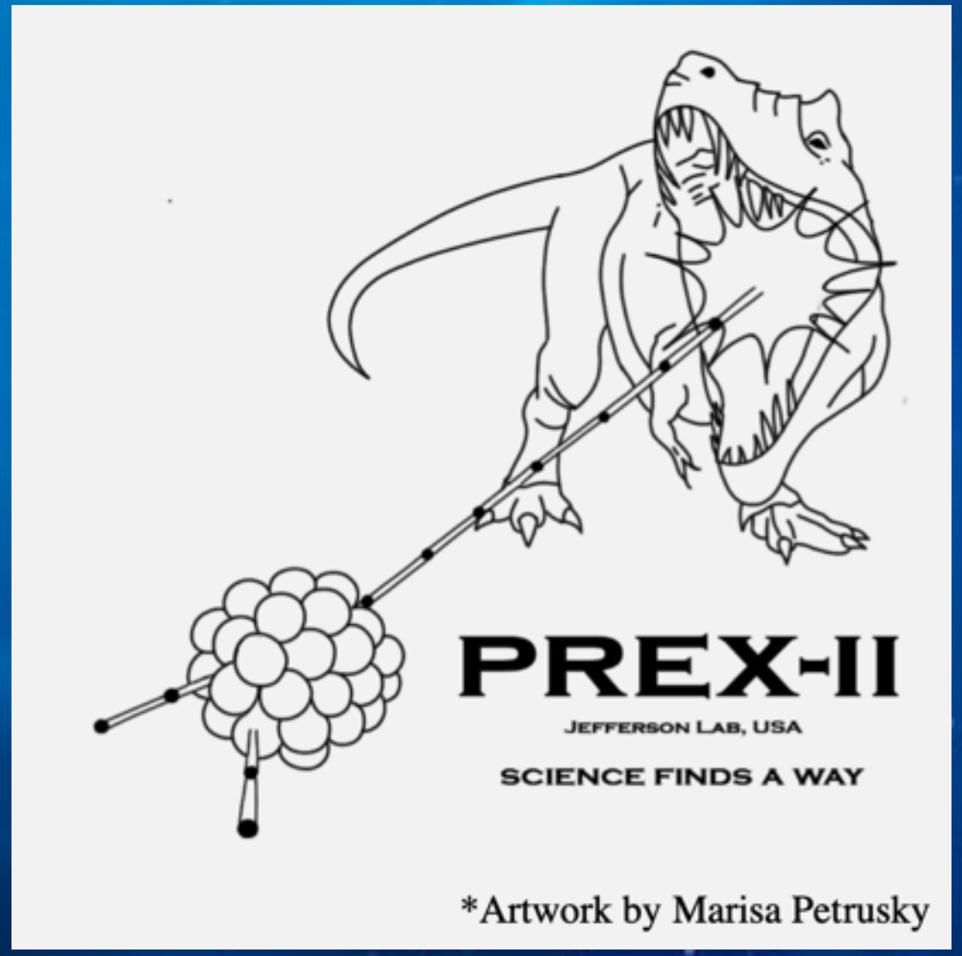
Dense matter in our minds, on earth, and in heaven: an overview of theory, experiment, and observation

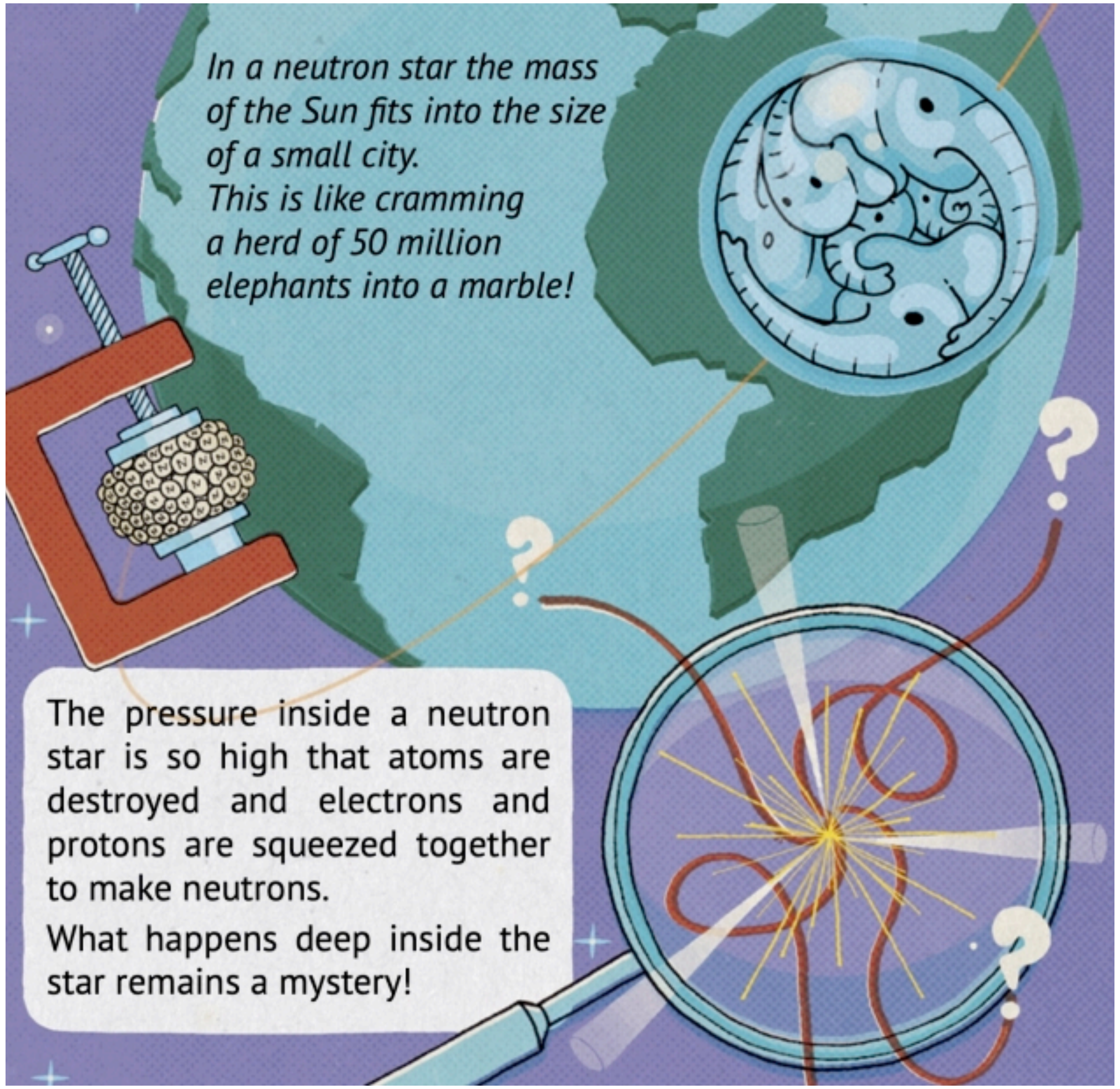


C. J. Horowitz, Colliders to Cosmos, INT Sep. 2025

Dense matter

- There is no evidence to suggest there were ever 50 million elephants on Earth at any point in history. While elephant populations have been significantly higher in the past than they are today, historical estimates do not reach 50 million. For example, it's estimated that Africa may have had over 20 million elephants before European colonization, and 1 million as recently as the 1970s.

AI responses may include mistakes.



Scope: NS mergers in a broad physical context.

This includes the following:

- [Heaven] Neutron star structure: masses and radii, magnetic fields, spins, temperatures, formation, death, populations... EM, neutrino, cosmic ray, and GW radiations of neutron stars.

- [Earth] Nuclear masses, abundances, and reactions

- [Mind] Dense matter **equation of state** [pressure as a function of density, temperature and composition, ie proton fraction]. **Transport coefficients**: thermal and electrical conductivities, shear and bulk viscosities, neutrino opacities…

What were the biggest breakthroughs this century?

- [Mind] Chiral EFT. Theoretical framework has great success predicting structure of many nuclei and the EOS at low densities.

- [Earth] Discovery of strongly interacting quark gluon plasma. Relativistic heavy ion collision experiments found a new nearly perfect fluid with a low shear viscosity.

- [Heaven] Observation of two solar mass neutron stars.

- What were the most important astronomical transients of the last 50 years?

  SN1987A, GW170817

- What taught us the most about cold dense matter? Two solar mass NS.

Questions

- What holds up a two solar mass neutron star?

- What are neutron stars made of? What are the phases of dense matter? What are the degrees of freedom?

- How does Chiral EFT break down? Example, it may break down because a whole series of three-body, four-body, five-body … interactions all start to contribute coherently. What is matter like just after Chrial EFT brakes down?

