.Phys.J.C80 (2020)]

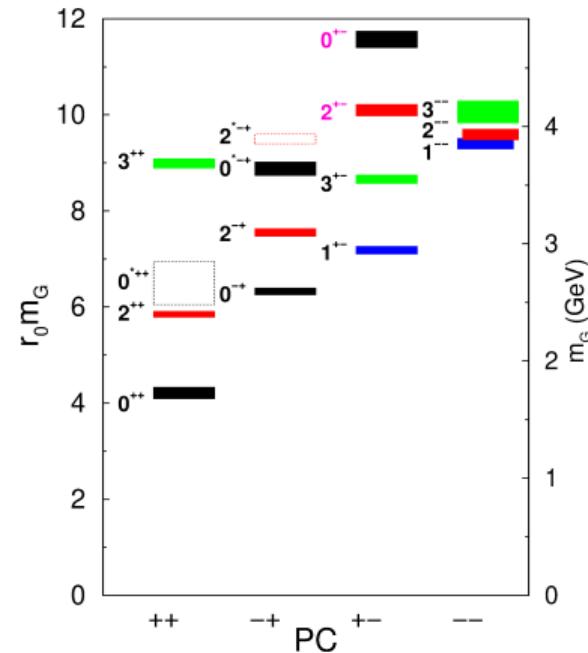
Glueball calculations: Lattice

Lattice methods

Pure gauge theory:

No dynamic quarks.
→ “Pure” glueballs

- [Morningstar, Peardon, Phys. Rev. D60 (1999)]: standard reference
- [Athenodorou, Teper, JHEP11 (2020)]: improved statistics, more states



[Morningstar, Peardon, Phys. Rev. D60 (1999)]

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“Real QCD”:

- [Gregory et al., JHEP10 (2012)]
- [Brett et al., AIP Conf. Proc. 2249 (2020)]
- [Chen et al., 2111.11929]
- [Review talk LATTICE2022 by D. Vadacchino]
- ...

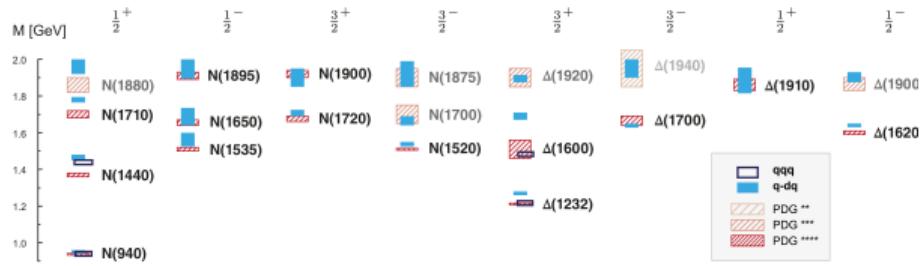
Challenging:

- Much higher statistics required (poor signal-to-noise ratio)
- Continuum extrapolation and inclusion of fermionic operators still to be done
- Mixing with $\bar{q}q$ challenging
- $m_\pi = 360 \text{ MeV}$
- Small unquenching effects found

No quantitative results yet.

Functional spectrum calculations

Functional methods successful in describing many aspects of the hadron spectrum qualitatively and quantitatively!



[Eichmann, Sanchis-Alepuz, Williams, Alkofer, Fischer, Prog.Part.Nucl.Phys. 91 (2016); Eichmann, Few Body Syst. 63 (2022)]

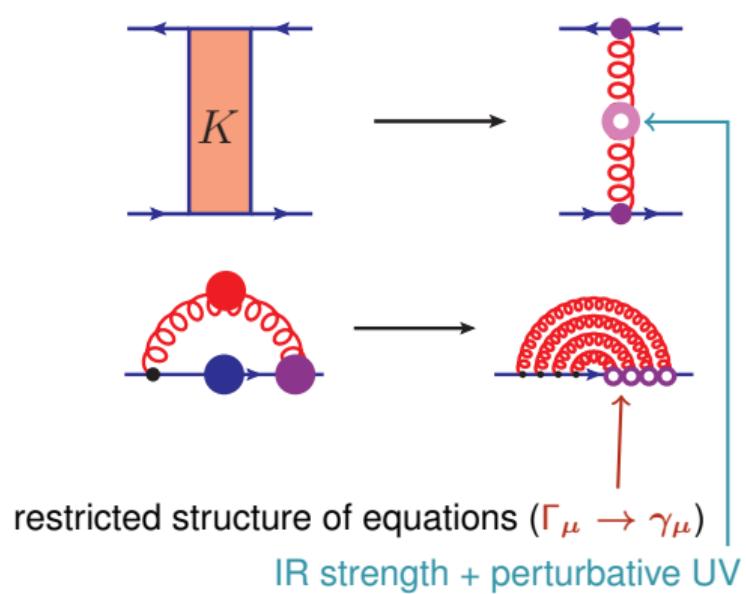
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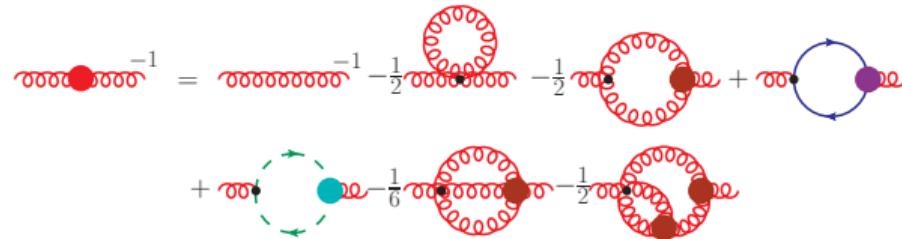
Workhorse for more than 20 years: Rainbow-ladder truncation with an effective interaction, e.g., Maris-Tandy (or similar).



Results for mesons beyond rainbow-ladder, e.g., [Williams, Fischer, Heupel, Phys.Rev.D 93 (2016)].

Functional glueball calculations

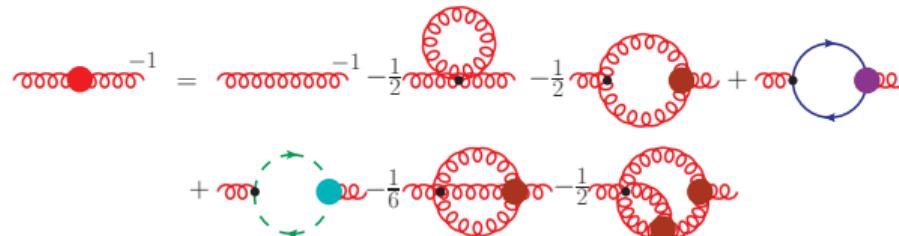
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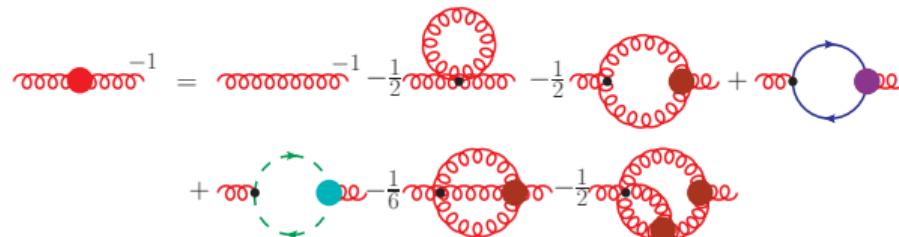
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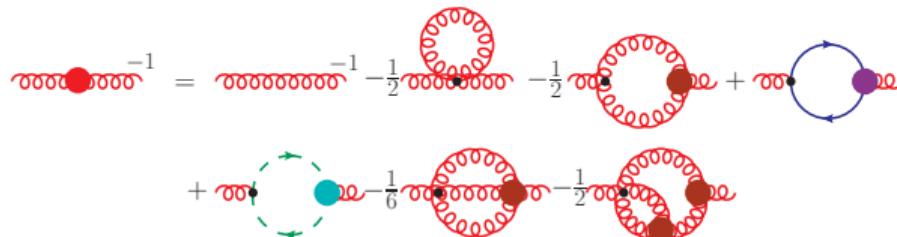


Model based BSE calculations
 $(J = 0)$:

- [Meyers, Swanson, Phys.Rev.D87 (2013)]
- [Sanchis-Alepuz, Fischer, Kellermann, von Smekal, Phys.Rev.D92, (2015)]
- [Souza et al., Eur.Phys.J.A56 (2020)]
- [Kaptari, Kämpfer, Few Body Syst.61 (2020)]

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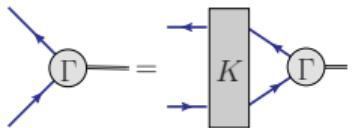
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- [Souza et al., Eur.Phys.J.A56 (2020)]
- [Kaptari, Kämpfer, Few Body Syst.61 (2020)]

Alternative: Calculated input [MQH, Phys.Rev.D 101 (2020)]

- $J = 0$: [MQH, Fischer, Sanchis-Alepuz, Eur.Phys.J.C80 (2020)]
- $J = 0, 2, 3, 4$: [MQH, Fischer, Sanchis-Alepuz, Eur.Phys.J.C81 (2021)]

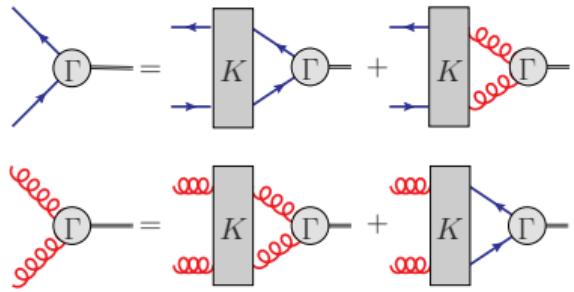
Extreme sensitivity on input!