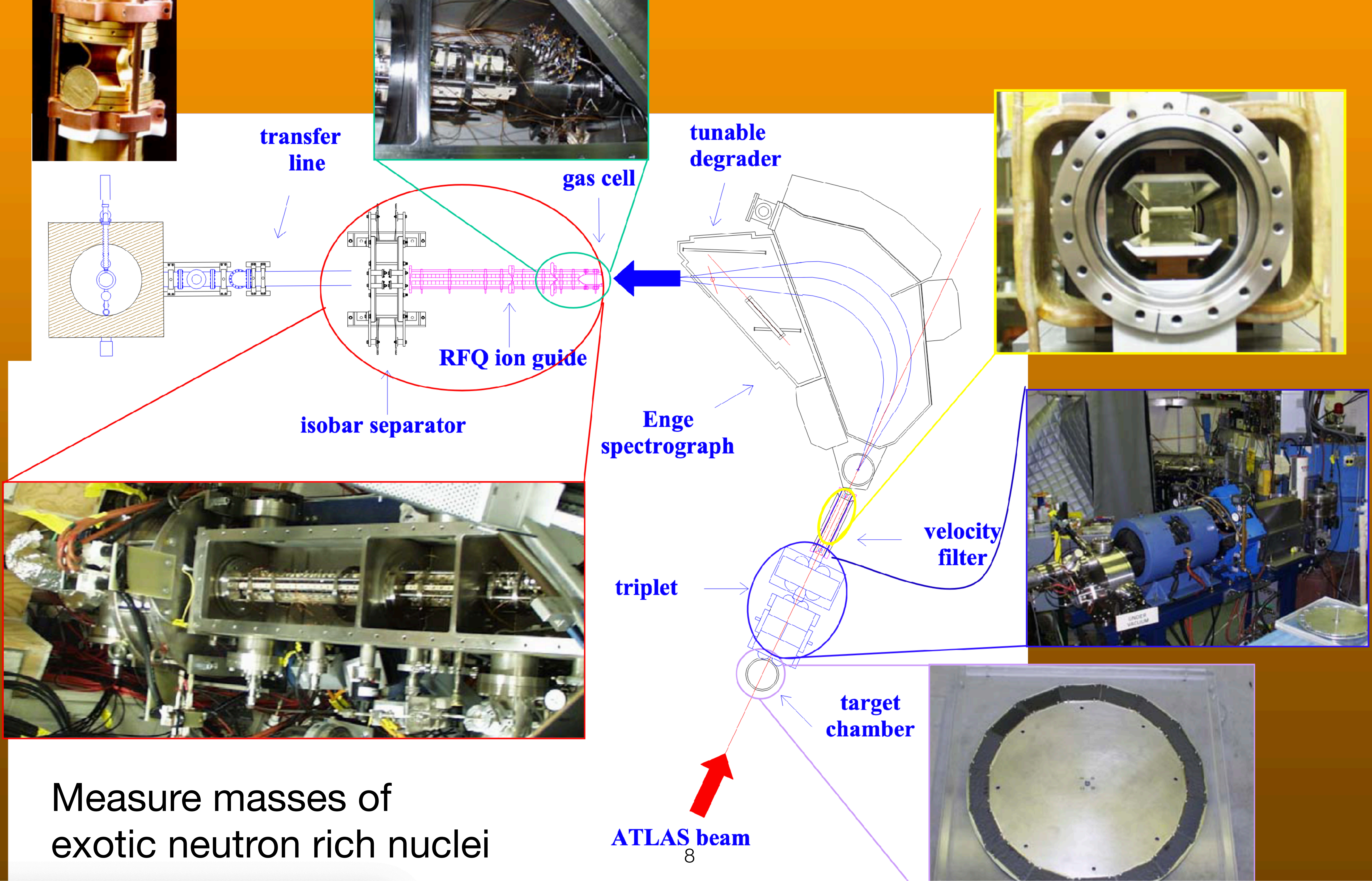
- Is there more than one equation of state? Because of different compositions (quark stars, twin stars, …). Is the composition equilibrated? (bulk viscosity)

- Do neutron stars have significant dark matter? How might dark matter show up?

What holds up $2M_{sun}$ NS?

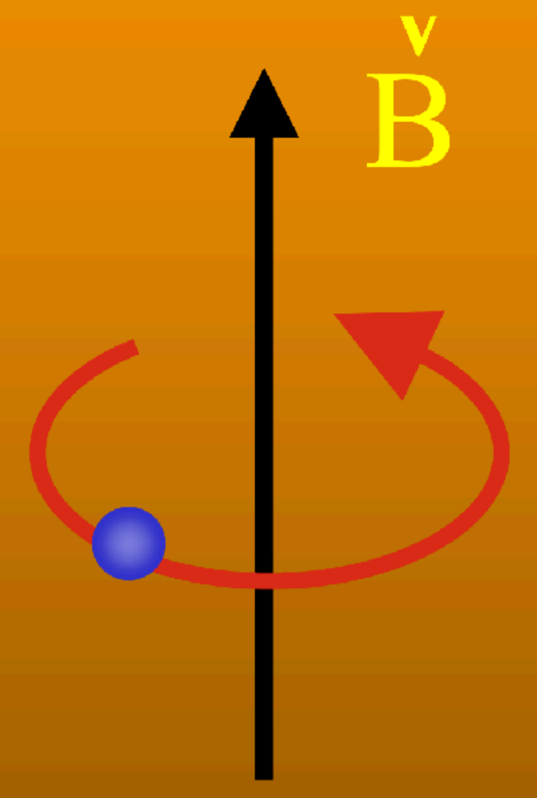
- The pressure is high at high densities. **Why?**
- Chiral EFT has this counter term with a large value. **Why?**
- The $^1S_0$ phase shift turned repulsive (negative) at high energies. **Why?** Note, why is a nucleon-nucleon scattering experiment in the laboratory correlated with radio observations of a massive pulsar?
- Nucleons have “hard cores” at short distances. **Why?**
- Quarks “from” different nucleons can’t be in the same quantum state? **Why?** [Note there are many spin, flavor, and color degrees of freedom] **Why** (are the quarks in the same state)?…

Nuclear experiment



Measure masses of
exotic neutron rich nuclei

How a Penning Trap works-1

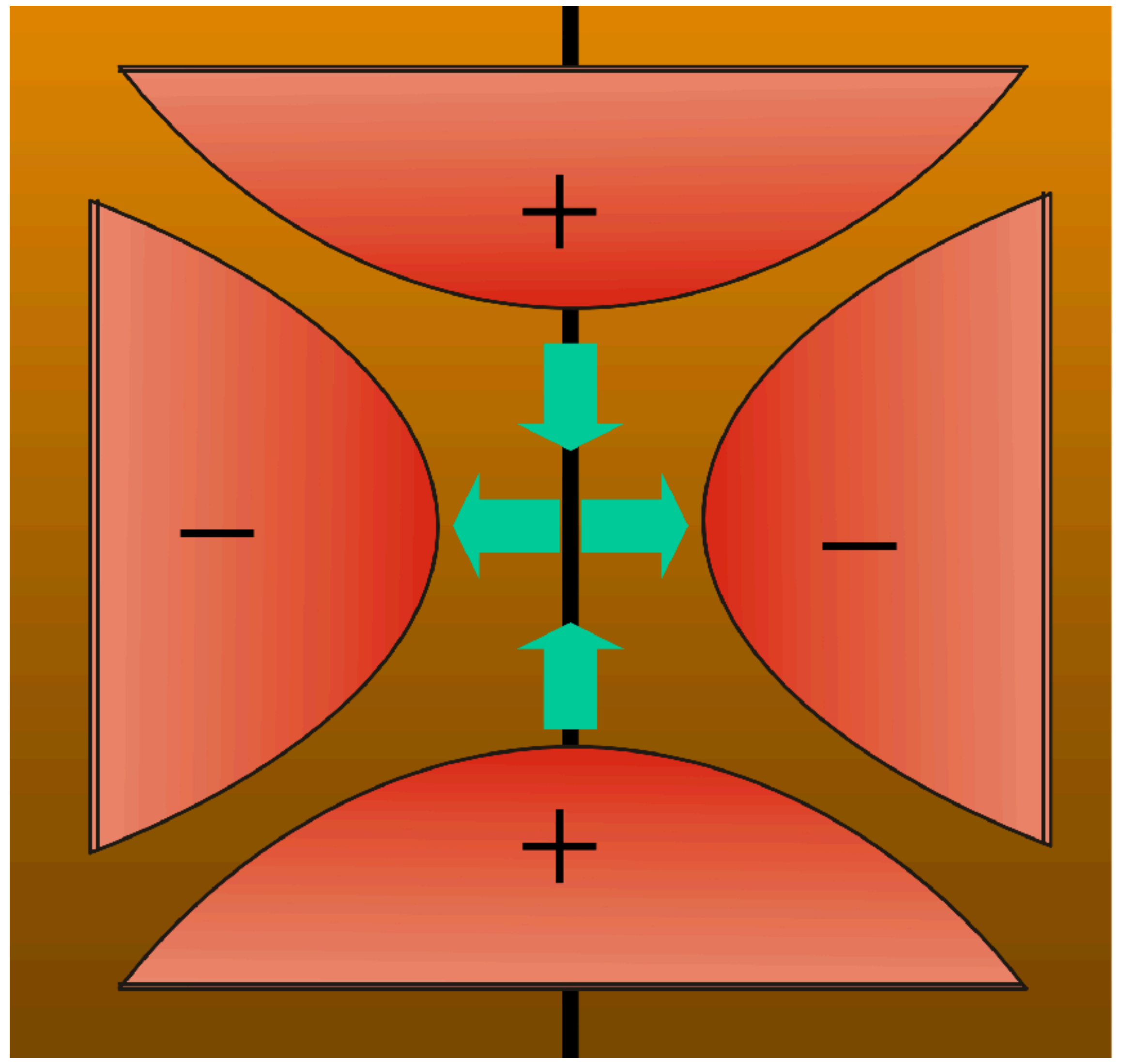


- Constant axial magnetic field
- particle orbits in horizontal plane

$$\omega_c = \frac{qB}{m}$$

- free to escape axially

In addition to B field, an electric quadrupole field is used to confine the ions in the z direction



Nuclear masses

- $m(Z,A) = (A-Z)m_n + Zm_p - BE(Z,A)$

- Important for nucleosynthesis. System at temp. T in nuclear statistical equilibrium has abundances related to Exp(BE/kT).

- Semiempirical mass formula has volume a_v, surface a_s, coulomb a_c, symmetry a_{sym} and pairing a_p terms.

- $BE(Z,A) = a_v A - a_s A^{2/3} - a_c Z^2/A^{1/3} - a_{sym} (N-Z)^2/A + \delta\, a_p A^{-3/4}$

- Pairing: $\delta = +1$(N and Z even), 0 (A odd), -1 (N and Z odd)

Symmetry Energy in Infinite Matter

- BE/A, neglect Coulomb and take A-> Infinity

- $-BE/A = -a_v + a_{sym} (1-2x_p)^2$ with $x_p = Z/A$ proton fraction.

- Density dependence: $-BE/A = -a_v(\rho) + S(\rho)\,(1-2x_p)^2$. S=symmetry energy and a_{sym} is symmetry coefficient. Expand $S(\rho)$ around ρ_0.

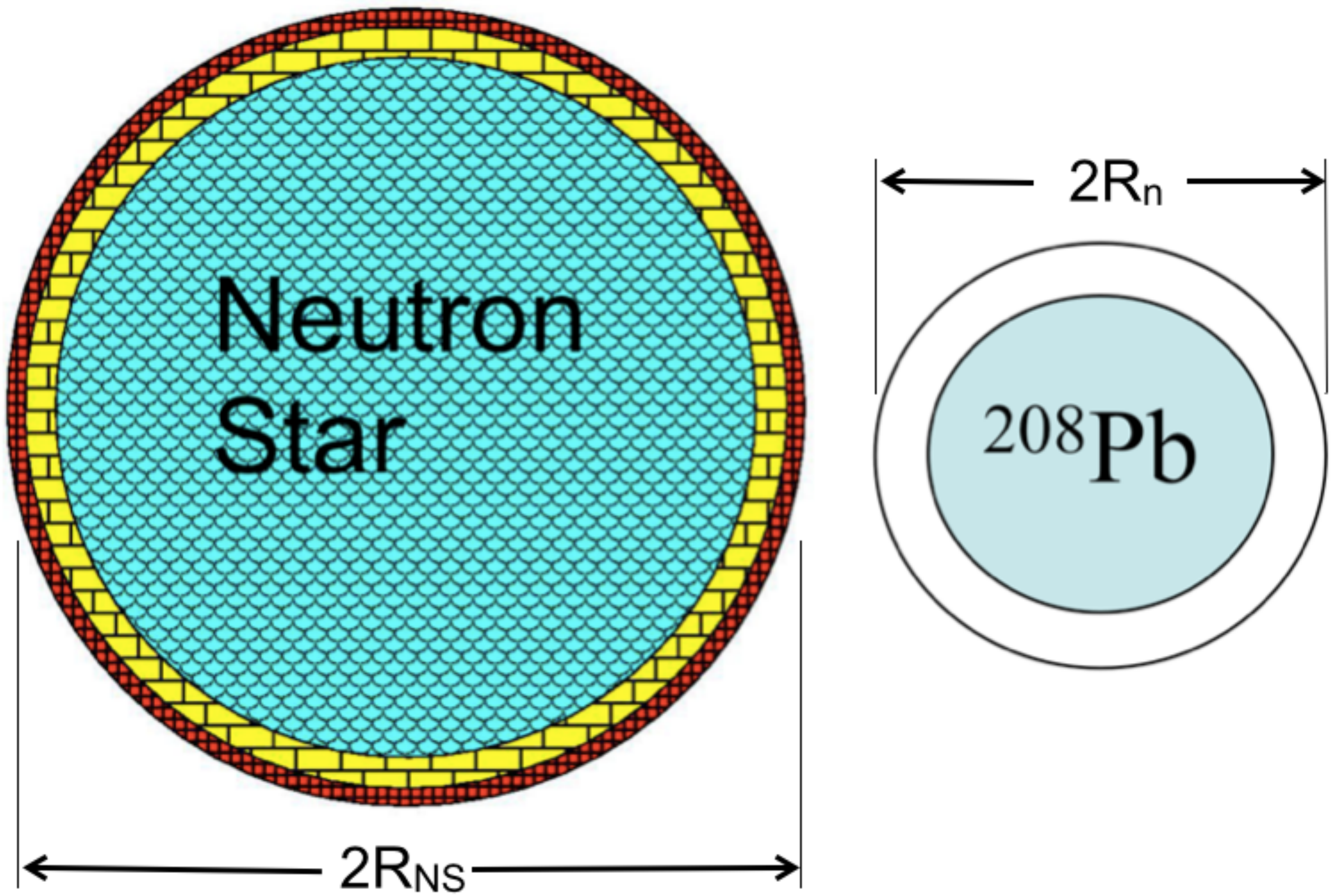
- $S(\rho) = J + L/3\,(\rho-\rho_0)/\rho_0 + \ldots$ with $L = 3\rho_0\,(dS/d\rho)|_{\rho 0}$

- $(E/A)_{neutron} \simeq (E/A)_{nuclear} + S(\rho)$. Pressure of neutron matter at ρ_0 is

- $P(\rho_0) = L\,(\rho_0/3)$

Radii of ^{208}Pb and Neutron Stars

- Pressure of neutron matter pushes neutrons out against surface tension ==> R_n-R_p of ^{208}Pb correlated with P of neutron matter.
- Radius of a neutron star also depends on P of neutron matter.
- Measurement of R_n (^{208}Pb) in laboratory has important implications for the structure of neutron stars.



Neutron star is 18 orders of magnitude larger than Pb nucleus but both involve neutron rich matter at similar densities with the same strong interactions and equation of state.

Pressure of neutron rich matter in laboratory

- Pressure is force/area. Measure force with spring and a ruler.
- For pressure of nuclear matter: spring is known surface tension and ruler is the PREX experiment.
- PREX ran at Jefferson Laboratory in Virginia

PREX uses Parity V. to Isolate Neutrons

- In Standard Model Z^0 boson couples to the weak charge.
- Proton weak charge is small:

$$Q_W^p = 1 - 4\sin^2\Theta_W \approx 0.05$$

- Neutron weak charge is big:

$$Q_W^n = -1$$

- Weak interactions, at low Q^2, probe neutrons.
- Parity violating asymmetry A_{pv} is cross section difference for positive and negative helicity electrons

- A_{pv} from interference of photon and Z^0 exchange.
- Determines weak form factor

$$F_W(q) = \frac{1}{Q_W} \int d^3r j_0(qr) \rho_W(r)$$

- Model independently map out distribution of weak charge in a nucleus.
- **Electroweak reaction free from most strong interaction uncertainties.**