Bound state equations for QCD



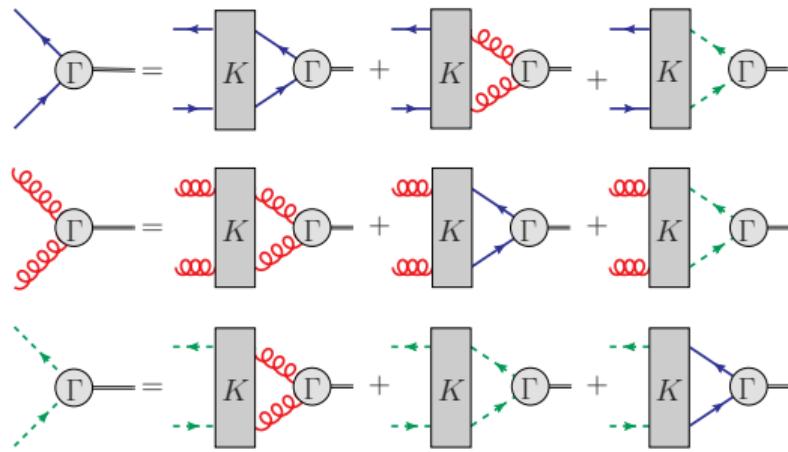
- Require scattering kernel K and propagator.

Bound state equations for QCD



- Require scattering kernels K and propagators.
- Quantum numbers determine which amplitudes Γ couple.

Bound state equations for QCD



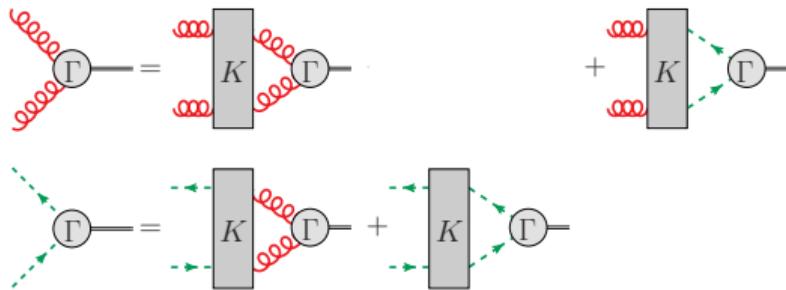
- Require scattering kernels K and propagators.
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- **Ghosts** from gauge fixing

One framework

- Natural description of **mixing**.
- Similar equations for hadrons with more than two constituents

Bound state equations for QCD

Focus on pure glueballs.



- Require scattering kernels K and propagators.
- Quantum numbers determine which amplitudes Γ couple.
- **Ghosts** from gauge fixing

One framework

- Natural description of **mixing**.
- Similar equations for hadrons with more than two constituents

Kernels

Systematic derivation from 3PI effective action: [Berges, Phys. Rev. D 70 (2004); Carrington, Gao, Phys. Rev. D 83 (2011)]

Self-consistent treatment of 3-point functions requires 3-loop expansion.

$$K = \text{[Diagram 1]} + \frac{1}{2} \text{[Diagram 2]} + \frac{1}{2} \text{[Diagram 3]} - \text{[Diagram 4]} + \text{[Diagram 5]} + \frac{1}{2} \text{[Diagram 6]}$$

$$K = \text{[Diagram 7]} + \frac{1}{2} \text{[Diagram 8]} + \frac{1}{2} \text{[Diagram 9]}$$

$$K = \text{[Diagram 10]} + \frac{1}{2}$$

$$K = \text{[Diagram 11]} + \frac{1}{2} \text{[Diagram 12]} + \frac{1}{2} \text{[Diagram 13]}$$

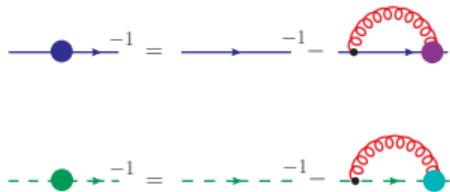
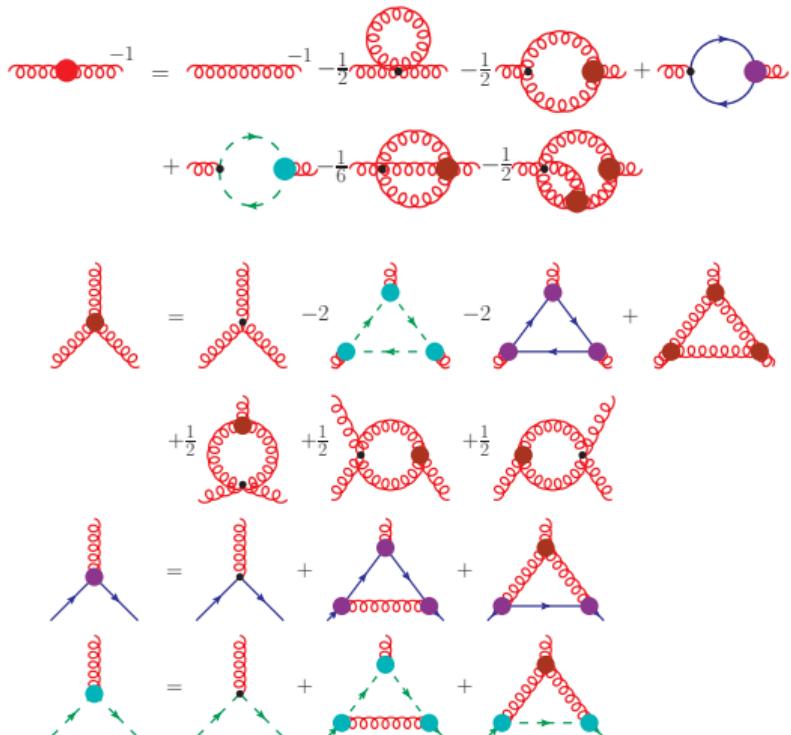


[Fukuda, Prog. Theor. Phys 78 (1987); McKay, Munczek, Phys. Rev. D 40 (1989); Sanchis-Alepuz, Williams, J. Phys: Conf. Ser. 631 (2015); MQH, Fischer, Sanchis-Alepuz, Eur.Phys.J.C80 (2020)]

Correlation functions of quarks and gluons

Equations of motion: 3-loop 3PI effective action

→ [Review: MQH, Phys.Rept. 879 (2020)]