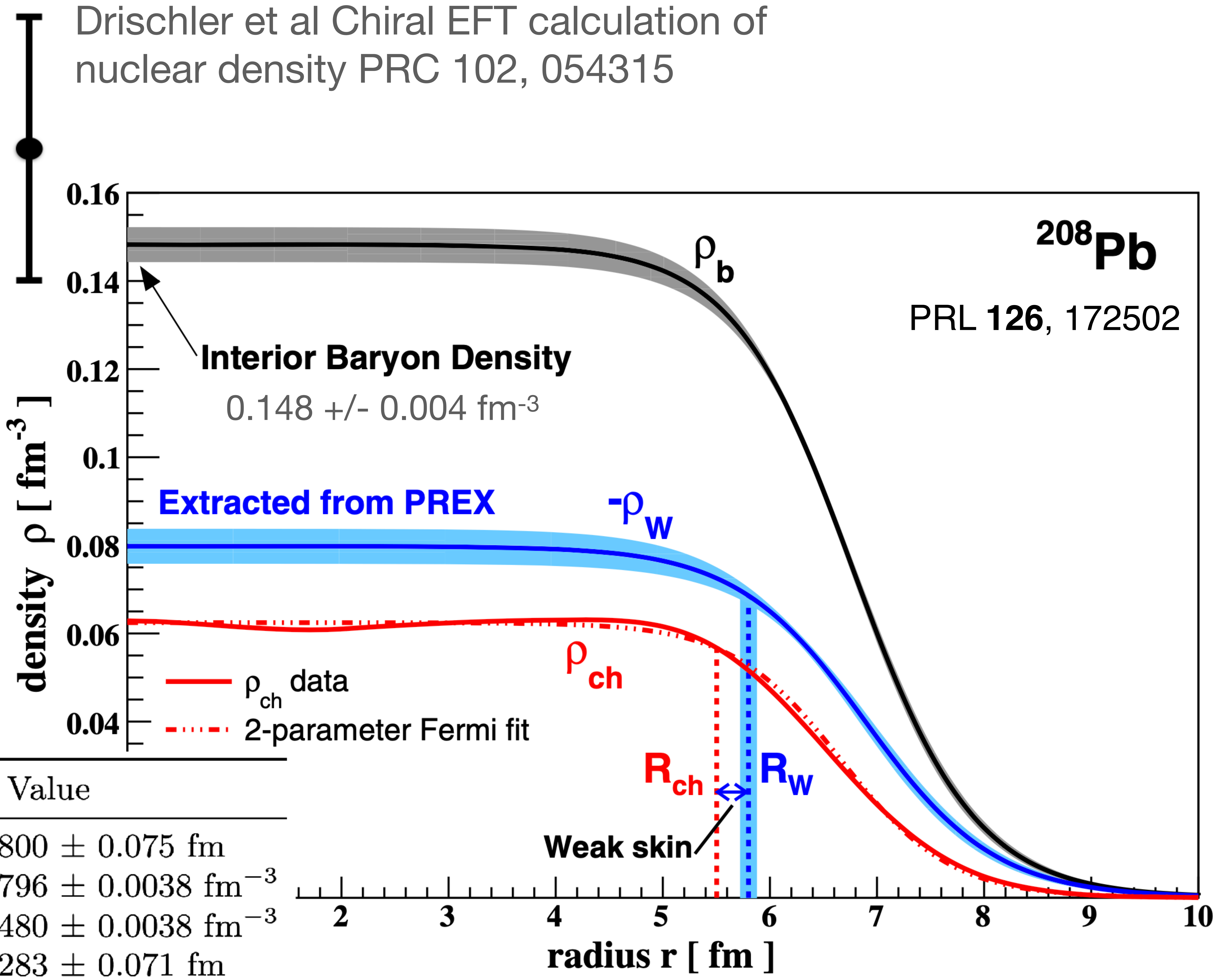
$$A_{pv} = \frac{d\sigma/d\Omega_+ - d\sigma/d\Omega_-}{d\sigma/d\Omega_+ + d\sigma/d\Omega_-} \approx \frac{G_F Q^2 |Q_W|}{4\pi\alpha\sqrt{2}Z} \frac{F_W(Q^2)}{F_{ch}(Q^2)}$$

High Resolution Spectrometers in Hall A at Jefferson LAB

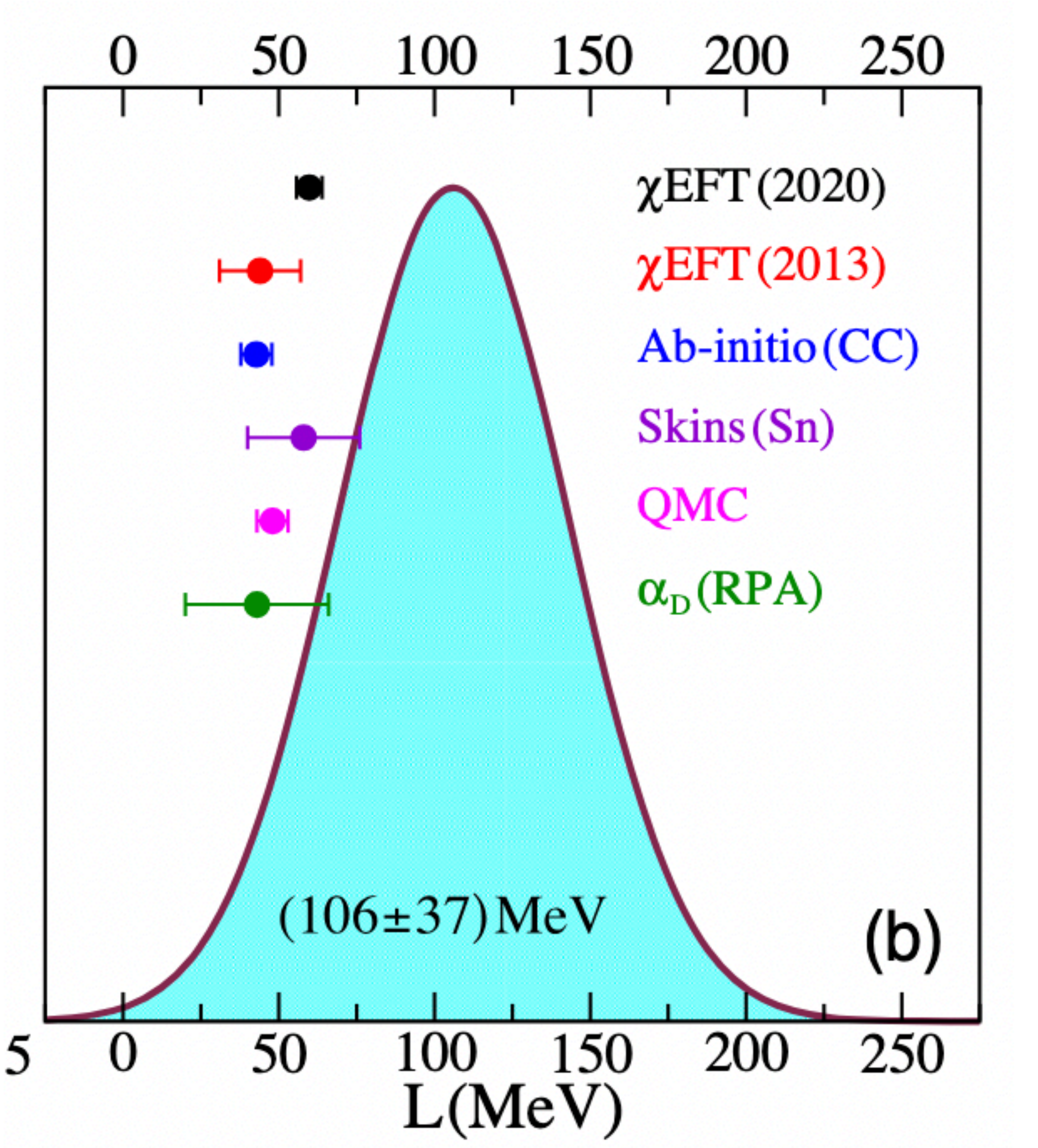
PREX results

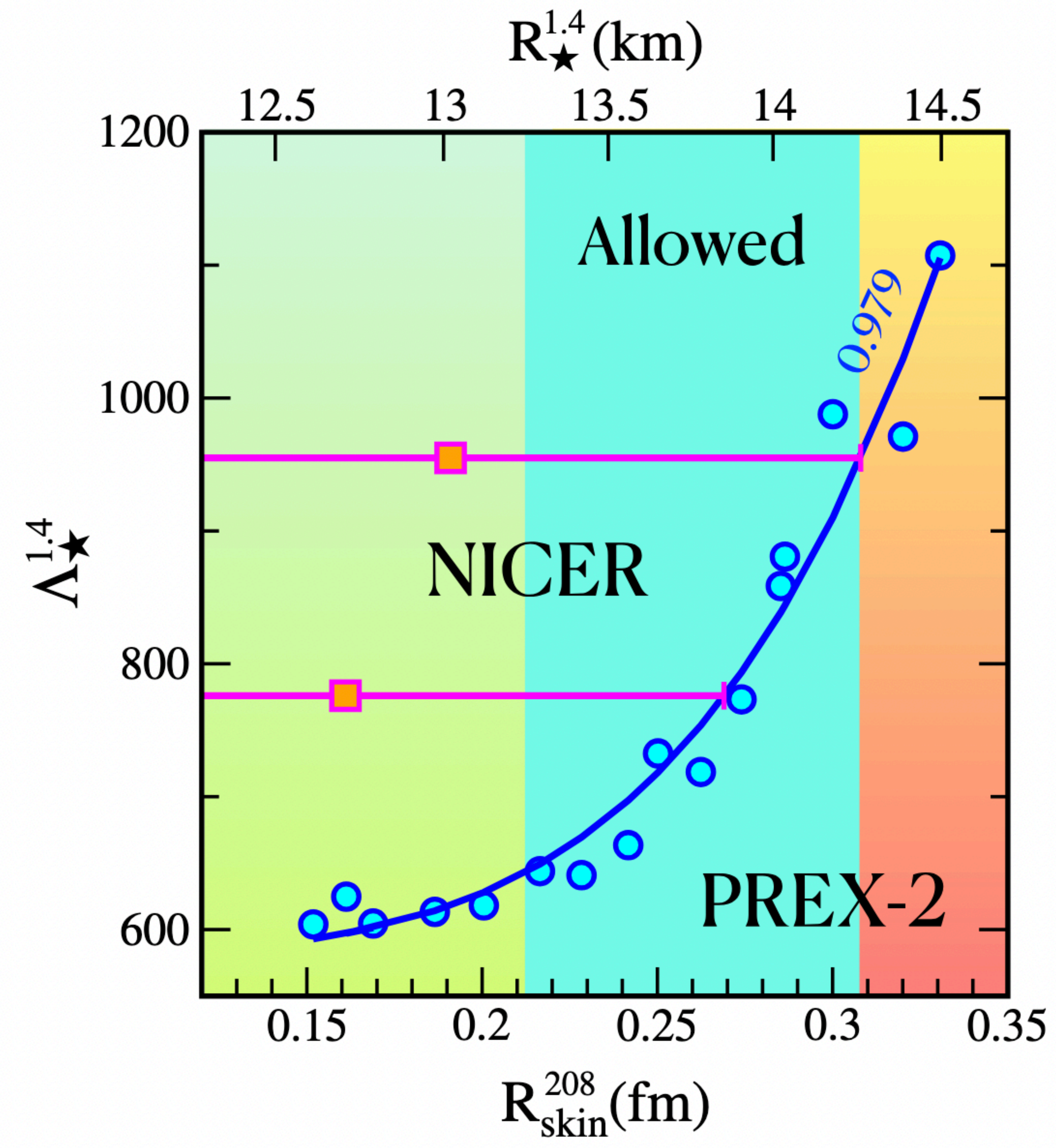
$$A_{pv} = \frac{d\sigma/d\Omega_+ - d\sigma/d\Omega_-}{d\sigma/d\Omega_+ + d\sigma/d\Omega_-}$$

$$A_{\rm PV}^{\rm meas} = 550 \pm 16\,({\rm stat}) \pm 8\,({\rm syst})\ {\rm ppb}$$



^{208}Pb Parameter	Value
Weak radius (R_W)	5.800 ± 0.075 fm
Interior weak density (ρ_W^0)	$-0.0796 \pm 0.0038\ \mathrm{fm}^{-3}$
Interior baryon density (ρ_b^0)	$0.1480 \pm 0.0038\ \mathrm{fm}^{-3}$
Neutron skin ($R_n - R_p$)	0.283 ± 0.071 fm