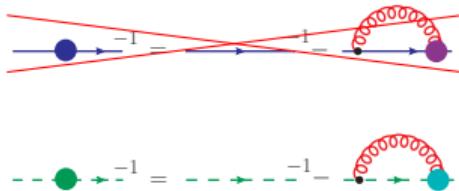
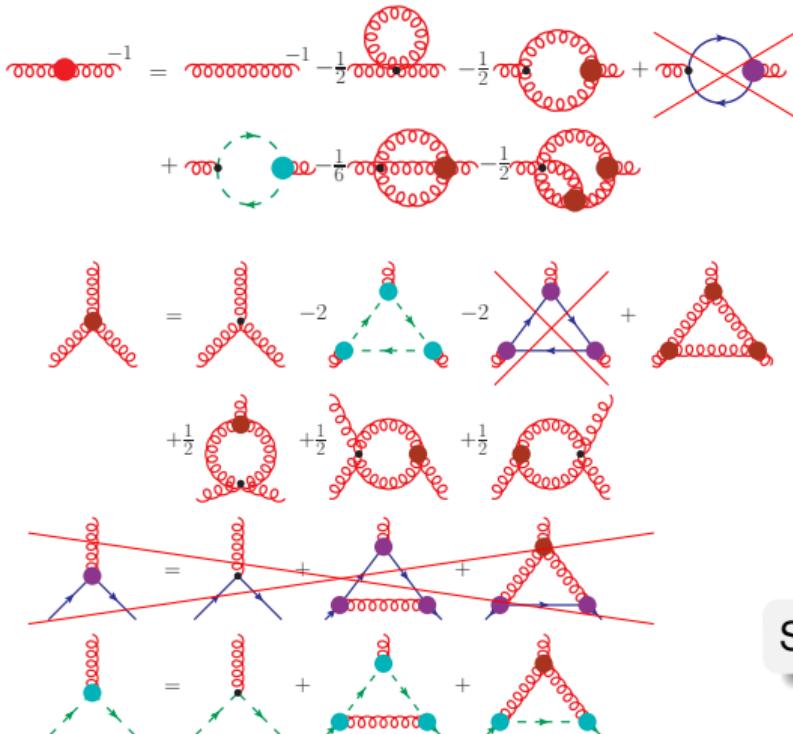


- Conceptual and technical challenges: nonperturbative renormalization, two-loop diagrams, convergence, size of kernels, ...
- Self-contained: Only parameters are the **strong coupling and the quark masses!**
- Long way, e.g., ghost-gluon vertex, three-gluon vertex, four-gluon vertex, ...
- → MQH, Phys.Rev.D 101 (2020)

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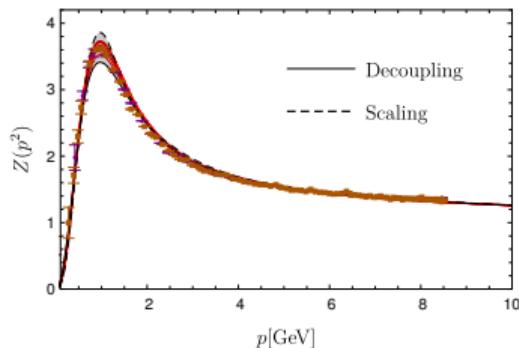
Start with **pure gauge theory**.

Landau gauge propagators

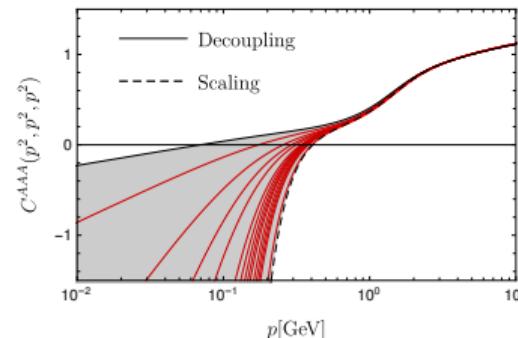
Self-contained: Only external input is the coupling!

[MQH, Phys.Rev.D 101 (2020)]

Gluon dressing function:



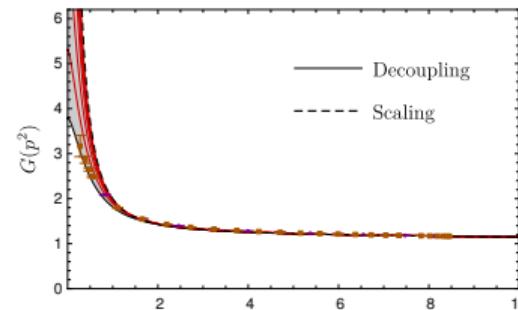
Three-gluon vertex:



Family of solutions [von Smekal, Alkofer, Hauck, PRL79 (1997); Aguilar, Binosi, Papavassiliou, Phys.Rev.D 78 (2008); Boucaud et al., JHEP06 (2008); Fischer, Maas, Pawłowski, Ann.Phys. 324 (2008); Alkofer, MQH, Schwenzer, Phys. Rev. D 81 (2010)]

Nonperturbative completions of Landau gauge [Maas, Phys. Lett. B 689 (2010)]?

Ghost dressing function:

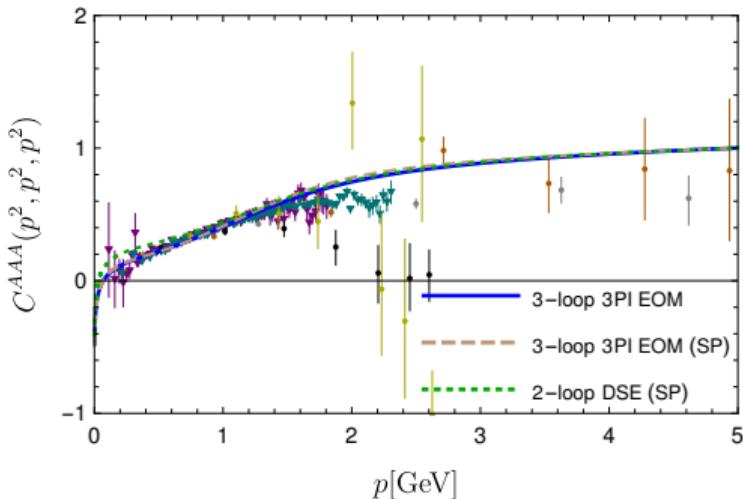


Stability of the solution

- Agreement with lattice results. ✓

Stability of the solution

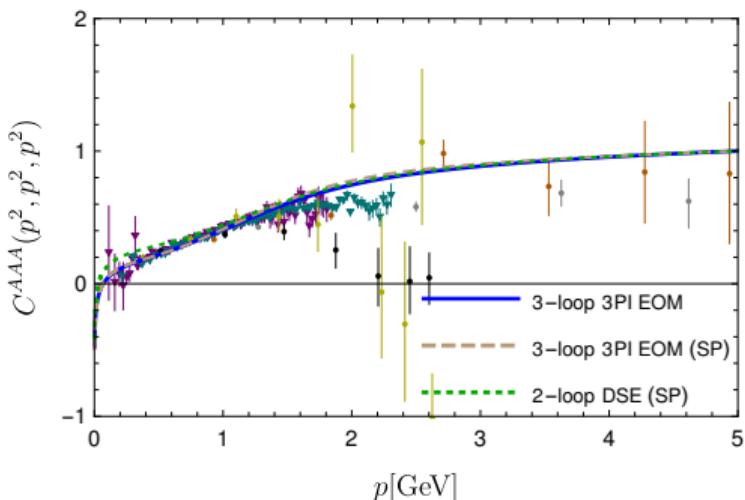
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- Concurrence between functional methods:
3PI vs. 2-loop DSE:



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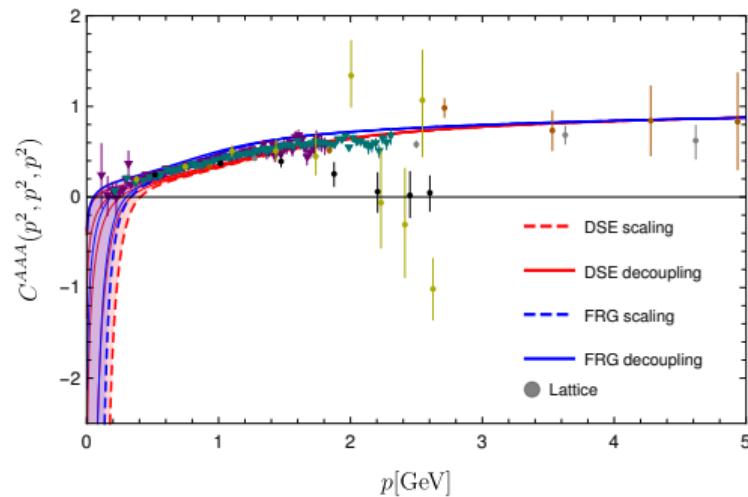
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3PI vs. 2-loop DSE:



[Cucchieri, Maas, Mendes, Phys.Rev.D77 (2008); Sternbeck et al., Proc.Sci. LATTICE2016 (2017); Cyrol et al., Phys.Rev.D 94 (2016); MQH, Phys.Rev.D101 (2020)]

DSE vs. FRG:

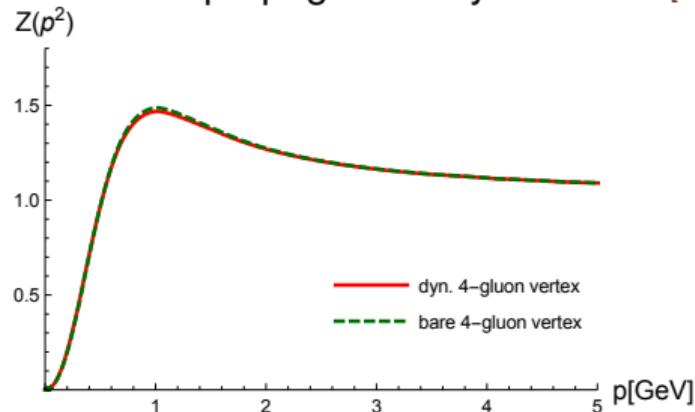


Stability of the solution: Extensions

- Three-gluon vertex: Tree-level dressing dominant, others subleading [Eichmann, Williams, Alkofer, Vujinovic, Phys.Rev.D89 (2014); Pinto-Gómez et al., 2208.01020] ✓

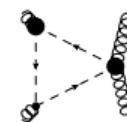
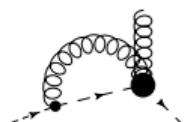
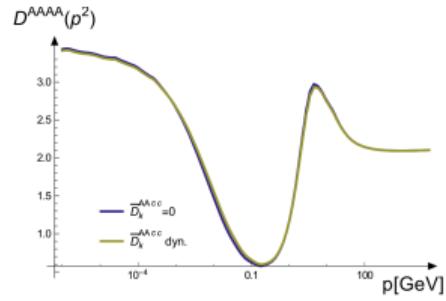
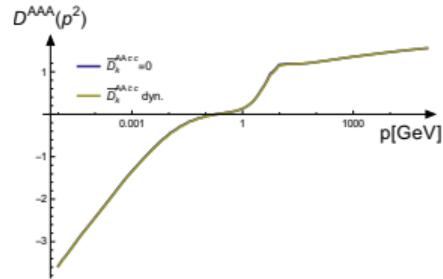
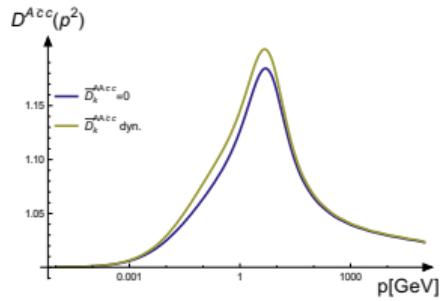
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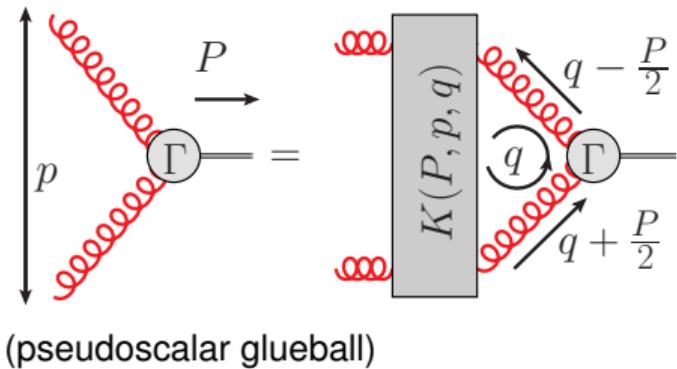


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- Four-gluon vertex: Influence on propagators tiny for $d = 3$ [MQH, Phys.Rev.D93 (2016)]
- Two-ghost-two-gluon vertex [MQH, Eur. Phys.J.C77 (2017)]: ✓
(FRG: [Corell, SciPost Phys. 5 (2018)])



Correlation functions for complex momenta

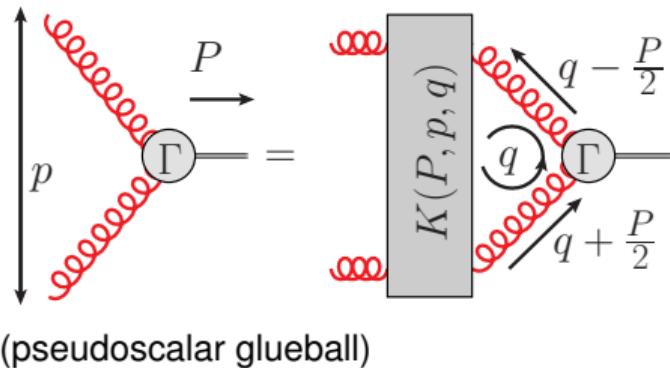


$$\lambda(\mathbf{P})\Gamma(P) = \mathcal{K} \cdot \Gamma(P)$$

→ Eigenvalue problem for $\Gamma(P)$:

- ➊ Solve for $\lambda(P)$.
- ➋ Find P with $\lambda(P) = 1$.
 $\Rightarrow M^2 = -P^2$

Correlation functions for complex momenta



$$\lambda(\mathbf{P})\Gamma(P) = \mathcal{K} \cdot \Gamma(P)$$

→ Eigenvalue problem for $\Gamma(P)$:

- ① Solve for $\lambda(P)$.
- ② Find P with $\lambda(P) = 1$.
 $\Rightarrow M^2 = -P^2$

However:

Propagators are probed at $\left(q \pm \frac{P}{2}\right)^2 = \frac{P^2}{4} + q^2 \pm \sqrt{P^2 q^2} \cos \theta = -\frac{M^2}{4} + q^2 \pm i M \sqrt{q^2} \cos \theta$
 \rightarrow Complex for $P^2 < 0$!

Time-like quantities ($P^2 < 0$) → Correlation functions for complex arguments.

Extrapolation of $\lambda(P^2)$

Extrapolation method

- Extrapolation to time-like P^2 using Schlessinger's continued fraction method (proven superior to default Padé approximants) [Schlessinger, Phys.Rev.167 (1968)]
- Average over extrapolations using subsets of points for error estimate

$$f(x) = \frac{f(x_1)}{1 + \frac{a_1(x-x_1)}{1 + \frac{a_2(x-x_2)}{1 + \frac{a_3(x-x_3)}{\dots}}}}$$

Coefficients a_i can
determined such that
 $f(x)$ exact at x_i .

Extrapolation of $\lambda(P^2)$

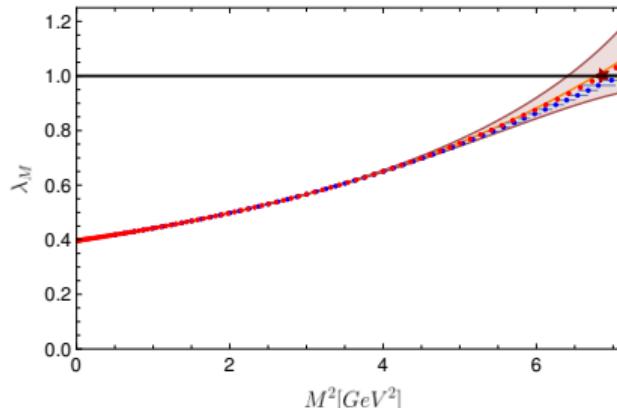
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Test extrapolation for solvable system:
Heavy meson [MQH, Sanchis-Alepuz, Fischer, Eur.Phys.J.C 80 (2020)]

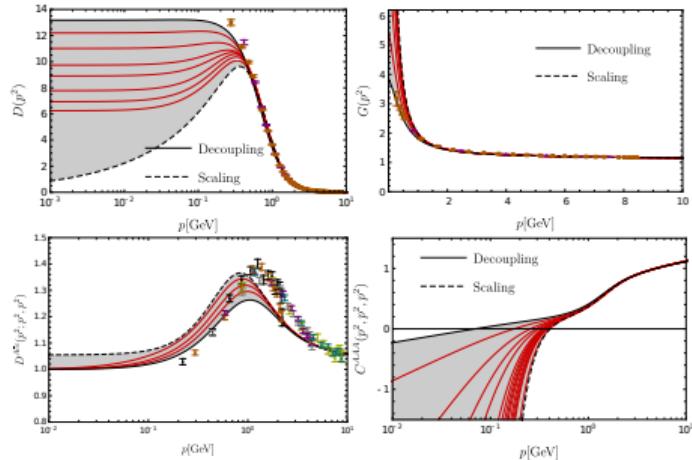
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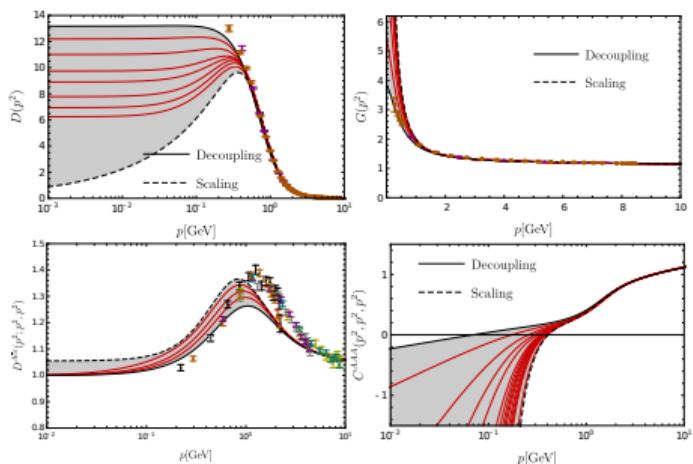
Glueball results J=0