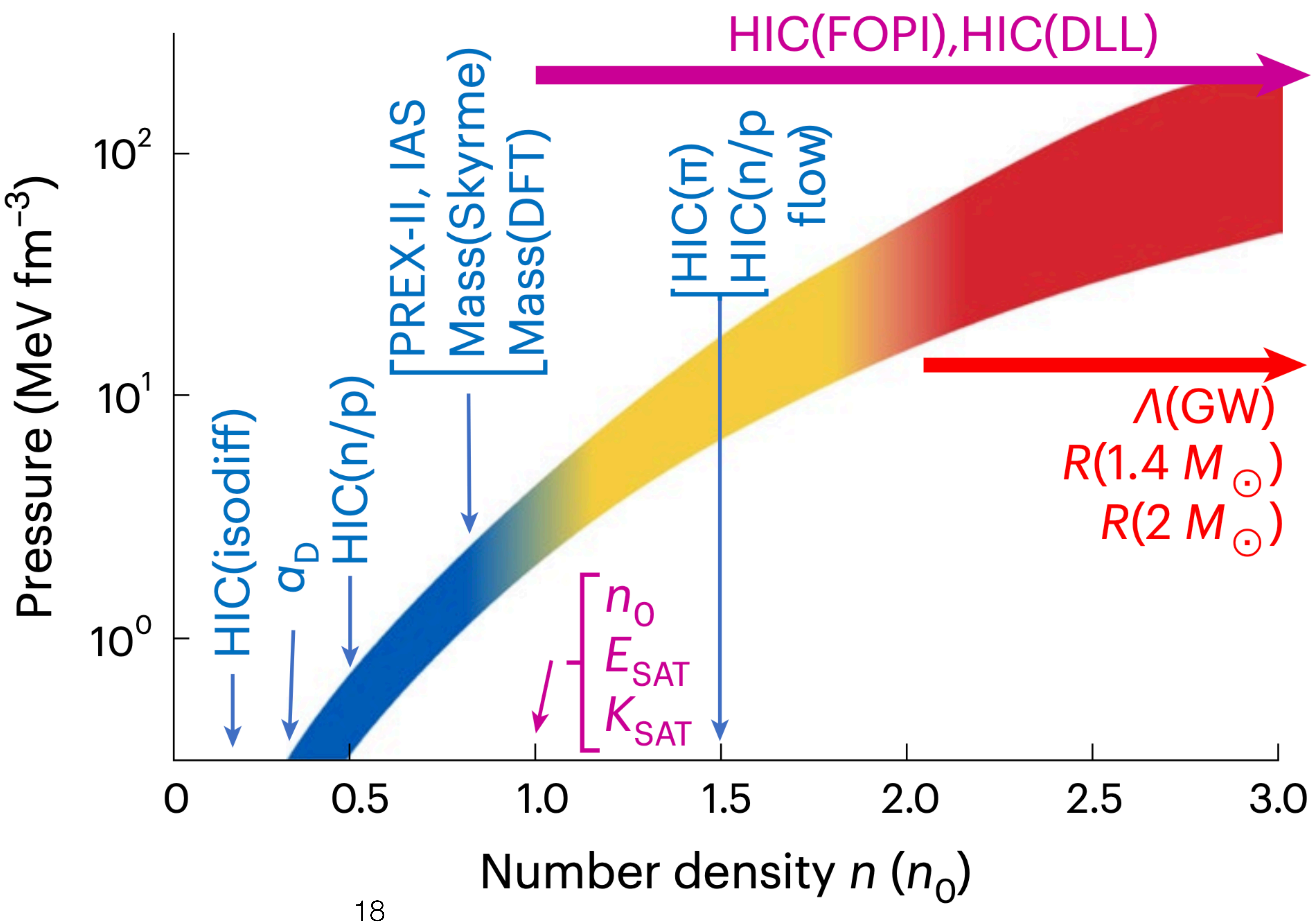


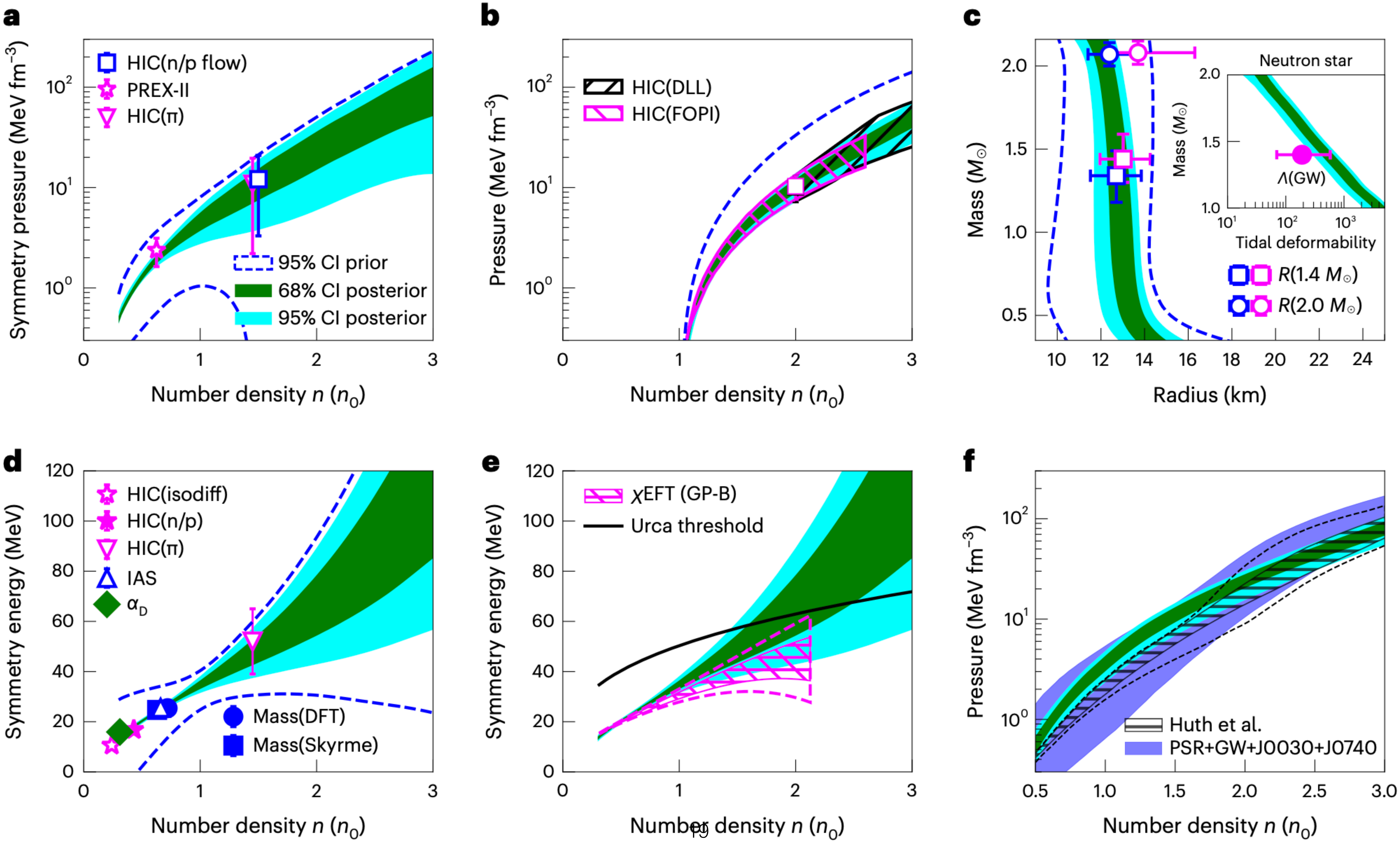
Brendan T. Reed, F. J. Fattoyev, CJH and J. Piekarewicz, PRL **126**, 172503 (2021).

See Tianqi Zhao's talk tomorrow

Determination of the equation of state from nuclear experiments and neutron star observations

- Chun Yuen Tsang, ManYee Betty Tsang, William G. Lynch, Rohit Kumar, CJH, Nature Astronomy **8**, 328 (2024).





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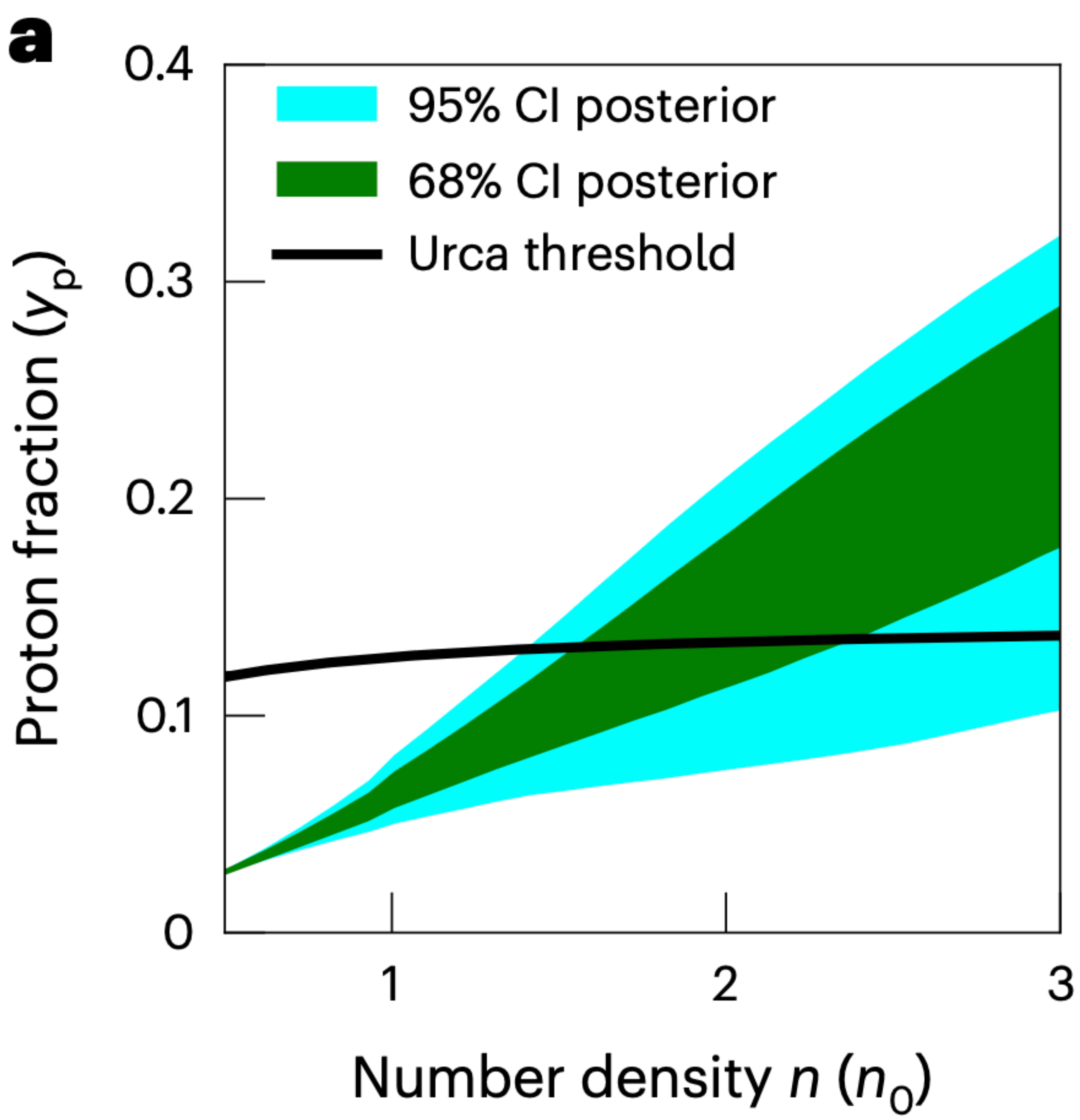
Symmetry pressure (MeV fm^{-3})
HIC(n/p flow)
PREX-II
HIC(π)
95% CI prior
68% CI posterior
95% CI posterior
Number density n (n_0)
b
Pressure (MeV fm^{-3})
HIC(DLL)
HIC(FOPI)
Number density n (n_0)
c
Mass ($M_\odot$)
Neutron star
Mass ($M_\odot$)
Λ(GW)
Tidal deformability
R(1.4 $M_\odot$)
R(2.0 $M_\odot$)
Radius (km)
d
Symmetry energy (MeV)
HIC(isodiff)
HIC(n/p)
HIC(π)
IAS
α_D
Mass(DFT)
Mass(Skyrme)
Number density n (n_0)
e
Symmetry energy (MeV)
χEFT (GP-B)
Urca threshold
Number density n (n_0)
f
Pressure (MeV fm^{-3})
Huth et al.
PSR+GW+J0030+J0740
Number density n (n_0)

Determine proton fraction

- Measure the pressure of *symmetric* nuclear matter at super nuclear densities with medium/ low E heavy ion collisions in addition to astronomical observations of pressure of neutron matter.

- Infer symmetry energy at super nuclear densities and proton fraction of matter in beta equilibrium.

- Large proton fraction can open rapid direct Urca cooling of NS: n->p e nu-bar followed by e+p->n nu



Pairing

$BE(Z,A) = a_v A - a_s A^{2/3} - a_c Z^2/A^{1/3} - a_{sym} (N-Z)^2/A + \delta a_p A^{-3/4}$

Pairing: δ= +1(N and Z even), 0 (A odd), -1 (N and Z odd)

- [Theory] At very high densities quarks should pair into a **color superconductor**.
- [Experiment] Abundance of even Z elements larger (in general) than odd Z. There are no stable odd Z, odd N isotopes heavier than ^{14}N. Beta decay can convert an odd odd nucleus into an even even nucleus and gain the pp or nn pairing interaction.
- [Observation] Neutron superfluidity important for thermal and spin evolution of NS.

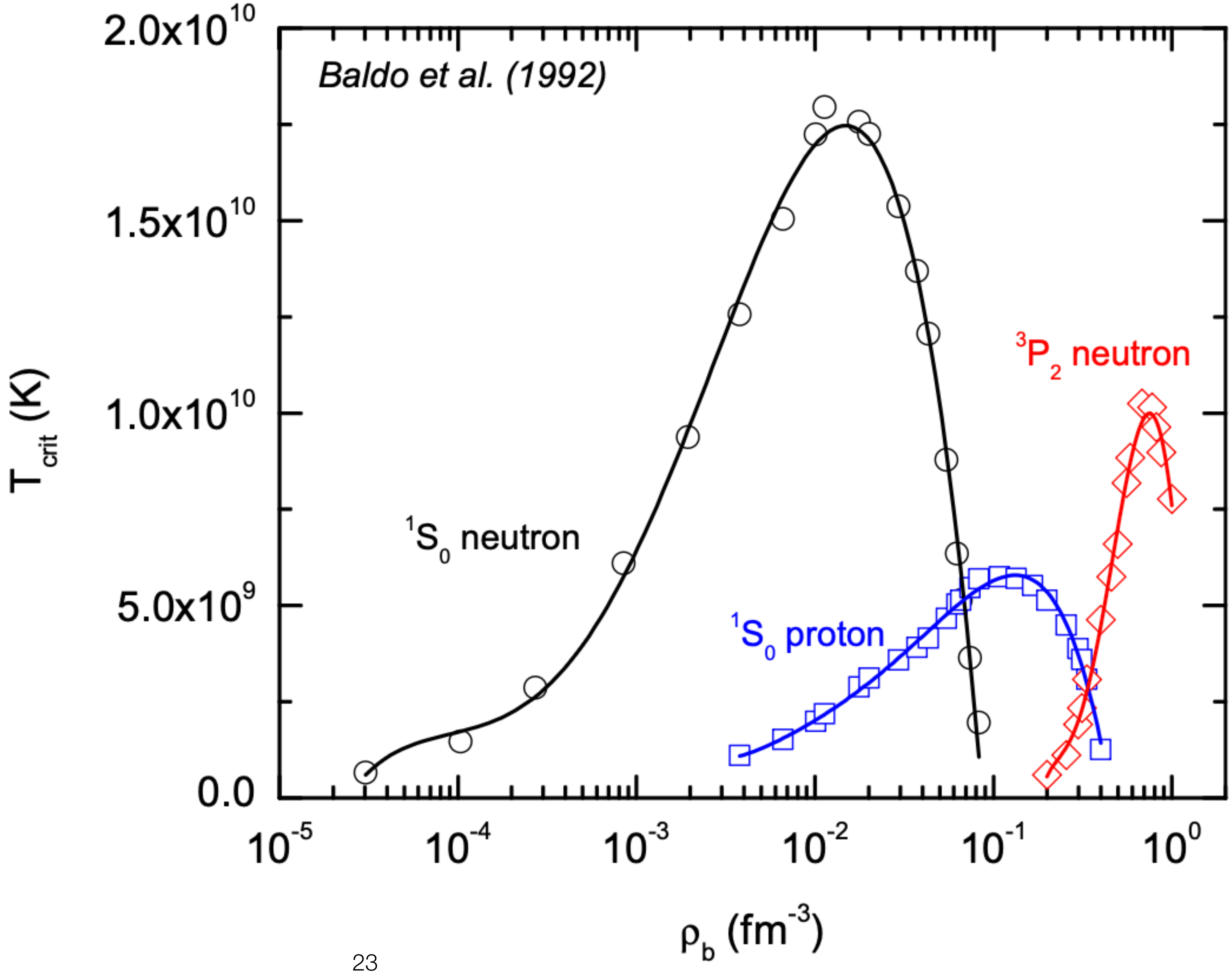
Cooper pairing instability

- **In 2 dimensions** an attractive potential, no matter how weak, produces a bound state. In 3 dim. must exceed a threshold to have bound state. np is bound while pp and nn are unbound even though nn interaction is attractive.

- Particles on the 2 dim surface of the Fermi sea can form a bound state (called a Cooper pair) as long as there is even a very weak attractive interaction.

- BCS theory of superconductivity (Noble 1972): One electron slightly deforms crystal lattice attracting a 2nd electron. Form Cooper pair with small binding energy or gap Δ. Critical temp of superconductor order Δ.

One model of pairing gaps in neutron star matter versus density

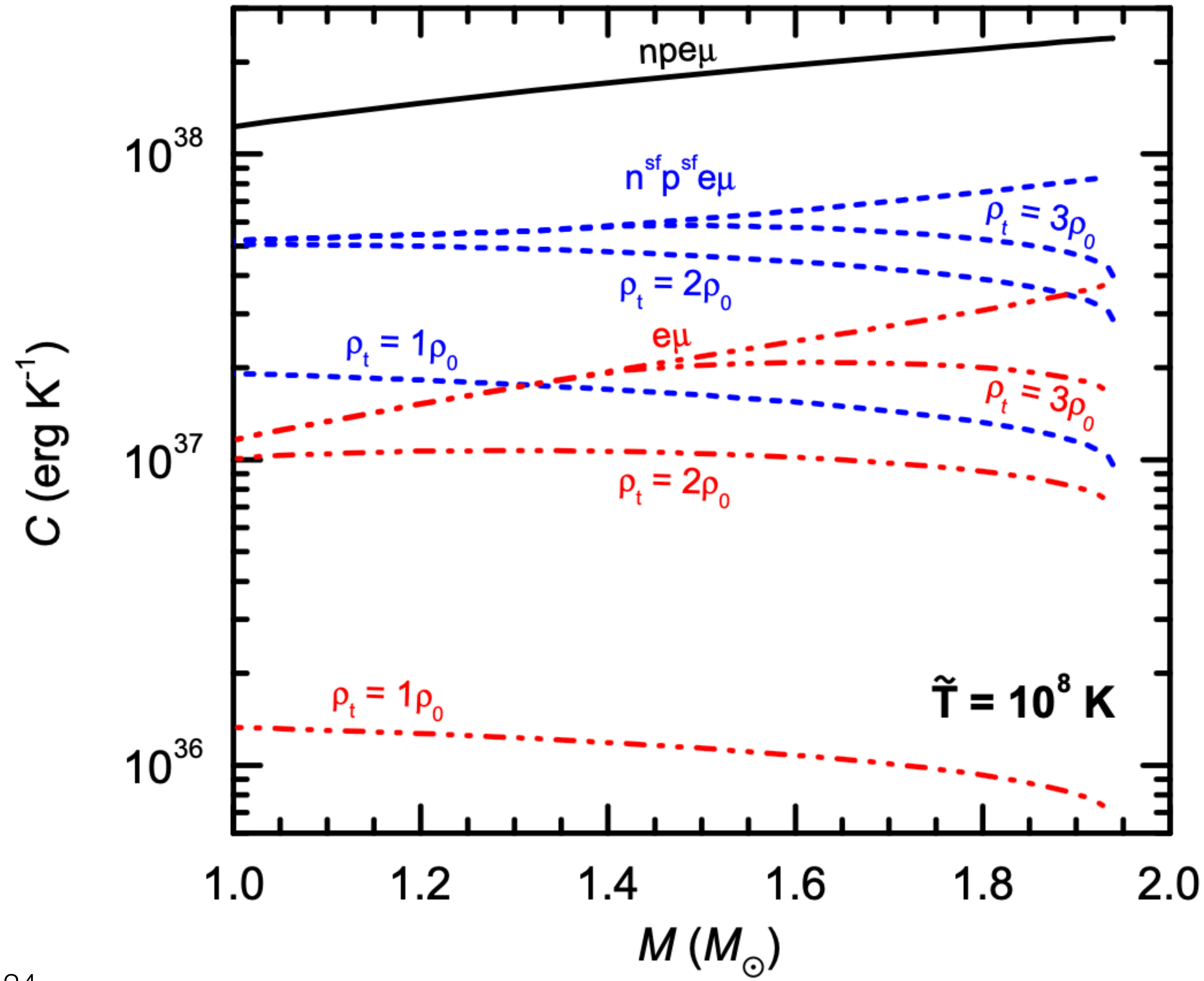


PHYSICAL REVIEW C **95**, 025806 (2017)

Lower limit on the heat capacity of the neutron star core

Andrew Cumming,[1,*] Edward F. Brown,[2,†] Farrukh J. Fattoyev,[3,‡] C. J. Horowitz,[3,§] Dany Page,[4,‖] and Sanjay Reddy[5,¶]