Gauge-variant correlation functions:



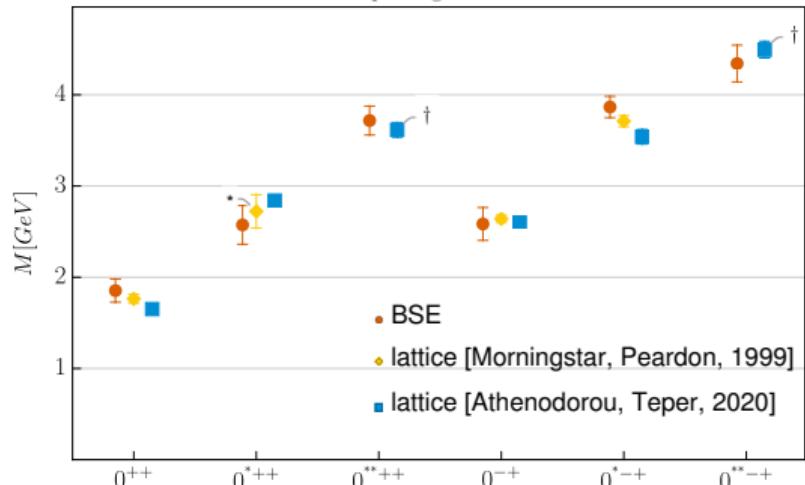
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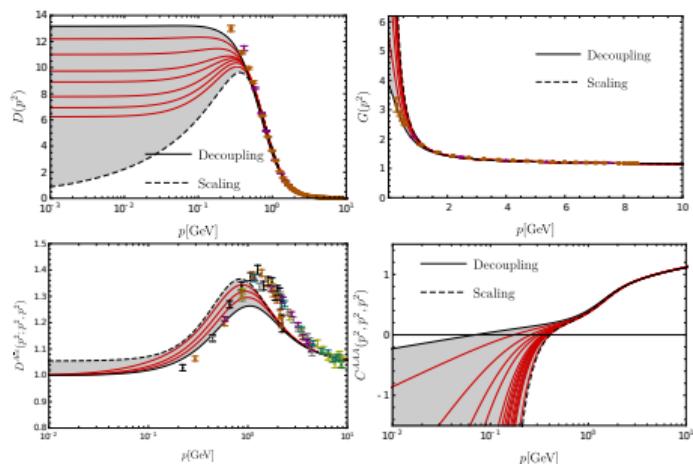
Unique physical spectrum:

Spin-0 glueballs

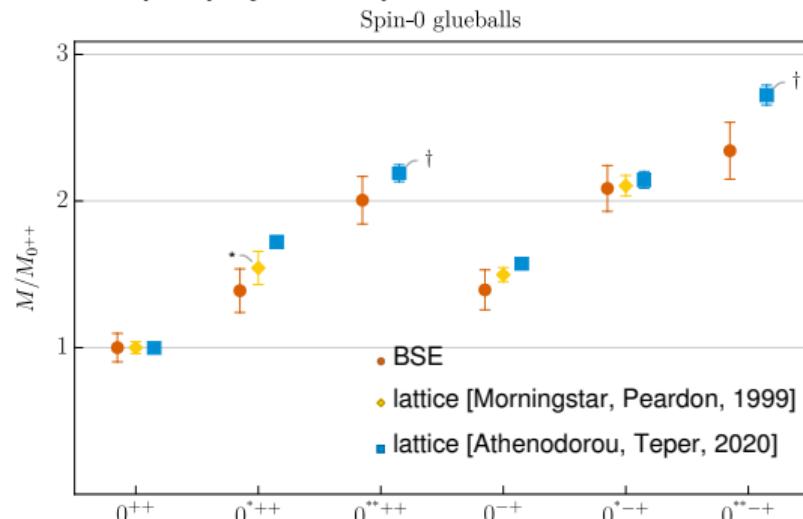


Glueball results J=0

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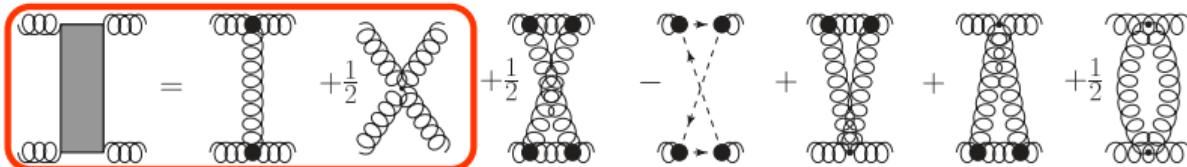


Spectrum independent! → Family of solutions yields the same physics.

All results for $r_0 = 1/418(5)$ MeV.

[MQH, Fischer, Sanchis-Alepuz, Eur.Phys.J.C80 (2020)]

Higher order diagrams



One-loop diagrams only:

[MQH, Fischer, Sanchis-Alepuz, Eur.Phys.J.C80 (2020); MQH, Fischer, Sanchis-Alepuz, Eur.Phys.J.C81 (2021)]

Two-loop diagrams: subleading effects

- 0^{-+} : none

[MQH, Fischer, Sanchis-Alepuz, EPJ Web Conf. 258 (2022)]

- 0^{++} : < 2%

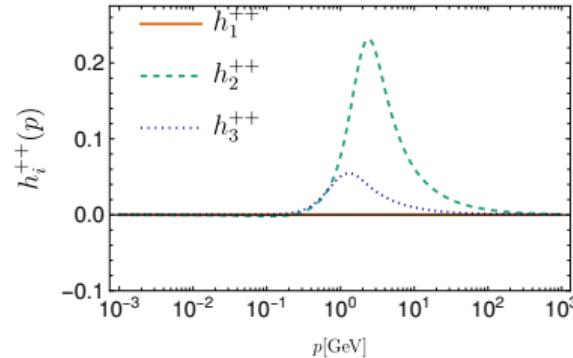
[MQH, Fischer, Sanchis-Alepuz, HADRON2021, arXiv:2201.05163]

Amplitudes

Information about significance of single parts.

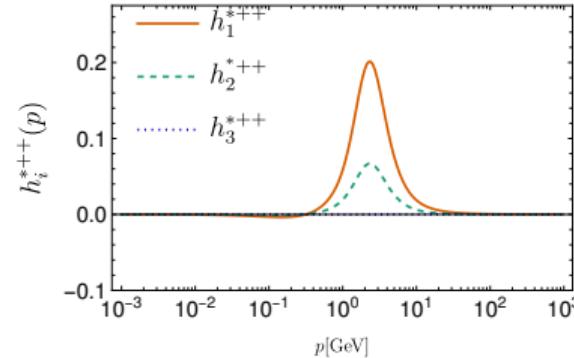
Ground state scalar glueball:

Amplitudes 0^{++}



Excited scalar glueball:

Amplitudes 0^{*++}

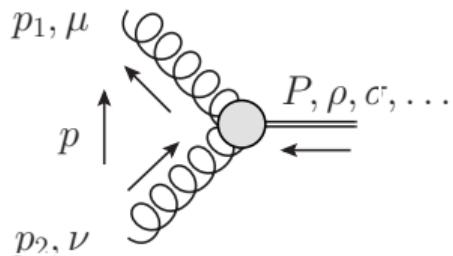


- Amplitudes have different behavior for ground state and excited state. Useful guide for future developments.
- Meson/glueball amplitudes: **Information about mixing.**

Glueball amplitudes for spin J

[MQH, Fischer, Sanchis-Alepuz, Eur.Phys.J.C81 (2021)]

$$\Gamma_{\mu\nu\rho\sigma\dots}(p_1, p_2) = \sum \tau^i_{\mu\nu\rho\sigma\dots}(p_1, p_2) h_i(p_1, p_2)$$



Increase in complexity:

- 2 gluon indices (transverse)
- J spin indices (symmetric, traceless, transverse to P)

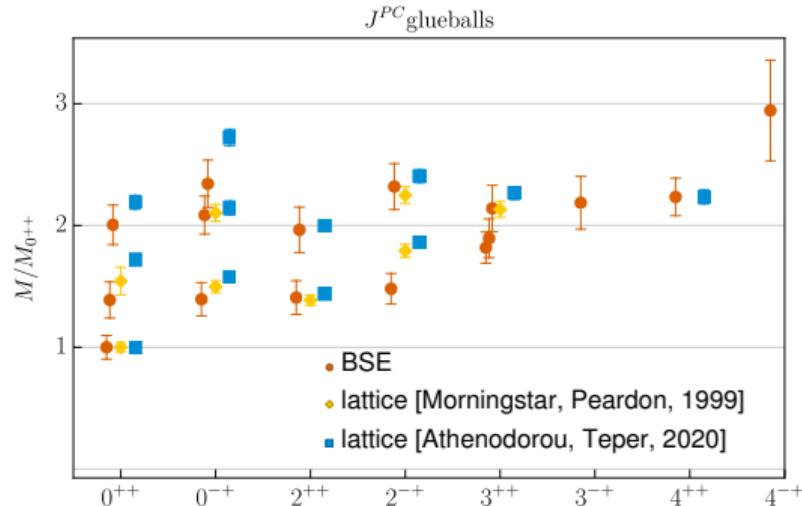
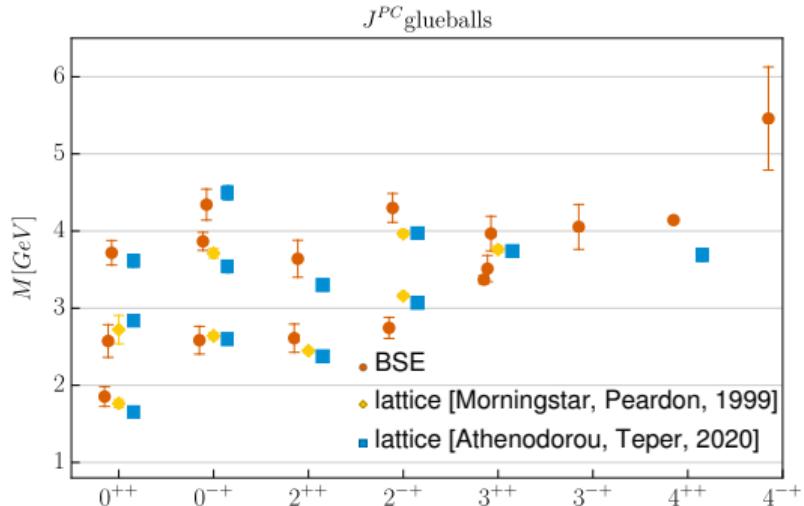
Numbers of tensors:

J	$P = +$	$P = -$
0	2	1
1	4	3
>2	5	4

Low number of tensors, but high-dimensional tensors!

→ Computational cost increases with J .

Glueball results



[MQH, Fischer, Sanchis-Alepuz, Eur.Phys.J.C81 (2021)]

- Agreement with lattice results
- New states: 0^{**++} , 0^{**-+} , 3^{-+} , 4^{-+}

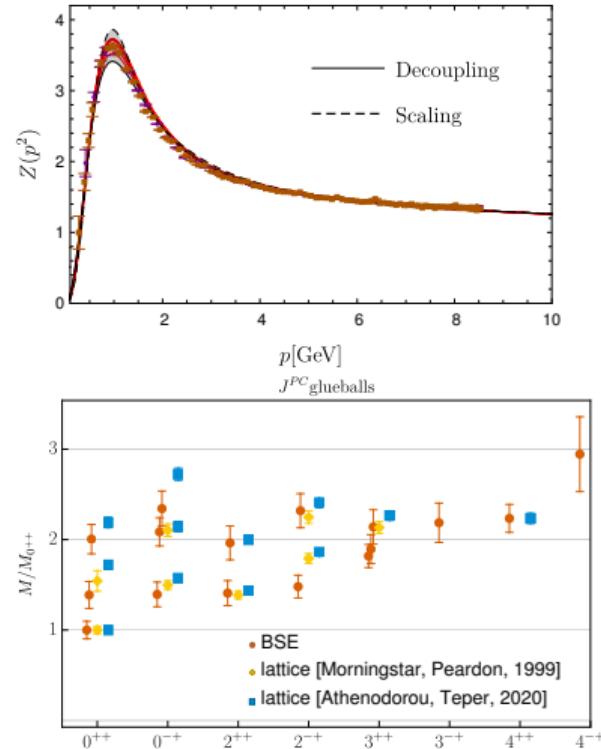
Summary and outlook

- Alternative to models in bound state equations: **Direct calculation** of input.
- Large system of equations may be necessary.
- Independent tests:
 - Agreement with other methods:
lattice + continuum
 - Extensions

Pure glueball spectrum from **first principles**.

Future:

- +quarks \rightarrow QCD
- three-body bound state eq. $\rightarrow C = -1$



Summary and outlook

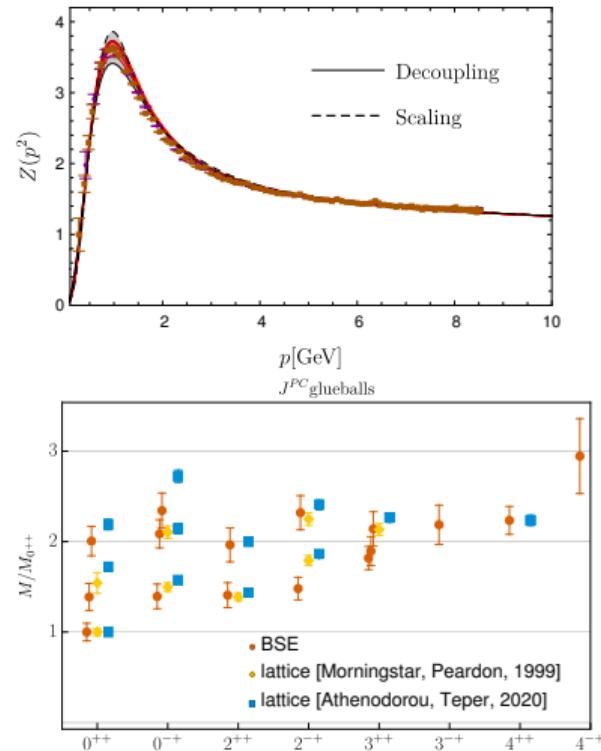
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Pure glueball spectrum from **first principles**.

Future:

- +quarks \rightarrow QCD
- three-body bound state eq. $\rightarrow C = -1$

Thank you for your attention.



$J = 1$ glueballs

Landau-Yang theorem

Two-photon states cannot couple to $J^P = \mathbf{1}^\pm$ or $(2n+1)^-$

[Landau, Dokl.Akad.Nauk SSSR 60 (1948); Yang, Phys. Rev. 77 (1950)].

(→ Exclusion of $J = 1$ for Higgs because of $h \rightarrow \gamma\gamma$.)

Applicable to glueballs?

- Not in this framework, since gluons are not on-shell.
- Presence of $J = 1$ states is a dynamical question.

$J = 1$ not found here.

Glueballs as bound states

Hadron masses from correlation functions of color singlet operators.

Glueballs as bound states

Hadron masses from correlation functions of color singlet operators.

Example: For $J^{PC} = 0^{++}$ glueball take $O(x) = F_{\mu\nu}(x)F^{\mu\nu}(x)$:

$$D(\mathbf{x} - \mathbf{y}) = \langle O(x)O(y) \rangle$$

Glueballs as bound states

Hadron masses from correlation functions of color singlet operators.

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Lattice: Mass exponential Euclidean time decay:

$$\lim_{t \rightarrow \infty} \langle O(x)O(0) \rangle \sim e^{-tM}$$

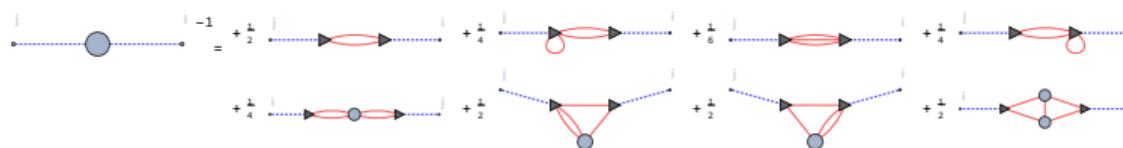
Glueballs as bound states

Hadron masses from correlation functions of color singlet operators.

Example: For $J^{PC} = 0^{++}$ glueball take $O(x) = F_{\mu\nu}(x)F^{\mu\nu}(x)$:

$$D(x - y) = \langle O(x)O(y) \rangle$$

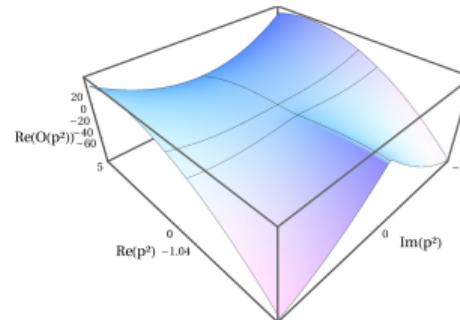
Functional approach: Complicated object in a diagrammatic language, 2-, 3- and 4-gluon contributions [MQH, Cyrol, Pawłowski, Comput.Phys.Commun. 248 (2020)]



+ 3-loop diagrams

Leading order:

[Windisch, MQH, Alkofer, Phys.Rev.D87 (2013)]



Glueballs as bound states

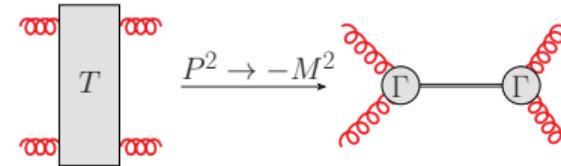
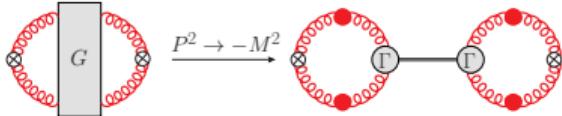
Hadron masses from correlation functions of color singlet operators.

Example: For $J^{PC} = 0^{++}$ glueball take $O(x) = F_{\mu\nu}(x)F^{\mu\nu}(x)$:

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Put total momentum **on-shell** and consider individual 2-, 3- and 4-gluon contributions. →
Each can have a pole at the glueball mass.

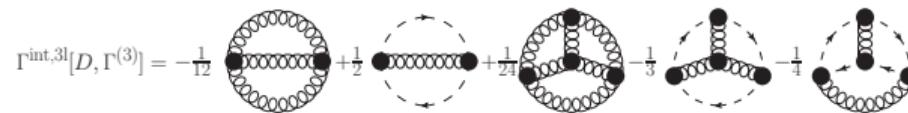
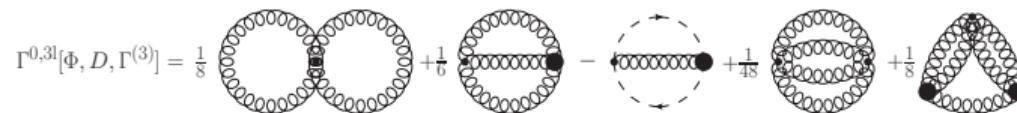
A^4 -part of $D(x - y)$, total momentum on-shell:



Kernel construction

From 3PI effective action truncated to three-loops: [Berges, Phys. Rev. D 70 (2004); Carrington, Gao, Phys. Rev. D 83 (2011)]

$$\Gamma^{3l}[\Phi, D, \Gamma^{(3)}] = \Gamma^{0,3l}[\Phi, D, \Gamma^{(3)}] + \Gamma^{\text{int},3l}[\Phi, D, \Gamma^{(3)}]$$



Kernels constructed by cutting two legs:
gluon/gluon, ghost/gluon, gluon/ghost, ghost/ghost

[Fukuda, Prog. Theor. Phys 78 (1987); McKay, Munczek, Phys. Rev. D 40 (1989); Sanchis-Alepuz, Williams, J. Phys: Conf. Ser. 631 (2015); MQH, Fischer, Sanchis-Alepuz, Eur.Phys.J.C80 (2020)]

Charge parity

Transformation of gluon field under charge conjugation:

$$A_\mu^a \rightarrow -\eta(a) A_\mu^a$$

where

$$\eta(a) = \begin{cases} +1 & a = 1, 3, 4, 6, 8 \\ -1 & a = 2, 5, 7 \end{cases}$$

Color neutral operator with two gluon fields:

$$A_\mu^a A_\nu^a \rightarrow \eta(a)^2 A_\mu^a A_\nu^a = A_\mu^a A_\nu^a.$$

$$\Rightarrow C = +1$$

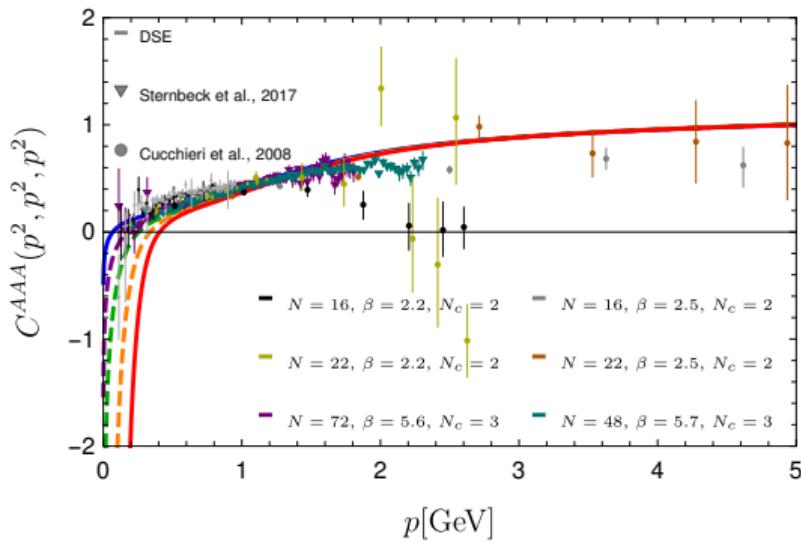
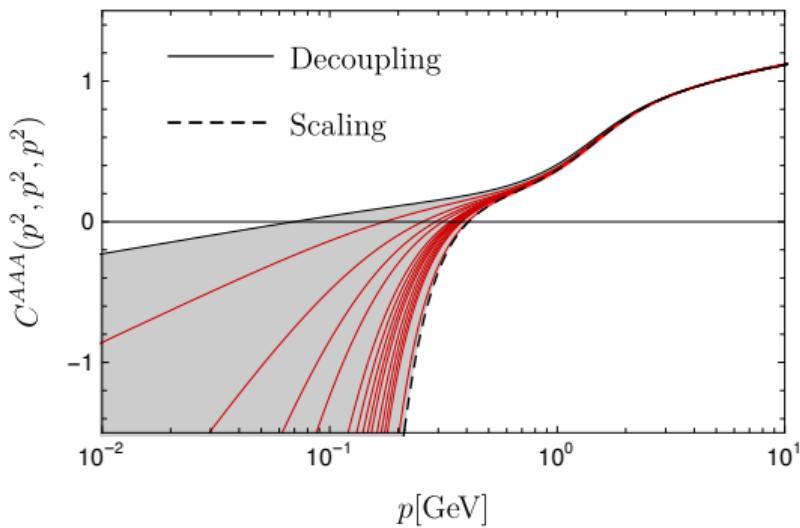
Negative charge parity, e.g.:

$$\begin{aligned} d^{abc} A_\mu^a A_\nu^b A_\rho^c &\rightarrow -d^{abc} \eta(a) \eta(b) \eta(c) A_\mu^a A_\nu^b A_\rho^c = \\ &-d^{abc} A_\mu^a A_\nu^b A_\rho^c. \end{aligned}$$

Only nonvanishing elements of the symmetric structure constant d^{abc} : zero or two indices equal to 2, 5 or 7.