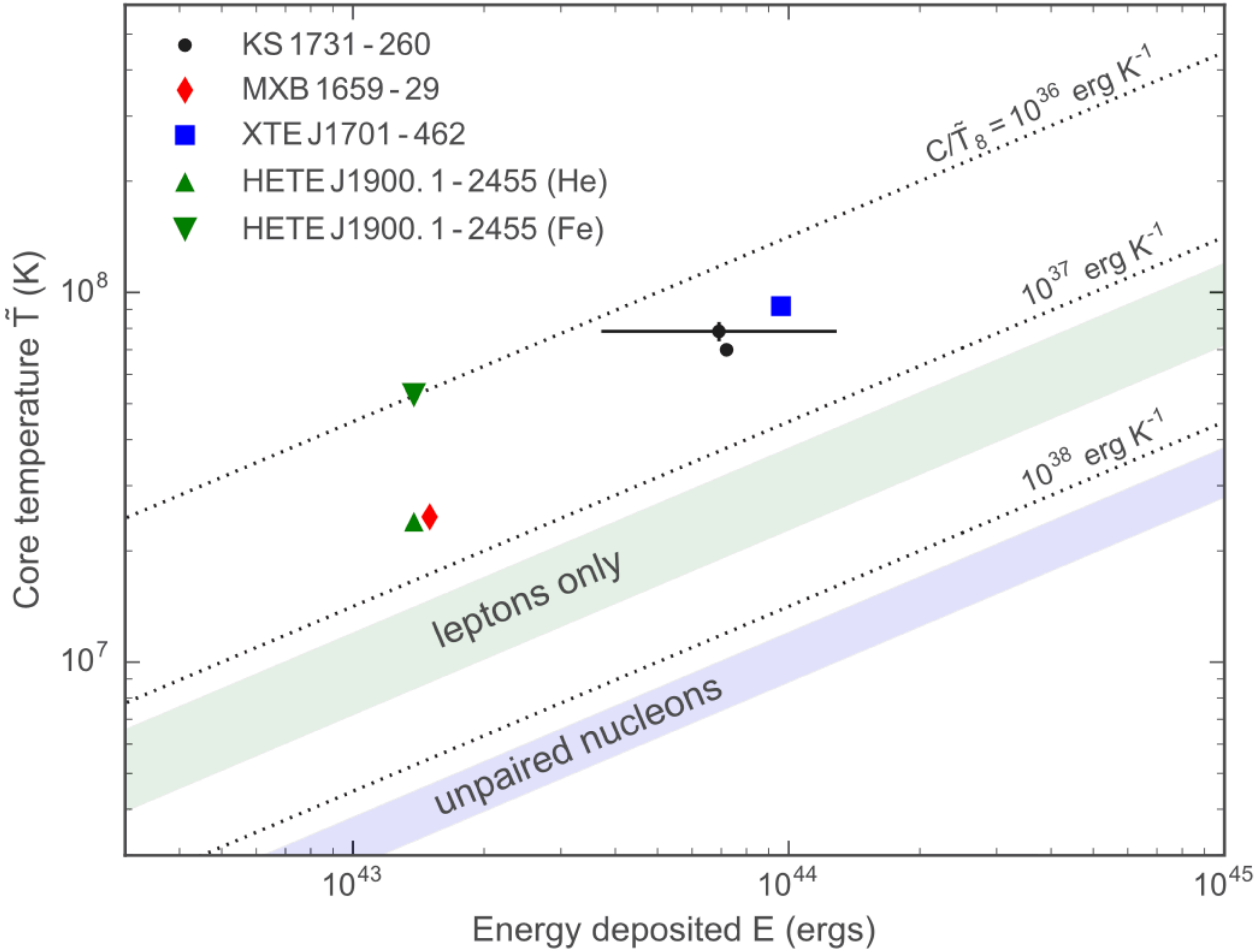
Heat capacity of a neutron star for different compositions and pairing gaps



Color superconductor: at very high densities quark fermi gas unstable to quark pairing from attractive one gluon exchange interactions. Most extreme color flavor locked phase (CFL) with u, d and s quarks and **no** electrons.

Limit on Heat Capacity of LMXB

- Accretion episode dumps significant energy into the core of a NS in a LMXB.
- Infer the final core temperature from measuring the final surface temperature.
- The initial temperature of core was at least 0. This allows one to set a lower limit on the heat capacity of a NS.
- Observed lower limit consistent with conventional NS composition and rules out the most extreme CFL color superconductor with a very low transition density.



Theory

Theory pre Chiral EFT

- Ab initio “From first principles”

Calculate properties of finite nuclei or nuclear matter using a Hamiltonian with only two-nucleon potentials where the potential was fit to nucleon-nucleon scattering data and then no parameters were (re-)adjusted to properties of nuclear systems.

- Bonn One-Boson Exchange potential

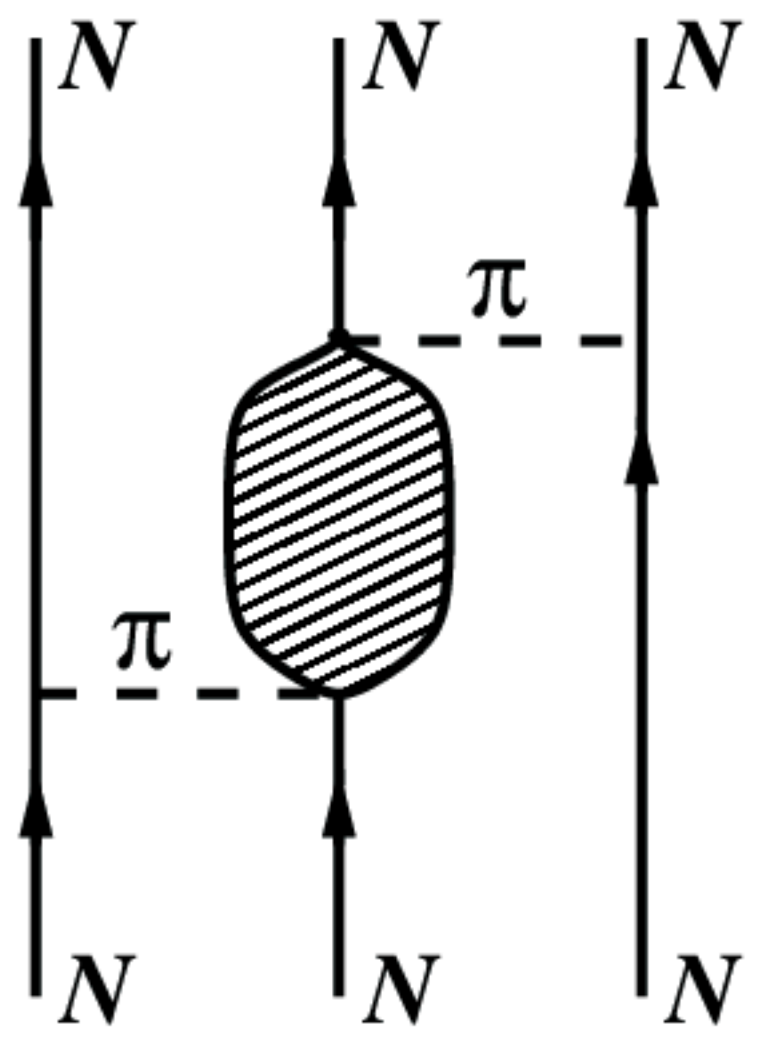
Describes the NN interaction as a sum of single boson exchanges of “mesons” of increasing mass including lowest mass (longest range) contribution from one pion exchange (pion 138 MeV). Sigma meson (scalar meson with a ~500 MeV mass) providing intermediate range attraction that leads to nuclear binding and short-range omega exchange (783 MeV vector 1- meson) that gives rise to short range repulsion (hard core). Fit coupling constants and some masses to NN scattering data.

Need for Three nucleon forces

This *ab initio paradigm was broken* by Ben Day in the 1980s. His beautiful calculations showed that two-nucleon forces alone saturate nuclear matter at too high a density (up to a factor of two). [Nuclear density is about 0.16 fm^{-3}] Ben was denied tenure at Argonne. “He wasted all this time calculating imaginary infinite nuclear matter quantities and did not even get the correct answer.”

- Three nucleon potentials

Nuclear saturation requires in addition to 2N potentials, 3N potentials where the interaction depends on coordinates of three nucleons at a time. For example, one N emits a pion that excites a second nucleon to a Delta baryon (spin 3/2 excited state with a 1232 MeV mass). This Delta de-excites by emitting a pion which is finally absorbed by a third nucleon.



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Need for Chiral EFT

- Simple 3N potentials with a few parameters could be fit in *ad hoc* ways to the binding energy of ^{3}He or some other properties of 3 or 4 N systems.

- There were no practical ways to systematically fit all the parameters of detailed 3N forces. Fitting phase shifts of p-D system did not work.

- A systematic way to build 3 N forces and determine their parameters was needed. *Chiral EFT solved this fundamental problem (good news).*

- *Bad news: Chiral EFT does not converge at high densities and because you need it, when it diverges you are lost.*

Effective field theory

Based on separation of scales. Use a cutoff (say 500 MeV) to separate high momentum physics that is not explicitly included from the low momentum physics that is. Mesons with larger masses are not included (for example the omega at 783 MeV). Instead, the physics at higher scales is included with a series of counter terms (contact terms) with values fit to data.

- Chiral EFT

Because of chiral symmetry the pion (lowest QCD excitation) is a pseudoscalar 0- meson. This couples to a nucleon with the pseudoscalar operator σ . q where σ is the nucleon spin and q is pion momentum.

- Chiral expansion

One can organize all diagrams in the effective theory in powers of q over the cutoff scale. At low momentum this expansion converges and allows one to calculate quantities at momenta below the cutoff scale in terms of a few coupling constants and many contact terms.