Three-gluon vertex

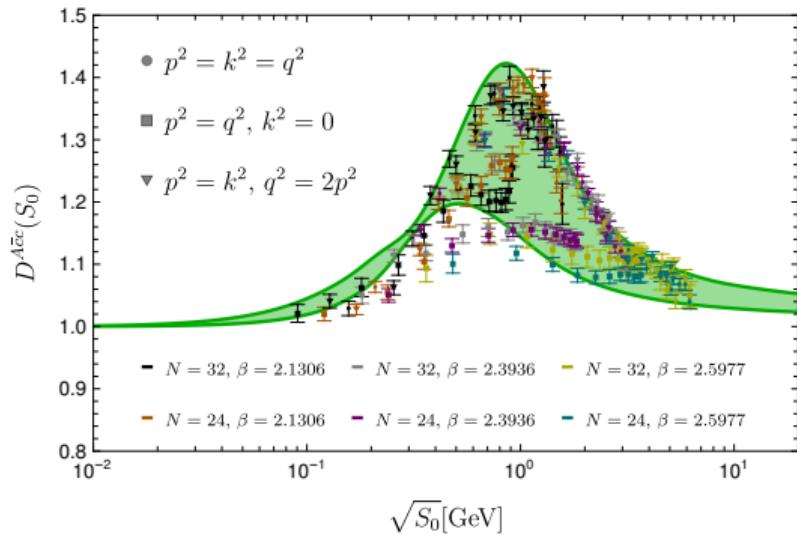
[Cucchieri, Maas, Mendes, Phys. Rev. D 77 (2008); Sternbeck et al., 1702.00612; MQH, Phys. Rev. D 101 (2020)]



- Simple kinematic dependence of three-gluon vertex (only singlet variable of S_3)
- Large cancellations between diagrams

Ghost-gluon vertex

Ghost-gluon vertex:



[Maas, SciPost Phys. 8 (2019);
MQH, Phys. Rev. D 101 (2020)]

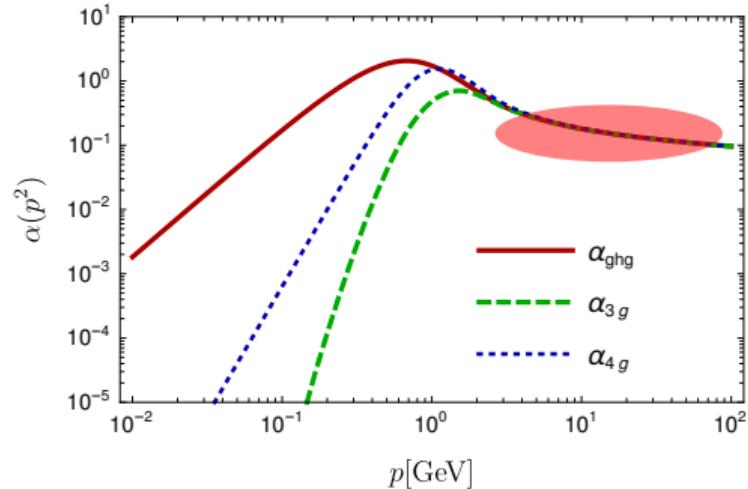
- Nontrivial kinematic dependence of ghost-gluon vertex
- Qualitative agreement with lattice results, though some quantitative differences (position of peak!).

Gauge invariance

[MQH, Phys. Rev. D 101 (2020)]

Couplings can be extracted from each vertex.

- Slavnov-Taylor identities (gauge invariance): Agreement perturbatively (UV) necessary.
[Cyrol et al., Phys. Rev. D 94 (2016)]
- Difficult to realize: Small deviations → Couplings cross and do not agree.
- Here: Vertex couplings agree down to GeV regime (IR can be different).



Landau gauge propagators in the complex plane

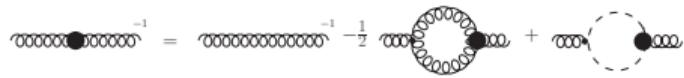
Simpler truncation:

$$\text{Diagram with two external wavy lines and one internal loop}^{-1} = \text{Diagram with two external wavy lines and one internal loop}^{-1} - \frac{1}{2} \text{Diagram with two external wavy lines and one internal loop} + \text{Diagram with two external wavy lines and one internal loop}$$

[Fischer, MQH, Phys.Rev.D 102 (2020)]

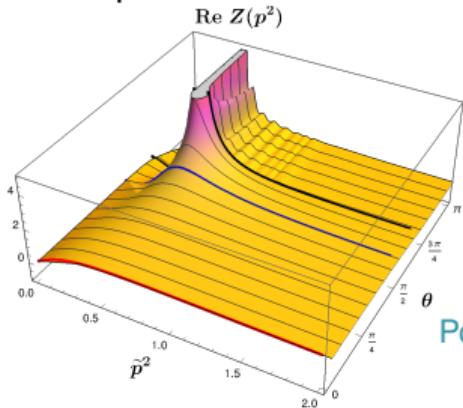
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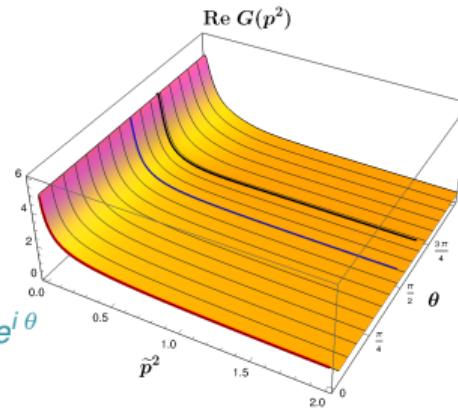


[Fischer, MQH, Phys.Rev.D 102 (2020)]

Ray technique for self-consistent solution of a DSE:



$$\text{Polar coordinates: } p^2 = \tilde{p}^2 e^{i\theta}$$



- Current truncation leads to a pole-like structure in the gluon propagator.
- Analyticity up to 'pole' confirmed by various tests (Cauchy-Riemann, Schlessinger, reconstruction)

Landau gauge propagators in the complex plane

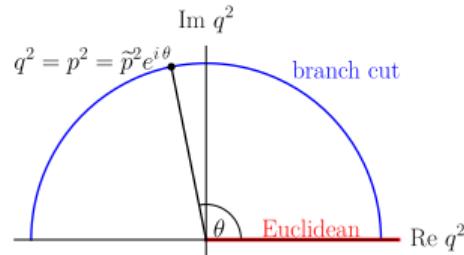
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Landau gauge propagators in the complex plane

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$$\text{propagator}^{-1} = \text{propagator}^{-1} - \frac{1}{2} \text{loop} + \text{loop}$$

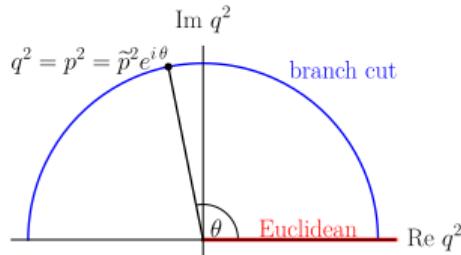


→ Opening at $q^2 = p^2$.

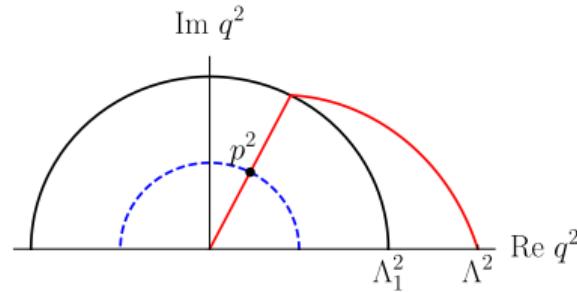
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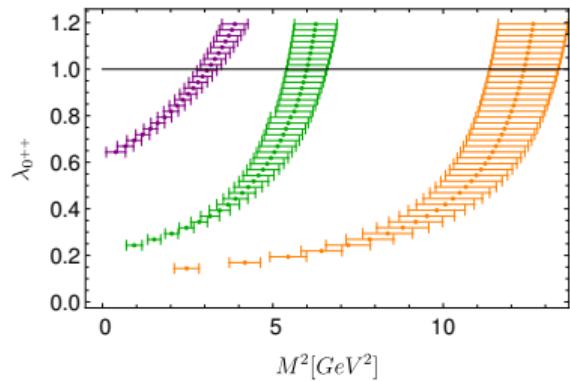
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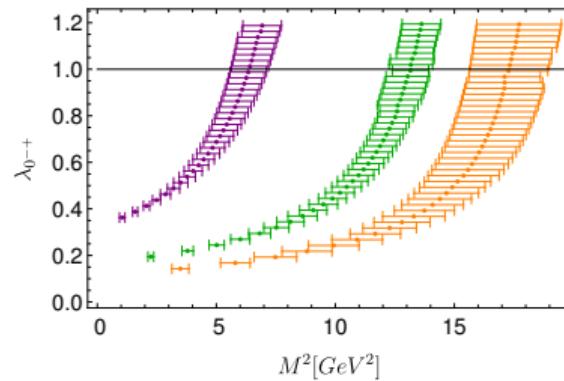
Appearance of branch cuts for complex momenta forbids integration directly to cutoff.

Extrapolation for glueball eigenvalue curves

0^{++} :



0^{-+} :



Several curves: ground state and excited states.